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Relationship between Czech and European developed stock markets: DCC MVGARCH analysis

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IES Working Paper: 9/2010

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Bibliographic information:
Princ, M. (2010). "Relationship between Czech and European developed stock markets: DCC MVGARCH analysis" IES Working Paper 9/2010. IES FSV. Charles University.

This paper can be downloaded at: http://ies.fsv.cuni.cz

Relationship between Czech and European developed stock markets: DCC MVGARCH analysis

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May 2010

Abstract:
The study concentrates on an analysis of the Czech stock market performed by an application of DCC MV GARCH model of Engle (2002). Data sample including years from 1994 to 2009 is represented by daily returns of Prague Stock Exchange index and other 11 major stock indices. There is found an existence of increasing trend in conditional correlations among a whole European region. The trend reveals breakpoints splitting a data series into three phases of development. The analysis includes a composition of returns adjusted by exchange rates capturing a point of view of global investors. The Czech Koruna exchange rate effects in a conjunction with equity returns are identified as a possible risk aversion instrument. Granger causality concept is added in order to find a development of data flow directions in a perspective of the Czech market. Results show that unidirectional influence of foreign markets affecting Czech market occurs in data series.

Keywords: stock market integration, multivariate analysis, dynamic modelling, conditional correlation

JEL: C32, E44, G14, G15, F36

1. Introduction

The structure of stock markets has changed all over the world in latest years. This is also a case of the Czech stock market, which changed its image from an artificial emerging market established during an economic transition period in 1992 to designated offshore securities market approved by U.S. Security Exchange Commission in 2004. The evolution of the Czech stock market meant a changing degree of integration into other markets, which can be characterized as an increase in dependences of various data series.

Reasons for increased market dependencies and an occurrence of a similar behaviour could be different. International spillovers may be associated in cases of cross-listed securities in various markets, which is analogous for an increasing number of abroad listed depositary receipts representing domestic securities as researched in TSOLOV (2005), where correlations between ADR/GDR and underlying stocks traded on PSE were observed . The international trade can affect the correlations of consumption and business cycles across countries. This will enhance the level of economic and financial integration process as was described in NG (2000), which suggested stronger links in regional markets and also described possible volatility transmissions in case of local developed and emerging markets.

A concept of volatility estimations based on GARCH modelling developed by BOLLERSLEV (1986) is a useful guide through shocks absorbed into market data series. In a case of stock market data captured in the Czech Republic GARCH model was identified to be an appropriate model describing data series in a comparison to random walk test as was proved in VOŠVRDA AND ŽIKEŠ (2004). It was also chosen in general as a model with an efficient goodness of fit or appropriate forecasting abilities well specified even in comparison to more complicated asymmetric models such as APARCH or EGRACH models as proved in LUNDE AND HANSEN (2005).

A correlation overview is important for investors realizing investments in all markets and also raise a lot of questions about interdependences among markets. Thus when some markets experience nearly simultaneous shocks, which seriously influencing risk bore by local or global investors, it can be well described by conditional correlations observed through DCC MV GARCH modelling developed by ENGLE (2002).

An increase in a degree of market integration into international structures can be a significant event, which can change a correlation among interconnected markets as was shown in CAPPIELLO ET AL. (2006). Furthermore also periods of crises tightened interlinks between equity markets as showed SALEEM (2008) or NG (2000), which described a precise turbulent events resulting from a contagion of equity markets. All these information and relations can be powerful tools, which can be useful in case of a search for different stages of development especially in the Czech Republic.

Useful aspects for a country's stage of a liberalization process and a common evolution of the equity market can be described in a point of view of conditional correlations. This can be related to a situation of PSE, which dramatically changed from its beginning to the status in the 3rd millennium. The study will investigate whether a development and a strong integration processes have affected forces guiding volatility and cross-market correlations at PSE in comparison with other developed markets. Namely the models offer to trace back an intensity of transmission mechanisms on a basis of a dynamic analysis. The research opens a possibility of perception of interlinks between PSE and other developed capital markets, which can also answer whether or when PSE became a part of global markets. It can be also determine at what extent it occurred, which can be concluded from a comparison with similar studies researching the theme of market relations as in CAPPIELLO ET AL. (2006), DIEBOLD (2007) etc.

The study is capturing an evolution of the integration process with proposed measures. It is divided into two main parts, the first will include dynamic analysis of conditional correlations using DCC MV GARCH modelling, which proved to be a reliable estimate of conditional correlations. A second part will add a complementary and auxiliary analysis of data flow directions based on Granger causality concept used in Granger (1969), which can enrich the analysis by additional information about origins of shocks and advanced analysis of interdependences. The synergy effects of proposed analysis with appropriate data samples will provide a complex historical overview of the Czech market evolution process including points of view of global and local investors.

2. DCC MV GARCH Analysis

2.1. Methodology

One of the most popular multivariate GARCH models is a constant conditionally correlation multivariate GARCH model proposed in Bollerslev (1990), which can be defined in a following way[1]:

$H_t = D_t R D_t$,

where $D_t = diag\left\{\sqrt{h_{i,j}}\right\}$

$E_{t-1}\left(\varepsilon_t \varepsilon_t'\right) = D_t^{-1} H_t D_t^{-1}$

$\varepsilon_t = D_t^{-1} r_t$

$r_t | \Psi_{t-1} \sim N(0, H_t)$

R denotes a correlation matrix, which contains conditional correlations, *r* stands for random variables, which are assumed to be normally distributed, and *h* are standalone univariate GARCH models. Although the model itself allow more precise analysis there are also drawbacks included in the model. It means a strict assumption of a nature of computed conditional correlations, which lie in a band of confidence, and it disallows to perceive precise changes of conditional correlations during estimated time period. Thus a generalization of CCC MV GARCH was proposed in order to eliminate these flaws, which enabled a dynamization of the conditional correlations and resulted in the dynamic conditional correlation MVGARCH model.

One of the sophisticated econometric models, which is able to show an evolving degree of integration across different countries in selected data sample, is DCC MVGARCH model described by Engle (2002).

The model is defined as follows see also Engle (2002):

$H_t = D_t R D_t$

$r_t | \Psi_{t-1} \sim N(0, H_t)$ (1)

$D_t^2 = diag\{\omega_i\} + diag\{\kappa_i\} r_{t-1} r'_{t-1} + diag\{\gamma_i\} D_{t-1}^2$ (2)

$\varepsilon_t = D_t^{-1} r_t$ (3)

$Q_t = S(\iota\iota' - A - B) + A \varepsilon_{t-1} \varepsilon'_{t-1} + B Q_{t-1}$ (4)

1 This is proposed form of the CCC MVGARCH model used in Engle (2002) for further generalisation into DCC MV GARCH.

$$R_t = diag\{Q_t\}^{-1} Q_t diag\, Q_t^{-1} \quad (5)$$

A relation (1) describes an assumption of normality. An equation (2) expresses the assumption that each subset follow an univariate GARCH process. (3) describes behaviour of residual terms and finally (4) and (5) describe matrix composition necessary for the estimation and iteration processes. Without the assumption of normality in (1), the estimator would be only QME. The log likelihood for the estimator is following:

$$\log(L) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log|D_t| + r_t' D_t^{-1} D_t^{-1} r_t - \varepsilon_t'\varepsilon_t + \log|R_t| + \varepsilon_t' R_t^{-1}\varepsilon_t\right)$$

which is being maximised through estimated parameters. The log-likelihood can be further divided into two parts

$$\log(L)(\theta,\phi) = \log(L_V)(\theta) + \log(L_C)(\theta,\phi)$$

$$\log(L_V)(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}\left(\log(2\pi) + \log(h_{i,t}) + \frac{r_{i,t}^2}{h_{i,t}}\right),$$

which shows that this part reflecting volatility is a sum of individual univariate GARCH log-likelihoods, which can be maximized separately. This emphasize a need of prior estimations of all involved univariate GARCH models. While a second term describing conditional correlation parameters is maximized individually meaning a two stage estimation.

$$\hat{\theta} = arg\ max\left\{L_V(\theta)\right\}$$

$$max\ \phi\left\{L_C(\hat{\theta},\phi)\right\}$$ [2]

These definitions can be adjusted to fit into elliptical distribution, which includes other nested distributions i.e. normal, Student, LaPlace and exponential power distributions; as used in Pelagatti and Rondena (2004), who incorporated this in their MultiGARCH library[3]. The elliptical distribution has following likelihood function[4]:

$$l(\theta) = \sum_{t=1}^{T}\left\{\log c_m - \frac{1}{2}\log|\Sigma_t| + \log g\left(r_t \Sigma_t^{-1} r'_t\right)\right\} \quad (6)$$

Because their results stated that normal distribution performed very well, in a comparison to other distributions, it was used in estimations. A final estimation of the model consists of three steps. In the first step univariate GARCH models are estimated for each data set and the resulting coefficients ω, α, β of equation (2) are used for next step as starting values. Next step begins recursion and following estimation of (3) and also residuals estimated in step 1 are used as estimate of matrix S in equation (4). Finally in a third step dynamic conditional

2 Log-likelihood maximization method of the conditional correlation part is described in Appendix III.
3 MultiGARCH library is a package used for DCC MVGARCH estimation.
4 The estimation process is divided into original code and redesigned routines of MultiGARCH package, which improve various output abilities.

correlations are computed. All steps are performed by MultiGARCH library using BFGS optimization method[5] proposed by Pelagatti and Rondena (2004).

2.2. Data Description

The main goal of the analysis is to describe stages of PSE development and its relations to other European advanced equity markets on a basis of conditional correlations, which could indicate whether or when PSE became also a part of developed markets. The European region is enriched by two other important markets (USA, Japan) representing a benchmark of the analysis, which can check, whether higher integration into European structures occurred in the sample.

Because different indices listed in one country would not significantly enhance outcomes of the analysis, only one representative index is chosen from each country i.e. ATX in Austria, BEL 20 in Belgium, CAC 40 in France, FTSE 100 in Great Britain, DAX 30 in Germany, NIKKEI 225 in Japan, AEX in Netherlands, IGBM in Spain, OMX SPI in Sweden, SMI in Switzerland, NYSE 100 in USA and finally PX index traded in the Czech Republic on Prague Stock Exchange, which is clearly irreplaceable in the analysis. This means that a whole data sample includes 12 national indices dating from 5th April 1994[6] until 30th March 2009 and thus an analyses of many important events of a recent economical history are available. For the purpose of clarity the names of variables are described by abbreviations of names of states instead of indices.

Data estimated in the routine were calculated in following form:

$$R_t = \log(P_t / P_{t-1}) \times 100 \ ,$$

where P_t stands for closing value of computed index. This means that input values of national stock indices were transformed into daily returns R_t computed as Close-to-Close value in percentages. When a expression daily return is mentioned it is important also to clarify from which point of view they are computed to be net, because there are two basic choices. The first one take into account only daily returns of local investor, who invests into national stocks and thus in my case into a particular national index. On the other hand there is another option, which takes into account real daily returns adjusted by exchange rate effects, which are important for Czech investors investing on global markets or global investors interested in returns in CZK, who utilise benefits from international diversification. This means that they are interested in strategies incorporating also a currency risk, which is significantly affecting a success of their strategies.

It is common to use daily returns denominated in local-national currencies as in Diebold (2007) or Cappiello et al. (2006). However it is possible also to test dynamic conditional correlation among currencies as in Kitamura (2007) was presented, which implies that a synthesis of these analyses would result into a point of view of fully informed investor, who is able to modify his strategy according to all available data. Thus a following analysis is conducted for both types of data i.e. daily index returns and also adjusted daily index returns[7].

5 The particular algorithm used in the library is described in Appendix III.
6 Initial date was set as a beginning day of Czech national index PX, which is the latest stock exchange index in the sample. At 5th April 1994 the value of PX 50 was set to initial value 1000.
7 Adjusted daily index returns include exchange rate effects and thus can be qualified as real daily returns.

Because of a lack of data sample synchronization[8] an original samples obtained from data servers were sorted by a custom programmed routine[9]. The algorithm approves only opening dates common for all countries, in order to capture comparable correlations among all markets and to set a common window of a trading period. The data sample restriction was chosen rather than a possible expansion with zero values to dates with no input, which would cause a probable underestimation of conditional variances estimated by individual GARCH (1,1) processes.

The asynchronization problem is common in DCC MVGARCH studies, which use rather weekly or averaged weekly data free of 'holiday-gaps'. But the data sample based on weekly data would offer only 780 samples for 15 years, which is approximately 4 times less than was achieved with a sorting procedure, which resulted in 3174 samples. This implies that the precision of the output should be higher than e.g. in Cappiello et al. (2006) or Diebold (2007). So although the routine removed some samples its data loss is only 14%, which is significantly less than a 80% data loss caused by a usage of weekly data sets. All values of daily returns and adjusted daily returns, which were used in the analysis, are depicted in Appendix I. All graphs offer an easy way to compare input data series among all researched markets, their values are normalized. An initial base value 1 means a 100% of particular index values on 5th April 1994.

