Spatial Analysis of Market Linkages in North Carolina Using Threshold Autoregression Models with Variable Transaction Costs

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Abstract

In North Carolina, where soybeans and corn are the two primary crops, the recent increase in the demand for U.S. corn has triggered a shift of farm acreage from soybeans to corn, leading to a rapid rise in prices of both commodities. However, the rate of the price changes, as well as the price level, is significantly different in markets that are located in different parts of the state. This study extends the literature that examines linkages between spatially separated markets by using a threshold autoregressive model with a less restrictive assumption for estimating the transaction cost neutral band – the band within which trade is not This generalization allows the neutral band of transactions costs to change according to various external factors, including fuel costs and seasonality. The estimation results indicate that for longer time series data, variable thresholds models statistically outperform the constant thresholds specification, and may provide a better representation of corn and soybean price data. Additionally, impulse response functions that use the asymmetric variable threshold model parameters indicate that the magnitude of the shock as well as the time-to-price-parity-equilibrium in the linked markets may be underestimated if a constant thresholds specification is implemented.

KEYWORDS: threshold autoregression, spatially separated markets, impulse response, neutral band

JEL classification codes: Q11, Q13

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In recent years there has been a significant increase in the price volatility of corn and soybean markets, due primarily to an increase in demand for ethanol based biofuels. With the increased demand for ethanol, the demand for corn has become much more inelastic, causing wider price movements in response to changes in quantity supplied. In general, these broad price movements have been observed across all markets; however, there exist significant differences in the price adjustment paths of individual markets. By examining the linkage structures between individual markets, it can be possible to estimate the price transmission behavior within the environment of highly volatile changes in price.

There are a number of studies that have examined market linkages through price transmission patterns (for example, see Goodwin and Piggott 2001; Bessler, Park, and Mjelde 2007). These works implement threshold autoregressive (TAR) models to estimate a neutral band within which prices follow a random walk process. The neutral band represents transaction costs that occur due to the spatial separation of any two markets. These transaction costs might often include expenditures on fuel, time and effort to coordinate the shipment and pick-up of transported commodities, synchronization of buyers and sellers, and knowledge of local highway laws for transporting grains. Additionally, transaction costs can vary seasonally – decreasing during harvest (when locally grown commodities become available), and increasing during months when the commodity must be imported

to meet local demand. Typically, prices of a commodity in any two linked markets differ by the amount of the inherent transaction costs that are required to ship the commodity from one market to the other. However, a shock to the price of one market may cause the price difference to be more than the transaction costs (the price difference falls outside of the transaction costs band), which would make it profitable to purchase the commodity in one market, incur the associated costs of transporting to the other, and sell the commodity for a higher price. This type of arbitrage behavior would continue until the prices in the linked markets re-adjust such that their difference is once again equal to the transaction costs that are associated with transporting the commodity from one market to the other.

The focus of this study is to examine the effects that price movements might have on the threshold values of the neutral band. Previous studies assume that the neutral band remains constant; however, due to the recent rapid rise of transportation costs as well as general seasonal effects, it is necessary to consider whether the assumption of a constant neutral band must be relaxed. By allowing the thresholds to vary according to relevant exogenous factors it may be possible to better understand the effects that external economic conditions might have on price discovery in linked markets. We develop an empirical framework that can appropriately model a variable neutral transaction costs band, and use the results to investigate differences with the model that restricts the neutral band to be constant.

Using the threshold cointegration methods of Balke and Fomby (1997), this study evaluates the linkages of North Carolina corn and soybean markets. Tests for the presence of threshold effects are performed following Tsay (1989). Upon

confirmation of threshold effects, a grid-search technique is used to determine the thresholds. The estimation uses a large collection of daily corn and soybean price data for markets in North Carolina. We find that variable threshold models statistically outperform the constant threshold specification. Additionally, the variable thresholds model provides a richer environment for examining impulse response functions. In general, our results are consistent with past studies, indicating that price behavior exhibits long-run market integration. However, we find that the constant threshold model typically underestimates the time-to-convergence as well as the magnitude of the effect that a shock can have on prices in linked markets.

The analysis is organized as follows: first, we present the methodology that is used for evaluating threshold autoregressive models, incorporate transaction costs that can be affected by exogenous factors; next, a description of the data and preliminary analyses are presented. Daily corn and soybean data are used to estimate threshold effects among spatially linked North Carolina markets, investigate the findings, and use impulse response functions to simulate price responsiveness. Concluding remarks are offered in the final section.

Econometric Specification of the Threshold Autoregressive Model

Following the specification developed by Balke and Fomby (1997), we build upon the threshold autoregression model, which defines a correspondence between error correction models that represent cointegrating relationships and autoregressive models of an error correction term. In this manner, it is possible to account for the transaction costs that might inhibit transmission of prices across spatially separated markets.

In general, threshold models can be viewed as a regime switching framework, in which a different regime is triggered when the variable of focus crosses the particular threshold. In the case of this study, the regime switch occurs if the parity relationship between commodity prices at linked locations becomes greater than or less than some value. A common parity relationship that is used to represent spatial integration among markets can be described as a simple autoregressive structure of price differences:¹

$$\tilde{P}_t = \delta \tilde{P}_{t-1} + \nu_t, \tag{1}$$

where $\tilde{P}_t = (P_t^1 - P_t^2)$, P_t^i is the price at location i at time t, and ν_t is a white-noise error term. To characterize the regime switching framework, we follow Balke and Fomby to define δ as:

$$\delta = \begin{cases} \delta^{(1)} & \text{if } |\tilde{P}_{t-1}| \le c \\ \delta^{(2)} & \text{if } |\tilde{P}_{t-1}| > c \end{cases}, \tag{2}$$

where c is the threshold value that causes a regime switch. Specifically, it is assumed that when $|\tilde{P}_{t-1}| \leq c$ holds, $\delta^{(1)} = 1$. This implies that the parity relationship follows a random walk when there are small deviations of price differences. However, a large deviation, such as a shock to the price in either market, will trigger the condition $|\tilde{P}_{t-1}| > c$, causing $\delta = \delta^{(2)}$. Under the

This specification can also be expressed as $\Delta P_t = (\delta - 1)P_{t-1}$.

assumption that a stable equilibrium between prices at the two spatially separated locations exists, $\delta^{(2)} < 1$, implying that the price differential process is stationary and shocks to P_t^1 or P_t^2 will die out over time.

Threshold autoregressive models represent price adjustments as a process that can be inhibited by transaction costs. As in Goodwin and Piggott (2001), we assume that there exists a band of transaction costs in which small deviations in a price pair difference, \tilde{P}_{t-1} , do not trigger a regime under which prices adjust back to an equilibrium. However, if the price difference exceeds the bounds of the transaction cost band, prices in the two linked markets will continue to adjust until \tilde{P} is no longer outside of the bounds of the neutral band. An example of this is as follows:

$$\tilde{P}_t = \theta[\delta^{(1)} \cdot \tilde{P}_{t-1}] + (1 - \theta)[\delta^{(2)} \cdot \tilde{P}_{t-1}] + \varepsilon_t,$$
(3)

where δ^1 and δ^2 are defined in equation (2). In this threshold autoregressive model there is a symmetric transaction costs band, such that $\theta = 0$ if $|\tilde{P}_{t-1}| > \tau$, and τ represents the transaction costs threshold. As discussed above, when the price difference for two markets does not exceed τ , then we set $\delta^{(1)} = 1$ (under the assumption that \tilde{P} behaves as a random walk). Otherwise, $|\delta^{(2)}| < 1$, and an adjustment back to an equilibrium occurs.

