

Technology Adoption by Heterogeneous Producers to Regulate a Stock Externality*

Àngels Xabadia
Department of Economic Analysis
University of Zaragoza
Gran Via 2-4, 50005 Zaragoza, Spain
axabadia@unizar.es

Renan U. Goetz §
Department of Economics, University of Girona

David Zilberman
Department of Agricultural and Resource Economics
University of California, Berkeley

May 9, 2005

Abstract

This paper presents a framework to determine optimal resource allocation over time for the production of a good by heterogeneous producers who generate a stock externality. We analyze the optimal intertemporal and quality-specific combination of abatement strategies at the source given by a change in the intensity of production and in the chosen technology, and/or removal of existing pollution stock. The results show how the specifications of the production and the emission functions affect technology adoption and the design of the optimal intertemporal combination of source and stock abatement strategies. Moreover, the paper shows that regulation at the intensive margin cannot be considered as a substitute for a regulation at the extensive margin. The paper employs the so-called two-stage solution approach for solving the resulting quality distributed-intertemporal optimal control problem.

JEL Classification: Q28 and H21

Key words: technology adoption, nonpoint-source pollution, optimal control, producer heterogeneity

Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Providence, Rhode Island, July 24-27, 2005

Copyright 2005 by Xabadia, Goetz and Zilberman. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided that this copyright notice appears on all such copies.

*The authors gratefully acknowledge the financial support of the Ministry of Science and Technology through contract number AGL 2001-23333-C02-01 and the support of the Programa Gaspar de Portolà of the Catalan Government. Research leading to this paper was partially supported by Grant R 8296122002 AR STAR from the U.S. Environmental Protection Agency.

1. Introduction

Major environmental policy problems, including climate change and water-quality deterioration, are stock externality problems (Farzin, 1996; Baudry, 2000). The accumulating pollutants are frequently the result of application of inputs (water, chemicals, fossil fuels) by heterogeneous producers. Emission of small producers and large factories contribute to climate change, and runoff of both family and corporate farms contribute to the contamination of bodies of water. Thus, the design of policies to control stock externalities should consider both time and heterogeneity dimensions of these problems and the technologies that affect accumulation of pollutants.

The buildup of the stock externality can be modified either through changes in production practices (source abatement) or, when possible, by removal of existing pollution stock (stock abatement). Source abatement can be achieved by reducing input use levels (control at the intensive margin), by retiring production units (extensive margin), and through adoption of modern technologies (extensive margin) (Khanna and Zilberman, 1997). The latter category of source abatement consists of technologies that improve technical efficiency of variable inputs and either reduce or enhance the associated pollution (Fuglie and Kascak, 2001). Examples include insulation, fuel-efficient engines and stoves (Edwards et al., 2004), and improved quality fuels that reduce the pollution intensity of energy generation, transportation, temperature control, or cooking. Drip, sprinkler, and computerized irrigation and high precision chemical applications are examples of agricultural technologies that improve productivity and reduce damaging residues. Stock abatement reduces pollution once it has been generated, for example, by catalytic converters, barriers (e.g., plants, containing walls in the case of water pollution), or by sequestration of the pollutant (e.g., forest management in the case of CO₂ emissions).

There are three different strands of literature addressing stock pollution problems. The first investigates whether and when to adopt a modern technology in solving stock externality problems. Examples include the use of efficiency-enhancing technologies in the energy-generating sector (Siegel and Temchin, 1991; Chakravorty et al., 1997; Khanna and Zilberman, 1999), energy-saving appliances (Hausman, 1979; Jaffe and Stavins, 1995), irrigation technologies (Khanna et al. 2002), and variable input application rates and soil testing (Wu and Babcock, 1998). A second strand of papers focuses on economics of reduction of pollution stocks. For example, Hongli et al. (2002) identify conditions when the use of minimum tillage increases carbon sequestration rates, which, in turn moderates global warming. A third line of research analyzes the optimal

combination of source abatement versus stock abatement. Shah et al. (1995) present a dynamic framework to analyze the optimal combination of on-farm and off-farm pollution abatement strategies for waterlogging problems. Similarly, Farzin (1996) in a more general context develops a dynamic framework to analyze modification of static policy instruments in the presence of a stock externality.

The existing papers either establish the optimal intertemporal policy assuming homogeneity of the production units, or neglect the intertemporal aspect of the pollution problem if they consider heterogeneity. In this paper we integrate both aspects, developing a model that incorporates both heterogeneity and time.

The paper modifies the two-stage optimal control approach of Goetz and Zilberman (2000), which is similar to the dynamic optimization technique in Segerstrom (1999). The first stage consists of a static analysis of choices at the extensive and intensive margins by heterogeneous firms. The aggregate outcomes of this analysis are then utilized to determine resource allocation and pricing over time. With this approach, it is possible to derive optimal policies that determine the timing and use of stock abatement and affect the firms' decisions at the intensive margin (choices of variable inputs) and the extensive margin (adoption of modern technologies and retirement of units). Our results suggest that policies, which target exclusively the reduction of input use, are in general not optimal since they produce a distortion at the extensive margin. In contrast to Pan and Hodge (1994) or Glaeser and Shleifer (2001), we show that regulations at the extensive and intensive margins should not be considered as substitutes but, rather, as indispensable complements. We show that the distortions of pollution control policies that target only reduction of variable input use have to be corrected by the design of economic incentives that trigger the adoption of clean technologies and discourage the adoption of dirty technologies. The results also show how the specifications of the production and pollution-generating technologies affect the pattern of adoption of modern technologies and how they affect the design of dynamic environmental policies.

The results show that the temporal aspect of the regulation is of great importance, since it determines the optimal mix and degree of severity of the policy measures. A late intervention, when the stock of pollution is above its steady-state value, drastically reduces production intensity below its steady-state level and then increases this intensity over time. Moreover, the dynamic framework allows for the possibility of removing the pollutant once it has been generated (stock abatement). In this way it is possible to evaluate the incentives for source abatement versus stock abatement. If abatement cost is highly convex, most likely it is optimal to rely on

abatement at the source, according to the specific conditions of each producer, and to have little stock abatement. However, if the marginal stock abatement cost increases slowly, the optimal intertemporal policy is characterized by high stock abatement. As a general result, this paper offers formulations of individually tailored dynamic policies to induce socially optimal behavior by the individual agents taking into account the specification of available technologies.

The paper is organized as follows. Section 2 describes the basic features of the model. Section 3 establishes the optimal environmental policy, consisting of the optimal static and intertemporal solution. Section 4 defines individually tailored and intertemporal policies with respect to the level of input and the choice of technologies that can establish the social optimum. Section 5 concludes the paper.

2. The modeling framework

Consider a competitive industry made of heterogeneous production units (fields or plants), which produces a good using fixed assets (land or machines) and variable inputs (water, chemicals, fossil fuels) based on different technologies. The production units differ by quality of the asset ϵ , $\epsilon \in [0, 1]$, where a higher ϵ corresponds to higher quality. In the case of land, ϵ measures the site productivity and environmental vulnerability of the location. For instance, ϵ may stand for the capacity of land to retain inputs (water or chemicals). In this way, higher ϵ results in higher productivity and lower residues. Similarly, machines with higher ϵ may be of improved vintage, with higher input-use efficiency and lower leakage coefficients. The asset can be used with different technologies. For simplicity, we concentrate on the case where only two alternative technologies i , $i = 1, 2$, are available. The variable $i = 1$ stands for the modern technology and $i = 2$ for the traditional one. The modern technologies alter the functional relation between input and output, and input and emissions. First, modern technology may be embodied in equipment. For instance, better cooking stoves improve fuel efficiency, reduce in-house air pollution, and reduce contribution to climate change (Edwards et al., 2004). Likewise in agriculture, modern irrigation technologies increase efficiency of water-use and reduce leaching of pollutants. Alternatively, the modern technology may also be embodied in extra effort. A key element of Integrated Pest Management is the monitoring of pest populations to increase the precision of chemical applications (Committee on the Future Role of Pesticides in US Agriculture, 2000). Finally, modern technology may be embodied by a higher quality input. For instance, the use of cleaner fuels in the transportation sector reduces urban pollution and greenhouse gas emissions

(Nakata, 2000). Let $x_1(t, \epsilon)$ denote the share of the fixed asset with quality ϵ utilized with modern technology ($i = 1$), and $x_2(t, \epsilon)$ in the form of traditional technology ($i = 2$), at each moment of time t . Hence, we have $\sum_{i=1}^2 x_i(t, \epsilon) \leq 1$. Thus, according to this notation, the complete retirement of production units takes place when $\sum_{i=1}^2 x_i(t, \epsilon) = 0$. Production functions for every ϵ exhibit constant returns to scale. The total amount of assets available in the industry is given by X . The amount of asset of quality ϵ in relation to the total amount of assets is given by the density function $l(\epsilon)$, with $\int_{\epsilon_0}^{\epsilon_1} l(\epsilon) d\epsilon = 1$, and $l(\epsilon) > 0, \forall \epsilon \in [\epsilon_0, \epsilon_1]$. Thus, the amount of asset available with quality ϵ is given by $X l(\epsilon)$.

