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A Diamond-Dybvig model without bank run: the power of signaling*

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Abstract

This paper introduces the possibility of signaling into a finite-depositor version of the Diamond-Dybvig model. More precisely, the decision to keep the funds in the bank is assumed to be unobservable, but depositors are allowed to make it observable by signaling, at a cost. Depositors decide consecutively whether to withdraw their funds or continue holding balances in the bank, and they choose if they want to signal the latter decision. If the cost of signaling is moderate, then bank runs do not occur. Moreover, no signals are made, so the unconstrained-efficient allocation is implemented without any costs.

JEL codes: C72, D82, G21

Keywords: bank run, sequential game, signaling, iterated deletion of strictly dominated strategies, coordination

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1 Introduction

During the recent financial turmoil several banks in developed countries have experienced runs. In 2007, the bank run on Northern Rock in the UK heralded the oncoming crisis, and several other banks suffered runs, such as the Bank of East Asia in Hong Kong and the Washington Mutual in the USA. Non-bank institutions, like investment funds, have also experienced massive withdrawals very similar to bank runs. Examples include the collapse of Bear Stearns and the temporary suspension of redemptions in the Spanish real estate investment fund, Banif Inmobiliario.

Media coverage that made observable the lines in front of the bank offices might have influenced the evolution of these runs. In general, the information that depositors have about the underlying situation seems to be crucial to understand how bank runs emerge. Descriptions of the banking panics in the nineteenth century (Sprague (1910)) or in the 1930's (Friedman and Schwartz (1971), Wicker (2001)) indicate that panic episodes lasted for months and withdrawals did not start at once in each panic-stricken region, so depositors might have information about what happened elsewhere. Starr and Yilmaz (2007) analyze a bank-run episode which affected Turkey's Islamic financial houses in 2001. They study the behavior of depositors of different size (small, medium and large) and find that depositors were responsive to their peers and to the observable behavior of depositors of other groups. Iyer and Puri (2008) examine depositor-level data for a bank that faced a run in India in 2001 and find that social network effects were important regarding the depositors' decision-making. This evidence suggests that information about other depositors' choices is important to understand how bank runs arise.

However, the idea of having information about other depositors' decisions is mostly absent in the theoretical literature. In the seminal paper by Diamond and Dybvig (1983), depositors play a simultaneous-move game, without knowing anything about other depositors' decisions. There are two equilibria: one without bank run and another in which all depositors (independently of their liquidity needs) rush to withdraw their funds. If the bank applies suspension of convertibility, then bank runs can be eliminated. Suspension of convertibility allows the bank to suspend the payment to withdrawing depositors if their number surpasses a certain threshold. By suspending the payment, the bank guarantees that there will be sufficient funds to pay a high consumption in the next period relative to the available immediate consumption. Therefore, depositors without immediate liquidity need (called patient depositors) have no incentives to withdraw and a run will never start. Only depositors that have urgent liquidity need (called impatient depositors) withdraw their funds. Ennis and Keister (2009a) show that suspension of convertibility may fall prey to time inconsistency and ex post it is not an efficient instrument to prevent bank runs. Thus, it is still an open question if coordination failures leading to bank runs in the Diamond-Dybvig model can be avoided or not.[1]

In this paper, we show that bank runs in the Diamond-Dybvig model may be prevented by enhancing the observability of depositors' actions. Following Diamond and Dybvig (1983), bank-run models generally use a simultaneous-move framework, implying that depositors do not observe any decisions. Nevertheless, banks are able to observe to some extent depositors' decisions. Peck and Shell (2003) claim that the most natural assumption is that only withdrawals are observed by the bank, since depositors do not go to the bank to tell that they do not want to withdraw. Green and Lin (2000, 2003) assume that each depositor contacts the bank and communicates her decision of withdrawal or keeping the money deposited. We combine these two views and suppose that withdrawals are observable, whereas waitings are not.[2] However, waitings can be made observable, at a cost. Thus, a depositor who decides to wait can send a signal to the bank that reveals her decision. Important for our purpose, the bank upon observing the decision of a depositor communicates it to those who have not decided yet. Our approach is in line with Nosal and Wallace (2009) who consider a general information setup in which depositors do not only know their liquidity preferences, but any information that the bank chooses to communicate to them (e.g. the depositor's place in the sequence of decisions or decisions of the preceding depositors).

