

The Dixit-Stiglitz-Krugman Trade Model: A Geometric Note

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Abstract

In this note, we briefly review the now standard Dixit-Stiglitz-Krugman trade model of monopolistic competition. Furthermore, we propose a convincing graphical exposition that emphasizes the firms' entry-exit process.

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1 Introduction

Among several competing trade models, the model of monopolistic competition à la Dixit-Stiglitz-Krugman (Krugman 1979, 1980, 1981; Dixit and Norman 1980; Helpman and Krugman 1985) provides an elegant account of intra-industry trade and plays a major role in the recent literature.¹

In his influential survey, Matsuyama (1995, p. 701) provides the following definition of monopolistic competition:

1. The products are differentiated. Each firm, as the sole producer of its own brand, is aware of its monopoly power and sets the price of its product.
2. The number of firms (and products) is so large that each firm ignores its strategic interactions with other firms; its action is negligible in the aggregate economy.
3. Entry is unrestricted and takes place until the profits of incumbent firms are driven down to zero.

This model is also attractive because increasing returns are internal to the firms, so the problem of multiple equilibria does not arise (as it did in the

¹ See Helpman (1990), Baldwin *et al.* (2003, ch. 2), Combes, Mayer and Thisse (2008, chs. 3–4), and Feenstra (2004, ch. 5) for surveys. In a series of articles, Neary (2001, 2004, 2009) provides an excellent overview of the literature.

models of external economies). Furthermore, as Matsuyama has pointed out, by assuming firms are very small, we don't have to worry about strategic interactions between firms that make any general treatment of oligopolies impossible. Although this type of model relies heavily on specific functional forms (e.g., CES utility), it remains appropriate to model global phenomena using the monopolistic competition model.

In this note, we present the now standard Dixit-Stiglitz-Krugman trade model of monopolistic competition. Furthermore, we propose a convincing graphical exposition that emphasizes the firms' entry-exit process. The next section presents the basic model. The nature of the trading equilibrium is considered in Section 3. The effects of factor mobility are briefly reviewed in Section 4, followed by concluding remarks in Section 5.

2 The Model

Suppose that there are two countries: Home and Foreign. Home (resp. Foreign) is endowed with L (L^*) units of labor, which is the only primary factor of production. The countries have identical tastes and technologies.

Each country produces two consumption goods, Good X and Good Y . Goods Y is sold in a perfectly competitive market, while Good X is sold in a monopolistically competitive market. Good Y is produced under constant

returns using only labor; units are chosen such that one unit of labor produces one unit of output. Wage rates are normalized to unity.

In each country, agents have the following utility function:

$$u = X^\mu Y^{1-\mu}, \quad 0 < \mu < 1, \quad (1)$$

where Y is the consumption level of Good Y and X is a Good X aggregate, given by

$$X = \left[\sum_{i=1}^n (c_i)^\rho \right]^{1/\rho}, \quad 0 < \rho < 1, \quad (2)$$

where consumption of each variety is given by c_i , n is the number of product varieties produced in Home, and $\sigma \equiv 1/(1 - \rho) > 1$ is the elasticity of substitution between every pair of Good X varieties, respectively. A lower value of σ implies that consumers value product diversity more.

The consumer's utility maximization problem can be solved in two steps.² First, for a given allocation of spending across goods, maximize X subject to total spending on the differentiated products, E_X . Second, determine spending on Good X and Good Y .