Let $u_i(t, \epsilon)$ be the variable input per unit of fixed asset (pesticides per acre, fuel per unit of machine capacity). Output per unit of fixed asset utilized in the form of technology i is $y_i = h_i(\epsilon) f(u_i(t, \epsilon))$, $i = 1, 2$, with $f_{u_i} > 0$ and $f_{u_i u_i} < 0$, where the subscript of a function with respect to a variable denotes its partial derivative. We assume that asset quality and the way that it is utilized affect productivity through a multiplicative fixed asset effect represented by $h_i(\epsilon)$, where $h(\cdot)$ is C^2 , and $h_i(0) = 0$. For simplicity, we assume for the traditional technology that $h_2(\epsilon) = \epsilon, \forall \epsilon$. For each technology, assets of higher quality are more productive, $\frac{dh_i}{d\epsilon} > 0$, and adoption of modern technology tends to increase fixed asset productivity, $h_1(\epsilon) > h_2(\epsilon)$; for, $0 < \epsilon < 1$. We distinguish between two specifications of the technology impacts on the asset productivity. The first is the case of *technology and asset quality substitution*, TAS, where $\frac{d^2 h_1}{d\epsilon^2} \leq 0$ and $h_1(1) = h_2(1) = 1$. In this case the new technology is effective of augmenting the quality of lower quality assets, but the augmentation declines with ϵ . For instance, the advantage of modern agricultural irrigation techniques compared to traditional irrigation techniques diminishes with land quality (water-holding capacity) as land quality approaches 1, since the loss of salts and minerals is cut back. With supplementary devices for combustion engines, the increase in fuel efficiency is reduced as the vintage of the engine is of a more recent vintage. The second is the case of *technology and asset quality complementarity*, TAC, where $\frac{d^2 h_1}{d\epsilon^2} \geq 0$ and thus $h_1(1) \gg 1$. In this case the multiplicative effect of the new technology is exacerbating the difference in productivity among assets. For example, when firms differ in their human capital, those with highly qualified human capital are likely to gain more from the use of computer software than those with less qualified human capital. The gain from agricultural pest management techniques increases as the quality of the land improves.

Output price is denoted by p , and it is assumed to be constant. Input price is denoted by c_i , $i = 1, 2$. We assume that $c_1 \geq c_2$, that is, input price is higher if modern technology is embodied in the applied input, e.g., higher seed price in the case of cultivation of transgenic

crops. Each form of the utilization of the fixed asset differs in its operational costs per unit of fixed asset, denoted by I_i . Operational costs include the costs of inputs such as labor (e.g., costs of extra monitoring in the case of Integrated Pest Management), the rental or annualized costs of equipment (e.g., to employ the services of contractors or purchase equipment that can be resold), and the cost of technology licensing or other fees associated with improved input quality. Furthermore, we assume that $I_1 > I_2$, i.e., the modern technology is more costly.

The pollutant generated in the production process accumulates over time. Following Millock et al. (2002), we consider two formulations of the pollution generation function $g_i(u_i(t, \epsilon), \epsilon)$. The first is *pollution as an input externality*. In this case the pollution is assumed to be emanating from the use of an input, e.g., fertilizer residue, and the pollution function is convex in u_i with $g_{i u_i} > 0$, $g_{i u_i u_i} \geq 0$. Since higher quality assets have higher input use efficiency, they generate less residue, and pollution decreases with quality, i.e., $g_{i \epsilon} < 0$ and $g_{i u_i \epsilon} < 0$. We assume that the modern technology in the case of an input externality is a precision technology, reducing pollution given input use and asset quality. Specifically, we have that $g_1(u_1, \epsilon) < g_2(u_2, \epsilon)$, $\forall u_1 = u_2 > 0$ and $\epsilon < 1$; and $g_i(u_i, 1) = 0$. In the second formulation we assume *pollution as an output externality* where it is proportional to output (e.g., output contains a toxic material or causes environmental damage), that is, $g_i(u_i(t, \epsilon), \epsilon) = \alpha y_i$, where α is the pollutant generated per unit of output. Therefore, the pollution function is strictly concave in u_i with $g_{i u_i} > 0$, $g_{i u_i u_i} < 0$, and $g_{i u_i \epsilon} > 0$. In this situation, modern technology is more polluting than the traditional technology, that is, $g_1(u_1, \epsilon) = \alpha h_1(\epsilon) f(u_1, \epsilon) > \alpha h_2(\epsilon) f(u_2, \epsilon) = g_2(u_2, \epsilon)$, $\forall u_1 = u_2 > 0$ and $\epsilon > 0$; and $g_1(u_1, 0) = g_2(u_2, 0) = 0$.

The aggregate pollution stock at time t is $s(t)$, and the temporal economic loss of pollution stock per period is given by the monetary damage function $m(s(t))$, with $m(0) = 0$, $m_s > 0$, and $m_{ss} > 0$. The pollution stock may be reduced by various abatement activities. Let $\eta(t)$ denote the amount of stock abatement at time t , and $k(\eta(t), s(t))$ stock abatement cost. We assume that marginal cost of stock abatement is positive, that is, $k_\eta > 0$ and $k_s > 0$, and jointly convex in η and s . In particular, we consider the case of a cleanup technology where the marginal abatement cost with respect to η is independent of s , and where it is decreasing in the pollution stock, i.e., $k_{\eta s} \leq 0$. Finally, we allow also for the case where marginal abatement cost increase with s , i.e., $k_{\eta s} > 0$.

The dynamics of the pollutant stock can be stated as

$$\dot{s}(t) = \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 g_i(u_i(t, \epsilon), \epsilon) x_i(t, \epsilon) \right) X l(\epsilon) d\epsilon - \eta(t) - \zeta s(t), \quad (1)$$

where a dot over a variable denotes the operator $\frac{d}{dt}$. The parameter ζ , $0 < \zeta < 1$ represents the natural decay rate of the pollutant stock.

3. The dynamics of the pollution stock problem

A social planner is assumed to maximize the present discount value of net benefits from production minus the social cost of the pollutant stock.¹ Thus, the optimization problem is given by

$$\max_{u_i(t,\epsilon), x_i(t,\epsilon), \eta(t)} \int_0^\infty \exp^{-\delta t} \left[\int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 (p h_i(\epsilon) f(u_i(t, \epsilon)) - c_i u_i(t, \epsilon) - I_i) x_i(t, \epsilon) \right) X l(\epsilon) d\epsilon - \left(m(s(t)) + k(\eta(t), s(t)) \right) \right] dt, \quad (S)$$

subject to

$$\begin{aligned} \dot{s}(t) &= \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 g_i(u_i(t, \epsilon), \epsilon) x_i(t, \epsilon) \right) X l(\epsilon) d\epsilon - \eta(t) - \zeta s(t), \\ s(0) &= s_0, \quad u_i(t, \epsilon) X l(\epsilon) \geq 0, \quad i = 1, 2, \quad x_i(t, \epsilon) X l(\epsilon) \geq 0, \quad i = 1, 2, \\ \left(1 - \sum_{i=1}^2 x_i(t, \epsilon) \right) X l(\epsilon) &\geq 0, \quad \eta(t) \in [0, s(t)], \end{aligned}$$

where s_0 denotes the pollution stock at time 0 and $\delta > 0$ is the social discount rate. Utilizing Pontryagin's Maximum Principle, the current Hamiltonian of the optimal pollution restoration strategy (S) is given by

$$\begin{aligned} \mathcal{H} \equiv & \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 (p h_i(\epsilon) f(u_i(t, \epsilon)) - c_i u_i(t, \epsilon) - I_i) x_i(t, \epsilon) \right) X l(\epsilon) d\epsilon - \left(m(s(t)) + k(\eta(t), s(t)) \right) \\ & - \mu \left(\int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 g_i(u_i(t, \epsilon), \epsilon) x_i(t, \epsilon) \right) X l(\epsilon) d\epsilon - \eta(t) - \zeta s(t) \right). \end{aligned}$$

¹We assume that the output price is not influenced by the production of the externality. We also assume that the utility function of the consumers is quasilinear with respect to the traded goods and the externality. Thus, the optimal level of the externality is independent of the consumers' expenditures, and it is possible to derive a utility function which depends only on the externality s (Mas-Colell et al., 1995). To discuss the results of our model in a practical setting, we propose that the derived utility function be represented by the damage function $m(s(t))$ and the stock abatement cost function $k(\eta(t), s(t))$. The assumption made with respect to the quasilinearity of the utility function helps to keep the model simple, and it allows us to concentrate on our analysis to answer the question of whether or not it is socially optimal to abate at the source or to abate the pollution stock, and which is the optimal policy to achieve the socially optimal outcome.