In our model, depositors decide in a consecutive way according to an exogenously given sequence of decisions.[3] Each depositor can either withdraw, wait and signal or wait without signaling. Sending the signal is costly, but a signal of waiting may induce subsequent patient depositors to wait as well. We show that as the game unfolds, for any patient depositor signaling strictly dominates withdrawal. As a consequence, patient depositors know that no other patient depositor would withdraw given the information sets that may arise, so they choose to wait without signaling. Therefore, the unconstrained-efficient allocation is implemented without costs. The intuition behind the result is that signaling is needed to make withdrawal a strictly dominated action, but once it is strictly dominated signaling becomes strictly dominated as well.

Our assumption about signaling the decision of waiting fits into the existing literature of bank runs, as explained before. Signaling - as seen in this paper - is not a standard practice in financial intermediation. However, with recent technological advances it may not just be a theoretical instrument but a practical one in the future. Signaling can also be seen as a metaphor of intense communication between the bank and its depositors.

[1] In Section 1.1 we review in depth the literature and claim that this is not clear not only in the Diamond-Dybvig model, but also in the literature.

[2] We use "keeping the money deposited" and "waiting" in an interchangeable manner.

[3] This assumption is usual in the literature. See Green and Lin (2000, 2003), Andolfatto et al. (2007) or Ennis and Keister (2009b).

1.1 Related literature

In this section, we survey the literature on bank runs focusing on the information that the depositors and the bank have.

As indicated before, in Diamond and Dybvig (1983) depositors decide simultaneously and then those who want to withdraw have the possibility to contact the bank in a random sequence of decisions. Bank runs may occur in equilibrium, unless a suspension-of-convertibility clause is included in the demand-deposit contract. This clause is dynamically inconsistent (Ennis and Keister (2009a)), leaving open the question if bank runs are avoidable in the Diamond-Dybvig setup.

Compared to Diamond and Dybvig (1983), the main modification by Peck and Shell (2003) is that the share of liquidity types (patient vs. impatient) is not fixed, but the realization of types is independent across depositors. Hence, there is aggregate uncertainty regarding the number of patient and impatient depositors. Depositors do not have any information about other depositors' decisions, they only know their own liquidity type. In this environment, bank runs constitute an equilibrium outcome. Both Diamond and Dybvig (1983) and Peck and Shell (2003) assume that only depositors who wish to withdraw contact the bank. In this paper, patient depositors who decide to wait have the opportunity to contact the bank.

Green and Lin (2000, 2003) build a model with aggregate uncertainty about liquidity types and introduce two novel elements. First, each depositor is assumed to contact the bank during the early period according to an exogenous sequence of decisions, not only those who attempt to withdraw. Second, depositors have information about their position in the sequence of decisions. These changes allow to show that bank runs do not occur in equilibrium. Notice that in spite of knowing the position in the sequence of decisions, the game is simultaneous in the game-theoretical sense.

Andolfatto et al. (2007) write down a model in the spirit of Green and Lin (2003) with an essential modification. The bank informs each depositor of the complete history of actions taken by the preceding depositors. Using the independence assumption about type realization, they show that any allocation that is implementable is also strictly implementable, so bank runs do not arise. However, in Andolfatto et al. (2007) observing previous choices is not important, because any patient depositor prefers to keep her money deposited if all subsequent patient depositors do so. Hence, whether previous withdrawals were due to real liquidity need or to panicking patient depositors does not affect the optimal decision. Even if all previous patient depositors have withdrawn waiting is optimal for a patient depositor provided that the remaining patient depositors wait. In our paper, optimal choice depends on the history and a patient depositor who infers that withdrawals by patient depositors occured may find it optimal to withdraw.

