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**Working Paper**

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## Legislative Bargaining and the Dynamics of Public Investment

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Abstract

## Legislative Bargaining and the Dynamics of Public Investment

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We present a legislative bargaining model of the provision of a durable public good over an infinite horizon. In each period, there is a societal endowment which can either be invested in the public good or consumed. We characterize the optimal public policy, defined by the time path of investment and consumption. In each period, a legislature with representatives of each of $n$ districts bargain over the current period's endowment for investment in the public good and transfers to each district. We analyze the Markov perfect equilibrium under different voting q–rules where q is the number of yes votes required for passage. We show that the efficiency of the public policy is increasing in q because higher q leads to higher investment in the public good and less pork. We examine the theoretical equilibrium predictions by conducting a laboratory experiment with five–person committees that compares three alternative voting rules: unanimity (q=5); majority (q=3); and dictatorship (q=1).

*Keywords: Dynamic political economy, voting, public goods, bargaining, experiments*

*JEL classification: D71, D72, C78, C92, H41, H54*

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# 1    Introduction

A central role of government is the provision of public goods to its citizenry. Most public goods provided by governments are durable, and hence dynamic in nature. It takes time to accumulate them, and they depreciate slowly, projecting benefits for many years. Prominent examples are national defense, environmental protection and public infrastructure. Although a large literature has studied public good provision and public policy formation by governments or legislatures in static models, both theoretically and empirically, much less is known about dynamics of public investments. Two political economy questions immediately come to mind. First, can we say anything about the efficiency of dynamic public investment by legislatures or governing bodies operating under democratic rules? Second, to what extent does the efficiency depend on the specific voting rules under which these governing bodies operate? Again, little is known about these effects, except for highly specialized static environments, where only a single decision is taken (for example, Baron and Ferejohn [1989], Volden and Wiseman [2005]).

In this work, we make a first attempt to answer such questions by proposing a new theoretical framework for studying the political economy of dynamic public good provision. We analyze a legislative bargaining model under alternative voting rules and examine its predictions in a laboratory experiment. The basic environment we study consists of many citizens who live in $n$ equal-sized districts and can invest resources over time to accumulate a stock of durable public good (roads, bridges, sewers, etc.). An investment policy is taken each period by a central governing body, the *Legislature*, composed of a single representative from each district, and operating under a procedure consisting of an agenda setting stage and a voting rule. In each period, the legislature has the power to allocate a fixed budget of resources between investment in the public good (whose utility is enjoyed by all citizens of all districts) and targeted transfers between districts.[1] Representatives bargain in the legislature over the budget allocation. We characterize the trajectory of public policies that would result from a symmetric Markov perfect equilibrium for any q rule adopted by the legislature, where a q rule requires q out of n yes votes for passage of policy for the current period. We compare these equilibrium trajectories with the optimal public policy (i.e. the policy that maximizes the welfare of the citizens).

The equilibrium generates clear predictions about how the dynamics of investment are affected by the voting rule the legislature uses to make its decisions. The model implies that a stricter requirement for passage (i.e., higher q) will generate a higher level of investment

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[1]That is, in each period, public policy actually has three components: public investment, taxation, and redistribution.

and a higher steady state of the public good. For any voting rule, investment should continue until a steady state is reached, and the equilibrium steady states are monotonically increasing in q. For any q rule except unanimity, this steady state is lower than in the optimal solution. Furthermore, for any voting rule including unanimity, the *speed* of investment along the transition path to the steady state is slower than in the optimal solution, as proposers with agenda control have an incentive to skim off resources along the way.

We examine the predictions of the theory by conducting a carefully designed series of controlled laboratory experiments. In the experiments, we focus on three alternative voting rules: a dictatorship rule (D), a simple majority rule (M), and a unanimity rule (U). There are several reasons for examining the theory with data from controlled experiments. First, some kind of data is needed to discipline the theory. It is important to identify whether the theory is at all reasonable from a behavioral or empirical standpoint. Second, if one is going to take the model to data, there are clear identification advantages for experimental studies when studying a highly structured dynamic environment such as the one in this paper. In our model, strategic behavior can be clearly identified only if there is a precise measurement of certain state variables as well as the actions available to the players, and for this purpose data from laboratory experiments that control key variables in observable ways have some obvious advantages over field data. The control of laboratory experiments allows us to directly test the main comparative static implication of the theory: Do higher q rules lead to more efficient investment paths?

