UNIVERSIDAD CARLOS III DE MADRID working papers

Working Paper
Economic Series 10-34
November 2010

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 916249875

House Prices, Sales, and Time on the Market: A Search-Theoretic Framework (Supplementary material) *

Antonia Díaz
Universidad Carlos III de Madrid

Belén Jerez†
Universidad Carlos III de Madrid

Abstract

In this document we present additional results for our main paper. First, we prove some properties of the steady state of our benchmark economy. Then, we present an alternative calibration.

Keywords: House prices, sales, time on the market, search frictions, competitive search.
JEL Classification: R31, D83, D40

* We thank Melvyn Coles, Juanjo Dolado, Jan Eackhout, Andrés Erosa, Miquel Faig, Javier Fernández Blanco, Ed Green, Tim Kehoe, María J. Luengo-Prado, Facundo Piguillem, Fabian Postel-Vinay, Guillaume Rocheteau, Peter Rupert, and Manuel Santos for their comments, as well as participants of the SED Conference at MIT 2008, III REDg DGE Macroeconomics Workshop at Madrid 2008, XIII Workshop on Dynamic Macroeconomics at Vigo 2008, SED Conference at Istanbul 2009, and seminars at Banco de España, University of Bristol, Universidad Carlos III, University of Essex, Universidad de Salamanca and IESE. Díaz thanks the Spanish DGCYT (project SEJ2007-65169) for financial support. Jerez thanks fundación Ramón Areces and Spanish DGCYT (project SEJ2007-66820 and Ramón y Cajal Program). Errors are ours.

† Corresponding author: Belén Jerez, Departament of Economics. Universidad Carlos III de Madrid, Calle Madrid, 126, Getafe 28903 Madrid, Spain. E-mail: mjerez@eco.uc3m.es. Ph: +34-91-624-9633.

1 The steady state

We start by characterizing the steady state of our benchmark economy in section 1.1. In section 1.2 we show some properties of the steady states.

1.1 Characterization of the steady state

In Díaz and Jerez (2010) we show that the steady state is characterized by the following system of equations:

$$\begin{aligned}
s = & \quad H - N, && (1.1)\\
q\, m(\theta) = & \quad \alpha \left(\frac{2\,N - H}{H - N} - \theta \right), && (1.2)\\
b = & \quad \theta s, && (1.3)\\
N = & \quad n + b + s, && (1.4)\\
(1-\beta) W_n = & \quad \overline{v} - \beta\, \alpha\, (W_n - W_b)\,, && (1.5)\\
(1-\beta) W_b = & \quad \underline{v} + \frac{q\, m(\theta)}{\theta}\, \eta\,(\theta)\, [\overline{v} - \underline{v} + \beta\,(W_n - W_b)]\,, && (1.6)\\
(1-\beta) W_s = & \quad \overline{v} + q\, m(\theta)\, (1 - \eta\,(\theta))\, [\overline{v} - \underline{v} + \beta\,(W_n - W_b)]\,, && (1.7)\\
\frac{\overline{v} - \underline{v} - p + \beta\,(W_s - W_b)}{p + \beta\,(W_n - W_s)} = & \quad \frac{\eta(\theta)}{1 - \eta(\theta)}, && (1.8)\\
\eta(\theta) = & \quad \frac{m'(\theta)\theta}{m(\theta)}. && (1.9)
\end{aligned}$$

Equation (1.2) pins down the unique value of θ that ensures that the composition of population is constant at the steady state.[1] Given θ and s, we obtain the rest of the equilibrium variables using (1.3)-(1.9).

It is easy to check combining (1.5) and (1.6) that, at the steady state, the value of non traders is higher than the value of buyers:

$$W_n - W_b = \frac{1 - \frac{qm(\theta)}{\theta}\eta(\theta)}{1 - \beta + \beta\left(\frac{qm(\theta)}{\theta}\eta(\theta) + \alpha\right)}\,(\overline{v} - \underline{v}) > 0. \tag{1.10}$$

(since $\pi_b(\theta) = \frac{qm(\theta)}{\theta} \in [0,1]$, $\eta(\theta), \beta \in [0,1]$ and $\overline{v} > \underline{v}$). This implies that the bilateral surplus is positive:

$$S = \bar{v} - \underline{v} + \beta(W_n - W_b) > 0. \tag{1.11}$$

It also implies that the value of sellers is highest: $W_s > W_n$ (see (1.5) and (1.7)).

The equilibrium price is given by the sum of the fraction of the bilateral surplus appropriated

[1] Remember that $m(\theta)$ is strictly increasing.

by sellers and their reservation price:

$$p = \beta\left(W_s - W_n\right) + \left(1 - \eta\left(\theta\right)\right) S; \tag{1.12}$$

Using (1.5) and (1.7), the seller's reservation price can be written as

$$p_s = \beta\left(W_s - W_n\right) = \frac{\beta}{1-\beta}\left[qm(\theta)(1-\eta(\theta))S + \beta\alpha(W_n - W_b)\right] \tag{1.13}$$

We now conduct some comparative statics exercises to identify the key factors which underly changes in prices, sales and time on the market across steady states. We first rewrite (1.2) as

$$q\, m(\theta) = \alpha\left(\frac{1}{V} - 2 - \theta\right), \tag{1.14}$$

where $V = \frac{H-N}{H}$ is the vacancy rate.[2] It is direct to check from (1.14) that the steady-state buyer-seller ratio θ increases with α and decreases with V and q (since $m(\theta)$ is increasing). In words, the buyer-seller ratio is higher when there are fewer vacancies, and also when mismatch shocks are more frequent and when buyers face more severe matching frictions in locating a suitable unit (since in both instances there are more buyers around). Because they affect θ, these variables affect prices, sales, and time on the market. Note that changes in the flow gain from switching to a suitable unit when mismatched, $\bar{v} - \underline{v}$, do not affect sales nor time on the market because they do not affect θ. They only affect the size of the bilateral surplus and the price (see (1.10)-(1.12)).

1.2 Across steady states comparative statics

The following propositions describe the effect of these exogenous changes on the steady state.

Proposition 1. *For a given population size N, changes in the vacancy rate $V = \frac{H-N}{H}$ (equivalently, in the housing stock H) have the following effects on a steady-state equilibrium:*

$$\begin{aligned}
&\frac{\partial\theta}{\partial V} < 0,\ \frac{\partial\pi_s}{\partial V} < 0,\ \frac{\partial\pi_b}{\partial V} > 0,\\
&\frac{\partial s}{\partial V} > 0,\ \frac{\partial b}{\partial V} < 0,\quad \frac{\partial n}{\partial V} < 0 \ \textit{if}\ \theta_s < \theta,\ \frac{\partial n}{\partial V} > 0\ \textit{if}\ \theta > \theta_s,\\
&\frac{\partial(\pi_s s)}{\partial V} < 0\ \textit{if}\ \theta_s < \theta,\ \frac{\partial(\pi_s s)}{\partial V} > 0\ \textit{if}\ \theta > \theta_s,\\
&\frac{\partial S}{\partial V} < 0,\ \textit{and}\ \frac{\partial p}{\partial V} < 0.
\end{aligned}$$

Here the threshold value θ_s satisfies $\theta_s = (1+\theta_s)\eta(\theta_s)$ (e.g. $\theta_s = 1.1416$ for an urn-ball matching function).

