

**A dynamic risk optimization model for evaluating profitable and feasible  
water management plans**

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# **A dynamic risk optimization model for evaluating profitable and feasible water management plans**

## **ABSTRACT**

Currently the South African government is advocating the cultivation of high-valued crops and more efficient use of available water resources through the adoption of more efficient irrigation technology and irrigation scheduling. A requirement of the National Water Act (Act 36 of 1998) is the compilation of water management plans. The main objective of this paper is to develop a multiperiod mathematical risk programming model able to assist water user associations with the compilation of water management plans that are both profitable and feasible. Special care was taken to represent canal capacities and irrigation system application rates in the model. Risk simulation procedures are used to generate an appropriately correlated inter- and intra-temporal risk matrix for the programming model. A combination of subjectively elicited distributions of crop yield and objective data on crop prices were used to characterise risk. The model was applied to a representative flood irrigation farm in the Vaalharts irrigation scheme South Africa to demonstrate the capability of the model to optimise agricultural water usage over a 15 year planning horizon. Model results clearly indicate the potential of high-value crops and more efficient irrigation technology to reduce the impact of water restrictions. Furthermore infrastructure, the financial position of the farmer and the level of risk averseness have significantly impacted on the results. Policy makers and government authorities should take cognisance of these factors when evaluating water use efficiency and water management plans of different water user associations. Improvements to the adopted modelling procedure are also suggested.

JEL classification: C6, Q15, Q12

Keywords: Dynamic Linear Programming, risk, irrigation, feasibility, South Africa

## INTRODUCTION

The National Water Act (Act 36 of 1998) requires the development of a National Water Resources Strategy (NWRS) as part of the implementation of the Act. This strategy describes how the water of South Africa will be protected, used, developed, conserved and managed. The NWRS seeks to identify opportunities where water can be made available for productive livelihoods and also to give the support and assistance needed for the efficient use of the water. Furthermore, South Africa is in a process of water allocation reform in order to promote equity, address poverty, generate economic growth and create jobs (DWAF, 2004). Politicians recognize that the allocation process should allow for the sustainable use of water and that it must promote efficient and non-wasteful use of water. The National Water Conservation and Demand Management Strategy (WCDMS) forms an integral part of the NWRS. The agricultural sectoral strategy was recently finalized and endeavours to provide a supportive and enabling framework to improve irrigation efficiency. By law each water user association (WUA) is required to develop and submit a water management plan as part of the WCDMS. Emphasis is placed on the cultivation of high-value crops and the adoption of more efficient irrigation technology.

In South Africa deterministic dynamic linear programming (DLP) is applied frequently as a method to assist water managers with optimal water usage over the long term (Backeberg, 1997; Haile *et al.*, 2003; Maré, 1995; Van Schalkwyk & Louw, 2004; Viljoen *et al.*, 1992). A problem with the application of mathematical programming models is that overspecialization occurs if important constraints and costs are not appropriately specified, resulting in unrealistic model results. Louw (2002) used positive mathematical programming to overcome these problems. Louhichi *et al.* (2004) cautioned that the integration of land, as well as technical, agronomical, economic and financial constraints in the cost function, might

become highly complicated. Louhichi *et al.* (2004) propose the use of risk programming as an alternative to positive mathematical programming.

The main objective of this paper is to develop a multiperiod mathematical risk programming model able to assist water user associations with the compilation of water management plans that are both profitable and feasible. Representing infrastructural and water supply capacity limitations in the programming model are seen as critical. Subjective and objective data are combined through the use of risk simulation procedures to generate an appropriately correlated inter- and intra-temporal risk matrix for the programming model. The applicability of the model is demonstrated by applying the model to a representative irrigation farm to evaluate the risk return tradeoffs of alternative management options when water allocations are restricted by 20%.

#### **DYNAMIC RISK PROGRAMMING MODEL**

A disequilibrium known life type of DLP model (McCarl & Spreen, 2003) was developed to optimize water usage over a period of 15 years. Known life means that resources and fund flows are committed for a fixed period of time, whereas disequilibrium implies that the same activity does not need to follow the previous activity and can be replaced with another activity. MOTAD was used to incorporate risk into the DLP model. A more detailed description of the model follows, with capital letters representing variables. All the input parameters were discounted to present values before entering the optimization model, and therefore no discounting is shown when the model is specified.