A third motivation for pursuing the experiments is that the predictions and assumptions of the model seem unintuitive and in many ways implausible. The legislators in the model are assumed to be completely rational and selfish, with rational expectations and perfect foresight about the entire infinite equilibrium path of public policies. Any behavioral limitations, bounded rationality, or other-regarding preferences are completely assumed away, and in principle the presence of these factors could drastically change the predictions of the model. Thus, because there is considerable evidence from both controlled experiments and casual observation that people are neither perfectly rational, purely selfish, nor perfectly clairvoyant, there is good reason to be skeptical of the predictions that come out of such a model. This leads to the second basic question we ask: Are the predictions of the model robust to behavioral factors and limitations on perfect rationality that we know exist but are assumed away in the model?

Finally, it is important to note that our model's predictions only apply to a very small subset of the huge set of subgame perfect equilibria in the infinite game. In particular, there generally exist non-Markov equilibria that lead to efficient investment paths *independently of the q rule*. Such equilibria closely resemble cooperative equilibria in repeated games, such

at the prisoner's dilemma. We know from experiments that groups are highly successful at achieving efficient outcomes in repeated prisoner's dilemma and similar games (Dal Bo [2005]). Hence the third basic question: Can the inefficiencies be overcome? Do efficient investment paths emerge in these legislative bargaining games, possibly due to the repeated game effects created by the dynamic environment?

The effect of q on the efficiency of public policy is confirmed by the experimental data. A higher q rule leads to better public policies in the form of significantly greater public good investment. Second, besides these qualitative findings, the public good levels are also quantitatively close to the predictions of the Markov perfect equilibrium. Also consistent with this equilibrium, players choices reflect forward looking decision making: the theoretical expected continuation value functions are the most important significant variables explaining voting behavior. Thus the behavior in these complicated environments appear to be largely robust to factors outside the model such as bounded rationality and non-selfish preferences. Of course the data is not a perfect match to the theory. We observe some differences between the finer details of the theoretical predictions and the data. The clearest such deviation is a statistically significant overinvestment in the early rounds under all voting rules. This phenomenon is reminiscent of the finding in experiments on static public good provision (by a centralized legislature or in a voluntary provision setting), but it is more complex in our dynamic setting: we observe a large initial overinvestment in the early rounds, however, this initial overinvestment is to a large extent "undone" in later rounds, as it is usually followed by significant disinvestment (in D and M) or lack of further investment (in U) approaching the equilibrium steady state.

We discuss some related literature in the next section. The formal model is explained in section 3. Section 4 fully characterizes the dynamically efficient public policy. Section 5 characterizes the Markov perfect equilibrium of the legislative bargaining game. Section 6 describes the experimental design and procedures. Section 7 presents the results of the experiment.

# 2   Related Literature

This work contributes primarily to the literature on impact of institutions on the dynamics of public good provision, which includes the important special case of common-pool resources management. Walker et al. [2000], Ostrom [1990], and Ostrom et al [1994] examine both theoretically and empirically how communication, voting rules, and other institutions for collective decision making affect the outcome of commons dilemma games; Olson [1993] and McGuire and Olson [1996] compare the negative externalities a democratic majority and an

autocrat might impose on society; Harrison and Hirschleifer [1989], Duffy et al. [2007], Choi et al. [2008,2011] examine sequential contribution mechanisms for provision of a public good[2]; Boylan and McKelvey [1995] and Boylan et al [1996] study a one-sector model of economic growth in which decisions about capital accumulation are made by a political process. In contrast to the present paper, in all these studies the actual dynamics are rudimentary or nonexistent; either the public good is not durable, in the sense of projecting a stream of benefits that changes over time along the investment path, the decision on its accumulation path is taken once and for all at the beginning of the game, or payoffs are one-shot, and depend only on the sum of investments over the entire horizon.