Proof. Since $V = 1 - N/H$ and N is given, it is equivalent to consider the sign effects of changes in H. Since $s = H - N$, $\frac{ds}{dH} > 0$. From (1.2),

$$\frac{d\theta}{dH} = \frac{-\alpha N}{(H-N)^2[\alpha + q\, m'(\theta)]} < 0, \tag{1.15}$$

[2] $V \in (0, 1/2)$ since $H \in (N, 2N)$.

since $m(\theta)$ is increasing. Hence, $\frac{d\pi_s}{dH} < 0$ and $\frac{d\pi_b}{dH} > 0$. Since $b = \theta(H - N)$, taking derivatives, using (1.15) and the definition of $\eta(\theta)$, and rearranging gives:

$$\frac{db}{dH} = \frac{-N + \theta(H-N)[1 + \frac{qm(\theta)}{\alpha\theta}\eta(\theta)]}{(H-N)[1 + \frac{qm(\theta)}{\alpha\theta}\eta(\theta)]}. \tag{1.16}$$

In (1.16) the denominator is positive, whereas the numerator can be written as

$$-(H-N)(1 + \frac{m(\theta)q}{\alpha}(1 - \eta(\theta))) < 0, \tag{1.17}$$

using (1.2). So $\frac{db}{dH} < 0$. Since $\alpha n = \pi^s s$, the sign effect on n and on sales is the same. Sales are given by $qm(\theta)(H - N)$. It is direct to show that the derivative of this term with respect to H has the same sign as the term $\frac{\eta(\theta)}{\theta}(H-N)\frac{d\theta}{dH} + 1$, which in turn may be written as

$$\frac{[\frac{\theta}{\eta(\theta)} - (1+\theta)]s}{(H-N)[\frac{\theta}{\eta(\theta)} + \frac{q}{\alpha}m(\theta)]}, \tag{1.18}$$

using (1.2), (1.3), (1.4), and (1.15). Since the denominator in (1.18) is positive, sales increase with H if $\frac{\theta}{\eta(\theta)} > (1+\theta)$, and they decrease with H if $\frac{\theta}{\eta(\theta)} < (1+\theta)$. Equivalently, since $\eta(\theta)$ is a decreasing function of θ, sales increase with H if $\theta > \theta_s$, and they decrease if $\theta < \theta_s$ where θ_s satisfies $\theta = (1+\theta)\eta(\theta)$. From (1.10)-(1.11) and (1.15), $\frac{dS}{dH} < 0$ since both $m(\theta)/\theta$ and $\eta(\theta)$ are decreasing functions. Given the above and using (1.13), it is also direct to check that the seller's reservation price p_s falls, and so does the price: $\frac{dp}{dH} < 0$. Since S falls, the buyer's reservation price falls more than the seller's reservation price. □

When the number of vacancies is higher (keeping the population size fixed), there are fewer buyers per seller, so it takes less time to buy and more time to sell units. Note that, while housing supply is higher, the effect on sales is ambiguous since it is now harder for sellers to find a buyer. We find that sales are lower when θ is below the threshold θ_s; otherwise they are higher. Intuitively, when θ is below θ_s the probability of realizing a sale is already low in the initial steady state, so the fact that this probability falls further out-weights the fact that these are more units for sale. Another way to see this is to note that, in the model, an increase in supply s lowers steady-state demand b. Since the increase in supply lowers the buyer-seller ratio, the number of buyers falls because buyers trade faster. Because sales are given by $q\mathcal{M}(b, s)$, they may be lower in the new steady state. Since it takes less time to buy a unit, the loss of being mismatched is lower and so is the total bilateral surplus. The seller's reservation price is also lower when there are more vacancies, and so is the seller's share of the surplus. Hence prices are lower. For values of $\theta \leq \theta_s$ prices and sales are then lower and average time on the market is higher when there are more vacancies (see our quantitative exercises in Section 3).

Proposition 2. *Changes in the probability α of becoming mismatched have the following effects on a steady-state equilibrium:*

$$\frac{\partial\theta}{\partial\alpha} > 0,\ \frac{\partial\pi_s}{\partial\alpha} > 0,\ \frac{\partial\pi_b}{\partial\alpha} < 0,\ \frac{\partial b}{\partial\alpha} > 0,\ \frac{\partial s}{\partial\alpha} = 0,\ \frac{\partial n}{\partial\alpha} < 0,\ \text{and}\ \frac{\partial(\pi_s s)}{\partial\alpha} > 0.$$

The effects on p and S are ambiguous in general.

Proof. From (1.2), $\frac{d\theta}{d\alpha} > 0$, so $\frac{d\pi_s}{d\alpha} > 0$ and $\frac{d\pi_b}{d\alpha} < 0$. Since $s = H - N$ is independent of α, $\frac{d\pi_s s}{d\alpha} < 0$. Also, (1.3) implies $\frac{db}{d\alpha} > 0$. From (1.4), $\frac{dn}{d\alpha} < 0$. On the one hand, a higher α has a direct negative effect on S (see (1.10)-(1.11)). On the other hand, since θ increases, it has an indirect positive effect on S. The same is true about the effect of α on p_s. In general, while $1 - \eta(\theta)$ increases, the effect on S and p_s is ambiguous, and so is the effect on p. □

A higher probability of becoming mismatched (say due to higher job mobility) generates the opposite effects as a higher vacancy rate in that there are more buyers around (i.e., housing demand is higher), and it takes more time to buy and less time to sell in the new steady state. Sales are always higher, and the number of non traders is lower in this case. The sellers's share of the surplus is higher, but in general the effect on the bilateral surplus and the price is ambiguous. In our calibration exercises we find that for reasonable parameter values both the price and the surplus are higher when α is higher.

Proposition 3. *Changes in the probability q that a buyer likes a unit visited at random have the following effects on a steady-state equilibrium:*

$$\frac{\partial\theta}{\partial q} < 0,\ \frac{\partial\pi_s}{\partial q} > 0,\ \frac{\partial\pi_b}{\partial q} > 0,\ \frac{\partial b}{\partial q} < 0,\ \frac{\partial s}{\partial q} = 0,\ \frac{\partial n}{\partial q} > 0,\ \frac{\partial(\pi_s s)}{\partial q} > 0,\ \frac{\partial S}{\partial q} < 0.$$

The effect on p is ambiguous.

Proof. From (1.2), $\frac{d\theta}{dq} < 0$, so $\frac{d\pi_b}{dq} > 0$. Since s is independent of q, (1.3) then implies $\frac{db}{dq} < 0$. From (1.4), $\frac{dn}{dq} > 0$, $\frac{d\pi_s s}{dq} > 0$, and (since s is constant) $\frac{d\pi_s}{dq} > 0$. From (1.10)-(1.11), $\frac{dS}{dq} < 0$ since both $m(\theta)/\theta$ and $\eta(\theta)$ are decreasing functions. The effect on p_s is ambiguous in general. It is easy to see from (1.13) that a higher q has a direct positive effect on p_s. On the other hand, since θ falls and S falls it has an indirect negative effect on p_s. Hence, while $1 - \eta(\theta)$ and S fall, the effect on p is ambiguous. □

When q is higher, matching frictions are less severe, so it is easier for buyers to locate the units they like. The effect is the opposite as that of a higher α in that there are more non traders and fewer buyers (i.e., housing demand is lower), who now trade faster, and sales are higher.[3] However, time on the market is lower, which is intuitive. The loss from being mismatched, the bilateral surplus and the seller's share are also lower, which tends to lower prices. However, the fact that the seller's trading probability $qm(\theta)$ is higher tends to raise prices (see (1.13)). The price response is ambiguous in general, and it is hump-shaped in our quantitative exercise (increasing for low values of q and decreasing above a certain threshold).

Proposition 4. *Changes in the mismatched households' flow gain from switching to a suitable unit have no effect on the distribution of the population, trading probabilities, or sales. They only affect the bilateral surplus and prices:* $\frac{\partial S}{\partial(\bar{v}-\underline{v})} > 0$ *and* $\frac{\partial p}{\partial(\bar{v}-\underline{v})} > 0$.

Remember that all matches where the buyer likes the unit result in trade, independently of the value of $\bar{v} - \underline{v}$. Changes in $\bar{v} - \underline{v}$ then do not affect the extensive margin when H is fixed. They only affect the bilateral surplus and the price. The same is true about changes in the discount factor β.

[3] This is not obvious since the probability that a buyer likes the house is higher, but sellers now meet buyers less frequently. Yet the first effect dominates over the second one.

