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**"Government Policy in
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Government Policy in Monetary Economies

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Abstract

I study how the specific details of a micro founded monetary economy affect the determination of government policy. I consider three variants of the Lagos-Wright monetary framework: a benchmark were all markets are competitive; a case which allows for financial intermediaries; and a case with trading frictions. Although intitutions/frictions are shown to have a significant structural impact in the determination of policy, the calibrated artificial economies are observationally equivalent in steady state. The policy response to aggregate shocks is qualitatively similar in the variants considered. However, there are significant quantitative differences in the response of government policy to productivity shocks, mainly due to the idiosyncratic behavior of money demand. The variants with no trading frictions display the best fit to U.S. post-war data.

Keywords: government policy; lack of commitment; financial intermediation; trading frictions; micro founded models of money.

JEL classification: E13, E52, E62, E63

1 Introduction

Monetary theorists have long stressed the importance of analyzing monetary policy in the context of environments that have an explicit, micro founded role for money (e.g., see Wallace, 1998, 2001 and Williamson and Wright, 2010 for recent expositions). Following the work by Kocherlakota (1998), it is agreed that these environments should feature a lack of double coincidence of wants, imperfect record keeping and limited commitment. Beyond this set of minimal frictions, there does not appear to be any guide as to which other details or frictions we should attribute to artificial economies when studying government policy. For example, do we assume competitive markets or bilateral meetings? Do we allow for financial intermediaries?

Due to the interaction between fiscal and monetary policies,[1] it seems reasonable to expect that the specific details of a monetary economy may alter our analysis and conclusions regarding the (endogenous) determination of government actions. However, it is not immediately apparent what results are altered by which details and whether these effects, if present, are quantitatively significant.

In this paper, I analyze how the specific details of a micro founded monetary economy affect the determination of government policy, both in the long-run and in response to aggregate shocks. To this end, I study the monetary economy proposed by Lagos and Wright (2005), with the addition of a benevolent government that cannot commit to future policy choices and uses money, nominal bonds and distortionary taxes to finance the provision of a valued public good. As shown in Martin (2009, 2010a,b), lack of commitment on the part of the government provides a mechanism that explains the level of debt[2] (and by extension, other policy variables) and allows the response of government policy to aggregate shocks to display realistic features.

I consider three variants of the underlying monetary economy: "competitive markets", assumes all markets are perfectly competitive; "financial intermediation", assumes the existence of a technology that records financial (but not goods) transactions, which allows for the intermediation of fiat money, as in Berentsen, Camera and Waller (2007); and "trading frictions", assumes decentralized exchange in some markets and introduces an inefficiency due to bargaining over the terms of trade. The set of model variants considered here, although not exhaustive, is fairly representative of the type of micro founded monetary economies we would adopt as a benchmark to study the determination of government policy. The three environments only differ in the number of frictions that are present. The case with financial intermediation has only those frictions necessary to make a medium of exchange essential, as enumerated above; the case with competitive markets adds a financial friction which precludes the intermediation of fiat money in some markets; finally, the case with trading frictions features the most number of frictions, both financial and trading.

There are three main results on the determination of long-run policy. First, at the steady

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[1]This link is both theoretically and empirically relevant. Ohanian (1998) provides a thorough historical account for the U.S. economy. Sargent and Wallace (1981) first showed how the effects of monetary policy are affected by a given fiscal policy. Lucas (1986) famously postulated a set of principles for optimal fiscal and monetary policy. See Martin (2009) and Martin (2010b) for further discussion.

[2]Some alternative mechanisms that explain the level of debt have been proposed. Battaglini and Coate (2008) show that inefficiencies due to pork-barrel spending provide an explanation for the *distribution* of (real) debt in the long-run. Diamond (1965), Aiyagari and McGrattan (1998) and Shin (2006) provide a role for debt by using it to reduce some dynamic inefficiency.

state endowing the government with the ability to commit has no effect on government policy. This applies to all three cases and is a generalization of the result in Martin (2010b). Second, the response of long-run policy variables to permanent changes in fundamentals is largely similar across environments, both qualitatively and quantitatively. Third, resolving trading frictions (i.e., movings towards competitive markets) implies higher long-run debt and inflation, whereas resolving financial frictions (i.e., allowing for financial intermediation) has the opposite effect; both effects are quantitatively significant. The overall conclusion from these results is that although intitutions/frictions matter for the determination of policy, the calibrated artificial economies are observationally equivalent in steady state.

To evaluate the response of government policy to aggregate shocks, I consider two sources of aggregate fluctuations at annual frequencies: shocks to the marginal value of the public good ("expenditure" shocks) and shocks to the productivity of labor. The simulated economies match basic time-series properties of the post-war U.S. economy. The policy response to aggregate shocks is qualitatively similar in the three variants considered. However, there are some significant quantitative differences in the response of government policy to productivity shocks. In particular, the most significant difference between variants is in the behavior of debt and inflation in response to productivity shocks.

To further compare the three environments and assess their relative empirical plausibility, I evaluate each variant's implications along three dimensions: the persistence of policy variables; the relationship between the nominal interest rate and velocity of circulation (i.e., the money demand function); and the relationship between inflation and GDP (i.e., the Phillips curve). For all three tests, the case with competitive markets provides the best fit to the data, while the variant with trading frictions features the worse fit. The difference in performance across monetary economies stems mainly from the idiosyncratic behavior of money demand in response to productivity shocks.

The paper is organized as follows. Section 2 presents the basic environment and its three variants. Section 3 compares the properties of long-run policy across environments. Section 4 compares the properties of government policy in the presence of aggregate shocks. Section 5 concludes.

2   Monetary Economies without Aggregate Uncertainty

In this section, I present three variants of the Lagos-Wright monetary economy. I start by describing the basic environment, which is common for all cases. Next, I consider the three variants in sequence: first, the benchmark case where all markets are competitive; second, I add financial intermediaries; and third, I allow for decentralized trade and trading frictions due to bargaining between buyers and sellers. For each case, I derive the government budget constraint in a monetary equilibrium, which constitutes the constraint in the government's problem. The case with competitive markets is used as a benchmark for two practical reasons. First, it is the environment with the least detail, i.e., it is mathematically the most straightforward. Second, it is an economy that has been studied extensively in Martin (2010b, a) and for which we have theoretical results that will guide the analysis here.[3]

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[3]Alternatively, we could use the number of frictions to select the benchmark economy. In this sense, the case with financial intermediation has the least frictions while the case with trading frictions has the most.

2.1 Basic environment

There is a continuum of infinitely lived agents. Each period, two markets open in sequence: a day and a night market. In each stage a perishable good is produced and consumed. At the beginning of each period, agents receive an idiosyncratic shock that determines their role in the day market. With probability $\eta \in (0,1)$ an agent wants to consume but cannot produce the day-good, x, while with probability $1-\eta$ an agent can produce but does not want consume. A *consumer* derives utility $u(x)$, where u is twice continuously differentiable, with $u_x > 0$ and $u_{xx} < 0$. A *producer* incurs in utility cost ϕx, where $\phi > 0$.[4]

Assume agents lack commitment and are anonymous, in the sense that private trading histories are unobservable. Thus, credit transactions between agents are not possible. Since the day market features lack of double coincidence of wants, some medium of exchange is essential for trade to occur.[5]

At night, all agents can produce and consume the night-good, c. The production technology is assumed to be linear in hours worked, n. Utility from consumption is given by $U(c)$, where U is twice continuously differentiable, with $U_c > 0$, $U_{cc} < 0$. Disutility from labor is given by αn, where n is hours worked and $\alpha > 0$.

There is a benevolent government that supplies a valued public good g at night. To finance its expenditure, the government may use proportional labor taxes τ, print fiat money at rate μ and issue one-period nominal bonds, which are redeemable in fiat money. The public good is transformed one-to-one from the night-good. Agents derive utility from the public good according to $v(g)$, where v is twice continuously differentiable, with $v_g > 0$, $v_{gg} < 0$.

The government can commit to policies within the period, but lacks the ability to commit to future policy choices. It announces period policy $\{B', \mu, \tau, g\}$ at the beginning of the day, before agents' preference shocks are realized. The government only actively participates in the night-market, i.e., taxes are levied on hours worked at night and open market operations are conducted in the night market. As in Aruoba and Chugh (2008), Berentsen and Waller (2008) and Martin (2010b), public bonds are book-entries in the government's record. Since bonds are not physical objects and the government does not participate in the day market (i.e., cannot intermediate or provide third-party verification), bonds are not used as a medium of exchange in the day market and thus, money is essential.[6]

All nominal variables—except for bond prices—are normalized by the aggregate money stock. Thus, today's aggregate money supply is equal to 1 and tomorrow's is $1 + \mu$. The government budget constraint is

$$1 + B + pg = p\tau n + (1 + \mu)(1 + qB'), \tag{1}$$

where B is the current aggregate bond-money ratio, p is the—normalized—market price of the night-good c, and q is the price of a bond that earns one unit of fiat money in the following night market. "Primes" denote variables evaluated in the following period. Thus, B' is tomorrow's

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[4]A linear production cost is typically adopted in calibrated applications of the Lagos-Wright framework and is made here to simplify exposition. Most results can be obtained with a general convex production function.

[5]See Kocherlakota (1998), Wallace (2001) and Shi (2006).

[6]Illiquid bonds are ex-ante socially optimal since they allow the government to trade-off distortions across periods. See Martin (2010b).

aggregate bond-money ratio.

2.2 Competitive Markets

Assume that the day and night markets are perfectly competitive. An agent arrives to the night market with individual money balances m and government bonds b. Since bonds are redeemed in fiat money at par, the composition of an agent's nominal portfolio at the beginning of the night is irrelevant. Let $z \equiv m + b$, i.e., total—normalized—nominal holdings. The budget constraint of an agent at night is

$$pc + (1 + \mu)(m' + qb') = p(1 - \tau)n + z. \tag{2}$$

Let $V(m, b)$ be the value of entering the day market with money balances m and bond balances b, and let $W(z)$ be the value of entering the night market with total nominal balances z. After solving n from (2), the problem of an agent in the night market is

$$W(z) = \max_{c,m',b'} U(c) - \frac{\alpha c}{(1 - \tau)} + \frac{\alpha(z - (1 + \mu)(m' + qb'))}{p(1 - \tau)} + v(g) + \beta V(m', b').$$

The first-order conditions are

$$U_c - \frac{\alpha}{(1 - \tau)} = 0 \tag{3}$$

$$-\frac{\alpha(1 + \mu)}{p(1 - \tau)} + \beta V'_m = 0 \tag{4}$$

$$-\frac{\alpha q(1 + \mu)}{p(1 - \tau)} + \beta V'_b = 0. \tag{5}$$

Focusing on a symmetric equilibrium, we can follow Lagos and Wright (2005) to show that (4) and (5) imply all agents exit the night market with the same money and bond balances.[7] Furthermore, the value function W is linear, $W_z = \frac{\alpha}{p(1-\tau)}$ and thus, $W(z) = W(0) + \frac{\alpha z}{p(1-\tau)}$. We also get

$$q = \frac{V'_b}{V'_m}. \tag{6}$$

The ex-ante value for an agent that enters the day market is $V(m, b) = \eta V^c(m, b) + (1 - \eta)V^p(m, b)$, where V^c and V^p are the values of being a consumer and a producer in the day market, respectively.

A consumer faces a day-budget constraint, $\tilde{p}x \leq m$, where $\tilde{p}$ is the—normalized—market price of good x. Using ξ as the Lagrange multiplier associated with this constraint, the problem of a consumer can be written as

$$V^c(m, b) = \max_x u(x) + W(0) + \frac{\alpha(m + b - \tilde{p}x)}{p(1 - \tau)} + \xi(m - \tilde{p}x).$$

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[7]Since V is linear in b, a non-degenerate distribution of bonds is possible in equilibrium. Here, we focus on symmetric equilibria. See Aruoba and Chugh (2008) and Martin (2010b) for related discussions.

The first-order condition implies

$$\xi = \frac{u_x}{\tilde{p}} - \frac{\alpha}{p(1-\tau)}. \tag{7}$$

From the envelope condition we get $V_m^c = \frac{u_x}{\tilde{p}}$ and $V_b^c = \frac{\alpha}{p(1-\tau)}$.