This difference is due to the different nature of the unconstrained-efficient allocations in models with and without aggregate uncertainty. When the share of different liquidity needs in the population is uncertain, then the bank takes into account each additional information that is revealed by the depositors' action. Thus, the bank "reoptimizes" the allocation after each decision and depositors of the same liquidity type end up with different consumptions, depending on their position and the earlier choices. As a consequence, optimal decision depends only on subsequent depositors' choices. In the other case (as Diamond and Dybvig (1983) or this paper), the unconstrained-efficient allocation is independent of the choices, depositors of the same liquidity type receive the same consumption (unless the bank runs out of funds). Therefore, upon observing many withdrawals a patient depositor may infer that the number of those who wait will not be sufficient to yield a period-2 consumption that is higher than the consumption related to immediate withdrawal. In this case, it is optimal to withdraw.

Gu (2010) incorporates the idea of observability into her model, and focuses on a signal extraction problem in which depositors try to find out whether the bank has fundamental problems or not. She disregards bank runs that are due to coordination failures, and studies the cases when previous withdrawals (possibly made by sophisticated depositors) are a signal of bad fundamentals. Our interest lies in investigating if some information structures eliminate the coordination problems that result in bank run, so our focus is different from Gu's.

The remainder of the paper is organized as follows. Section 2 describes the model, illustrates the main idea through an example and derives the result. Section 3 concludes.

2 The model

Our model builds on the seminal paper by Diamond and Dybvig (1983). There are three time periods denoted by $t = 0, 1, 2$ and a finite set of depositors denoted by $I = \{1, ..., N\}$, where $N > 2$. Depositor i's consumption in period t is denoted by $c_{t,i} \in \mathbb{R}_+$, and her liquidity type by θ_i. It is a binomial random variable with support given by the set of liquidity types $\Theta = \{0, 1\}$. If $\theta_i = 0$, depositor i is called *impatient*, who only cares about consumption at $t = 1$. If $\theta_i = 1$, depositor i is called *patient*. Depositor i's utility function is given by

$$u(c_{1,i}, c_{2,i}, \theta_i) = u(c_{1,i} + \theta_i c_{2,i}). \tag{1}$$

It is assumed to be strictly increasing, strictly concave, twice continuously differentiable and to satisfy the Inada conditions. The relative risk-aversion coefficient $-c_i u''(c_i)/u'(c_i) > 1$, for every $c_i \in \mathbb{R}_+$, and all $i \in N$.

The number of patient depositors is assumed to be constant and given by $p \in [1, N]$. The remaining depositors are impatient. Hence, there is no aggregate uncertainty about types in this model, and the number of patient and impatient depositors is assumed to be common knowledge.

At $t = 0$, each depositor $i \in I$ has one unit of a homogeneous good which she deposits in the bank, to be defined below. The bank has access to a constant-return-to-scale productive technology which pays a gross return of one unit for each endowment liquidated at $t = 1$, and a fixed return of $R > 1$ for each endowment liquidated at $t = 2$.

2.1 The efficient allocation and the bank

If a benevolent social planner observed each depositor's liquidity type, then she could maximize the sum of depositors' utilities with respect to $c_{1,i}$ and $c_{2,i}$ subject to a resource constraint and p. Since depositors differ only in their types, in the optimum those of the same type receive the same consumption. Therefore, henceforth we supress the subindex i and use c_1 and c_2. This first-best allocation solves

$$\begin{aligned}&\max_{c_1,c_2} \ (N-p)u(c_1) + pu(c_2)\\ &\text{s. t.}\\ &(N-p)c_1 + [pc_2/R] = N.\end{aligned} \tag{2}$$

The solution to this problem is

$$u'(c_1^*) = Ru'(c_2^*), \tag{3}$$

which - as in Diamond and Dybvig (1983) - implies that $R > c_2^* > c_1^* > 1$. Therefore, patient depositors receive a higher consumption than impatient ones. This solution is the unconstrained-efficient allocation. It offers liquidity insurance, because the amount of consumption given to an impatient depositor is higher than that in autarky.[4]