For the first step, one can check that the demand function for variety i

² See, for example, Helpman and Krugman (1985, ch. 6).

can be written as³

$$\begin{aligned} c_i &= \frac{p_i^{-\sigma}}{(P_X)^{1-\sigma}} E_X \\ &= \left(\frac{p_i}{P_X} \right)^{-\sigma} \left(\frac{E_X}{P_X} \right), \end{aligned} \quad (3)$$

where P_X is the price index of Good X , which is dual to X :⁴

$$P_X = \left[\sum_{i=1}^n (p_i)^{\rho/(\rho-1)} \right]^{(\rho-1)/\rho} = \left[\sum_{i=1}^n (p_i)^{1-\sigma} \right]^{1/(1-\sigma)}. \quad (4)$$

Now we turn to the problem of finding the optimal spending on Good X , E_X . E_X can be obtained by solving the following problem:

$$\begin{aligned} \max u &= X^\mu Y^{1-\mu}, \\ \text{s.t. } P_X X + Y &= E, \end{aligned}$$

where E represents national income. Then, one can obtain

$$E_X = \mu E. \quad (5)$$

Substituting this back into (3), one can obtain the demand function:

$$c_i = \frac{p_i^{-\sigma}}{(P_X)^{1-\sigma}} \mu E. \quad (6)$$

³ Note that this function is log-linear in own price, p_i , and total spending on Good X , E_X , both deflated by a price index of Good X .

⁴ Note that P_X is defined in terms of *negative* exponents ($\sigma > 1$). See, Neary (2001, p. 537) on this point.

It is important to note that the demand function perceived by the typical firm is not (6) but rather:⁵

$$c = \phi p^{-\sigma}, \quad \phi = \mu E (P_X)^{\sigma-1}, \quad (7)$$

with the intercept ϕ assumed to be taken as given by the firm.⁶ Figure 1(a) shows the constant-elasticity demand curve described by equation (7).

Note also that we can express maximized utility as a function of income and the price index for Good X , giving the indirect utility function:

$$V = \mu^\mu (1 - \mu)^{1-\mu} \frac{E}{(P_X)^\mu} = \mu^\mu (1 - \mu)^{1-\mu} \frac{E}{P}. \quad (8)$$

The term

$$P \equiv (P_X)^\mu$$

is the cost-of-living index in Home.⁷

Now turn to the production of each variety. Each product is supplied by a monopolistically competitive firm. Before starting production, α units of labor are required as a fixed cost of production. Then, β units of labor are required as a marginal cost of production. Thus, the total cost function of

⁵ Hereafter, the subscript i is dropped for simplicity.

⁶ Neary (2001, p. 538) and Helpman (2006, p. 593).

⁷ Fujita, Krugman, and Venables (1999, p. 48). Baldwin *et al.* (2003, p. 15) call it a “perfect” price index in that real income defined with P is a measure of utility.

the typical firm becomes⁸

$$TC = \alpha + \beta x, \tag{9}$$

where x is the output level. This implies a horizontal marginal cost (MC) curve at the level β , and an average cost (AC) curve which is a rectangular hyperbola with respect to the vertical axis and the marginal cost curve. These curves are also illustrated in Figure 1(a).

Given a Dixit-Stiglitz specification with constant elasticity σ , each firm sets its price as

$$p = \frac{\sigma}{\sigma - 1} \beta. \tag{10}$$

With free entry and exit, the level of output that generates zero profits is given by

$$\bar{x} = \frac{\alpha}{\beta} (\sigma - 1). \tag{11}$$

It is important to note that the (long-run) equilibrium output of each firm is constant.

Now let us add one more panel for a better understanding. Figure 1(b) depicts the relationship between the total number of varieties, n , and the demand level for each variety, c . In the present setting the total expenditure

⁸ Note that the wage rate is normalized to unity.

for Good X is constant:⁹

$$npc = \mu E = \mu L. \quad (12)$$

Substituting the pricing rule (10) into this and rearranging, one can obtain the following relationship:

$$c = \frac{1}{n} \frac{(\sigma - 1)}{\sigma} \frac{\mu L}{\beta}. \quad (13)$$

This demand condition (i.e., budget constraint) is depicted as hyperbola CC in panel (b).

