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## House Prices, Sales, and Time on the Market: A Search-Theoretic Framework<sup>\*</sup>

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### Abstract

We build a search model of the housing market which captures the illiquidity of housing assets. In this model, households experience idiosyncratic shocks over time which affect how much they value their residence (e.g. the location of their job could change). When hit by a shock, households become mismatched and seek to buy a new home. Yet they take time to locate an appropriate housing unit and to sell their current home. Competitive forces are present in the housing market since, by posting lower prices, sellers increase the average number of buyer visits they get and sell their property faster. We characterize a stationary equilibrium for a fixed housing stock. We then calibrate a stochastic version of the model to reproduce selected aggregate statistics of the U.S. economy. The model is consistent with the high volatility of prices, sales and average time on the market, the positive correlation of prices and sales, and the negative correlation of prices and average time on the market observed in the data. This is not the case when we consider the perfectly competitive version of the model.

Keywords: House prices, sales, time on the market, search frictions, competitive search.

JEL Classification: R31, D83, D40

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# 1 Introduction

A household's residence is usually its largest single asset. In the US, household real net housing wealth is about half of the level of tangible financial assets held by households on average.<sup>1</sup> Despite its importance, our understanding of many aspects of the housing sector is at best incomplete. Specifically, the literature has emphasized the following stylized facts. First, prices are more volatile than GDP, displaying positive auto-correlation at annual and higher frequencies. Second, sales are even more volatile and co-move with prices (see Figure 1). Attempts to build a model of the housing market which is consistent with these facts have proven difficult. One reason in our view is that some key features of this market are hard to reconcile with the Walrasian paradigm.

First of all, houses are *illiquid* assets. They are costly to buy and sell, can be partially financed, and are not movable.<sup>2</sup> Incorporating frictions along these lines into the Walrasian set-up helps to match the observed price volatility (e.g. see Nakajima 2005, VanNieuwerburgh and Weill 2006, Sanchez-Marcos and Ríos-Rull 2007). Yet it is still hard to explain the positive co-movement of prices and sales (see Sanchez-Marcos and Ríos-Rull 2007). Moreover, average time on the market is also highly volatile and is negatively correlated with prices (Figure 1). This additional stylized fact suggests that adjustments in the housing market take place not only through prices and quantities, but also through the degree of liquidity. As noted by Krainer (2001, 2008), in “hot” real estate markets prices and sales rise and time on the market falls, and the opposite happens in “cold” real estate markets.

Search theory is a natural paradigm to study housing markets since it captures the aforementioned illiquidity of houses and the fact that the degree of liquidity may vary with market conditions. Anyone who has gone through the process knows it takes time and resources to buy and sell houses. There are costs of acquiring relevant information in each potential transaction. Buyers' valuations typically depend not only on attributes of the houses which are observable or easily verifiable (e.g. through the phone), but also on idiosyncratic features that can only be verified by visiting a house. For instance, according to the National Association of Realtors (NAR) data for 2007, on average a typical buyer took about 8 weeks to purchase a home and visited 10 units beforehand.

The search equilibrium model of owner-occupied housing in this paper builds on Wheaton

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<sup>1</sup>See the Board of Governors of the Federal Reserve System's data for 2006.

<sup>2</sup>See Díaz and Luengo-Prado (2008) for a model of tenure choice with housing adjustment costs and collateralized borrowing constraints.

(1990). In our model economy, households experience idiosyncratic preference shocks which affect how much they value their residence (e.g. the location of their job could change). When hit by a shock, households become mismatched and seek to buy a new home and sell the current one. There does not exist a centralized market where agents trade instantaneously at the market price. Instead, agents meet randomly, so sellers take time to find potential buyers and vice versa. Also, not all the units in the market are suitable for a buyer, so she may visit several units before purchasing one. We adopt the standard competitive equilibrium notion for search environments (see Montgomery 1991, Peters 1991, Moen 1997, Shimer 1996, and Burdett, Shi, and Wright 2001, among others). This is unlike Wheaton (1990) and most of the housing search literature where search is random and prices are determined by Nash bargaining or take-it-or-leave-it offers by one of the parties.<sup>3</sup> Competitive forces seem to be present in the housing market. For instance, Merlo and Ortalo-Magné (2004) provide evidence for the UK that, by listing lower prices, sellers increase the number of visits and offers they get, and sell their property faster. Similar evidence for the US is provided by Anglin, Rutherford, and Springer (2003). The competitive search equilibrium notion is consistent with this finding.

To shed light on the economic mechanism underlying movements in prices, sales and time on the market, we first consider an economy with no aggregate uncertainty and a fixed housing stock. We characterize a stationary search equilibrium, and calibrate the model economy to reproduce some selected statistics of the U.S. economy. We find, for instance, that an increase in the probability of becoming mismatched (e.g. higher mobility), which increases housing demand, increases prices and sales and reduces average time on the market. A fall in the number of homes for sale has the same effect for reasonable parameter values. Moreover, the effect on the price is quite large in both cases. Interestingly, in the latter case, sales increase even though there are fewer units for sale. The reason is that those units sell faster because time on the market falls. Notice the crucial difference with the standard demand and supply competitive model, where supply changes tend move prices and sales in opposite directions.

To study the dynamic behavior of sales, price and time on the market, we construct a stochastic version of the model where the composition of the population (mismatched versus matched households), the instantaneous utility that matched households derive from housing services and the per capita housing stock fluctuate over time. Since we do not model construction of new houses,

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<sup>3</sup>Exceptions are Carrillo (2010), and Albrecht, Gautier, and Vroman (2010).

we assume that the population and the housing stock grow stochastically in our economy so that the per capital supply of houses in our model economy mimics the properties of the time series of vacancies in the U.S. at the quarterly frequency. We take the stand that when the economy is booming, the marginal utility of owner occupied housing services increases so that buyers are willing to pay more for a suitable unit. The opposite occurs in recessions. We further assume that mobility is higher in booms than in recessions. The preference shocks faced by households are an ad-hoc way of modeling the effects of households decisions which are out of the scope of this paper. Since there is no counterpart of these demand parameters in data, our choice is to calibrate the probability of becoming mismatched and the instantaneous utility of being matched so that their statistical properties match those of durable expenditures on household equipment in the data. One reason for using household equipment instead of GDP is that our model period is a month (although we present results at the quarterly frequency) and we do not have data on GDP at the monthly frequency.

We find that the price and sales have a strong and positive correlation, whereas time on the market is negatively correlated with the price. The contemporaneous correlation of the price and sales is very close to that observed in the data, 0.51 versus 0.49. Moreover, as in the data, a high price today leads to high sales in the next period. This effect is due to two facts. First, there is a direct effect coming from the demand side. A higher probability of becoming mismatched rises the number of households seeking to buy a new home. Because there are more buyers around willing to pay a higher price for a suitable housing unit, the price and sales rise simultaneously. Moreover, the existence of search frictions implies that buyers experience longer trading delays (and sellers trade faster), so that this comovement propagates over time. Second, there is an indirect effect coming from the supply side. A decrease in the per capita supply of houses rises the price and produces a contemporaneous decrease in sales. Yet it also implies that buyers take longer to trade and sellers trade faster, and this increases the fraction of mismatched households in the future. The result is an increase in sales afterwards, and a positive correlation of current price and next period sales. Thus, a supply side shock mimics, in a way, a demand side shock. The combination of both effects is what makes the correlation of the price and sales positive and persistent over time. The combination of demand and supply changes also implies that time on the market has a negative and strong correlation with the price, as observed in the data.

The cyclical behavior of sales, price and time on the market resembles that of their counterparts

in the data: Time on the market has higher volatility than sales, and sales fluctuate more than the price. In particular, the volatility of the price is about 38 percent that of sales, whereas in the data this ratio is 26 percent. Moreover, the volatility of the price relative to that of time on the market is 15 percent in the model, as opposed to 22 percent in the data. The volatility of sales and time on the market is due to the movement in the per capita supply of houses of houses and the probability of becoming mismatched. The volatility of the price, however, is due to the existence of search and matching frictions. Indeed in the (frictionless) Walrasian version of the economy the volatility of sales is similar, but the price does not change. Finally, to check the robustness of our results we also calibrate a version of the model where prices are determined by Nash bargaining. The results are similar, except that for the price volatility which is substantially lower. This is intuitive. In our benchmark competitive search economy (which assumes an urn-ball matching process) the sharing rule depends on positively on the buyer-seller ratio. Our quantitative exercises show that the sharing rule is very sensitive to relatively small changes in this ratio. With Nash bargaining, however, the surplus sharing rule is constant.

The literature that incorporates search frictions to study the housing market is large.<sup>4</sup> Krainer (2001) builds a model where agents are also subject to idiosyncratic preference shocks and aggregate demand shocks. The model can generate a positive correlation in prices, sales and selling probabilities. A key modeling difference with respect to Krainer (2001)—aside from the competitive pricing mechanism that we assume—is that we also study the effect of aggregate shocks in the per capita supply of houses. This allows us to uncover the feedback effect (described above) that a supply shock has in the economy. Namely, a reduction in the current supply of houses is initially followed by an increase in the price and a reduction in sales, but it produces an increase in sales in the near future (as in the data). Novy-Marx (2009) builds a search model which generates a co-movement in prices, sales and selling probabilities across steady states, and where demand shocks generate a similar feedback effect. A symmetric channel is at work there, as an increase in demand decreases the number of sellers over time, reducing the speed at which buyers trade and thus increasing future demand. This channel is not present in our model because the per capita supply of houses changes exogenously. Instead it is captured in an ad-hoc way because the per capita supply of houses has a

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<sup>4</sup>See, for instance, Yavaş (1992), Williams (1995), Arnold (1999), Albrecht, Anderson, Smith, and Vroman (2007), Yui and Zhang (2007), Hendel, Nevo, and Ortalo-Magné (2009), and Albrecht, Gautier, and Vroman (2010). One key difference with this literature—and also with standard labor search models—is that in Wheaton (1990) moving involves buying a house and selling another so households are affected by frictions on both sides of the market (see also Anglin 2004). A related line of work by Ngai and Tenreyro (2009) uses a search model to explain seasonal fluctuations in prices and transactions in the U.K. and the U.S.

small negative correlation with the number of households that became mismatched in the previous period, as vacancies and durable expenditures in household equipment (and GDP) have in the data.

This paper abstracts away from many important aspects of the housing market, including credit frictions (see, for instance, Stein 1995, Genesove and Mayer 1997 and Magné and Rady 2006), real state agents (e.g. see Hendel, Nevo, and Ortalo-Magné 2009), construction (see Davis and Heathcote 2007, Glaeser and Gyourko 2006 and Head, Lloyd-Ellys, and Sun 2010), and rental housing markets. We view our model as a first step in building a theory of housing markets that is consistent with some of the market’s key stylized facts. The point we want to make is that, even if one ignores other potentially important factors, search frictions may play a key role in explaining these facts.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 characterizes the steady state. In section 4 we calibrate our basic model economy and conduct some across steady states exercises. In section 5 we present the stochastic version of the model and discuss the cyclical properties of sales, price and time on the market. Finally, section 6 concludes.

## 2 The model economy without aggregate uncertainty

### 2.1 Population, preferences and endowments

There is a measure  $N$  of infinitely-lived symmetric households. Time is discrete. Households derive utility from the services of housing units, and discount the future at rate  $\beta \in (0, 1)$ . Houses are durable assets which are indivisible (e.g. one either owns a unit or does not, but cannot own “half a unit”). Households consume the services of a single unit each period, and assign zero utility to additional units. As is standard in search and matching models, utility is transferable.

We consider an endowment economy, without aggregate uncertainty (for now). There is an exogenous housing stock with measure  $H \in (N, 2N)$ . As in Wheaton (1990), this stock is owned by the households, and each household owns either one or two units.<sup>5</sup> The measure of households who own two units is then  $H - N$ . These households will seek to sell their second unit, which yields

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<sup>5</sup>We assume that no one can own more than two units, and that the utility of being homeless is  $-\infty$ . Also, in period 1,  $H$  is distributed so some households own one and others own two units. Then that is also the case in all the periods that follow (since no one can buy a third unit and no one owning a single unit will sell it and become “homeless” no matter how high its price is).

no value to them. For simplicity, we abstract away from the rental housing market and from credit constraints, and assume that units do not depreciate over time.

The reason why households trade is that they are subject to idiosyncratic preference shocks which make them want to switch dwellings (e.g. the location of the household's job could change by a sufficient commuting distance). For simplicity, we assume that the flow (per period) utility  $v_t$  a household derives from its residence can be either high or low:  $v_t \in \{\underline{v}, \bar{v}\}$  where  $0 < \underline{v} < \bar{v}$ . We say that households are *matched* if they live in a unit which yields utility  $\bar{v}$ , and are *mismatched* otherwise (e.g. households are matched if their commuting costs are low). Each period a fraction of the matched households are hit by an idiosyncratic shock as a result of which they become mismatched. We introduce these shocks by assuming that matched households who own one unit become mismatched with probability  $\alpha \in (0, 1)$  each period.<sup>6</sup> Shocks are realized in such a way that the Law of Large Numbers holds, so  $\alpha$  is also the fraction of these households who become mismatched each period.

Households can then be in three individual states each period: matched with one unit, mismatched with one unit (seeking to buy an appropriate unit),<sup>7</sup> and matched with two units (seeking to sell the unit they do not value). We denote the measure of households in each state by  $n_t, b_t$  and  $s_t$ , and refer to these households as *non traders*, *buyers* and *sellers* respectively. Our benchmark model assumes that non traders do not participate in the housing market. This highly simplifies the equilibrium characterization because it implies that buyers are homogeneous. In Appendix A we relax this assumption. We prove in the case of a steady-state equilibrium and check in all our quantitative exercises that non traders do not participate in the housing market for reasonable parameter values. The reason is that the bilateral trade surplus is higher when sellers trade with mismatched (type- $b$ ) rather than with matched (type- $n$ ) buyers. For reasonable parameter values this difference is sufficiently high so sellers optimally choose not to trade with the  $n$ -types.

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<sup>6</sup>Households who own two units are always matched (see also Wheaton (1990)). This assumption highly simplifies the model. It also implies that households may want to hold two units to insure against the preference shocks; e.g. in addition to the current residence one may want to own an apartment at some other location if one thinks that one may eventually move there. In our quantitative exercises this insurance motive is not significant, however. The reason is that  $\alpha$  is calibrated to a very low value and it is always optimal to sell the second unit as soon as a trading opportunity arises.