The objective of the model is to maximize the present value of after-tax cash surpluses at the end of the planning horizon, plus terminal values for any activity beyond the planning horizon, minus a risk aversion parameter ( $\alpha$ ), multiplied by the approximate standard error (SE) of the solution.

$$CF_{15} + \sum_i^3 \sum_c^5 \sum_{it}^{15} Q_{i,c,it} qt_{i,c,it} + \sum_i^3 \sum_t^{15} IR_{i,it} irt_{i,it} - \sum_c^5 \sum_{it}^{15} P_{c,it} pt_{c,it} - \sum_i^3 \sum_{it}^{15} IL_{i,it} ilt_{i,it} - \alpha SE \quad (1)$$

$CF_{15}$  cash flow in year 15

$Q_{i,c,it}$  quantity of crop  $c$  established in year  $it$  utilizing irrigation system  $i$

$qt_{i,c,it}$  terminal value associated with cropping activities established in year  $it$

$IR_{i,it}$  investment in irrigation system  $i$  in year  $it$

$irt_{i,it}$  terminal value associated with irrigation investment  $i$  in year  $it$

$P_{c,it}$  production loan for financing production cost of crop  $c$  in year  $it$

$pt_{it}$  terminal value associated with production loan in year  $it$

$IL_{i,it}$  borrowed capital to finance irrigation system  $i$  in year  $it$

$ilt_{i,it}$  terminal value associated with borrowed capital in year  $it$

$\alpha$  risk aversion parameter

$SE$  approximate standard error

The normative approach proposed by Rae (1970) is used to account for any cash flow streams beyond the planning horizon. With the normative approach a terminal value is calculated for each activity as the present value of future net revenue discounted from infinity for an assumed replacement cycle, given the planning horizon, is exceeded. Terminal values ensure that capital investments with cash flow streams beyond the planning horizon are not penalized. The following two equations are used to calculate the cash surpluses in each year of the planning horizon:

$$\begin{aligned} & \sum_i^2 \sum_c^5 \sum_{it}^{15} Q_{i,c,it} pi_{i,c,t,it} + B_t(1 + ri_t) - \sum_c^5 \sum_{it}^{15} P_{c,it} pay_{c,t,it} + \sum_i^3 \sum_{it}^{15} IR_{i,it} sal_{i,t,it} - fix_t \\ & - \sum_i^3 \sum_{it}^{15} IL_{i,it} ipay_{i,t,it} - lc_t - TI_t tax = CS_t \end{aligned} \quad (2)$$

$$\begin{aligned} & \sum_i^3 \sum_c^5 \sum_{it}^{15} Q_{i,c,it} (pi_{i,c,t,it} - pc_{i,c,t,it}) - \sum_i^3 \sum_{it}^{15} IR_{i,it} (dep_{i,t,it} - sal_{i,t,it}) + Bri_t \\ & - \sum_c^5 \sum_{it}^{15} P_{c,it} pay_{c,t,it} - \sum_i^3 \sum_{it}^{15} IL_{i,it} ipay_{i,t,it} - fix_t - TT_{t-1} + TT_t = TI_t \end{aligned} \quad (3)$$

$pi_{i,c,t,it}$	production income in year $t$ of crop $c$ established in year $it$ utilizing irrigation system $i$
$pc_{i,c,t,it}$	production cost in year $t$ of crop $c$ established in year $it$ utilizing irrigation system $i$
$B_t$	money in the bank in year $t$
$ri_t$	interest on money in the bank in year $t$
$pay_{c,t,it}$	instalment in year $t$ to finance production cost of crop $c$ established in year $it$
$ipay_{c,t,it}$	instalment in year $t$ to finance irrigation system $i$ established in year $it$
$payi_{c,t,it}$	interest portion of instalment in year $t$ to finance production cost of crop $c$ established in year $it$
$ipayi_{c,t,it}$	interest portion of instalment in year $t$ to finance irrigation system $i$ established in year $it$
$fix_t$	overheads in year $t$
$lc_t$	living expenses in year $t$
$tax$	marginal tax rate
$dep_{i,t,it}$	parameter specifying the tax deductions in year $t$ associated with irrigation system $i$ established in year $it$
$sal_{i,t,it}$	salvage value in year $t$ of irrigation system $i$ purchased in year $it$
$TI_t$	taxable income in year $t$
$TT_t$	taxable income transferred in year $t$ due to a negative taxable income
$CS_t$	cash surplus in year $t$