2.3. Result analysis

Using an programmed procedures and the econometric software[10] univariate GARCH(1,1) processes were computed for each particular national index using both data sets, which is depicted in Table 1 and Table 2[11]. As was already mentioned this is an essential basis for a next step of DCC MVGARCH analysis. At this stage results in both tables confirmed that all estimated models fulfilled necessary conditions for both data sets of daily returns and adjusted daily returns - parameters were positive $\omega, \alpha_i \geq 0, \beta_i \geq 0$ and also all processes were stationary $\alpha + \beta < 1$. From this point the result analysis is divided into two parts i.e. the analysis of daily returns and the analysis of adjusted daily returns.

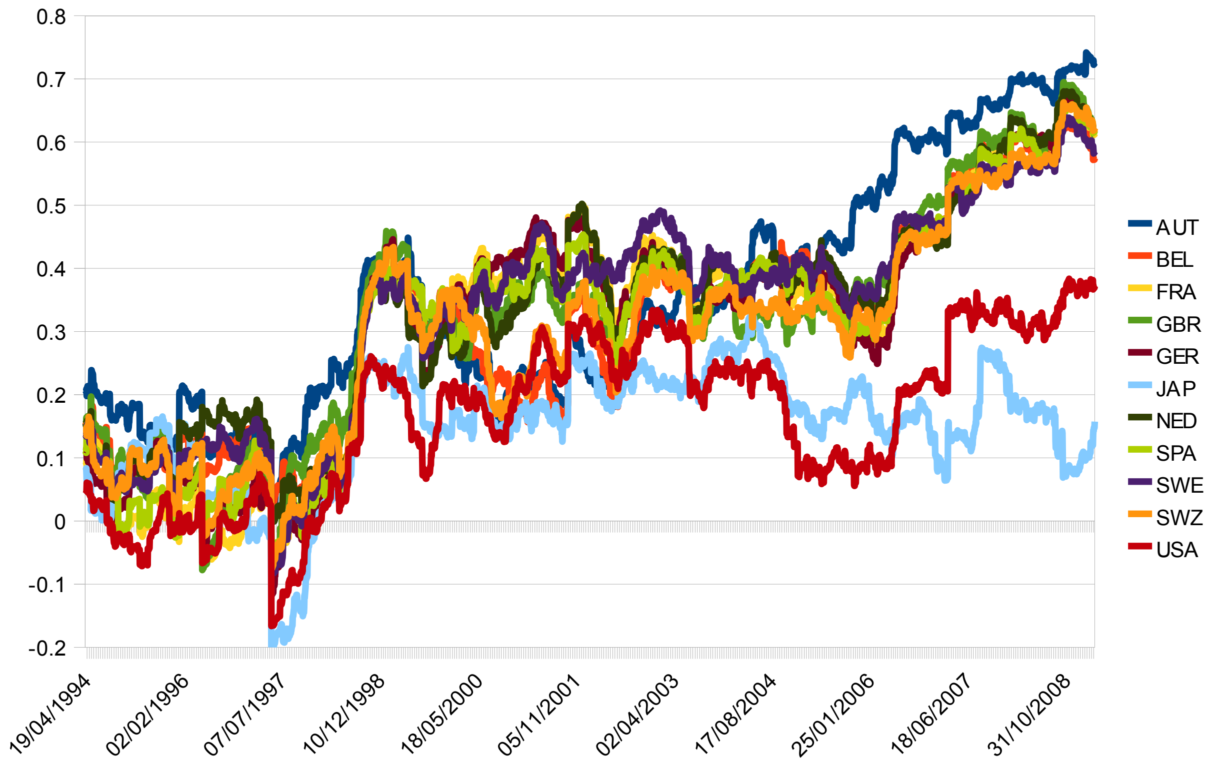
2.3.1. Daily Returns

The DCC MVGARCH model was successfully estimated and all necessary conditions were fulfilled, the convergence of the model would was successfully achieved[12]. Conditional correlations estimated by DCC MVGARCH in Graph 1 model shows a gradual increasing trend of interdependencies of Czech capital market among nearly all perceived data sets. This can be interpreted as a gradually increasing integration of Czech stock market to developed markets. A very interesting consequence of the output shows that this gradual integration of Czech stock exchange is common for all remaining data sets including relatively far Sweden, which is not even a part of EMU similarly to Switzerland and Great Britain. This proves that capital market interrelations are deepening without regards to membership in EMU. However there are two exceptions. Japan and USA indices behave differently and stay in a -0.15

8 i.e. that it is common that some exchanges close on holidays, which are unique in their countries and thus list of dates, when are stock exchanges open, is specific for a particular country.
9 Programmed in OxEdit 5.10.
10 OxEdit 5.10 including package G@rch 4.2 and package MultiGarch 0.3
11 See on pages 22 to 23.
12 A convergence of BFGS method, which is sensitive to input data, has to be ensured. The model estimation method is suitable just for percentage changes of particular indices.

to 0.4 band of correlation for all the time, this can be perceived in individual graphs of conditional correlation in Appendix II also with individual conditional covariances.

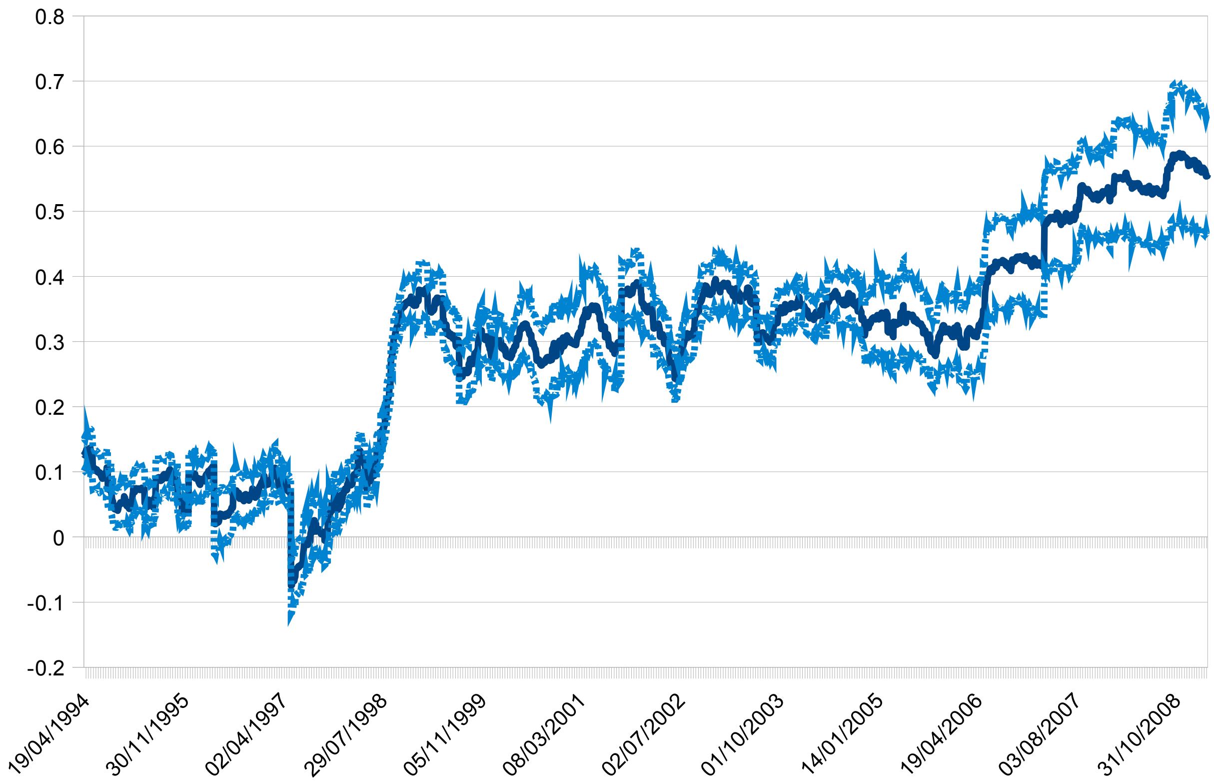
Graph 1: Aggregated conditional correlations - Daily returns



The most illustrative picture of a typical behaviour of the correlation can be achieved through a computation of the expected value based on values of all estimated correlations. This approach is similar to Cappiello et al. (2006), where average correlations are computed for particular regions. Thus if the average of all estimated correlations is computed, the result is an average correlation to all markets in the sample from a point of view of the Czech Republic. The final outcome of the average correlation is in Graph 2, which is even amended with its band of confidence calculated for 95% level of confidence and based on the Student distribution[13].

13 The band of confidence requires an assumption of a normal distribution of individual conditional correlations and was computed with 10 degrees of freedom.

Graph 2: Average of conditional correlations with band of confidence - Daily returns



When the band of confidence was computed, it is also possible to compare, which national indices get off the band at most and whether they belong to more integrated group with higher conditional correlation or vice versa. Following Table shows these statistics in a clear and straightforward aggregate form.

Table 3: Summary statistics of computed conditional correlations

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
Above average	75%	77%	73%	79%	68%	10%	90%	75%	88%	61%	0%
Out of band	89%	42%	45%	43%	32%	92%	36%	32%	40%	15%	96%

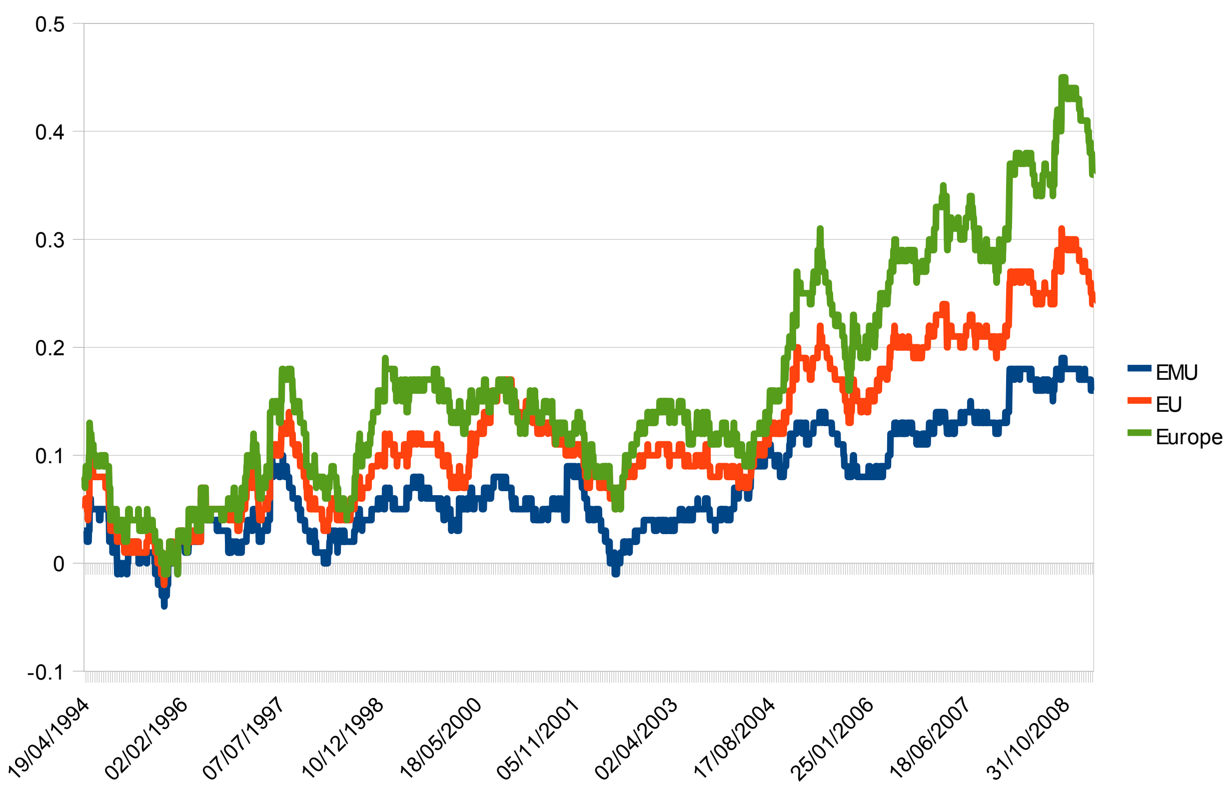
The Table 1 implies that Dutch AEX and British FTSE and Swedish OMX SPI can be referred as markets with higher integration to the Czech market in average. Austrian ATX over excesses the band most of the time and thus can be referred as the market with a high interdependence to PSE, which is also mostly different from an overall trend. On the other hand indices of USA and Japan under excess the band and it implicates that markets out of the Europe have lower interconnections with the PSE. A comparison of conditional correlation last values of USA and Japanese indices finally reveals that recently the USA equity market is more interlinked to PSE than the Japanese market, but during the whole researched period USA conditional correlations still remained under a value of the average correlations described in Graph 2.

