RISK INSOLVABILITY MANAGEMENT THROUGH OPTIMIZING INSURANCE PORTFOLIO – MATHEMATICAL CALCULATIONS

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1. Introduction

Starting with the second half of the year 2007, the international financial markets indicate new essential features, influenced by the issues existing on the high-risk mortgage market of the USA (known as subprime crisis) most of them generating a wrong perception of the investors' risk and a liquidity diminution.

However, the insurance market, in full expansion and finalizing the stage of harmonization with the European legislation, was not troubled by these issues existing on the financial markets, due to the insurance features and to their development and integration with external financial markets.

The main aspects which are essential for a careful observation are the increase of the indemnification installment for general insurances and the return dynamics of the personal asset portfolio for life insurances.

It is important for the insurance company to calculate in a very strict manner the premiums owed by the insured persons in order to create an adequate insurance fund necessary to cover the indemnifications, diminishing thus the insolvability risk.

The structure of the subscription portfolio should be elaborated by means of the return analysis as well as the financial stability analysis of the insurance activity. Thus, the insurers should find the adequate methods of substantiating the premium installments, the adequate ways of attracting insurances in order to achieve the right structure of the portfolio and the desired level of financial stability within the company.

The present paper views the analysis of the subscription portfolio within an insurance company considering the number of estates comprised by the insurance and the net premium installment in order to obtain a certain level of financial stability. This analysis represents an essential instrument for each insurer as the optimization of the subscription portfolio generates more important returns for the company; it also improves the financial stability and reduces the possibility of insolvency. Using mathematical calculation, different solution may be given in order to optimize this portfolio, determining thus its adequate structure to a certain level of stability planned by the company. Thus, the mathematical calculation shown below within this paper may be applied in practice and improved.

2. Material and method

The solvency represents an important aspect of the operational regulation within an insurance company, thus, for every year of the next period, the manager is concerned with the knowledge of the following aspects: the level of the indemnifications to pay; the level of the indemnifications compared to level of the net premium paid for each estate category; the possibility that the

amount of indemnifications exceeds the amount of paid premiums $P_{(D>PnT)}$.

Compared to the multiannual average of the risk indicators, considered for the net premium[1] substantiation, there are errors reflected by the dissimilarities between the indemnifications to pay currently and those belonging to the considered period.

In order to establish the value of the difference between the indemnifications which ought to be paid currently and the indemnifications registered during the analyzed period, **mean squared deviation calculations** are used: $\sigma = S \cdot \sqrt{n \cdot q(1-q)}$, (1)

where: S – insured sum of an indemnified estate; n – number of insured estates; q – probability of damage occurrence; (1-q) - probability of having no damage occurring

For an estate, $q = \frac{P_n/b}{S}$, (2)

where: b = number of estates; P_n/b - net premium for an estate.

For the total amount of estates, $q = \frac{Pnt}{S_t} = \frac{P_n/b \cdot n}{S \cdot n}$, (3)

where: *Pnt* – total net premium.

The interval meant for the indemnification variation is given by the relation: $D : [Pnt - \sigma; Pnt + \sigma]$ (4)

The **financial stability level** is given by the value of the K coefficient: $K = \frac{\sigma}{Pnt}$. (5)

As the K coefficient is lower, the financial stability level is higher (the deviation is reduced).

In order to establish the **number of years counted until a favorable year occurs**, "a" is determined as it follows:

$$a = \frac{100}{\frac{K}{2} \cdot 100} = \frac{1}{P_{(D>Pnt)}}. \quad (6)$$

The following conditions are necessary, in order to obtain an **improvement of the financial stability level**: a large number of insured estates; the increase of the net premium rate; the cession to reinsurance.

For knowing if the cession for reinsurance is required, we calculate the maximum insured sum for each insured risk (X), this calculation should be maintained by the insurer, in order to obtain an adequate stability level:

$X = 2K^2 \cdot Pnt$, (7)

where *K* is the average coefficient of financial stability, for all risks, and *Pnt* is the net premium within the insurance company.

The insurance companies resort to reinsurance for protecting its clients, the insured persons, whenever the assumed risks are too important. Through reinsurance, an insurance company gets a higher financial stability and, in addition to this, a higher ability of dealing with the new risks. The reinsurance interferes whenever the insured sum allotted to a risk or a group of risks exceeds the limit that an insurance company is able to sustain without affecting the protection of the other insured persons. One of the reinsurance functions is constrained by the monitoring and control institutions of the insurance market in each country by imposing a minimum level of solvency.

In order to apply this mathematical calculation, we consider an insurance company for which we determine the financial stability level according to the variation of the net premium rate and to the number of insured estates. Thus, the insurance company underwrites 920 estates belonging to a certain category, being insured for an average sum of 23,000 euros, with a premium rate of 1.3%.