As noted by Goodwin and Piggott (2001), research that uses threshold autoregressive models to analyze price transmissions in spatially separated markets usually assumes a constant neutral band of transaction costs (for example, see Obstfeld and Taylor 1997; Goodwin and Grennes 1998; and Goodwin and Piggott 2001). Fackler and Goodwin (2001) discuss the implications for the validity of

empirical tests of spatial price analysis if this assumption is made. Additionally, Li and Barrett (1999) point out that the neutral band may not be constant or stationary in the long run. We attempt to relax the condition of a constant neutral band by allowing the threshold variable, τ , to vary according to exogenous factors. This is as follows:

$$\tau = \alpha_o + \alpha_1 F_t + \alpha_2 S_t^1 + \alpha_3 S_t^2, \tag{4}$$

where F_t reflects fuel prices (fuel price index), and S_t^1 and S_t^2 are seasonality components that follow a first order Fourier approximation to an unknown seasonal function. Specifically, $S_t^1 = \sin(2\pi d_t/260)$ and $S_t^2 = \cos(2\pi d_t/260)$, where d_t represents a weekday of the year $(d_t = 1, 2, ..., 260)$.

Testing for threshold effects is performed by implementing a general nonparametric test for the nonlinearity implied by thresholds in an autoregressive series, a technique developed by Tsay (1989). To construct the test, consider a simple autoregressive equation, as follows:

$$y_t = \alpha + \phi y_{t-1} + e_t. \tag{5}$$

Each combination of y_t and y_{t-1} is denoted as a 'case' of the data. These cases are ordered according to the variable that is relevant to the threshold behavior—in this case, y_{t-1} . Then, recursive residuals are generated by estimating the autoregressive model for an initial sample² and then for sequentially updated samples, which are obtained by adding a single observation. A test of nonlinearity

²We denote the first 1% of the data as the initial sample.

is given by the F-statistic from the regression of the recursive residuals on the explanatory variables (y_{t-1}) . The test is run with both increasing and decreasing ordering in the arranged autoregression.³

To summarize, the estimation methodology is as follows. The time series properties of the data are evaluated using augmented Dickey-Fuller unit-root tests. In addition, ordinary least squares estimates of cointegrating relationships (following Engle and Granger (1987)) are performed. Next, we test for the presence of threshold effects using a nonparametric test for the nonlinearity implied by thresholds in an autoregressive series. If the presence of thresholds is determined, we use a grid search approach to estimate the specific thresholds. Following the technique that was proposed by Balke and Fomby (1997), the grid search is used to find the threshold that minimizes a sum of squared errors criterion. We estimate two alternative specifications: the first assumes a constant transaction costs neutral band, while the second allows thresholds to vary according to equation (4). We perform the latter by estimating both symmetric and asymmetric thresholds. A symmetric threshold assumes that for any two locations, transaction costs for moving a commodity are the same in either direction. However, this assumption can be relaxed by estimating asymmetric thresholds, which allows for the transaction costs associated with transporting the commodity to the central location to be different from the transaction costs that arise with transporting the commodity from the central site. The parameters of the symmetric and asymmetric threshold functions are estimated using a four- and

³The alternative ordering of the data allows for additional power to discern thresholds for which data are concentrated in a particular regime at either end of the series. Only the more significant of the two ordered tests is reported.

eight-dimensional grid searches, respectively.⁴ Then, error correction models are estimated, conditional on the estimated threshold parameters. These are defined as follows:

$$\Delta \tilde{P}_t = \lambda \Delta \tilde{P}_{t-1} + \nu_t, \tag{6}$$

where $\tilde{P}_t = P_t^c - P_t^j$ is the difference of prices between a central market and market j.

It is helpful to test the statistical significance of the differences in parameters across alternative regimes. For instance, a conventional Chow test might be used to test the parameter differences across regimes. However, in this analysis, the parameters of the alternative regimes are not identified under the null hypothesis of no threshold effects, which causes the conventional test statistics to have non-standard distributions. In order to adjust for this complication, we employ the approach of Hansen (1982) for testing the statistical significance of threshold effects. Specifically, we run a number of simulations in which the dependent variable is replaced by draws from the standard normal distribution, and a grid search is used to identify the optimal thresholds. Then, a standard Chow-type test is used to test the significance of the threshold effects. The simulated sample of test statistics is used to approximate the asymptotic p-value by calculating the percentage of test statistics for which the test value that is taken from the estimation sample exceeds the observed test statistic.

⁴In all cases, the grid search is restricted to ensure that there are a sufficient number of observations for estimating the parameters of each regime. At least 1% of the total number of observations are required for each estimation.

Empirical Application to North Carolina Markets

Data

In this study we use daily cash prices for corn and soybeans that are reported by grain elevators and processors in North Carolina. Specifically, we choose corn markets in Cofield, Candor, Nashville, and Statesville, and soybean markets Candor, Greenville, Lumberton, and Fayetteville. To calculate price pairs, we select a central location based on the smallest average road distance among all pairs. Accordingly, we select Candor (corn) and Fayetteville (soybeans) as the central locations. Additionally, we use New York Harbor spot prices for number 2 low sulfur diesel as a proxy for transportation costs. The data set spans the range between 01 January, 2000 and 24 July, 2008. Some of the observations within the data are missing primarily due to holidays and days during which the elevators and processors did not report the cash prices. We exclude dates for which there are missing data in all locations, and use an exponential spline method to interpolate values for all other unreported data points.

Summary statistics for the data are presented in table 1, and the time series plots for commodity and diesel prices are shown in figures 1, 2, and 3.

Next, figures 4 and 5 illustrate time series plots of price pairs, \tilde{P}_t , for corn and soybeans. Finally, several basic time series tests are performed. For all market pairs, the results of the augmented Dickey-Fuller unit-root test, presented in table 2, support the assumption that price differences are stationary. Additionally, ordinary least squares estimates of the cointegrating relationship, shown in table 3, indicate that in all cases the intercept term is close to zero and the slope parameter

is close to one. This may suggest a strong interrelationship among prices in linked markets.⁵

Table 1: Summary Statistics: Price Pairs^a for Selected N.C. Locations

| Market Location | Obs. | Min. | Max. | Mean | Std. Dev. | $\mathbf{Distance}^b$ |
|-------------------------|------|----------|---------|---------|-----------|-----------------------|
| | | | | | | |
| Corn | | | | | | |
| Cofield-Candor | 2217 | -0.1182 | 0.19416 | 0.05662 | 0.03997 | 216 |
| Nashville-Candor | 2217 | -0.14492 | 0.17352 | 0.02105 | 0.02489 | 129 |
| Statesville–Candor | 2217 | -0.12204 | 0.24464 | 0.04035 | 0.05732 | 95.4 |
| | | | | | | |
| Soy beans | | | | | | |
| Greenville-Fayetteville | 2220 | -0.10279 | 0.1789 | 0.04577 | 0.02349 | 113 |
| Cofield-Fayetteville | 2220 | -0.1092 | 0.1242 | 0.02985 | 0.0244 | 173 |
| Lumberton-Fayetteville | 2220 | -0.10279 | 0.1789 | 0.04208 | 0.02294 | 32.9 |

 $[^]a$ All prices are logged and differenced.