On the other hand, the zero-profit condition implies that each firm must sell at least \bar{x} in the long run. This is depicted as the horizontal line ZZ . In equilibrium, then, the following condition must hold for each variety:

$$c = \bar{x}. \quad (14)$$

By combining these conditions, the equilibrium number of varieties is obtained:

$$n^A = \frac{\mu L}{\alpha \sigma}, \quad (15)$$

where the superscript A represents the autarky (i.e., no international trade) equilibrium value. Thus the autarky equilibrium value of the cost-of-living

⁹ Since free entry ensures that profits will be zero in the long run, the national income consists only of wage income.

index becomes:

$$\begin{aligned}
 P^A &= (n^A)^{\mu/(1-\sigma)} p = \left(\frac{\mu L}{\alpha \sigma}\right)^{\mu/(1-\sigma)} \left(\frac{\sigma \beta}{\sigma - 1}\right)^\mu, \\
 -\left(\frac{L}{P^A}\right) \left(\frac{dP^A}{dL}\right) &= \frac{\mu}{\sigma - 1}.
 \end{aligned} \tag{16}$$

It is important to note that the cost-of-living index is a *decreasing* function of the labor endowment: the larger country can support a greater number of varieties of differentiated products than the smaller country.¹⁰ Note also that as the share of Good X , μ , becomes larger and/or product differentiation matters more (i.e., σ is smaller), the impact of a change in labor endowment on the price index becomes larger.

In panel (b), the autarky equilibrium is obtained as the intersection of curve CC and curve ZZ , point A . This graphical exposition provides a easier understanding for comparative statics analysis. Let us consider, for example, an increase in the labor endowment, L . In this case, the hyperbola CC moves upward to $C'C'$. Then, in the short run, each firm can sell more than the zero-profit output \bar{x} : each firm earns positive profits. This situation is depicted as point A' .

However, responding to positive profits, new firms enter into the Good X sector. Since consumers spread their income among every variety, demand for each variety becomes lower. This change is shown by the arrow in panel

¹⁰ Fujita, Krugman and Venables (1999, pp. 56–57) call this the *price index effect*.

(b). In the long run, each firm sells \bar{x} again: changes in the level of the labor endowment L lead to adjustments in industry output via changes in the number of firms only.¹¹

3 Trading Equilibrium

Suppose that the two countries open their goods markets: the effect will be the same as if *each* country had experienced an increase in its labor force.¹²

The product market equilibrium requires that the demand for each product is equal to the zero-profit output level:

$$c + c^* = \bar{x}, \quad (17)$$

where c^* represents the demand for a Home product in Foreign. Adding (13) and its Foreign counterpart, the LHS of (17) can be obtained as follows:

$$c + c^* = \frac{\mu(L + L^*)}{(n + n^*)p}. \quad (18)$$

Substituting this and (10) into (17), one can obtain the total number of varieties in the trading equilibrium, which is the sum of the number of varieties in the autarky equilibrium,

$$N^T \equiv n^T + n^{*T} = \frac{\mu(L + L^*)}{\alpha\sigma} = n^A + n^{*A}, \quad (19)$$

¹¹ Neary (2001, p. 539).

¹² See, Krugman (1979) on this point.

where superscript T indicates a trading equilibrium value. Opening trade can be interpreted as an expansion of market size.

Now we can show the impact of trade liberalization in Figure 2. Let us take the case of $L = L^*$. Panel (b) shows the relationship between N and c , while panel (a) is its Foreign counterpart. As in the case of autarky, the demand condition (i.e., budget constraint) is depicted by hyperbolas CC and C^*C^* . The production equilibrium in each country is depicted by point A and point A^* , respectively.