[1] ***Net insurance premium*** serves to the creation of the fund needed for the indemnification or the insurance compensation payment. For obtaining the total premium owed by insured persons, to the net premium is added the s***upplement or additional premium***, wihich covers the insurer's purchase and administration fees, as well as a return achievement

3. Results and discussions

Considering the above presented data, the insurance company is able to create an analysis of its financial stability variation, for different situations that may occur in practice.

1) When the net premium rate increases, for example 1.6%, the financial stability level of the company is changed.

According to the relation (5), the calculation of the coefficient of financial stability indicating the financial stability level can be known: $K = \frac{\sigma}{Pnt}$.

For the initial situation,

$$K_0 = \frac{\sigma_0}{Pnt_0} = \frac{S \cdot \sqrt{n \cdot q_0(1-q_0)}}{q_0 \cdot S \cdot n} =$$

$$= \frac{23{,}000 \cdot \sqrt{920 \cdot 0.013 \cdot (1-0.013)}}{0.013 \cdot 23{,}000 \cdot 920} = 0.287$$

When the net premium rate increases, the coefficient of financial stability becomes:

$$K_1 = \frac{\sigma_1}{Pnt_1} = \frac{S \cdot \sqrt{n \cdot q_1(1-q_1)}}{q_1 \cdot S \cdot n} =$$

$$= \frac{23{,}000 \cdot \sqrt{920 \cdot 0.016 \cdot (1-0.016)}}{0.016 \cdot 23{,}000 \cdot 920} = 0.259$$

The change index for the coefficient of financial stability is determined as it follows:

$$I_K = \frac{K_1}{K_0} \cdot 100 = \frac{0.259}{0.287} \cdot 100 = 90.24\% \Rightarrow$$

$$\Rightarrow \%\Delta_K = \left|90.24 - 100\right| = 9.76\%$$

Therefore, the ***increase of the net premium rate*** from 1.3% to 1.6%, ***considering the fact that the number of insured estates and the insured value do not change, leads to a growth of the financial stability level*** of 9.76%, this percentage is given by the diminution of the financial stability coefficient, K, from 0.287 to 0.259. Of course, this situation is recorded only if the market, the competition respectively, allows it.

2) When the number of insured estates increases with 50%, and the net premium rate of 1.3%, as well as the insured value remain unchanged, the financial stability level of the insurance company withstands modifications.

The initial number of insured estates, n_0, was 920; its rise to 50% is expected, obtaining thus n_1, which becomes 1.380 estates.

The modification of the financial stability coefficient is given by the following formula:

$$I_K = \frac{K_1}{K_0} \cdot 100 \Rightarrow$$

$$\Rightarrow I_K = \frac{S \cdot \sqrt{n_1 \cdot q \cdot (1-q)}}{q \cdot S \cdot n_1} \cdot \frac{q \cdot S \cdot n_0}{S \cdot \sqrt{n_0 \cdot q \cdot (1-q)}} \Rightarrow$$

$$\Rightarrow I_K^2 = \frac{n_1 q \cdot (1-q) \cdot n_0^2}{n_1^2 \cdot n_0 \cdot q \cdot (1-q)} = \frac{n_0}{n_1} = \frac{920}{1{,}380} = 0.666 \Rightarrow$$

$$\Rightarrow I_K = \sqrt{0.6666} \cdot 100 = 0.8165 \cdot 100 = 81.65\% \Rightarrow$$

$$\Rightarrow \%\Delta_K = \left|81.65 - 100\right| = 18.35\%$$

If the competition does not allow the increase of the net premium rate, the insurance company will make efforts for the growth of the number of customers, raising the number of insured estates. ***The underwriting of over 50% of the initial number of insured estates leads to an increase of 18.35% of the financial stability level,*** creating thus a favorable situation for the insurance company.

3) In order to increase the financial stability level to a certain percentage, for example 40%, compared to the initial situation, then what is the number of estates needed to be underwritten?