Results for Empirical Application⁶

The first part of the empirical estimation is the identification of the appropriate transaction cost bands for each market combination. Since there are numerous costs relevant to spatial arbitrage and trade, it is virtually impossible to directly measure the transaction costs that affect the transfer of a particular commodity between two locations. Given these difficulties, we use the modeling techniques described above to estimate the transaction cost bands. First, we

^b Road distance between markets, in miles.

⁵However, these results should be considered with caution, because of the nonstationary nature of the price data.

⁶All estimations were performed using the SAS v9.1.3 analysis software.

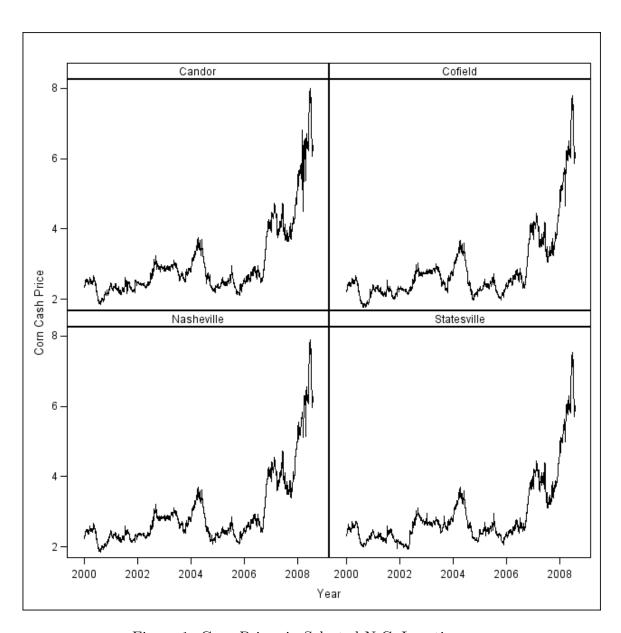


Figure 1: Corn Prices in Selected N.C. Locations

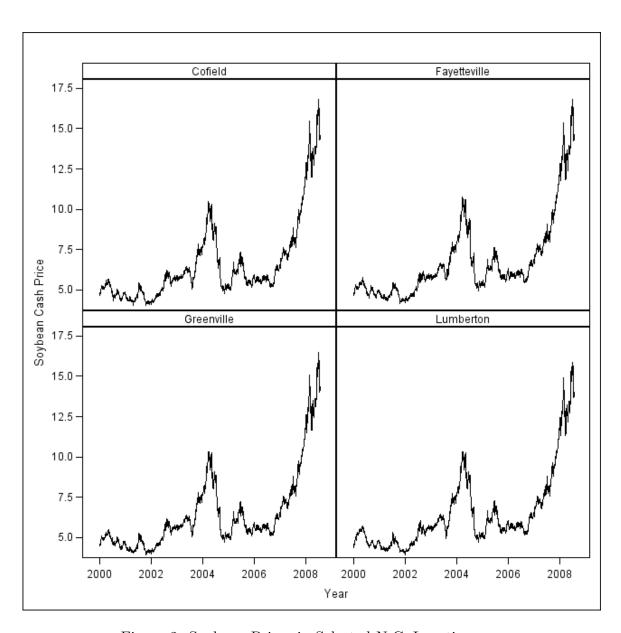


Figure 2: Soybean Prices in Selected N.C. Locations

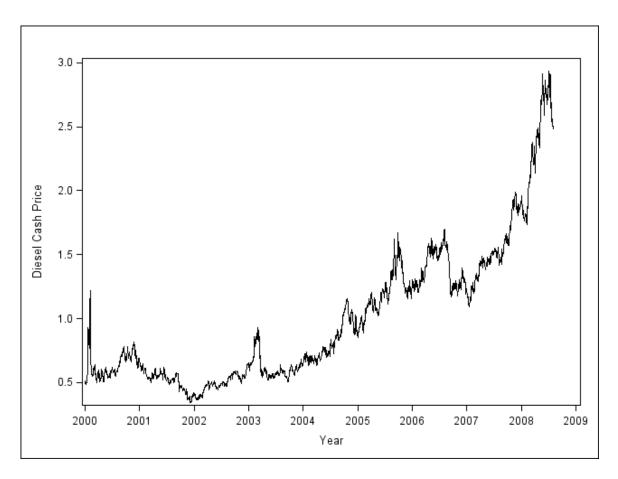


Figure 3: Diesel Fuel Price

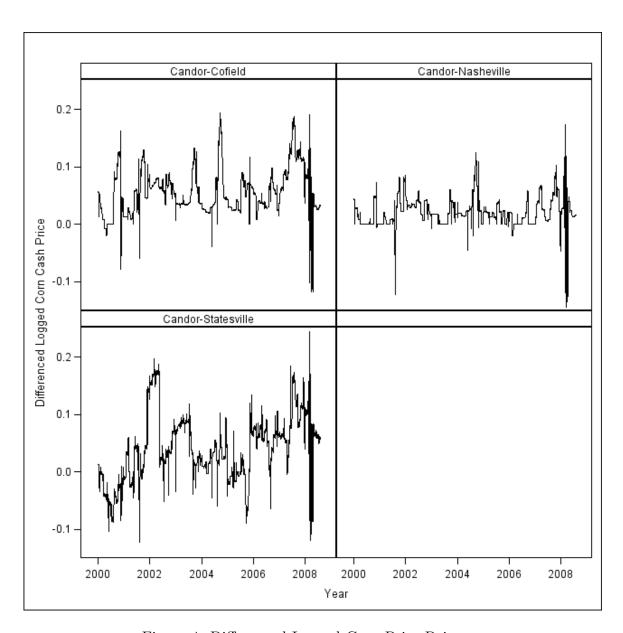


Figure 4: Differenced Logged Corn Price Pairs

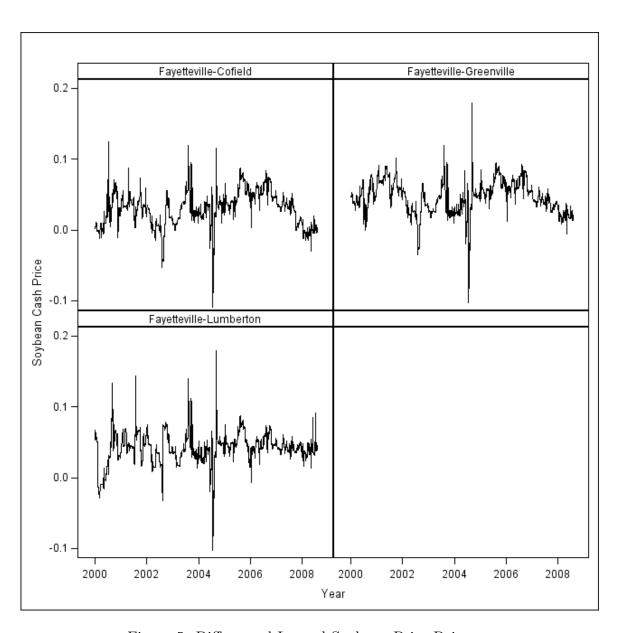


Figure 5: Differenced Logged Soybean Price Pairs

Table 2: Augmented Dickey-Fuller Test: Price Pairs for Selected N.C. Locations

| Market Location | Lags | au | p-value | F-value | p-value |
|-------------------------|-------------------|--------|---------|---------|---------|
| Corn | | | | | |
| | | | | | |
| Cofield-Candor | 0 | -8.45 | 0.0001 | 35.74 | 0.001 |
| | 1 | -6.88 | 0.0001 | 23.67 | 0.001 |
| | 2 | -5.7 | 0.0001 | 16.24 | 0.001 |
| Nashville-Candor | 0 | -13.07 | 0.0001 | 85.47 | 0.001 |
| | 1 | -11.39 | 0.0001 | 64.84 | 0.001 |
| | 2 | -9.23 | 0.0001 | 42.58 | 0.001 |
| Statesville-Candor | 0 | -7.61 | 0.0001 | 28.93 | 0.001 |
| 10 0000 000 0000 | 1 | -6.06 | 0.0001 | 18.36 | 0.001 |
| | 2 | -4.87 | 0.0001 | 11.84 | 0.001 |
| | | | | | |
| Soy beans | | | | | |
| Greenville–Fayetteville | 0 | -7.73 | 0.001 | 29.87 | 0.001 |
| v | 1 | -7.18 | 0.001 | 25.79 | 0.001 |
| | 2 | -7.02 | 0.001 | 24.64 | 0.001 |
| Cofield-Fayetteville | 0 | -7.49 | 0.001 | 28.02 | 0.001 |
| V | 1 | -6.74 | 0.001 | 22.73 | 0.001 |
| | 2 | -6.57 | 0.001 | 21.56 | 0.001 |
| Lumberton-Fayetteville | 0 | -8.58 | 0.001 | 36.79 | 0.001 |
| V | 1 | -8.2 | 0.001 | 33.66 | 0.001 |
| | $\stackrel{-}{2}$ | -8.51 | 0.001 | 36.22 | 0.001 |
| | | | | | |

Table 3: OLS Estimates of Cointegrating Relationships: $P_t^1 = C + \beta P_t^2$

| Market Location | C | β | Model adj - R^2 |
|---------------------------|-------------------------|----------------------|---------------------|
| | | | |
| Corn | | | |
| Cofield – Candor | -0.03578 $(0.0032)^a$ | 0.9805 (0.00289) | 0.9811 |
| Nashvile – Candor | -0.00933 (0.002) | 0.98904 (0.0018) | 0.9927 |
| Statesville – Candor | 0.04517 (0.00424) | 0.91994 (0.00383) | 0.9631 |
| | | | |
| Soy beans | | | |
| Greenville – Fayetteville | -0.10095 (0.00285) | 1.02969 (0.00151) | 0.9952 |
| Cofield – Fayetteville | -0.06986 (0.00309) | 1.02153 (0.00164) | 0.9943 |
| Lumberton – Fayetteville | -0.0521 (0.00301) | 1.00539 (0.0016) | 0.9944 |

 $[^]a$ Standard errors in parentheses. *** indicates significance at the 1% level, ** indicates significance at the 5% level, * indicates significance at the 10% level

estimate unrestricted and restricted forms of a first-order threshold autoregressive specification (as defined in equation (6)), for which we assume constant transaction cost bands. The unrestricted model estimates separate autoregressive parameters for the regime that corresponds to price differences that are less than the transaction cost band, and the regime corresponding to price differences exceeding the band. This model is used to test for the significance of threshold effects by using Hansen's testing procedure. The restricted specification follows Obstfeld and Taylor (1997), restricting the within-band parameter to be zero, which corresponds to a random walk for price differences that do not exceed the transactions cost band. Then, we estimate the alternative models that allow the thresholds to vary according to transportation costs and seasonal factors, as in equation (4). Both symmetric and asymmetric variable thresholds are estimated.