The emphasis on the comparative performance of different voting rules ties the paper to a large literature going back to Rousseau [1762], Condorcet [1785], Wicksell [1896], Buchanan and Tullock [1962] and, more recently, Austen-Smith and Banks [1996], Cox [1987], Messner and Polborn [2004], Harstad [2005], and many others. Contrary to our analysis, all these papers focus on static settings and, with the exception of Harstad [2005], deal with purely distributive politics or reforms with unknown winners (rather than with investment in a public project). The debate on the economic consequences of different voting rules has important practical implications, as we observe a large variety of majority rules in the real world. In a parliamentary assembly, motions on the floor are usually passed by a simple majority. However, a broader consent is required in many other contexts. In the European Union's council of ministers, some proposals require only a simple majority, some a supermajority and some unanimous consent. Explicit supermajorities are required in most countries for a constitutional reform, and sixteen US states (Arizona, Arkansas, California, Colorado, Delaware, Florida, Kentucky, Louisiana, Michigan, Mississippi, Missouri, Nevada, Oklahoma, Oregon, South Dakota, Washington) require a two-thirds supermajority legislative vote in order to increase taxes. Our finding that a higher majority requirement leads to higher long-run public investment and hence more dynamically efficient public policy provides a formal rationale for Wicksell [1896], who, more than a century ago, advocated unanimity as the only rule guaranteeing Pareto improvements. However, in our dynamic setting, the impact of the voting rule on the economic outcomes is more complex (and perhaps surprising): even with unanimity, the accumulation path is predicted to be inefficiently slow and, when the marginal value of investment is high, a higher majority rule increases, rather than reducing, the amount of rents an agenda setting proposer can guarantee to the district he represents.

This paper is also the first experimental study of the dynamic accumulation process of a durable public good by a legislature. Our findings, thus, extend the recent experimental literature on legislative bargaining models to dynamic settings. McKelvey [1991] is the

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[2]See also the references in those papers.

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first experimental study of legislative bargaining models à la Baron and Ferejohn [1989]. Diermeier and Gailmard [2006], Diermeier and Morton [2006], Frechette, Kagel, and Lehrer [2003], and others have also reported laboratory experimental studies of legislative bargaining, but only in a *static* setting with purely distributive policies. Recently, Frechette, Kagel, and Morelli [2011] extend the experimental analysis to policy spaces with public goods using a model based on Volden and Wiseman [2006]. All of these works, however, limit the analysis to static environments in which only a single policy outcome is decided. Battaglini and Palfrey [2011] study a simple *dynamic* model of legislative bargaining, but limit the analysis to purely distributive policies in which public goods cannot be accumulated and redistribution across periods is not possible.

Finally, this paper contributes to a growing theoretical literature on dynamic political economy. The most closely related paper in this branch of the literature is the theoretical paper by Battaglini and Coate [2006][3], who study public good accumulation in a different political-economic environment, and consider a different bargaining protocol. Their results are different from ours because the budget in each period is endogenously determined, there is distortionary taxation, and preferences are stochastic.

# 3 The Model

Consider an economy in which a continuum of infinitely lived citizens live in $n$ districts and each district contains a mass one of citizens. There are two goods: a private good $x$ and a public good $g$. An allocation is an infinite nonnegative sequence of public policies, $z = (x_\infty, g_\infty)$ where $x_\infty = (x_1^1, ..., x_1^n, ..., x_t^1, ..., x_t^n, ...)$ and $g_\infty = (g_1, ..., g_t, ...)$. We refer to $z_t = (x_t, g_t)$ as the *public policy* in period $t$. The utility $U^j$ of a representative citizen in district $j$ is a function of $z^j = (x_\infty^j, g_\infty)$, where $x_\infty^j = (x_1^j, ..., x_t^j, ...)$. We assume that $U^j$ can be written as:

$$U^j(z^j) = \sum_{t=1}^{\infty} \delta^{t-1} \left[ x_t^j + u(g_t) \right],$$

where $u(\cdot)$ is continuously twice differentiable, strictly increasing, and strictly concave on $[0, \infty)$, with $\lim_{g \to 0^+} u'(g) = \infty$ and $\lim_{g \to \infty^+} u'(g) = 0$. The future is discounted at a rate $\delta$.