To facilitate the interpretations of the costate variable μ , it has been multiplied by minus one. The arguments ϵ and t of the variables and the Lagrange multipliers to be introduced later will be suppressed to simplify the notation unless it is required for an unambiguous notation. Taking account of the constraints leads to the Lagrangian: $\mathcal{L} \equiv \mathcal{H} + (\omega_1 u_1 + \omega_2 u_2 + \omega_3 x_1 + \omega_4 x_2 + \omega_5(1 - x_1 - x_2))Xl + \omega_6 \eta + \omega_7(s - \eta)$, where $\omega_1, \dots, \omega_7$ denote Lagrange multipliers. The solution of problem (S) has to satisfy the following necessary conditions² stated in accordance with Theorem 1 in Seierstad and Sydsæter, (1987, p. 276)

$$\mathcal{L}_{u_i} \equiv (ph_i f_{u_i} - c_i - \mu g_{i u_i})x_i + \omega_i = 0, \quad (2)$$

$$\mathcal{L}_{x_i} \equiv py_i - c_i u_i - I_i - \mu g_i + \omega_{i+2} - \omega_5 = 0, \quad (3)$$

$$\mathcal{L}_\eta \equiv -k_\eta + \mu + \omega_6 - \omega_7 = 0, \quad (4)$$

$$\dot{\mu}(t) = \delta\mu + \mathcal{H}_s = \mu(\delta + \zeta) - m_s - k_s + \omega_7, \quad (5)$$

$$\begin{aligned} \dot{s}(t) &= \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 g_i x_i \right) X l d\epsilon - \eta - \zeta s, \\ s(0) &= s_0. \end{aligned} \quad (6)$$

Since the analytical solution of the necessary conditions (2) - (6) is difficult to obtain, we propose to solve problem (S) by a two-stage solution technique described in the following proposition.

Proposition 1: *The solution of the optimization problem S is equivalent to the solution of two sequential problems, denoted by S1 and S2. In the first stage (problem S1), the social net benefits are maximized over ϵ given a prespecified level of aggregate emissions z . The solution consists of the optimal trajectories of $u_i(\epsilon)$ and $x_i(\epsilon)$, $i = 1, 2$. In the second stage (problem S2), the prespecified level of aggregate emissions z becomes a decision variable; and the optimal value function of the first stage is optimized over time yielding the optimal path of $s(t)$, $z(t)$, $\eta(t)$, and consequently of $u_i(t, \epsilon)$, and $x_i(t, \epsilon)$, $i = 1, 2$.*

Due to the fact that the state variable of problem (S) depends exclusively on time and not on ϵ , one is able to decompose part of the problem into a static optimization problem over quality, and another part into a dynamic control problem. In the first-stage problem, the use of resources over the heterogeneous characteristic of the production units is optimized, i.e., for every quality ϵ , we determine the optimal amount of variable input and the way the fixed asset should be utilized, including the option not to utilize it at all. The value function associated

²We assume that the solution of the necessary conditions is the global optimum, in particular, for the case of an output externality where the emission function is concave.

with the first stage problem is then plugged into the dynamic control problem to determine the optimal combination of the different abatement options.

3.1. The solution to the optimization problem over quality

In the first stage the solution of the social planner's decision problem (S1) is given by the value function $V(z)$ defined as:³

$$V(z) \equiv \max_{u_i(\epsilon), x_i(\epsilon)} \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 (p h_i(\epsilon) f(u_i(\epsilon)) - c_i u_i(\epsilon) - I_i) x_i(\epsilon) \right) X l(\epsilon) d\epsilon, \quad (S1)$$

subject to

$$z = \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 g_i(u_i(\epsilon), \epsilon) x_i(\epsilon) \right) X l(\epsilon) d\epsilon,$$

$$u_i(\epsilon) X l(\epsilon) \geq 0, \quad i = 1, 2, \quad x_i(\epsilon) X l(\epsilon) \geq 0, \quad i = 1, 2, \quad \left(1 - \sum_{i=1}^2 x_i(\epsilon) \right) X l(\epsilon) \geq 0,$$

where z denotes the aggregate emissions over the entire range of ϵ , i.e., from ϵ_0 to ϵ_1 . As in section 3, the argument ϵ of the variables and the Lagrange multipliers v_i , $i = 1, \dots, 5$, to be introduced later, will be suppressed to simplify the notation unless required for an unambiguous notation.

Taking account of the constraints on the control variables leads to the Lagrangian

$$\begin{aligned} \mathcal{L}_1 \equiv & \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 (p h_i f(u_i) - c_i u_i - I_i) x_i \right) X l d\epsilon + \lambda \left(z - \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 g_i(u_i, \epsilon) x_i \right) X l d\epsilon \right) \\ & + \left(v_1 u_1 + v_2 u_2 + v_3 x_1 + v_4 x_2 + v_5 (1 - x_1 - x_2) \right) X l. \end{aligned}$$

A solution of the problem has to satisfy the following necessary conditions:

$$\mathcal{L}_{1u_i} \equiv (p h_i f_{u_i} - c_i - \lambda g_{i u_i}) x_i + v_i = 0, \quad (7)$$

$$\mathcal{L}_{1x_i} \equiv p y_i - c_i u_i - I_i - \lambda g_i + v_{i+2} - v_5 = 0, \quad (8)$$

$$\mathcal{L}_{1\lambda} \equiv z - \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^2 g_i x_i \right) X l d\epsilon = 0. \quad (9)$$

The Lagrange multiplier λ is interpreted as the shadow cost of the prespecified level of aggregate emissions z over the entire range of quality ϵ . Thus, z does not depend on ϵ , and consequently λ is constant over quality. For an interior solution, given quality ϵ and given a

³We assume that V is a concave C^2 function.

particular technology, necessary condition (7) indicates that the value of the marginal product of applied input per unit of asset should equal the sum of the marginal cost of input use and the marginal cost of pollution per unit of asset. In the case of a boundary solution, the Lagrange multiplier of the binding constraint reflects the difference between the value of the marginal product and the sum of the marginal costs. The necessary condition (8) indicates that the marginal net benefits of production per unit of asset with quality ϵ , given a particular technology, should equal the marginal cost of pollution per unit of asset. However, since both the production and emission functions are linear in the fixed asset, the technology which has the maximal social net returns, Π_i^* ,⁴ defined as $\Pi_i^* \equiv \mathcal{L}1_{x_i} - v_{i+2} + v_5 \equiv py_i(u_i^*) - c_i u_i^* - I_i - \lambda g_i(u_i^*, \epsilon)$, will be completely preferred to the technology with the lower social net returns, implying that the entire asset with quality ϵ should be utilized in the process that yields the highest social net returns. Hence, we obtain corner solutions where either the production units are retired and $x_i(\epsilon) = 0$, $i = 1, 2$ when the social net returns for both technologies are negative, or where $x_i(\epsilon) = 1$ for the technology which has the highest positive social net returns. However, the maximal social net returns for technology i , Π_i^* depends on the asset quality and, thus, it will change over ϵ .

The next proposition explains how the optimal levels of the key variables changes with a change in quality ϵ .

Proposition 2: *Input use and output increase with asset quality, and the social net return does not decrease with asset quality.*

$$\frac{\partial u_i^*}{\partial \epsilon} > 0, \quad \frac{\partial y_i^*}{\partial \epsilon} > 0, \quad \frac{\partial \Pi_i^*}{\partial \epsilon} \geq 0.$$

The proof is presented in the Appendix. Because of the multiplicative effect of technology, higher asset quality has the same effect as a higher output price and leads to an increase in input use and output. Proposition 2 suggests that this multiplicative effect is not negated by the externality cost under our assumptions, and therefore the social net returns are likely to increase with asset quality as well.

The modern technology will be adopted if its social net returns are positive and higher than that of the traditional technology. The difference in social net returns per unit of asset with quality ϵ is:

⁴The asterisk, as a superscript of a decision variable, indicates its optimal value, and as a superscript of a function, it indicates that the function is evaluated at the optimal value of its arguments.

$$\Pi_1^* - \Pi_2^* = \left(p\Delta y^* - c_1\Delta u^* - \Delta cu_2^* - \Delta I - \lambda\Delta g^* \right), \quad (10)$$

where Δ represents the difference in the level of the different variables with the two technologies. Equation (10) shows that the difference in social net returns depends on the impact of both technologies on the revenues, input use, operational costs, and emissions. When the pollution problem is an input externality, a precision technology will be adopted if the gain from the higher output and lower emissions is greater than the extra variable and operational costs. In the case of an output externality, adoption is optimal if the gain from higher output is higher than the extra variable, operational and pollution costs.

The pattern of technology use with respect to quality depends on whether the modern technology is a substitute (TAS) or a complement (TAC) to the asset quality. In the case of TAS, the traditional technology is adopted at the higher end of the quality asset range, and the modern technology at the second tier of asset quality. In the case of TAC, the modern technology is adopted at the higher end of asset quality, and traditional technology at the lower end - see Figure 1a and 1b.