Let κ be an individual producer's output of the day-good. The problem of a producer is

$$V^p(m,b) = \max_{\kappa} -\phi\kappa + W(0) + \frac{\alpha(m+b+\tilde{p}\kappa)}{p(1-\tau)}.$$

The first-order condition is

$$-\phi + \frac{\alpha\tilde{p}}{p(1-\tau)} = 0. \tag{8}$$

The envelope condition implies $V_m^p = V_b^p = \frac{\alpha}{p(1-\tau)}$. We can now derive:

$$V_m = \frac{\eta u_x}{\tilde{p}} + \frac{(1-\eta)\alpha}{p(1-\tau)} \tag{9}$$

$$V_b = \frac{\alpha}{p(1-\tau)}. \tag{10}$$

The day aggregate resource constraint is $\eta x = (1-\eta)\kappa$. A standard result in monetary economies is that consumers spend all their money in the day market. The day market clearing condition is then $\eta = (1-\eta)\tilde{p}\kappa$, which implies $\tilde{p} = \frac{1}{x}$. Substitute this expression into (8) and get

$$\phi x = \frac{\alpha}{p(1-\tau)}. \tag{11}$$

At night, all agents choose the same money and bond holdings; thus, $m' = 1$ and $b' = B'$. The night aggregate resource constraint is $c + g = n$, where n is aggregate labor. Note that private consumption c and public consumption g are the same for all agents, whereas individual labor depends on whether an agent was a consumer or a producer during the day.

Using $\tilde{p} = \frac{1}{x}$ and (11), equations (9) and (10) imply $V_m = x(\eta u_x + (1-\eta)\phi)$ and $V_b = \phi x$. Furthermore, (7) can be written as $\xi = x(u_x - \phi)$; in a monetary equilibrium $\xi \geq 0$, which implies $u_x - \phi \geq 0$, i.e., $V_m \geq V_b$.

We can now collect the remaining equations that summarize agents' behavior in any given period. After some rearrangement we can write equations (3), (4), (6) and (11) as

$$\mu = \frac{\beta x'(\eta u_x' + (1-\eta)\phi)}{\phi x} - 1 \tag{12}$$

$$\tau = 1 - \frac{\alpha}{U_c} \tag{13}$$

$$p = \frac{U_c}{\phi x} \tag{14}$$

$$q = \frac{\phi}{\eta u_x' + (1-\eta)\phi}. \tag{15}$$

We can now use the night-resource constraint and equilibrium conditions (12)—(15) to write the government budget constraint (1) in a monetary equilibrium as

$$(U_c - \alpha)c - \alpha g + \beta\eta x'(u_x' - \phi) + \beta\phi x'(1+B') - \phi x(1+B) = 0. \tag{16}$$

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2.3 Financial intermediation

One important inefficiency in the benchmark economy described above is due to the inability of producers in the day to lend their idle cash balances. In this section, we consider a monetary economy that resolves this *financial* friction by assuming the existence of a technology that record financial transactions, as suggested by Berentsen, Camera and Waller (2007). Financial intermediation is conducted by perfectly competitive banks, which accept nominal deposits and make nominal loans. Banks are endowed with a technology that allows them to record financial histories at zero cost. However, trading histories cannot be recorded. Banks cannot issue their own notes, nor can they provide third-party verification for government bonds in transactions between agents. Thus, money is still used as the only medium of exchange in the day market. The added feature is that now, at the beginning of each period, sellers can deposit their money holdings at banks, and buyers can borrow money from banks. Deposits and loans mature at night. Perfect competition in the banking sector implies that the deposit and loan interest rates are equal. Let $i \geq 0$ be the bank nominal interest rate. Assume perfect enforcement and no borrowing constraints in financial markets.

An agent enters the night market with nominal balances z, which may include fiat money, government bonds and bank claims. These bank claims may be positive (deposits) or negative (loans) and include the accrued interest. Thus, the problem of the agent at night remains functionally the same as before.

A consumer in the day market enters the period with m units of fiat money and b units of government bonds. Being generally cash-constrained, he borrows ℓ units of fiat money from the bank with the obligation to repay $(1+i)\ell$ units of money at night. The consumer then uses $m+\ell$ to buy x goods at price $\tilde{p}$. Thus, his starting nominal balances at night—net of loan obligations—are equal to $m + b - \tilde{p}x - i\ell$. Using ξ as the Lagrange multiplier associated with the budget constraint, the problem of a consumer is

$$V^c(m,b) = \max_{x,\ell} u(x) + W(0) + \frac{\alpha(m + b - \tilde{p}x - i\ell)}{p(1-\tau)} + \xi(m + \ell - \tilde{p}x).$$

The first-order conditions imply

$$\xi = \frac{u_x}{\tilde{p}} - \frac{\alpha}{p(1-\tau)}$$

$$i = \frac{u_x p(1-\tau)}{\alpha\tilde{p}} - 1.$$

Note that $i = 0$ if and only if $\xi = 0$.

A producer has no use for cash and thus, deposits his money holdings at the bank. If he starts the period with m units of money and b units of government bonds, deposits d units of money and sells κ units of the day-good at price $\tilde{p}$, his starting nominal balances at night—including deposit claims—are $m + b + \tilde{p}\kappa + id$. The problem of a producer can be written as

$$V^p(m,b) = \max_{\kappa,d} -\phi\kappa + W(0) + \frac{\alpha(m + b + \tilde{p}\kappa + id)}{p(1-\tau)} + \xi_d(m - d),$$

where ξ_d is the Lagrange multiplier associated with the constraint that states that an agent cannot

deposit more than his fiat money holdings. The first-order conditions imply

$$\phi = \frac{\alpha \tilde{p}}{p(1-\tau)}$$

$$\xi_d = \frac{\alpha i}{p(1-\tau)}.$$

The second equation shows that producers deposit all their money holdings if $i > 0$. Without loss of generality, assume that they also deposit all their money holdings when $i = 0$.

The market clearing conditions are $\eta \ell = (1-\eta)d$ and $(1-\eta)\tilde{p}\kappa = \eta(1+\ell)$. The first equation states that the total amount of money borrowed from banks has to equal the total amount of money that was deposited at banks. The second equation states that the nominal value of total output sold by producers has to equal total money holdings—including loans—of buyers. Note that since producers deposit all their money holdings, $d = 1$ and thus, $\ell = \frac{1-\eta}{\eta}$, which implies $(1-\eta)\tilde{p}\kappa = 1$. Using the day-resource constraint, $\eta x = (1-\eta)\kappa$, we get $\tilde{p} = \frac{1}{\eta x}$. Thus, the equilibrium in the day market is characterized by

$$i = \frac{u_x}{\phi} - 1$$

$$\eta \phi x = \frac{\alpha}{p(1-\tau)}.$$

We also get $V_m = \eta u_x x$ and $V_b = \eta \phi x$. In a monetary equilibrium, $\xi \geq 0$, which implies $u_x - \phi \geq 0$ (i.e., $V_m \geq V_b$), as in the case with no financial intermediation. We can now collect the equations characterizing agents' behavior in equilibrium:

$$\mu = \frac{\beta u_x' x'}{\phi x} - 1 \tag{17}$$

$$\tau = 1 - \frac{\alpha}{U_c} \tag{18}$$

$$p = \frac{U_c}{\eta \phi x} \tag{19}$$

$$q = \frac{\phi}{u_x'}. \tag{20}$$

Note that $q = \frac{1}{1+i}$. The government budget constraint in a monetary equilibrium is

$$(U_c - \alpha)c - \alpha g + \beta \eta x'(u_x' - \phi) + \beta \eta \phi x'(1 + B') - \eta \phi x(1 + B) = 0. \tag{21}$$

2.4 Trading frictions

Consider the benchmark economy without financial intermediation, but assume now that the day-good is traded in a decentralized market. I abstract from search frictions, i.e., the possibility that an agent does not meet someone with whom to trade in the day-market.[8] Thus, let $\eta = \frac{1}{2}$ and assume all agents in the day are matched in consumer-producer pairs. In these bilateral meetings,

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[8]Note that this is a standard assumption when these type of models are calibrated.

consumers and producers bargain over the terms of trade: a quantity x and a monetary transfer (normalized by the aggregate money stock) δ. Here, we need to make the additional assumption $u(0) = 0$ to ensure the trading surplus is positive. In terms of the bargaining problem, I follow the analysis in Aruoba, Rocheteau and Waller (2007) and adopt the proportional solution due to Kalai (1977). In Appendix A, I consider the Nash (1950) bargaining solution and show why it is not suitable for policy analysis in this context.

Suppose the terms of trade agreed in a bilateral meeting are $\{x, \delta\}$. A consumer that starts the period with nominal holdings $\{m, b\}$ gets $u(x) + W(m - \delta + b) = u(x) + W(0) + \frac{\alpha(m+b-\delta)}{p(1-\tau)}$; similarly, a producer starting with the same nominal holdings gets $-\phi x + W(m + \delta + b) = -\phi x + W(0) + \frac{\alpha(m+b+\delta)}{p(1-\tau)}$. If no agreement is reached, both agents get $W(m + b) = W(0) + \frac{\alpha(m+b)}{p(1-\tau)}$.

Given consumer's bargaining weight $\theta \in (0, 1]$ and consumer's money holdings m, the proportional solution is given by

$$\{x, \delta\} = \underset{x, \delta \leq m}{\operatorname{argmax}} \, u(x) - \frac{\alpha\delta}{p(1-\tau)}$$

subject to $(1 - \theta)\left(u(x) - \frac{\alpha\delta}{p(1-\tau)}\right) = \theta\left(-\phi x + \frac{\alpha\delta}{p(1-\tau)}\right)$. Following standard arguments, we can show that in a monetary equilibrium $\delta = m = 1$ and

$$-h(x) + \frac{\alpha}{p(1-\tau)} = 0, \tag{22}$$

where $h(x) \equiv (1 - \theta)u(x) + \theta\phi x$.

The terms of trade are not affected by the money holdings of a producer. Thus, $V_m^p = W_z = \frac{\alpha}{p(1-\tau)}$, i.e., $V_m^p = h(x)$ by (22). On the other hand, $V_m^c = u_x \frac{\partial x}{\partial m} + \frac{\alpha}{p(1-\tau)}(1 - \frac{\partial \delta}{\partial m})$. If a consumer brings one more unit of money to the match, then x increases by $\frac{\alpha}{p(1-\tau)h_x}$ and δ goes up by 1. Thus, using (22), we get $V_m^c = \frac{h(x)u_x}{h_x}$, and so $V_m = \frac{h(x)(u_x+h_x)}{2h_x}$. The value of starting the period with one more unit of bonds is still W_z, regardless of whether the agent is a consumer or a producer. Thus, $V_b = h(x)$. No arbitrage implies $V_m \geq V_b$, i.e., $u_x - h_x \geq 0$, which simplifies to $u_x - \phi \geq 0$, as in the case without trading frictions.

In a monetary equilibrium we get

$$\mu = \frac{\beta h(x')(u_x' + h_x')}{2h(x)h_x'} - 1 \tag{23}$$

$$\tau = 1 - \frac{\alpha}{U_c} \tag{24}$$

$$p = \frac{U_c}{h(x)} \tag{25}$$

$$q = \frac{2h_x'}{u_x' + h_x'}. \tag{26}$$

The government budget constraint in a monetary equilibrium is

$$(U_c - \alpha)c - \alpha g + \frac{\beta h(x')(u_x' - h_x')}{2h_x'} + \beta h(x')(1 + B') - h(x)(1 + B) = 0. \tag{27}$$

9

3 Government Policy without Aggregate Uncertainty

3.1 Problem of the government

To characterize government policy with lack of commitment, I adopt the notion of Markov-perfect equilibrium, i.e., where policy is a function of fundamentals only.[9] For each of the environments characterized above, the government budget constraint in a monetary equilibrium is a function of B, B', x, x', c and g—see (16), (21) and (27). Note that x' is implemented by the government tomorrow, depending on the inherited level of debt. Thus, let $x' = \mathcal{X}(B')$, where $\mathcal{X}$ is the *policy* that the current government expects its future-self to follow. Thus, regardless of the environment, we can write the government budget constraint compactly as

$$\varepsilon(B, B', x, \mathcal{X}(B'), c, g) = 0. \tag{28}$$

Another constraint in the government's problem is the non-negativity constraint, $u_x - \phi \geq 0$, which is derived from the no-arbitrage condition $V_m \geq V_b$.

Notice that from equations (12), (17) and (23), for a given $x' = \mathcal{X}(B')$, a higher μ implies a lower x in all three environments. In other words, given current debt policy and future monetary policy, the allocation of the day-good is a function of *current* monetary policy. In the discussion that follows, I will refer interchangeably to variations in the day-good allocation and variations in current monetary policy.

Martin (2010*b*) analyzes government policy at any level of debt for the case with competitive markets and shows several relevant properties of the equilibrium. Among other results, Proposition 2 in that paper establishes that for all $B \geq -1$: *(i)* $B' > -1$; *(ii)* $\mathcal{X}'_B < 0$; and *(iii)* the non-negativity constraint, $u_x - \phi \geq 0$, does not bind. Applying similar arguments, we can obtain these results for the environments with financial intermediation and trading frictions.[10] A formal proof is a straightforward application of the proof in Martin (2010*b*), so it is omitted here to save space. Result *(i)* allows us to restrict attention to $B \geq -1$, i.e., positive net nominal government obligations, which is convenient given that the focus here is on the empirical predictions of the model. Since we focus on $B \geq -1$, *(ii)* rules out the possibility of $\mathcal{X}_B = 0$ in equilibrium (i.e., monetary policy not reacting to inherited debt), while *(iii)* allows us to ignore the non-negativity constraint when formulating the government's problem.