The estimates of the threshold band are presented in table 4 and table 5. For corn, the neutral band that represents the smallest price differences is at about 8.9–10.2% (Candor–Nashville), while for soybeans, the smallest neutral band is about 6.9–9.2% (Fayetteville–Greenville). The largest is at about 22.4–24.3% for corn (Candor–Statesville) and at about 13.1% for soybeans (Fayetteville–Lumberton).⁷ These relationships can be used to indicate linkage strengths in each market pair because the neutral band reflects the price differences that are required to trigger equilibrating conditions. For example, price differences of soybeans between Fayetteville and Greenville need to exceed only 9.2% in order

⁷It is somewhat surprising that the Candor–Statesville market pair indicates the largest neutral band even though these are the geographically closest corn markets in the analysis. This might be due to various reasons: we do not have information about the volume of trade, which, if low, can contribute to the large neutral band; a large body of water separates the two markets; there are various grain transportation laws, which could be enforced with greater strictness near Statesville.

to trigger conditions that will drive prices back to the market pair equilibrium, while for Fayetteville and Lumberton, the price differences would need to exceed 13.1%.

Also in tables 4 and 5, we present the parameter estimates of the variable threshold autoregressive models. For the symmetric and asymmetric cases, we used a grid search to determine parameters, which does not permit for a direct statistical significance inference of the parameter estimates. However, bootstrapping was performed to determine the standard errors of each value.⁸ Using the results of the grid search estimates, it is possible to understand the overall effects that each component of the threshold model has on the transaction cost neutral band.

In both the symmetric and asymmetric specifications, it is not surprising to find that, typically, diesel prices have a significant effect on the thresholds. Additionally, in almost all cases (the exception being the Candor–Statesville market pair), both the symmetric and asymmetric variable threshold models indicate that higher fuel prices imply a wider neutral band. This is intuitive in the sense that higher transportation costs would cause price pairs to increase. The coefficients for the seasonality components, in most cases, also have significant effects on the neutral band. However, the direction and magnitude of these effects varies across market pairs and across commodities.⁹

The comparison of the constant, symmetric, and asymmetric thresholds models

⁸Estimation of grid searches for large data sets requires a significant amount of computing power. Due to this, bootstrapping for the symmetric and asymmetric variable threshold models was restricted to 200 iterations.

⁹It should be noted that it is difficult to fully identify deterministic seasonal components and the effects of diesel prices because the fuel prices are likely influenced by seasonality as well.

Table 4: Threshold Band Parameter Estimates

| | \mathbf{U} - $\mathbf{T}\mathbf{A}\mathbf{R}^a$ (Constant) | R-TAR (Constant) | Symmetric Thresholds $(\tau_t = \alpha_o + \alpha_1 F_t + \alpha_2 S_t^1 + \alpha_3 S_t^2)$ | $+8 + \alpha_3 S_t^2)$ | | |
|-------------------------------------|--|-------------------------|---|--------------------------------|-------------------------------|---------------------|
| | | | α_o | $lpha_1$ | α_2 | α_3 |
| Corn Candor-Cofield | 0.1837*** | 0.1846*** | 3.76*** | -0.7449*** | -2.1234*** | -0.2833 |
| Candor-Nashville | 0.102** | 0.0892*** | 3.399*** 0.878) | (0.4116) -0.5427 (0.3556) | -2.1318*** -0.1318*** | 0.3324 |
| Candor-Statesville | (0.0205) $(0.2428***$ (0.0205) | 0.2241*** (0.0297) | $\begin{array}{c} (0.055) \\ 3.744^{***} \\ (0.9658) \end{array}$ | (0.9590) $-1.2014*$ (0.9107) | (0.0300) -1.3306 (1.1672) | 0.3022 (1.1123) |
| Soybeans Fayetteville-Greenville | 0.0919*** | 0.0919*** | 1.3308* (1.064) | 1.4281* (1.0137) | 1.0907* (0.75) | 1.7724* (1.2024) |
| Fayetteville-Cofield | 0.0965*** (0.0084) | 0.0964^{***} (0.0115) | $0.5257 \\ (2.2811)$ | 1.9232** (1.1528) | -0.0688 (1.1623) | 2.101 (1.8533) |
| Fayetteville-Lumberton | 0.1307*** (0.004) | 0.1307*** (0.0073) | 2.1242** (1.1656) | 0.5574 (1.3112) | 0.8543 (0.7335) | 1.6881** (1.0158) |