<sup>7</sup>As in Wheaton (1990), we assume that households buy a new unit before selling the old one. We have also considered a variant of the model where these households must sell before they buy (in the interim while they do not own a unit they pay a cost in terms of rent or hotel fees) obtaining similar qualitative results.

Distribution of population across types satisfies

$$N = n_t + b_t + s_t, \tag{2.1}$$

and the distribution of the housing stock among the households implies

$$H = n_t + b_t + 2 s_t. \tag{2.2}$$

Equivalently,  $s_t = H - N$ , so there is a constant measure  $H - N$  of units for sale each period. The vacancy rate is the number of units for sale as a fraction of the housing stock,  $(H - N)/H$ , and will be one of the key variables in our analysis.<sup>8</sup>

## 2.2 Search and matching frictions in the housing market

To capture the illiquidity of housing assets, we assume that buyers and sellers meet bilaterally and at random in the housing market. Sellers then take time to find potential buyers, and vice versa. For simplicity, we drop all time subscripts since the matching process is the same every period.

We assume that buyers and sellers experience at most one match each period. A standard matching function,  $\mathcal{M}(b, s)$ , determines the measure of bilateral matches as a function of the measures of buyers and sellers in the market. As usual,  $\mathcal{M} : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$  is increasing in both arguments, strictly concave, homogeneous of degree one, and continuously differentiable. The total number of matches cannot exceed the number of traders in the short side of the market so  $\mathcal{M}(b, s) \leq \min\{b, s\}$ . In particular,  $\mathcal{M}(0, s) = \mathcal{M}(b, 0) = 0$ .

Since  $\mathcal{M}$  exhibits constant returns to scale, the probabilities with which buyers and sellers meet each other depend on the buyer-seller ratio (or market tightness level):

$$\theta \equiv b/s. \tag{2.3}$$

Specifically, by the Law of Large Numbers, the probability that a seller meets a buyer is

$$m(\theta) \equiv \frac{\mathcal{M}(b, s)}{s} = \mathcal{M}(\theta, 1), \tag{2.4}$$

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<sup>8</sup>Since  $H \in (N, 2N)$ ,  $(H - N)/H \in (0, 1/2)$ .



where  $m : \mathbb{R}_+ \rightarrow (0, 1)$  is strictly increasing, strictly concave and continuously differentiable, with  $m(0) = 0$  and  $\lim_{\theta \rightarrow \infty} m(\theta) = 1$ . Likewise, the probability that a buyer meets a seller is

$$m(\theta)/\theta = \frac{\mathcal{M}(b, s)}{b} = \mathcal{M}(1, \theta^{-1}), \quad (2.5)$$

The function in (2.5) is strictly decreasing and continuously differentiable, with  $\lim_{\theta \rightarrow 0} m(\theta)/\theta = 1$  and  $\lim_{\theta \rightarrow \infty} m(\theta)/\theta = 0$ . Intuitively, the higher the buyer-seller ratio, the easier it is for sellers to meet buyers and the harder it is for buyers to meet sellers. As  $\theta$  goes to infinity (zero) the probability that a seller meets a buyer goes to one (zero), and the probability that a buyer meets a seller goes to zero (one).

We assume that not all the units that buyers visit in the course of their search suit their needs. Specifically, in every random match between a buyer and a seller, the buyer's flow valuation for the seller's unit is an i.i.d. random variable which measures the quality of the match between the buyer and the unit. This valuation is realized ex-post, after the match takes place and the buyer visits the unit. Intuitively, in addition to characteristics which are easily verified over the phone or via e-mail (e.g. size, number of bedrooms/bathrooms, neighborhood, proximity to public transportation, year built, floor, ...), buyers' valuations depend also on idiosyncratic features that can only be verified by visiting and inspecting the units. For instance, some buyers may want to avoid sources of noise such as heavy traffic and loud neighbors, while other buyers may care more about light than noise. For simplicity, buyers' valuations have two possible realizations:  $\bar{v}$  with probability  $q \in (0, 1)$  (the buyer likes the unit) and 0 with probability  $1 - q$  (the buyer does not like the unit). Buyers will then visit several units— $1/q$  on average—until they find one that suits their idiosyncratic tastes.<sup>9</sup> The parameter  $q$ , together with the probability  $\alpha$  of becoming mismatched, captures the extent of the matching frictions in the housing market.

Each period buyers and sellers then experience a trading opportunity with probability

$$\pi_b(\theta) = \frac{q m(\theta)}{\theta}, \quad (2.6)$$

$$\pi_s(\theta) = q m(\theta), \quad (2.7)$$

respectively. Buyers contact a seller with probability  $m(\theta)/\theta$ , and like her unit with probability  $q$ .

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<sup>9</sup>A higher  $q$  could reflect a general improvement in the search technology like the internet, or (in an ad-hoc way) better access to credit.

Sellers contact a buyer with probability  $m(\theta)$ , who likes their unit with probability  $q$ .

### 2.3 Price determination: Competitive search

We adopt the standard notion of competitive equilibrium for search economies (see Montgomery (1991), Peters (1991), Moen (1997) and Shimer (1996), among others). The idea of this equilibrium notion is that sellers can attract more buyers and increase their matching probability by posting low prices. Specifically, before the matching process takes place, sellers compete by simultaneously posting and committing to a price. Buyers observe all the posted prices and direct their search to those sellers posting the most attractive price (possibly randomizing if they are indifferent). The set of sellers posting the same price and the set of buyers directing their search towards them then form a submarket where they meet randomly according to the matching function  $\mathcal{M}$ . When a buyer and a seller meet in a given submarket, the buyer's valuation is realized, and the buyer chooses whether to buy the unit at the posted price or not (there is no renegotiation).

As shown by Moen (1997), with ex-ante symmetric buyers and sellers and a constant return to scale differentiable matching function, the competitive search equilibrium notion is equivalent to adopting the bargain rule in Hosios (1990).<sup>10</sup> This rule says the buyer gets a share of the bilateral surplus which is equal to the elasticity of the seller's matching probability:

$$\eta(\theta) = \frac{m'(\theta)\theta}{m(\theta)}, \tag{2.8}$$

and the seller gets a share  $1 - \eta(\theta)$ . As is standard,  $\eta(\theta) \in [0, 1]$  is assumed non-increasing, so the higher the buyer-seller ratio the higher the fraction of the surplus received by sellers in equilibrium.<sup>11</sup>

While we have assumed a general matching function, the urn-ball matching process is specially

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<sup>10</sup>The derivation for our particular environment is in the Appendix. See also Díaz and Jerez (2009).

<sup>11</sup>Hosios (1990) shows that this rule attains an efficient division of the surplus in search models with endogenous entry of buyers and sellers because it allocates each trader a share of the surplus that measures their contribution to the matching process (internalizing the search externalities). In our model there are no entry decisions, and no endogenous search efforts or acceptance thresholds in a match. All matches where the buyer's realized valuation is  $\bar{v}$  will result in trade each period, as we shall see, because the total bilateral surplus is positive. Total sales are then  $q\mathcal{M}(b_t, s_t)$ , and  $b_t$  and  $s_t$  are deterministic sequences given an initial population  $(n_0, b_0, s_0)$ . Hence, the equilibrium allocation is the same regardless of the sharing rule. This will no longer be the case in extensions of the model which introduce any of the above endogenous margins.

suitable for our environment and is the process we use in the calibration of our benchmark economy:

$$m(\theta) = 1 - e^{-\theta}, \text{ and } \eta(\theta) = \frac{\theta e^{-\theta}}{1 - e^{-\theta}}. \quad (2.9)$$

Peters (1991) provides strategic micro-foundations for this matching process by showing it emerges endogenously in a finite game of competitive search as the number of traders gets large. In the limiting game, buyers get a share  $\eta(\theta) = \frac{\theta e^{-\theta}}{1 - e^{-\theta}}$  of the surplus.

A caveat of the competitive search notion is that it makes a strong commitment assumption that all transactions take place at the posted price, and there is no ex-post renegotiation. There is evidence that competitive forces are present in the housing market. Merlo and Ortalo-Magné (2004) provide evidence for the UK that, by listing lower prices, sellers increase the number of visits and offers they get. Yet there seems to be some bargaining going on as well. In their sample, for instance, bargaining sometimes reduces the sale price relative to the listed price—properties sell at about 96 percent of their current listed price. The National Association of Realtors provides a similar number for the US. Our goal in this paper is to build a simple model that we can take to the data, and is also sufficiently stylized and parsimonious to shed light on the basic economic mechanisms that underly movements in prices, sales and average time on the market in the presence of search frictions. We must then pay the price of abstracting away from the rich details of the strategic interaction between buyers and sellers.<sup>12</sup>

We do conduct some robustness checks. In particular, we calibrate the Nash bargaining version of the model (see Sections 4 and 5). While the results are similar, the benchmark model generates higher price volatility in the presence of aggregate shocks. As we have already noted, the reason is that the surplus sharing rule depends on  $\theta$  and hence on whether the housing market is a “buyer’s market” or a “seller’s market”, while with Nash bargaining this rule is constant.

A particularly interesting feature of the competitive search equilibrium notion is that, when search and matching frictions vanish, the economy limits to a Walrasian economy. This allows us to identify what search frictions add to an otherwise competitive world in terms of the magnitude of the volatilities and the co-movements of prices, sales, and time on the market.

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<sup>12</sup>See, however, Albrecht, Anderson, Smith, and Vroman (2007), Merlo, Ortalo-Magne, and Rust (2007), Caplin and Leahy (2008), Piazzesi and Schneider (2009), and Albrecht, Gautier, and Vroman (2010).

## 3 Search equilibrium

### 3.1 Characterization of equilibrium

In this Section we characterize a search equilibrium in the absence of aggregate uncertainty. First, we state the law of motion of the population distribution. Then we write down the value functions for non traders, buyers and sellers. Finally, we derive the equilibrium prices.

#### 3.1.1 The law of motion of population

Suppose that each period the total bilateral trade surplus is positive (as is the case in our quantitative exercises). Since buyers and sellers get a positive share of this surplus, there will be trade each period. Given the distribution of the population in period  $t$ ,  $(n_t, b_t, s_t)$ , the measure of non traders in period  $t + 1$  is

$$n_{t+1} = (1 - \alpha)n_t + \pi_s(\theta_t) s_t. \quad (3.1)$$

This includes those households who were non traders in period  $t$  and continue to be matched at the start of  $t + 1$  (e.g. they are not hit by a shock), as well as those sellers who sold their vacant unit in period  $t$ . Similarly, the measure of buyers in period  $t + 1$  is

$$b_{t+1} = (1 - \pi_b(\theta_t)) b_t + \alpha n_t. \quad (3.2)$$

This includes those buyers who did not trade in period  $t$  (and continue to search for a suitable unit in  $t + 1$ ), as well as those households who were non traders in period  $t$  and become mismatched at the start of  $t + 1$ . Remember that the measure of sellers is constant:

$$s_{t+1} = (1 - \pi_s(\theta_t)) s_t + \pi_b(\theta_t) b_t = H - N. \quad (3.3)$$

Those sellers who did not trade in period  $t$  and those households who bought a second unit in period  $t$  are sellers in period  $t + 1$ . Dividing (3.2) by  $s_t$ , we obtain

$$\theta_{t+1} = (1 - \pi_b(\theta_t)) \theta_t + \alpha \left( \frac{2N - H}{H - N} - \theta_t \right). \quad (3.4)$$

Knowing the law of motion (3.4) of  $\theta_t$  and the population distribution in period 0 is then enough to track down the evolution of the population distribution over time (since  $s_t = H - N, b_t = \theta_t(H - N)$  and  $n_t = N - b_t - s_t$ ). In what follows we refer to the function in (3.4) as the law of motion  $h(\cdot)$ .<sup>13</sup>

In period  $t$  sales are given by  $\pi_s(\theta_t)s_t = \pi_s(\theta_t)(H - N)$ , while average time on the market is  $1/\pi^s(\theta_t)$ . Since  $H - N$  is fixed, a trivial feature of the search model is that if average time on the market falls, sales rise and vice versa.

### 3.1.2 Value functions

We use a recursive formulation to describe the value functions and the problem solved by each agent type. The buyer-seller ratio,  $\theta$ , summarizes is the economy wide state provided there is trade every period. Denote the value functions of buyers, sellers, and non traders by  $W_b, W_s$  and  $W_n$ , respectively. Since buyers and sellers are symmetric, all bilateral transactions in the housing market are identical, so there will be a single equilibrium price  $p(\theta)$  at which all units are traded each period.

The Bellman equation of a non-trader is

$$\begin{aligned} W_n(\theta) &= \bar{v} + \beta [(1 - \alpha)W_n(\theta') + \alpha W_b(\theta')] \\ \theta' &= h(\theta). \end{aligned} \tag{3.5}$$

These households get flow utility  $\bar{v}$  from their unit and do not trade in period  $t$ . Yet they may become mismatched at the start of  $t + 1$  and seek to buy a unit during that period.

The Bellman equation of a buyer is

$$\begin{aligned} W_b(\theta) &= \pi_b(\theta) [\bar{v} - p(\theta) + \beta W_s(\theta')] + (1 - \pi_b(\theta)) [\underline{v} + \beta W_b(\theta')] \\ \theta' &= h(\theta). \end{aligned} \tag{3.6}$$

If buyers locate and purchase a suitable unit, they get flow utility  $\bar{v}$ , pay a price  $p(\theta)$ , and become sellers in  $t + 1$ . Otherwise, they get utility  $\underline{v}$  from their current unit and continue to be buyers in  $t + 1$ .

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<sup>13</sup>If there where no trade in period  $t$  then  $n_{t+1} = (1 - \alpha)n_t, b_{t+1} = b_t + \alpha n_t, s_{t+1} = H - N$ , so  $\theta_{t+1} = \theta_t + \alpha \left( \frac{2N-H}{H-N} - \theta_t \right)$ .

Finally, the Bellman equation of a seller is

$$\begin{aligned} W_s(\theta) &= \bar{v} + \pi_s(\theta) [p(\theta) + \beta W_n(\theta')] + (1 - \pi_s(\theta)) \beta W_s(\theta') \\ \theta' &= h(\theta). \end{aligned} \tag{3.7}$$

Sellers always get flow utility  $\bar{v}$  from the unit where they reside, whether they trade or not. If they meet a buyer who likes their vacant unit and sell the unit, they receive the price  $p(\theta)$  and become non traders in  $t + 1$ . Otherwise, they continue to be sellers in  $t + 1$ .