First consider the case of an input externality where the modern technology is a precision technology. The reason for the different pattern is that the adoption of the modern technology increases output and reduces pollution under both TAS and TAC while increasing variable and operational costs, but the gain from adoption occurs at different asset qualities. In the case of TAS, the modern technology does not provide yield gain or pollution saving at the highest quality ($\epsilon = 1$), but it still entails extra costs. Thus, $\Pi_2^*(\epsilon = 1) > \Pi_1^*(\epsilon = 1)$ and the traditional technology is superior. Because of the concavity of the difference $h_1(\epsilon) - h_2(\epsilon) > 0$, the gain from the adoption of modern technology increases within a range as quality asset quality declines below 1. Thus, there may exist an asset quality ϵ^S , with $\Pi_1^*(\epsilon^S) = \Pi_2^*(\epsilon^S)$, which separates a segment $\epsilon^S \leq \epsilon \leq 1$ where the traditional technology is optimal and a segment of lower asset qualities where the modern technology is optimal. Non negativity constraints may set a lower bound ϵ_1^L to the previous segment, where ϵ_1^L is defined by $\Pi_1^*(\epsilon_1^L) = 0$, and the segment of the modern technology is limited by $\epsilon_1^L \leq \epsilon \leq \epsilon_1^S$.⁵

Figure 1a and 1b

⁵In some cases where the difference between the costs of the different technologies are sufficiently high, there may exist another quality ϵ^{SS} with $\Pi_1^*(\epsilon^{SS}) = \Pi_2^*(\epsilon^{SS})$ separating the range of ϵ in three segments. Thus, it is optimal to use the traditional technology at the lower and higher segments, $\epsilon_2^L \leq \epsilon \leq \epsilon^{SS}$ and $\epsilon^S \leq \epsilon \leq 1$, where ϵ_2^L is defined by $\Pi_2^*(\epsilon_2^L) = 0$, and the modern technology will be adopted at the middle segment, $\epsilon^{SS} \leq \epsilon \leq \epsilon^S$.

In the case of TAC where h_1 is convex, the gain in productivity of the new technology is increasing with asset quality, and thus if adoption occurs it will be on the range of higher asset quality. In this case, it is likely to have a two segment solution. As depicted in Figure 1b, the traditional technology is adopted at $\epsilon_2^L \leq \epsilon \leq \epsilon^S$, where ϵ_2^L is defined by $\Pi_2^*(\epsilon_2^L) = 0$, and the modern technology is adopted at $\epsilon^S \leq \epsilon \leq 1$.

Similar patterns of adoption can also be found for the case of an *output externality*. However, the only gain from the adoption of modern technology is higher output associated with higher variable, operational and pollution costs. Thus, the segments where it is optimal to adopt the modern technology contracts relative to the case of an input externality presented in Figure 1a and 1b.

3.2. The optimal dynamic solution

To analyze how the optimal solution is affected over time, we maximize the value function V , obtained in the first stage, over time. Hence, the social planner's decision problem is given by:

$$\max_{z(t), \eta(t)} \int_0^{\infty} \exp^{-\delta t} \left(V(z(t)) - m(s(t)) - k(\eta(t), s(t)) \right) dt, \quad (S2)$$

subject to

$$\dot{s}(t) = z(t) - \eta(t) - \zeta s(t), \quad s(0) = s_0, \quad \eta(t) \in [0, s(t)].$$

The variable z of the first-stage problem still denotes aggregate emissions and becomes the decision variable in the second stage. However, it now depends on t . Thus, the decision variables in the intertemporal allocation problem are $z(t)$ and the stock abatement $\eta(t)$. Hence, we will be able to analyze the optimal mix of source abatement versus stock abatement. The current value Hamiltonian of the second stage is given by: $\mathcal{H}2 \equiv V(z(t)) - m(s(t)) - k(\eta(t), s(t)) - \varphi(z(t) - \eta(t) - \zeta s(t))$, where φ denotes the costate variable. It indicates the ‘‘user cost’’ of the pollution stock, i.e., it reflects the marginal cost of reducing the pollution stock and the value of pollution in production over time. Taking account of the constraints leads to the Lagrangian: $\mathcal{L}2 = \mathcal{H}2 + v_6 \eta(t) + v_7 (s(t) - \eta(t))$. The first-order conditions read as follows:

$$\mathcal{L}2_z \equiv V_z - \varphi = 0, \quad \Rightarrow \lambda = \varphi \quad (11)$$

$$\mathcal{L}2_\eta \equiv -k_\eta + \varphi + v_6 - v_7 = 0, \quad (12)$$

$$\dot{\varphi} = \delta\varphi + \mathcal{H}2_s = (\delta + \zeta)\varphi - m_s - k_s + v_7, \quad (13)$$

$$\dot{s} = z - \eta - \zeta s, \quad s(0) = s_0. \quad (14)$$

Equation (11) states that the marginal value of aggregate emissions to producers should equal the temporal shadow cost of the pollution stock φ . By the Envelope Theorem, a change in the value function as a result of a change in the right-hand side value, z , of the constraint of problem (S1) is equal to λ . Thus, we have $V_z = \lambda$ for a change in z , and therefore the shadow prices of the aggregate pollution in problems (S1) and (S2) are identical, i.e., $\lambda = \varphi$. Equation (12) indicates, for an interior solution, that the marginal cost of stock abatement should equal the shadow cost of pollution stock. However, two different boundary solutions are possible. If the marginal stock abatement cost is greater than the shadow cost of the pollution stock, i.e., $k_\eta > \varphi, \forall \eta$, stock abatement will be equal to zero. In this case, it is optimal to reduce pollution exclusively at the source. On the contrary, if the marginal stock abatement cost is lower than the shadow cost of the pollution stock, i.e., $k_\eta < \varphi, \forall \eta$, it is optimal to abate the entire pollution stock, that is, $\eta(t) = s(t)$. Equation (13) suggests that the cost of a one-period delay in generating a marginal unit of pollutant stock will be the extra discounting and forgone depreciation benefits $(\delta + \zeta)\varphi$ minus the temporal marginal social cost of the pollutant stock m_s and the marginal effect of pollutant stock on stock abatement cost k_s .

For a sustainable environmental policy, the social planner is particularly interested in the achievement of a steady state, defined by equations (13) and (14) with $\dot{\varphi} = \dot{s} = 0$. For any initial value of s within the neighborhood of s^∞ , where the superscript ∞ indicates the steady-state equilibrium value, it is possible to find a corresponding value of the shadow cost, which assures that the optimal environmental abatement policy leads toward the long-run optimum.⁶

The description of the characteristics of the steady state presented in the main body of the paper is based on the case where the marginal abatement cost are nonincreasing, i.e., $k_{\eta s} \leq 0$. The mathematical analysis for this case, as well as the case where the cleanup technology is characterized by $k_{\eta s} \geq 0$ is presented in the section, Analysis of the Steady State, in the Appendix, and shows that the qualitative characteristics of the steady state are likely to be identical.

⁶This result holds only for values within a certain neighborhood of the steady state, as our steady-state analysis has local character.

For $k_{\eta s} \leq 0$, the analysis shows that the steady-state equilibrium is locally characterized by a saddle point. The isoclines of the phase diagram in the (s, φ) space are given by

$$\left. \frac{d\varphi}{ds} \right|_{\dot{\varphi}=0} = -\frac{\frac{\partial \dot{\varphi}}{\partial s}}{\frac{\partial \dot{\varphi}}{\partial \varphi}} > 0, \quad \left. \frac{d\varphi}{ds} \right|_{\dot{s}=0} = -\frac{\frac{\partial \dot{s}}{\partial s}}{\frac{\partial \dot{s}}{\partial \varphi}} < 0. \quad (15)$$

The resulting phase diagram is depicted in Figure 2.⁷ However, It shows that the stable path leading to the steady state is upward sloping, while the unstable path is downward sloping and, thus, the pollution stock and its shadow cost evolve in the same direction. Therefore, any pollution abatement policy is characterized by a decrease in the shadow cost.

Figure 2

The fact that the pollution stock and its shadow cost evolve in the same direction over time allows us to derive the optimal intertemporal combination of source abatement versus stock abatement. Moreover, it allows us to determine the evolution of the optimal input use over time. The results for an interior solution are summarized in the following proposition.

Proposition 3: *Given that the initial stock of pollution, s_0 , is greater (smaller) than the steady-state stock of pollution, s^∞ , the optimal dynamic policy consists of:*

- (a) *choosing the aggregate emissions $z(0)$ and the input use $u_i(0, \epsilon)$, $i = 1, 2$, initially below (above) their steady-state values z^∞ and u_i^∞ , and in gradually increasing $z(t)$ and $u_i(t, \epsilon)$, $i = 1, 2$, until z^∞ and u_i^∞ are reached, and*
- (b) *in choosing the initial stock abatement $\eta(0)$ above (below) its steady-state value η^∞ , and in gradually decreasing (increasing) $\eta(t)$ until η^∞ is reached.*

The proof of Proposition 3 (a) is presented in the Appendix. Suppose that the initial pollution stock, s_0 , is greater than its steady-state value, s^∞ , and the implementation of a pollution abatement policy is required. Given the fact that the stable path leading to the steady state is upward sloping, the initial shadow cost, $\varphi(0)$, is also greater than its steady-state value. According to equations (11) and (12), the optimal initial values of the emissions and the stock abatement are determined by the initial shadow cost. As the stock of pollution decreases over time, the shadow cost has to decrease as well. Consequently, by equation (12) and the convexity of the abatement cost function, k , one can conclude that stock abatement decreases over time. Moreover, lower shadow cost provokes an increase in the intensity of production leading to a

⁷The curvature of the isoclines depends on the third derivatives of the functions. However, since they have not been specified, we have drawn for simplicity the isoclines as linear functions.

higher level of aggregate emissions. Therefore, an intertemporally and quality-differentiated optimal pollution abatement policy, for $s_0 > s^\infty$, can be characterized by choosing the levels of applied input initially below their steady-state values. As time passes, they increase until their steady-state values are reached. Figure 3 depicts the evolution of the stock abatement and aggregate emissions over time.