^a U-TAR represents the unrestricted threshold autoregressive model. R-TAR represents the threshold autoregressive model with the within-band parameter held at zero, which corresponds to a random walk.

 b Numbers in parentheses represent bootstrapped standard errors. *** indicates significance at the 1% level, ** indicates significance at the 5% level, * indicates significance at the 10% level

Table 5: Threshold Band Parameter Estimates (continued)

| Ą | Asymmetric Thresholds $(au_t^{(1)} = lpha_o + lpha_1 F_t + lpha_2 S_t^1 + lpha_3 S_t^2)^a$ | is $\frac{1}{t} + \alpha_2 S_t^1 + \alpha_3 S_t^2$ | $(a,b)^a$ | | $(au_t^{(2)}=lpha_4$ | $(au_t^{(2)} = lpha_4 + lpha_5 F_t + lpha_6 S_t^1 + lpha_7 S_t^2)$ | $S_t^1 + \alpha_7 S_t^2)$ | |
|-------------------------|---|--|------------|-----------|----------------------------------|---|---------------------------|------------|
| | α_o | α_1 | α_2 | α3 | α_4 | α_5 | α_6 | α 7 |
| Corn | ***&GU& U | **67660 | 0.9961** | 0.0705** | 77 87 87 87 87 87 | . 7510*** | ****** | **0077 |
| | $(0.1003)^b$ | (0.0987) | (0.1686) | (0.1228) | (0.1568) | (0.2064) | (0,0897) | (0.1952) |
| Candor-Nashville | 0.1209*** | 0.1185*** | 0.0303 | -0.0398 | 0.1239*** | 0.1015** | -0.1122** | 0.0587 |
| | (0.03896) | (0.03866) | (0.04687) | (0.07522) | (0.0548) | (0.0429) | (0.0579) | (0.0965) |
| Candor-Statesville | 0.4358** | 0.4137*** | 0.2567*** | 0.4451*** | 0.3514** | 0.4035** | 0.439** | 0.2751** |
| | (0.1581) | (0.1406) | (0.0897) | (0.1831) | (0.1665) | (0.193) | (0.1981) | (0.1333) |
| Soybeans | | | | | | | | |
| Fayetteville-Greenville | 0.3579*** | 0.3717*** | 0.2159 | 0.2129** | 0.3511*** | 0.3957** | 0.1036 | 0.1492 |
| | (0.1166) | (0.177) | (0.1927) | (0.1295) | (0.1228) | (0.2209) | (0.275) | (0.228) |
| Fayetteville-Cofield | 0.438** | 0.3977** | 0.3825*** | 0.4457** | 0.3022** | 0.5078*** | 0.2522*** | 0.4019*** |
| | (0.2616) | (0.1875) | (0.1502) | (0.2036) | (0.1366) | (0.1248) | (0.0599) | (0.1219) |
| Fayetteville-Lumberton | 0.37*** | 0.3517*** | 0.4227*** | -0.2153 | 0.2843*** | 0.3608*** | -0.2137 | 0.1711 |
| | (0.1339) | (0.1453) | (0.1744) | (0.216) | (0.0747) | (0.1845) | (0.179) | (0.2737) |

 a $\tau_t^{(1)}$ represents the upper threshold and $\tau_t^{(2)}$ the lower threshold.

^b Numbers in parentheses represent bootstrapped standard errors.

*** indicates significance at the 1% level, ** indicates significance at the 5% level, * indicates significance at the 10% level

indicates that there exist similarities among the constant and asymmetric variable threshold models. For the constant and asymmetric thresholds specifications, the estimates imply that the variation that exists in the variable thresholds is concentrated around the thresholds implied by a constant threshold model. However, the parameters of the symmetric variable threshold model are questionable, implying a transaction cost band that is unreasonably wide. This may be due to the inability of the symmetric variable threshold specification to appropriately account for different price parity behavior at the endpoints of the observed time series. An asymmetric variable thresholds model, however, is more flexible, which is supported in all cases by its better fit to the data (see table 6) as well as a more intuitive representation of the transactions costs band. In light of this, the asymmetric variable thresholds model is preferred over the symmetric variable thresholds model.

Figure 6 illustrates the threshold bands that are estimated by each model for corn and soybeans.¹² Although the asymmetric variable thresholds are concentrated around the constant thresholds, in almost all cases the variable thresholds indicate a widening of the band toward the end of the time series. This may indicate that in longer time series data, the variable thresholds models can better represent long-run behavior of the neutral transaction band. Additionally, the lower threshold may be less concentrated around the constant threshold than

¹⁰We compare the thresholds estimates that are produced by the restricted model. Estimates from the unrestricted constant thresholds model are insignificantly different.

¹¹In an attempt to decrease the influence of endpoints on the estimation of the symmetric variable thresholds specification, we restricted the data set to 99%, 95%, 90%, and 85% of all available data points. However, in all cases, the symmetric variable thresholds model exhibits the same abnormal behavior.

¹²In figure 6a we present the threshold bands implied by all three specifications in order to illustrate the poor fit of the symmetric variable thresholds model.

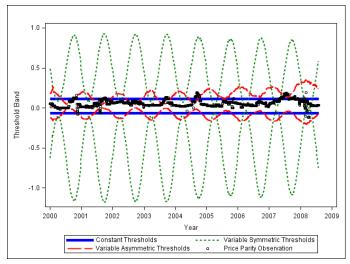
the associated upper threshold, and vice versa. This is most evident in the case of the Fayetteville-Cofield and Fayetteville-Lumberton soybean price pairs.¹³ This might imply that for longer time series, estimates of the autoregressive parameters are likely to be sensitive to the assumption of constant thresholds, which has been noted by Barrett (2001).

Table 6: Comparisons of Sum of Squared Errors for Alternative Models

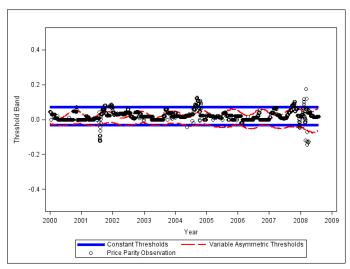
| | $\mathbf{A}\mathbf{R}$ | | | |
|-------------------------------------|------------------------|---------|---------|---------|
| Corn Candor-Cofield | 0.40621 | 0.40082 | 0.25741 | 0.24912 |
| Candor-Nashville | 0.34607 | 0.34051 | 0.24297 | 0.23885 |
| Candor-Statesville | 0.67721 | 0.61355 | 0.57279 | 0.5477 |
| Soybeans Fayetteville-Greenville | 0.12468 | 0.12223 | 0.12445 | 0.12392 |
| Fayetteville-Cofield | 0.12468 | 0.12456 | 0.12226 | 0.12052 |
| Fayetteville-Lumberton | 0.1429 | 0.14168 | 0.14097 | 0.13871 |

In general, the estimates of the asymmetric variable thresholds models indicate that the band is typically smaller (narrower) later in the calendar year around the time that the new crop harvest in North Carolina becomes available. In general, this conforms with intuition because new harvest induces less intra-state trading,

¹³This might be an indication of the different ways that transportation costs and seasonality factors affect transaction costs, depending on the direction in which the commodity is transported.