Suppose that the opening of trade does not affect the production structure. On the other hand, since consumers now face twice as many product varieties (from n^A to $N^T = 2n^A$), demand for each product becomes halved (the increase from \bar{x} to $\bar{x}/2$). Because each country specializes in a different range of differentiated products, intra-industry trade in Good X occurs. Home consumers' consumption point moves from point A to point B . Thus, the total import volume of Foreign varieties is shown by the shaded rectangle. Although the (wage) income level in terms of the numeraire remains unchanged, an increase in the number of product varieties implies that the cost-of-living index becomes lower:

$$P^T = (P_X^T)^\mu = (N^T)^{\mu/(1-\sigma)} p^\mu < (n^A)^{\mu/(1-\sigma)} p^\mu = (P_X^A)^\mu = P^A. \quad (20)$$

Note that an increasing availability of differentiated products leads to a lower

cost of obtaining each unit of utility, u , although the price of each product remains constant.

4 Factor Mobility

Now suppose that there are impediments to trade in goods, but economic integration makes it possible for some workers to migrate across countries.¹³

Workers migrate toward the country where the equilibrium real wage is higher. Using (16), one can define the real wage rate in one country,

$$\frac{1}{P} = \left(\frac{\mu L}{\alpha \sigma}\right)^{\mu/(\sigma-1)} \left(\frac{\sigma-1}{\sigma\beta}\right)^{\mu}. \quad (21)$$

That is, in the presence of internal scale economies, a larger country offers a greater number of differentiated products and thus the real wage rate becomes higher than in the smaller country.

In this setting, workers migrate from the smaller country to the larger country. Thus, the size of the larger country will expand, while the size of the smaller country will shrink. The point is that there will be a *cumulative process* in which the wide range of differentiated products attracts workers, and immigration will enhance further expansion of the range of differentiated products.

¹³ Krugman (1979, pp. 477–478), Helpman and Krugman (1985, ch. 11), Matsuyama (1995, pp. 712–713).

Figure 3 illustrates the allocation of labor between countries. The horizontal axis represents the total labor force in the world economy, $L + L^*$. The quantity of labor employed in Home (resp. Foreign) is measured from the left (resp. right). The left (resp. right) vertical axis shows the real wage rate (21) in Home (resp. Foreign). Initially, in the autarkic equilibrium with identical labor endowments ($L = L^*$), wage rates are equalized between countries. The relationship in Home between the total labor force and the real wage rate is depicted with the curve ω :

$$\omega(L) \equiv \frac{1}{P} = \left(\frac{\mu L}{\alpha \sigma}\right)^{\mu/(\sigma-1)} \left(\frac{\sigma-1}{\sigma \beta}\right)^{\mu}. \quad (22)$$

Likewise, the relationship in Foreign is depicted with the curve ω^* .

Now let us describe the process of labor movement. If some workers move from Foreign to Home, it raises the real wage rate in Home, while lowering the real wage rate in Foreign. This wage gap further stimulates labor movement from Foreign to Home. Note that this movement hurts those left behind in Foreign (i.e., the smaller country). While the Home wage rate increases along the ω curve, the Foreign counterpart decreases along the ω^* curve. This provides a striking contrast with the case of trade in goods, in which all workers gain and those in the small country gain the most.¹⁴

Until now, we have concentrated on the case with identical technologies

¹⁴ Matsuyama (1995, p. 712).

between countries. Now let us briefly review what happens if both fixed and variable costs are higher in one country.¹⁵ In this case, it is clearly desirable that all worker should move to the other country. But if the inferior country starts with a large enough share of the labor endowment, migration may move in the wrong direction.¹⁶ As in the case of external economies, the world economy may be trapped into a Pareto inferior situation.¹⁷

5 Concluding Remarks

In this paper, we have briefly reviewed the now standard Dixit-Stiglitz-Krugman model of monopolistic competition. In particular, we have proposed a convincing graphical exposition that emphasizes the firms' entry-exit process, which facilitates the understanding of several topics such as determinants of equilibrium and existence of intra-industry trade. Although this tractable model of monopolistic competition relies heavily on specific functional forms, it will remain as one of the key ingredients of trade models for

¹⁵ Krugman (1979, p. 478). Matsuyama and Takahashi (1998) present a model of two regional economies with similar features.