A cash surplus in any given year exists if the sum of production income, money in the bank account (including interest earnings) and any salvage income is more than the sum of all overhead expenses, loan repayments, living expenses and tax liabilities. Equation (2) does not account for operating capital, as the bank balance is net of operating capital. Taxable income is a function of production income, operating expenses, salvage income, overheads, interest and depreciation deductions, as well as any losses transferred from the previous year. The DLP model has the unique ability to defer tax payments until a positive taxable income is calculated. Equations (4) to (7) are used to determine how the generated cash surplus of the previous year will be utilized in the current production year.

$$B_t + \sum_c CP_{c,t} + CI_t - CS_{t-1} \leq 0 \quad (4)$$

$$\sum_i^3 \sum_{it}^{15} Q_{i,c,it} pc_{i,c,t,it} - CP_{c,t} - P_{c,t} \leq 0 \quad (5)$$

$$\sum_i^3 \sum_{it}^{15} IR_{i,it} inv_{i,t,it} - \sum_i^3 IL_{i,t} - CI_t \leq 0 \quad (6)$$

$$\sum_i^3 IL_{i,t} + \sum_{\substack{it \\ it < t}} IL_{i,it} ipayo_{i,t,it} \leq icf_t \quad (7)$$

$$\sum_c P_{c,t} + \sum_{\substack{it \\ it < t}} P_{c,it} payo_{c,t,it} \leq cf_t \quad (8)$$

$CP_{c,t}$  money used to finance production cost of crop  $c$  in year  $t$

$CI_t$  money used to finance investments in irrigation systems in year  $t$

$inv_{i,t,it}$  investment cost in year  $t$  of irrigation system  $i$  established in year  $it$

$payo_{c,t,it}$  outstanding capital year  $t$  of production loan used to finance production cost of crop  $c$  established in year  $i$

$ipayo_{i,t,it}$  outstanding capital year  $t$  of borrowed capital used to finance irrigation system  $i$  established in year  $it$

$cf_t$  credit facility for financing production costs in year  $t$

$icf_t$  credit facility for financing irrigation investment cost in year  $t$

Cash surpluses from the previous year can be used to purchase new irrigation technology and/or to finance operating expenses with any surplus deposited in a bank account. The model furthermore allows for the use of production loans as a means to finance production cost, and borrowed capital to finance irrigation investments. The amount of money that might be borrowed in any given year is limited by the credit facilities and the amount outstanding. The following constraints are used to determine land occupation by irrigation system and crop:

$$\sum_c^3 \sum_{it}^{15} Q_{i,c,it} lo_{c,t,it} - \sum_{it}^{15} IR_{i,it} io_{i,t,it} \leq 0 \quad (8)$$

$$\sum_{it}^{15} IR_{i,it} io_{i,t,it} \leq land_i \quad (9)$$

$$\sum_c^3 \sum_{it}^{15} Q_{i,c,it} ru_{r,i,c,t} \leq ra_{r,t} \quad (10)$$

$io_{i,t,it}$  land occupation in year  $t$  of irrigation system  $i$  established in year  $it$

$lo_{c,t,it}$  land occupation in year  $t$  of crop  $c$  established in year  $it$

$land$  available land resources

$ru_{r,i,c,t}$  use of resource  $r$  by crop  $c$  planted with irrigation technology  $i$  in year  $t$

$ra_{r,t}$  availability of resource  $r$  in year  $t$

Equation (8) is used to ensure that an investment in an irrigation system is made first, before any cropping activities can take place. Thus the cultivation of a specific crop is linked to the availability of a specific irrigation technology. The total irrigation development is restricted to available land resources with equation (9). Equation (10) ensures that resource use is less than resource availability in any time period.

## DATA

Objective and subjective data were combined to develop a risk matrix for the multiperiod MOTAD model through the use of risk simulation procedures. Deflated historical prices (NDA, 2005) after the deregulation of South Africa's markets were used to characterize price risk as empirical distributions. None of the deflated prices exhibit significant trends at a  $p=0.05$  level. Crop yield variability associated with flood and pivot irrigation technologies were subjectively estimated using the triangular distribution for which the cumulative probability distribution,  $F(x)$ , is completely defined in terms of the minimum ( $a$ ), maximum ( $b$ ), and the most probable value (mode) ( $m$ ) (Hardaker *et al.*, 1997). Eighteen irrigation farmers were asked to specify these parameters for each of the irrigation technologies. These distributions were aggregated by taking 100 random draws from each of



these triangular distributions and then using the 1800 values to represent an empirical distribution of crop yields.