Although the average correlation behaviour can be simply smoothed with a rising linear trend, it is not perfectly linear and several important leaps can be perceived in the estimation. The average correlation behaviour can be divided into three different periods of time. The first period lasts from an establishment of the PX[14] index until a half of the year 1998, when the average correlation stayed in a band from -0.1 to 0.15. It indicates very low or even zero correlation between PSE and other markets, which implies that PSE was in a position typical for unintegrated emerging markets as described Khalid and Rajaguru (2007) or Hyde et al. (2008). A second period is characterised by a significant increase in a correlation, which lies between 0.2 and 0.45, lasting until 2006. This means that the correlation is significantly positive and it fills the gap between periods of low and high correlations, which occurred in the last period. The final period starts in 2006 and remains until nowadays. The main characteristic is a continual increase in correlation up to values around 0.6, which is typical to developed and integrated states of EU according to Cappiello et al. (2006).

When the analysis is enriched by important economic events it can reveal the spirit of a development of PSE. This means that Czech stock market was rather "stand-alone" than integrated into Europe in the first period, which is typical for emerging markets. When a following development is researched year 1998 shows very important change, which can be associated with various economic events. According to Saleem (2008) this change could be related to Russian crisis, which occurred during the same period of time, however there is possible also another explanation.

Cappiello et al. (2006) suggests that during 1998 Euro had already effects on financial markets. This implicates that the correlation with EMU should be increased from 1998 or 1999, when compared to a rest of the sample. Thus a Graph 3 was made, which compare average correlations of states represented in the EMU subsample[15] with EU subsample[16] and finally European subsample[17] with an average correlations of a rest of the whole sample for each subsample. The idea of subsample creation is inspired by Cappiello et al. (2006), where individual conditional correlations were sorted into different classes according to a size of researched states.

Graph 3: Difference between average correlations designed subsamples vs. rest of the data – Daily returns



The Graph 3 shows that the difference between average correlations of defined subsamples was often positive. In case of EMU subsample which indicates stronger interlinks with EMU countries, but there was no significant increase during 1998 or 1999, which would confirm a hypothesis of an importance of Euro adoption in context to the Czech stock market. This concludes that during 1998 correlation with all market indices stood up steeply, because the Russian crisis contagion, but lasted for longer period of time, which is consistent with Saleem (2008). This sudden difference in co-movements is typical for emerging markets in a case of period of Russian crisis as was researched in Caporale et al. (2006).

A next important event, which affected the Czech market was an accession to EU in May 2004. A development of the average correlations suggests that integration of PSE strengthened over the time, but it is possible to analyse correlations similarly as in a case of Euro adoption, which was analysed by Cappiello et al. (2006). The Graph 3 shows that before 2004, the difference between EU and a rest of the sample was positive in terms of correlations, but from 2004 the difference increased significantly and exceeded a band of previous values and the difference is even more visible in a case of comparison of European subsample with a rest of the sample. The result suggests that the EU enlargement was an important event, which increased a degree of PSE interlinks to European markets and allowed PSE to become a developed market with a full-fledged integration in following years. Although it is probable that the Czech market can be generally labelled as a post-emerging market, according to a level of integration to other markets it can be regarded as developed or a part of developed markets. The particular date of a new stage of a development can be perceived in year 2004, in a case of analysis of differences among subsamples, or in year 2006, when the average correlation amongst all markets increased, but in both cases the date is after the accession announcement and even the accession itself, which suggests that the EU enlargement was rather a reason for a change than an anticipated event.

Finally it is possible to interpret an impact of a global financial crisis in 2008 on PSE in terms of a degree of interdependence. The outcomes indicate that a financial crisis in 2008 did not affect a steady trend, which started during 2006 and lasted until the end of a data sample in March 2009. There is no sudden change in a correlation development, which means that although correlations increased in 2008 on PSE a trend remained the same.[18] This offers a conclusion that the global financial crisis did not affected significantly a degree of integration of PSE into developed market indicating a stable evolution of interrelations between the Czech and other analysed equity markets.

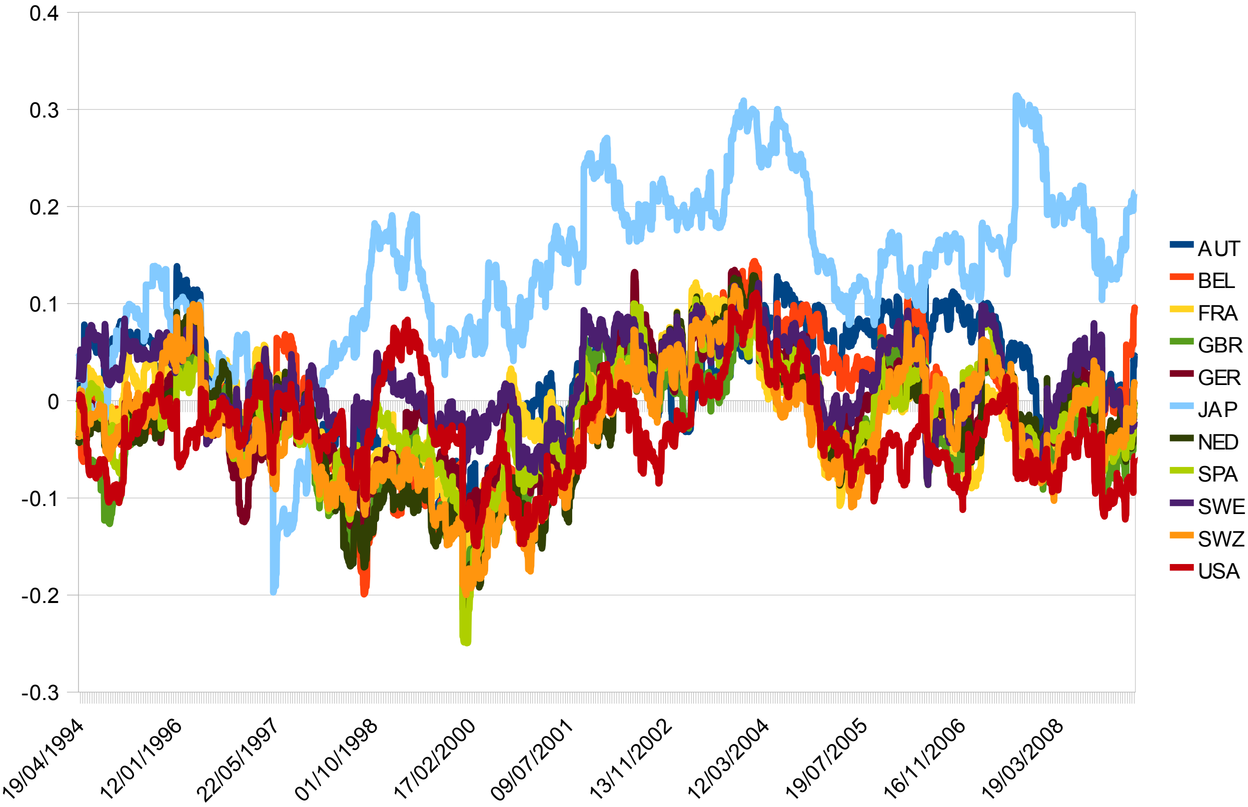
2.3.2. Adjusted Daily Returns

As in a previous analysis of daily returns the estimation of DCC MVGARCH was computed using adjusted daily returns, which incorporate an exchange rate effects. All returns were weighted by CZK, which was chosen as a basis for a comparison. The result of the model is depicted in a Graph 4, which shows co-movements were not significant during the whole period of time, all values remained in a band from -0.25 to 0.3. This indicates that although significant integration was perceived in case of daily returns, which analyse a situation from point of view of a local investor or a global investor interested only in returns in a same currency as is denominated the index, the degree of integration remained oscillating close to a zero value during the researched period. A good signal for a global investor,

18 This statement can be supported by a fact that correlation over 50% can be perceived from year 2007, which is not regarded as a time of a global financial crisis.

who is interested in investments with low correlations, which would offer a maximum diversification effect[19]. Returns weighted in CZK realized on foreign markets remained almost uncorrelated to returns of PSE meaning that exchange rate effect can be important factor, when an international portfolio is composed.

Graph 4: Aggregated conditional correlations - Adjusted daily returns



The Graph 5 shows that an average correlation of PSE among the world sample remained even in band bordered by values -0.15 and 0.15, which is more typical for CCC MVGARCH model, because a correlation stayed almost constant. This outcome shows that adjusted daily returns would be only little affected by excessive volatility in last years of crisis and thus it can be assumed that volatility spillovers or market contagions have low effects.

19 Amount of the diversification effect arises from a degree of co-movement and thus also correlations, higher correlations imply lower diversification effect and on contrary lower correlations mean higher diversification effect.

Graph 5: Average of conditional correlations with band of confidence - Adjusted daily returns

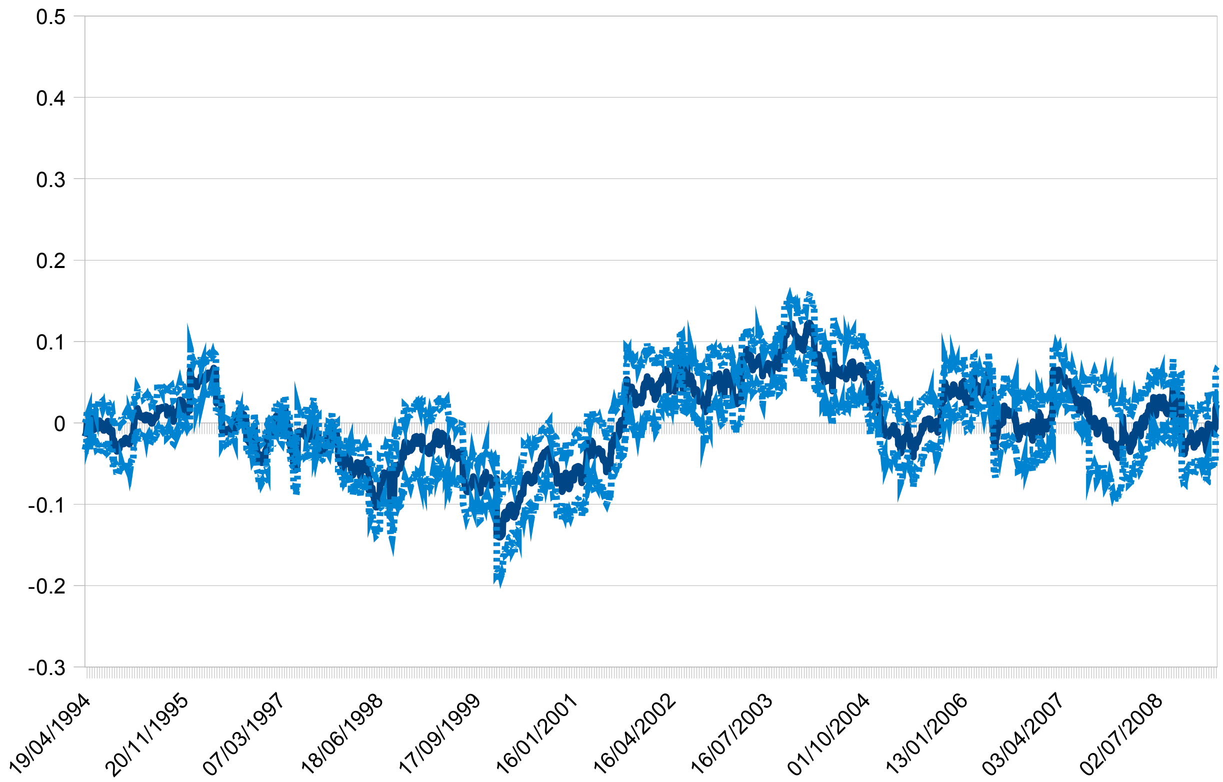
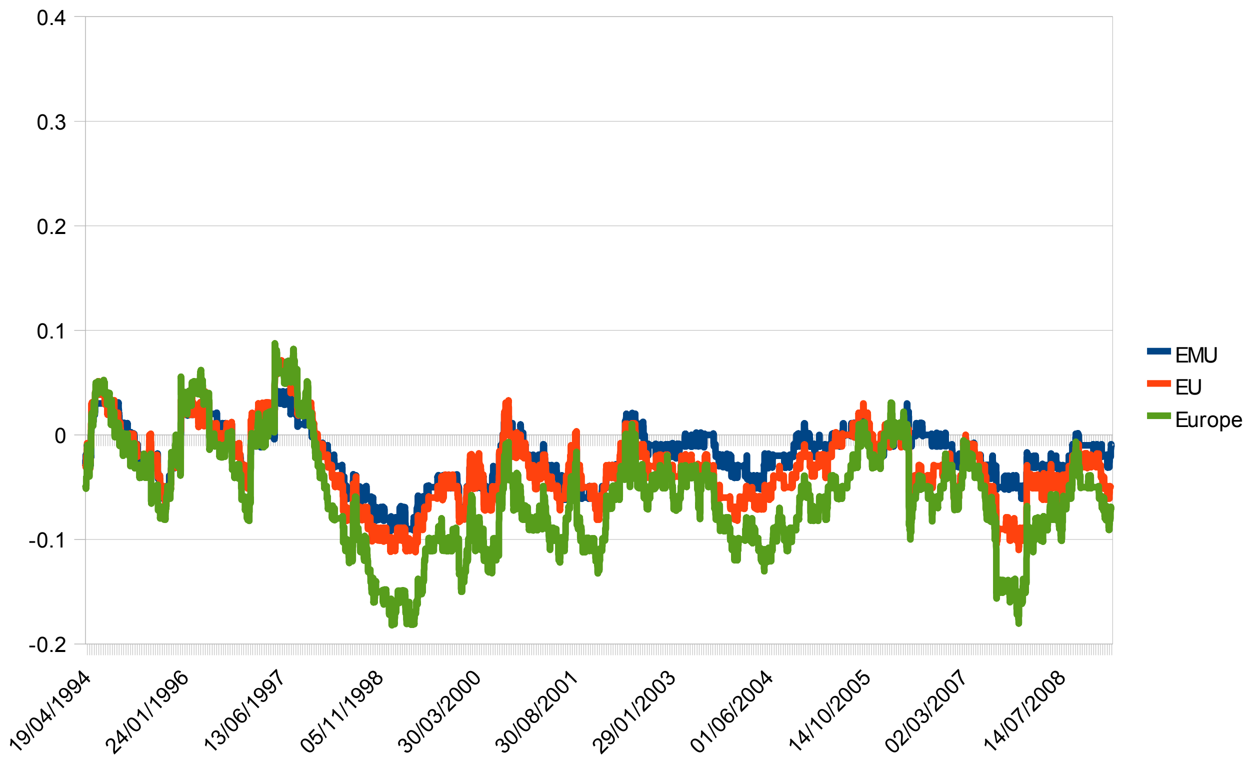