Given $B \in \Gamma \equiv [-1, \bar{B}]$, where $\bar{B} \in (0, \infty)$, and the perception that future governments implement $\mathcal{X}(B)$, the problem of the current government is

$$\mathcal{V}(B) = \max_{B' \in \Gamma, x, c, g} \eta(u(x) - \phi x) + U(c) - \alpha(c + g) + v(g) + \beta \mathcal{V}(B')$$

subject to (28). Note that we use the day and night resource constraints to simplify the expression above. Also, it is understood that $\eta = \frac{1}{2}$ for the case with trading frictions.

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[9]See Maskin and Tirole (2001) for a definition and justification of this solution concept. For recent applications to dynamic policy games see Ortigueira (2006), Klein, Krusell and Ríos-Rull (2008), Díaz-Giménez, Giovannetti, Marimón and Teles (2008), Martin (2009, 2010*b*) and Azzimonti, Sarte and Soares (2009), among others.

[10]Results *(ii)* and *(iii)* do not hold in general if we assume the Nash solution to the bargaining problem. See Appendix A for further discussion.

Definition 1 *A Markov-Perfect Monetary Equilibrium (MPME) is a set of functions* $\{\mathcal{B}, \mathcal{X}, \mathcal{C}, \mathcal{G}, \mathcal{V}\}$:
$\Gamma \to \mathbb{R}^5$, *such that for all* $B \in \Gamma$:

(i) $\{\mathcal{B}(B), \mathcal{X}(B), \mathcal{C}(B), \mathcal{G}(B)\} = \underset{B' \in \Gamma, x, c, g}{\operatorname{argmax}} \ \eta(u(x) - \phi x) + U(c) - \alpha(c + g) + v(g) + \beta\mathcal{V}(B')$

subject to $\varepsilon(B, B', x, \mathcal{X}(B'), c, g) = 0$; *and*

(ii) $\mathcal{V}(B) = \eta\big(u(\mathcal{X}(B)) - \phi\mathcal{X}(B)\big) + U(\mathcal{C}(B)) - \alpha(\mathcal{C}(B) + \mathcal{G}(B)) + v(\mathcal{G}(B)) + \beta\mathcal{V}(\mathcal{B}(B))$.

Assume the policy function $\mathcal{X}(B)$ followed by future governments is differentiable.[11] Further-more, let $\bar{B}$ be large enough so that $\mathcal{B}(B) < \bar{B}$ for all $B \in \Gamma$. Using λ as the Lagrange multiplier associated with (28), the first-order conditions are

$$\lambda(\varepsilon_{B'} + \varepsilon_{x'}\mathcal{X}'_B) + \beta\lambda'\varepsilon'_B = 0 \tag{29}$$
$$\eta(u_x - \phi) + \lambda\varepsilon_x = 0 \tag{30}$$
$$U_c - \alpha + \lambda\varepsilon_c = 0 \tag{31}$$
$$-\alpha + v_g + \lambda\varepsilon_g = 0, \tag{32}$$

where the partial derivatives of (28) are derived in Appendix B for each environment.

Given $\mathcal{X}'_B < 0$, it follows that $\lambda > 0$. To see this, suppose $\lambda = 0$. Then, from (30)—(32) we get $u_x = \phi$, $U_c = v_g = \alpha$. For all three environments, we get $B' > B$ from (28) and $\lambda' = 0$ from (29), since $\varepsilon'_B > 0$. Then, $x = x'$ and thus, $\mathcal{X}'_B = 0$, a contradiction.

Thus, for $B \geq -1$ the first-best is not implemented in a MPME. It is easy to see now that the non-negativity constraint $u_x - \phi \geq 0$ does not bind for $B \geq -1$. For $\lambda > 0$, given that $\varepsilon_x < 0$ for $B > -1$ in all environments, (30) implies $u_x - \phi > 0$. If $B = -1$, then $\varepsilon_x = 0$ and $u_x - \phi = 0$; the non-negativity constraint is satisfied with equality, but is still slack.

The interaction between debt and monetary policy is captured by equations (29) and (30). Note that $\varepsilon_{B'} = -\beta\varepsilon'_B$ in all three environments (see Appendix B) and thus, (29) can be written as

$$\varepsilon_{B'}(\lambda - \lambda') + \lambda\varepsilon_{x'}\mathcal{X}'_B = 0. \tag{33}$$

The term $\varepsilon_{B'}(\lambda - \lambda')$ in (33) states the standard trade-off between distortions today and tomorrow, and displays the government's incentive to smooth these distortions over time. The term $\lambda\varepsilon_{x'}\mathcal{X}'_B$ appears due to the lack of commitment friction and reflects how a change in current debt affects the government budget constraint, through its effect on future (monetary) policy. Debt increases or decreases depending on the direction of this effect, i.e., the sign of $\varepsilon_{x'}\mathcal{X}'_B$. Debt increases, i.e., distortions are pushed to the future ($\lambda < \lambda'$) when the costs of this policy can be offset by a relax-ation of the government budget constraint ($\varepsilon_{x'}\mathcal{X}'_B > 0$). Similarly, debt decreases ($\lambda > \lambda'$) when $\varepsilon_{x'}\mathcal{X}'_B < 0$. Thus, how a government that lacks commitment trades-off distortions intertemporally depends crucially on how future (monetary) policy reacts to inherited debt. Debt policy is then purely determined by the time-consistency problem between successive governments.

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[11]This is a refinement that rules out equilibria where the discontinuities in policy are not rooted in the environment fundamentals, but are rather an artifact of the infinite horizon. For an analysis and discussion of non-differentiable Markov-perfect equilibria see Krusell and Smith (2003) and Martin (2009). See also Martin (2010*b*) for further discussion in a similar context.

Given that $\mathcal{X}'_B < 0$, current debt increases or decreases depending on the sign of $\varepsilon_{x'}$. This term measures how changes in future monetary policy (due to changes in debt today as captured by $\mathcal{X}'_B$) affect the current government budget constraint. The effects of future monetary policy come through two channels: changes in the current demand for money and changes in the financial cost of issuing debt.

Intuitively, an increase in future inflation (due to higher inherited debt) increases the cost of issuing debt (i.e., lowers q) and lower marginal value of money for future producers; both effects tighten the government budget constraint today. On the other hand, higher inflation tomorrow increases the marginal value of money for future consumers and thus, has a positive impact on the current demand for money, which lowers the size of distortions associated with government policy. If this latter effect dominates, then the current government budget constraint is relaxed and there is an incentive to increase the level of debt. The benefit of this policy is offset with increased distortions tomorrow, due to the higher financial burden of debt.

One critical difference across monetary economies is the determination of the demand for fiat money, which as argued above, affects how the current government internalizes the effects of its debt policy on future monetary policy. This feature will account for most of the difference in how the three model variants perform in the presence of aggregate shocks—see Section 4 below.

Consider now the intratemporal trade-off implied by monetary policy, as displayed in equation (30). For $B > -1$, $\varepsilon_x < 0$, i.e., the government has an incentive to lower the day-good allocation to relax its budget constraint. Intuitively, the government wants to reduce the financial burden of debt using monetary policy to increase the price level. However, this implies distorting the allocation of the day-good, i.e., $u_x - \phi > 0$. Note that in all environments, $d\varepsilon_x/dB < 0$, which implies that the incentives to use inflation increase with the level of debt.

Given that $\varepsilon_g = -\alpha$ in all three environments, (32) implies $\lambda = \frac{v_g}{\alpha} - 1$ in all cases. Also note that $\varepsilon_c = U_c - \alpha + U_{cc}c$ in all three cases. Thus, a MPME is characterized by (28) and

$$\varepsilon_{B'}(v_g - v'_g) + (v_g - \alpha)\varepsilon_{x'}\mathcal{X}'_B = 0 \tag{34}$$

$$\alpha\eta(u_x - \phi) + (v_g - \alpha)\varepsilon_x = 0 \tag{35}$$

$$v_g(U_c - \alpha) + (v_g - \alpha)U_{cc}c = 0, \tag{36}$$

where the expressions for $\varepsilon_{B'}$, ε_x and $\varepsilon_{x'}$ depend on the environment being considered.

The following result states under what conditions the environments with financial intermediation and trading friction coincide with the benchmark case with competitive markets.

Proposition 1 *Equivalence between environments: (i) the MPME with financial intermediation approaches the MPME without financial intermediation as $\eta \to 1$; and (ii) the MPME with trading frictions is equivalent to the MPME without trading frictions if $\eta = \frac{1}{2}$ and $\theta = 1$.*

Proof. Part (i). As $\eta \to 1$ equations (28), (34) and (35) converge for the cases with and without financial intermediation. Equation (36) coincides for both cases for all η.

Part (ii). Suppose $\theta = 1$ and $\eta = \frac{1}{2}$ which implies $h(x) = \phi x$ and $h_x = \phi$. Thus, (27) is identical to (16). Given that the objectives are the same as well, the problems of the government with and without trading frictions are identical. ∎

3.2 Long-run debt

In steady state, (34) simplifies to $\varepsilon_{x'} = 0$, since $\mathcal{X}'_B < 0$ for all $B \geq -1$ as mentioned above. The steady state $\{B^*, x^*, c^*, g^*\}$ is characterized by

$$\varepsilon^*_{x'} = 0 \tag{37}$$
$$\alpha\eta(u^*_x - \phi) + (v^*_g - \alpha)\varepsilon^*_x = 0 \tag{38}$$
$$v^*_g(U^*_c - \alpha) + (v^*_g - \alpha)U^*_{cc}c^* = 0 \tag{39}$$
$$\varepsilon(B^*, B^*, x^*, x^*, c^*, g^*) = 0. \tag{40}$$

Focus on (37). Although $\mathcal{X}^*_B < 0$, i.e., small changes in debt choice at B^* have an effect on future policy, the positive and negative effects of these changes on the current government budget constraint are balanced out. In other words, the time-consistency problem, which is driving the change in debt, cancels out at the steady state. It follows that if the governments starts at B^*, it will stay there, regardless of its ability to commit. The following proposition generalizes the result in Martin (2010b).

Proposition 2 *Irrelevance of commitment at B^*. Suppose initial debt is equal to B^*; then, a government with commitment and a government without commitment will both implement the allocation $\{x^*, c^*, g^*\}$ and choose debt level B^* in every period.*

Proof. See Appendix C. ∎

Thus, the steady state in all three environments is constrained-efficient, since endowing the government with commitment at B^* would not affect the allocation. This is an important property of this class of monetary economies: lack of commitment by the government provides a mechanism that explains the level of debt and thus, policy in general, but is not a primary concern in terms of welfare.

The following series of propositions derive relevant properties of long-run debt in each of the monetary economies being considered.

Proposition 3 *Long-run debt with competitive markets: (i) $B^* > 0$ only if $\frac{-x^* u^*_{xx}}{u^*_x} > 1$; and (ii) as $\eta \to 1$, $B^* > 0$ if and only if $\frac{-x^* u^*_{xx}}{u^*_x} > 1$.*

Proof. Part (i). From (37),
$$B^* = -\frac{\eta(u^*_x + u^*_{xx}x^*)}{\phi} - 1 + \eta. \tag{41}$$

Thus, $B^* > 0$ implies $-1 - \frac{u^*_{xx}x^*}{u^*_x} > (\frac{1-\eta}{\eta})(\frac{\phi}{u^*_x})$. The right-hand side of this inequality is positive; thus, $-\frac{u^*_{xx}x^*}{u^*_x} > 1$ is a necessary condition for $B^* > 0$.

Part (ii). As $\eta \to 1$, $B^* \to -\frac{u^*_x + u^*_{xx}x^*}{\phi}$. Given $\phi > 0$, $B^* > 0$ iff $u^*_x + u^*_{xx}x^* < 0$. ∎

As we can see, critical for long-run debt are the curvature of the utility function for the day-good, $u(x)$, and the measure of buyers in the day market, η. Both these elements determine how

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future monetary policy is internalized by the current government and thus, affect the incentives to increase or decrease debt. Given part *(i)* of Proposition 3, if $u(x)$ is CES, then its curvature needs to be higher than log for debt to be positive in the long-run; from part *(ii)*, this condition is also sufficient as the measure of buyers approaches one.