### 3.1.3 Trade surpluses and price determination

The Bellman equations (3.5)-(3.7) can be written as

$$W_n(\theta) = \bar{v} + \beta W_n(\theta) - \beta \alpha (W_n(\theta') - W_b(\theta')), \tag{3.8}$$

$$W_b(\theta) = \underline{v} + \beta W_b(\theta') + S_b(\theta), \tag{3.9}$$

$$W_s(\theta) = \bar{v} + \beta W_s(\theta') + S_s(\theta), \tag{3.10}$$

where  $\theta' = h(\theta)$ , and  $S_b(\theta)$  and  $S_s(\theta)$  represent the expected trade surpluses for buyers and sellers in period  $t$ :

$$S_b(\theta) = \pi_b(\theta) [\bar{v} - \underline{v} - p(\theta) + \beta (W_s(\theta') - W_b(\theta'))], \tag{3.11}$$

$$S_s(\theta) = \pi_s(\theta) [p(\theta) + \beta (W_n(\theta') - W_s(\theta'))]. \tag{3.12}$$

When buyers trade, their flow utility increases by  $\bar{v} - \underline{v}$  and they pay the price  $p(\theta)$ . Since they become sellers in  $t + 1$  their continuation utility also changes. Similarly, sellers receive the price  $p(\theta)$  when they trade and their continuation value changes since they become non traders in  $t + 1$ . The expressions of the expected trade surpluses in equations (3.11) and (3.12) highlight the fact that traders face a trade-off between the transaction price and the expected time it takes them to trade. Buyers prefer low prices and low levels of  $\theta$ , while sellers prefer high prices and high levels of  $\theta$ . This trade-off between prices and trading delays is key in the division of the surplus between buyers and sellers under competitive search (see Peters 1991 and Moen 1997). Note that traders care also about how their continuation values change when they trade.

When the buyer likes the unit, the bilateral surplus is the sum of the ex-post surplus of the

buyer and the seller:

$$S(\theta) = \bar{v} - \underline{v} + \beta (W_n(\theta') - W_b(\theta')), \theta' = h(\theta). \quad (3.13)$$

This surplus includes the instantaneous gain from a match,  $\bar{v} - \underline{v}$ , as well as the gain in terms of the expected change in the discounted continuation utilities of buyers and sellers (i.e., when they trade the buyer becomes a seller and the seller becomes a non trader).

In a competitive search equilibrium, the price  $p(\theta)$  is pinned down by the Hosios rule, so buyers get a share  $\eta(\theta)$  of the surplus and sellers get a share  $1 - \eta(\theta)$ :

$$\frac{\bar{v} - \underline{v} - p(\theta) + \beta (W_s(\theta') - W_b(\theta'))}{p(\theta) + \beta (W_n(\theta') - W_s(\theta'))} = \frac{\eta(\theta)}{1 - \eta(\theta)}. \quad (3.14)$$

In our model, prices will change with the economy wide state  $\theta$ . This does not imply that buyers will only buy when prices are “low” and sellers will sell when prices are “high”. As we have already noted, in all our quantitative exercises all agents choose to trade when a trading opportunity arises because the bilateral surplus is always positive. This is intuitive and has to do with the above mentioned trade-off between prices and trading delays. Even if the current price is low, a seller may choose to sell when a trading opportunity arises instead of waiting for a future higher price. If the seller waits, there is a possibility that she will not meet a buyer who likes the unit when the price is high.<sup>14</sup> Moreover, in the model, moving involves buying a unit and selling another. Periods when prices are high may be a good time to sell but they are a bad time to buy, and the opposite is true when prices are low. The option value of waiting is incorporated in the seller’s reservation price,  $p_s = \beta (W_s(\theta') - W_n(\theta'))$ . This term incorporates also the value of the vacant house as an insurance mechanism against the risk of becoming mismatched. Similarly, the buyer’s reservation price is  $p_b = \bar{v} - \underline{v} + \beta (W_s(\theta') - W_b(\theta'))$

### 3.1.4 Competitive search equilibrium

We are now ready to define a search equilibrium.

**Definition 1.** *A competitive search equilibrium is a sequence of functions  $\{\pi_b, \pi_s, W_n, W_b, W_s, S_b, S_s\}$ ,*

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<sup>14</sup>This is true even in the model with aggregate uncertainty (Section 5), where sellers locate buyers faster when prices are higher.

a law of motion  $h$  for the level of market tightness  $\theta$ , and a price function  $p$  satisfying (2.6), (2.7), (3.4), (3.8)-(3.12), and (3.14).

**Remark 1.** *It is straightforward to write the Nash bargaining version of the model. Simply replace the competitive search pricing equation (3.14) by its generalized Nash bargaining counterpart:*

$$\frac{\bar{v} - \underline{v} - p(\theta) + \beta (W_s(\theta') - W_b(\theta'))}{p(\theta) + \beta (W_n(\theta') - W_s(\theta'))} = \frac{\phi}{1 - \phi}; \quad (3.15)$$

*i.e.,  $\phi \in (0, 1)$  denotes the buyers' bargaining weight and  $1 - \phi$  is the sellers' bargaining weight.*

### 3.2 The steady state

To define a steady state we need to add to Definition 1 the condition that the composition of population is constant over time; i.e.,  $\theta = h(\theta)$ . The steady state is characterized by the following system of equations

$$s = H - N, \quad (3.16)$$

$$q m(\theta) = \alpha \left( \frac{2N - H}{H - N} - \theta \right), \quad (3.17)$$

$$b = \theta s, \quad (3.18)$$

$$N = n + b + s, \quad (3.19)$$

$$(1 - \beta)W_n = \bar{v} - \beta \alpha (W_n - W_b), \quad (3.20)$$

$$(1 - \beta)W_b = \underline{v} + \frac{q m(\theta)}{\theta} \eta(\theta) [\bar{v} - \underline{v} + \beta (W_n - W_b)], \quad (3.21)$$

$$(1 - \beta)W_s = \bar{v} + q m(\theta) (1 - \eta(\theta)) [\bar{v} - \underline{v} + \beta (W_n - W_b)], \quad (3.22)$$

$$\frac{\bar{v} - \underline{v} - p + \beta (W_s - W_b)}{p + \beta (W_n - W_s)} = \frac{\eta(\theta)}{1 - \eta(\theta)}, \quad (3.23)$$

$$\eta(\theta) = \frac{m'(\theta)\theta}{m(\theta)}. \quad (3.24)$$

All equations are self-explanatory, except perhaps for (3.17). This equation pins down the unique value of  $\theta$  that ensures that the composition of population is constant.<sup>15</sup> It says that the measure of sellers who become non traders each period,  $q m(\theta)s$ , is equal to the measure of non traders who become buyers,  $\alpha n$  (i.e., the flows in and out of the non-trading state are equal). In (3.17) both

<sup>15</sup>Remember that  $m(\theta)$  is strictly increasing with  $m(0) = 0$  and  $\lim_{\theta \rightarrow \infty} m(\theta) = 1$ , and  $N < H < 2N$ . A sufficient condition for existence is  $\alpha \frac{2N-H}{H-N} \leq q$ . The parameters calibrated in Section 4 satisfy this condition.



measures are represented as a fraction of  $s$ . Given  $\theta$  and  $s$ , we obtain the rest of the equilibrium variables using (3.18)-(3.24).

It is easy to check using (3.20) and (3.21) that the value of non traders is higher than the value of buyers at the steady state:<sup>16</sup>  $W_n > W_b$ . Equation (3.13) then implies that the bilateral surplus is positive, as we have assumed. Also, from (3.20)-(3.22), the value of sellers is highest:  $W_s > W_n$ . The equilibrium price can be expressed as

$$p = \beta (W_s - W_n) + (1 - \eta(\theta)) S(\theta); \quad (3.25)$$

i.e., the fraction of the bilateral surplus appropriated by sellers plus their reservation price.

### 3.3 The frictionless Walrasian economy

Consider now the limiting perfectly competitive situation where search frictions vanish. In this case, buyers can perfectly identify suitable units so  $q = 1$ , and matching is frictionless so

$$\mathcal{M}(b, s) = \min\{b, s\}, \quad \pi_s(\theta) = \min\{\theta, 1\}, \quad \pi_b(\theta) = \min\{1, \theta^{-1}\}. \quad (3.26)$$

This means that traders on the short side of the market trade with probability one, while traders on the long side of the market are rationed. Also, market clearing prices are such that entire bilateral surplus goes to the former traders, while the latter get zero surplus and thus are indifferent between trading or not.<sup>17</sup> The equilibrium is defined as before, except that now trading probabilities are given by (3.26) and the pricing equation (3.14) does not apply.

Consider a steady state. There are two possible cases (see Díaz and Jerez 2010 for the details). If  $\theta < 1$ , there are fewer buyers than sellers so buyers trade with probability one and sellers are rationed ( $\pi_b = 1$  and  $\pi_s = \theta$ ). Sales are then given by  $\theta(H - N)$  and time on the market is  $1/\theta$ . Since buyers are the short side of the market they appropriate the entire bilateral surplus, so the equilibrium price is  $p = \beta (W_s - W_n)$ . It can be shown that the value of buyers, non traders and sellers is equal,  $W_b = W_n = W_s$ . In particular, being mismatched has no cost since buyers can find

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<sup>16</sup> $W_n - W_b = \frac{1 - \frac{q}{\theta} \eta(\theta)}{1 - \beta + \beta \left( \frac{q}{\theta} \eta(\theta) + \alpha \right)} (\bar{v} - \underline{v}) > 0$ .

<sup>17</sup>This economy is a simple dynamic example of Gretsky, Ostroy, and Zame's (1992) competitive equilibrium formulation of the Shapley-Shubik assignment model.

a suitable property instantaneously. Also, for this reason, a second housing unit has no insurance value. Thus, the bilateral surplus is  $S = \bar{v} - \underline{v}$ , and the price is  $p = 0$ . If instead  $\theta > 1$ , buyers are rationed and sellers get all the surplus ( $\pi_s = 1$ ,  $\pi_b = \theta^{-1}$ ,  $S_s = S$  and  $S_b = 0$ ). Sales are given by  $H - N$  and time on the market is 1. In this case,  $S = \frac{\bar{v} - \underline{v}}{1 - \beta(1 - \alpha)}$  and  $p = \left(\frac{1 + \alpha\beta^2}{1 - \beta}\right)S$ .<sup>18</sup> In sum,  $\theta > 1$  is the high-price, high-surplus scenario (the case of a seller’s market), while  $\theta < 1$  is the low-price, low-surplus scenario (the case of a buyer’s market). Everything else fixed, sales are higher and time on the market is lower in the high-price scenario. To switch between a high-price and a low-price steady state,  $\theta$  must cross the threshold 1. Intuitively, in the Walrasian model there is a stark discontinuity, as the seller’s share jumps from 1 to 0 when  $\theta$  moves from values above to values below 1 but does not change otherwise. The competitive search model can be thought as “smooth” version of the Walrasian model where the sharing rule changes continuously with  $\theta$ .

## 4 Quantitative properties of the steady state

Díaz and Jerez (2009) and Díaz and Jerez (2010) study the across steady state comparative statics of our benchmark economy in detail. Here we briefly report some quantitative properties of the steady state.

### 4.1 Calibration

We assume that the model period is a month. As we have already noted, in our benchmark economy we assume an urn-ball matching process, so  $\pi_s(\theta) = q(1 - e^{-\theta})$ , and  $\pi_b = \pi_s(\theta)/\theta$ . We normalize the population size to  $N = 100$ . In the model, the average number of months it takes to buy a unit is  $1/\pi_b(\theta)$ . Moreover, after some calculations, it can be shown that average tenure length is  $(1 - \alpha)/\alpha + 1/\pi_b(\theta)$  months. The National Association of Realtors conducts an annual survey, the *Profile of Home Buyers and Sellers*. There it is reported that buyers typically search for 2 months for a home and that they plan to stay in that home for about 10 years. Also, according to the American Housing Survey the average of the ratio of houses for sale to the total stock of owner occupied housing is 1.6 percent per quarter. We use the above three observations to calibrate  $q$ ,  $\alpha$  and  $V = (H - N)/H$ . The calibration is shown in Table 1. We find that  $H = 101.61$ ,  $q = 0.79$ , and  $\alpha = 0.0084$ . We set  $\beta$  so that the discount factor is 0.96 in annual terms. Since we have no

<sup>18</sup>If  $\theta = 1$  then  $\pi_s = \pi_b = 1$  and prices are indeterminate. But this is a measure zero event in our large economy.

guidance to set values of the utility when matched and mismatched, we assume that  $\bar{v} = 1$  and  $\underline{v} = 0.1$ .

For comparison purposes, we also consider three alternative economies. In the first one, labeled as the *CSE-CD* economy, we assume a Cobb-Douglas (instead of an urn-ball) matching function:<sup>19</sup>

$$m_s(\theta) = \min \{A \theta^{1-\eta}, 1\}, m_b(\theta) = \min \{A \theta^{-\eta}, 1\}, \text{ and } \eta(\theta) = \eta \quad (4.1)$$

Unlike in the benchmark economy, the sharing rule is now constant and exogenous, so the economy is equivalent to a Nash bargaining economy. We set the value of  $\eta$  so that this rule is the same in both economies at the steady state. In the second economy, labeled as the *Nash-CD* economy, we assume  $\eta = 0.5$  instead (symmetric Nash bargaining). Given the value of  $q$  above, in each economy the value of  $A$  is set so that the model delivers the observed average number of periods it takes to buy a home, 2 months. Finally, we consider a *Walrasian* economy. In this economy  $q = 1$ , and  $\alpha$  is chosen to match the assumed tenure length. The values of the parameters of the four economies considered are shown in Table 1.

The first column of Table 2 labeled as *CSE (UB and CD)*, shows the steady state in our benchmark economy, as well as in the *CSE-CD* economy (both economies are calibrated to produce the same steady state). Note that once we calibrate to match the observed average time to buy, time on the market is implied by the assumed vacancy rate and tenure length. This is so because the law of motion of  $\theta$  in (3.18) ties the vacancy rate, the expected tenure length and time on the market. Indeed (3.18) can be written as  $\pi_s = \alpha \left( \frac{1}{V} - 2 - \frac{\pi_s}{\pi_b} \right)$ . (This is why  $\theta$  and time on the market are the same in the three search economies). Time on the market is 1.98 months. The equilibrium price, in flow terms, is 73.56 percent of the instantaneous gain of becoming matched,  $\bar{v} - \underline{v}$ . Sellers get 42 percent of total bilateral surplus.