At $t = 0$, the depositors form a bank by pooling their initial endowments. The bank insures against the privately observed liquidity risk, which is only realized at the beginning of $t = 1$, by offering a simple demand-deposit contract that implements the unconstrained-efficient allocation, as is shown by Diamond and Dybvig (1983). The simple demand-deposit contract offers to pay c_1^* to any depositor i who withdraws at $t = 1$ as long as the bank has funds. Any patient depositor i who waits until $t = 2$ receives a pro rata share of the funds available then. Let $\eta \in [0, p]$ be the number of depositors who wait at $t = 1$. Given η, depositor i's consumption at $t = 2$,

[4] In autarky, an impatient depositor earns the unit gross return at $t = 1$, while a patient depositor earns R at $t = 2$.

$$c_2(\eta) = \begin{cases} \max\{0, \frac{R(N-(N-\eta)c_1^*)}{\eta}\} \text{ if } \eta > 0 \\ 0 \text{ if } \eta = 0 \end{cases} \tag{4}$$

If $\eta = p$, that is, only impatient depositors withdraw at $t = 1$, then $c_2(\eta) = c_2^*$ and patient depositors enjoy a higher consumption than impatient ones.

However, if η is too low, then to withdraw at $t = 1$ is better also for patient depositors since to wait until $t = 2$ yields them strictly less than c_1^*. That is, if the number of patient depositors who keep the money in the bank is below $\bar{\eta}$, a threshold value for η, then their period-2 consumption is strictly below c_1^*. The threshold value $\bar{\eta}$ is derived formally in Lemma 1 whose proof is given in Appendix A.

Lemma 1 *There exists* $1 \leq \bar{\eta} \leq p$ *such that for all* $i \in N$,

$$\begin{gathered} c_2(\bar{\eta}-1) < c_1^*, \ \textit{for any } \eta \leq \bar{\eta}-1, \ \textit{and} \\ c_1^* \leq c_2(\bar{\eta}), \ \textit{for any } \eta \geq \bar{\eta}. \end{gathered} \tag{5}$$

Note that η is known only at the end of period 1, after each depositor has decided. Yet depositors have to guess its value as it is their turn to choose, based on their available information.

2.2 Decisions and signaling

Depositors decide in an exogenously given sequence of decisions. Let $\Theta^N = \{0,1\}^N$ be the set of all possible sequences of depositors and let $\theta^N = (\theta_1, ..., \theta_N) \in \Theta^N$ denote the realized sequence. There are $\binom{N}{p}$ possible sequences of length N with p patient depositors. Suppose that each of them is selected by a random process with some probability. The realized sequence is unobserved both by the depositors and by the bank. Neither do depositors know their position in the sequence. As usual in the literature (Wallace 1988), depositors are isolated and no trade can occur among them in period 1.

We assume waitings to be unobservable (as in Peck and Shell (2003)) but we allow (but do not require) patient depositors to signal their waiting. The available actions are withdraw (w), wait without signaling (k), wait and signal (r).[5] The difference between the last two lies in the observability. When a depositor signals, her decision to wait becomes visible to the bank, and in turn to the depositors, since the bank shares the available information with them. Since signaling to the bank in period 1 is not related to consumption,

[5] Ocassionally, to the last action we will simply refer as signaling.

we allow for the possibility that it is costly. There is a nonnegative and uniform signaling cost in utility terms and it is denoted by ξ.[6]

Assumption 1: $u(c_2^*) - u(c_1^*) > \xi$.

If the opposite held, then the cost would be so high that it does not compensate for the potential gain in utility. To make signaling a real option we use assumption 1 throughout the paper. Intuitively, a patient depositor would like to signal, because sending this signal could induce subsequent patient depositors not to withdraw, and have a high period-2 consumption.

Note that to signal and withdraw does not make sense, because withdrawal implies immediate consumption and signaling does not affect the amount of this consumption. Moreover, it is costly. For this reason, we disregard the possibility of withdrawing and signaling.