Figure 3

The curvature of the abatement cost function determines the speed of the decrease in stock abatement. Given an interior solution, equation (12) requires that the marginal stock abatement cost is equal to the shadow cost. The shadow cost decreases, along the optimal path, if $s_0 > s^\infty$. Therefore, the marginal stock abatement cost has to decrease as well. The decrease of the shadow cost translates to slower decrease in stock abatement the more convex the stock abatement cost function is. Thus, along the optimal path we see that stock abatement is decreased, and abatement effort at the source is increased.

Given the dynamic setting of the social planner's decision problem, the technology adoption pattern will change over time. As shown in the previous section, the shadow price of emissions decreases (increases) over time if the steady-state value of the pollutant is above (below) its initial value. In the case of an input externality and independently of whether we have the case of TAS or TAC, a decrease in the shadow price over time leads to a higher increase in the social net return function of the traditional technology than that of the modern technology given that $g_1(u_1, \epsilon) < g_2(u_2, \epsilon)$, $\forall u_1 = u_2 > 0$, and $\epsilon < 1$. Hence, by graphical analysis, one can see from Figures 1a and 1b that the range of the quality of the asset where the traditional technology is adopted expands and the range where the precision technology is adopted contracts. In the case of an output externality, the dynamic technology adoption pattern is reversed since $g_1(u_1, \epsilon) > g_2(u_2, \epsilon)$, $\forall u_1 = u_2 > 0$ and $\epsilon > 0$. On the other hand, if the shadow price increases over time, we obtain an increase in the adoption of modern technology in the case of an input externality and a reduction in the case of an output externality.

4. Optimal quality differentiated and intertemporal policies

The social optimum, characterized by the equations (7) - (9), is not equivalent to the private optimum since producers do not consider the externality. At each period of time, the private decision problem of the producers can be expressed as a private net-returns maximization prob-

lem. It is given by

$$\max_{x_i, u_i} \int_{\epsilon_0^P}^{\epsilon_1^P} \left(\sum_{i=1}^2 (p h_i f(u_i) - c_i u_i - I_i) x_i \right) X^P l^P d\epsilon, \quad (P)$$

subject to

$$u_i X^P l^P \geq 0, \quad i = 1, 2, \quad x_i X^P l^P \geq 0, \quad i = 1, 2, \quad \left(1 - \sum_{i=1}^2 x_i \right) X^P l^P \geq 0,$$

where $[\epsilon_0^P, \epsilon_1^P]$ denotes quality range of the assets available to an individual producer, l^P is the density function of the asset quality (with $\int_{\epsilon_0^P}^{\epsilon_1^P} l^P(\epsilon) = 1$), and X^P is the amount of assets available to the producer.

The optimality conditions for the private decision problem are similar to the first-order conditions (7) and (8) of problem (S1). The only difference is that the shadow price of pollution stock, φ , is zero. Thus, producers choose the level of inputs where the value of the marginal product is equal to its price (equation (7) with $\varphi = 0$). Adoption of modern technologies occurs, independent from the marginal cost of pollution, when the marginal net value of adoption is greater than its marginal costs. In reality, we observe that modern technologies are adopted to a substantial extent, even though producers do not face the shadow cost of the pollution stock. Thus, if producers were faced with this cost, producers owning assets with a certain quality would adopt in addition to the producers that already have adopted.

Since prices are constant over time, individual producers solving problem (P) will choose the same level of inputs and the same technology at every moment of time. However, the pollution stock changes over time and, without stock abatement, it may actually be growing fast. Thus, explicit pricing of the pollution is triggering a gradual adoption process over asset quality and time by the individual firm.

The solution of problem (P) leads to a private behavior where aggregate emissions are above the socially optimal level. Therefore, government intervention is indicated, for instance, in the form of a first-best policy by a tax on individual emissions. However, individual emissions often cannot be observed due to high costs or technical infeasibility (Knopman and Smith, 1993) and, therefore, policymakers must resort to other policy measures where the key variables are observable and correlate as close as possible to individual emissions (Braden and Segerson, 1993). These selection criteria are met by individually tailored input taxes supported by individually tailored technology taxes or subsidies. Since the pollution function is linear with respect to the fixed asset, the following proposition establishes policies that lead to the optimal level of input

and technology choice.

Proposition 4: *Provided that input use and technology choices can be observed at each unit with quality ϵ , an optimal policy can be obtained by*

- a quality differentiated input tax τ_i , $i = 1, 2$, given by $\tau_i(\epsilon) = \lambda^* g_{i u_i}(u_i^*(\epsilon), \epsilon)$, $i = 1, 2$, together with
- a quality differentiated technology subsidy or tax per unit of asset σ_i , $i = 1, 2$, given by $\sigma_i(\epsilon) = -\tau_i(\epsilon) u_i^*(\epsilon) + \lambda^* g_i(u_i^*(\epsilon), \epsilon) \gtrless 0$.

The proof is presented in the Appendix. An input tax alone, however, is not sufficient to achieve the social optimum since it only establishes equation (7) but not equation (8). That is, the introduction of a tax on the intensive margin causes a distortion on the extensive margin. To establish the socially efficient allocation of technologies, the input tax needs to be complemented by a technology subsidy/tax. The yet undetermined sign of σ_i , $i = 1, 2$, determines when we have a technology subsidy or tax. In the case where it is positive, we have a technology tax. If it is negative, we have, in fact, a subsidy. To determine the sign of σ_i , $i = 1, 2$, we substitute the value of the quality-differentiated input tax τ_i , $i = 1, 2$, into σ_i , $i = 1, 2$, and obtain:

$$\sigma_i = \lambda^* (g_i(u_i^*, \epsilon) - g_{i u_i}(u_i^*, \epsilon) u_i^*) \gtrless 0. \quad (16)$$

Employing the Mean Value Theorem (Theorem 2.17 in de la Fuente, 2000, p. 258), we know that $g_i(u_i, \epsilon)$ is strictly convex in u_i if $g_{i u_i}(u_i^*, \epsilon) u_i^* > g_i(u_i^*, \epsilon)$. Hence, if the marginal contribution of applied input to pollution is increasing, σ_i , $i = 1, 2$, is negative. In other words, if $g_i(\cdot)$ is convex, σ_i , $i = 1, 2$, turns into a technology subsidy. However, if $g(\cdot)$ is strictly concave, that is, $g_{i u_i}(u_i^*, \epsilon) u_i^* < g_i(u_i^*, \epsilon)$; σ_i , $i = 1, 2$, turns into a technology tax, and if $g(\cdot)$ is linear, that is, $g_{i u_i}(u_i^*, \epsilon) u_i^* = g_i(u_i^*, \epsilon)$; σ_i , $i = 1, 2$, is zero. The latter case implies that a quality-differentiated input tax alone is able to establish the social optimum and does not need to be complemented by a technology tax or subsidy.

In the case of an input externality, the pollution function may be linear or strictly convex. If the pollution function is linear in the input, then a tax on input use is equivalent to a tax on individual emissions. Therefore, no additional taxes or subsidies are needed on the extensive margin. However, if the pollution function is strictly convex, the introduction of an input tax alone leads to a change in the optimal intensity which, in turn, distorts the decision of technology adoption. As a result of these two adjustments, the resulting amount of pollution is not socially optimal. Thus, input taxes need to be complemented by technology subsidies that promote

the adoption of precision technologies. In this case, the optimal policy consists of decreasing the intensity of production and expanding the extensive margin. On the contrary, in the case of an output externality, the pollution function is strictly concave, and input taxes need to be complemented by technology taxes to moderate the adoption of modern technology. The reduction at the intensive margin is complemented by a reduction at the extensive margin, thus, both margins act complementary.

The specific design of policy instruments based on input and/or technology choice has to simultaneously take into account the varying quality of the asset and the aspect of time. In this way the policies can be adjusted according to the characteristics of the potential emissions of the production unit. Moreover, technology and input use are easy to monitor so that the policies can be enforced in practice as well. These taxes are adjusted over time in line with the changes of the shadow cost of the pollutant that varies according to the development of the stock of pollutant over time.

Proposition 4 also demonstrates the importance of an early regulation. If the regulator designs a policy when the pollution stock is smaller than its steady-state value, the initial policy is smooth and, since the shadow cost increases over time, it becomes more restrictive till the steady state is reached. On the contrary, if pollution problems were ignored for a long time and the intervention occurred at a crisis situation where the pollution stock is greater than its steady-state level, the policymaker needs to impose draconian measures in the short run that will be reduced till the steady state is attained.