Since the values of  $\theta$  and  $s$  are the same in the three search economies, the *Nash-CD* economy differs only in the price and in the size of the bilateral surplus. Since the sellers' share of the surplus is higher, the loss from becoming mismatched, the total surplus, and the price are also higher.

In the Walrasian economy the number of buyers is much smaller (and lower than the number of sellers, which is the same). This is because buyers always find the property they want and trade

<sup>19</sup>Because the total number of matches cannot exceed the number of agents in the short side of the market,  $\mathcal{M}(b, s) = Ab^\eta s^{1-\eta}$  if  $Ab^\eta s^{1-\eta} \leq \min\{b, s\}$ . Otherwise,  $\mathcal{M}(b, s) = \min\{b, s\}$ .

with probability one when they are on the short side of the market. Since buyers get all the surplus, the equilibrium price is zero so sellers indifferent between trading or not. The loss from becoming mismatched is much lower and so is the bilateral surplus.

## 4.2 Changes across steady states

To shed some light on the effects of search frictions, we now briefly report the long run effects of changes in the vacancy rate and mobility in the economies we consider.<sup>20</sup>

### 4.2.1 Changes in the vacancy rate

Table 3 shows the effect of an increase in the vacancy rate (i.e., an increase in the per capita number of houses) from 1.58 to 4 percent. In our benchmark economy, *CSE-UB*, the price drops from 73.56 to 4.73 percent of the instantaneous utility gain of becoming matched. There are two effects. On the one hand, given the number of buyers, an increase in the number of sellers reduces the buyer-seller ratio  $\theta$ , which in turn reduces the seller's share of the surplus. This static effect is reinforced by a second dynamic effect, which is due to the fact that an increase in the number of sellers reduces buyer time on the market. As a result, there are fewer buyers each period, which further reduces  $\theta$ , and thus the seller's share of the surplus (which falls from 42 to 13 percent). The total bilateral surplus also falls. Notice that sales fall. This may seem surprising since there are more units for sale, but note that units now take longer to sell. Another way to see this is to remember that sales are given by  $q\mathcal{M}(b, s)$  and that the increase in the number of sellers  $s$  lowers the steady-state number of buyers  $b$  from 1.63 to 1.15 percent. This is why sales fall. Figures 2 and 3 show that, in general, sales are a non-monotone function of the vacancy rate. Yet when the vacancy rate exceeds a certain threshold (1.2 percent), the fact that units take longer to sell dominates, so sales decrease with the vacancy rate. The present analysis then suggests that whenever the vacancy rate is above some threshold we can observe a positive comovement of sales and prices, as well as a negative comovement of prices and time on the market at higher frequencies.

Table 3 considers also the other two search economies. Notice that the price falls even further in the *CSE-CD* economy. This is because buyer time on the market falls more than in the benchmark

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<sup>20</sup>In Díaz and Jerez (2009) and Díaz and Jerez (2010) we show that these are the key parameters which generate co-movements in prices, sales and selling probabilities across steady states in our model.

economy (so there are fewer buyers in the new steady state). Also, in this case, the seller's share of the surplus is reduced to zero. The reason is that, with a Cobb-Douglas matching function, the search frictions faced by buyers disappear when  $\theta$  is below a certain threshold (satisfying  $A\theta^{-\eta} = 1$ ), and on that range buyers appropriate the entire surplus. This is not the case with an urn-ball matching function. Unlike in the Walrasian economy, the price is not zero because the informational friction faced by buyers, captured by the probability  $q$ , does not vanish. In the *Nash-CD* economy, the number of buyers also falls from 1.63 to 1.00, again inducing a larger fall in  $\theta$  than in our benchmark economy. Yet the percentage drop in the price is lower than in the *CSE-CD* economy. The reason is that traders continue to split the bilateral surplus equally. In this economy, the search frictions faced by buyers also disappear when  $\theta$  is below a certain threshold and at this point the seller's share is reduced to zero, but this threshold is lower than in the *CSE-CD* economy. Figure 2 shows that, in general, prices are more responsive to changes in the vacancy rate in the benchmark economy than in the other two search economies as long as the sharing rule remains constant in the latter. It also shows that, for a sufficiently high vacancy rate, the equilibrium price in the three search economies drops to the Walrasian price.

Table 3 shows that, in the Walrasian economy, the price is not affected by the increase in the vacancy rate. This is because buyers are on the short side of the market when the vacancy rate is 1.59 percent, and this continues to be so when the number of seller rises. Sales fall slightly and this is again because time on the market rises. The fact that the Walrasian price does not move for this range of the vacancy rate does not imply that it does not change across steady states. Figure 3 shows that, for values of  $V$  below a certain threshold, sellers become the short side of the market and appropriate the entire surplus.<sup>21</sup> This analysis may suggest that the equilibrium price can potentially fluctuate more in a Walrasian framework than in a search environment. Nevertheless, we need to keep in mind that the Walrasian price fluctuates when the vacancy rate is lower than 0.9 percent, which is far below the average 1.59 observed in the U.S. economy. We will discuss this issue in Section 5. Also, on that range sales *increase* with the vacancy rate, so they no longer comove with prices (see Díaz and Jerez (2010)).

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<sup>21</sup>The results hinge on the assumption that, in all transactions where the bilateral surplus positive, buyers have identical valuations. This keeps the analysis simple. With a continuum of valuations, the equilibrium price is such that the threshold type that trades gets zero surplus. This type, and thus prices and sales, may change after exogenous demand and supply changes. Yet a standard feature of the Walrasian model is that valuations are private information and all the relevant information is conveyed through prices. In recent work, Albrecht, Gautier, and Vroman (2010) build a more involved competitive search model with private valuations. See also ?).

## 4.2.2 Changes in the tenure length

Let us turn to study the effect of changes in tenure length. Table 4 shows the new steady state when  $\alpha$  increases so that the average tenure length drops from 10 to 7 years. Note first that this increases the number of mismatched households every period, that is the number of buyers. The increase in the number of buyers today increases the buyer-seller ratio and is propagated over time through the search frictions, as sellers trade faster and buyers trade more slowly. In the three search economies, the qualitative effect is the same: a fall in time on the market and an increase in sales and prices. Yet the price rises more in our benchmark economy, from 73.56 percent to 551.29 percent of the instantaneous utility gain of becoming matched. The reason is that the increase in the steady-state number of buyers and buyer time on the market is larger in this economy. In addition, the seller's share of the surplus (which is constant in the other two search economies) rises sharply from 42 to 75 percent. The bilateral surplus and the seller's reservation price are then also much higher. In the Walrasian economy, the increase in the number of buyers is again not enough for the price to change as buyers still are on the short side of the market.

## 5 Business cycle fluctuations

In this section we apply our framework to study the cyclical behavior of the housing market.

### 5.1 The facts

It is well documented that the housing market experiences cyclical movements in prices and sales, as well as time on the market and vacancies. Table 5 shows selected statistics of these variables. We report the standard deviation of each variable, in percentage terms, as well as its correlation with *GDP*. We also report their correlation with a component of durable consumption, household equipment, labeled as *e*. As a measure of vacancies we use the quarterly series labeled *for sales only* reported in the *Housing Vacancy Survey* conducted by the *U.S. Census Bureau*. This figure comprises all the vacant units out of the total housing inventory that are for sale at every quarter.<sup>22</sup> As an index of house prices we use the (seasonally adjusted) All Transactions Index reported by the *Federal Housing Finance Agency*. The measure of sales is the *Existing Single-Family Home Sales*

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<sup>22</sup>We could have used vacancies as a fraction of the stock but the properties of both time series are almost identical.

monthly figure reported by the *National Association of Realtors* and aggregated to the quarterly frequency. Finally, we take as a proxy of Time on the Market in the data the figure *Median Number of Months for Sale* reported by the *U.S. Census Bureau*. This figure is the median number of months it takes to sell a newly built housing unit. Our model cannot account for this number since there is no construction. Thus, we take the view that the volatility of this measure is a good approximation for average time to sell second hand housing units.

Notice that the price has lower volatility than vacancies, sales and time on the market (TOM hereafter) and household equipment, but higher volatility than *GDP*. The volatility of TOM is almost 7 times that of the price, while sales are about 4 times more volatile than the price. Price and sales are positively correlated with *GDP* and household equipment, their correlation being larger with the latter, 0.61 and 0.58, respectively. TOM has a negative correlation of about the same magnitude with either *GDP* and household equipment, consistently with the view that expansions coincide with “hot markets” and recessions with “cold markets”, as noted by Krainer (2008). Notice also that the price and sales have a strong positive correlation, whereas the price and TOM are negatively correlated. Let us turn now to vacancies. Notice that vacancies have a small but significant negative contemporaneous correlation with either *GDP* and household equipment. This negative correlation may be due to the fact that sales lead somewhat the cycle. Although not shown in the table the correlation of sales with next period *GDP* and household equipment is, respectively, 0.67 and 0.69. Notice also that vacancies are almost 3 times more volatile than the price but sales and TOM have higher volatility. The price and sales are both negatively correlated with vacancies, whereas TOM is positively correlated. We also report the forward correlation of vacancies with price, sales and TOM since it will be useful to compare the data with our results in that dimension.

In what follows we modify our basic environment to study cyclical fluctuations. Our goal is to assess the ability of our search framework to account for the observed cyclical behavior of sales, prices and TOM.

## 5.2 A stochastically growing economy

### 5.2.1 The structure of uncertainty

We assume that the probability of becoming mismatched,  $\alpha$ , changes stochastically. High  $\alpha$  implies that more households become mismatched every period, which increases the demand of houses. We also assume that the instantaneous flow utility of matched households,  $\bar{v}$ , varies over time according to a random process. Thus, these two shocks together imply that more households become mismatched and they obtain higher flow utility when matched if  $\alpha$  and  $\bar{v}$  are high.

Next, we assume that the vacancy rate—the per capita number of houses—changes stochastically. Specifically, we assume that both the size of population,  $N_t$ , and the stock of houses,  $H_t$ , increase randomly over time in the following way. New sellers and new non traders are born stochastically each period (but no one dies in our economy). A two dimensional random variable  $(\epsilon_t^n, \epsilon_t^s)$  specifies the number of sellers and non traders that arrive at period  $t$  as a fraction of previous population,  $N_{t-1}$ . This random variable is such that sellers and non traders never arrive at the same time (e.g. if  $\epsilon_t^n > 0$  then  $\epsilon_t^s = 0$  and vice versa). Since sellers arrive with two units and non traders arrive with one unit, the vacancy rate rises when sellers arrive and it falls when non traders arrive. This is an ad-hoc way to model the fact that new housing units arrive to the economy without modeling the construction sector.<sup>23</sup> We could have assumed that, instead of non-traders, mismatched households arrive to the economy as this also leads to a reduction in the vacancy rate. In that case, the buyer-seller ratio would be higher because the fraction of sellers in the population is lower but also because the fraction of buyers is higher, amplifying the effect on prices. We choose to be conservative and consider instead a situation where all agents arriving to the economy are matched. The realization of all aggregate shocks is full information.

### 5.2.2 Search equilibrium

At the start of period  $t$ , prior to the realization of aggregate uncertainty, the distribution of the population is given by  $(n_t, b_t, s_t)$ . Then the population and housing shocks are realized, so the

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<sup>23</sup>Ours is a model of the second hand market of houses, which do not depreciate. We are implicitly assuming that there is a market of new houses where newly arrived agents buy either one or two houses. See Head, Lloyd-Ellys, and Sun (2010) for a search model which explicitly models construction.



distribution of the population becomes:

$$\tilde{n}_t (1 + \epsilon_t^n + \epsilon_t^s) = n_t + \epsilon_t^n, \quad (5.1)$$

$$\tilde{b}_t (1 + \epsilon_t^n + \epsilon_t^s) = b_t, \quad (5.2)$$

$$\tilde{s}_t (1 + \epsilon_t^n + \epsilon_t^s) = s_t + \epsilon_t^s. \quad (5.3)$$

Simultaneously, households learn the realization of the probability of becoming mismatched at the beginning of period  $t + 1$ ,  $\alpha_t$ , as well as the realization of the utility flow of being matched,  $\bar{v}_t$ . The housing market opens and trades take place. Households have rational expectations about the buyer-seller ratio, so  $\theta_t = \tilde{b}_t/\tilde{s}_t$ . Assuming that there is trade each period, as will be the case in our calibrated economy, the distribution of population at the beginning of period  $t + 1$  is

$$n_{t+1} = (1 - \alpha_t) \tilde{n}_t + \pi_s(\theta_t) \tilde{s}_t, \quad (5.4)$$

$$b_{t+1} = \alpha_t \tilde{n}_t + (1 - \pi_b(\theta_t)) \tilde{b}_t, \quad (5.5)$$

$$s_{t+1} = \tilde{s}_t. \quad (5.6)$$

The household's problem can be written recursively. The economy wide state variable is composed by the distribution of population, summarized by the fractions of buyers and sellers,  $(b, s)$ , the shocks that affect the demand of houses,  $\Delta = (\alpha, \bar{v})$ , and the shocks that affect the per capita number of houses,  $\Sigma = (\epsilon^n, \epsilon^s)$ . Define  $\Gamma = (\Delta, \Sigma)$ . From (5.1)-(5.3), the buyer-seller ratio is a function of  $(b, s)$  and  $\Sigma$ , denoted by  $\theta = h((b, s), \Sigma)$ . We denote the law of motion of  $(b, s)$  implied by (5.1)-(5.6) by  $(b', s') = H((b, s), \Gamma)$ .