Table 4: Summary statistics of computed conditional correlations - Adjusted daily returns

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
Above average	69%	39%	38%	13%	22%	93%	23%	19%	74%	12%	21%
Out of band	51%	42%	39%	48%	15%	95%	48%	19%	37%	48%	65%

When summary statistics is constructed, it indicates that on contrary to previous findings the most correlated market appeared to be Nikkei 225 representing Japan, but even its correlations are significantly lower than in a case of European markets computed from daily returns series.

Graph 6: Difference between average correlations designed subsamples vs. rest of the data – Adjusted daily returns



Graph 6 shows that when similar subsamples are analysed as in a case of daily returns no important difference can be perceived. In this case the EMU subsample shows to behave almost in same manner as a rest of the sample, which results into a minimal variations. The role of the Czech Koruna appears to be very significant in a case of possible volatility transmissions based on conditional correlation estimations and lowers potential risk bore by a global investor receiving returns in the Czech national currency. This offer opportunities to invest into the Czech Koruna in order to minimize possible risk in portfolios, which can earn high diversification effect due to very low correlations.

3. Granger Causality Test

3.1. Methodology

Although previous chapters clarified changes in correlations among various markets, the directions of possible spillovers remained unsolved. An idea of the volatility research accompanied by a direction of data flows is inspired by Mathur and Subrahmanyam (1990). The study suggested the Granger causality analysis as a toll, which can reliably determine directions of interdependencies.

Causality test employed by Granger (1969) is a relatively easy concept using standard Fisher test to find whether a zero hypothesis can or cannot be rejected. Granger causality test uses lag variables to find interconnections between researched data series. Due to lower complexity it is possible to do cross tests between all markets, however this is not a purpose of the work and thus only relations between Czech Republic and other markets are deeply analysed. The Granger causality is researched using a two-variable interdependence model described as follows:

$$\begin{aligned} x_t &= \sum_{i=1}^{n} \alpha_{1,i} x_{t-i} + \sum_{i=1}^{n} \beta_{1,i} y_{t-i} + \varepsilon_{1,t}, \\ y_t &= \sum_{i=1}^{n} \alpha_{2,i} x_{t-i} + \sum_{i=1}^{n} \beta_{2,i} y_{t-i} + \varepsilon_{2,t}, \end{aligned},$$

where x_t and y_t denote individual time series, which are mutually compared from Granger causality perspective.

A relation assuming $x_t \rightarrow y_t$, where is y_t dependent on x_t in a sense of Granger causality, can be computed through testing zero hypothesis $H_0 : \alpha_{2,i} = 0$, *for i = 1,...,n*, which rejection indicates that *y* is caused by *x* in terms of Granger causality, while opposing relation assuming $y_t \rightarrow x_t$ involves testing hypothesis $H_0 : \beta_{1,i} = 0$ *for i = 1,...,n*.

As it was shown in previous equations, if the Granger causality e.g. in case of x_t variable is intended to be computed, it is necessary to use its own lagged variables x_{t-i}, *for i = 1,...,n* in the model in order to compare a benefit of new data series y_{t-i}, *for i = 1,...,n*, which is regarded as a 'Granger origin'.

3.1.1. Akaike Information Criterion

A need for a proper definition of the Granger causality test brings a question "How many lagged variables should be used in the estimation?", which can be answered with a usage of Akaike information criterion. AIC can determine the optimal number of independent variables in the Granger causality model. AIC was proposed in Akaike (1974), it is a relative measure of the information lost when a given model is used for a purpose to describe a reality. The basic idea is to determine the relation between a precision and a complexity of the model. Akaike's test suggest to choose a model with the lowest possible AIC value. It compares benefits of additional variables with their total amount, the definition is as follows:

$$AIC = 2k - 2\ln(L)$$

where *k* is a number of parameters in the model and $\ln(L)$ is the value of maximized log-likelihood function for the estimated model. There was used another option how to compute AIC in this case. Under an assumption that errors of a model are normally, independently and identically distributed sum of squared residuals were computed:

$$SSR = \sum_{i=1}^{n} \hat{\varepsilon}_i^2$$

which can lead into another form of AIC test statistic:

$$AIC = 2\text{k} + n\left[\ln\left(SSR/n\right)\right]$$

This equation can be interpreted as a preference of lower sum of squared residuals, because also lower AIC means better outcome. While higher number of parameters *k* imposes penalty to estimated model in terms of AIC.

The AIC values were computed for all models characterized by previous hypothesis $H_0 : \beta_{1,i} = 0$ and thus a number of lagged variables in tested alternative were set to same amount. The maximum number of lags checked through AIC sorting algorithm were 10 lagged variables. This was conducted in order to achieve the best restricted model so resulting p-values reveal the Granger causality with a substantial elimination of possible spurious outcomes, which would resulted from an inappropriate model definition.

3.2. Estimations of Tests

For a purpose of more precise calculations, the whole data series, which starts on 5th April 1994 and ends on 31st March 2009, was divided on a basis of whole years into 15 periods as is depicted in following tables[20]. The reason for the division was an assumption, that Granger causality could differ during a long term. Finally because of a dual analysis of market interrelations based on both daily returns and adjusted daily returns, also all results involving Granger causality and AIC comparison have to be conducted two times.

Table 5 shows advised number of lagged variables according to the lowest AIC based on daily returns for each country, while Table 6 shows advised number of lags based on values including exchange rate effects for each country.[21]

In Tables 7, 8, 9 and 10 the computed p-values of F-tests testing depicted zero hypothesis are shown. The names of tables indicate, which direction of Granger causality is tested. Resulting p-values describe at which level of confidence a hypothesis of a non-existence of Granger causality can be rejected.

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20 Period 1994 starts on 5th April 1994 and ends on 31st December 1994, all periods from 1995 to 2007 starts on 1st January and ends on 31st December of depicted years, finally period 2008 starts on 1st January 2008 and ends on 31st March 2009.

21 Number of advised lagged variables is the same for the Czech republic in both estimations. This is caused, because values of indices are weighted by real returns in CZK.

Table 7: Granger Causality P-Values for Daily Returns - Direction of Causality from Foreign Countries to the Czech Republic:

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
1994	0.26	0.85	0.88	0.54	0.45	0.46	0.35	0.7	0.43	0.07	0.76
1995	0.66	0.07	0.19	0.47	0.07	0.79	0.01	0.04	0.02	0.02	0.75
1996	0.12	0.13	0.09	0.06	0.57	0.58	0.14	0.06	0.04	0.1	0.24
1997	0.11	0.35	0.09	0.03	0.09	0.29	0.19	0.13	0.29	0.33	0.01
1998	0.6	0.86	0.46	0.43	0.74	0.03	0.23	0.52	0.03	0.41	0.04
1999	0.06	0.56	0.84	0.37	0.11	0.17	0.55	0.4	0.78	0.37	0.57
2000	0.04	0.07	0.04	0.69	0.24	0.01	0.01	0.38	0.24	0.04	0.01
2001	0.39	0.72	0.43	0.29	0.19	0.44	0.74	0.35	0.35	0.35	0.07
2002	0.19	0.31	0.17	0.15	0.9	0.66	0.08	0.02	0.25	0.21	0.01
2003	0.52	0.49	0.65	0.6	0.97	0.33	0.97	0.45	0.78	0.81	0.05
2004	0.51	0.76	0.56	0.76	0.58	0.02	0.82	0.86	0	0.99	0
2005	0.01	0.2	0.02	0	0.03	0	0.01	0.03	0.11	0.17	0
2006	0.83	0.45	0.35	0.6	0.31	0	0.26	0.6	0.23	0.85	0
2007	0.33	0.01	0	0.01	0	0	0	0.15	0	0	0
2008	0.12	0	0.27	0.21	0	0	0.13	0.2	0	0.23	0

Table 8: Granger Causality P-Values for Daily Returns- Direction of Causality from the Czech Republic to Foreign Countries

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
1994	0.53	0.99	0.95	0.15	0.64	0.16	0.76	0.88	0.83	0.58	0.1
1995	0.03	0.32	0.14	0.21	0.17	0.98	0.29	0.68	0.35	0.48	0.48
1996	0.52	0.5	0.81	0.68	0.65	0.07	0.68	0.98	0.27	0.33	0.57
1997	0.18	0.51	0.08	0.26	0.74	0.66	0.22	0.1	0.98	0.51	0.58
1998	0.71	0.9	0.91	0.44	0.29	0.13	0.9	0.26	0.05	0.82	0
1999	0.19	0.49	0.17	0.05	0.12	0.9	0.23	0.03	0.12	0.4	0.98
2000	0.87	0.15	0.29	0.23	0.74	0.19	0.45	0.57	0.92	0.04	0.02
2001	0.96	0.53	0.24	0.02	0.23	0.17	0.44	0.69	0.47	0.71	0.41
2002	0.7	0.98	0.24	0.36	0.4	0.02	0.27	0.58	0.54	0.69	0.91
2003	0.28	0.22	0.27	0.44	0.09	0.39	0.57	0.34	0.63	0.19	0.31
2004	0.15	0.64	0.9	0.45	0.96	0.75	0.48	0.98	0.28	0.86	0.6
2005	0.7	0.25	0.65	0.78	0.66	0.08	0.79	0.66	0.41	0.62	0.75
2006	0.28	0.3	0.16	0.8	0.02	0.62	0.16	0.03	0.07	0.73	0.03
2007	0.44	0.14	0.25	0.19	0.2	0.47	0.17	0.35	0.74	0.07	0.75
2008	0.78	0	0	0	0	0.04	0.02	0	0.08	0	0.08

Table 9: Granger Causality P-Values for Adjusted Daily Returns - Direction of Causality from Foreign Countries to the Czech Republic

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
1994	0.47	0.5	0.83	0.66	0.55	0.72	0.19	0.6	0.29	0.03	0.46
1995	0.36	0.12	0.71	0.14	0.5	0.46	0.2	0.56	0.68	0.74	0.99
1996	0.93	0.64	0.06	0.91	0.54	0.31	0.31	0.65	0.58	0.43	0.64
1997	0.01	0.29	0.25	0.36	0.15	0.18	0.3	0.1	0.65	0.12	0.42
1998	0	0	0	0	0	0.2	0	0	0	0	0
1999	0	0	0	0	0	0.09	0	0	0	0	0.15
2000	0.07	0.16	0	0	0	0.02	0	0	0	0.05	0
2001	0.01	0	0	0	0	0.3	0	0	0	0	0
2002	0	0	0	0	0	0.97	0	0	0	0	0
2003	0	0	0	0	0	0.1	0	0	0	0	0
2004	0	0	0	0	0	0.01	0	0	0	0	0
2005	0	0	0	0	0.01	0	0	0.01	0	0	0.89
2006	0	0	0	0	0	0	0	0	0	0	0.01
2007	0	0	0	0	0	0	0	0	0	0	0
2008	0	0	0	0	0	0	0	0	0	0	0