There is a linear technology by which the private good can be used to produce public good, with a marginal rate of transformation $p$ equal to 1. The private consumption good is nondurable, while the public good is durable.[4] Thus, if the level of public good at time $t-1$

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[3]Other recent contributions in dynamic bargaining are Baron [1996], Battaglini and Coate [2008], Barseghyan, Battaglini, and Coate 2010], Baron, Diermeier and Fong [2011], Duggan and Kalandrakis [2010], Kalandrakis [2004, 2009], and Penn [2009].

[4]We assume that the stock of the public good does not depreciate over time, independently from the

is $g_{t-1}$ and the investment in the public good is $I_t$, then the level of public good at time $t$ will be:

$$g_t = g_{t-1} + I_t.$$

Because all citizens in district $j$ are identical, we refer collectively to the "behavior of a district" as described by the behavior of a representative citizen $j$. Henceforth we will simply refer to district $j$. In period $t$, the economy is endowed with $W_t$ units of private good, where $W_t = W \;\; \forall t$. The initial stock of public good is $g_0 \geq 0$, exogenously given.

The *public policy* in period $t$ is required to satisfy three feasibility conditions:

$$
\begin{aligned}
x_t^j &\geq& 0 \;\; \forall j \\
I_t + g_{t-1} &\geq& 0 \;\; \forall t \\
I_t + \sum_{j=1}^{n} x_t^j &\leq& W_t \;\; \forall t
\end{aligned}
$$

The first two conditions guarantee that allocations are nonnegative. We assume that public investment can be scaled back in the future at no cost. The third condition requires that the current budget is balanced. These conditions can be rewritten slightly. If we denote $y \equiv g_t = g_{t-1} + I_t$ as the new level of public good after an investment $I_t$ when the last period's level of the public good is $g_{t-1}$, then the public policy in period $t$ can be represented by a vector $(y, x_t^1, ..., x_t^n)$. Dropping the $t$ subscripts and substituting $y$, the budget balance constraint $I_t + \sum_{j=1}^{n} x_t^j \leq W_t$ can be rewritten as:

$$\sum_{j=1}^{n} x^j + [y - g] \leq W,$$

recalling that we use $y$ to denote the post-investment level of public good attained in period $t$, and $g$ for the pre-investment level of public good inherited from period $t-1$. The one-shot utility to district $j$ from this public policy, $(y, x^1, ..., x^n)$, is $U^j = x^j + u(y)$.

Our interest in this paper is to analyze the performance of a class of political procedures in building public infrastructure, i.e., generating a feasible sequence of public policies, $z$. We consider a legislature where representatives of each district bargain with each other to decide how to divide the current period's societal endowment between public investment and private transfers to each district. We will consider procedures that are time independent and have no commitment. That is, the voting procedures is the same in every period, and the

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investment decision. One possible extension is the case in which the public good depreciates at a rate $d > 0$.

outcome of the procedure is a public policy for only the current period. The level of the state variable $g$, however, creates a dynamic linkage across policy making periods. In such procedures, we will characterize the outcomes associated with symmetric Markov perfect equilibria.

# 4   The Optimal Public Policy

As a benchmark with which to compare the equilibrium allocations by a legislature, we first analyze the sequence of public policies that maximizes the sum of utilities of the districts. This is the *optimal public policy*. This optimization problem has a recursive representation in which $g$ is the state variable, and the value function $v_O(g)$ can be represented recursively as:

$$v_O(g) = \max_{y,x} \left\{ \begin{array}{c} \sum_{j=1}^{n} x^i + nu(y) + \delta v_O(y) \\ s.t \ \ \sum_{j=1}^{n} x^i + y - g \leq W, \ x^i \geq 0 \ \forall i, \ y \geq 0 \end{array} \right\} \tag{1}$$