2 Across steady states comparative statics in the Walrasian economy

The comparative static results below focus on the effects on the Walrasian equilibrium allocation of changes in the vacancy rate and the probability of becoming mismatched for $\theta > 1$ and $\theta < 1$ respectively.[4]

Proposition 5 (*High price scenario*)**.** *Assume $\theta > 1$ at the steady state. For fixed N, changes in the vacancy rate have the following effects on a steady-state equilibrium:*

$$\frac{\partial \theta}{\partial V} < 0,\ \frac{\partial \pi_s}{\partial V} = 0,\ \frac{\partial \pi_b}{\partial V} > 0,\ \frac{\partial b}{\partial q} < 0,\ \frac{\partial s}{\partial V} = 0,\ \frac{\partial n}{\partial V} > 0,\ \frac{\partial (\pi_s s)}{\partial V} > 0,\ \frac{\partial S}{\partial V} = 0,\ \frac{\partial p}{\partial V} = 0.$$

The effect of changes in the probability of becoming mismatched are:

$$\frac{\partial \theta}{\partial \alpha} > 0,\ \frac{\partial \pi_s}{\partial \alpha} = 0,\ \frac{\partial \pi_b}{\partial \alpha} < 0,\ \frac{\partial b}{\partial q} > 0,\ \frac{\partial s}{\partial \alpha} = 0,\ \frac{\partial n}{\partial \alpha} < 0,\ \frac{\partial (\pi_s s)}{\partial \alpha} = 0,\ \frac{\partial S}{\partial \alpha} < 0,\ \frac{\partial p}{\partial \alpha} < 0.$$

Proof. Since $\theta > 1$, $\pi^b = \theta^{-1}, \pi^s = 1, S^b = S$ and $S^s = 0$, so the steady state is characterized by:

$$\begin{aligned}
s &= H - N, & (2.1)\\
b &= \theta s, & (2.2)\\
n &= 2N - H - \theta(H - N), & (2.3)\\
\alpha n &= s, & (2.4)\\
(1-\beta)W^b &= \underline{v}, & (2.5)\\
(1-\beta)W^s &= \bar{v} + S, & (2.6)\\
(1-\beta)W^n &= \bar{v} - \beta\alpha(W^n - W^b), & (2.7)\\
S &= \bar{v} - \underline{v} + \beta(1-\alpha)(W^n - W^b), & (2.8)\\
p &= \bar{v} - \underline{v} + \beta(W^s - W^b). & (2.9)
\end{aligned}$$

Combining (2.5), (2.7) and (2.8),

$$W^n - W^b = S = \frac{\bar{v} - \underline{v}}{1 - \beta(1-\alpha)}. \tag{2.10}$$

Consider an increase of the housing stock H. From (2.1) and (2.4), both s and n increase, so b and hence θ must fall. Then π^b increases while π^s remains constant. Sales $\pi^s s$ increase. From (2.10) and (2.5)-(2.7) and (2.9), W^b, W^s, W^n, S and p are all independent of H.

Suppose now that α increases. From (2.4), n must fall (since s is constant), and so b must increase. Then θ increases, so π^b falls, while π^s remains constant. Sales $\pi^s s$ also remain constant. From (2.10), $W^n - W^b$ and S fall. Then W^s falls from (2.6) and, since W^b remains constant (equation (2.5)), W^n falls as well. Finally, (2.9) implies that p falls. □

Time on the market does not respond to supply changes when sellers are on the short side of

[4] Assuming these changes are small so this is also the relevant scenario in the new steady state.

the market. Sales increase, but prices do not change either. On the other hand, demand changes driven by an increase in the probability of becoming mismatched raise the bilateral surplus and the price, but do not affect neither sales nor time on the market.

Proposition 6 (*Low price scenario*)**.** *Assume $\theta < 1$ at the steady state. For fixed N, changes in the vacancy rate have the following effects on a steady-state equilibrium:*

$$\frac{\partial \theta}{\partial V} < 0, \frac{\partial \pi_s}{\partial V} < 0, \frac{\partial \pi_b}{\partial V} = 0, \frac{\partial b}{\partial V} < 0, \frac{\partial s}{\partial V} > 0, \frac{\partial n}{\partial V} < 0, \frac{\partial (\pi_s s)}{\partial V} < 0, \frac{\partial S}{\partial V} = 0, \frac{\partial p}{\partial V} = 0.$$

The effect of changes in the probability of becoming mismatched are:

$$\frac{\partial \theta}{\partial \alpha} > 0, \frac{\partial \pi_s}{\partial \alpha} > 0, \frac{\partial \pi_b}{\partial \alpha} = 0, \frac{\partial b}{\partial \alpha} > 0, \frac{\partial s}{\partial \alpha} = 0, \frac{\partial n}{\partial \alpha} < 0, \frac{\partial (\pi_s s)}{\partial \alpha} > 0, \frac{\partial S}{\partial \alpha} = 0, \frac{\partial p}{\partial \alpha} = 0.$$

Proof. When $\theta < 1$, $\pi^b = 1, \pi^s = \theta, S^b = S$ and $S^s = 0$, so the steady state is characterized by:

$$\begin{aligned}
s &= H - N, && (2.11)\\
b &= \theta s, && (2.12)\\
n &= 2N - H - \theta(H - N), && (2.13)\\
\alpha\, n &= \theta\, s, && (2.14)\\
(1-\beta)W^b &= \underline{v} + S, && (2.15)\\
(1-\beta)W^s &= \bar{v}, && (2.16)\\
(1-\beta)W^n &= \bar{v} - \beta\alpha(W^n - W^b), && (2.17)\\
S &= \bar{v} - \underline{v} + \beta(1-\alpha)(W^n - W^b). && (2.18)\\
p &= \beta(W^s - W^n) && (2.19)
\end{aligned}$$

Combining (2.11), (2.13), and (2.14),

$$\frac{\alpha}{1+\alpha}\left(\frac{2N - H}{H - N}\right) = \theta \text{ or } \frac{\alpha}{1+\alpha}\left(\frac{1}{V} - 2\right) = \theta. \quad (2.20)$$

Combining (2.15)-(2.18), $W^n = W^b = W^s = \frac{\bar{v}}{1-\beta}$. So $S = \bar{v} - \underline{v}$, and $p = 0$.

Consider an increase of the housing stock H or the vacancy rate V. From (2.20), θ falls, so π^s falls while π^b remains constant. From (2.20), sales as a fraction of the stock $\pi^s V$ are given by $\frac{\alpha}{1+\alpha}(1 - 2V)$, so they also fall. As noted above, W_b, W_s, W_n, S and p are all independent of V.

Suppose now that α increases. From (2.20), θ increases, so π^s increases, while π^b remains constant. Sales $\pi^s s$ also increase (since s is constant). Again, W_b, W_s, W_n, S and p are independent of α. □

In the low price scenario, both sales and time on the market respond to both demand and supply changes, but prices remain constant.

For prices, sales, and selling probabilities to increase when α increases, θ must cross the threshold 1. Say the economy is initially in a low-price steady state ($\theta < 1$). If the increase α raises the value of θ above 1, it raises prices, sales and selling probabilities. On the other hand, if a fall in the housing stock raises the value of θ above 1, it raises prices and selling probabilities. Yet sales may rise or fall, since they switch from $\theta(H-N)$ to $H-N$. The logic is as in the search model: there are fewer units for sale but (as seller's rationing decreases) units sell faster. We have conducted some quantitative exercises and find that sales fall. That is, the effect of fall in the vacancy rate on time on the market is not strong enough to generate an increase in sales in the Walrasian model. In the next section we also show that the steady-state value of θ is always below 1 for reasonable parameter values, so the Walrasian economy is always in the high-price steady state. This is why in the frictionless version of the economy, prices do now respond to demand and supply changes (see Díaz and Jerez (2010)).

3 Quantitative properties of stationary economies

Here we study quantitatively the properties of our stationary economy. In Díaz and Jerez (2010) we discuss the properties of the data that we want to match with our calibration, shown in Table 1.

3.1 Changes across steady state

In this section we want to study the long run effects of changes in the vacancy rate, mobility (as captured by the mismatch probability α) and the extent of matching frictions (captured by q) to shed some light on the effects of search frictions as well as the assumed market arrangement (i.e., competitive search with an urn-ball matching function).

Changes in the vacancy rate Let us study the steady state effect of a change in the vacancy rate; e.g., an increase in the number of houses. Table 3 shows the effect of an increase in the vacancy rate from 1.58 to 4 percent. In our benchmark economy, *CSE-UB*, the price drops from 73.56 to 4.73 percent of the present discounted value of being matched instead of mismatched. There are two effects. On the one hand, given the number of buyers, an increase in the number of sellers reduces the buyer-seller ratio θ, which in turn reduces the seller's share of the surplus. This static effect is reinforced by a second dynamic effect, which is due to the fact that an increase in the number of sellers reduces buyer time on the market. As a result, the steady-state number of buyers b falls from 1.63 to 1.15 percent. This in turn further reduces θ, and thus the seller's share of the surplus (which falls from 42 to 13 percent). The total bilateral surplus also falls. Even though there are more units for sale, sales fall because units now take longer to sell. Figures 2 and 4 show that, in general, sales are a non-monotone function of the vacancy rate. Yet when the vacancy rate exceeds a certain threshold (1.2 percent), the fact that units take longer to sell outweighs the fact that there are more units for sale, so sales decrease with the vacancy rate. The present analysis then suggests that whenever the vacancy rate is above this threshold we can observe a positive comovement of sales and prices, as well as a negative comovement of prices and time on the market at higher frequencies.