The value function of a non trader is

$$\begin{aligned} W_n((b, s), \Gamma) &= \bar{v} + \beta E_{(\Gamma'/\Gamma)} [(1 - \alpha)W_n((b', s'), \Gamma') + \alpha W_b((b', s'), \Gamma')] \\ &\quad (b', s') = H((b, s), \Gamma), \\ &\quad \Gamma = (\Delta, \Sigma). \end{aligned} \quad (5.7)$$

The value function of a buyer is

$$\begin{aligned}
W_b((b, s), \Gamma) &= (1 - \pi_b(\theta)) [\underline{v} + \beta E_{(\Gamma'/\Gamma)} W_b((b', s'), \Gamma')] + \\
&\quad \pi_b(\theta) [\bar{v} - p((b, s), \Gamma) + \beta E_{(\Gamma'/\Gamma)} W_s((b', s'), \Gamma)] \\
\theta &= h((b, s), \Sigma), \\
(b', s') &= H((b, s), \Gamma), \\
\Gamma &= (\Delta, \Sigma).
\end{aligned} \tag{5.8}$$

The value function of a seller is

$$\begin{aligned}
W_s((b, s), \Gamma) &= (1 - \pi_s(\theta)) [\bar{v} + \beta E_{(\Gamma'/\Gamma)} W_s((b', s'), \Gamma')] + \\
&\quad \pi_s(\theta) [p((b, s), \Gamma) + \beta E_{(\Gamma'/\Gamma)} W_n((b', s'), \Gamma)], \\
\theta &= h((b, s), \Sigma), \\
(b', s') &= H((b, s), \Gamma), \\
\Gamma &= (\Delta, \Sigma).
\end{aligned} \tag{5.9}$$

As in our economy without aggregate uncertainty, sellers get a share  $1 - \eta(b, s)$  of the bilateral surplus, so that the equilibrium price satisfies

$$\frac{\bar{v} - \underline{v} - p((b, s), \Gamma) + \beta E_{(\Gamma'/\Gamma)} [W_s((b', s'), \Gamma') - W_b((b', s'), \Gamma')]}{p((b, s), \Gamma) + \beta E_{(\Gamma'/\Gamma)} [W_n((b', s'), \Gamma') - W_s((b', s'), \Gamma')]} = \frac{\eta(b, s)}{1 - \eta(b, s)}. \tag{5.10}$$

### 5.3 Calibrating the cycle

Remember that our model period is a month. Thus, we are going to calibrate our economy at the monthly frequency and aggregate to the quarterly frequency. We will assume that the flow utility  $\bar{v}$ , and probability of becoming mismatched,  $\alpha$ , change according to an autoregressive process. We take this process to be the same for both variables:

$$\log(\bar{v}_{t+1}) = (1 - \rho_\Delta) \log(\bar{v}_{ss}) + \rho_\Delta \log(\bar{v}_t) + \varepsilon_t, \tag{5.11}$$

$$\log(\alpha_{t+1}) = (1 - \rho_\Delta) \log(\alpha_{ss}) + \rho_\Delta \log(\alpha_t) + \varepsilon_t. \tag{5.12}$$

The idea is that, when the economy is booming, the marginal utility of owner occupied housing services increases so that buyers are willing to pay more for a suitable unit. The opposite occurs

in recessions. We further assume that mobility is also higher in booms than in recessions. The preference shocks faced by households are an ad-hoc way of modeling the effects of households decisions which are out of the scope of this paper. We could think that in booms more households may choose to switch to a new home because of higher incomes or better access to credit.

We estimate the process in (5.12) so that it has the properties of the time series of the consumption item household equipment. We proceed this way because we think that household equipment is a reasonable proxy for housing demand. We could have used *GDP* or residential investment but, in both cases, there is no data at the monthly frequency. Figure 4 shows *GDP*, household equipment and residential investment at the quarterly frequency. Residential investment leads a bit the cycle and has the highest volatility. Thus, by using household equipment we think that we capture the main properties of the cycle without introducing too much volatility. We have detrended monthly household equipment with the Hodrik-Prescott filter using  $\lambda = 100000$  (as Shimer 2005), and find that its standard deviation is 3.21 percent and the estimated autocorrelation coefficient is  $\rho_{\Delta} = 0.94$ .<sup>24</sup> Following Tauchen and Hussey (1991), we approximate this process with a Markov process with six grid points. We set the stationary value  $\bar{v}_{ss} = 1$ , and  $\alpha_{ss} = 0.0084$ , as in Section 4. The standard deviation of the process implies relatively small changes in average tenure length at the steady state (ranging from about 9 years when  $\alpha = 0.0093$  to about 11 years for  $\alpha = 0.0076$ ). Perhaps one could argue that we are increasing the size of fluctuations in our economy by detrending with  $\lambda = 100000$  instead of using the more common value  $\lambda = 14400$ . Notice though that, while we calibrate our economy to the monthly frequency, we aggregate our results to the quarterly frequency to compare them with Table 5. Figure 5 shows the detrended time series of household equipment at the quarterly frequency. It also shows the monthly series of household equipment detrended with  $\lambda = 100000$  and aggregated to quarterly frequency, and those obtained using  $\lambda = 14400$ . Notice that the time series detrended with  $\lambda = 100000$  tracks almost exactly the evolution of the quarterly data. This is also the case for all durable expenditures and vacancies.

The last key ingredient in our analysis is the movement in vacancies. In our model the per capita supply of houses is equal to the fraction of sellers in the population,  $s/N = (H - N)/N$ . We shall assume that the exogenous processes of population and housing stock growth are such that the fraction of sellers in the population at the beginning of any period  $t$ , shown in (5.3),

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<sup>24</sup>Using the Dickey-Fuller test, we reject the existence of a unit root in all the autoregressive processes considered.

follows a stationary process. To estimate this process we use our quarterly series of vacancies (before detrending) and impute it to the monthly frequency, since we do not have monthly data. We detrend the vacancies time series with the Hodrik-Prescott filter using  $\lambda = 100000$ . We regress vacancies on household equipment and find the coefficient of the linear regression to be  $\phi = -0.3192$ . Then we take the residuals of the regression and estimate an autoregressive process. The standard deviation of the residual is 6.65 percent and its estimated autocorrelation coefficient is  $\rho_\vartheta = 0.8564$ . Thus, vacancies are represented by the process

$$\log(s_{t+1}) = \log(s_{ss}) + \phi \log\left(\frac{\bar{v}_t}{\bar{v}_{ss}}\right) + \vartheta_t, \quad (5.13)$$

$$\vartheta_{t+1} = \rho_\vartheta \vartheta_t + \epsilon_t. \quad (5.14)$$

We approximate the autocorrelated component with a Markov chain with five points. The implied change in the vacancy rate ranges from almost 1.3 to about 2 percent of the housing stock.

## 5.4 Results

To understand the effect of search frictions on the cyclical properties of our economy, we compare it with a Walrasian economy. It will be illustrative to first study an environment where there are only demand shocks, and another where there are only supply shocks.

**The effect of demand shocks** Table 6 shows the cyclical properties of the two economies. Given our calibration, in the Walrasian economy buyers are always on the short side of the market. Hence, the volume of sales is equal to the number of buyers. Note that the volatility of sales is almost identical to that of  $\alpha$ . This is so because the number of buyers is just the number of non traders that become mismatched. In this economy, buyers extract all the surplus and pay the sellers' reservation price. Since we have assumed that the utility of a second housing unit does not change (e.g. it is zero), this price is constant over time.

In our benchmark search economy, the volatility of the number of buyers is 1.62 times that in the Walrasian economy. The reason is that the probability of buying displays substantial volatility in the former economy (and it is constant, and equal to one, in the latter). However, the volatility of sales is similar in both economies. This is because the probability of buying is counter cyclical

in the search model, which dampens the volatility of sales. Also, in the presence of demand shocks, the search model delivers a ratio of sales volatility to price volatility of 2.7, whereas in the data this ratio is 3.8. Note also the positive correlation of the demand proxy,  $\alpha$ , with prices and sales and its negative correlation with time on the market, all much higher than in the data. The price is positively correlated with sales because higher sales are a result of higher mismatch. Intuitively, higher  $\alpha$  implies that more households become mismatched, which rises sales and the price. Since it gets harder for buyers to find a suitable housing unit, this increase in demand is propagated to the following periods. This produces a high and persistent positive correlation between the price and sales.

Table 6 shows that, on average, the seller’s reservation price is about 99 percent of the equilibrium price, and that it displays a similar (though slightly lower) volatility. Thus, the most important source of price volatility is the changes in the seller’s reservation price. The volatility of the bilateral surplus,  $S$ , and the seller’s share,  $1 - \eta(\theta)$ , are higher, 9.21 and 2.12 percent, respectively. Notice that in the Walrasian economy, this reservation price is constant. This is because sellers do not gain anything by postponing trade as the risk of becoming mismatched has no cost.

**The effect of supply shocks** Table 6 also shows the properties of both economies when there are only supply shocks. In the Walrasian economy, an increase in the current fraction of sellers has a negligible impact on per capita sales today. Yet it also implies that the fraction of mismatched households will be lower in the future. Hence, an increase in current supply implies lower sales next period—which is line with the data, as shown in Table 5. Finally, a volatility of the per capita supply of houses of 7.84 percent produces a volatility of per capita sales of 1.53 percent. Again, the price is equal to the sellers’ reservation price and does not change.

In the search economy, when the fraction of sellers in the population raises there is an increase in sales today because the buyer-seller ratio decreases, and this decreases buyer time on the market. This effect, coupled with the fact that the higher supply of houses implies fewer mismatched households in the future, implies lower sales in the future (as in the data). The volatility of sales is 3.75 times that of the price, so it is higher than in the Walrasian economy. As we have already noted, in the Walrasian economy, sales fluctuate because a change in vacancies today changes the number of mismatched households tomorrow. In the search economy, sales fluctuate for this reason, but also because the increase in vacancies today implies shorter trading delays for buyers, which

also propagate over time.

Finally, notice that supply shocks produce a negative contemporaneous correlation of the price and sales. This is the typical supply side effect. Yet the correlation of the price and sales of the following period is positive. This is due to the feedback effect of the supply shock on the fraction of buyers in the economy.

**Correlated demand and supply shocks** Table 7 shows the properties of the search and the Walrasian economy in the presence of both demand and supply shocks. In the Walrasian economy the volume of sales equals the number of buyers. Thus, the correlation of sales with the demand shock is almost one and the correlation of sales and vacancies mimics the correlation of the demand shock and the supply shock. The model produces a correlation of sales and the demand shock, 0.91, much higher than the correlation of sales and household equipment in the data, 0.58. This is not the case in our search economy, where the correlation of sales and the demand shock is 0.68. Remember that sales not only depend on the number of buyers but also on buyer time on the market, which dampens the movement in sales. In the Walrasian economy, the correlation of sales and vacancies is -0.14, almost the assumed correlation between the supply and the demand shock, -0.16. In our search economy, this contemporaneous correlation is positive for the reasons already mentioned. Note that the Walrasian price does not move, as again buyers are always on the short side of the market.<sup>25</sup>

In the search economy, the volatility of sales is a bit higher, 4.75 versus 4.16, whereas the volatility of the price is 1.82. Sales are then 2.6 times more volatile than the price, whereas in the data they are 4 times more volatile than the price. Moreover, time on the market is almost 4.6 times more volatile than the price, as opposed to the factor of 7 in the data. The correlation of sales and the price is 0.51, while it is 0.49 in the data. The correlation of the price today and sales tomorrow is too high compared to the data, 0.72 versus 0.37. The reason is that sales are too persistent in our model. The fact that buyers either like the unit or not implies that, as long as the equilibrium price is lower than the buyers' reservation price, a high price does not discourage demand. As noted above, the correlation of sales and  $\alpha$  is 0.68 in our search economy, and 0.58

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<sup>25</sup>Modifying the calibration so, when the economy is booming, the vacancy rate drops below an unrealistically low threshold can generate price movements. Yet, when we do so, the model generates a negative correlation between the price and both sales today and sales tomorrow suggesting that the Walrasian model has a hard time explaining the positive correlation of the price and sales. See Díaz and Jerez (2010). Whether one can modify the Walrasian model (e.g. by adding other frictions) to explain the facts we focus on is an open question.

in the data. The Walrasian economy, while producing similar volatilities in sales and time on the market, produces zero price volatility and a too high correlation between sales and  $\alpha$ .

We have also analyzed the search economy with a Cobb-Douglas matching function (shown in Table 1). The main difference is that price volatility is much lower; i.e., 0.86 instead of 1.82 when  $\eta = 0.5$ . Our results are then robust to different specifications of the matching process, but the benchmark model yields higher price volatility. It also relies on solid micro-foundations of a competitive search market.

In summary, in our model search frictions by themselves do not add extra volatility to sales with respect to the Walrasian economy, since typically number of buyers and time to buy should be negatively correlated over the cycle. The key effect is on the price. In our benchmark economy, price volatility is about 40 percent that of sales, whereas in the data this ratio is about 26 percent. With a Cobb-Douglas matching function, this ratio goes down to 18.15 percent. In the absence of search frictions, it is zero. Search frictions do not only rise price volatility with respect to a Walrasian setting but, more importantly, they help to explain the observed significant positive and persistent correlation between sales and prices. The key factor to explain this fact is that cyclical movements are propagated over time through two different channels. First, search frictions imply that an increase in number of buyers will reduce the probability of buying (and increase the probability of selling), which introduces persistence in the demand side. Second, these frictions also imply that a fall in vacancies today will eventually raise the number of buyers and hence housing demand.

## 6 Final comments

In this paper we have studied the dynamics of house prices, sales and time on the market in a search model of the housing market where households experience idiosyncratic preference shocks that generate turnover. Competitive forces are present in the housing market since, by posting lower prices, sellers increase the average number of buyer visits they get and sell their property faster. We have characterized a stationary equilibrium for a fixed housing stock. We then have calibrated a stochastic version of the model to reproduce selected aggregate statistics of the U.S. economy. The model is consistent with the high volatility of prices, sales and average time on the market, the positive correlation of prices and sales, and the negative correlation of prices and average time

on the market observed in the data. This is not the case when we consider the Walrasian version of the model. This is so because there are no other frictions in our economy aside from search and matching frictions, which are absent in a Walrasian framework. One could argue that we have given the Walrasian framework little chance to match the features of the housing market in the data, but we have isolated search and matching frictions from any other friction that may operate in the economy to better understand their effects.

To focus our attention on the price formation mechanism, we have assumed that the per capita supply of houses changes exogenously. We also have also ignored the fact that households may have ex-ante heterogeneous preferences. Future extensions of the model could incorporate construction and heterogeneity. Our simple model brings to light how search and matching frictions produce delays in the trading process which affect the volatility of prices, sales and time on the market and the co-movement of these variables.



## Appendix

### A Allowing non traders to participate in the housing market

In this section, we let non traders become potential buyers. We refer to them as type- $n$  buyers, and to the mismatched buyers as type- $b$  buyers. To simplify the notation, we omit time subscripts and use primes to denote the following period.