By standard methods (see Stokey and Lucas [1989]) we can show that a continuous, strictly concave and differentiable $v_O(g)$ that satisfies (1) exists and is unique. The optimal policies have an intuitive characterization.[5] When the accumulated level of public good is low, the marginal benefit of investing in $g$ is high, and it is efficient to invest as much as possible: in this case $y_O(g) = W + g$ and $\sum_{j=1}^{n} x^i = 0$. When $g$ is high, it is efficient to reach the level of public good $y_O^*(n)$ that solves the unconstrained optimization problem in (1): i.e. $nu'(y_O^*(n)) + \delta v_O'(y_O^*(n)) = 1$. As it can be shown:

$$y_O^*(n) = [u']^{-1}\left(\frac{1-\delta}{n}\right) \tag{2}$$

For $g_O \geq y_O^*(n) - W$, therefore, investment will stop, $y_O(g_O) = y_O^*(n)$ and, without loss of generality, $x^i = x^o = (W + g - y)/n$.[6]

The policy and the investment functions, therefore, have the following simple structure:

$$y_O(g) \ = \ \min\{W + g, y_O^*(n)\} \tag{3}$$

$$I_O(g) \ = \ \min\{W, y_O^*(n) - g\} \tag{4}$$

This investment function implies that, in the optimal solution, the stock of the public good converges to a unique steady state, $y_O^o = y_O^*(n)$. In $y_O^o$, the per agent level of private consumption is positive: $x^* = W/n > 0$. Figure 1 provides a representation of the optimal

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[5]A formal derivation of the properties discussed in this sections is available in the appendix.

[6]The optimal solution does not depend on the distribution of private consumption.
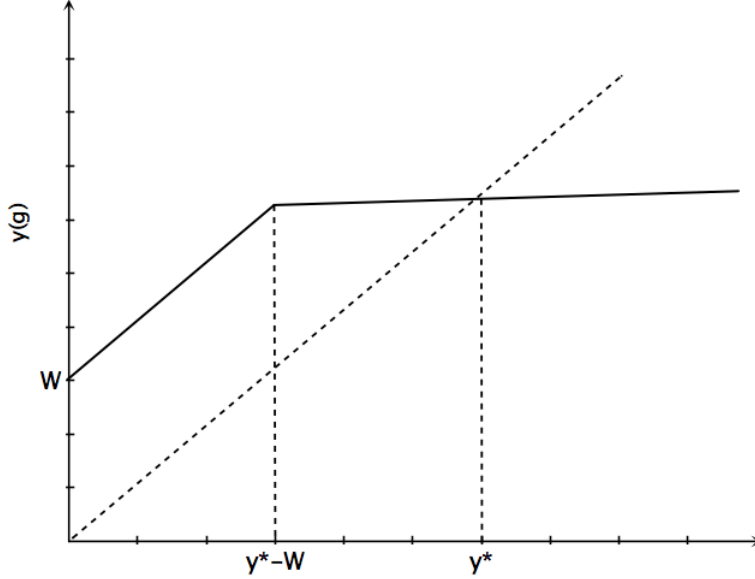
Figure 1: The Optimal Public Policy

investment path.

# 5    The Political Equilibrium

We now consider a legislature, composed of a representative from each district, which bargains over the allocation of the economy-wide resources among private goods and public good investment. In this mechanism, in each period, the legislature decides on a level of investment in the public good. The legislative policy also includes an allocation of the budgetary surplus (endowment minus investment) to the districts, which is non-negative for all districts, but not necessarily uniform. Investment can be negative, but the amount of negative investment cannot exceed the current stock of public good. Thus, we can represent a policy by the legislature at time $t$, by a public policy $(x_t^1, ..., x_t^n, y_t)$ that satisfies the same feasibility constraints as in the previous section. The bargaining protocol with which a public policy is chosen in a legislature is as follows. At the beginning of each period an agent is chosen by nature to propose a policy $(x^1, ..., x^n, y)$. Each legislator has the same probability to be recognized as proposer. If at least $q \in \{1, 2, ..., n\}$ legislators vote in favor of the proposal, it passes and it is implemented. The legislature then adjourns and meets in the following period with a new level of public good $y$. If instead the policy does not receive a qualified majority, then the status quo policy is implemented. We assume that the status quo is zero investment in public goods and $x^j = W/n$ for all $j$. The legislature, moreover, adjourns and meets in the following period with a new level of public good $g$.