¹⁶ Related to this, in the case of trade in goods, Lancaster (1980, pp. 167–168) notes that a size difference between countries may become a source of “false comparative advantage.”

That is, autarky relative prices do not serve as reliable predictors of trade patterns.

¹⁷ Note also that this model is similar to the models of standard setting in the Industrial Organization literature. See, for example, Chou and Shy (1990).

internal scale economies.¹⁸ Note that, since this model is quite special, one should view it as a complement rather than a substitute for the other models of trade (e.g., trade models for external economies).

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¹⁸ In his influential contribution, Melitz (2003) has proposed an extension of the Dixit-Stiglitz-Krugman model that makes it possible to work with heterogeneous firms in terms of their marginal labor input requirement. See Helpman (2006) for a survey of the relevant literature.

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Figure 1

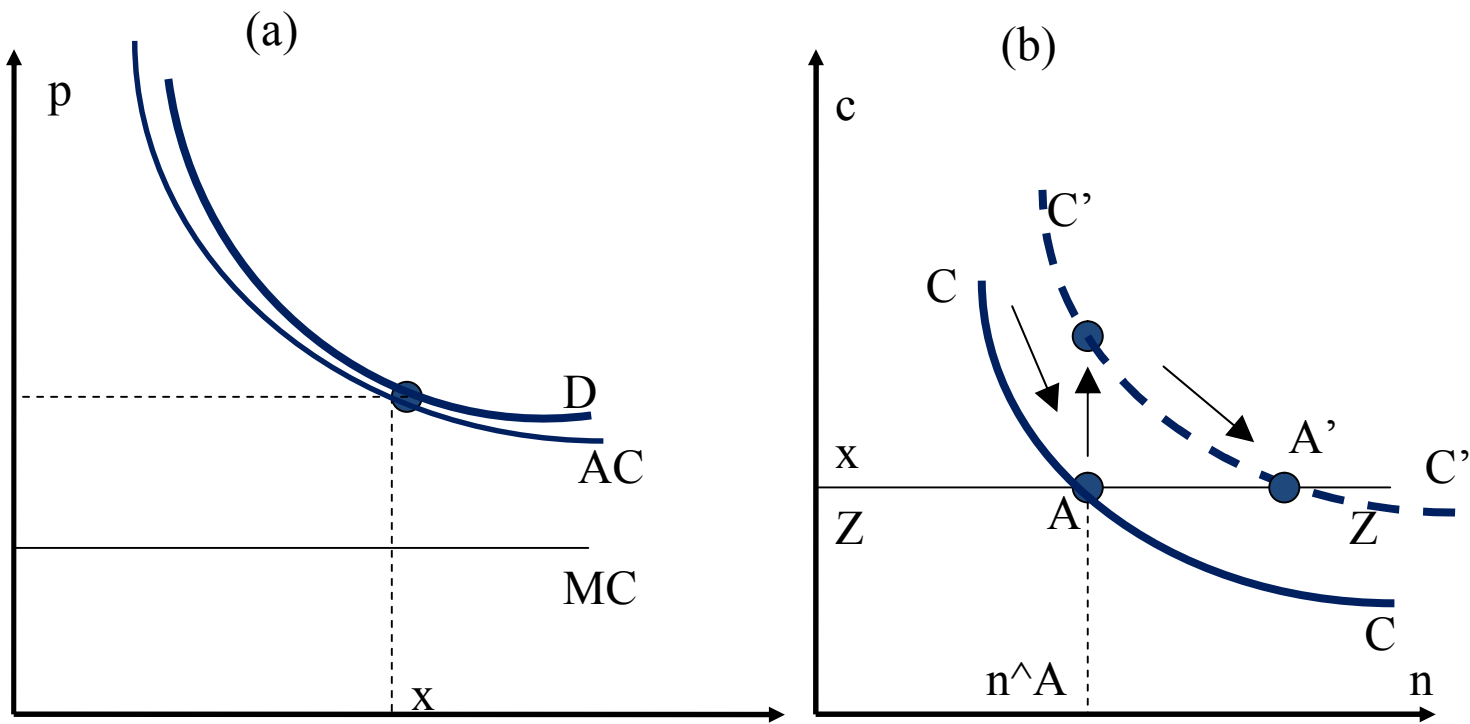


Figure 2

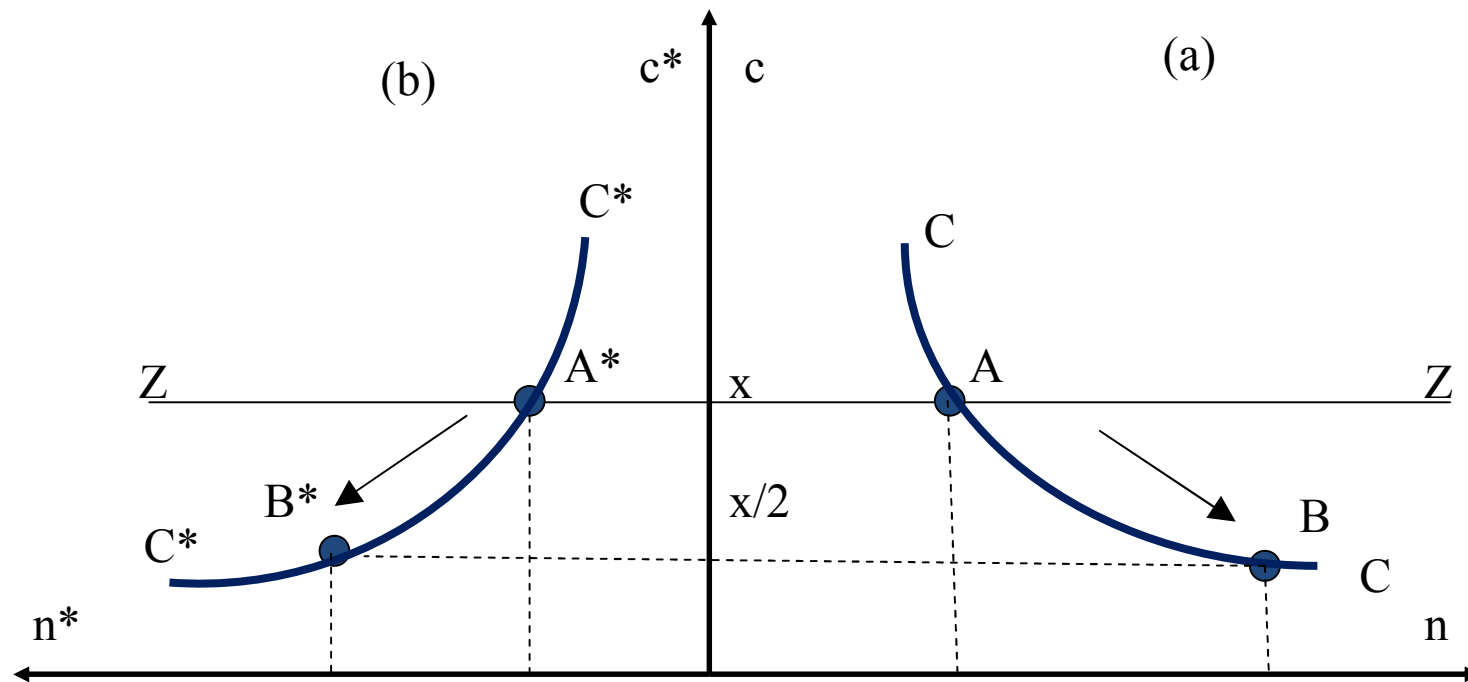


Figure 3