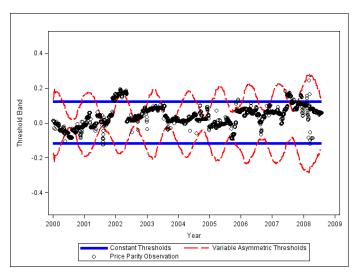


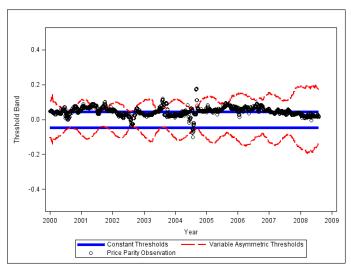
(a) Thresholds Model Estimates – Candor-Cofield (Corn)



 ${\rm (b)\ Thresholds\ Model\ Estimates-Candor-Nashville\ (Corn)}$

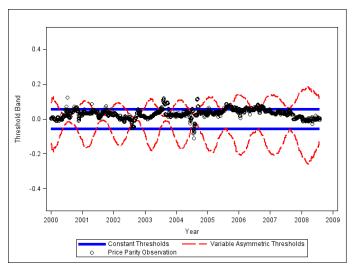
Figure 6: Comparison of Threshold Model Estimates



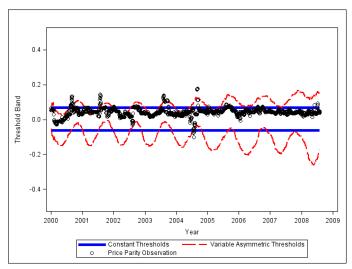


(d) Thresholds Model Estimates – Fayetteville-Greenville (Soybeans)

Figure 6: Continued



(e) Thresholds Model Estimates – Fayetteville-Cofield (Soybeans)



 $\begin{array}{ll} (f) \ Thresholds \ Model \ Estimates - Fayetteville-Lumberton \\ (Soybeans) \end{array}$

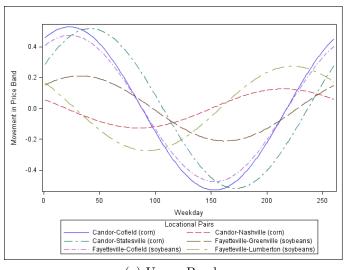
Figure 6: Continued

since locally produced commodities are in use. Conversely, the band increases (broadens) earlier in the calendar when intra-state trading is more prominent. These effects are confirmed in figure 7, which plots the seasonality component of price bands over 260 weekdays (a calendar year).

Table 7 presents the autoregressive parameter estimates and the associated half-lives¹⁴ for the alternative models. Half-lives are a measure of market integration in that their values indicate the degree to which the price pairs move toward equilibrium after a shock. In almost every case, the models that do not incorporate threshold effects imply longer half-lives, which suggests that ignoring thresholds will bias the adjustment parameters toward zero, and variable threshold specifications indicate even smaller half-lives in four of the six market pairs. In general, for corn, half-lives are smaller (twice as small in many cases) than the halflives for soybeans, implying a faster adjustment of price parities – and stronger market integration – in North Carolina corn markets. Thus, if threshold effects are not taken into account, price parity models can incorrectly imply a lower degree of market integration. Additionally, table 8 contains Tsay's test for threshold effects and Hansen's test for differences in parameters across different regimes. In every case, both test statistics are highly significant, which indicates a strong presence of threshold effects and difference of parameters across regimes.

Overall, this analysis confirms the presence of threshold effects in price linkages that exist in corn markets and soybean markets within North Carolina. In this data set, we find that the asymmetric variable threshold model best fits

¹⁴Half-lives represent the period of time that is required for one-half of a deviation from price parity to be eliminated. The half-life for an estimated adjustment coefficient, $\hat{\lambda}$, is $-\ln(2)/\ln(1+\hat{\lambda})$.



(a) Upper Band

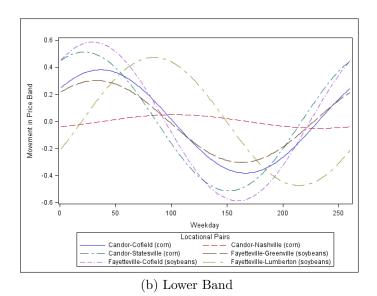


Figure 7: Seasonality Component of Asymmetric Variable Thresholds Model

the data, and the variability is closely clustered around the constant thresholds. However, in this rich time series data set, the asymmetric variable threshold model estimates capture the widening of the transaction cost band around and after year 2005. These changes correspond to contemporaneous rises in fuel prices, increased variability of corn and soybean prices, and enactment of the Energy Policy Act of 2005 and Energy Independence and Security Act of 2007¹⁵ The potential effects on corn and soybean prices that may be triggered by external shocks are a motivation for analyzing impulse responses.

¹⁵The Energy Policy Act of 2005 increased the standards for the use of ethanol-based fuels in the United States. This resulted in a rise in the demand for ethanol-based fuel production, causing a rise in the price of corn, and an associated increase of soybean prices. Similarly, the Energy Independence and Security Act of 2007 appropriated taxpayer funding for promoting the production of biofuels in the following 15 years.

Table 7: Threshold Autoregressive Model Estimates

| ${f AR}$ Half Life a U-T ${f AR}$ U-T | ife a U-TAR $^{-1}$ | · | U-1 | $\textbf{U-TAR}\\ \lambda_{out}$ | Half Life | $\mathbf{TAR} \\ \lambda_{out} \\ (\mathrm{Const.})$ | Half Life | $\frac{\mathbf{TAR}}{\lambda_{out}}$ (Symm.) | Half Life | $\frac{\mathbf{TAR}}{\lambda_{out}}$ (Asym.) | Half Life |
|--|------------------------|----------------------------------|-----|----------------------------------|-----------|--|-----------|--|-----------|--|-----------|
| 2.6152 | | -0.3513 | | 0.4451 | 1.1769 | 0.3592 | 1.5575 | -0.3108 | 1.8622 | 0.2573 | 2.3302 |
| | | (0.02030) -0.3719 | | (0.05527) 0.2743 (0.04729) | 2.1619 | (0.0011) | 2.1703 | (0.01247) -0.2899 | 2.0247 | (0.1193) -0.3114 | 1.8578 |
| |) (| (0.02019) -0.37823 (0.02128) | | 0.40586 0.04789 | 1.3313 | 0.4843 (0.0633) | 1.0467 | (0.01503) -0.2209 (0.0144) | 2.7769 | (0.0221) -0.1736 (0.0488) | 3.6352 |
| 6.0655 | 10 | -0.18782 | | -0.0599 | 11.2216 | -0.108 | 6.0649 | -0.1053 | 6.2296 | -0.1267 | 5.1164 |
| 4.8105 | ,0 | (0.0502) -0.178 (0.05436) | | (0.02911) -0.0699 (0.0909) | 9.5655 | (0.02937) -0.0699 (0.0409) | 9.5655 | (0.0370) -0.145 | 4.4247 | (0.0405) -0.1555 | 4.1012 |
| $\begin{array}{cccc} 0.02094) & (0.02450) \\ -0.08591 & 7.7165 & -0.10676 \\ 0.02108) & (0.02575) \end{array}$ | | (0.02450) -0.10676 (0.02575) | | (0.0396) -0.1164 (0.0371) | 5.6012 | (0.0403) -0.1165 (0.0373) | 5.5960 | (0.01779) -0.1008 (0.03475) | 6.5238 | (0.0319) -0.1787 (0.0347) | 3.5209 |

 a Half-lives indicate the weekdays required for one-half of a deviation from equilibrium to be eliminated. b Numbers in parentheses are standard errors.