Proposition 4 ***Long-run debt with financial intermediation:*** *(i) for all $\eta \in (0, 1)$, the steady state allocation $\{x^*, c^*, g^*\}$ and taxes τ^* are the same in the environments with and without financial intermediation, whereas debt B^* and inflation μ^* are higher for the case with financial intermediation; and (ii) $B^* > 0$ if and only if $-\frac{x^* u_{xx}^*}{u_x^*} > 1$.*

Proof. Part (i). From (37) we get

$$B^* = -\frac{u_x^* + u_{xx}^* x^*}{\phi}. \tag{42}$$

We have $\varepsilon_x = -\phi x(1 + B)$ for the case with competitive markets and $\varepsilon_x = -\eta\phi x(1 + B)$ for the case with financial intermediation. Using (41) and (42), respectively, we get that ε_x^* yields the same expression in both cases, i.e., (38) is the same in both environments; similarly with (40). Also note that (39) is the same in both cases and does not depend on B^*. Thus, $\{x^*, c^*, g^*\}$ is the same in both environments. Taxes are equal to $1 - \frac{\alpha}{U_c}$ in both cases, so they are also the same in steady state. Next, comparing equations (37) and (42), we get $B^* = \frac{1 + B^{NF}}{\eta} - 1 > B^{NF}$, since $B^{NF} > -1$, where B^{NF} is the steady state for the case with no financial intermediation. For steady state inflation, we get $\mu^* = \frac{\mu^{NF} - \beta}{\eta} + 1 > \mu^{NF}$, since $\mu^{NF} > \beta$.

Part (ii). From (42), since $\phi > 0$, $B^* > 0$ if and only if $u_x^* + u_{xx}^* x^* < 0$. ∎

The result above states that resolving the financial friction in the benchmark environment does not alter the long-run allocation. However, both debt and inflation increase. The difference in debt between the two environments is equal to $(1 - \eta)(1 + B^*)$, where B^* is the steady state with financial intermediation. Similarly, the difference in long-run inflation is $\frac{\beta(1 - \eta)(u_x^* - \phi)}{\phi}$. For both debt and inflation, the measure of day-good buyers η has a first-order negative effect on the size of the differences between the two cases. The reason is that, with financial intermediation the costs associated with future monetary policy are lower since producers can lend their cash balances to buyers (note the term $(1 - \eta)\phi$ in $\varepsilon_{x'}$ for the case with competitive markets, which is absent for the case with financial intermediation—see Appendix B); thus, the incentives to issue debt are higher. Given the larger debt, long-run inflation ends up being higher, due to the higher financial burden (i.e., a movement up, along the money growth function).

With financial intermediation, if $u(x)$ is CES, then higher than log curvature is a necessary and sufficient condition for long-run debt to be positive. This is a stronger result than for the case with no financial intermediation, where the same condition was necessary, but not sufficient.

The steady state for the economy with trading frictions is less tractable than in the other two environments. The following result suggests that the bargaining power, θ needs to be high enough to obtain a realistic level of debt in the long-run, which is a point I revisit in the numerical section below.

Proposition 5 *Long-run debt with trading frictions: as $\theta \to 0$, monetary policy approximates the Friedman rule for all B and $B^* \to -1$.*

Proof. From (37) we get

$$B^* = -\frac{1}{2h_x^*}\left\{u_x^* + h_x^* + \frac{\theta\phi h(x^*)u_{xx}^*}{h_x^{*2}}\right\}.\tag{43}$$

As $\theta \to 0$, $h(x) \to u(x)$ and thus, $u_x \to h_x$, $u_{xx} \to h_{xx}$. From (26), $q \to 1$ for all B, i.e., the Friedman rule. From (43), $B^* \to -1$. ∎

3.3   Calibration

Let us calibrate the steady state of each environment. Consider the following functional forms:

$$
\begin{aligned}
u(x) &= \begin{cases} \frac{x^{1-\sigma}}{1-\sigma} & \text{if} \quad \bar{x}=0 \\ \frac{(x+\bar{x})^{1-\sigma}-\bar{x}^{1-\sigma}}{1-\sigma} & \text{if} \quad \bar{x}>0 \end{cases} \\
U(c) &= \frac{c^{1-\rho}}{1-\rho} \\
v(g) &= \psi \ln g.
\end{aligned}
$$

When there are no trading frictions we set $\bar{x} = 0$, so that $u(x)$ is standard CES; for the case with trading frictions we need $\bar{x} > 0$ to satisfy the assumption $u(0) = 0$. For now, normalize ψ to 1 and set $\eta = \frac{1}{2}$. The parameters left to calibrate are α, β, ρ, σ, ϕ, θ and $\bar{x}$.

Define nominal GDP as the sum of nominal output in the day and night markets. Let Y be nominal GDP normalized by the aggregate money stock, i.e., $Y \equiv \eta\tilde{p}x + p(c + g)$. Note that by the equation of exchange, Y is also equal to velocity of circulation. For the case with competitive markets, $\tilde{p}x = 1$ and thus, $Y = \eta + p(c + g)$. With financial intermediation, $\tilde{p}x = \frac{1}{\eta}$ and thus, $Y = 1 + p(c + g)$. With trading frictions, note that the implicit price in all bilateral meetings is $\frac{1}{x}$; thus, Y is the same as with competitive markets. In the benchmark calibration (see details below), the night market is 91% of total GDP for the cases without financial intermediation; with banks, the relative size of the night market drops to 82%.

Calibration targets are taken from 1962-2006 averages for the U.S. economy. Period length is set to a year. Government in the model corresponds to the federal government. The calibration targets are: debt over GDP, annual inflation, interest payment over GDP, outlays (excluding interest) over GDP and revenues over GDP. Inflation is measured from the CPI, while the rest of the variables are taken from the Congressional Budget Office. Government debt is defined as debt held by the public, excluding holdings by the Federal Reserve system.

Next, we need to specify the model steady state statistics that correspond to the selected calibration targets. For debt over GDP use $\frac{B(1+\mu)}{Y}$, since debt is measured at the end of the period in the data. Let π be annual inflation in the model, which in steady state is equal to μ. Interest payments over GDP are defined as $\frac{B(1+\mu)(1-q)}{Y}$. Given that debt over GDP is already targeted, this implies a target for the nominal interest rate i, where $i = \frac{1}{q} - 1$. Interest payments are 2.1% of GDP in the data, which implies a target nominal interest rate of 7.3% annual. Outlays and revenues are defined as $\frac{pg}{Y}$ and $\frac{p\tau n}{Y}$, respectively, where $n = c + g$ from the night-resource constraint. For the case

with trading frictions, the typical approach is to include an additional target, the price-to-marginal cost ratio or *markup*.[12] Since the markup in the night-market is zero, the markup is equal to the share of the day-market output in GDP times the day-market markup. Let ω be the markup, where $\omega \equiv \frac{\eta(1-\theta)}{Y}\left(\frac{u(\bar{x})}{\phi x} - 1\right)$. I use the usual target adopted by the literature, 10%. Table 1 summarizes the target statistics.

Table 1: Target statistics

$\frac{B^*(1+\mu^*)}{Y^*}$	π^*	i^*	$\frac{p^*\tau^*n^*}{Y^*}$	$\frac{p^*g^*}{Y^*}$	ω^*
0.308	0.044	0.073	0.182	0.182	0.100

Note: ω is a calibration target only for the case with trading frictions.

Table 2 shows the parameters that match the calibration targets for each of the environments considered. As mentioned above, for the cases with no trading frictions, $\bar{x} = 0$. For the case with trading frictions, $\bar{x} = 0.5$ and θ is set equal to 0.8739 to match the markup, ω. The value of $\bar{x}$ is much higher than the typically found in the literature (which is close to zero, say $\bar{x} = 0.0001$). The reasons is that lower values of $\bar{x}$ imply higher values of θ to match the markup target (see the expression for ω above); e.g., if $\bar{x} = 0.25$ then $\theta = 0.9284$, and if $\bar{x} = 0.005$ then $\theta = 0.9989$. The benchmark value for $\bar{x}$ is a compromise between making the environment with trading frictions sufficiently different from the competitive markets case and not deviating too much from a standard CES utility specification. It is important to point out that the quantitative results reported in the sections below are not affected by the choice of $\bar{x}$. In particular, setting $\bar{x} = 0.5$ (and recalibrating) for the cases without trading frictions or setting $\bar{x} = 0.005$ (and recalibrating) for the case with trading friction have only minor quantitative effects that are not sufficiently significant to overturn any of the conclusions.

Table 2: Benchmark calibration

Parameters	Competitive Markets	Financial Intermediation	Trading Frictions
α	4.9801	4.1722	5.1648
β	0.9728	0.9728	0.9728
ρ	7.3670	8.1879	6.4633
σ	4.6965	2.5084	5.8405
ϕ	1.3332	4.8290	1.3701
ψ	1.0000	1.0000	1.0000
η	0.5000	0.5000	—
θ	—	—	0.8739
$\bar{x}$	0.0000	0.0000	0.5000

The steady state allocation $\{B^*, x^*, c^*, g^*\}$ is found by solving numerically the system of equations (37)—(40). Table 3 shows the solutions for all the cases considered.

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[12]See Lagos and Wright (2005) and Aruoba, Waller and Wright (2008) for further discussion.

Table 3: Steady state variables for benchmark calibration

	Competitive Markets	Financial Intermediation	Trading Frictions
B^*	1.619	1.619	1.619
x^*	0.914	0.519	0.422
c^*	0.780	0.814	0.749
g^*	0.195	0.233	0.188

3.4 Comparative statics

To further understand the differences in long-run policy across cases, we can analyze the response of steady state statistics to changes in parameter values. Each parameter is perturbed by $\pm 10\%$. Next, the percentage change in a steady state statistic is divided by the percentage change in the parameter value, to measure the elasticity of the statistic to changes in parameters. Table 4 presents the results, highlighting elasticities between 0.25 and 1, and elasticities above 1, plus the sign of the change in statistics.[13] See Appendix D for a table with the actual figures.

Table 4: Parameter-elasticity of steady state statistics

	Competitive Markets						Financial Intermediation						Trading Frictions				
	$\Delta\alpha$	$\Delta\phi$	$\Delta\eta$	$\Delta\rho$	$\Delta\sigma$	$\Delta\psi$	$\Delta\alpha$	$\Delta\phi$	$\Delta\eta$	$\Delta\rho$	$\Delta\sigma$	$\Delta\psi$	$\Delta\alpha$	$\Delta\phi$	$\Delta\rho$	$\Delta\sigma$	$\Delta\psi$
$\frac{B^*(1+\mu^*)}{Y^*}$	−	+	++	−	++		−	+	+		++		−	−	−	++	
π^*	−		++	−−	++	++	−			−−	++	++	−	−	−−	++	++
τ^*	−					+	−					+	−				+
$\frac{p^*\tau^*n^*}{Y^*}$	−					+	−					+	−				+
$\frac{p^*g^*}{Y^*}$	−					+	−					+	−				+

Note. Each parameter is increased and then decreased by 10%. *Elasticity is measured as the percentage change in a statistic divided by* $1.1/0.9 - 1$, *where the change in statistic corresponds to* $\pm 10\%$ *change in parameter value. A positive (negative) sign implies the statistic increases (decreases) with an increase in parameter value. A single sign implies the elasticity is higher than* 0.25 *but lower than* 1; *a double sign implies the elasticity is equal to or higher than* 1. *For the case with trading frictions,* $\Delta\eta$ *is omitted since it is always assumed that* $\eta = 0.5$.

Let us first focus on the case with competitive markets. As we can see, only debt and inflation feature parameter-elasticities greater than 1: both variables are increasing in η and σ; in addition, inflation also increases significantly with reductions in ρ and increases in ψ. The remaining effects feature lower parameter-elasticities. In this sense, increases in α reduce debt, inflation, taxes, expenditure and revenue; increases in ϕ increase debt; and increases in ψ increase taxes, expenditure and revenue.

The other two monetary economies feature some notable similarities with the benchmark case. Most notably, the effect of changes in α and ψ go in the same direction for all three variants; quantitatively, the effect is also very similar across model variants—see Appendix D. In all three

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[13]The cut-off point of 0.25 is somewhat arbitrary. The idea is to focus on changes in long-run statistics which are sufficiently significant, given that the change in parameter values is $\frac{1.1}{0.9} - 1 \approx 0.222$.

cases, an increase in σ implies a large increase in both debt and inflation. Quantitatively, the effect is significantly larger for the case with trading frictions.

There are two differences in comparative statics worth highlighting. For the case with financial intermediation, changes in η affect debt, but not inflation. For the case with trading frictions, an increase in ϕ decreases both debt and inflation, in sharp contrast with the other two cases, which feature an increase in debt and no change in inflation.

3.5 Frictions and long-run policy

The three monetary economies studied in this paper all share the fundamental frictions that make a medium of exchange essential: lack of commitment, anonymity and lack of double coincidence of wants. The differences lie in the additional trading and financial frictions that may afflict the environment. The case with competitive markets resolves trading frictions, whereas the case with a banking sector further resolves financial frictions. Here, we quantify the effects of these frictions on long-run government policy.