The increase of the financial stability level with 40%, implies the reduction of the *K* coefficient with 60%, resulting I_k, as it follows:

$$I_K = 60\% \Rightarrow I_k = 0.60 \Rightarrow I_K^2 = 0.36 \Rightarrow \frac{n_0}{n_1} = 0.36 \Rightarrow$$

$$\Rightarrow n_1 = \frac{n_0}{0.36} = \frac{920}{0.36} = 2{,}555 \text{ estates.}$$

The result can be verified as it follows:

$$K_1 = \frac{\sigma_1}{Pnt_1} = \frac{S \cdot \sqrt{n_1 \cdot q\ (1-q\ )}}{q\ \cdot S \cdot n_1} =$$

$$= \frac{23{,}000 \cdot \sqrt{2{,}555 \cdot 0.013 \cdot (1-0.013)}}{0.013 \cdot 23{,}000 \cdot 2{,}555} = 0.172$$

$$\%\Delta_K = \left|\frac{K_1}{K_0} \cdot 100 - 100\right| = \left|\frac{0.172}{0.287} \cdot 100 - 100\right| = 40\%$$

For increasing the financial stability level with 40%, the insurance company is forced to increase the number of insured estates about three times, from 920 insured estates,to 2,555 estates.

4) If it is not possible to increase the number of insured estates (the market is saturated or other different reasons), for increasing the financial stability level with 40% compared to the initial circumstances, the insurer may resort to the increase of the net premium rate. ***The question is then, which is the level of the net premium rate in order to provide a certain coefficient of the financial stability?***

As in the previous case, the growth of 40% of the financial stability level implies a reduction of 60% of the *K* coefficient, given by I_k, as it follows:

$$I_K = 60\% \Rightarrow I_k = 0.60 \Rightarrow I_K^2 = 0.36.$$

When only the net premium rate is changed, I_k^2 becomes:

$$I_K^2 = \frac{n. \cdot q_1 \cdot (1-q_1) \cdot q_0^2 \cdot}{q_1^2 \cdot n \cdot q_0 \cdot (1-q_0)} = \frac{q_0. \cdot (1-q_1)}{q_1. \cdot (1-q_0)} =$$

$$= \frac{0.013 \cdot (1-q_1)}{q_1 \cdot (1-0.013)} = 0.36 \Rightarrow$$

$$\Rightarrow 0.013 \cdot (1-q_1) = 0.36 \cdot q_1 \cdot (1-0.013) \Rightarrow$$

$$\Rightarrow 0.013 = 0.013q_1 + 0.36q_1 - 0.00468q_1 \Rightarrow$$

$$\Rightarrow q_1 = 0.0353 = 3.53\%.$$

So, ***in order to allow the growth of the financial stability level with 40%, the insurance company has to increase the net premium rate about three times***, from 1.3% to 3.53%, fact that leads to a pretty high value considering the market average. As, there is the addition which represents the insurer's expenses and profit, the insurance company is not able to maintain the number of insured estates unchanged on a competitive market, in terms of an important growth of the net premium rate.

A synthesis of the four cases leads to the following results obtained by an insurance company, as shown in the Table 1.

Possible cases registered within an insurance company

Table 1

Case	No. of insured estates	Net premium rate	Value of the K coefficient
Ini□ială	**920**	**1.3%**	**0.287**
1	920	1.6%	0.259
2	1,380	1.3%	0.234
3	2,555	1.3%	0.172
4	920	3.53%	0.172

Analyzing these cases, for a higher financial stability level, the insurer has to make his option for the increase of the net premium rate to a certain level, taking into account the competition on the market, and for the increase of the number of insured estates belonging to the same category of risk.

In practice, the portfolio of insurance companies, for the same category of insured estates, includes several risk groups.

In addition to the initial case mentioned above, belonging to a certain category of risk (for example risk category type I), the insurance company disposes for the same class of three risk categories (see Table 2), thus the manager may adopt decisions according to a series of eventual situations.

The portfolio of an insurance company for a class of three risk categories

Table 2

Risk category	No. of insured estates	Average insured sum (euro)	Net premium rate (%)
I	920	23,000	1.3
II	1,200	20,000	1.1
III	1,360	15,000	0.9

In order to establish the financial stability level for the whole insured class, respectively for the

entire insurance company, it is important to determine first the financial stability coefficient for the other risk categories (risk category type II and III).

For the risk category type II:

$$K_{II}=\frac{\sigma_{II}}{Pnt_{II}}=\frac{S_{II}\cdot\sqrt{n_{II}\cdot q_{II}(1-q_{II})}}{q_{II}\cdot S_{II}\cdot n_{II}}=$$

$$=\frac{20{,}000\cdot\sqrt{1{,}200\cdot 0.011\cdot(1-0.011)}}{0.011\cdot 20{,}000\cdot 1{,}200}=\frac{72{,}263}{264{,}000}=0.274$$

For the risk category type III:

$$K_{III}=\frac{\sigma_{III}}{Pnt_{III}}=\frac{S_{III}\cdot\sqrt{n_{III}\cdot q_{III}(1-q_{III})}}{q_{III}\cdot S_{III}\cdot n_{III}}=$$

$$=\frac{15{,}000\cdot\sqrt{1{,}360\cdot 0.009\cdot(1-0.009)}}{0.009\cdot 15{,}000\cdot 1{,}360}=\frac{52{,}242}{183{,}600}=0.285$$

The ***financial stability level, for the whole class*** or for the entire portfolio of the insurance company is determined as it follows:

$K_T=\frac{\sigma_T}{Pn_T}$,where: $\sigma_T=\sqrt{\sum_{i=1}^{n}\sigma_i^2}$;

$Pn_T=\sum_{i=1}^{n}Pnt_i$; *i* – number of risk categories.