Table 10: Granger Causality P-Values for Adjusted Daily Returns - Direction of Causality from the Czech Republic to Foreign Countries:

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA
1994	0.86	0.98	0.64	0.47	0.8	0.22	0.93	0.93	0.84	0.96	0.14
1995	0.82	0.42	0.74	0.77	0.31	0.76	0.98	0.34	0.18	0.86	0.32
1996	0.64	0.97	0.88	0.65	0.64	0.25	0.87	0.96	0.83	0.59	0.98
1997	0.1	0.41	0.01	0.05	0.3	0.57	0.13	0	0.29	0.44	0.07
1998	0.7	0.96	0.32	0.74	0.88	0.21	0.95	0.85	0.17	0.74	0.56
1999	0.17	0.67	0.84	0.33	0.26	0.91	0.22	0.93	0.55	0.69	0.82
2000	0.79	0.56	0.81	0.59	0.72	0.67	0.54	0.69	0.9	0.87	0.32
2001	0.63	0.56	0.36	0.01	0.17	0.25	0.41	0.62	0.78	0.15	0.98
2002	0.19	0.15	0.27	0.25	0.63	0.02	0.33	0.79	0.23	0.26	0.49
2003	0.4	0.96	0.18	0.99	0.71	0.85	0.53	0.34	0.76	0.76	0.62
2004	0.04	0.86	0.73	0.75	0.76	0.93	0.66	0.46	0.69	0.37	0.73
2005	0.5	0.94	0.55	0.43	0.56	0.12	0.67	0.25	0.84	0.87	0.89
2006	0.01	0.32	0.14	0.3	0.03	0.27	0.06	0.03	0.13	0.12	0.68
2007	0.24	0.82	0.37	0.74	0.61	0.42	0.62	0.37	0.55	0.59	0.92
2008	0.79	0.03	0	0.02	0	0.15	0	0	0.34	0.22	0.05

3.3. Results Analysis

Lower p-values indicate that the market is affected by Granger causality, while high p-values reject the causality relation. When results of daily returns are analysed and a level of confidence is set to 5%, it can be stated that the Czech market is dependent on other countries in Granger sense[22] since 2004, when the occurrence of lower p-values is more often, but this relationship was only unidirectional. A bidirectional relation can be dated only in year 2008, when 70% of the countries was dependent on the PSE. Before year 2004 the dependences are only sporadic, which is consistent with results of DCC MVGARCH, which revealed that from year 2004 PSE can be marked as integrated market. That also confirms that year 2004 was important for the Czech market and thus an accession of the Czech Republic improved an integration of PSE to other markets. The process of integration seems to be still in progress, because results from the latest year 2008 show that the interlinks are bidirectional.

Results of adjusted daily returns implies, that Granger causality occurred even earlier, but it could not be perceived through daily returns, because since 1998 p-values are near zero for most of the indices in the sample. This also confirms that relations between PSE and other markets were almost always unidirectional and in addition the dependences can be perceived through data including exchange rate effects. The outcomes confirm findings of Trešl and Blatná (2008), where was also a significant influence of Western European stock markets on the Central Europe perceived.

However it cannot be clearly answered whether the Granger causality is solely connected with exchange rates and thus the impact of equity market could be marginal. A comparison of the Granger causality with DCC MVGARCH estimates can conclude, that in a case of adjusted daily returns the dependence occurred only in terms of Granger causality, but conditional correlation was not approved. This means that on a daily basis

Alas the results of the Granger causality cannot give unambiguous answers, but they offer a useful outlook to interdependencies of PSE to other markets and it supports findings that years 1998 and 2004 were important milestones in history of the Czech equity market, because during that time p-values were significantly lowered. The outcomes also show that the Granger causality was only unidirectional in a history of PSE, but it can be assumed that this will change in a near future, because year 2008 already recorded bidirectional relations.

4. Conclusion

The assumed stages of PSE development during its existence, which would follow the most important milestones identified according previously mentioned studies, were confirmed and specified in the analysis. The DCC MV GARCH model demonstrated that straightforward outcomes can be received after a comparison of PSE with different markets. The dynamic model marked two important events in the history of the Czech equity market, i.e. year 1998 and the Asian/Russian crisis and also year 2004 and the accession of the Czech Republic into European Union. Before year 1998 PSE had all signs of a typical emerging market. In 1998 the awareness about the Czech market was spread out and an intermediate period began. It meant that an integration of PSE into European developed markets stood up to higher level. The intermediate period is typical with a mediocre interlinks to developed markets. Finally year 2004 was a very important event for PSE, a reason is not only the accession into EU, but also a full membership in the Federation of European Stock Exchanges and a granted status 'designated offshore securities market' to PSE from U.S. Securities and Exchange Commission within the meaning of Rule 902(b) of Regulation S under the Securities Act of 1933. This 'invitation' to a club of developed markets was 'accepted' by PSE and furthermore proved during the analysis. The outcomes showed that from year 2004 PSE reached a new stage, which is typical for other developed

22 Further mentioned dependencies are assumed to be in sense of Granger causality.

exchanges. Dvořák and Podpiera (2006) confirmed that a behaviour of investors changed among investors towards markets of new accession states into EU, however their study proposed that the change occurred immediately after an announcement of the enlargement. This contradicts my findings indicating that the accession was not anticipated by market agents and rather was a reason for a change of behaviour, which is a result implied by outputs of used models estimations.

Alas a membership in a 'developed markets club' also brought costs, which were fully counted during the global financial crisis in 2008, although an adverse impact of the crisis on the Czech stock market could be anticipated due to its global nature. A severity of the impact indicated by a high degree of interdependence was substantial and thus a probability of a shock-transmission was also high. Results also indicate that increasing conditional correlations were part of a long-lasting process, where the crisis was not its sudden cause, but rather its steady outcome.

Outcomes of Babetskii, Komárek, Komárková (2007) are comparable to results obtained from the analysis of adjusted daily returns data series. Their high degree of beta convergence, capturing long term periods, is similar to high degree of the Granger causality dependence, which was confirmed as unidirectional - a significant flow from foreign countries to the Czech Republic. In a case of the sigma convergence analysis, describing immediate changes, their findings imply that volatility spillovers slightly decreased over time, which conflicts behaviour similar to CCC MVGARCH model identified in the study showing a stable pattern oscillating nearly zero conditional correlation, which implies low probability of possible contagions. In contrary to simple outcomes of Babetskii, Komárek, Komárková (2007), the study identifies significantly different outcomes for unadjusted and adjusted daily returns. The possible volatility transmissions between the Czech and other European markets caused by high interdependences seems to a serious threat in latest years.

The effect of the Czech Koruna in conjunction with equity markets returns showed that although the high correlations to foreign markets can be a serious issue for the Czech market causing excessive volatilities spillovers, daily outcomes of possible contagions are minimized through exchange rates. This can cause PSE more attractive, when Czech national currency still exists, because a low correlation among investments is a desirable condition for investors. Moreover the Czech Koruna itself can be regarded as a complementary investment asset, which can minimize risks of investors on a daily frequency. Alas Granger causality tests reveal that although on a daily basis adjusted returns are uncorrelated, according DCC MV GARCH model, in a longer period information is transferred from foreign to Czech market. A deeper currency analysis should be conducted in order to depict real exchange rate effects.

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http://www.cnb.cz/ - Česká národní banka - Czech National Bank

http://yahoo.finance.com - Yahoo! Finance

http://www.kurzy.cz - Kurzy.cz - Financial site for experts and laity

List of Abbreviations:

AIC - Akaike information criterion

AUT - Austria

BEL - Belgium

BFGS - Broyden–Fletcher–Goldfarb–Shanno

CCC - constant conditional correlation

CNB - Czech National Bank

CZE - Czech Republic

CZK - Czech Crown/Czech Koruna

DCC - dynamic conditional correlation

EMU - European Monetary Union

EU - European Union

FESE - Federation of European Securities Exchanges

FRA - France

GARCH - generalised autoregressive conditional heteroskedasticity

GBR - Great Britain

GER - Germany

i.i.d - independent and identically distributed

JAP - Japan

MV - multivariate

NED - Netherlands

OPG - outer product of gradient

QMLE - quasi-maximum likelihood estimator

SPA - Spain

SWE - Sweden

SWZ - Switzerland

USA - United States of America

Table 1: Parameters of univariate GARCH estimations - daily returns

0.060	ω	AUT
0.133	α	AUT
0.844	β	AUT
0.066	ω	BEL
0.183	α	BEL
0.793	β	BEL
0.033	ω	FRA
0.101	α	FRA
0.888	β	FRA
0.018	ω	GBR
0.112	α	GBR
0.882	β	GBR
0.037	ω	GER
0.107	α	GER
0.884	β	GER
0.039	ω	JAP
0.105	α	JAP
0.888	β	JAP
0.031	ω	NED
0.130	α	NED
0.866	β	NED
0.033	ω	SPA
0.094	α	SPA
0.894	β	SPA
0.032	ω	SWE
0.080	α	SWE
0.911	β	SWE
0.046	ω	SWZ
0.124	α	SWZ
0.855	β	SWZ
0.018	ω	USA
0.082	α	USA
0.909	β	USA
0.114	ω	CZE
0.145	α	CZE
0.822	β	CZE

Table 2: Parameters of univariate GARCH estimations - Adjusted daily returns.

0.042	ω	AUT
0.078	α	AUT
0.899	β	AUT
0.022	ω	BEL
0.117	α	BEL
0.877	β	BEL
0.015	ω	FRA
0.068	α	FRA
0.928	β	FRA
0.012	ω	GBR
0.070	α	GBR
0.926	β	GBR
0.023	ω	GER
0.076	α	GER
0.917	β	GER
0.046	ω	JAP
0.072	α	JAP
0.917	β	JAP
0.022	ω	NED
0.090	α	NED
0.905	β	NED
0.013	ω	SPA
0.052	α	SPA
0.944	β	SPA
0.017	ω	SWE
0.057	α	SWE
0.939	β	SWE
0.022	ω	SWZ
0.080	α	SWZ
0.910	β	SWZ
0.013	ω	USA
0.042	α	USA
0.952	β	USA
0.064	ω	CZE
0.144	α	CZE
0.837	β	CZE

Table 5: Number of Lagged Variables Suggested by AIC - Daily Returns

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA	CZE
1994	8	4	2	1	1	1	2	9	4	2	1	10
1995	1	1	1	1	1	1	1	1	1	4	1	1
1996	4	1	2	1	8	1	4	4	1	1	3	1
1997	1	1	3	1	1	2	1	2	1	9	10	7
1998	1	1	2	2	2	2	2	1	2	1	1	1
1999	1	9	4	4	4	1	10	1	1	5	3	4
2000	1	1	1	3	1	4	1	1	1	1	1	1
2001	1	8	5	3	5	1	5	1	2	1	1	1
2002	1	1	7	7	7	1	7	7	1	6	1	1
2003	2	1	8	2	1	1	5	1	1	1	1	1
2004	5	2	1	1	2	1	2	1	2	1	1	6
2005	1	1	2	1	2	1	1	2	1	1	2	1
2006	9	6	6	1	6	6	1	6	7	1	6	1
2007	1	1	1	1	1	1	1	1	1	1	1	1
2008	1	5	5	5	5	2	9	5	1	5	2	1