Depositors are called by nature one-by-one to decide according to θ^N. Depositors only observe the information that the bank provides about previous choices that can be observed. We suppose that the time elapsed in period 1 is not informative. As a consequence neither the bank, nor the depositors can find out the number of depositors who have waited without signaling based on the elapsed time and the number of withdrawals.

To illustrate the game consider the following example.

2.3 An example

There are four depositors, three patients and an impatient one. Suppose that all patient depositors have to keep the money in the bank to make waiting worthwhile ($\bar{\eta} = 3$). Since waiting without signaling is unobservable, there is uncertainty about the position in the sequence. Suppose that

$$\begin{aligned} u(c_2(\eta)) > u(c_1^*) \text{ for } \eta = 3, \\ u(c_1^*) > u(c_2(\eta)) \text{ for } \eta \leq 2. \end{aligned} \tag{6}$$

so patient depositors only prefer not to withdraw if all the other patient depositors do so as well.

Consider the observable history (r) that is compatible with being at position 2 and 3. A patient depositor observing it may believe the following: (i) she is at position 3 and - besides the depositor who signaled - she was preceded by a patient depositor who waited without signaling, (ii) the observed history coincides with the true history, so she is the second to decide. Clearly, if the history contained also an unobserved waiting,

[6] How are signaling costs in real life? Our guess is that they are rather small as a consequence of technological advances, like Internet banking. Notice that in Green and Lin (2003) each depositor has to contact the bank (even if she waits) and contacting is not costly.

then for a patient depositor the best response is to wait without signaling. In the other case signaling strictly dominates withdrawal, because the last patient depositor would observe two signals which would make her wait and the signaling depositor would have $u(c_2^*) - \xi > u(c_1^*)$. Therefore, a patient depositor observing a signal would not withdraw. As a consequence, when observing (r, w) any depositor knows that the withdrawal must have been due to the impatient depositor. Hence, for a patient depositor observing this history signaling strictly dominates withdrawal. Since no patient depositor withdraws when observing (r, w), the best response is to wait without signaling. Anticipating this decision, a patient depositor's best response observing (r) is also to wait without signaling.

Let us see what happens if a patient depositor observes (w). We have seen that when the history begins with a signal, then no subsequent patient depositor will withdraw.[7] Consequently, for a patient depositor who observes nothing signaling strictly dominates withdrawal, so this depositor will not withdraw. Therefore, if an observable history begins with a withdrawal, it must have been the choice of the impatient depositor. When observing (w, r) signaling strictly dominates withdrawal, since when there are two signals in any observable history, then the next patient depositor (if there is any) will wait without signaling. Again, since the unique impatient depositor has already withdrawn and no patient depositor observing (w, r) withdraws, the best response is to wait without signaling. It implies also that when observing (w) signaling strictly dominates withdrawal, because the ensuing information sets surely lead to higher consumptions than c_1^*. Moreover, waiting without signaling is the best response, because when observing a withdrawal a patient depositor knows that it was done by the impatient depositor and if there are any subsequent patient depositors, then those depositors will not withdraw.

As we have seen, if a patient depositor does not observe anything, then she will not withdraw. Nor will she signal, since for a patient depositor the best response to the observable history (w) is to wait without signaling. As a consequence, the best response to observing nothing is to wait without signaling, because it leads to the unconstrained-efficient allocation and does not entail costs. Hence, when observing either $(\emptyset)$ or (w) the best response is to wait without signaling, so as the game unfolds patient depositors wait without signaling and the first best obtains.

The intuition behind the result is that signaling is needed to make withdrawal a dominated action, but once it was strictly dominated signaling becomes dominated as well.

[7] A patient depositor would best respond by withdrawing to an observable history (r, w, w), but by our previous arguments it cannot arise.

2.4 The general case

The information set consists of the own type and the history which is observable. Due to the unobservability of waitings, a depositor observing any history does not know her position with certainty. If she observes ω withdrawals and ρ signals, then she just knows that she is at least in position $\omega + \rho + 1$ and at most in position $\omega + p$. The range of possible positions is $p - \rho - 1$ which makes position uncertainty eventually quite large.