Denote the fraction of type- $j$  agents who trade in the current period by  $\pi_j$  for  $j = n, b, s$ . The law of motion of  $n$  in (3.1) is replaced by:

$$n' = (1 - \alpha)(1 - \pi_n)n + \pi_s s,$$

since the flow out of this state now includes the mass of type- $n$  agents who buy a second unit. The law of motion of  $b$  is as before and again  $s = H - N$ .

The description of a competitive search equilibrium is as in Section 2.3 (see also Moen 1997). Yet now more than one submarket may be active in equilibrium, so not all units will necessarily be traded at the same price. Each submarket is characterized by a pair  $(\theta, p)$ , where  $\theta$  is the buyer-seller ratio and  $p$  is the price posted by sellers in that submarket. Matching is anonymous, so if different buyer types choose to trade in the same submarket they will be matched with identical probability.

Let  $\Omega \subset R_+ \times R$  denote the set of submarkets that are active in equilibrium, with generic element  $\omega = (\theta, p)$ . Since some agents may choose not to trade in equilibrium, we introduce a fictional submarket  $\omega^0 = (\theta^0, p^0)$ . Choosing to participate in this submarket is equivalent to not trading (e.g.  $p^0 = 0$  and  $m(\theta^0) = m(\theta^0)\theta^0 = 0$ ). The set  $\Omega$  then includes the submarkets which are joined by positive masses of both buyers and sellers (e.g. with  $\theta > 0$ ) and, if a positive mass of agents choose not to trade, it also includes  $\omega^0$ .<sup>26</sup> The key to the competitive search process is that all agents have rational expectations about the measure of buyers that will be attracted by each posted price  $p$ , and hence about the buyer-seller ratio  $\theta$  the offer will generate. In a competitive search equilibrium the offers posted by sellers must be such that no seller has an incentive to post a deviating offer (see below).

The value functions of the agents depend on the submarket they visit in equilibrium, but for notational convenience this dependency is suppressed. Equilibrium will force the value of all active submarkets to be equal for each agent type. The Bellman equation of type- $n$  agents is

$$W_n = \bar{v} + \frac{qm(\theta)}{\theta}[-p + \beta W'_s] + \left(1 - \frac{qm(\theta)}{\theta}\right) \beta[(1 - \alpha)W'_n + \alpha W'_b].$$

These agents get flow utility  $\bar{v}$  for their residence. If they visit submarket  $(\theta, p)$ , they trade with probability  $qm(\theta)\theta$ , in which case they pay  $p$ , get zero value for the unit and become sellers next period. Otherwise, with probability  $\alpha$ , they become mismatched next period. We may rewrite  $W_n$

<sup>26</sup>Note that if sellers trade in more than one submarket  $\omega \in \Omega$ , then by the Law of Large Numbers the fraction of sellers that trade in equilibrium is  $\pi_s = \frac{\sum_{\omega \in \Omega} \pi_{s\omega} s_\omega}{H - N}$  where  $\pi_{s\omega}$  is the probability with which sellers trade in submarket  $\omega$  and  $s_\omega$  is the measure of sellers in that submarket. Similarly for the other agents.

as

$$W_n = \bar{v} + \beta[(1 - \alpha)W'_n + \alpha W'_b] + S_n(\theta, p),$$

where is the expected trade surplus of a type- $n$  who chooses to trade in submarket  $(\theta, p)$ :

$$S_n(\theta, p) = \frac{qm(\theta)}{\theta} [-p + \beta(W'_s - (1 - \alpha)W'_n - \alpha W'_b)]. \quad (\text{A.1})$$

Similarly, for type- $b$  buyers and sellers,

$$W_b = \underline{v} + \beta W'_b + \max_{(\theta, p) \in \Omega} S_b(\theta, p),$$

$$W_s = \bar{v} + \beta W'_s + \max_{(\theta, p) \in \Omega} S_s(\theta, p), \quad (\text{A.2})$$

$$S_b(\theta, p) = \frac{qm(\theta)}{\theta} [\bar{v} - \underline{v} - p + \beta(W'_s - W'_b)], \quad (\text{A.3})$$

$$S_s(\theta, p) = qm(\theta) [p + \beta(W'_n - W'_s)]. \quad (\text{A.4})$$

It is convenient to decompose  $\Omega$  into two subsets describing the sets of submarkets where the two buyer types participate:  $\Omega = \Omega_b \cup \Omega_n$ . As shown by Peters (1997), in equilibrium, different buyer types must trade in different submarkets if their marginal rates of substitutions between  $\theta$  and  $p$  are different<sup>27</sup>. This is the case in our model, where type- $b$  buyers are willing to pay a higher price than type- $n$  buyers to marginally increase the probability of being matched (see below). Hence, in equilibrium,  $\Omega_b|\{\omega^0\}$  and  $\Omega_n|\{\omega^0\}$  are disjoint sets.

In a competitive search equilibrium  $\Omega_b$  and  $\Omega_n$  must satisfy the following. First, since all sellers are ex-ante identical and are free to choose the submarket they visit (including  $\omega^0$ ), they must receive a common non-negative expected surplus  $\bar{S}_s$  in all submarkets. The same is true for all type- $n$  and type- $b$  buyers, who receive  $\bar{S}_n$  and  $\bar{S}_b$  respectively. Second, each submarket  $\omega^j = (\theta^j, p^j) \in \Omega_j|\{\omega^0\}$  must solve the following program:

$$\bar{S}_s = \max_{(\theta^j, p^j) \in R_+ \times R} S_s(\theta^j, p^j) \text{ s.t. } S_j(\theta^j, p^j) = \bar{S}_j, \quad (\text{A.5})$$

for  $j = b, n$ . In words, if both sellers and type- $j$  buyers choose submarket  $\omega^j \neq \omega^0$ , then  $\omega^j$  must maximize the expected surplus of the sellers while ensuring that type- $j$  buyers get their common expected surplus  $\bar{S}_j$ . This follows from a standard arbitrage argument, which is omitted here (e.g. see Peters (1997)).

We focus on the interesting case where  $\bar{S}_j, \bar{S}_s > 0$ , so there is trade between sellers and type- $j$  buyers in equilibrium. Then (A.5) is a convex program with a unique and interior solution (i.e.,  $\Omega_j$  is a singleton) characterized by the tangency condition:<sup>28</sup>

$$\frac{\frac{\partial S_j(\theta^j, p^j)}{\partial \theta^j}}{\frac{\partial S_j(\theta^j, p^j)}{\partial p^j}} = \frac{\frac{\partial S_s(\theta^j, p^j)}{\partial \theta^j}}{\frac{\partial S_s(\theta^j, p^j)}{\partial p^j}}. \quad (\text{A.6})$$

<sup>27</sup>That is,  $\frac{\partial S_b(\theta, p)/\partial \theta}{\partial S_b(\theta, p)/\partial p} \neq \frac{\partial S_n(\theta, p)/\partial \theta}{\partial S_n(\theta, p)/\partial p}$ .

<sup>28</sup> $\bar{S}_j > 0$  in equilibrium only if program (A.5) is solvable. Substituting  $p^j$  as a function of  $\theta^j$  from the constraint into the objective function yields a strictly concave function (since  $m(\theta)\theta$  is strictly concave). Since  $\bar{S}_s, \bar{S}_j > 0$ , the unique solution must satisfy  $0 < \theta^j < \infty$

For  $j = b, n$ , differentiating (A.1), (A.3) and (A.4), substituting into (A.6), and rearranging yields

$$\frac{\bar{v} - \underline{v} - p^b + \beta(W'_s - W'_b)}{p^b + \beta(W'_n - W'_s)} = \frac{\eta(\theta^b)}{1 - \eta(\theta^b)}, \quad (\text{A.7})$$

$$\frac{-p^n + \beta(W'_s - (1 - \alpha)W'_n - \alpha W'_b)}{p^n + \beta(W'_n - W'_s)} = \frac{\eta(\theta^n)}{1 - \eta(\theta^n)}. \quad (\text{A.8})$$

It is direct to check that the ratio of the left-hand side of (A.7) and (A.8) is equal to the ratio of the marginal rates of substitution of type- $b$  and type- $n$  buyers, and is greater than one. Hence, there are at most two active submarkets,  $\omega^b$  and  $\omega^n$ , and in both the division of the surplus is determined by the Hosios rule. The total bilateral surplus in the two submarkets is:

$$S^{sb} = \bar{v} - \underline{v} + \beta(W'_n - W'_b), \quad (\text{A.9})$$

$$S^{sn} = \beta\alpha(W'_n - W'_b). \quad (\text{A.10})$$

If  $W'_n - W'_b > 0$  then  $S^{sb} > S^{sn} > 0$ , so there is trade in at most two submarkets and the gains from trade are higher in submarket  $\omega^b$ . Remember that, for both submarkets to be active, sellers must get the same expected surplus in both submarkets (see below). If  $\frac{-(\bar{v}-\underline{v})}{\beta} < W'_n - W'_b \leq 0$ , then  $S^{sb} > 0 \geq S^{sn}$ , so only  $\omega^b$  is active.<sup>29</sup> If  $W'_n - W'_b \leq \frac{-(\bar{v}-\underline{v})}{\beta}$ , then  $S^{sb}, S^{sn} < 0$  so there is no trade.

The following result is intuitive. If type- $n$  buyers trade in equilibrium so do type- $b$  buyers.

**Lemma App. 1.** *Suppose  $S^{sb}, S^{sn} > 0$ . If  $\Omega^n \setminus \{\omega^0\}$  is not empty then so is  $\Omega^b \setminus \{\omega^0\}$*

*Proof.* Suppose not. Then the sellers' expected surplus is  $qm(\theta^n)(1 - \eta(\theta^n))S^{sn}$  where  $\theta^n = \frac{n}{H-N}$ . Suppose a small (negligible) mass of sellers post a deviating offer  $p^b$  which implies a negligible but positive ex-post surplus for type- $b$  buyers. That is,  $p^b$  slightly below but close to  $S^{sb} + \beta(W'_s - W'_n)$ . The deviating offer would attract all type- $b$  buyers (who currently get zero surplus). It would not attract type- $n$  buyers because it involves a higher price  $p^b > p^n = (1 - \eta(\theta^n))S^{sn} + \beta(W'_s - W'_n)$  and a negligible probability of trading. The deviating sellers would then trade with probability close to one and get almost the entire surplus, so their expected payoff is close to  $qS^{sb} > qm(\theta^n)(1 - \eta(\theta^n))S^{sn}$ ; a contradiction.  $\square$

Formally, the set of active submarkets  $\Omega \subset R_+ \times R$  is characterized as follows,

$$\Omega = \begin{cases} \{\omega^0\} & \text{if } W'_n - W'_b \leq \frac{-(\bar{v}-\underline{v})}{\beta} \\ \{\omega^b, \omega^n\} & \text{if } W'_n - W'_b > 0 \text{ and} \\ & (1 - \eta(\theta^b))[\bar{v} - \underline{v} + \beta(W'_n - W'_b)] = (1 - \eta(\theta^n))\beta\alpha(W'_n - W'_b) \\ \{\omega^b, \omega^0\} & \text{otherwise} \end{cases}$$

The first case is the autarky case. The third case is the one in the benchmark model:  $\theta^b = \frac{b}{H-N}$  and  $p^b = \beta(W'_s - W'_n) + (1 - \eta(\theta^b))[\bar{v} - \underline{v} + \beta(W'_n - W'_b)]$ . In the second case, the seller's indifference

<sup>29</sup>If the total bilateral surplus is zero, we assume agents choose not to trade.

condition must hold, and  $\omega^b$  and  $\omega^n$  are characterized by

$$\begin{aligned}\theta^j &= \frac{j}{s^j}, j = n, b, \text{ where } s^b + s^n = H - N, 0 < s^b < H - N, \\ p^b &= \beta(W'_s - W'_n) + (1 - \eta(\theta^b))[\bar{v} - \underline{v} + \beta(W'_n - W'_b)], \\ p^n &= \beta(W'_s - W'_n) + (1 - \eta(\theta^n))\beta\alpha(W'_n - W'_b).\end{aligned}$$

Since  $p^b \neq p^n$ ,  $\theta$  must be higher in the submarket with lower prices. The reason some sellers join this submarket is that average time on the market is also lower there.

## A.1 The steady state

We now derive conditions under which only type- $b$  buyers trade in the steady state. We thus ignore the autarky case. A steady state is as an array  $\{b, n, s^n, s^b, \theta^b, \theta^n, S^{sb}, S^{sn}, W_s, W_b, W_n, p^b, p^n\} \in \mathfrak{R}_+^{13}$  which satisfies the following system of equations:

$$N = n + b + s^n + s^b \quad (\text{A.11})$$

$$s^n + s^b = H - N \quad (\text{A.12})$$

$$\theta^j = \begin{cases} \frac{j}{s^j} & \text{if } s^j > 0 \\ \theta^0 & \text{otherwise} \end{cases} \quad j = b, n. \quad (\text{A.13})$$

$$\alpha n + (1 - \alpha)n \frac{qm(\theta^n)}{\theta^n} = qm(\theta^n)s^n + qm(\theta^b)s^b \quad (\text{A.14})$$

$$(1 - \beta)W_s = \bar{v} + qm(\theta^j)(1 - \eta(\theta^j))S^{sj} \text{ if } s^j > 0 \quad (\text{A.15})$$

$$(1 - \beta)W_b = \underline{v} + \frac{qm(\theta^b)}{\theta^b}\eta(\theta^b)S^{sb} \quad (\text{A.16})$$

$$(1 - \beta)W_n = \bar{v} - \beta\alpha(W_n - W_b) + \frac{qm(\theta^n)}{\theta^n}\eta(\theta^n)S^{sn} \quad (\text{A.17})$$

$$S^{sb} = \bar{v} - \underline{v} + \beta(W_n - W_b) \quad (\text{A.18})$$

$$S^{sn} = \beta\alpha(W_n - W_b) \quad (\text{A.19})$$

$$p^b = (1 - \eta(\theta^b))S^{sb} + \beta(W_s - W_n) \quad (\text{A.20})$$

$$p^n = (1 - \eta(\theta^n))S^{sn} + \beta(W_s - W_n) \quad (\text{A.21})$$

$$\eta(\theta) = \frac{-m'(\theta)\theta}{m(\theta)} \quad (\text{A.22})$$

If  $s^n = 0$ , this system reduces to that in Section 3.

We first show that the gains from trade are positive for the two buyer types.