Table 8: Tests for Thresholds Effects and Nonlinearity

Tsay's Test Hansen's Test

| Corn | | |
|-------------------------|--------------|----------|
| Candor-Cofield | 7.51E4 | 297.4437 |
| | $[0.0001]^a$ | [0.0001] |
| Candor-Nashville | 3.45 E5 | 349.2374 |
| | [0.0001] | [0.0001] |
| Candor-Statesville | 1.59E5 | 218.8642 |
| | [0.0001] | [0.0001] |
| Soybeans | | |
| Fayetteville-Greenville | 8.68E4 | 43.9998 |
| | [0.0001] | [0.0001] |
| Fayetteville-Cofield | 4.34E5 | 57.3766 |
| | [0.0001] | [0.0001] |
| Fayetteville-Lumberton | 2.95 E5 | 46.8915 |
| | [0.0001] | [0.0001] |

 $^{^{}a}$ Numbers in brackets are probability values associated with test statistics.

Impulse Response Analysis

A useful approach to analyzing the dynamic relationship among market price pairs is through impulse response functions, which can be used to examine the responses of prices and price pairs to shocks. For instance, it might be of interest to observe the effects on prices if there is a decision to build an ethanol-fuel production facility near one of the corn markets, such as the 110 million gallon corn-ethanol plant proposed to be built by East Coast Ethanol, LLC in 2008. The plant is proposed to be constructed in Northampton county, North Carolina – 36 miles west of Cofield, NC and 53 miles north-east of Nashville, NC. Due to the proximity of the ethanol plant site to the two corn processor sites, it is expected that a rise in the demand for corn¹⁶ will trigger an associated rise in prices, which may impact, through market linkages, prices in other North Carolina corn markets.

Similarly, shocks to the poultry industry in eastern North Carolina may lead to associated demand and price responses in North Carolina soybean markets. The economic recession of 2008-09 has placed significant financial pressures on major poultry processors such as Pilgrim's Pride, causing the company to file for bankruptcy in December, 2008 and cutting 50 growers in North Carolina. Because soybeans are an important input to poultry production, incidents such as the Pilgrim's Pride bankruptcy may lead to an associated decrease in the demand for soybeans in eastern North Carolina. However, because North Carolina soybean markets are linked (as shown above), the price shocks in a particular geographical region may be transmitted to soybean markets in other parts of the state. By

 $^{^{16}}$ On average, 2.8 bushels of corn are required to produce 1 gallon of corn-ethanol. This implies that for a 110 million gallon corn-ethanol plant, 308 million bushels of corn are required to maintain full production capacity.

implementing impulse response functions, it is possible to learn about such price transmission behavior within interrelated markets.

We examine post-shock response of both the price parity relationship and individual prices in linked markets. A nonlinear impulse response function is used, which defines a response, Ω_{t+r} , as a function of all previously observed data (i_t, i_{t-1}, \ldots) and a shock (ψ) . Thus, for both markets we select the last observation in our data set (31 July, 2008) to determine the responses to negative and positive shocks.¹⁷ This approach is consistent with previous studies that examine price linkage behavior in agricultural markets, such as Goodwin and Piggott (2001) and Balagtas and Holt (2009). Specifically,

$$\Omega_{t+r} = E[I_{t+r}|I_t = i_t + \psi, \ I_{t-1} = i_{t-1}, \ \ldots] - E[I_{t+r}|I_t = i_t, \ I_{t-1} = i_{t-1}, \ \ldots].$$
 (7)

It is necessary to note the nonstationarity of price data as well as the error correction properties. Due to these factors, shocks may elicit responses that are temporary, such that there is a return to the initial time path of the variables, or permanent, causing a persistent shift in the time path. For all analyses, we used a one-half standard deviation as the shock amount.

Figure 8 illustrates the responses to positive and negative shocks of the price parity relationships between a central market and an auxiliary market j. For corn,

¹⁷As discussed in Gallant, Rossi, and Tauchen (1993), Potter (1995), and Koop, Pesaran, and Potter (1996), an alternative approach is to observe the effects of a particular shock on all possible histories. The difficulty, however, is appropriately summarizing the information attained by applying a shock to the various historical data. A frequent method is to average the outcomes; however, this may result in a loss of important information. For example, averaging can difference out discrepancies that might exist in the various impulse responses or weaken the effects of asymmetric shocks.

the shock responses end between 10 and 20 weekdays (approximately one to two weeks), while all of the shock responses in the soybean markets last under one week. Additionally, five of the six market pairs exhibit a movement back toward the original price parity relationship; only in the Candor–Cofield market pair, the resulting price pair relationship is greater than the initial shock amount.

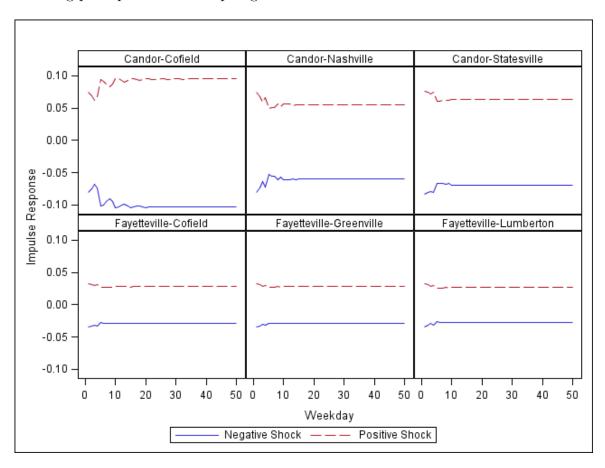


Figure 8: Long-run Impulse Response Functions: Asymmetric Variable Thresholds Model

In addition to examining the impulse responses of long-run price pair relationships, we use a generalized threshold autoregressive model, which allows for short-run components of price interrelationships. Using the generalized threshold autoregressive specification, we can attempt to better capture the dynamic aspects of linked market price pairs after a shock to the price of a particular market. Unlike the long-run impulse functions, which can be used to examine the post-shock path of the price pair, short-run impulse response functions can be used to observe the individual price paths in each market. Specifically, we consider the following model,

$$\tilde{\mathbf{P}}_{\mathbf{t}} = \theta \left[\alpha^{(1)} + \mathbf{\Theta}^{(1)} \Delta \mathbf{P}_{\mathbf{t}-\mathbf{1}} + \lambda^{(1)} z_{t-1} \right]
+ (1 - \theta) \left[\alpha^{(2)} + \mathbf{\Theta}^{(2)} \Delta \mathbf{P}_{\mathbf{t}-\mathbf{1}} + \lambda^{(2)} z_{t-1} \right] + \varepsilon_{\mathbf{t}},$$
(8)

where $\tilde{\mathbf{P}}_{\mathbf{t}}$ is an $(n \times 1)$ vector of price differences, such that the first element (\tilde{P}_{1t}) represents price differences in the central market and the second element (\tilde{P}_{2t}) represents the price differences in the j^{th} market. Additionally, α is an $(n \times 1)$ vector of constants, $\boldsymbol{\Theta}$ is an $(n \times n)$ matrix of coefficients on the differences of lagged prices, and λ is an $(n \times 1)$ vector of coefficients on the error vector correction term.¹⁸ Finally, ε is an $(n \times 1)$ vector of error terms.