We denote by $H^{obs}_{\omega_j,\rho}$ the set of observed histories containing any permutation of $\omega \in \{0, 1, 2, ...n - 1\}$ withdrawals and $\rho \in \{0, 1, 2, ...p - 1\}$ signals. An element in this set is denoted by $h^{obs}_{\omega,\rho}$. Notice that it is possible that two (or even more) patient depositors observe the same observable history.

A pure strategy for an depositor is a map $\mathbf{s}(\boldsymbol{\theta}, H^{obs}) : \{0, 1\} \times H^{obs} \rightarrow \{w, k, r\}$, where $H^{obs} = \times(H^{obs}_{\omega,\rho})_{\substack{\omega\in\{0,1,2,...n-1\}\\ \rho\in\{0,1,2,...p-1\}}}$ is the set of all possible observable histories. Therefore, each depositor has to specify what to do when observing any possible history and being of either type. We have suppressed the subindex i to stress that position is unknown. We focus on patient depositors, because impatient depositors always withdraw.

We show that the game has a unique outcome using iterated deletion of dominated strategies.

Proposition 1 *For patient depositors signaling dominates withdrawal for any observable history starting with $\rho \in [0, p - 1]$ signals and followed by $\omega \in [1, n - p]$ withdrawals.*

Proof. See Appendix B. ■

The proof results from the interaction of two elements: truthful histories and histories that start with signals. An observable history is truthful if no patient depositor has withdrawn.

If a history begins with $p-1$ signals, then a patient depositor knows that she is the last patient depositor in the sequence and her dominant strategy is to wait. Moreover, for patient depositors waiting is the dominant strategy for any history that contains $p-1$ signals. Note that a patient depositor observing any history with $p - 1$ signals and $\omega \in [1, n - p]$ withdrawals infers that the history is truthful.

Now consider a history that begins with $p - 2$ signals. A patient depositor observing this history knows that she is the $(p - 1)$th patient depositor in the sequence. For this patient depositor signaling dominates withdrawal, because $u(c^*_2) - \xi > u(c^*_1)$.[8] If a patient depositor observes a history that begins with $p - 2$ signals and is followed by a withdrawal, then she knows that the withdrawal has been due to an impatient depositor and that she is the $(p - 1)$th patient depositor in the sequence. Signaling dominates withdrawal,

[8] Note that by signaling she induces the last patient depositor to wait, since the last patient depositor will observe $p-1$ signals and her best response is to wait. As a consequence, all patient depositors wait, yielding to all of them period-2 consumption c^*_2.

since it yields a truthful history with $p-1$ signals that induces the last patient depositor to wait. Given this argument, upon observing a history that begins with $p-2$ signals and is followed by two withdrawals a patient depositor infers that only impatient depositors have withdrawn and that she is the $(p-1)$th patient depositor in the sequence. Signaling dominates withdrawal for the same reasons as before. Applying this reasoning repeatedly, we conclude that for any history beginning with $p-2$ signals for a patient depositor signaling dominates withdrawal. Furthermore, for any truthful history containing $p-2$ signals signaling dominates withdrawal. This is the case because by signaling a truthful history with $p-1$ signals is generated and by previous results we know that then no patient depositor withdraws.

Consider a history that begins with $p-3$ signals. A patient depositor observing this history knows that she is the $(p-2)$th patient depositor in the sequence. For this patient depositor signaling dominates withdrawal, because that would yield a truthful history with $p-2$ signals that - by previous arguments - leads to the utility of $u(c_2^*)-\xi > u(c_1^*)$. Then, when observing a history that begins with $p-3$ signals and is followed by a withdrawal a patient depositor knows that it is a truthful history, so signaling dominates withdrawal. This line of reasoning results in the conclusion that for any history beginning with $p-3$ signals for a patient depositor signaling dominates withdrawal. Moreover, for any truthful history containing $p-3$ signals signaling dominates withdrawal.

By repeating the same procedure with histories that begin with less and less signals, we obtain Proposition 1.