Table 3 considers also the other two search economies. Notice that the price falls even further in the *CSE-CD* economy. This is because buyer time on the market falls more than in the benchmark economy (so there are fewer buyers in the new steady state). Also, in this case, the seller' share of the surplus is reduced to zero. The reason is that, with a Cobb-Douglas matching function, the search frictions faced by buyers disappear when θ is below a certain threshold (satisfying $A\,\theta^{-\eta} = 1$), and on that range buyers appropriate the entire surplus. This is not the case with an urn-ball matching function. Unlike in the Walrasian economy, the price is not zero because the matching friction faced by buyers, captured by the probability q, does not vanish. In the *Nash-CD* economy, the number of buyers also falls from 1.63 to 1.00, again inducing a larger fall in θ than in our benchmark economy. Yet the percentage drop in the price is lower than in the *CSE-CD* economy. The reason is that traders continue to split the bilateral surplus equally. In this economy, the search frictions faced by buyers also disappear when θ is below a certain threshold and at this point the seller's share is reduced to zero, but this threshold is lower than in the *CSE-CD* economy. Figure 2 shows that, in general, prices are more responsive to changes in the vacancy rate in the benchmark economy than in the other two search economies as long as the sharing rule remains constant in the latter. It also shows that, for a sufficiently high vacancy rate, the equilibrium price in the three search economies drops to the Walrasian price.

Table 3 shows that, in the Walrasian economy, the price is not affected by the increase in the vacancy rate. This is because buyers are on the short side of the market when the vacancy rate is 1.59 percent, and this continues to be so when the number of seller rises. Sales fall slightly and this is again because time on the market rises. The fact that the Walrasian price does not move for this range of the vacancy rate does not imply that it does not change across steady states. Figure 4 shows that, for values of V below a certain threshold, sellers become the short side of the market and appropriate the entire surplus. This analysis may suggest that the equilibrium price can potentially fluctuate more in a Walrasian framework than in a search environment. Nevertheless, we need to keep in mind that the Walrasian price fluctuates when the vacancy rate is lower than 0.9 percent, which is far below the average 1.59 observed in the U.S. economy. Also, on that range sales *increase* with the vacancy rate, so they no longer comove with prices (see below).

Changes in the tenure length Let us turn to study the effect of changes in tenure length. Table 4 shows the new steady state when α increases so that the average tenure length drops from 10 to 7 years. Note first that this increases the number of mismatched households every period, that is the number of buyers. The increase in the number of buyers today increases the buyer-seller ratio and is propagated over time through the search frictions, as sellers trade faster and buyers trade more slowly. In the three search economies, the qualitative effect is the same: a fall in time on the market and an increase in sales and prices. Yet the price rises more in our benchmark economy, from 73.56 percent to 551.29 percent of the present discounted value of being matched instead of mismatched. The reason is that the increase in the steady-state number of buyers and buyer time on the market is larger in this economy. In addition, the seller's share of the surplus (which is constant in the other two search economies) rises sharply from 42 to 75 percent. The bilateral surplus and the seller's reservation price are then also much higher. In the Walrasian economy, the increase in the number of buyers is again not enough for the price to change as buyers still are on the short side of the market.

Changes in the extent of matching frictions Figures 7 and 8 show the steady state effects of a change in q. The higher q the lower the extent of matching frictions. In other words, buyers are more likely to find a home that suits them. This is why the buyer-seller ratio θ falls when q increases. Notice that the effect on the price is not monotonous. There are two opposing effects. On the one hand, an increase in q increases the probability of finding a suitable trading partner. This implies an increase in effective demand, which increases the price. This effect is static. The second effect is dynamic: a higher q reduces the number of mismatched households over time, which reduces the equilibrium price. The first positive effect is stronger for low levels of q, while the second effect dominates for high levels of q. Thus, if matching frictions are sufficiently severe, a higher q implies higher demand, higher price, higher sales and lower time on the market. This result is reminiscent of that found by Ngai and Tenreyro (2009), who exploit this positive relation between prices and sales to study seasonal fluctuations in the housing market. Their key modeling assumption is that buyers are more likely to find a suitable home when there are more houses for sale (q is higher when s is higher). This assumption captures a thick-markets effect. As noted above, our framework delivers similar results when q is sufficiently small.

4 Alternative calibrations

4.1 Matching time to sell

In our benchmark economy we have calibrated the vacancy rate to match that observed in the data. As a result, average time to sell at the steady state is about 2 months, which may seem low. The U.S Census Bureau reports the *Median Number of Months for Sale* for the period 1965-2009, which is 5.42 months on average. As we argued in Díaz and Jerez (2010), regardless of the matching function used, it is not possible to match independently the vacancy rate, time to buy and time to sell if we want to match also the average tenure length. Table 5 shows an alternative calibration where we set H so that time to sell is 5.42 months, while also targeting the observed time to buy and average tenure length (2 months and 10 years, respectively). As a result, the implied vacancy rate is larger, 4 percent. Moreover, the extent of matching frictions must also be higher—the calibrated value of q decreases from 0.7940 in our benchmark calibration to 0.5979. Likewise, the parameter A in the Cobb-Douglas matching function is lower.

Table 6 shows the steady state with this alternative calibration. The main difference with our benchmark calibration is that the buyer-seller ratio is lower, and so is the bilateral surplus and the price. We have also calibrated to this case the stochastically growing economy described in Section 5.2 in Díaz and Jerez (2010). The results are found in Table 10. Comparing this table with Table 9 (which shows the results of our benchmark calibration), the main difference is that volatility of the price is about half of that in our benchmark economy. This is so because the average number of sellers is larger now. (Remember that when the vacancy rate is sufficiently high the search economy behaves much like the Walrasian economy).

4.2 A very low vacancy rate

It could be argued that we are not giving any chance to the Walrasian economy to account for the observed fluctuations in the housing market. This is why we have conducted the following experiment. We have kept our benchmark calibration except for the average vacancy rate, which has been reduced to 0.8 percent. We have done this because the steady state comparative statics exercises conducted above suggest that the Walrasian price fluctuates very much when θ crosses the threshold 1 (so buyers and sellers alternate between the short and the long side of the market). The results are shown in Table 11. As expected, the Walrasian price fluctuates very much, its standard deviation being 4.73 percent. This is so because the price is "jumping" from the seller's reservation price to the buyer's reservation price. Moreover, the price is positively correlated with α and negatively correlated with the number of vacancies. Sales, however, have a counterfactual zero correlation with α. This is so because sellers are often on the short side of the market, so changes in the number of buyers do not affect sales. Note that time on the market is essentially constant over the cycle, and so its correlation with the price is zero. This is also why sales are so strongly correlated with vacancies. Notice, though, that the price is negatively correlated with sales. This is so because increases in the vacancy rate tend to lower prices but, for such low values of the vacancy rates, they also tend to raise sales.

Let us turn now to the search economy. It turns out that for such low values of the vacancy rate the search economy is behaves similarly to the Walrasian economy. In particular, changes in α hardly affect time on the market when there are so few houses for sale (as these houses are essentially being sold with probability one). Also, as discussed in Díaz and Jerez (2010), α and V are negatively correlated, and so sales and α display a (low) negative correlation in this case. Also, sales are again strongly correlated with vacancies. While in our benchmark economy more vacancies today imply fewer sales next period (because time on the market falls), this is not the case here because the additional units sell almost instantaneously. This is why the contemporaneous and the forward correlation of price and sales is negative, as in the Walrasian economy.