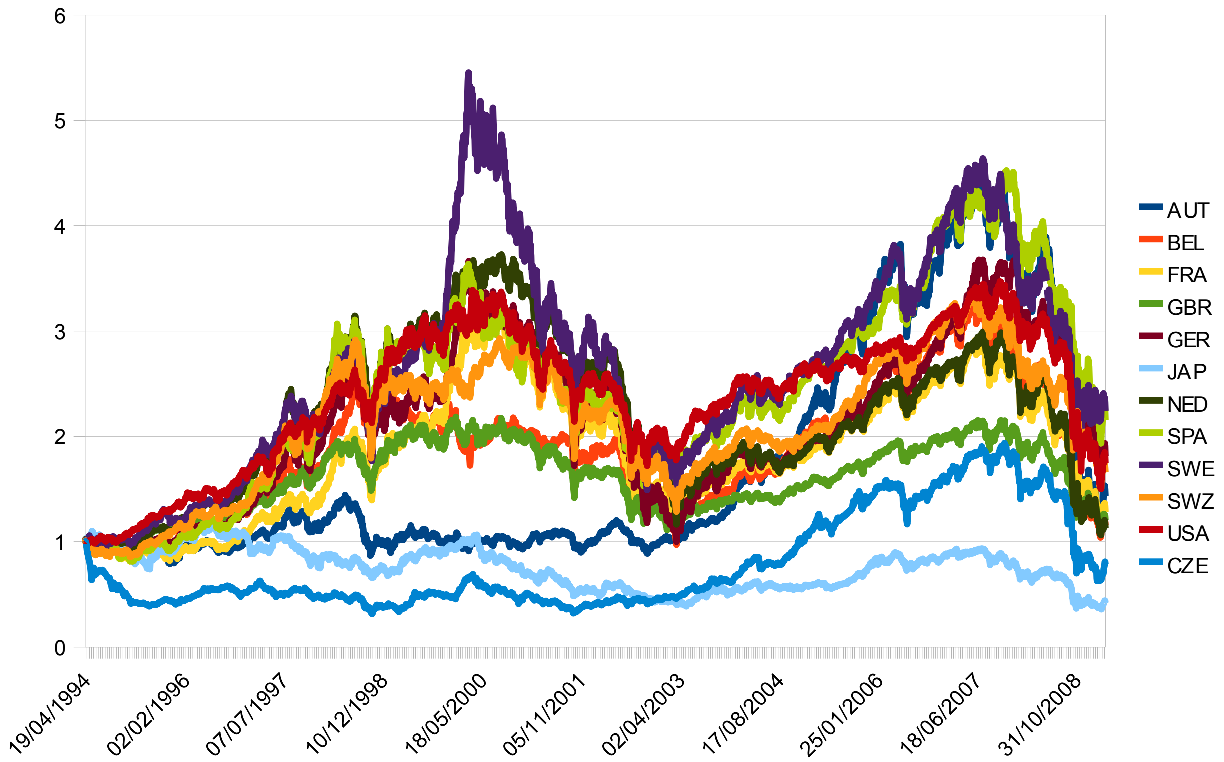
Table 6: Number of Lagged Variables Suggested by AIC - Adjusted Daily Returns

	AUT	BEL	FRA	GBR	GER	JAP	NED	SPA	SWE	SWZ	USA	CZE
1994	10	4	9	1	1	7	1	10	4	10	1	10
1995	1	1	3	4	1	1	1	1	2	5	3	1
1996	4	1	1	1	4	1	1	4	1	1	1	1
1997	1	1	3	1	1	2	1	5	1	10	1	7
1998	1	1	4	1	1	7	1	1	1	1	1	1
1999	3	1	1	5	4	1	6	1	1	6	2	4
2000	1	2	1	3	8	3	8	1	1	1	1	1
2001	1	8	5	3	8	1	5	1	1	2	1	1
2002	1	1	7	7	7	2	7	7	1	1	1	1
2003	2	1	8	3	1	1	1	1	1	9	1	1
2004	5	2	2	1	2	1	2	2	2	1	1	6
2005	1	1	1	7	1	1	1	1	1	1	1	1
2006	9	6	6	1	6	4	1	6	6	1	2	1
2007	1	1	1	1	1	8	1	1	1	4	1	1
2008	1	7	6	7	8	2	9	5	2	1	2	1

Appendix I

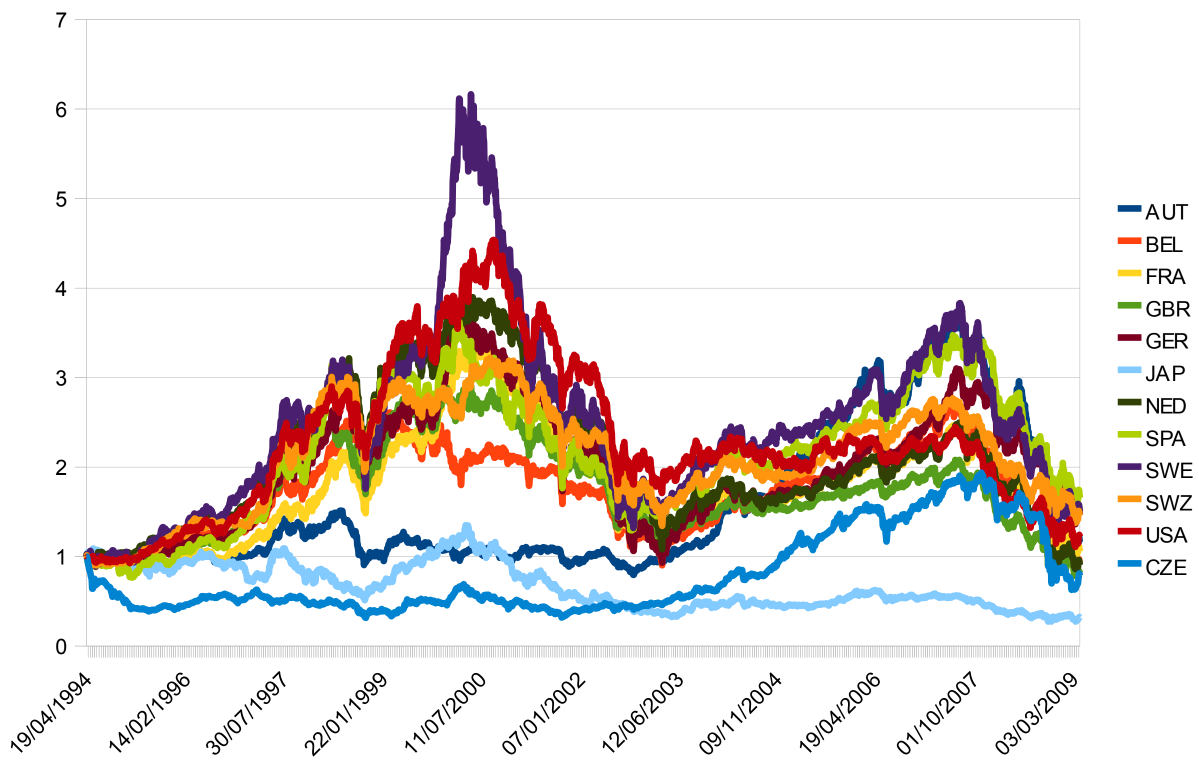
Daily Returns of Particular Indices (Normalized outcomes - initial value in 1994)

Data source: yahoo.finance.com



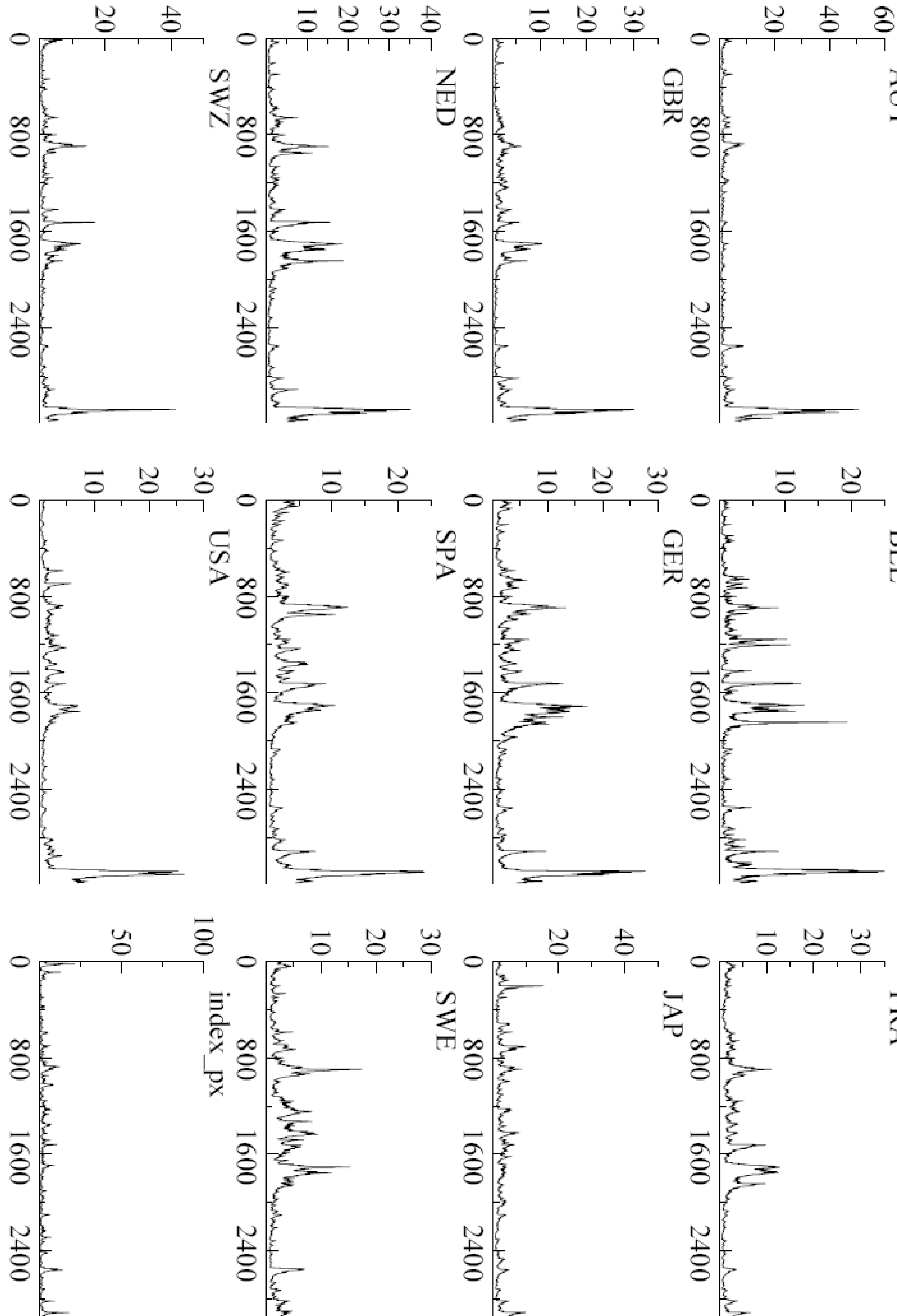
Adjusted Daily Returns of Particular Indices (Normalized outcomes - initial value in 1994)

Data source: yahoo.finance.com + Czech National Bank database (www.kurzy.cz)