A direct consequence of the proposition is the following theorem.

Theorem 1 *The unconstrained-efficient allocation is strongly implementable.*

Proof. *See Appendix C.* ■

Proposition 1 implies that a patient depositor does not withdraw when observing zero signal followed by $\omega \in [1, n-p]$ withdrawals. As a consequence, in whatever position the first patient depositor arrives, she will not withdraw. She will not signal either, because even if the next patient depositor only observes the withdrawals of the impatient depositors, she will not withdraw either. In fact, this is the case for any subsequent patient depositor, so the optimal decision is to wait without signaling.

The proposition predicts a unique outcome of the game in which patient depositors do not signal. Signaling makes withdrawal a strictly dominated strategy, and once withdrawals can only be due to impatient depositors there is no need to incur the cost of signaling. The possibility of signaling can be seen as richer communication between the bank and its depositors. This result is in line with the findings of Iyer and Puri (2008) which state that the longer and deeper the relation ship between a depositor and the bank, the less likely it is that the depositor participates in a run.

3 Conclusion

Most of the literature on bank runs uses a simultaneous-move approach to model depositors' decisions in spite of contrary empirical evidence. To make the informational structure richer, we introduce two elements. We allow the bank to share information with depositors about previous decisions and we allow depositors who decide to wait to signal their decision to the bank at a cost (and through the bank to subsequent depositors). We find that in our environment bank runs do not occur. Moreover, in the unique outcome no signals are made, so the unconstrained-efficient allocation obtains.

Although we do not study explicitly policy issues, our result has a clear policy message. Observing other decisions matters in depositors' decision-making and communication structures allowing better information flow may help to avoid unjustified bank runs.

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5 Appendix

5.1 Appendix A

Lemma 1: *There exists a $1 \leq \bar{\eta} \leq p$ such that for all $i \in N$,*

$$\begin{gathered} c_2(\bar{\eta}-1) < c_1^* \text{ for any } \eta \leq \bar{\eta}-1 \\ \text{and} \\ c_1^* \leq c_2(\bar{\eta}) \text{ for any } \eta \geq \bar{\eta}. \end{gathered} \tag{7}$$

Proof. *Note that $\left\lfloor \frac{n}{c_1^*} \right\rfloor$, that is the integer part of $\frac{n}{c_1^*}$, is the maximum number of depositors to whom the bank is able to pay c_1^*. Since $1 < c_1^*$, we have that $\left\lfloor \frac{n}{c_1^*} \right\rfloor < n$. That is, the bank cannot pay in period 1 to all depositors $1 < c_1^*$, since it has only n units of deposits. Hence, for any $\eta < n - \left\lfloor \frac{n}{c_1^*} \right\rfloor$, $c_2(\eta) = 0$. Thus, if the number of withdrawals is too high, then the bank runs out of funds and cannot pay anything to those who have waited.*

On the other hand, $c_2^* = c_2(p)$ and $c_2(x) > c_2(x-1)$ for any $n - \left\lfloor \frac{n}{c_1^*} \right\rfloor < x - 1 < p$, so given

$$c_2(\eta) < c_1^* < c_2^* = c_2(p) \text{ for } \forall\ \eta < n - \left\lfloor \frac{n}{c_1^*} \right\rfloor \tag{8}$$

there is a unique $\bar{\eta}$ such that for any $\bar{\eta} \leq \eta$ we have $c_1^* \leq c_2(\eta)$, whereas for any $\eta < \bar{\eta}$ we have $c_2(\eta) < c_1^*$.

■

5.2 Appendix B

The following definition will prove convenient for the proof.

Definition 1 *An observable history is truthful if no patient depositor has withdrawn.*

First, we show that if for a patient depositor upon observing a truthful history with x signals signaling dominates withdrawal, then it holds also when observing a truthful history with $x-1$ signals.