Table 1: Benchmark calibration

Param.	Observation	Value
N	-	100.0000
H	Vacancy rate = 1.59%	101.6117
q	Time to buy = $\frac{1}{q\, m^b(\theta)}$ = 2 months	0.7940
α	Tenure = $\frac{1-\alpha}{\alpha}$+ time to buy = 10 years	0.0084
β	0.96 in annual terms	0.9966
$\overline{v}$	-	1.0000
$\underline{v}$	$0.1\,\overline{v}$	0.1000
Cobb-Douglas matching function and Nash bargaining, $m(\theta) = \min\left\{A\,\theta^{1-\eta}, 1\right\}$		
η	Share of the seller	0.5000
A	Time to buy = $\frac{1}{q\, m^b(\theta)}$ = 2 months	0.6326

Table 2: Steady State (Benchmark calibration)

	CSE (UB and CD)	Nash-CD	Walrasian
Tightness θ	1.01	1.01	0.51
Sellers s	1.61	1.61	1.61
Buyers b	1.63	1.63	0.82
Non-traders n	96.76	96.76	97.57
Sales $q\, m\,(\theta)\; s$	0.81	0.81	0.82
Time to sell $\frac{1}{q\, m(\theta)}$	1.98	1.98	1.97
Time to buy $\frac{\theta}{q\, m(\theta)}$	2.00	2.00	1.00
Price $\frac{p\,(1-\beta)}{(\overline{v}-\underline{v})}$ (%)	73.56	100.21	0.00
Reser. price $\frac{p_s\,(1-\beta)}{(\overline{v}-\underline{v})}$ (%)	73.08	99.56	0.00
$S^s/S = 1-\eta\,(\theta)$	0.42	0.50	0.00
Bilateral surplus S	3.02	3.48	0.90

Table 3: Steady state when $V = 4$ % (instead of 1.59 %) (Benchmark calibration)

	CSE-UB	CSE-CD	Nash-CD	Walrasian
Tightness θ	0.28	0.24	0.24	0.19
Sellers s	4.17	4.17	4.17	4.17
Buyers b	1.15	1.00	1.00	0.80
Non-traders n	94.69	94.83	94.83	95.03
Sales $q\, m\,(\theta)\; s$	0.80	0.80	0.80	0.80
Time to sell $\frac{1}{q\, m(\theta)}$	5.24	5.23	5.23	5.22
Time to buy $\frac{\theta}{q\, m(\theta)}$	1.44	1.26	1.26	1.00
Price $\frac{p\,(1-\beta)}{(\overline{v}-\underline{v})}$ (%)	4.73	0.21	25.24	0.00
Reser. price $\frac{p_s\,(1-\beta)}{(\overline{v}-\underline{v})}$ (%)	4.65	0.21	24.82	0.00
$S^s/S = 1-\eta\,(\theta)$	0.13	0.00	0.50	0.00
Bilateral surplus S	1.48	1.13	2.23	0.90

Table 4: Steady-state when average tenure is 7 years (instead of 10, Benchmark calibration)

	CSE-UB	CSE-CD	Nash-CD	Walrasian
Tightness θ	2.35	1.88	2.06	0.74
Sellers s	1.61	1.61	1.61	1.61
Buyers b	3.78	3.03	3.33	1.19
Non-traders n	94.60	95.35	95.06	97.20
Sales $q\, m\,(\theta)\; s$	1.14	1.17	1.14	1.17
Time to sell $\frac{1}{q\, m(\theta)}$	1.39	1.38	1.39	1.36
Time to buy $\frac{\theta}{q\, m(\theta)}$	3.27	2.60	2.86	1.00
Price $\frac{p\,(1-\beta)}{(\overline{v}-\underline{v})}$ (%)	551.29	134.07	178.17	0.00
Reser. price $\frac{p_s\,(1-\beta)}{(\overline{v}-\underline{v})}$ (%)	611.03	133.51	197.14	0.00
$S^s/S = 1-\eta\,(\theta)$	0.75	0.42	0.50	0.00
Bilateral surplus S	9.99	3.84	4.80	0.90

Table 5: Calibrating to time to sell in the data

Param.	Observation	Value
N	-	100.0000
H	Time to sell $= \frac{1}{q\, m^s(\theta)} = 5.42$ months	104.2873
q	Time to buy $= \frac{1}{q\, m^b(\theta)} = 2$ months	0.5979
α	Tenure $= \frac{1-\alpha}{\alpha} +$ time to buy $= 10$ years	0.0084
β	0.96 in annual terms	0.9966
$\overline{v}$	-	1.0000
$\underline{v}$	$0.1\,\overline{v}$	0.1000
Cobb-Douglas matching function and Nash bargaining, $m(\theta) = \min\left\{A\,\theta^{1-\eta}, 1\right\}$		
η	Share of the seller	0.5000
A	Time to buy $= \frac{1}{q\, m^b(\theta)} = 2$ months	0.5080

Table 6: Steady State with a large vacancy rate

	CSE (UB and CD)	Nash-CD	Walrasian
Tightness θ	0.37	0.37	0.19
Sellers s	4.29	4.29	4.29
Buyers b	1.58	1.58	0.80
Non-traders n	94.13	94.13	94.92
Sales $q\, m\,(\theta)\; s$	0.79	0.79	0.80
Time to sell $\frac{1}{q\, m(\theta)}$	5.42	5.42	5.38
Time to buy $\frac{\theta}{q\, m(\theta)}$	2.00	2.00	1.00
Price $\frac{p\,(1-\beta)}{(\overline{v}-\underline{v})}$ (%)	19.98	49.70	0.00
$S^s/S = 1-\eta\,(\theta)$	0.17	0.50	0.00
Bilateral surplus S	2.14	3.48	0.90

Table 7: Steady state when $V = 1.58$ % (instead of 4 %, alternative calibration)

	CSE-UB	CSE-CD	Nash-CD	Walrasian
Tightness θ	1.81	1.25	2.63	0.51
Sellers s	1.61	1.61	1.61	1.61
Buyers b	2.90	2.00	4.23	0.82
Non-traders n	95.49	96.39	94.17	97.57
Sales $q\,m\,(\theta)\ s$	0.80	0.81	0.79	0.82
Time to sell $\frac{1}{q\,m(\theta)}$	2.00	1.98	2.03	1.96
Time to buy $\frac{\theta}{q\,m(\theta)}$	3.62	2.47	5.34	1.00
Price $\frac{p\,(1-\beta)}{(\overline{v}-\underline{v})}$ (%)	316.09	38.31	255.76	0.00
$S^s/S = 1-\eta\,(\theta)$	0.65	0.17	0.50	0.00
Bilateral surplus S	8.29	2.63	8.64	0.90

Table 8: Steady-state when average tenure is 7 years (instead of 10, alternative calibration)

	CSE-UB	CSE-CD	Nash-CD	Walrasian
Tightness θ	0.58	0.56	0.73	0.27
Sellers s	4.29	4.29	4.29	4.29
Buyers b	2.48	2.42	3.15	1.14
Non-traders n	93.24	93.29	92.56	94.57
Sales $q\,m\,(\theta)\ s$	1.12	1.12	1.12	1.14
Time to sell $\frac{1}{q\,m(\theta)}$	3.81	3.81	3.84	3.76
Time to buy $\frac{\theta}{q\,m(\theta)}$	2.20	2.15	2.82	1.00
Price $\frac{p\,(1-\beta)}{\overline{v}}$ (%)	33.37	24.63	85.53	0.00
$S^s/S = 1-\eta\,(\theta)$	0.26	0.17	0.50	0.00
Bilateral surplus S	2.60	2.29	4.74	0.90

Table 9: Business cycle properties of the benchmark economy and the Walrasian economy

	Benchmark economy				Walrasian economy			
	$\sigma_x(\%)$	$\rho(\alpha,x)$	$\rho(V,x)$	$\rho(V,x_{+1})$	$\sigma_x(\%)$	$\rho(\alpha,x)$	$\rho(V,x)$	$\rho(V,x_{+1})$
e, $\overline{v}$, α	3.49	1.00	-0.16	-0.15	3.49	1.00	-0.16	-0.15
Buyers	9.12	0.71	-0.46	-0.68	4.16	0.91	-0.14	-0.26
V	7.78	-0.16	1.00	0.76	7.78	-0.16	1.00	0.76
p	1.82	0.88	-0.58	-0.49	-	-	-	-
p_s	1.70	0.90	-0.56	-0.46	-	-	-	-
$1-\eta(\theta)$	4.90	0.53	-0.83	-0.83	-	-	-	-
S	15.01	0.73	-0.75	-0.69	-	-	-	-
Sales	4.75	0.68	0.16	-0.24	4.16	0.91	-0.14	-0.26
TOM	8.43	-0.53	0.83	0.83	9.33	-0.54	0.90	0.75

	$\rho(p,y)$			$\rho(p,y)$		
	y_{-1}	y	y_{+1}	y_{-1}	y	y_{+1}
Sales	0.40	0.51	0.72	-	-	-
TOM	-0.68	-0.82	-0.83	-	-	-

Notes: The measure of buyers is the number of buyers as a fraction of the population. Sales is the per capita volume of sales, in percentage points.