First, suppose we resolve trading frictions. Thus, take the parameterization for the case with trading frictions and solve the case with competitive markets.[14] In steady state, debt over GDP decreases to 4.1%, annual inflation drops to 1.7%, while tax revenue and expenditure both increase slightly to about 19% of GDP. If we consider a higher bargaining power for consumers and recalibrate, the difference in statistics becomes even larger. The reason is that as we increase θ (lower $\bar{x}$), we need to reduce σ to match the target statistics for the case with trading frictions. From the analysis above, we know that B^* is increasing in σ; thus, when we switch to the case with competitive markets, long-run debt (and thus, inflation) decreases even more.

Second, suppose we shut down the banking sector. Thus, take the parameterization for the case with financial intermediation and solve the case with competitive markets. From Proposition 4 we know that in the long-run, only debt and inflation change; without banks, steady state debt over GDP decreases to 11.4%, while annual inflation drops to 0.8%. Note that the magnitude of these policy changes are inversely related to the assumed value for η. For example, set $\eta = 0.8$ and recalibrate the economy with financial intermediation; thus, $\phi = 2.2101$ while all other parameters remain at benchmark. If we now solve the steady state with competitive markets, we get debt over GDP of 25.7% and annual inflation of 3.0%; i.e., the changes are significant, but not quite as dramatic as when $\eta = 0.5$.

The analysis shows that resolving trading frictions reduces long-run debt and inflation, whereas resolving financial frictions has the opposite effect. The quantitative magnitude of these effects may be quite seizable. Given that with certain technological advances (e.g., electronic record keeping), both goods markets and financial markets have become more efficient, the results indicate that these improvement may have had a significant impact on government policy. Recovering the contribution of these changes from the data may prove difficult as they have canceling effects. In addition, these institutional changes to not occur in isolation; e.g., technological advances that alleviate trading and financial frictions are likely to also improve labor productivity.

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[14]Note that by Proposition 1 this exercise is equivalent to increasing θ to 1.

4 Government Policy and Aggregate Uncertainty

In this section, I study the differences between monetary economies in the presence of aggregate uncertainty. I consider shocks to government expenditure and aggregate productivity and compare the model variants along four dimensions: the response of government policy to aggregate shocks; the persistence of policy variables; the money demand; and the Phillips curve.

The analysis in this section is an extension of the work in Martin (2010a) where I evaluate the empirical plausibility of the variant with competitive markets along several non-calibrated dimensions. Here, the focus is on understanding how policy response to aggregate shocks depends on the details of the monetary economy.

4.1 Government policy in monetary economies with aggregate uncertainty

Assume there are two aggregate shocks: one to the marginal value of the public good (an "expenditure" shock) and one to the productivity of labor. We keep the assumption that $v(g) = \psi \ln g$, but now assume that ψ is a random variable. Let A be labor productivity, which affects both day and night output, and follows a random process. Thus, day-good producers incur a utility cost $\frac{\phi \kappa}{A}$ and night-output is equal to An.[15]

Let $s \equiv \{\psi, A\}$ follow a Markov process and let $E[s'|s]$ be the expected value of s' given s. The set of all possible realizations for the stochastic state is S. To simplify exposition, define $\phi_A \equiv \frac{\phi}{A}$ and $\alpha_A \equiv \frac{\alpha}{A}$.

For the respective cases of competitive markets, financial intermediation and trading frictions, the government budget constraint in a monetary equilibrium is

$$(U_c - \alpha_A)c - \alpha_A g - \phi_A x(1+B) + \beta E\left[\eta x'(u'_x - \phi'_A) + \phi'_A x'(1+B')\big|s\right] = 0$$

$$(U_c - \alpha_A)c - \alpha_A g - \eta\phi_A x(1+B) + \beta\eta E\left[x'(u'_x - \phi'_A) + \phi'_A x'(1+B')\big|s\right] = 0$$

$$(U_c - \alpha_A)c - \alpha_A g - h_A(x)(1+B) + \beta E\left[\frac{h_A(x')(u'_x - h'_{A,x})}{2h'_{A,x}} + h_A(x')(1+B')\bigg|s\right] = 0,$$

where $h_A(x) \equiv (1-\theta)u(x) + \phi_A x$ and $h_{A,x} \equiv (1-\theta)u_x - \phi_A$. Note that we can write the budget constraint compactly as

$$\zeta(B, x, c, g, s) + \beta E[\vartheta(B', \mathcal{X}(B', s'), s')|s] = 0. \tag{44}$$

Given debt level $B \in \Gamma$, current stochastic state $s \in S$ and the perception that future governments will implement $\mathcal{X}(B, s)$, the problem of the current government is

$$\mathcal{V}(B, s) = \max_{B', x, c, g} \eta(u(x) - \phi_A x) + U(c) - \alpha_A(c + g) + \psi \ln g + \beta E[\mathcal{V}(B', s')|s]$$

subject to (44).

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[15]To derive the day-utility cost, assume a production function $\kappa = Ae$ and linear disutility in effort, $-\phi e$.

After some work, the first-order conditions imply

$$E\left[\phi'_A x'\left(\frac{\psi}{g} - \frac{\alpha_A \psi'}{\alpha'_A g'}\right) + \left(\frac{\psi}{g} - \alpha_A\right)\vartheta'_x \mathcal{X}'_B \bigg| s\right] = 0 \tag{45}$$

$$\alpha_A \eta (u_x - \phi_A) + \left(\frac{\psi}{g} - \alpha_A\right)\zeta_x = 0 \tag{46}$$

$$\frac{\psi}{g}(U_c - \alpha_A) + \left(\frac{\psi}{g} - \alpha_A\right)U_{cc}c = 0, \tag{47}$$

where the expressions for ζ_x and ϑ'_x depend on the variant being considered. A MPME in a stochastic economy is a set of functions $\{\mathcal{B}(B,s), \mathcal{X}(B,s), \mathcal{C}(B,s), \mathcal{G}(B,s)\} : \Gamma \times S \to \mathbb{R}^5$ characterized by equations (44)—(47), for all $B \in \Gamma$ and $s \in S$.

4.2 Numerical evaluation

As a reference, Table 5 shows a summary of the time-series properties at annual frequencies of selected policy variables for the U.S. between 1962 and 2006. The table includes the variable dGDP, which is linearly-detrended (log) real GDP.

Table 5: Time-series properties of U.S. economy — 1962-2006

Variable	Mean	Std.Dev.	Autocorr.
Debt/GDP	0.308	0.078	0.967
Inflation	0.044	0.030	0.747
Revenue/GDP	0.182	0.010	0.653
Outlays/GDP	0.182	0.012	0.798
Deficit/GDP	0.001	0.018	0.743
dGDP	0.000	0.027	0.715

I keep the benchmark calibration from the previous section and assume the following:

$$\psi' = 1 - \varrho_g + \varrho_g \psi + \epsilon'_g$$
$$\ln A' = \varrho_A \ln A + \epsilon'_A,$$

where $\varrho_g, \varrho_A \in (0,1)$, $\epsilon_g \sim N(0, \sigma_g^2)$ and $\epsilon_A \sim N(0, \sigma_A^2)$. Note that both ψ and A average 1, as in the economies without aggregate uncertainty.

The model is solved globally using a projection method. See Appendix E for a description of the algorithm and other details of the numerical approximation.

There are many alternative ways to calibrate or estimate the stochastic processes for ψ and A. Here, I adopt an approach that allows for a single parameterization to offer empirically plausible dynamics in all three variants. Specifically, the stochastic process for ψ is set to match the autocorrelation and variance of government expenditure over GDP, assuming labor productivity is constant and equal to its long-run value; the process for A is set to match the autocorrelation and variance of detrended (log) real GDP (i.e., dGDP), assuming the marginal value for public good

consumption is fixed at its long-run value. In both cases, I target the statistics for the case with competitive markets, but the assumed processes also match the statistics for the other two cases, with only very minor deviations in the autocorrelation of expenditure and dGDP. The calibrated parameters are: $\varrho_g = 0.804$, $\varrho_A = 0.726$, $\sigma_g = 0.045$; $\sigma_A = 0.061$.

4.3 Policy response to aggregate shocks

The artificial economies are simulated for $1,000,000$ periods, starting from the non-stochastic steady state. Table 6 shows average, standard deviation and autocorrelation of selected policy variables. There are three different simulations for each of the three monetary economies: expenditure shocks only, productivity shocks only, and both shocks. The variable dy in the model corresponds to dGDP, i.e., linearly detrended (log) real GDP. See Appendix E for a description of how it was computed.

Table 6: Statistics for simulated economies

	Competitive Markets			Financial Intermediation			Trading Frictions		
	Mean	Std.Dev.	Autocorr.	Mean	Std.Dev.	Autocorr.	Mean	Std.Dev.	Autocorr.
Expenditure shocks									
$\frac{B(1+\mu)}{Y}$	0.305	0.040	0.989	0.305	0.037	0.989	0.305	0.039	0.989
π	0.044	0.009	0.882	0.044	0.008	0.871	0.044	0.009	0.875
$\frac{p\tau n}{Y}$	0.182	0.007	0.935	0.182	0.008	0.928	0.182	0.007	0.933
$\frac{pg}{Y}$	0.182	0.012	0.798	0.182	0.012	0.800	0.182	0.012	0.798
$\frac{p(g-\tau n)}{Y}$	0.000	0.007	0.703	0.000	0.006	0.698	0.000	0.006	0.700
dy	0.000	0.013	0.779	0.000	0.013	0.776	0.000	0.013	0.773
Productivity shocks									
$\frac{B(1+\mu)}{Y}$	0.310	0.009	0.921	0.311	0.018	0.933	0.310	0.042	0.942
π	0.046	0.031	0.312	0.046	0.038	0.082	0.047	0.057	-0.070
$\frac{p\tau n}{Y}$	0.182	0.001	0.906	0.182	0.002	0.945	0.182	0.004	0.975
$\frac{pg}{Y}$	0.182	0.012	0.717	0.182	0.011	0.720	0.182	0.011	0.720
$\frac{p(g-\tau n)}{Y}$	0.001	0.011	0.702	0.001	0.010	0.683	0.000	0.009	0.639
dy	0.000	0.027	0.715	0.000	0.030	0.714	0.000	0.029	0.707
Both shocks									
$\frac{B(1+\mu)}{Y}$	0.307	0.041	0.986	0.307	0.041	0.978	0.307	0.057	0.964
π	0.046	0.032	0.358	0.045	0.039	0.117	0.046	0.058	-0.046
$\frac{p\tau n}{Y}$	0.182	0.007	0.934	0.182	0.008	0.929	0.182	0.008	0.941
$\frac{pg}{Y}$	0.182	0.017	0.759	0.182	0.016	0.763	0.182	0.016	0.762
$\frac{p(g-\tau n)}{Y}$	0.001	0.013	0.703	0.000	0.012	0.687	0.000	0.011	0.661
dy	0.000	0.030	0.728	0.000	0.033	0.724	0.000	0.032	0.717

Artificial economies are simulated for $1,000,000$ periods.

Consider first the environment with competitive markets. Almost all of the volatility in debt and tax revenue is due to expenditure shocks, while most of the volatility in inflation is due to productivity shocks. The volatility of expenditure is similar for the two types of shocks and get compounded when including both. The autocorrelation of these four policy variables is higher

under expenditure shocks than productivity shocks, which is not surprising considering that expenditure shocks are more persistent than productivity shocks. The autocorrelation of inflation varies significantly with the type of shock (this feature is analyzed further below). The primary deficit is slightly more volatile with productivity shocks, but the autocorrelation is nearly identical under the two types of shocks. When we consider both shocks, the volatility and autocorrelation of policy variables are reasonably close to the data, except for the autocorrelations of inflation and tax revenues—see Table 5.

Let us now compare the behavior of policy across the different environments. The response of variables to government expenditure shocks is very similar in all cases. The response to productivity shocks features some important differences. Compared to the benchmark with competitive markets, introducing financial intermediation results in slightly higher volatility in debt and inflation; the autocorrelation of inflation drops quite significantly and the autocorrelation of tax revenues increases. Incorporating trading frictions results in larger increases in the volatility of debt and inflation; the autocorrelation of inflation changes sign (becomes negative) and the autocorrelation of tax revenue increases even more. There is also a significant drop in the autocorrelation of the deficit. When we consider both types of shocks, the most significant difference across cases is the volatility and autocorrelation of inflation. For the case with trading friction, we also have a higher volatility in debt and a lower autocorrelation in the deficit.

To further understand the differences in policy response across environments, Figure 1 displays the impulse-response functions of selected variables to expenditure and aggregate shocks. The responses are computed from a VAR estimated from the simulated data; the VAR consists of the following variables (in order): ψ, A, $\frac{p(g-\tau n)}{Y}$, dy, μ and $\frac{B(1+\mu)}{Y}$. This VAR specification allows for an easy and precise numerical approximation to the true impulse-response functions, which cannot be accurately computed directly given that the exogenous state space is discrete. As we can see, the qualitative responses of the primary deficit, money growth rate and debt are similar in all environments. The main difference across monetary economies is the *quantitative* response of debt and monetary policies to productivity shocks. This is the source of the differences in time-series statistics across cases, as reported in Table 6.