Lemma 2 *Suppose that for patient depositors signaling strictly dominates withdrawal when observing a truthful history with ρ signals. Then, signaling strictly dominates withdrawal when observing a truthful history with $\rho-1$ signals.*

Proof. *The assumption that signaling strictly dominates withdrawal when observing a truthful history with ρ signals implies that at the end of period 1 the amount of non-withdrawals is such that $u(c_2(\eta))-\xi > u(c_1^*)$. When a patient depositor observes a truthful history with $\rho-1$ signals, then by signaling she can generate a truthful history with ρ signals and as a consequence she can ensure to have utility $u(c_2(\eta))-\xi > u(c_1^*)$. Therefore, signaling in this case strictly dominates withdrawal.* ■

In the next step, we show how proceeding from the end of the sequence of decisions we can determine if a history is truthful or not.

Lemma 3 *Suppose that signaling strictly dominates withdrawal when observing a truthful history with ρ signals. Then, any history beginning with $\rho-1$ signals is a truthful history.*

Proof. *Consider first the history consisting of $\rho-1$ signals. By definition, it must be a truthful history. Furthermore, a patient depositor observing this history prefers signaling to withdrawal by lemma 2. Consider next the history that begins with $\rho-1$ signals followed by a withdrawal. Since a patient depositor would not withdraw upon observing $\rho-1$ signals, the withdrawal must be due to an impatient depositor, so this history is truthful as well. By applying lemma 2 we know that given this history signaling strictly dominates withdrawal. As a consequence, when observing the history that begins with $\rho-1$ signals followed by two withdrawals depositors infer that the withdrawals have been choices of impatient depositors, so this history is truthful as well. By repeating this reasoning, we find that any history that begins with $\rho-1$ signals and is followed by $\omega \in [1, n-p]$ withdrawals is a truthful history.* ■

We put now the two lemmas to work. Consider a patient depositor who observes any history that contains $p-1$ signals. The histories are truthful since all the other patient depositors have signalled and clearly signaling strictly dominates withdrawal. By lemma 2, for patient depositors signaling strictly dominates withdrawal when observing a truthful history with $p-2$ signals and by lemma 3 any history beginning with $p-2$ signals must be a truthful history. Therefore, for patient depositors signaling strictly dominates withdrawal for any history beginning with $p-2$ signals and followed by $\omega \in [1, n-p]$ withdrawals.

By applying lemma 2 again, for patient depositors signaling will strictly dominate withdrawal when observing a truthful history with $p-3$ signals and lemma 3 implies that any history beginning with $p-3$ signals must be a truthful history. Hence, for patient depositors signaling strictly dominates withdrawal for any history beginning with $p-3$ signals and followed by $\omega \in [1, n-p]$ withdrawals.

By applying the two lemmas repeatedly yields Proposition 1.

Proposition 1 *For patient depositors signaling dominates withdrawal for any observable history starting with $\rho \in [0, p-1]$ signals and followed by $\omega \in [1, n-p]$ withdrawals.*

5.3 Appendix C

We begin with the definition of strong implementability in our setup.

Definition 2 *The unconstrained-efficient allocation is strongly implementable if for all patient depositors, $\mathbf{s}(\boldsymbol{\theta} = \mathbf{1}, H^{obs}) = k$ is the unique strategy profile that survives the process of iterated deletion of dominated strategies.*

In the proof of the theorem, we show that as the game unfolds, patient depositors will face observable histories for which according to Proposition 1 signaling strictly dominates withdrawal. Patient depositors realize that by waiting without signaling all subsequent patient depositors will see observable histories that makes them not to withdraw. Hence, the optimal action is to wait without signaling.

Theorem 1: *The unconstrained-efficient allocation is strongly implementable.*

Proof. *By proposition 1, for any history beginning with $\rho \in [0, p-1]$ signals and followed by $\omega \in [1, n-p]$ withdrawals signaling strictly dominates withdrawal. As a consequence, if a patient depositor observes any of these histories the optimal decision for her is k, because even though subsequent patient depositors do not observe her signal, they will observe a history that makes her to wait without signaling. Hence, there is no need to incur the cost of signaling. As a result, the unconstrained-efficient allocation obtains.* ■