The inverse transform method (Rae, 1994) was used to transform the  $F(x) = p$  of the triangle and empirical distributions to  $x = f(p)$ . Since empirical distributions are presented as discrete points on a cumulative distribution function and therefore do not exhibit any closed form for  $F(x)$ , interpolation was used to determine the continue  $f(p)$  function. By substituting appropriately correlated inter- and intra-temporal uniform random values for  $p$ , it was possible to draw correlated random entities from the cumulative probability distributions characterizing risk. Procedures developed by Richardson (2004) were used to correlate the random numbers. Firstly, independent standard normal deviates (ISND) equal to the number of iterations are generated for each risk entity for each year in the planning horizon. Each year's ISNDs are correlated through the multiplication of the deviates with the Cholesky decomposition of the inter-temporal correlation matrix. A second multiplication is used to correlate the ISNDs between different years in the planning horizon. The following procedure is used to calculate the Cholesky matrix (Dagpunar, 1988:157):

$$\begin{aligned}
 c_{ii} &= \sqrt{V_{ii} - \sum_{m=1}^{i-1} c_{im}^2} \\
 c_{ij} &= (V_{ij} - \sum_{m=1}^{i-1} c_{im} c_{jm}) / c_{ii} \quad , j > i
 \end{aligned}
 \tag{3}$$

Through integration, the correlated standard normal deviates are transformed into correlated uniformly distributed values, which are then used in the inverse transform functions to simulate risk. The randomly generated prices and crop yields were integrated with enterprise budgets obtained from a local agricultural co-operative to generate distributions of gross margins for each crop over a 15 year period. The results of these simulations are given in Table 1.

TABLE 1: Present value of randomly generated gross margins over a 15 year planning horizon in constant 2003 price values.

<b>Crops</b>	<b>Mean</b>	<b>Standard Deviation</b>	<b>CV</b>	<b>Skewness</b>	<b>Kurtosis</b>	<b>Minimum</b>	<b>Maximum</b>	<b>Median</b>
<b>Large Pivot (R0.08/m<sup>3</sup>)</b>								
Lucerne	70523	10114	14	0.097	-0.051	46792	99317	70239
Maize	62615	10645	17	-0.325	-0.026	32307	84492	63181
Groundnuts	75638	15739	21	0.472	-0.001	46473	122101	74318
Wheat	43464	4312	10	0.325	0.047	34882	54372	43259
<b>Small Pivot (R0.12/m<sup>3</sup>)</b>								
Lucerne	61192	10114	17	0.097	-0.051	37461	89985	60908
Maize	58625	10645	18	-0.325	-0.026	28317	80502	59191
Groundnuts	71100	15739	22	0.472	-0.001	41935	117563	69780
Wheat	39287	4312	11	0.325	0.047	30705	50195	39082
<b>Flood</b>								
Pecans	156576	14232	9	0.181	-0.337	121571	192249	155208
Lucerne	66295	10253	15	0.104	-0.078	42916	95793	66186
Maize	42393	8820	21	-0.253	0.044	17815	61302	42428
Groundnuts	57168	14317	25	0.467	0.059	29958	98975	56009
Wheat	37128	4424	12	0.435	0.372	28408	49820	37158

From Table 1 it is clear that the relative risk of flood irrigation is higher than that of pivot irrigation. When irrigation cost is increased, the cumulative distribution shows a parallel shift to the left resulting in a lower mean, but with no change in the form of the distribution. The relative risk associated with the small pivot is therefore slightly higher. In general, pivot irrigation is more profitable than flood, with the large irrigation system being most profitable. More specifically, pecan trees are by far the most profitable crop over the long term; however, negative gross margins are realized for the first five years. Interesting is

the fact that flood-irrigated lucerne has a higher gross margin than lucerne irrigated with a small pivot. Thus, the relatively higher crop yields of pivot-irrigated lucerne compared to flood-irrigated lucerne did not compensate for the higher irrigation cost associated with the small pivot. The groundnut is the second most profitable crop, but also shows the highest relative risk, followed by maize, lucerne and wheat.