Similar to equation (3), there is a transaction costs band, such that

$$\theta = \begin{cases} 1 & \text{if } z_{t-1} \geq \tau_t \\ 0 & \text{otherwise} \end{cases} , \tag{9}$$

We allow for asymmetries in the transaction costs bands according to the threshold variable, τ_t , where τ is defined in equation (4). We estimate short-run impulse response functions for constant and asymmetric variable threshold models. In order to incorporate variable thresholds, we perform in-sample impulse response

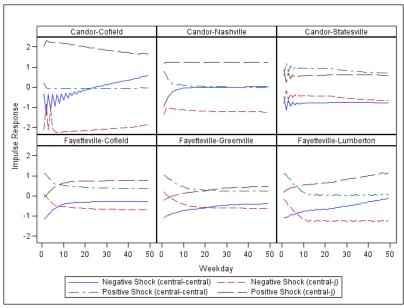
¹⁸We assume that price differences between markets that do not exceed the transaction cost band follow a random walk. This corresponds to following: $z_{t-1} = P_{1,t-1} - P_{2,t-1}$.

functions, which use parameters estimated in section.

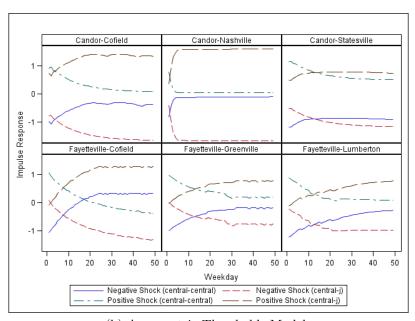
Figure 9 illustrates responses to positive and negative shocks to prices in the central market for the constant and variable thresholds models. Similarly, figure 10 shows responses to price shocks in the j^{th} market. In almost all cases, the initial price responses are as expected, in that the larger response is associated with the market in which the shock occurred. Additionally, the nonstationarity of the price series is reflected in the permanent shifts of the price paths after a shock. Generally, positive shocks lead to prices equilibrating at a higher level in both of the linked markets, and conversely, negative shocks lead to price equilibration at lower levels. This might imply that, although equilibration of the price pair relationship occurs, the market in which the price shock transpires influences the direction and level at which the price pair equilibrates. However, in some instances, we observe negative shocks leading to price equilibration at a level that is higher than the pre-shock price level. ¹⁹ This type of outcome is only evident in the richer asymmetric variable thresholds model; specifically, in the Fayetteville—Cofield market pair.

Also, although the direction and general time path of the impulse response functions are similar for markets across models, the magnitude of the impulse response as well as the time path of the price variable are noticeably different in the different specifications. First, it is almost always the case that when using the asymmetric variable threshold model parameters, shocks in the central and auxiliary markets lead to larger price movements, which often results in greater post-shock price differences. Additionally, unlike the time path of the

¹⁹Similarly, positive shocks can lead to price equilibration at a level that is lower than the price level before the shock.

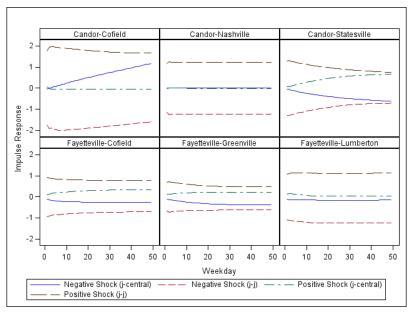


(a) Constant Thresholds Model

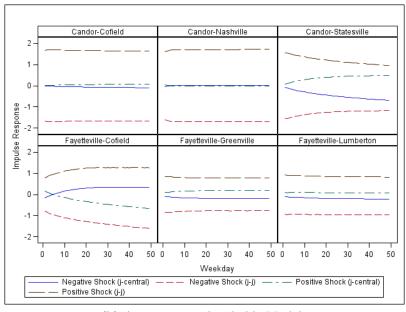


(b) Asymmetric Thresholds Model

Figure 9: Short-run Impulse Response Functions: Shock to the Central Market Price



(a) Constant Thresholds Model



(b) Asymmetric Thresholds Model

Figure 10: Short-run Impulse Response Functions: Shock to the j^{th} Market Price

price variable that is characterized by the constant thresholds model, the price movements in the asymmetric variable thresholds model often exhibit jumps across regimes. These differences can be attributed to allowing the neutral transaction costs band to vary according to external factors.

Despite the important differences that surface when the variable thresholds model is used, the price behavior that is exhibited in both specifications strongly supports long-run market integration. This result is consistent with the findings of Goodwin and Piggott (2001). Regardless of whether isolated price shocks occur in the central or auxiliary market, the impulse responses reflect behavior that is consistent with converging prices. In general, when a price in the central market is shocked, there is a longer time-to-convergence than that of a shock to the j^{th} auxiliary market. However, the time-to-convergence is typically longer in the asymmetric variable thresholds model, which might be due (as above) to the effects of external factors on the neutral transaction costs band. While responses to market shocks begin to expire in 10–15 weekdays in the constant thresholds specification, the responses in the variable thresholds model typically last 20–30 weekdays.

Overall, the comparison of impulse response functions using the alternate specifications indicates that the constant thresholds model may underestimate the time-to-convergence as well as the magnitude of the effect that a shock can have on prices in linked markets. Relaxing the assumption of a constant transaction costs band by allowing thresholds to vary according to external factors can lead to an improved representation of price parity relationships in interrelated markets. This can be crucial in examining the potential effects of policy as well as other

events that can trigger shocks to North Carolina corn and soybean markets.

Conclusion

This analysis examines spatial price linkages in North Carolina corn markets and soybean markets by using asymmetric, threshold autoregressive and error correction models. The primary motivation was to remove several restrictive assumptions that have been used in previous literature. Specifically, we allow thresholds to vary according to external factors, such as a fuel price index and seasonality effects, implying an analysis of linked price dynamics with a variable neutral transactions costs band. This extends the analyses within the existing literature, which restricts the band to be constant.

In general, our results confirm the findings of Goodwin and Piggott (2001). The variable thresholds models indicate that prices in North Carolina corn and soybean markets are highly interrelated, but the statistically significant presence of threshold effects may influence the price linkages in the spatially separated markets. However, relative to constant thresholds models, specifications that allowed for variable thresholds had a better fit to the data and implied faster adjustments to deviations from spatial equilibrium. Specifically, asymmetric variable thresholds model typically outperformed the alternative constant and symmetric variable thresholds specifications.

Additionally, we use nonlinear impulse response functions to evaluate the behavior of dynamic adjustments to localized price shocks. In both the constant and variable thresholds models, the responses strongly suggest high market integration and quick equilibration of price paths. However, in many cases the magnitude of the post-shock price change as well as the time-to-convergence are larger when the asymmetric variable thresholds model is used. This might imply that using a model that assumes a constant transaction costs band may lead to underestimating the overall post-shock price effects in North Carolina corn and soybean markets.

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