**Lemma App. 2.** *If  $S^{sb} > 0$  then  $S^{sb} > S^{sn} > 0$ .*

*Proof.* Suppose  $S^{sb} > 0 \geq S^{sn}$ . Then the equilibrium characterization is that in Section 3. There  $W_n - W_b > 0$ , which combined with (A.9) and (A.10), implies  $S^{sb} > S^{sn} > 0$ ; a contradiction.  $\square$

Suppose two submarkets are active in equilibrium:  $s^b, s^n > 0$ . From (A.11)-(A.13),

$$s^b = \frac{(H - N)\theta^n - (2N - H)}{\theta^n - \theta^b}, \quad (\text{A.23})$$

$$s^n = \frac{(2N - H) - (H - N)\theta^b}{\theta^n - \theta^b}. \quad (\text{A.24})$$

Plugging (A.13) for  $j = n, b$ , and (A.23)-(A.24) into condition (A.14) (which ensures that the flow in and out of the non-trading state are equal) gives:

$$\frac{\alpha[\theta^n - qm(\theta^n)]}{qm(\theta^b)} = \frac{(H - N)\theta^n - (2N - H)}{(2N - H) - (H - N)\theta^b}. \quad (\text{A.25})$$

Now, using (A.16)-(A.19), we obtain

$$W_n - W_b = \frac{(\bar{v} - \underline{v})[1 - \frac{qm(\theta^b)}{\theta^b}]\eta(\theta^b)}{1 - \beta(1 - \alpha) - \beta\alpha\frac{qm(\theta^n)}{\theta^n}\eta(\theta^n) + \beta\frac{qm(\theta^b)}{\theta^b}\eta(\theta^b)}. \quad (\text{A.26})$$

Substituting (A.18), (A.19) and (A.26) into the sellers' indifference condition implied by (A.15),

$$\frac{m(\theta^b)(1 - \eta(\theta^b))}{m(\theta^n)(1 - \eta(\theta^n))} \left[ \frac{1 + \beta\alpha[1 - \frac{qm(\theta^n)}{\theta^n}\eta(\theta^n)]}{\beta[1 - \frac{qm(\theta^b)}{\theta^b}\eta(\theta^b)]} \right] = \alpha. \quad (\text{A.27})$$

If there are finite values of  $\theta^b$  and  $\theta^n$  solving (A.25) and (A.27),  $s^b, s^n > 0$ .<sup>30</sup> Otherwise,  $s^n = 0$ .

Proposition 1 derives sufficient conditions for  $s^n = 0$ . Under these conditions, the seller's indifference condition in (A.27) does not hold even if  $\theta^n$  becomes arbitrarily large ( $s^n$  is close to zero) so sellers meet type- $n$  buyers almost instantaneously and extract almost all the surplus from them. The reason is that the total bilateral surplus in those transactions is too low compared to that generated in transactions with type- $b$  buyers.

**Proposition App. 1.** *Let  $\theta^*$  be the steady-state value of  $\theta$  in the benchmark model (i.e., solving (3.17)). If  $m(\theta^*)(1 - \eta(\theta^*)) > \alpha$  then type- $n$  agents do not trade in a steady state equilibrium.*

*Proof.* Sellers are indifferent between two active submarkets iff (A.27) holds. The first term in the left-hand side of (A.27) is increasing in  $\theta^b$  and decreasing in  $\theta^n$ . The second term (which is equal to  $\alpha S^{sb}/S^{sn}$ ) is decreasing in  $\theta^b$  and increasing in  $\theta^n$ . The left-hand side of (A.27) then is not monotone in  $\theta^b$  and  $\theta^n$ . To make sellers more willing to trade with type- $n$  buyers this expression should be as low as possible. On the one hand, decreasing  $\theta^b$  and increasing  $\theta^n$  makes the trading probability and the surplus extracted from these buyers relatively higher. On the other hand, increasing  $\theta^b$  and decreasing  $\theta^n$  makes the bilateral surplus of trading with these buyers relatively higher. Since  $\theta^b$  and  $\theta^n$  must satisfy the steady state condition (A.25), a sufficient condition for  $s^n = 0$  in the steady state is that the value of minimizing the left-hand side of (A.27) subject to (A.25) is higher than  $\alpha$ .

<sup>30</sup>Given these values,  $s^b, s^n, b, n, W_n - W_b, S^{sb}, S^{sn}, W_s, W_b, W_n, p^b$  and  $p^n$  are calculated recursively using (A.23),(A.24), (A.13), (A.26), (A.18), (A.19), (A.15), (A.16), (A.20) and (A.21).

A stronger sufficient condition can be derived as follows. First,

$$\frac{m(\theta^b)(1 - \eta(\theta^b))}{m(\theta^n)(1 - \eta(\theta^n))} \left( \frac{\alpha S^{sb}}{S^{sn}} \right) = \frac{m(\theta^b)(1 - \eta(\theta^b))}{m(\theta^n)(1 - \eta(\theta^n))} \left[ \frac{\bar{v} - \underline{v}}{\beta(W_n - W_b)} + 1 \right] > \frac{m(\theta^b)(1 - \eta(\theta^b))}{m(\theta^n)(1 - \eta(\theta^n))} \quad (\text{A.28})$$

because  $\frac{\bar{v} - \underline{v}}{W_n - W_b} > 0$ . Moreover,

$$\frac{m(\theta^b)(1 - \eta(\theta^b))}{m(\theta^n)(1 - \eta(\theta^n))} > \lim_{\theta^n \rightarrow \infty, \theta^b \rightarrow \theta^*} \frac{m(\theta^b)(1 - \eta(\theta^b))}{m(\theta^n)(1 - \eta(\theta^n))} = m(\theta^*)(1 - \eta(\theta^*)), \quad (\text{A.29})$$

where  $\theta^*$  solves (3.17). The inequality follows from the fact that  $\frac{m(\theta^b)(1 - \eta(\theta^b))}{m(\theta^n)(1 - \eta(\theta^n))}$  is minimized by letting  $s^n \rightarrow 0$  and  $s^b \rightarrow H - N$  so  $\theta^n \rightarrow \infty$  and  $\theta^b \rightarrow \theta^*$  from (A.11)-(A.13) (i.e., allocating a negligible mass of sellers to trade with type- $n$  buyers and the rest to trade with type- $b$  buyers). Hence it is enough to check that  $m(\theta^*)(1 - \eta(\theta^*)) > \alpha$ .  $\square$

It then suffices that in the steady-state of the benchmark model sellers get a non-negligible fraction of the surplus, and take a sufficiently short time to locate a buyer relative to the time it takes to become mismatched ( $\alpha$  is sufficiently low relative to  $m(\theta)$ ). This condition holds in our calibration in Section 4 because in the data the value of  $\alpha$  is very low. Only if the vacancy rate and thus time on the market are very large would type- $n$  agents trade in equilibrium (e.g. given our calibration the vacancy rate would have to reach almost 20 percent of the housing stock).

Table 1: Calibration of model parameters

Param.	Observation	Value
$N$	-	100.0000
$H$	Vacancy rate = 1.59%	101.6117
$q$	Time to buy = $\frac{1}{qm^b(\theta)} = 2$ months	0.7940
$\alpha$	Tenure = $\frac{1-\alpha}{\alpha}$ + time to buy = 10 years	0.0084
$\beta$	0.96 in annual terms	0.9966
$\bar{v}$	-	1.0000
$\underline{v}$	$0.1\bar{v}$	0.1000
Cobb-Douglas matching function and CSE, $m(\theta) = \min \{A\theta^{1-\eta}, 1\}$		
$\eta$	Share of the seller	0.4211
$A$	Time to buy = $\frac{1}{qm^b(\theta)} = 2$ months	0.6321
Cobb-Douglas matching function and Nash bargaining, $m(\theta) = \min \{A\theta^{1-\eta}, 1\}$		
$\eta$	Share of the seller	0.5000
$A$	Time to buy = $\frac{1}{qm^b(\theta)} = 2$ months	0.6326

Table 2: Steady State

	CSE (UB and CD)	Nash-CD	Walrasian
Tightness $\theta$	1.01	1.01	0.51
Sellers $s$	1.61	1.61	1.61
Buyers $b$	1.63	1.63	0.82
Non-traders $n$	96.76	96.76	97.57
Sales $qm(\theta)s$	0.81	0.81	0.82
Time to sell $\frac{1}{qm(\theta)}$	1.98	1.98	1.97
Time to buy $\frac{\theta}{qm(\theta)}$	2.00	2.00	1.00
Price $\frac{p(1-\beta)}{(\bar{v}-v)}$ (%)	73.56	100.21	0.00
Reser. price $\frac{p_s(1-\beta)}{(\bar{v}-v)}$ (%)	73.08	99.56	0.00
$S^s/S = 1 - \eta(\theta)$	0.42	0.50	0.00
Bilateral surplus $S$	3.02	3.48	0.90

Table 3: Steady state when  $V = 4$  % (instead of 1.59 %)

	CSE-UB	CSE-CD	Nash-CD	Walrasian
Tightness $\theta$	0.28	0.24	0.24	0.19
Sellers $s$	4.17	4.17	4.17	4.17
Buyers $b$	1.15	1.00	1.00	0.80
Non-traders $n$	94.69	94.83	94.83	95.03
Sales $qm(\theta)s$	0.80	0.80	0.80	0.80
Time to sell $\frac{1}{qm(\theta)}$	5.24	5.23	5.23	5.22
Time to buy $\frac{\theta}{qm(\theta)}$	1.44	1.26	1.26	1.00
Price $\frac{p(1-\beta)}{(\bar{v}-v)}$ (%)	4.73	0.21	25.24	0.00
Reser. price $\frac{p_s(1-\beta)}{(\bar{v}-v)}$ (%)	4.65	0.21	24.82	0.00
$S^s/S = 1 - \eta(\theta)$	0.13	0.00	0.50	0.00
Bilateral surplus $S$	1.48	1.13	2.23	0.90

Table 4: Steady-state when average tenure is 7 years (instead of 10)

	CSE-UB	CSE-CD	Nash-CD	Walrasian
Tightness $\theta$	2.35	1.88	2.06	0.74
Sellers $s$	1.61	1.61	1.61	1.61
Buyers $b$	3.78	3.03	3.33	1.19
Non-traders $n$	94.60	95.35	95.06	97.20
Sales $q m(\theta) s$	1.14	1.17	1.14	1.17
Time to sell $\frac{1}{q m(\theta)}$	1.39	1.38	1.39	1.36
Time to buy $\frac{\theta}{q m(\theta)}$	3.27	2.60	2.86	1.00
Price $\frac{p(1-\beta)}{(\bar{v}-v)}$ (%)	551.29	134.07	178.17	0.00
Reser. price $\frac{p_s(1-\beta)}{(\bar{v}-v)}$ (%)	611.03	133.51	197.14	0.00
$S^s/S = 1 - \eta(\theta)$	0.75	0.42	0.50	0.00
Bilateral surplus $S$	9.99	3.84	4.80	0.90

Table 5: Business cycle facts

	$\sigma_x(\%)$	$\rho(GDP, x)$	$\rho(GDP, x_{+1})$	$\rho(e, x)$	$\rho(e, x_{+1})$	$\rho(V, x)$	$\rho(V, x_{+1})$
GDP	1.53	1.00	0.86	0.87	0.82	-0.18	-0.24
Equipment $e$	3.26	0.87	0.75	1.00	0.88	-0.17	-0.24
Vacancies $V$	6.93	-0.18	-0.10	-0.17	-0.08	1.00	0.61
FHFA price	2.39	0.47	0.45	0.61	0.58	-0.13	-0.25
Sales	9.29	0.50	0.29	0.58	0.38	-0.41	-0.39
TOM	15.99	-0.60	-0.52	-0.61	-0.55	0.16	0.22

	$\rho(p, y)$		
	$x_{-1}$	$x$	$x_{+1}$
Sales	0.54	0.49	0.37
TOM	-0.33	-0.35	-0.32

Notes:  $GDP$ , household equipment, and vacancies are taken for the period 1965:1 to 2009:4. The FHFA price index goes from 1975:1 to 2009:4. NAR sales go from 1968:1 to 2009:4. Quarterly data, HP filtered with  $\lambda = 1600$ . Source: Census Bureau and NIPA.



Table 6: Only one shock

	Only demand shocks				Only supply shock							
	Search		Walrasian		Search			Walrasian				
	$\sigma_x(\%)$	$\rho(\alpha, x)$	$\sigma_x(\%)$	$\rho(\alpha, x)$	$\sigma_x(\%)$	$\rho(V, x)$	$\rho(V, x_{+1})$	$\sigma_x(\%)$	$\rho(V, x)$	$\rho(V, x_{+1})$		
$\bar{v}, \alpha$	3.49	1.00	3.49	1.00	-	-	-	-	-	-		
Buyers	6.22	0.93	3.84	0.98	5.94	-0.56	-0.88	1.53	0.02	-0.34		
$V$	-	-	-	-	7.84	1.00	0.76	7.84	1.00	0.76		
$p$	1.29	1.00	0.00	-	0.84	-0.99	-0.81	0.00	-	-		
$p_s$	1.22	1.00	0.00	-	0.74	-0.99	-0.79	0.00	-	-		
$1 - \eta(\theta)$	2.12	0.93	0.00	-	4.11	-0.91	-0.91	0.00	-	-		
$S$	9.21	0.99	0.00	-	10.27	-0.96	-0.88	0.00	-	-		
Sales	3.59	0.93	3.84	0.98	3.16	0.42	-0.18	1.53	0.02	-0.34		
TOM	3.59	-0.93	3.84	-0.98	7.12	0.91	0.91	7.96	0.98	0.81		
	$\rho(p, y)$			$\rho(p, y)$			$\rho(p, y)$			$\rho(p, y)$		
	$y$		$y_{+1}$		$y$		$y_{+1}$		$y$		$y_{+1}$	
Sales	0.94	0.99			-0.32	0.22						
TOM	-0.94	-0.99			-0.94	-0.91						

Notes: The measure of buyers is the number of buyers as a fraction of the population. Sales is the per capita volume of sales, in percentage points.