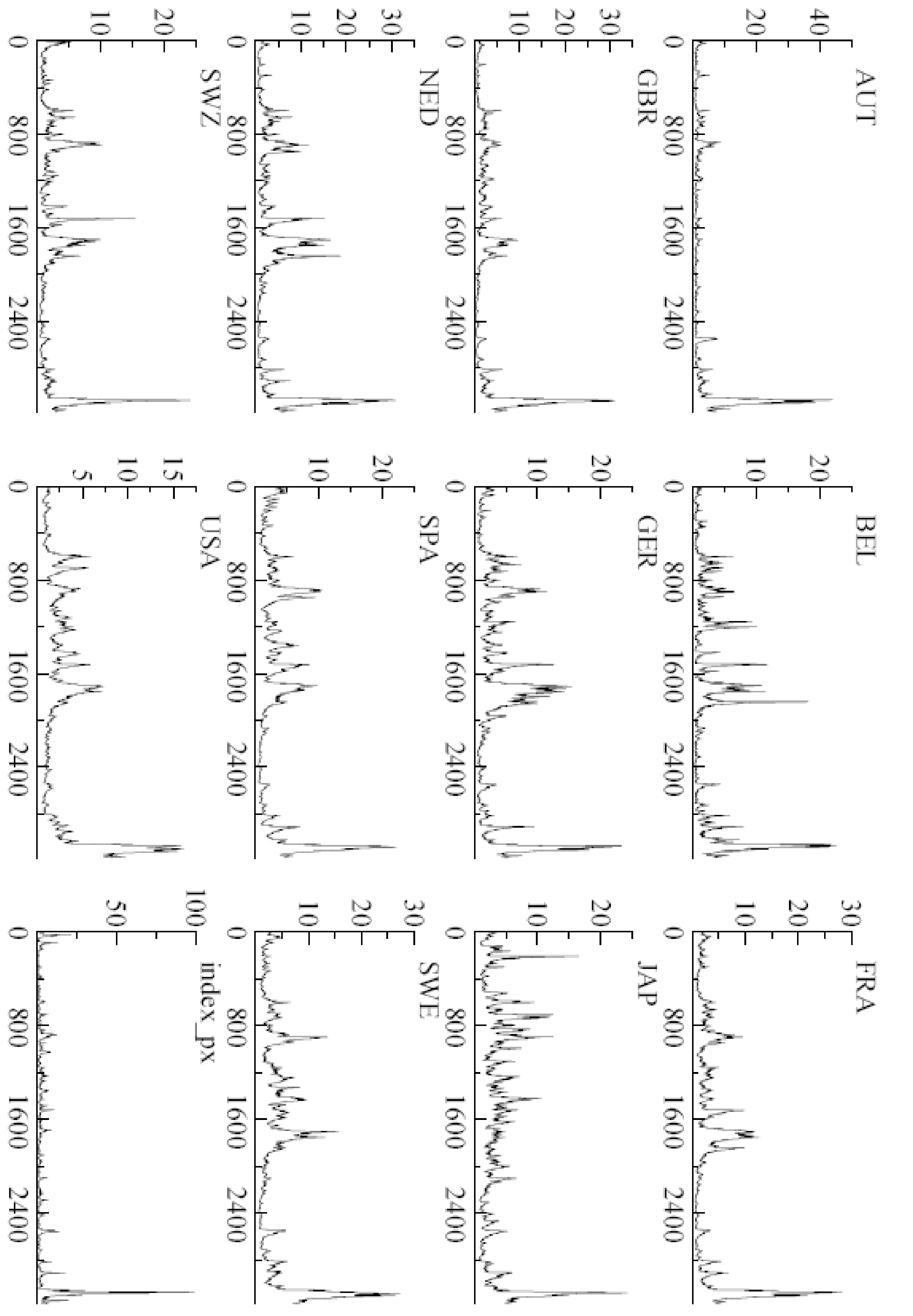


APPENDIX II

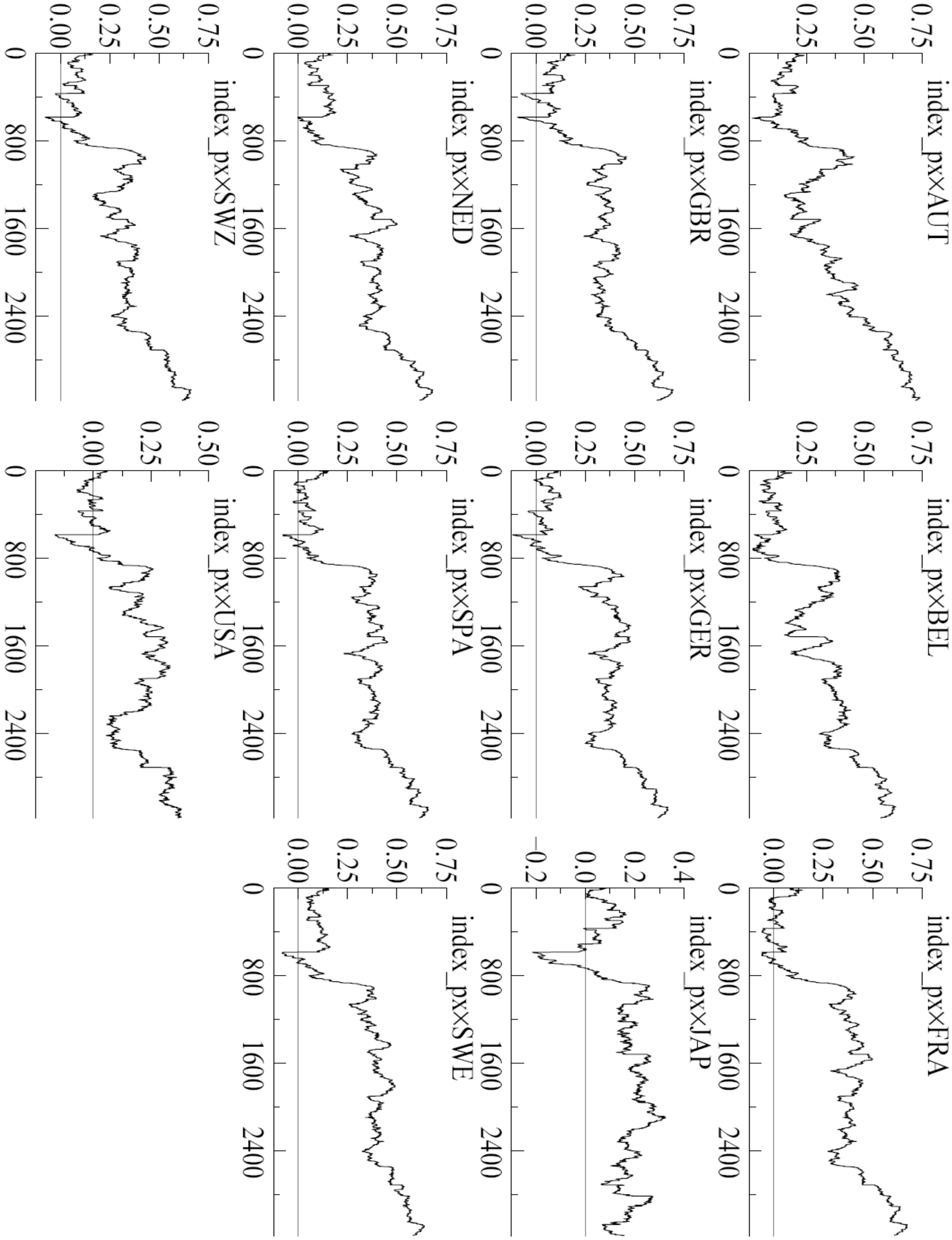
Variance Graphs - Daily Returns



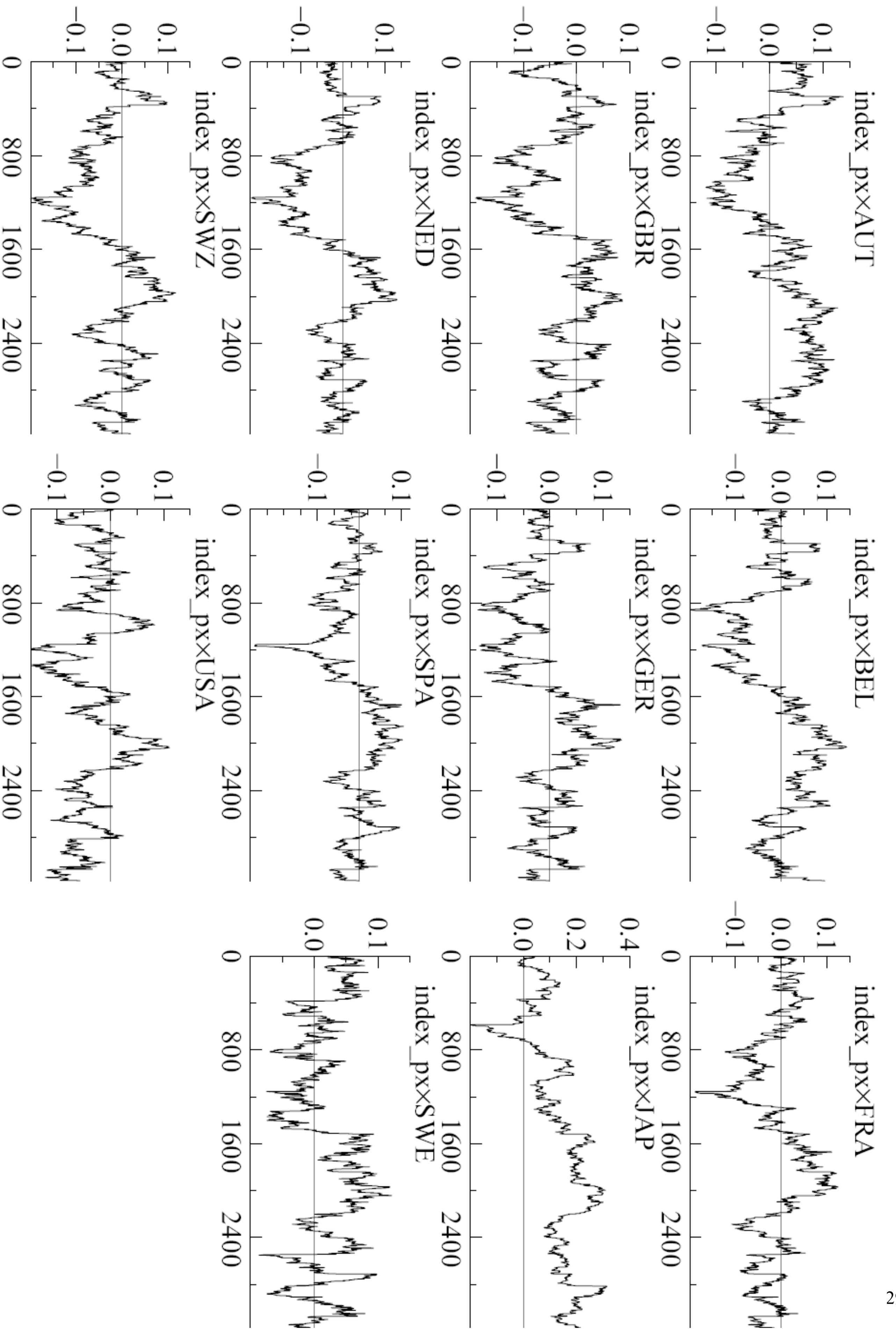
Variance Graphs - Adjusted Daily Returns



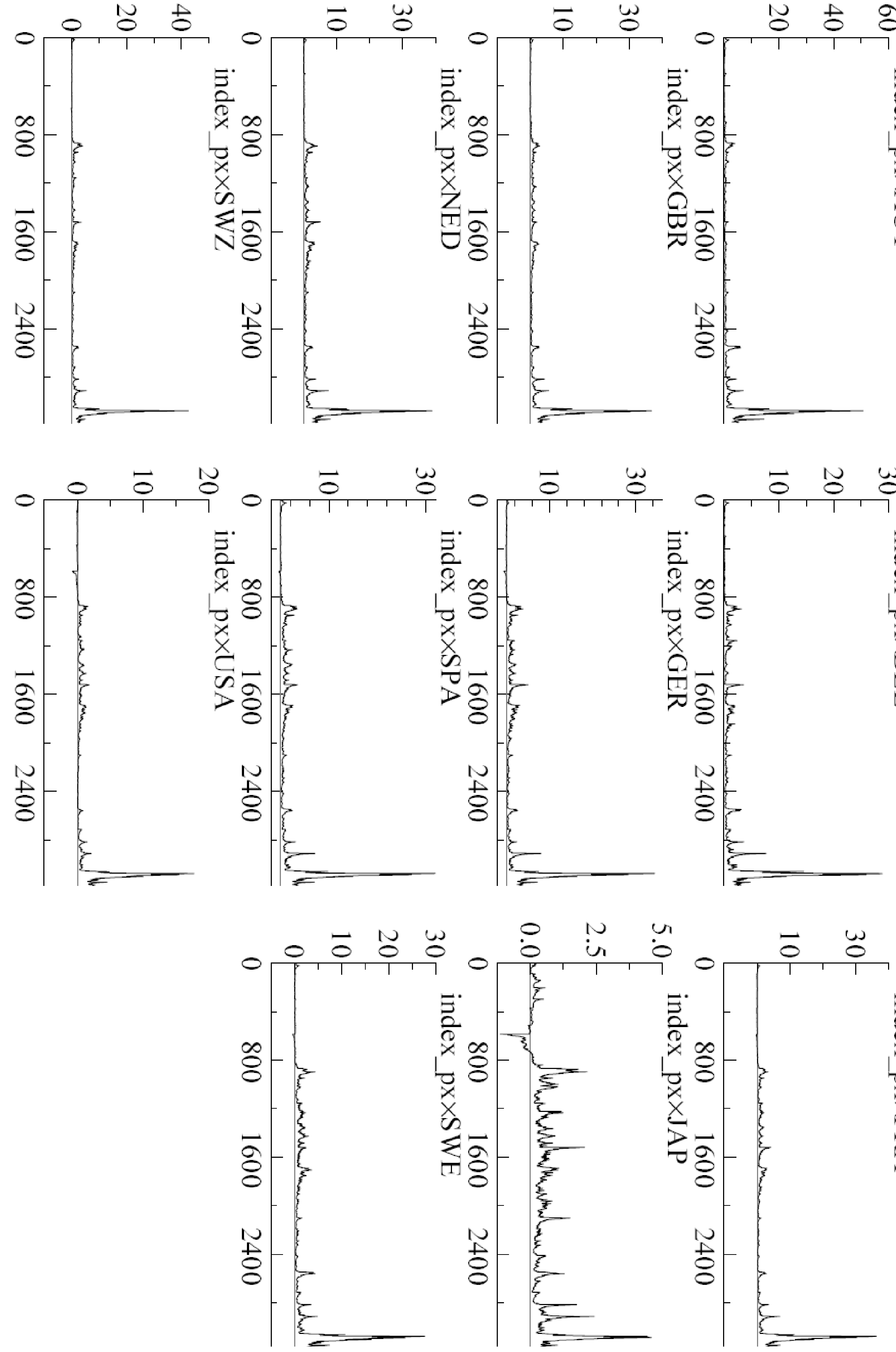
Conditional Correlation Graphs - Daily Returns