## **RESULTS**

Results are presented for the base case with 100% water allocation and a scenario where only 80% of the original water allocation of 9140 ha<sup>-1</sup> is available. The base case farm has the potential to grow lucerne, maize, groundnuts and wheat using 152 hectares of flood irrigation. The farm manager has a total of R300 000 start-up capital available at the beginning of the planning horizon. Two alternative scenarios to the base case are also presented. In the first scenario, pecan nuts are introduced as a higher value crop, while in the second scenario the possibility of converting to pivot irrigation is introduced. Figure 1 shows the tradeoffs between the different water allocation scenarios and the alternatives for coping with a water restriction of 20%.

From Figure 1 it is clear that the impact of a 20% reduction in the water allocation for the base case farm is severe, and the net worth is on average about R1 million less than the base case. However, this amount is an underestimation of the cost to the farm, since an additional R600 000 was needed as starting capital to secure a feasible model solution. The ability of pecan nuts to increase the overall net worth is also clearly shown in the graph. Compared to the full water allocation scenario, a higher net worth is realized with RAPs smaller than one when pecan nuts are introduced with a starting capital of R600 000 more than the base case. As risk aversion increases, the benefits of pecan nuts decrease; however, the curve remains above the flood tradeoff curve. When pivot irrigation is introduced, feasible answers are obtained when only R300 000 additional starting capital is needed (A in

graph). Due to the lower value, the tradeoff curve lies below that of pecan nuts. At RAPs greater than 1.5 the curve drops below that of the base case scenario with a 20% water restriction. If R600 000 starting capital is allowed, this alternative dominates all others (B in graph).

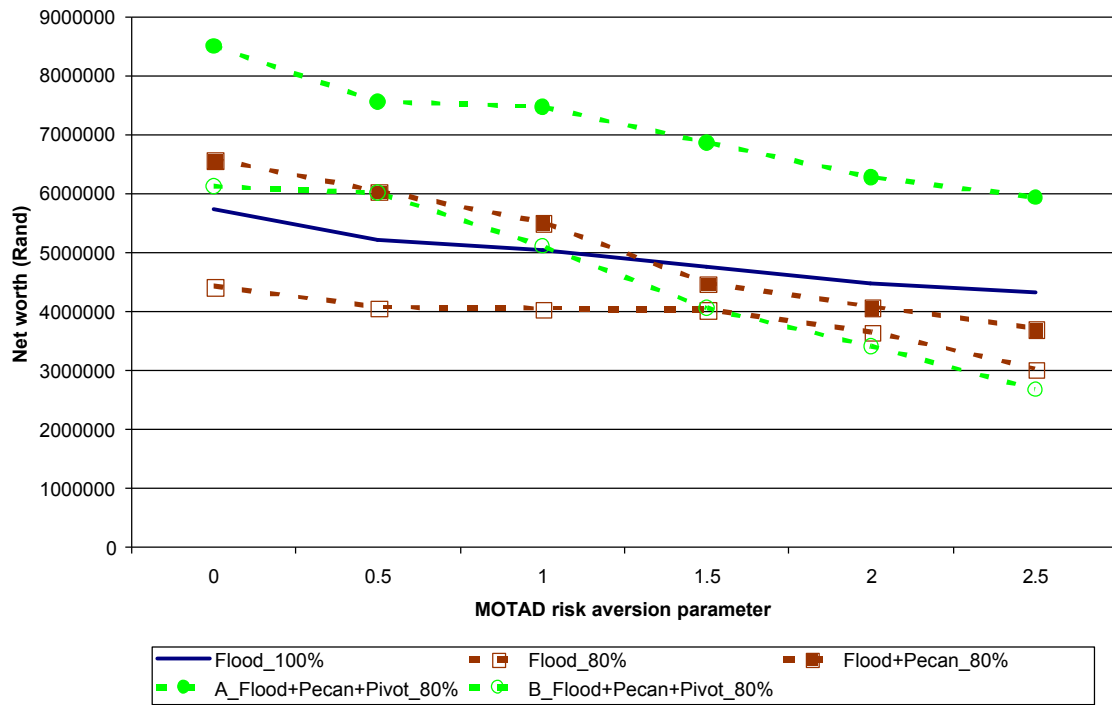


Figure 1: Present value of risk return tradeoffs of alternative options for combating water restrictions (2003)