Table 7: Business cycle properties of the benchmark economy and the Walrasian economy

	Benchmark economy				Walrasian economy					
	$\sigma_x(\%)$	$\rho(\alpha, x)$	$\rho(V, x)$	$\rho(V, x_{+1})$	$\sigma_x(\%)$	$\rho(\alpha, x)$	$\rho(V, x)$	$\rho(V, x_{+1})$		
$e, \bar{v}, \alpha$	3.49	1.00	-0.16	-0.15	3.49	1.00	-0.16	-0.15		
Buyers	9.12	0.71	-0.46	-0.68	4.16	0.91	-0.14	-0.26		
$V$	7.78	-0.16	1.00	0.76	7.78	-0.16	1.00	0.76		
$p$	1.82	0.88	-0.58	-0.49	-	-	-	-		
$p_s$	1.70	0.90	-0.56	-0.46	-	-	-	-		
$1 - \eta(\theta)$	4.90	0.53	-0.83	-0.83	-	-	-	-		
$S$	15.01	0.73	-0.75	-0.69	-	-	-	-		
Sales	4.75	0.68	0.16	-0.24	4.16	0.91	-0.14	-0.26		
TOM	8.43	-0.53	0.83	0.83	9.33	-0.54	0.90	0.75		
	$\rho(p, y)$			$\rho(p, y)$			$\rho(p, y)$			
	$y_{-1}$		$y$		$y_{-1}$		$y$		$y_{+1}$	
Sales		0.40	0.51	0.72		-	-	-		
TOM		-0.68	-0.82	-0.83		-	-	-		

Notes: The measure of buyers is the number of buyers as a fraction of the population. Sales is the per capita volume of sales, in percentage points.

Figure 1: The cyclical behavior of prices, sales and time on the market.

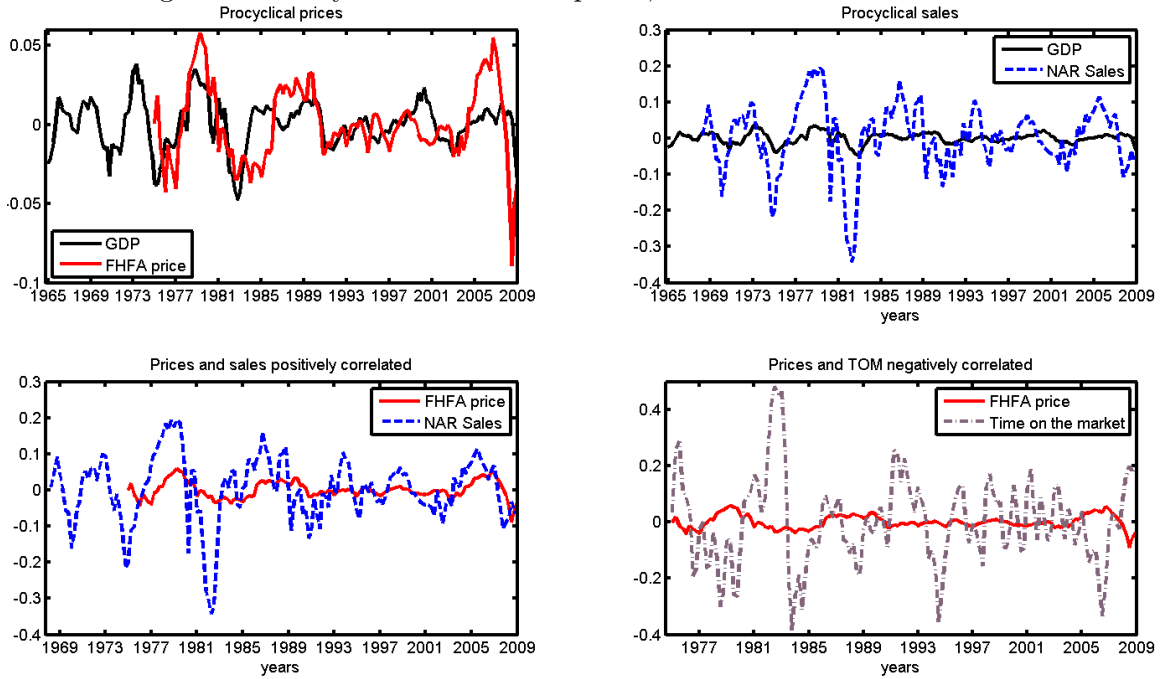


Figure 2: The steady state effect of a change in the vacancy rate (I).

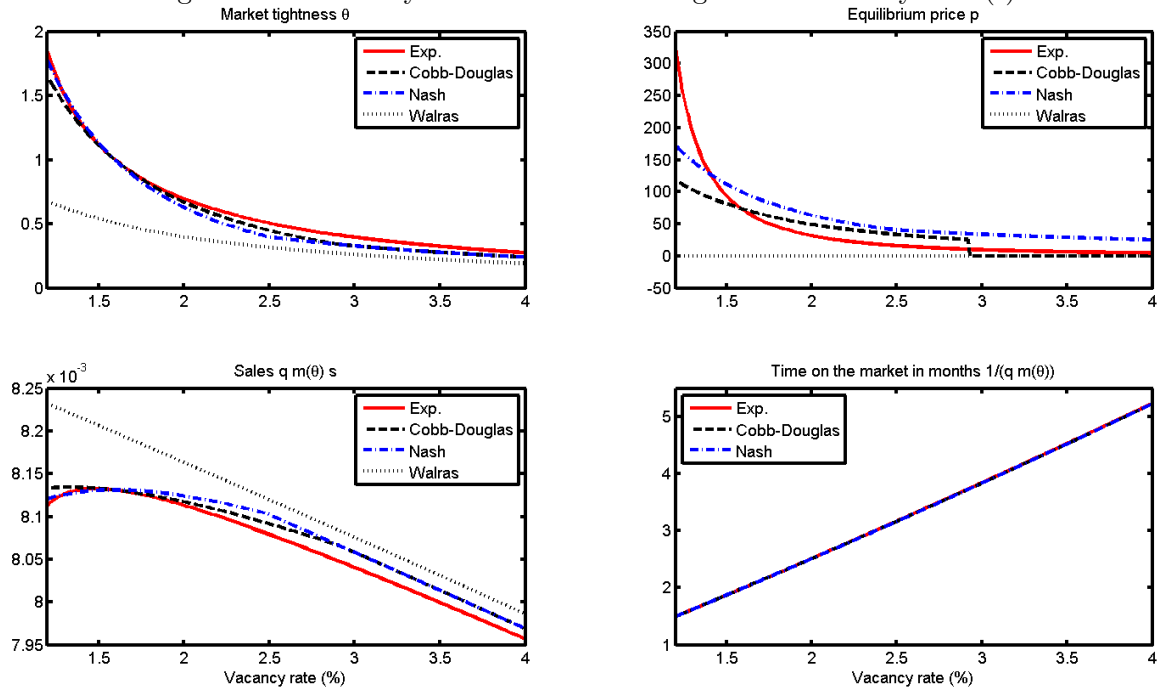


Figure 3: The steady state effect of a change in the vacancy rate (II).

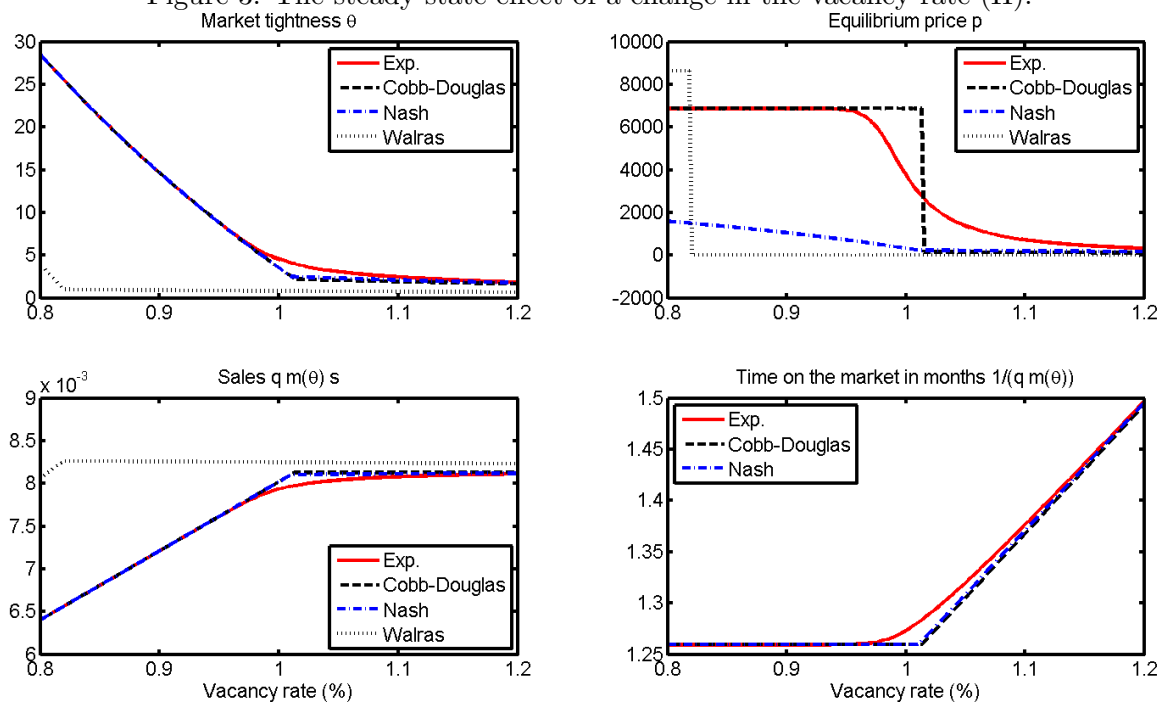


Figure 4: *GDP* and durable household equipment.

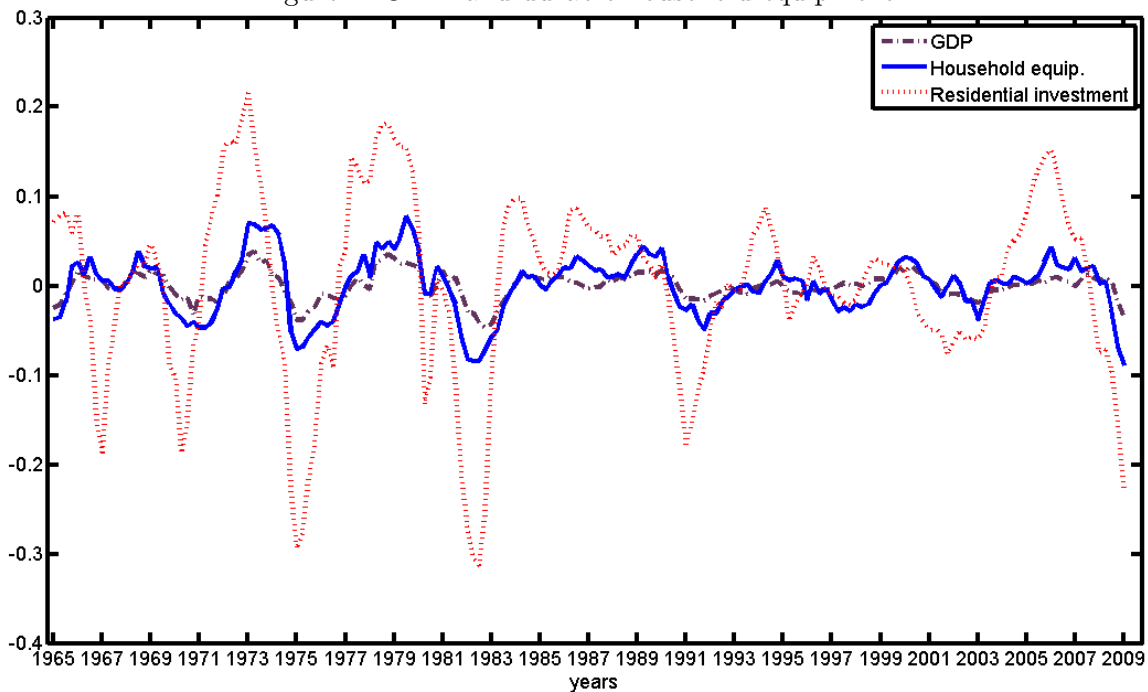
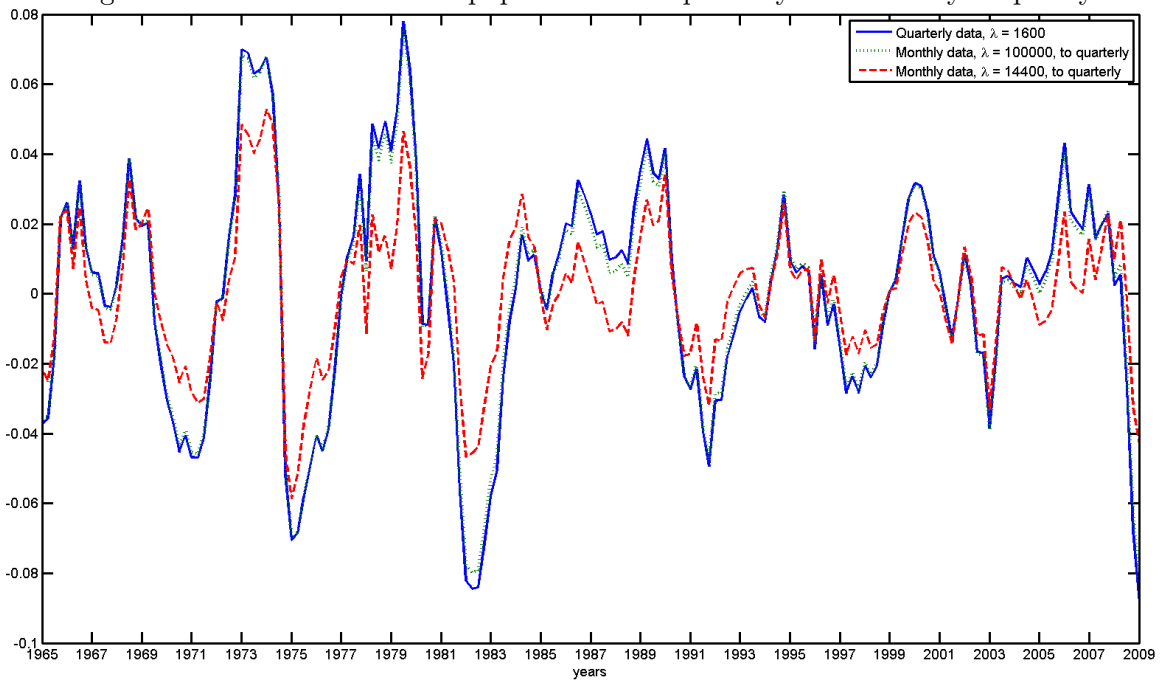


Figure 5: Durable household equipment at the quarterly and monthly frequency.



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