Thus, K_T becomes:

$$K_T=\frac{\sqrt{\sum_{i=1}^{n}\sigma_i^2}}{\sum_{i=1}^{n}Pnt_i}=\frac{\sqrt{\sigma_I^2+\sigma_{II}^2+\sigma_{III}^2}}{Pnt_I+Pnt_{II}+Pnt_{III}}=$$

$$=\frac{\sqrt{79{,}023^2+72{,}263^2+52{,}242^2}}{275{,}080+264{,}000+183{,}600}=\frac{119{,}146}{722{,}680}=0.165$$

In addition, the insurance company is able to determine the ***interval of years to which an unfavorable year is recorded for each risk category, respectively for the entire portfolio*** of the insurance company, considering the relation (7).

$$a=\frac{100}{\frac{K}{2}\cdot 100}=\frac{1}{P_{(D>Pnt)}},$$

$$a_I=\frac{100}{14{,}35}=6.96\approx 7 \text{ years}$$

$$a_{II}=\frac{100}{13{,}7}=7.29\approx 7 \text{ years}$$

$$a_{III}=\frac{100}{14{,}25}=7.01\approx 7 \text{ years}$$

$$a_T=\frac{100}{8{,}25}=12.12\approx 12 \text{ years}$$

In order to prevent losses hard to bear, the insurance company may cede a part of the concluded policies to reinsurance. Thus, in order to know whether the cession to reinsurance is imperious or not, it is necessary to compare the maximum insured value for each insured risk (X) to the insured sum meant for each risk. If the insured sum is lower than the maximum insured sum, the insurer is able to bear alone the insured risks and he is not forced to cede them to reinsurance (relation 7): $X=2K^2\cdot Pnt$.

$$X_I=2K_I^2\cdot Pnt_I=2\cdot 0.287^2\cdot 275{,}080=45{,}316$$

$$X_{II}=2K_{II}^2\cdot Pnt_{II}=2\cdot 0.274^2\cdot 264{,}000=39{,}640$$

$$X_{III}=2K_{III}^2\cdot Pnt_{III}=2\cdot 0.285^2\cdot 183{,}600=29{,}826$$

$$X_T=2K_T^2\cdot Pn_T=2\cdot 0.165^2\cdot 722{,}680=39{,}350 .$$

We notice that $X_I > S_I$ (45,316 > 23,000), $X_{II} > S_{II}$, $X_{III} > S_{III}$, $X_T > S_T$. Therefore, the insurance comapny is not forced to cede the policies to reinsurance because it is able to undertake all the insured risks.

Considering the crisis of the external financial markets and the control of financial stability, the AIG, the most important insurer of the world, has announced accountable reductions in assets because of the subprime crisis, of about 40 billion dollars, the most considerable loss in the insurance industry. One of the methods meant to protect the financial stability of the company is the reduction of premiums for the insurance policies of the trade estates, as an attempt to gain market share. Prices for this type of insurances diminished with 11% since June 2007 till June 2008.

4. Conclusions

Insurances represent products which provide protection against risks having the financial impact for insured persons. Thus, the consumers' trust in the financial stability of the insurance

sector is extremely important. The European Union proposed an indicator meant to measure the insurance company solvency, denominated as the solvency margin, being available for all the member countries.

Regarding Romania, the impact produced by the issues existing on the external financial markets over the insurance sector was limited, with no risks over its financial stability. The rate of growth specific to the underwriting activity in Romania was exceeded on the whole market by the evolution of indemnifications with 27.7% compared with 25% registered in the case of the underwritten premiums. This fact may generate negative implications over the financial stability of the insurance sector on middle term (Report of the financial stability, NBR, 2008). This evolution was generated by the auto insurances, because of the competition rise and the high costs meant for repairs, thus the requested premiums were inappropriate, unable to cover the underwritten risks.

We consider that this paper, the result of elaborated studies and analysis, represents an useful instrument for the insured persons, thus, they are able to choose the right type of insurance appropriate for their services, resting on its comparisons, analysis and conclusions, and for the insurance companies, being meant to improve their subscription and investment activity, as well as the financial stability.

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