The cumulative totals of the different crops grown and the irrigation technology used for different RAPs are given in Table 2. Generally, optimization models tend to give answers with a high level of specialization. From Table 2 it is clear that complete specialization does not occur when risk is ignored (RAP=0) due to the constraints in the model with regard to water supply capacities. However, some degree of specialization does take place, because in general too much groundnut crop and too little maize is produced. Thus, where risk is concerned, less groundnut crop and more maize crop are planted. Generally speaking, when risk aversion is increased, more wheat and maize are planted and fewer hectares of groundnut are planted. Lucerne seems to be decreasing at high levels of risk aversion; however, a strong

TABLE 2: Cumulative areas (ha) planted with different crops, as well as the irrigation technology used

	<b>Risk Aversion Level</b>					
	0	0.5	1	1.5	2	2.5
<b>Full Water Allocation Base Case</b>						
<b>Flood</b>						
Lucerne	104	119	118	116	111	107
Maize	0	458	441	483	504	536
Groundnuts	1158	689	673	641	622	611
Wheat	310	422	521	518	517	518
<b>80% of water allocation Base Case</b>						
<b>Flood</b>						
Lucerne	75	75	76	75	71	64
Maize	0	624	634	637	674	767
Groundnuts	1233	699	694	693	665	590
Wheat	110	238	244	255	263	260
<b>80% of water allocation Base Case + Pecan Nuts</b>						
<b>Flood</b>						
Pecan	35	33	27	16	11	6
Lucerne	26	28	38	46	49	57
Maize	0	586	573	728	684	708
Groundnuts	1344	914	869	671	663	632
Wheat	21	63	153	165	236	219
<b>80% of water allocation Base Case + Pecan Nuts + Pivot</b>						
<b>Flood</b>						
Pecan	16	16	7	4	0	0
Lucerne	0	0	0	6	16	44
Maize	0	54	99	103	153	296
Groundnuts	511	550	514	656	562	717
Wheat	221	258	318	296	300	238
<b>Pivot</b>						
Lucerne	52	44	48	47	37	8
Maize	0	181	399	497	494	349
Groundnuts	1111	851	634	363	394	212
Wheat	38	0	0	0	0	0

inverse relationship exists between lucerne and pecan nuts. In all cases, the hectares planted with pecan nuts decrease as the risk aversion increases, while the hectares planted with

lucerne increase. The main reason for the decrease in the number of hectares planted with pecan nuts is that higher risk aversion implies lower expected but less variable profit margins. As a result the cash surpluses generated are lower and therefore the establishment of pecan nuts is delayed.

Compared to the base case scenario under full water allocation, the crop hectares most severely affected by the decrease in water availability are lucerne and wheat.

## **CONCLUSIONS**

Model results indicate that the developed multiperiod risk optimization model is capable of modelling the complex dynamics of investment decisions satisfactorily when optimizing water use over the long term. Furthermore, the ability of the model to account for the risk tradeoffs between different crops, taking both intra- and inter-temporal correlations into account, was demonstrated. However, certain limitations must be raised regarding the modelling procedures.

The procedures used to correlate the stochastic entities hinge strongly on the ability to factor the correlation matrix. Since only a small number of cropping activities was included in the model, it was a fairly straightforward task to generate the risk matrix for the MOTAD specification. However, for larger regional models it may not be possible to factor the complete correlation matrix. The importance of starting capital and the ability of the adopted production processes to generate sufficient cash flow that can be reinvested in the farming business were emphasized by the risk return tradeoffs of pecan nuts. Furthermore, the variability of gross margins will necessarily impact significantly on cash flows. The problem is that the multiperiod optimization model optimizes these dynamics using expected values. Thus, on average, all these dynamics are accounted for. However, risk enters the model only as deviations from expected gross margins and not as deviations from expected cash flows at

the end of each year in the planning horizon. More research is necessary to account for these deviations endogenously.

The model results also have some implications for policy within the South African water sector. Currently government is advocating the cultivation of high-value crops and the more efficient use of available water resources through the adoption of more efficient irrigation technology and irrigation scheduling. From the analyses conducted in this research it is concluded that factors other than mere profitability of alternatives will play an important role in the adoption of more efficient practices. Due to the unique infrastructure of the evaluated irrigation scheme it is impossible to convert all the land to pivot irrigation. Other factors that are important are the financial situation and risk aversion of a specific farmer. Policy makers and government authorities should take cognizance of these factors when evaluating the water use efficiency and water management plans of different water user associations.

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