5. Summary and conclusions

This paper presents a modeling approach for the socially optimal management of an accumulating pollutant generated by heterogeneous producers. The paper considers source abatement by reduction of input use, retirement of production units ($x_1 = x_2 = 0$), choice of technology, and stock abatement. The proposed solution procedure decomposes the optimization problem over time and asset quality into two stages. In the first stage the optimal form of the utilization of the quality-differentiated asset is determined subject to an aggregate emission constraint. In the second stage, the socially optimal intertemporal equilibrium is determined by optimizing the solution of the first stage over time.

Due to the presence of an externality, the private net return-maximizing strategy of the producers does not produce the socially optimal outcome. Thus, environmental policies in the form

of individually tailored input taxes (intensive margin) and individually tailored technology taxes or subsidies (extensive margin) are proposed to induce individual differentiated responses rather than uniform responses. The results show that regulations at the extensive margin should not be considered as a substitute for regulations at the intensive margin but, rather, as indispensable complements. Moreover, if the emission function is concave, complementary regulations at the extensive margin require to impose a technology tax to achieve the socially optimal outcome that moderates adoption of modern technologies and, if the emission function is convex, the payment of subsidies will trigger adoption of precision technologies. Moreover, the specification of the relationship between technology and asset quality determines the adoption pattern of each technology over asset quality.

Considering the aspect of time and quality simultaneously permits formulating the necessary changes to transform an individually tailored optimal, yet static, environmental policy to an intertemporally and individually tailored optimal policy. In particular, the temporal aspect of the regulation is of great importance, since it determines how the optimal composition and the intensity of the regulation at the intensive and extensive margins change over time.

With the advent of geographic information systems, reduced computation cost, and improved monitoring technologies, the discriminatory policies presented here are becoming feasible. We show that optimality can be also attained by incentives, even without direct measurement of pollution at the microlevel. Good estimates of production and pollution-generation functions, and information on microlevel and input use at the microlevel, are sufficient to yield optimal outcomes.

The model presented here abstracts from some important issues that should be addressed in future research. Some can be incorporated into the existing framework without altering the main results of the paper. For example, learning by doing (reduction in operational costs of new technologies as manufacturers learn from experience) may be introduced by having $I_1(t)$ with $\partial I_1/\partial t < 0$, $\partial^2 I_1/\partial t^2 \geq 0$. Learning by using (improvement in the use of technology or users learning from their and others' experience) may be presented by a production function where the fixed-asset effect of the modern technology depends on time, i.e., $h_1(\epsilon, t)$ with $\partial h_1/\partial t > 0$, or where the fixed-asset effect depends on a second stock variable, denoted by L_1 , that measures the aggregate of the asset utilized with the modern technology, i.e., $h_1(\epsilon, L_1, t)$, with $\partial h_1/\partial t > 0$. Uncertainty and irreversibility of the emission or the performance of the technologies might have to be recognized using the Dixit-Pindyck real option model. The approach taken here may have some problems in situations where the cost of the reversal of the adoption process changes over

time. These costs can be accounted for by the introduction of a new state variable for capital that is distributed over quality. The extension of the model to a distributed optimal control problem is considered as a challenge of future research.

Appendix

Proof of Proposition 2

To determine the effect of a change in asset quality on the level of applied input, we differentiate equation (7) with respect to ϵ and solve for $\partial u_i^*/\partial\epsilon$. Hence, it results in:

$$\frac{\partial u_i^*}{\partial\epsilon} = -\frac{(ph_i'f_{u_i} - \lambda g_{i u_i \epsilon})}{ph_i f_{u_i u_i} - \lambda g_{i u_i u_i}}. \quad (\text{A.1})$$

For the case of an input externality, keeping in mind that $g_{i u_i \epsilon} < 0$, it is straightforward to see that $\frac{\partial u_i^*}{\partial\epsilon} > 0$. For the case of an output externality, substituting $g_i(u_i, \epsilon)$ by its value αy_i , equation (7) reads as:

$$\mathcal{L}1_{u_i} \equiv ((p - \lambda\alpha) h_i f_{u_i} - c_i)x_i + v_i = 0.$$

Differentiating the previous equation with respect to ϵ and solving for $\partial u_i^*/\partial\epsilon$ results in:

$$\frac{\partial u_i^*}{\partial\epsilon} = -\frac{h_i' f_{u_i}}{h_i f_{u_i u_i}} > 0.$$

Hence, $\partial u_i^*/\partial\epsilon$ is positive in both externality cases.

Since the function $f(u_i)$ has regular, neoclassical properties, we obtain for the case of an input and output externality:

$$\frac{\partial y_i^*}{\partial\epsilon} = h_i' f(u_i^*) + h_i f_{u_i} \frac{\partial u_i^*}{\partial\epsilon} > 0. \quad (\text{A.2})$$

The changes in the allocation of the technologies are determined in the case of an input externality by differentiating equation (8) with respect to ϵ :

$$\Pi_{i\epsilon}^* = ph_i' f(u_i^*) - \lambda g_{i\epsilon} > 0. \quad (\text{A.3})$$

In the case of an output externality, the social net returns are given by $\Pi_i^* \equiv (p - \lambda\alpha)h_i f(u_i^*) - c_i u_i^* - I_i$, since $g_i(u_i(\epsilon), \epsilon)$ is given by αy_i . Differentiation of the social net returns with respect to ϵ yields

$$\Pi_{i\epsilon}^* = (p - \lambda\alpha)h_i' f(u_i^*) \geq 0. \quad (\text{A.4})$$

Equation (A.4) is strictly positive for $(p - \lambda\alpha) > 0$. The inequality $(p - \lambda\alpha) < 0$ corresponds to a situation where the social net returns are negative for every quality of the asset, and hence no production takes place, that is, $u_i^* = 0, \forall \epsilon \in [0, 1]$. Therefore, $(p - \lambda\alpha) < 0$ implies that $f(u_i^*) = 0$ and consequently $\Pi_{i\epsilon}^* = 0$. ■

Analysis of the Steady State

Assuming an interior solution, equations (11) and (12) can be solved globally and uniquely by using Theorem 6 in Gale and Nikaidô (1965) for $z = \hat{z}(\varphi, s)$ and $\eta = \hat{\eta}(\varphi, s)$. By the implicit function theorem, we obtain

$$\begin{pmatrix} \mathcal{L}2_{zz} & \mathcal{L}2_{z\eta} \\ \mathcal{L}2_{\eta z} & \mathcal{L}2_{\eta\eta} \end{pmatrix} \begin{pmatrix} \frac{\partial \hat{z}}{\partial \varphi} & \frac{\partial \hat{z}}{\partial s} \\ \frac{\partial \hat{\eta}}{\partial \varphi} & \frac{\partial \hat{\eta}}{\partial s} \end{pmatrix} + \begin{pmatrix} \mathcal{L}2_{z\varphi} & \mathcal{L}2_{zs} \\ \mathcal{L}2_{\eta\varphi} & \mathcal{L}2_{\eta s} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \quad (\text{A.5})$$

The application of Cramer's rule yields that

$$\frac{\partial \hat{z}}{\partial \varphi} = \frac{1}{V_{zz}} \leq 0, \quad \frac{\partial \hat{z}}{\partial s} = 0, \quad \frac{\partial \hat{\eta}}{\partial \varphi} = \frac{1}{k_{\eta\eta}} \geq 0, \quad \frac{\partial \hat{\eta}}{\partial s} = -\frac{k_{\eta s}}{k_{\eta\eta}} \geq 0. \quad (\text{A.6})$$

For the purposes of a qualitative analysis, we reduce the necessary conditions (11) - (14) to a pair of differential equations in φ and s by substituting $z = \hat{z}(\varphi, s)$ and $\eta = \hat{\eta}(\varphi, s)$ into (13) and (14) to obtain

$$\dot{\varphi} = (\delta + \zeta)\varphi - m_s - k_s(\hat{\eta}(\varphi, s), s), \quad (\text{A.7})$$

$$\dot{s} = \hat{z}(\varphi, s) - \hat{\eta}(\varphi, s) - \zeta s, \quad s(0) = s_0. \quad (\text{A.8})$$

A linearization of the canonical system of differential equations around the steady-state values of φ and s results in

$$\begin{pmatrix} \dot{\varphi} \\ \dot{s} \end{pmatrix} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} & \frac{\partial \dot{\varphi}}{\partial s} \\ \frac{\partial \dot{s}}{\partial \varphi} & \frac{\partial \dot{s}}{\partial s} \end{pmatrix} \begin{pmatrix} \varphi - \varphi^\infty \\ s - s^\infty \end{pmatrix}. \quad (\text{A.9})$$

The implicit function theorem is also used to calculate the elements of the Jacobian matrix evaluated at the steady-state equilibrium with $\dot{\varphi} = \dot{s} = 0$, leading to

$$\tilde{J} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} = \delta + \zeta - \frac{k_{\eta s}}{k_{\eta\eta}} & \frac{\partial \dot{\varphi}}{\partial s} = \frac{(k_{\eta s})^2 - k_{\eta\eta} k_{ss}}{k_{\eta\eta}} - m_{ss} \\ \frac{\partial \dot{s}}{\partial \varphi} = \frac{1}{V_{zz}} - \frac{1}{k_{\eta\eta}} & \frac{\partial \dot{s}}{\partial s} = -\zeta + \frac{k_{\eta s}}{k_{\eta\eta}} \end{pmatrix}. \quad (\text{A.10})$$