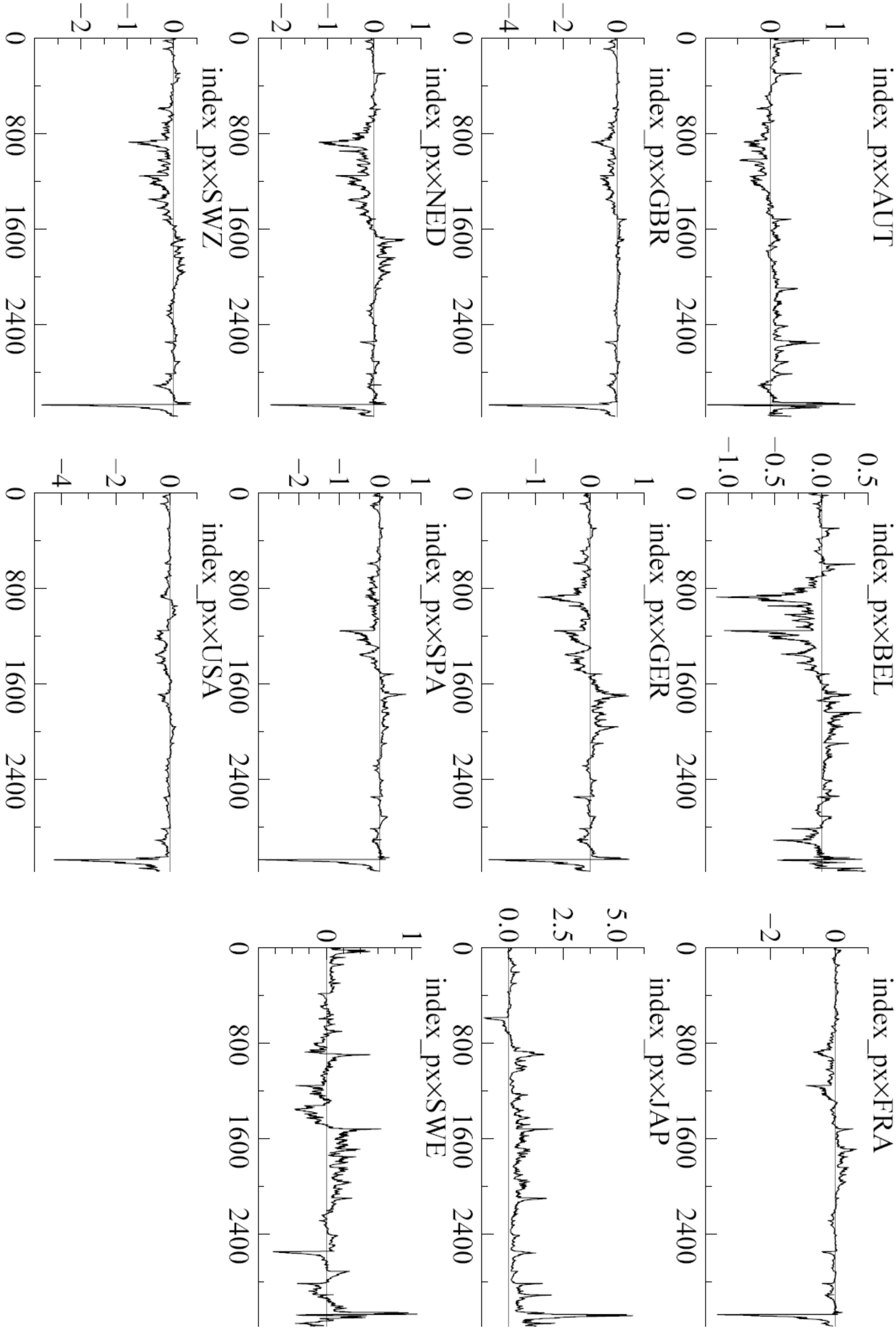
Conditional Correlation Graphs - Adjusted Daily Returns Data



Conditional Covariance Graphs - Daily Returns



Conditional Covariance Graphs - Adjusted Daily Returns



APPENDIX III

The elliptical DCC model as used in MultiGARCH library

PELAGATTI AND RONDENA (2004) shows in their OxMetrics library following relations, which lead to final estimation of the model[23]:

Let r_t be *k*-dimensional a vector process defined by

$$r_t / \Omega_{t-1} \sim EC_k(O, \Sigma_t, g) \quad (1)$$

where Ω_t is the filtration on which r_t is adapted and Σ_t is a positive definite Ω_{t-1} measurable dispersion matrix defined by

$$\Sigma_t = D_t R_t D_t \quad (2)$$

with D_t diagonal matrix defined by the recursion

$$D_t^2 = diag\{\omega_i\} + diag\{\kappa_i\} r_{t-1} r'_{t-1} + diag\{\lambda_i\} D_{t-1}^2 \quad (3)$$

$\circ$ representing element by element multiplication , and with R_t , conditional correlation matrix defined by the set of equations

$$\xi_t = D_t^{-1} r_t$$
$$Q_t = S(11' - A - B) + A \xi_{t-1} \xi'_{t-1} + B Q_{t-1} \quad (4)$$
$$R_t = diag\{Q_t\}^{-1/2} Q_t \, diag\{Q_t\}^{-1/2}$$

Equation (3) is a set of univariate GARCH models with parameters ω_i, κ_i *and* $\lambda_i, (i=1,\ldots,n)$, applied to every element of the vector r_t . Equation (4) controls the dynamics of the conditional correlation matrix R_t through the square symmetric matrices of parameters S, A and B . DING AND ENGLE (2001) show that if A, B and $(11' - A - B)$ are positive semi-definite and S is positive definite, then Q_t is also positive definite. In order to keep small the number of parameters to be simultaneously estimated, A and B are usually taken as scalars or set equal to $A = \alpha\alpha'$ and $B = \beta\beta'$, with α and β *k*-dimensional vectors of parameters. For the same reason, S , which can be shown to be the unconditional correlation matrix, is estimated using the sample correlation of the standardized residuals ξ_t .

If in equation (1) we take an elliptical distribution with density, then it is easy to build the log-likelihood function

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23 Following equations are described in a way that PELAGATTI AND RONDENA (2004) presented in their work in order to include all necessary prerequisites for analysis of the econometric approach used in the thesis.

$$l(\theta)=\sum_{t=1}^{T}\{\log c_m-\frac{1}{2}\log|\Sigma_t|+\log g(r_t\Sigma_t^{-1}r_t')\} \quad (5)$$

which, for a moderate number *k* of assets, may be maximized by numerical methods. When the number of assets, and with it, the number of parameters is too large, then a three steps estimation procedure may be exploited to obtain consistent, asymptotically normal, although inefficient, estimates of the parameters.

1st step

Since the marginals of an elliptical distribution are elliptical distributions of the same family (property P2.), the parameters ω_i, κ_i *and* λ_i of the sequence of univariate GARCH models in equation (3) may be estimated by maximizing the *k* univariate likelihoods $EC(0,\sigma_{ii},g)$, for $i=1,\ldots,k$. Through the recursion (3) the matrices D_t and the standardized residuals, $\xi_t=D_t^{-1}r_t$ may be estimated.

2nd step

The sample correlation matrix of the standardized residuals estimated in the first step is then used as estimate of the matrix S in equation (4).

3rd step

Using the estimated D_t and S , the likelihood

$$l(A,B)=\sum_{t=1}^{T}\{\log c_m-\frac{1}{2}\log|R_t|-\log|\hat{D}_t|+\log g(\hat{\xi}_t R_t^{-1}\hat{\xi}'_t)\}$$

is maximized with respect to the parameters in A and B (usually the two scalars α and β).

Consistency and asymptotic normality of the 3-step estimates may be demonstrated exploiting the same results of Newey and McFadden (1994) used by Engle and Sheppard (2001) and Pelagatti and Rondena (2004).

Let $\phi=(\omega_1,\kappa_1,\lambda_1,\ldots,\omega_k,\kappa_k,\lambda_k)'$ be the parameters' vector of the first step, $\rho=(s_{1,2},\ldots,s_{1,k},\ldots,s_{k,1},\ldots,s_{k,k-1})'$ contain the unique elements of matrix S , which are the 2nd step parameters, and $\psi=(\alpha,\beta)'$ be the vector of the parameters estimated in the 3rd step. Furthermore let

$$\begin{aligned} h^{(1)}(r,\phi) &= \nabla_\psi\{l_i(r_i,\omega_i,\kappa_i,\lambda_i)\}_{i=1,\ldots,k} \\ h^{(2)}(r,\phi,\rho) &= vech(\hat{\xi}\hat{\xi}'-S) \\ h^{(3)}(r,\phi,\rho,\psi) &= \nabla_\psi l_c(r,\phi,\rho,\psi) \end{aligned}$$

where $l_i(r_i,\omega_i,\kappa_i,\lambda_i)$ for $i=1,\ldots,k$, is the *t*-th contribution to the log-likelihood of the *i*-th univariate GARCH model (1st step) and $l_c(r,\phi,\rho,\psi)$ is the *t*-th contribution to the log-likelihood of the 3rd step. Letting

$\theta=(\psi', \rho', \phi')'$, the 3-step procedure can be cast in general method of moments form with sample "orthogonality" conditions

$$\bar{h}(\theta)=\frac{1}{T}\sum_{t=1}^{T} h(r_t,\theta)=0$$

where

$$h(r_t,\theta)=\begin{bmatrix} h^{(1)}(r,\psi) \\ h^{(2)}(r,\psi,\rho) \\ h^{(3)}(r,\psi,\rho,\phi) \end{bmatrix}$$

and the estimates are obtained by solving

$$\hat{\theta}_T=\{\theta:\min_{\theta}\bar{h}(\theta)'\bar{h}(\theta)\}$$

Since the system is just-identified with so many equations as parameters, the absolute minimum of the quadratic form (that is, 0) can be reached, and the orthogonality conditions relative to $h^{(i)}$ are independent of those relative to $h^{(i+j)}$ with j positive integer, the general method of moments estimate is equivalent to the 3-step estimate.

Now let

$$\begin{aligned}
H_{\phi}^{(1)} &= E\left[\nabla_{\phi} h^{(1)}(r,\phi_0)\right],\\
H_{\phi}^{(2)} &= E\left[\nabla_{\phi} h^{(2)}(r,\phi_{0,}\rho_0)\right],\\
H_{\rho}^{(2)} &= E\left[\nabla_{\rho} h^{(2)}(r,\phi_{0,}\rho_0)\right],\\
H_{\phi}^{(3)} &= E\left[\nabla_{\phi} h^{(3)}(r,\phi_{0,}\rho_{0,}\psi_0)\right],\\
H_{\rho}^{(3)} &= E\left[\nabla_{\rho} h^{(3)}(r,\phi_{0,}\rho_{0,}\psi_0)\right],\\
H_{\psi}^{(3)} &= E\left[\nabla_{\psi} h^{(3)}(r,\phi_{0,}\rho_{0,}\psi_0)\right],
\end{aligned}$$

the expected Jacobian matrix is given by

$$H=\mathrm{E}\left(\frac{\partial h(r,\theta)}{\partial\theta}\right)=\begin{pmatrix} H_{\psi}^{(1)} & 0 & 0 \\ H_{\psi}^{(2)} & H_{\rho}^{(2)} & 0 \\ H_{\psi}^{(3)} & H_{\rho}^{(3)} & H_{\phi}^{(3)} \end{pmatrix} \quad (6)$$

By adapting from Newey and McFadden (1994), under regularity conditions

$$\sqrt{T}(\hat{\theta}_T-\theta_0)\xrightarrow{D}\mathrm{N}(0,H^{-1}\Omega H^{-1}), \quad (7)$$

where

$$\Omega=\mathrm{E}\left[h(r,\theta_0)h(r,\theta_0)'\right]. \quad (8)$$

Consistent estimates of H and Ω may be obtained by substituting expectations with sample means:

$$\hat{\Omega}=\frac{1}{T}\sum_{t=1}^{T} h(r,\theta_0)h(r,\theta_0)'$$

and

$$\begin{aligned}
\hat{H}_{\phi}^{(1)} &= \frac{1}{T}\sum_{t=1}^{T}\left[\nabla_{\phi} h^{(1)}(r,\phi_0)\right], \\
&\cdots \\
\hat{H}_{\psi}^{(3)} &= \frac{1}{T}\sum_{t=1}^{T}\left[\nabla_{\psi} h^{(3)}(r,\phi_{0,}\rho_{0,}\psi_0)\right],
\end{aligned}$$

as blocks of $\hat{H}$.

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