Table 10: Business cycle properties of the economy with a large vacancy rate

	Data		Correl. supply and demand shocks		Indep. supply and demand shocks		Only supply shocks		Only demand shocks	
variable	$\sigma_x(\%)$	$\rho(\overline{v},x)$	$\sigma_x(\%)$	$\rho(\overline{v},x)$	$\sigma_x(\%)$	$\rho(\overline{v},x)$	$\sigma_x(\%)$	$\rho(V,x)$	$\sigma_x(\%)$	$\rho(\overline{v},x)$
$\overline{v},\alpha$	3.26	1.00	3.49	1.00	3.49	1.00	-	-	3.49	1.00
V	6.93	-0.17	7.78	-0.16	7.79	0.00	7.84	1.00	-	-
p	2.39	0.61	0.65	0.89	0.56	0.84	0.30	-0.99	1.29	1.00
Sales	9.29	0.58	4.54	0.78	4.50	0.78	2.59	0.18	3.59	0.93
TOM	15.99	-0.61	9.11	-0.52	8.60	-0.41	7.80	0.94	3.59	-0.93
	$\rho(p,x)$		$\rho(p,x)$		$\rho(p,x)$		$\rho(p,x)$		$\rho(p,x)$	
Sales	0.49		0.68		0.64		-0.07		0.94	
TOM	-0.35		-0.83		-0.81		-0.97		-0.94	

Table 11: Business cycle properties of the search economy and the Walrasian economy with a very low mean vacancy rate

	Benchmark economy				Walrasian economy			
	$\sigma_x(\%)$	$\rho(\alpha,x)$	$\rho(V,x)$	$\rho(V,x_{+1})$	$\sigma_x(\%)$	$\rho(\alpha,x)$	$\rho(V,x)$	$\rho(V,x_{+1})$
e, $\overline{v}$, α	3.49	1.00	-0.16	-0.15	3.49	1.00	-0.16	-0.15
Buyers	16.77	0.24	0.00	-0.14	57.91	0.38	-0.29	-0.42
V	7.78	-0.16	1.00	0.76	7.78	-0.16	1.00	0.76
p	0.04	0.87	-0.31	-0.31	4.73	0.78	-0.51	-0.49
Sales	7.78	-0.16	1.00	0.76	6.94	0.00	0.87	0.60
TOM	0.01	-0.17	0.28	0.31	-	-	-	-
		$\rho(p,y)$				$\rho(p,y)$		
		y_{-1}	y	y_{+1}		y_{-1}	y	y_{+1}
Sales		-0.28	-0.30	-0.11		-0.30	-0.28	-0.15
TOM		-0.50	-0.58	-0.61		-	-	-

Notes: The measure of buyers is the number of buyers as a fraction of the population. Sales is the per capita volume of sales, in percentage points.

Figure 1: The steady state effect of a change in the vacancy rate (I).

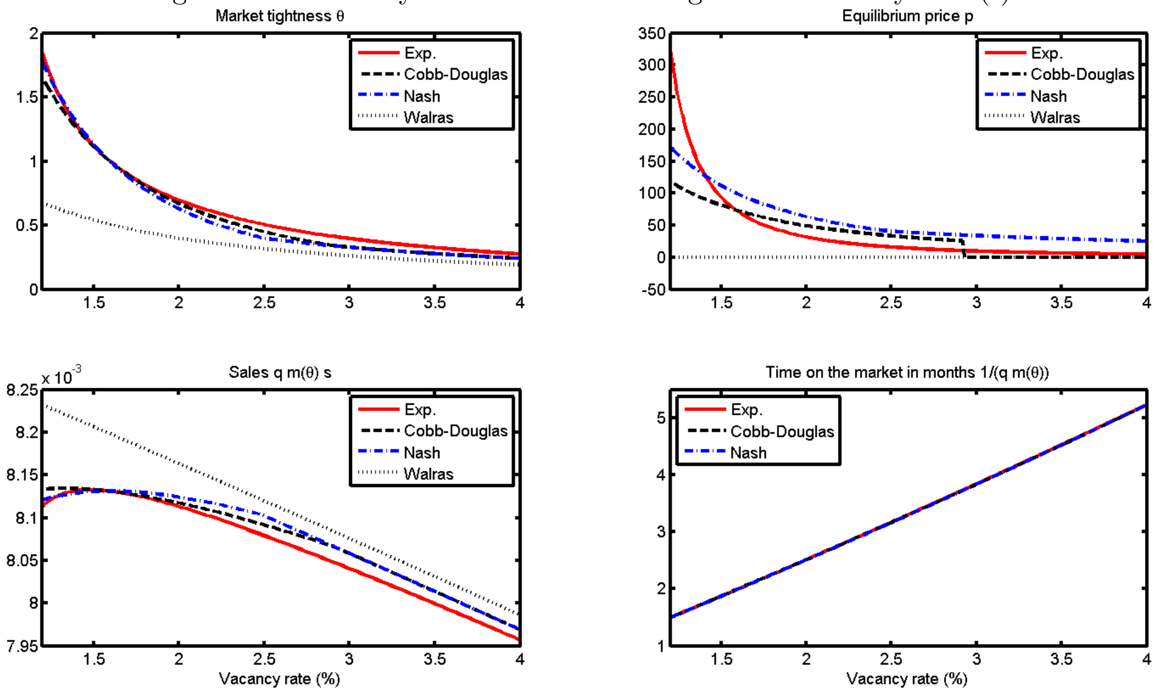


Figure 2: The steady state effect of a change in the vacancy rate (II).

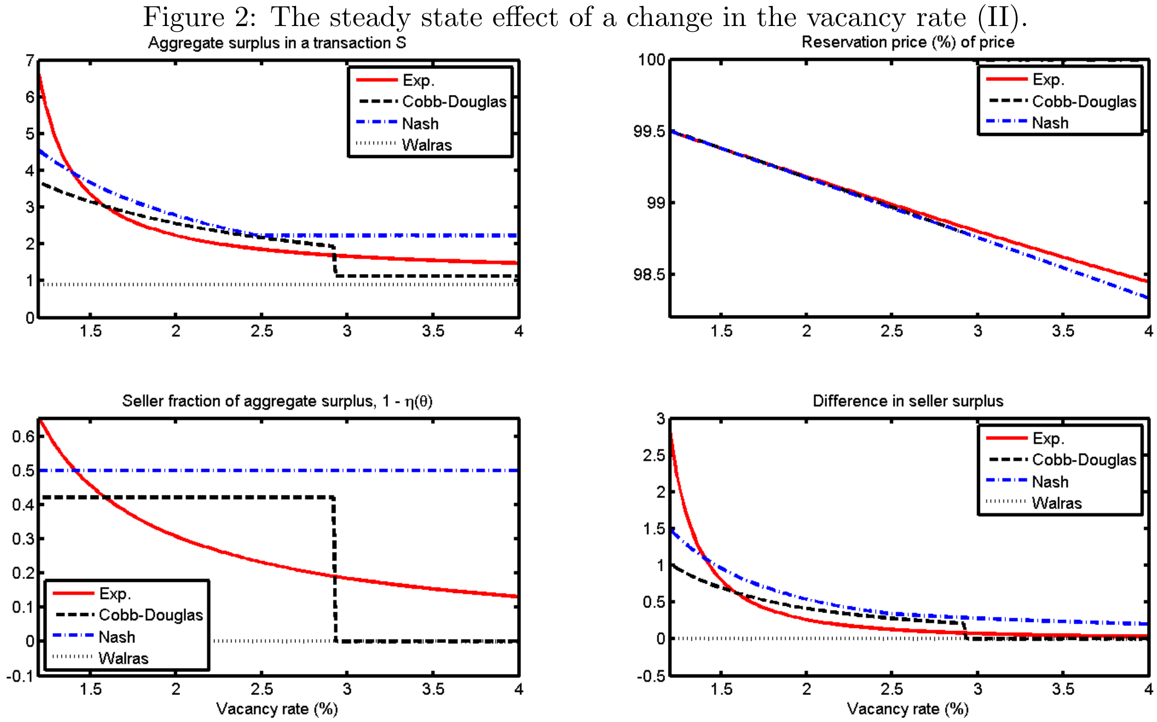


Figure 3: The steady state effect of a change in the vacancy rate (III).