To characterize behavior when policies are chosen by a legislature we look for a symmetric Markov perfect equilibrium. In this type of equilibrium, equilibrium strategies depend only on payoff-relevant information, and all representatives use the same strategy. Therefore, in a symmetric Markov perfect equilibrium, any representative selected to propose at some time $t$ uses the same strategy, and this depends only on the current stock of public good $(g)$. Similarly, the probability a legislator votes for a proposal depends only on the proposal itself and the state $g$. As is standard in the theory of legislative voting, we focus on weakly stage-undominated strategies, which implies that legislators vote for a proposal if and only if their expected utility (current payoff plus discounted continuation value) from the status quo is not greater than their expected utility from the proposal. Without loss of generality, we focus on an equilibrium in which proposals are accepted with probability one on the equilibrium path.

It is easy to verify that, in a symmetric Markov perfect equilibrium, a proposer would either make no monetary transfer to the other districts, or would make a transfer to exactly $q - 1$ legislators. An equilibrium can therefore be described by a collection of functions $\{y_L(g), s_L(g)\}$ that specifies the choice made by the proposer in a period in which the state is $g$. Here $y_L(g)$ is the proposed new level of public good and $s_L(g)$ is a transfer offered to the $q - 1$ other districts.[7] The proposer's district receives the surplus revenues $x_L(g) = W - y_L(g) + g - (q - 1)s_L(g)$. Associated with any symmetric Markov perfect equilibrium in the $L$ game is a value function $v_L(g)$ which specifies the expected continuation payoff of a legislator when the state is $g$ before the proposer is selected.

Contrary to the case of the previous section, the policy is now chosen by a self interested proposer who maximizes the utility of his own district. Given $v_L$, the proposer's problem is:

$$
\max_{x,y,s} \left\{
\begin{array}{c}
x + u(y) + \delta v_L(y) \\
s.t \\
(q - 1)s + x + y - g \leq W \\
x \geq 0, s \geq 0 \\
s + u(y) + \delta v_L(y) \geq \frac{W}{n} + u[g] + \delta v_L(g)
\end{array}
\right\}
\tag{5}
$$

where $x$ is the transfer to the proposer. This problem is similar to the efficient problem (1): the first inequality is the budget balance constraint, and the following two inequalities are the feasibility constraints.[8] The last inequality is however new: it is the incentive compatibility constraint that needs to be satisfied if a proposal is to be accepted by $q - 1$ other districts.

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[7]To ensure symmetry, $q - 1$ legislators are randomly selected by the proposer.

[8]Since $u'(0) = \infty$, the constraint $y \geq 0$ is never binding and therefore it can be ignored without loss of generality.

The solution to (5) is complicated by the fact that the set of binding constraints is state dependent and the value function is not typically concave in $g$. Despite this, the next result shows a sufficient condition for the existence of a Markov perfect equilibrium. We say that an equilibrium is *regular* if the associated value function is continuous, non decreasing, and almost everywhere differentiable, and leads to a strictly concave objective function in (5). We have:

**Proposition 1.** *There is a $\bar{\delta} < 1$ and a $\overline{W} > 0$ such that for all $\delta > \bar{\delta}$, and all $W > \overline{W}$ a regular Markov perfect equilibrium exists in which the public good level is given by*

$$
y_L(g) = \begin{cases} y_1^* & g \le g_1(y_1^*) \\ \widetilde{y}(g) & g \in (g_1(y_1^*), g_2(y_L^*)] \\ y_L^* & else \end{cases} \tag{6}
$$

*where $y_1^*$ and $y_L^*$ are constants with $y_L^* > y_1^*$; $g_1(y_1^*), g_2(y_L^*)$ are functions respectively of $y_1^*$ and $y_L^*$; and $\widetilde{y}(g)$ is an increasing function of $g$.[9]*