In all cases, the response of inflation to a productivity shock has an important difference with the response of the money growth rate. To see this, note that prices for the case with competitive markets are $\tilde{p} = \frac{1}{x}$ and $p = \frac{AU_c}{\phi x}$ in the day and night markets, respectively; taxes are $\tau = 1 - \frac{\alpha}{AU_c}$. As reported in Table 6, the volatility of taxes in response to productivity shocks is close to zero; thus, the volatility of AU_c is close to zero. When a positive innovation to A hits the economy, x increases, AU_c remains approximately the same and so, both $\tilde{p}$ and p decrease. This behavior is displayed despite the fact that μ actually increases in response to a positive innovation in A, as shown in Figure 1. In the subsequent periods after the shock, prices increase at a decreasing rate; i.e., after the initial period, inflation follows the behavior of the money growth rate. Thus, most of the difference in the autocorrelation of inflation between monetary economies is due to how large the initial drop in prices is.

The case with trading frictions presents an interesting oddity. The response of monetary policy to a productivity shocks is the smoothest of all cases, while the volatility of inflation is the highest. Relatively speaking, most of the volatility in inflation reported for this case does not stem from changes in the money supply, but from changes in the money demand, which in turn react to both the aggregate shocks and the endogenous policy response.

4.4 Policy persistence

To better measure the persistence of policy variables, Marcet and Scott (2009) suggest using the k-variance ratio, which is defined as $P_x^k = \frac{Var(x_t - x_{t-k})}{kVar(x_t - x_{t-1})}$. A variable is more persistent the longer it takes the k-variance ratio to converge to zero. Figure 2 compares the persistence of debt, inflation and the deficit, in the data and artificial economies.

In the data, the k-variance ratio for debt over GDP is increasing and only starts leveling off after 9 years at about 3.7. Inflation and the deficit are much less persistent and both series display a more or less decreasing P^k ratio. Furthermore, the persistence in inflation and the deficit are relatively similar. As we can see in Figure 2, all model variants match these qualitative features broadly. The P^k ratios for inflation and the deficit are quite close to the data in all cases, especially after a few periods. The big difference between variants is in the persistence of debt. The case with competitive markets provides the best fit to the data, whereas the other two cases match the qualitative shape of debt persistence, but underestimate it quantitatively.

4.5 Money demand

Let us evaluate the model's implication for the money demand, i.e., the relationship between the nominal interest rate and the inverse of velocity of circulation. Note that neither of these variables were calibration targets. For the U.S. data, define velocity of money as nominal GDP divided by average M_1, which is the measure typically adopted by the literature. For the interest rate, I use the 1-year treasury constant maturity rate published by the Federal Reserve, which is closely related to the nominal interest rate in the model. One issue with the data is that velocity of circulation has a secular trend whereas the interest rate does not. To remove this effect, I linearly detrend the series for the inverse of velocity. In the model, velocity of circulation is defined as (normalized) nominal aggregate output, Y, and the interest rate is $i = 1/q - 1$.

Consider the money demand regression $k_t = \gamma\, i_t + \varepsilon_t$, where k and i are the (detrended) inverse of velocity and the nominal interest rate, respectively. Table 7 reports the results of the money demand regressions in the data and the model. For the artificial economies, the γ coefficient is estimated using the simulated sample of 1,000,000 periods. This method provides an estimate of the "true" relationship between k and i in the model. Fit can be evaluated by checking whether the model estimate for γ falls within the one-standard error band in the data. Figure 3 complements the analysis with a graphical representation of the money demand curve in the data and the initial $10,000$ simulation periods in each monetary economy. Note that both the inverse of velocity and the nominal interest rate are presented as deviations from the mean, to facilitate the visual comparison across cases.

As we can see in Table 7, all cases feature a money demand curve with a negative slope. The model fit is very good for the cases with competitive markets and financial intermediation. The case with trading frictions features a poor fit, which as can be observed in Figure 3, is due to the relatively high volatility of the money demand (see Section 4.3 for a related discussion).

Table 7: Money demand regression: $k_t = \gamma\, i_t + \varepsilon_t$

	U.S. Data 1962-2006	Competitive Markets	Financial Intermediation	Trading Frictions
γ	-0.442	-0.432	-0.395	-0.107
	(0.057)	(0.000)	(0.000)	(0.001)
R^2	0.583	0.707	0.642	0.025

Note: standard errors are shown in parenthesis.

4.6 Phillips curve

Lastly, we compare the model variants by studying the implied relationship between inflation and output, i.e., a variant of the standard Phillips curve. For the U.S. annual data between 1962 and 2006, the regression $\pi_t = \gamma\, dy_t + \varepsilon_t$ implies $\gamma = 0.521$, with a standard error of 0.150. Table 8 displays the Phillips curve regression for the U.S. data and simulated economies. Figure 4 provides a graphical representation.

Table 8: Phillips curve regression: $\pi_t = \gamma\, dy_t + \varepsilon_t$

	U.S. Data 1962-2006	Competitive Markets	Financial Intermediation	Trading Frictions
γ	0.521	0.400	0.151	-0.276
	(0.150)	(0.001)	(0.001)	(0.002)
R^2	0.219	0.142	0.016	0.023

Note: standard errors are shown in parenthesis.

The case with competitive markets implies an estimated coefficient on dy_t of 0.402, which falls within the one standard error band of the data estimate. The positive correlation between inflation and GDP in the model obtains despite a negative policy trade-off between the two variables (since inflation is distortionary), and results from the interaction between policy and aggregate shocks over time. Figure 4 displays very clearly how locally negative policy trade-offs shift around with aggregate shocks, so that the long-run relationship becomes positive.

For the case with financial intermediation, the coefficient in the Phillips curve regression is also positive, but significantly lower than with competitive markets. For the case with trading frictions, the coefficient is actually negative. In both these variants, the R^2 is close to zero, i.e., real GDP has virtually no explanatory power for inflation. This result is also apparent from Figure 4.

The differences between economies follow from the behavior of inflation in the presence of productivity shocks—see Table 6. To smooth out any artifacts generated by the contemporaneous response of inflation, I run the regressions using 5-year moving averages.[16] As we can see in Table

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[16]I also conducted a similar exercise for the money demand regressions, but found no significant differences with the results presented in Table 7.

9 all variants now feature a positively-sloped Phillips curve, although the case with competitive markets still offers the closest fit to the data. Also note that the R^2 for all cases is significantly higher than in Table 8.

Table 9: Phillips curve regression II: $\pi_t = \gamma \; y_t + \varepsilon_t$, using 5-year moving averages

	U.S. Data 1962-2006	Competitive Markets	Financial Intermediation	Trading Frictions
γ	0.703	0.707	0.527	0.310
	(0.236)	(0.002)	(0.002)	(0.003)
R^2	0.186	0.660	0.441	0.106

Note: standard errors are shown in parenthesis.

4.7 Robustness

A legitimate concern is that the preceding analysis may be sensitive to the calibration adopted in Section 3. In particular, the values of η and $\bar{x}$ (and thus, θ) where arbitrarily chosen. In this section, I will verify the robustness of the results obtained above, by focusing on three key elements: (i) the time-series statistics in Table 6 with both shocks; (ii) the k-variance ratio for debt; and (iii) the money demand and Phillips curve regressions.[17]

First, let us analyze how the cases with competitive markets and financial intermediation are affected by changes in the measure of buyers, η. Lowering η for the case with competitive markets improves the model fit. For example, set $\eta = 0.3$ and recalibrate to match the target statistics. Then, we get the following improvements: standard deviation of debt increases to 0.043; standard deviation of inflation decreases to 0.030; autocorrelation of inflation increases to 0.487; k-variance ratio increases for all periods (after 10 periods it is equal to 3.84); and γ coefficient in the Phillips curve regression equal to 0.491. The other statistics do not change significantly, except for a lower coefficient in the money demand regression, -0.412. On the other hand, varying η (and recalibrating) for the case with financial intermediation has no significant effect on any the variables considered here; all statistics for the simulated economy look virtually identical for different values of η.

Second, consider the effects of the parameter $\bar{x}$ in the utility function for the day-good, $u(x)$. The calibration specifies $\bar{x} = 0$ for the case with competitive markets and $\bar{x} = 0.500$ for the case with trading frictions. Assume $\bar{x} = 0.500$ for the case with competitive markets and recalibrate parameters to match target statistics. Then, the results are virtually identical to those of the benchmark calibration. Suppose instead that we set $\bar{x} = 0.005$ for the case with trading frictions and recalibrate. Here, there are a few noticeable changes, but not significant enough to overturn any of the reported conclusions.

Third, since the reason why the economy with trading frictions does not fit the data as well as the other cases is the behavior of the money demand, consider lowering the size of the bargaining

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[17]To keep this section brief, I only report the results of the exercise. All supporting computations are available upon request.

frictions by reducing the targeted markup by half. In this case the fit improves significantly. We get: lower volatility and higher persistence of inflation (the standard deviation falls to 0.048 and the autocorrelation increases to 0.010), although the volatility of debt falls (standard deviation decreases to 0.051); increased persistence of debt, as measured by the k-variance ratio (about 15% higher than benchmark after 6 periods); closer estimates for the money demand and Phillips curve regressions (-0.242 and -0.052, respectively). Overall, lowering trading frictions improves model fit, but the economies with competitive markets and financial intermediation still outperform.

5   Concluding remarks

The results in this paper contribute to two strands of the macroeconomic literature. First, the analysis here complements studies in monetary theory by providing a theoretical treatment of how environment frictions affect the (endogenous) determination of government policy. It also presents tools for testing alternative variants of monetary economies by evaluating their performance relative to the data along several relevant dimensions. Second, the paper advances our understanding of government policy (both normative and positive) by identifying the details of micro founded monetary economies that affect specific results and also the results which appear unaltered across variants.

Appendix

A   Nash bargaining

Given consumer's bargaining weight $\theta \in (0,1]$ and consumer's money holdings m, the Nash (1950) solution to the bargaining problem is

$$\{x,\delta\} = \operatorname*{argmax}_{x,\delta \leq m} \left(u(x) - \frac{\alpha\delta}{p(1-\tau)} \right)^{\theta} \left(-\phi x + \frac{\alpha\delta}{p(1-\tau)} \right)^{1-\theta}.$$

A monetary equilibrium under the Nash solution looks identical to the proportional solution as derived in the paper, except for the expression for $h(x)$. With Nash, we get $h(x) \equiv (1-\Theta(x))u(x) + \Theta(x)\phi x$, where $\Theta(x) = \frac{\theta u_x}{\theta u_x + (1-\theta)\phi}$.

One feature of the Nash solution that sets it apart from all the other cases considered in this paper, is that we cannot guarantee that the non-negativity constraint, $u_x - h_x \geq 0$, will not bind at B^*. In other words, it may be possible that the equilibrium under the Nash solution features a zero nominal interest rate in the long-run. In fact, it is straightforward to construct examples. Consider the following: $u(x) = \sqrt{x}$, $\phi = 1$, $U(c) = \log c$, $v(g) = \log g$, $\alpha = 2$, $\beta = 0.9$ and $\theta = 0.4$. The steady state for this parametrization is $\{B^* = -0.889, x^* = 0.111, c^* = 0.299, g^* = 0.200\}$ and features $u_x^* = h_x^*$. Increasing the bargaining power θ, while keeping all other parameters fixed, alleviates this issue; e.g., with $\theta = 0.5$ the solution is interior.[18] Similarly, when applying the calibration with the proportional solution (see Table 2) to the Nash bargaining case, we obtain $u_x^* = h_x^*$. Increasing θ

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[18]A similar issue is reported by Aruoba and Chugh (2008) for the case with commitment.

to 0.999 resolves this issue but the steady state statistics are off target. Overall, it does not appear possible to calibrate the economy with the Nash bargaining solution to the U.S. economy.

See Aruoba, Rocheteau and Waller (2007) for further analysis of the differences between the Nash and Kalai bargaining solutions in the Lagos-Wright framework, for the case with exogenous government policy.

B   Partial derivatives of the government budget constraint

For the case with competitive markets, we get: $\varepsilon_B = -\phi x$; $\varepsilon_{B'} = \beta\phi x'$; $\varepsilon_x = -\phi(1+B)$; $\varepsilon_{x'} = \beta\{\eta(u'_x + u'_{xx}x') + (1 - \eta + B')\phi\}$; $\varepsilon_c = U_c - \alpha + U_{cc}c$; and $\varepsilon_g = -\alpha$.