We can distinguish three different cases, depending on the value of $k_{\eta s}/k_{\eta\eta}$; case A: $k_{\eta s}/k_{\eta\eta} \leq \zeta$, case B: $k_{\eta s}/k_{\eta\eta} \geq \delta + \zeta$, and case C: $\zeta < k_{\eta s}/k_{\eta\eta} < \delta + \zeta$. For case A where $k_{\eta s}/k_{\eta\eta} \leq \zeta$, the Jacobian Matrix is given by:

$$\tilde{J} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} > 0 & \frac{\partial \dot{\varphi}}{\partial s} < 0 \\ \frac{\partial \dot{s}}{\partial \varphi} < 0 & \frac{\partial \dot{s}}{\partial s} \leq 0 \end{pmatrix}. \quad (\text{A.11})$$

In this case, the determinant of the Jacobian matrix is negative. Moreover, since the trace of

the Jacobian matrix, trJ , is equal to $\delta > 0$, the eigenvalues have opposite signs. Therefore, the steady-state equilibrium is locally characterized by a saddle point. This will always be the case if $k_{\eta s} \leq 0$. The isoclines of the phase diagram in the (s, φ) space are given by

$$\left. \frac{d\varphi}{ds} \right|_{\dot{\varphi}=0} = -\frac{\frac{\partial \dot{\varphi}}{\partial s}}{\frac{\partial \dot{\varphi}}{\partial \varphi}} > 0, \quad \left. \frac{d\varphi}{ds} \right|_{\dot{s}=0} = -\frac{\frac{\partial \dot{s}}{\partial s}}{\frac{\partial \dot{s}}{\partial \varphi}} < 0, \quad (\text{A.12})$$

and the stable path leading to the steady state is upward sloping, while the unstable path is downward sloping.

However, if $k_{\eta s} \geq 0$, cases A, B, and C are possible. In the case where $k_{\eta s}/k_{\eta\eta} \geq \delta + \zeta$ (case B), the Jacobian matrix is given by:

$$\tilde{J} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} \leq 0 & \frac{\partial \dot{\varphi}}{\partial s} < 0 \\ \frac{\partial \dot{s}}{\partial \varphi} < 0 & \frac{\partial \dot{s}}{\partial s} > 0 \end{pmatrix}. \quad (\text{A.13})$$

Like in case A, the determinant of the Jacobian matrix is negative and the steady-state equilibrium is locally characterized by a saddle point. The slope of both isoclines is different, but the slope of the stable path is still positive.

Finally, in the case where $\zeta < k_{\eta s}/k_{\eta\eta} < \delta + \zeta$ (case C), the Jacobian matrix is given by:

$$\tilde{J} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} > 0 & \frac{\partial \dot{\varphi}}{\partial s} < 0 \\ \frac{\partial \dot{s}}{\partial \varphi} < 0 & \frac{\partial \dot{s}}{\partial s} > 0 \end{pmatrix}. \quad (\text{A.14})$$

In case C, the determinant of the Jacobian matrix might not be negative. If it is positive, two complex eigenvalues may result, where the real parts of the eigenvalues are given by $\delta/2$. Under these conditions, the “equilibrium” is characterized by an unstable spiral. However, if the Jacobian matrix and the discriminant of the characteristic equation are positive, the two eigenvalues are positive and real, leading to an “equilibrium” in the form of a source.

Yet, if the determinant of the Jacobian matrix is negative, the steady-state equilibrium is again locally characterized by a saddle point, however the slope of both isoclines is positive and the slope of the stable path is negative. Since case C is very specific and not likely to occur in reality as it requires a large δ , our analysis concentrates on cases A and B.

Proof of Proposition 3

To find the optimal intertemporal path of $\hat{z}(t)$ and $\hat{\eta}(t)$, we totally differentiate with respect to time, make use of equation (A.6), and obtain

$$\frac{d\hat{z}}{dt} = \frac{1}{V_{zz}} \frac{d\varphi}{dt}, \quad \frac{d\hat{\eta}}{dt} = \frac{1}{k_{\eta\eta}} \frac{d\varphi}{dt} - \frac{k_{\eta s}}{k_{\eta\eta}} \frac{ds}{dt}. \quad (\text{A.15})$$

Thus, the optimal path of $\hat{z}(t)$ and $\hat{\eta}(t)$ is determined by the path of φ and s . Taking into account that φ and s evolve according to Figure 2 in the same direction allows us to determine the sign of $d\hat{z}/dt$ and $d\hat{\eta}/dt$.

Additionally, we conduct a comparative static analysis to determine the effect of a change in the shadow cost on the level of input use. Since neither V nor λ depend on ϵ , we assume that the technologies are located optimally, and the amount of pollution is chosen optimally. The sign of $\partial u_i^*/\partial \lambda$ can be determined by solving the first-order equation (7) for $u_i = u_i^*(\lambda)$, $i = 1, 2$, obtaining, in the case of an input externality

$$\frac{\partial u_i^*}{\partial \lambda} = \frac{g_{i u_i}}{p h_i f_{u_i u_i} - \lambda g_{i u_i u_i}} < 0, \quad (\text{A.16})$$

and in the case of an output externality

$$\frac{\partial u_i^*}{\partial \lambda} = \frac{\alpha f_{u_i}}{(p - \lambda \alpha) f_{u_i u_i}} < 0. \quad (\text{A.17})$$

Using equations (A.15), (A.16), and the fact that $\lambda = \varphi$, allows to verify Proposition 3. ■

Proof of Proposition 4

When producers are facing a quality-differentiated input tax and a quality-differentiated technology subsidy/tax, their decision problem is given by

$$\max_{x_i, u_i} \int_{\epsilon_0^P}^{\epsilon_1^P} \left(\sum_{i=1}^2 (p h_i f(u_i) - c_i u_i - I_i) x_i - \sum_{i=1}^2 (\tau_i u_i x_i + \sigma_i x_i) \right) X^P l^P d\epsilon, \quad (\text{T})$$

subject to

$$u_i X^P l^P \geq 0, \quad i = 1, 2, \quad x_i X^P l^P \geq 0, \quad i = 1, 2, \quad \left(1 - \sum_{i=1}^2 x_i \right) X^P l^P \geq 0.$$

Taking account of the constraints on the control variables leads to the Lagrangian

$$\mathcal{L}^T \equiv \int_{\epsilon_0^P}^{\epsilon_1^P} \left(\sum_{i=1}^2 (p h_i f(u_i) - c_i u_i - I_i) x_i - \sum_{i=1}^2 (\tau_i u_i x_i + \sigma_i x_i) \right) X^P l^P d\epsilon + \left(v_1 u_1 + v_2 u_2 + v_3 x_1 + v_4 x_2 + v_5 (1 - x_1 - x_2) \right) X^P l^P.$$

The first-order conditions read as follows

$$\mathcal{L}_{u_i}^T \equiv (p h_i f_{u_i} - c_i - \tau_i) x_i + v_i = 0, \quad (\text{A.18})$$

$$\mathcal{L}_{x_i}^T \equiv p y_i - c_i u_i - I_i - (\tau_i u_i + \sigma_i) + v_{i+2} - v_5 = 0. \quad (\text{A.19})$$

Substituting the values of τ_i and σ_i into equations (A.18) and (A.19) leads to

$$\mathcal{L}_{u_i}^T \equiv (p h_i f_{u_i} - c_i - \lambda^* g_{i u_i}) x_i + v_i = 0, \quad (\text{A.20})$$

$$\mathcal{L}_{x_i}^T \equiv p y_i - c_i u_i - I_i - \lambda^* g_i(u_i, \epsilon) + v_{i+2} - v_5 = 0. \quad (\text{A.21})$$

The comparison of the necessary conditions (A.20) and (A.21) with the necessary conditions (7) and (8) of the social optimum shows that the input tax τ_i , $i = 1, 2$, together with the technology subsidy or tax σ_i , $i = 1, 2$, establish the quality-differentiated optimal input use and technology choice for every quality ϵ . ■