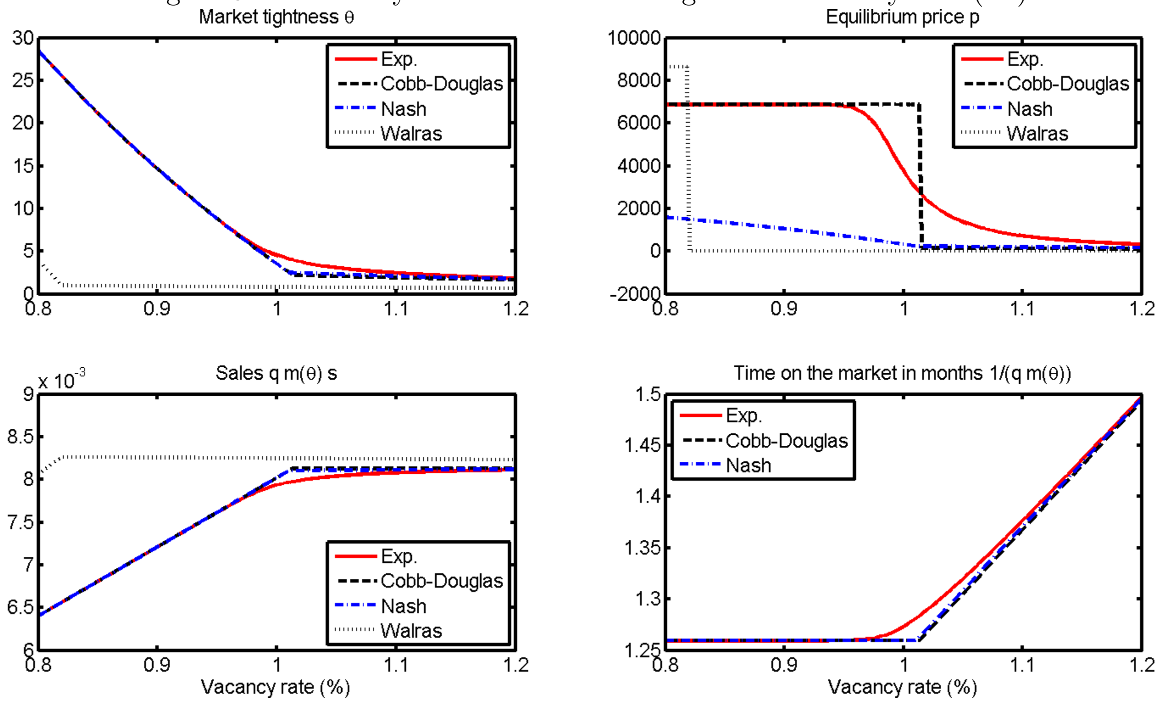


Figure 4: The steady state effect of a change in the vacancy rate (IV).

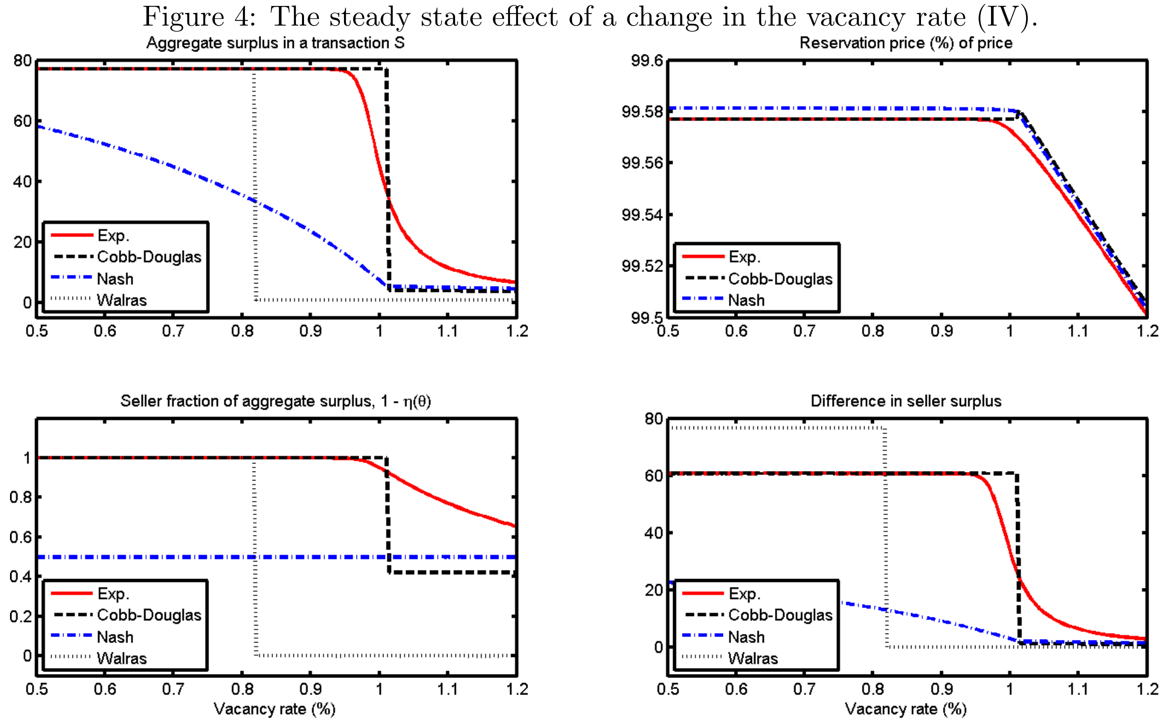


Figure 5: The steady state effect of a change in the tenure length (I).

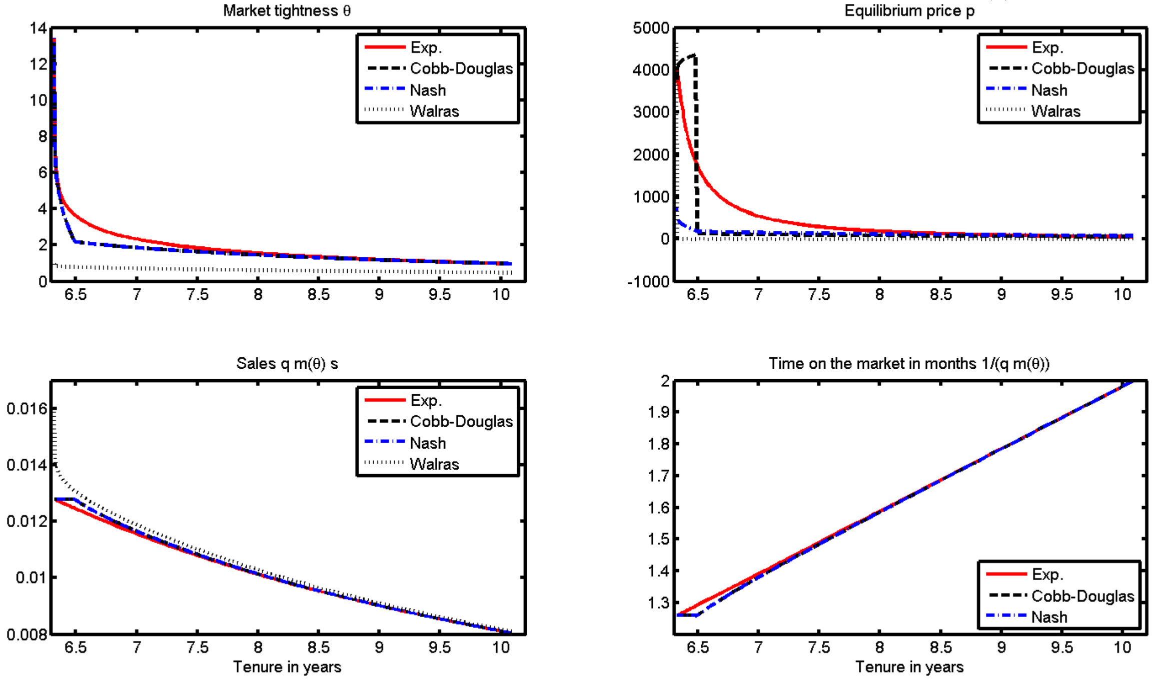


Figure 6: The steady state effect of a change in the tenure length (II).