For the case with financial intermediation, we get: $\varepsilon_B = -\eta\phi x$; $\varepsilon_{B'} = \beta\eta\phi x'$; $\varepsilon_x = -\eta\phi(1+B)$; $\varepsilon_{x'} = \beta\eta\{u'_x + u'_{xx}x' + \phi B'\}$; $\varepsilon_c = U_c - \alpha + U_{cc}c$; and $\varepsilon_g = -\alpha$.

For the case with trading frictions, we get: $\varepsilon_B = -h(x)$; $\varepsilon_{B'} = \beta h(x')$; $\varepsilon_x = -(1+B)h_x$; $\varepsilon_{x'} = \beta\{\frac{\theta(u'_x - \phi)}{2} - \frac{\theta\phi u'_{xx}h(x')}{2(h'_x)^2} + h'_x(1+B')\}$; $\varepsilon_c = U_c - \alpha + U_{cc}c$; and $\varepsilon_g = -\alpha$.

Two useful observations: *(i)* $\varepsilon_{B'} = -\beta\varepsilon'_B$; and *(ii)* both ε_c and ε_g have the same expression in all three environments.

C   Proof of Proposition 2

Let us consider the government problem with commitment, also known as the Ramsey problem. A standard result is that the sequence of government budget constraints collapses to a single "implementability" constraint. Start with (1). For every period, multiply this equation by $\frac{\beta^t U_{c,t}}{p_t}$ and sum over all periods. We get:

$$\sum_{t=0}^{\infty} \beta^t U_{c,t}\left\{\tau_t c_t - (1 - \tau_t)g_t + \frac{(1 + \mu_t)(1 + q_t B_{t+1}) - (1 + B_t)}{p_t}\right\} = 0.$$

Next, use the transversality condition, $\lim_{T\to\infty} \beta^T \frac{(1+\mu_T)(1+q_T B_{T+1})}{p_T} = 0$. In addition, use the monetary equilibrium condition, $\tau_t = 1 - \frac{\alpha}{U_{c,t}}$, which is the same for all three environments. The present value government budget constraint simplifies to

$$\sum_{t=0}^{\infty} \beta^t\{(U_{c,t}-\alpha)c_t-\alpha g_t\} - \frac{U_{c,0}(1 + B_0)}{p_0} + \sum_{t=1}^{\infty} \beta^{t-1}\left\{\frac{U_{c,t-1}(1 + \mu_{t-1})(1 + q_{t-1}B_t)}{p_{t-1}} - \frac{\beta U_{c,t}(1 + B_t)}{p_t}\right\} = 0.$$

The remaining step is to show that the whole third term can be simplified to an expression that only depends on $\{x_t\}_{t=0}^{\infty}$. We can rewrite this term as

$$\sum_{t=1}^{\infty} \beta^{t-1}\left\{\frac{U_{c,t-1}(1 + \mu_{t-1})}{p_{t-1}} - \frac{\beta U_{c,t}}{p_t} + \left(\frac{U_{c,t-1}(1 + \mu_{t-1})q_{t-1}}{p_{t-1}} - \frac{\beta U_{c,t}}{p_t}\right)B_t\right\}$$

Using the monetary equilibrium conditions for each environment, we can show $\frac{U_{c,t-1}(1+\mu_{t-1})q_{t-1}}{p_{t-1}} - \frac{\beta U_{c,t}}{p_t} = 0$ in all cases. The term $\frac{U_{c,t-1}(1+\mu_{t-1})}{p_{t-1}} - \frac{\beta U_{c,t}}{p_t}$ depends on the case; we get $\beta\eta x_t(u_{x,t} - \phi)$

with competitive markets *and* financial intermediation, and $\frac{\beta h(x_t)(u_{x,t}-h_{x,t})}{2h_{x,t}}$ with trading frictions. Note that in all cases the term is only a function of x_t; call it $\beta\Omega(x_t)$. Thus, the implementability constraint is

$$\sum_{t=0}^{\infty}\beta^t\{(U_{c,t}-\alpha)c_t-\alpha g_t+\Omega(x_t)\}-\Omega(x_0)-(1+B_0)\Phi(x_0)=0, \tag{48}$$

where $\Phi(x_0)\equiv\frac{U_{c,0}}{p_0}$, which is equal to ϕx_0 with competitive markets, $\eta\phi x_0$ with financial interme-diaries and $h(x_0)$ with trading frictions. Another useful expression is the period budget constraint, (28), which can now be written as

$$(U_{c,t}-\alpha)c_t-\alpha g_t+\beta\Omega(x_{t+1})+\beta\Phi(x_{t+1})(1+B_{t+1})-\Phi(x_t)(1+B_t)=0. \tag{49}$$

Given $B_0\geq-1$, the problem of the government is

$$\max_{\{x_t,c_t,g_t\}_{t=0}^{\infty}}\sum_{t=0}^{\infty}\beta^t\left\{\eta(u(x_t)-\phi x_t)+U(c_t)-\alpha(c_t+g_t)+v(g_t)\right\}$$

subject to (48). It is easy to verify that the non-negativity constraint, $u_{x,t}-\phi\geq0$, does not bind in any period. The first-order conditions are

$$\begin{aligned}
\eta(u_{x,0}-\phi)-\Lambda(1+B_0)\Phi_{x,0} &= 0, && \text{for } t=0\\
\eta(u_{x,t}-\phi)+\Lambda\Omega_{x,t} &= 0, && \text{for all } t\geq1\\
U_{c,t}-\alpha+\Lambda(U_{c,t}-\alpha+U_{cc,t}c_t) &= 0, && \text{for all } t\geq0\\
v_{g,t}-\alpha-\Lambda\alpha &= 0, && \text{for all } t\geq0,
\end{aligned}$$

where Λ is the Lagrange multiplier associated with (48). Note that c_t and g_t are constant for all $t\geq0$, while x_t is constant for all $t\geq1$ and may be different in the initial period. Call the corresponding allocation $\{x_0,x_1,c,g\}$. Thus, we can write (48) as $(U_c-\alpha)c-\alpha g+\beta\Omega(x_1)=(1-\beta)\Phi(x_0)(1+B_0)$. Plug this expression into (49) and we get $B_t=\frac{\Phi(x_0)(1+B_0)}{\Phi(x_1)}-1$ for all $t\geq1$, i.e., debt is constant after the initial period. Thus, $\{x_0,x_1,c,g\}$ solve

$$\begin{aligned}
\alpha\eta(u_{x,0}-\phi)-(v_g-\alpha)\Phi_{x,0}(1+B_0) &= 0\\
\alpha\eta(u_{x,1}-\phi)+(v_g-\alpha)\Omega_{x,1} &= 0\\
v_g(U_c-\alpha)+(v_g-\alpha)U_{cc}c &= 0\\
(U_c-\alpha)c-\alpha g+\beta\Omega(x_1)-(1-\beta)\Phi(x_0)(1+B_0) &= 0.
\end{aligned}$$

Suppose $B_0=B^*$. We now verify that $\{x^*,x^*,c^*,g^*\}$ solves the above system. We get

$$\begin{aligned}
\alpha\eta(u_x^*-\phi)-(v_g^*-\alpha)\Phi_x^*(1+B^*) &= 0 && (50)\\
\alpha\eta(u_x^*-\phi)+(v_g^*-\alpha)\Omega_x^* &= 0 && (51)\\
v_g^*(U_c^*-\alpha)+(v_g^*-\alpha)U_{cc}^*c^* &= 0 && (52)\\
(U_c^*-\alpha)c^*-\alpha g^*+\beta\Omega(x^*)-(1-\beta)\Phi(x^*)(1+B^*) &= 0. && (53)
\end{aligned}$$

Equations (52) and (53) are identical to the MPME steady state conditions, (39) and (40), re-spectively. Note that $\varepsilon_x=-\Phi_x(1+B)$; thus, (50) is identical to (38). Finally, note that $\varepsilon_{x'}=\beta(\Omega_x'+\Phi_x'(1+B'))$; since (37) states that $\varepsilon_x^*=0$, we get $\Omega_x^*=-\Phi^*(1+B^*)$ and (51) is identical to (50). Thus, given $B_0=B^*$, the solution to the Ramsey problem is $\{B_t=B^*,x_t=x^*,c_t=c^*,g_t=g^*\}$ for all $t\geq0$. ∎

D Parameter-elasticities for Table 4

	Competitive Markets						Financial Intermediation						Trading Frictions				
	$\Delta\alpha$	$\Delta\phi$	$\Delta\eta$	$\Delta\rho$	$\Delta\sigma$	$\Delta\psi$	$\Delta\alpha$	$\Delta\phi$	$\Delta\eta$	$\Delta\rho$	$\Delta\sigma$	$\Delta\psi$	$\Delta\alpha$	$\Delta\phi$	$\Delta\rho$	$\Delta\sigma$	$\Delta\psi$
$\frac{B^*(1+\mu^*)}{Y^*}$	−0.55	0.69	1.82	−0.38	2.22	−0.13	−0.43	0.47	0.80	−0.22	2.69	−0.21	−0.50	−0.72	−0.32	13.07	−0.19
π^*	−0.96	0.00	1.74	−1.51	2.04	1.45	−0.90	0.00	0.00	−1.39	1.89	1.31	−0.91	−0.85	−1.48	7.70	1.38
τ^*	−0.55	0.00	0.00	−0.09	0.00	0.73	−0.54	0.00	0.00	−0.05	0.00	0.71	−0.54	0.00	−0.11	0.00	0.73
$\frac{p^*\tau^*n^*}{Y^*}$	−0.55	−0.06	0.00	−0.15	0.00	0.83	−0.51	−0.10	−0.16	−0.10	−0.10	0.85	−0.54	−0.02	−0.17	−0.15	0.83
$\frac{p^*g^*}{Y^*}$	−0.51	−0.06	0.00	−0.07	0.00	0.77	−0.47	−0.10	−0.16	−0.03	−0.10	0.79	−0.50	−0.02	−0.10	−0.15	0.77

E Numerical approximation of stochastic economies

The monetary economies with aggregate uncertainty are solved globally using a projection method with the following algorithm:

(i) Define a grid of N_Γ points over Γ. The stochastic state space S is discretized in N_S states, using the method described in Tauchen (1986).[19] Create the indexed functions $\mathcal{B}^i(B)$, $\mathcal{X}^i(B)$, $\mathcal{C}^i(B)$, and $\mathcal{G}^i(B)$, for $i = \{1,\ldots,N_S\}$, and set an initial guess.

(ii) Construct the following system of equations: for every point in the debt and exogenous state grids, evaluate equations (44)—(47). Since (45) contains $\mathcal{X}^j(\mathcal{B}^i(B))$ (and its derivative) and $\mathcal{G}^j(\mathcal{B}^i(B))$, use cubic splines to interpolate between debt grid points and calculate the derivatives of policy functions.

(iii) Use a non-linear equations solver to solve the system in (ii). There are $N_\Gamma \times N_S \times 4$ equations. The unknowns are the values of the policy function at the grid points. In each step of the solver, the associated cubic splines need to be updated so that the interpolated evaluations of future choices are consistent with each new guess.

We could alternatively simplify step (iii) by using value function iteration: simply solve the maximization problem of the government at every grid point. Update the policy and value functions and iterate until convergence is achieved. This method is simpler to implement, but less precise. However, relative to the algorithm described above, it serves a dual purpose: first, it can be used to obtain a good initial guess for policy functions; and second, it provides a method to verify the numerical approximation found using the first-order conditions, as described above.

Each exogenous stochastic process is approximated by 7 discrete states, which implies $N_S = 49$. For debt, I set $N_B = 10$ and let $\Gamma = [-1.0, 3.5]$. There are a total of 196 functions to solve. Given that the debt grid has 10 points, we have to solve a system of $1,960$ equations. To measure the precision of the solution, I create a debt grid of $1,000$ points for Γ, evaluate (44)—(47) for all these debt points and all $s \in S$, and sum the squared residuals. For the case with competitive markets, the sum of squared residuals for each equation are, respectively: $2e^{-11}$, $2e^{-14}$, $3e^{-12}$ and $3e^{-13}$. The other two cases feature similar degrees of precision.

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[19] See Flodén (2008) for a recent comparison with alternative methods.

Finally, we need to construct measures of real GDP and the inflation rate. In the model, real GDP is measured using the non-stochastic steady state as the base period for prices. Thus, let $y_t = \ln(\tilde{p}^* x_t + p^*(c_t + g_t))$ be the measure of log real GDP in the artificial economy and let dy_t be log real GDP in period t minus its sample average. To calculate the inflation rate, define the aggregate (normalized) price level P as the weighted average of prices in the day and night markets. I.e., for any period t, let $P_t \equiv s_D \tilde{p}_t + s_N p_t$, where s_D and s_N are the expenditure shares for the day and night markets, respectively. Expenditure shares are constructed using the non-stochastic steady state statistics as the base period: $s_D \equiv \frac{\tilde{p}^* x^*}{Y^*}$ and $s_N \equiv \frac{p^*(c^* + g^*)}{Y^*}$. The inflation rate is defined as: $\pi_t \equiv \frac{P_t(1+\mu_{t-1})}{P_t - 1} - 1$.