References

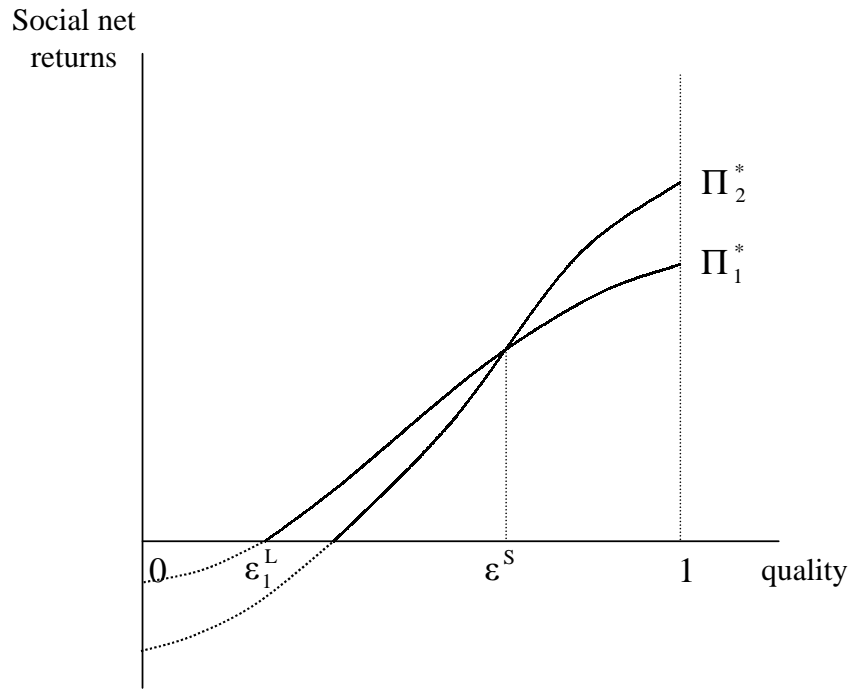
- Baudry, M. (2000). Joint management of emission abatement and technical innovation for stock externalities, *Environmental and Resource Economics* **16**: 161–183.
- Braden, J. and Segerson, K. (1993). Information problems in the design of nonpoint-source pollution policy, in C. Russel and J. Shogren (eds), *Theory, Modeling and Experience in the Management of Nonpoint-Source Pollution*, Kluwer, Boston, chapter 1, pp. 1–36.
- Chakravorty, U., Roumasset, J. and Tse, K. (1997). Endogenous substitution among energy resources and global warming, *Journal of Political Economy* **105**: 1201–34.
- Committee on the Future Role of Pesticides in US Agriculture (2000). *The Future Role of Pesticides in US Agriculture*, National Academy Press, Washington, D.C.
- de la Fuente, A. (2000). *Mathematical Methods and Models for Economists*, Cambridge University Press, Cambridge. United Kingdom.
- Edwards, R., Smith, K., Zhang, J. and Ma, Y. (2004). Implications of changes in household stoves and fuel use in china, *Energy Policy* **32**: 395–411.
- Farzin, Y. (1996). Optimal pricing of environmental and natural resource use with stock externalities, *Journal of Public Economics* **62**: 31–57.
- Fuglie, K. and Kascak, C. (2001). Adoption and diffusion of natural-resource-conserving agricultural technology, *Review of Agricultural Economics* **23**(2): 386–403.
- Gale, D. and Nikaidô, H. (1965). The Jacobian matrix and global univalence of mappings, *Mathematische Annalen* **159**: 81 – 93.
- Glaeser, E. and Shleifer, A. (2001). A reason for quantity regulation, *American Economic Review* **29**: 431–435.
- Goetz, R. and Zilberman, D. (2000). The dynamics of spatial pollution - the case of phosphorus runoffs from agricultural land, *Journal of Economic Dynamics and Control* **24**(1): 143–163.
- Hausman, J. (1979). Individual discount rates and the purchase and utilization of energy-using durables, *Bell Journal of Economics* **10**: 33–54.
- Hongli, F., Zhao, J. and Kling, C. (2002). The time path and implementation of carbon sequestration, *American Journal of Agricultural Economics* **84**: 134–149.

- Jaffe, A. and Stavins, R. (1995). Dynamic incentives of environmental regulations: The effects of alternative policy instruments on technology diffusion, *Journal of Environmental Economics and Management* **29**: 43–63.
- Khanna, M., Isik, M. and Zilberman, D. (2002). Cost-effectiveness of alternative green payment policies for conservation technology adoption with heterogeneous land quality, *Agricultural Economics* **27**: 157–174.
- Khanna, M. and Zilberman, D. (1997). Incentives, precision technology and environmental protection, *Ecological Economics* **23**: 25–43.
- Khanna, M. and Zilberman, D. (1999). Freer markets and the abatement of carbon emissions: the electricity-generating sector in india, *Resource and Energy Economics* **21**: 125–152.
- Knopman, D. and Smith, R. (1993). 20 years of the clean water act - has U.S. water quality improved?, *Environment* **35**: 16–41.
- Mas-Colell, A., Whinston, M. and Green, J. (1995). *Microeconomic Theory*, Oxford University Press, New York.
- Millock, K., Sunding, D. and Zilberman, D. (2002). Regulating pollution with endogenous monitoring, *Journal of Environmental Economics and Management* **44**: 221–241.
- Nakata, T. (2000). Analysis of the impact of hybrid vehicles on energy systems in Japan, *Transportation Research Part D* **5**: 373–83.
- Pan, J. and Hodge, I. (1994). Land use permits as an alternative to fertiliser and leaching taxes for the control of nitrate pollution, *Journal of Agricultural Economics* **45**(1): 102–112.
- Segerstrom, P. (1999). Endogenous growth without scale effects, *American Economic Review* **88**: 1290–1310.
- Seierstad, A. and Sydsæter, K. (1987). *Optimal Control Theory with Economic Applications*, North-Holland, Amsterdam.
- Shah, F., Zilberman, D. and Lichtenberg, E. (1995). Optimal combination of pollution prevention and abatement policies: The case of agricultural drainage, *Environmental and Resource Economics* **5**: 29–49.

- Siegel, J. and Temchin, J. (1991). Role of clean coal technology in electric power generation in the 21st century, in J. Tester, D. Wood and N. Ferrari (eds), *Energy and the Environment in the 21st Century*, MIT Press, Cambridge and London, pp. 623–30.
- Wu, J. and Babcock, B. (1998). The choice of tillage, rotation and soil testing practices: Economic and environmental implications, *American Journal of Agricultural Economics* **80**: 494–511.

Figure 1: Optimal Technology Choice for the Case of an Input Externality.

a) Technology asset substitution



b) Technology asset complementarity

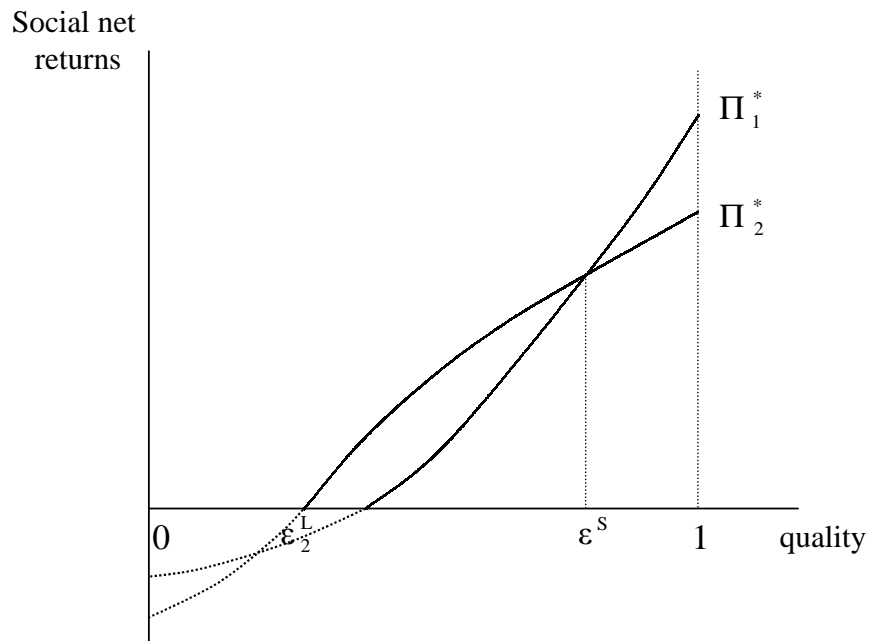


Figure 2: The Phase Diagram in the (s, φ) Space.

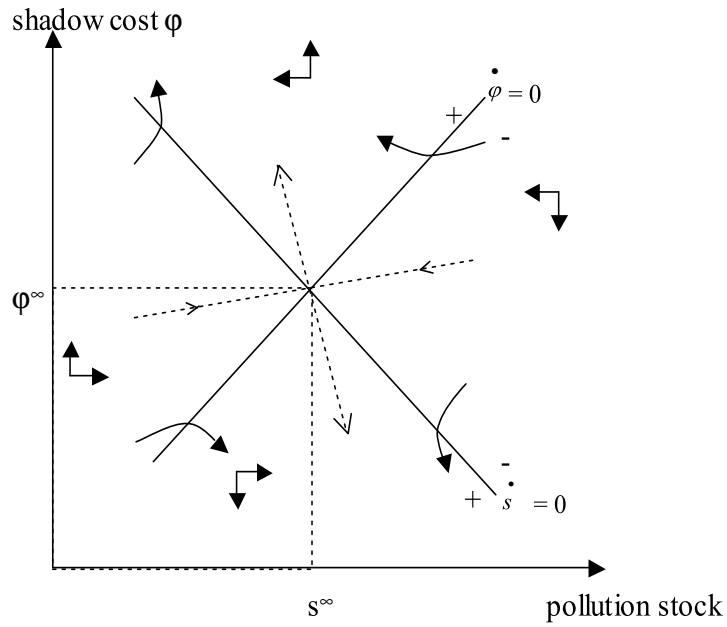


Figure 3: Optimal Intertemporal Abatement Policy.