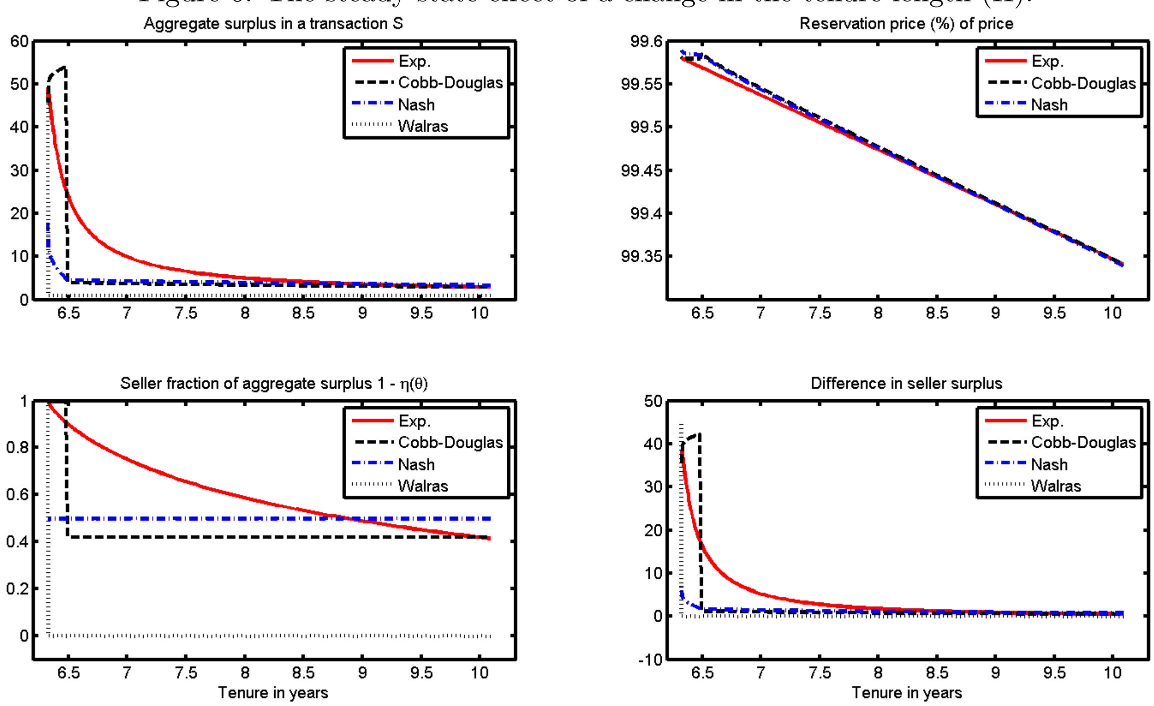


Figure 7: The steady state effect of a change in the parameter q (I).

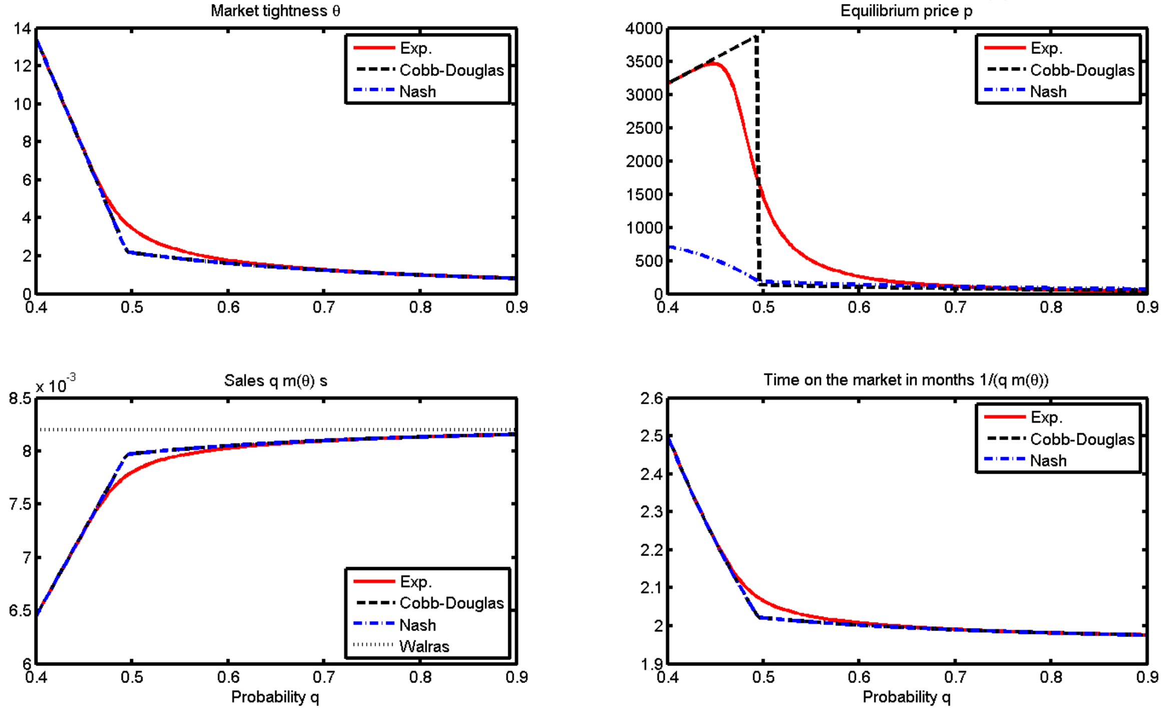
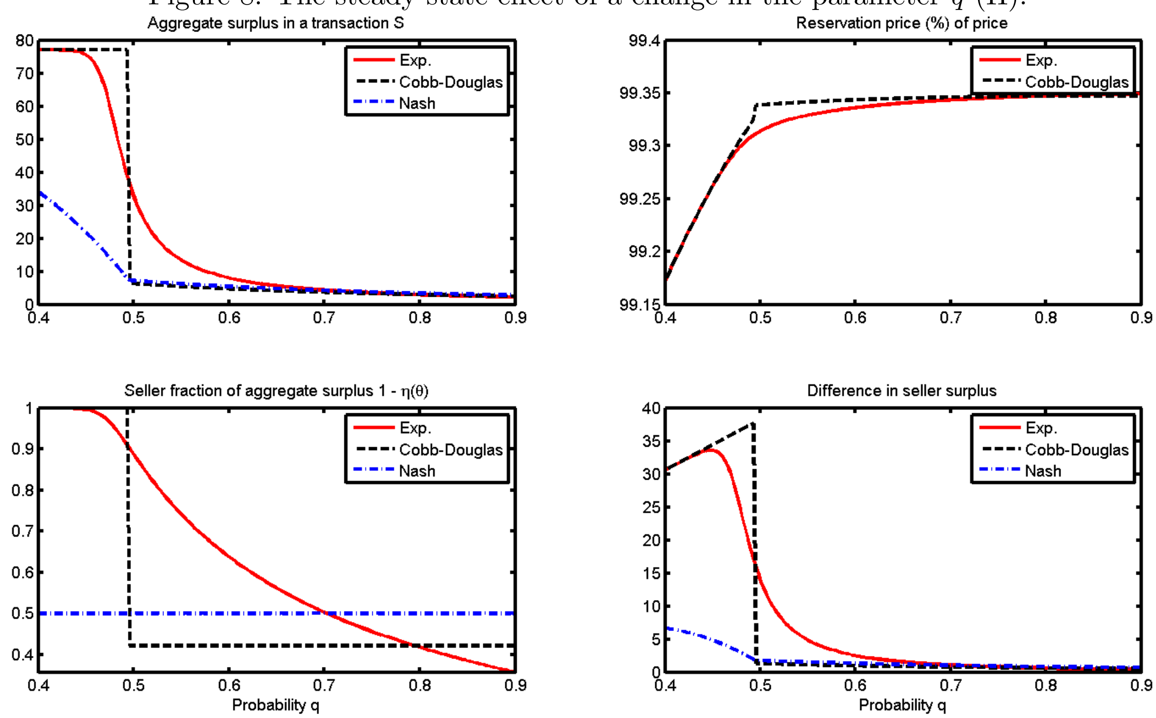


Figure 8: The steady state effect of a change in the parameter q (II).



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