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Figure 1: Impulse response functions

Response to one Cholesky s.d. expenditure innovation   **Response to one Cholesky s.d. productivity innovation**
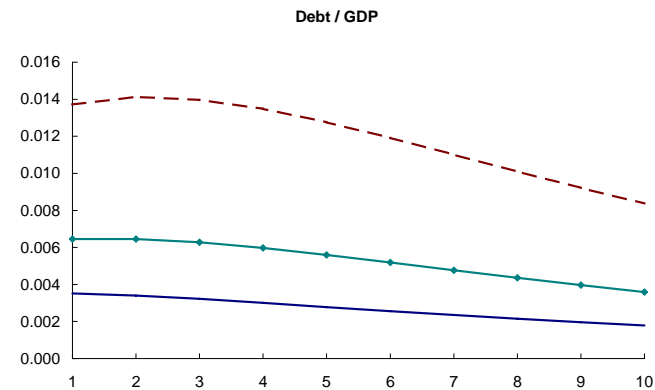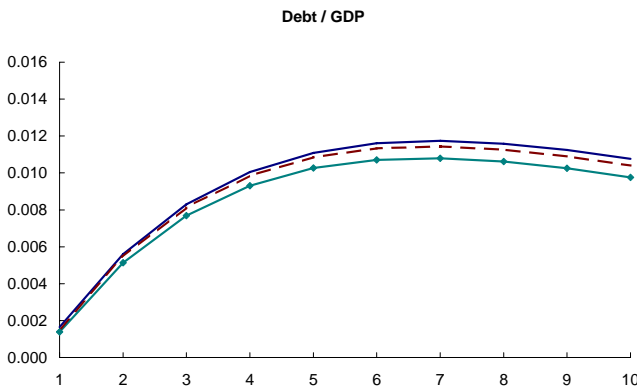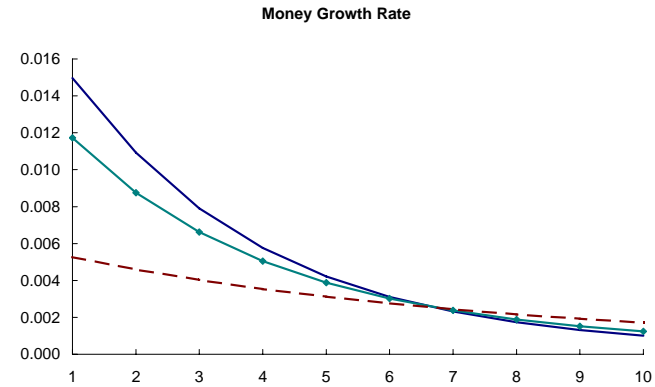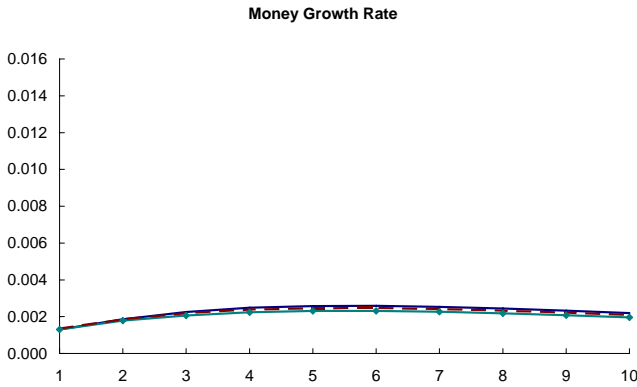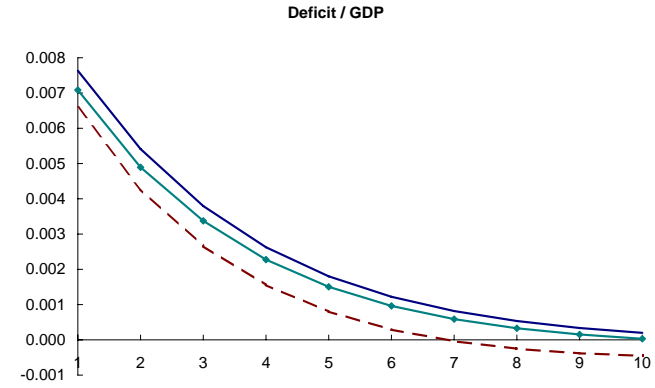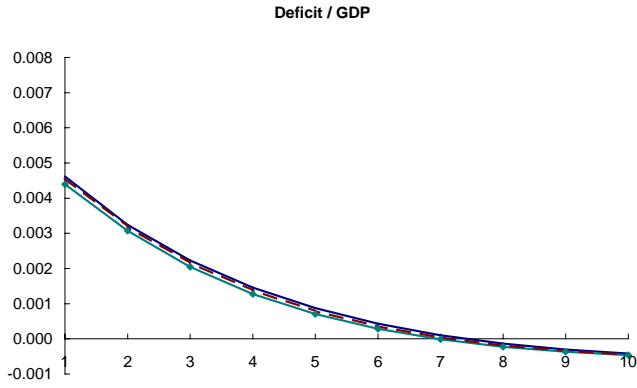
Note. Competitice Markets: solid lines; Financial Intermediation: solid lines with diamonds markers; Trading Frictions: dashed lines.
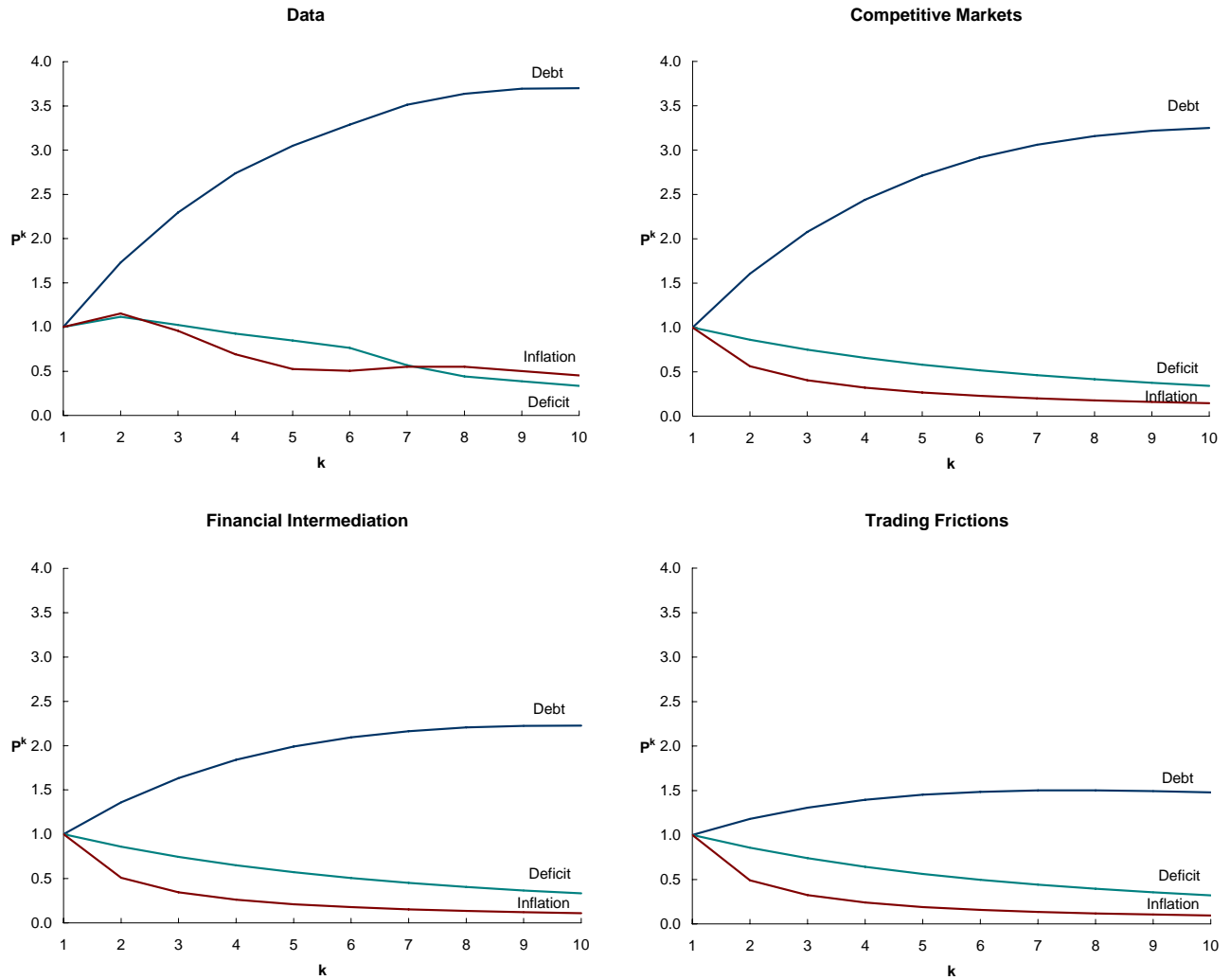
Figure 2: k-variance ratios of policy variables
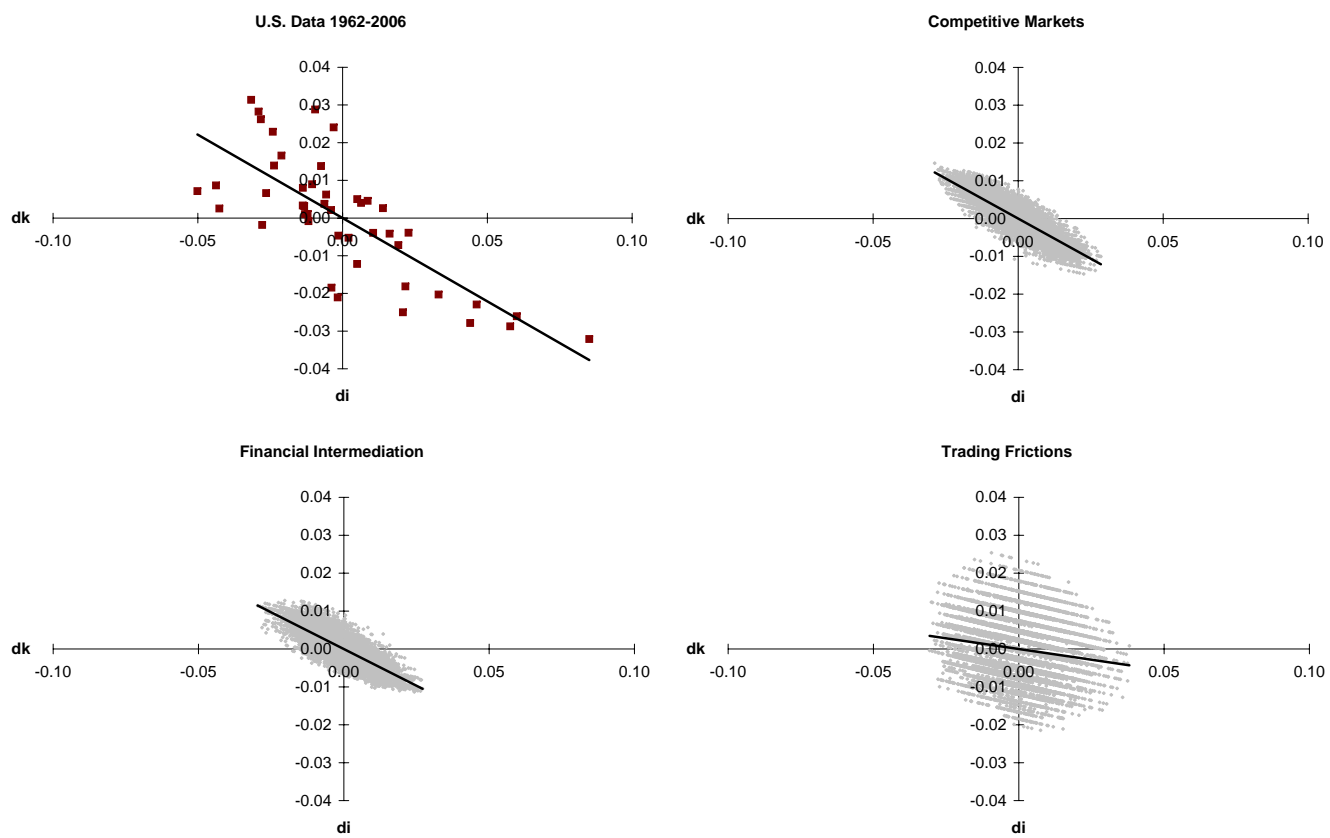
Figure 3: Money demand

Figure 4: Phillips curve