There is an intuitive explanation for the shape of the policy function (6). For $g \le g_1(y_1^*)$ the proposer acts as if the other districts did not exist: he diverts resources only toward his own district and chooses the investment without internalizing the other districts' welfare. This implies that the proposer can choose $y_1^*$ where:

$$
y_1^* \in \arg\max_y \{u(y) - y + \delta v_L(y)\} \tag{7}
$$

The other districts accept this policy because the investment $y_1^*$, is sufficiently high to make this policy better than the status quo. When $g \ge g_1(y_1^*)$, the proposer can not afford to ignore the other districts. He first finds it optimal to "buy" their approval by increasing $g$ and investing $\widetilde{y}(g) > y_1^*$ (in the interval $(g_1(y_1^*), g_2(y_L^*)]$): $\widetilde{y}(g)$ is chosen large enough to satisfy the incentive compatibility constraint as an equality. For $g > g_2(y_L^*)$, however, the proposer finds it optimal to provide pork to a minimal winning coalition of districts, and to invest $y_L^*$. In choosing $y$ now the proposer must internalize the utility of $q$ legislators, so:

$$
y_L^* \in \arg\max_y \{qu(y) - y + \delta q v_L(y)\} \tag{8}
$$

It is interesting to note that when the proposer's strategy is constant (at $y_1^*$ or at $y_L^*$) we have a *dynamic free rider problem*: an increase in investment above, say, $y_L^*$, at $t$ would induce a proportional reduction in investment at $t+1$, and so discourage public good accumulation.

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[9]Notice that $y_1^*$, $\widetilde{y}(g)$, and $y_L^*$ also depend on $\delta$, $n$, and $q$.

This is key to understanding underinvestment in the steady state. When $y_L(g) = \widetilde{y}(g)$ the dynamic free rider problem is mitigated because an increase in $g$ induces an increase in $\widetilde{y}(g)$. This occurs because the increase in $g$ makes the incentive constraint at $t+1$ more binding, so it forces the proposer in the following period to increase the investment in the public good.

The next result guarantees that the equilibrium outcome is unique:

**Proposition 2.** *For $\delta > \bar{\delta}$, and $W > \overline{W}$ as defined in Proposition 1, the legislative game has a unique regular equilibrium steady state, $y_L^*(q,n) = [u']^{-1}\left(\frac{n/q-\delta}{n}\right)$.*
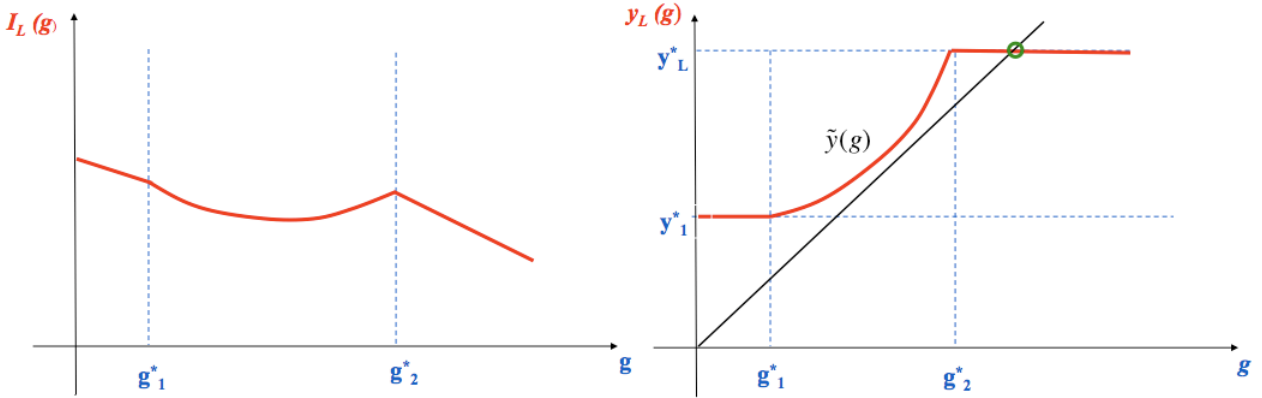


Figure 2: Legislative Game Equilibrium, I(g) and y(g)

Figure 2 provides a representation of the equilibrium. The first panel of Figure 2 represents the investment function $I_L(g)$:

$$I_L(g) = \begin{cases} y_1^* - g & g \le g_1 \\ \widetilde{y}(g) - g & g \in (g_1, g_2] \\ y_L^* - g & else \end{cases} \tag{9}$$

(where for simplicity $g_1$ is the equilibrium value $g_1(y_1^*)$, and similarly for $g_2$). It is interesting to note that while in the optimal solution $I_O(g)$ is a monotonically (weakly) decreasing function, in the political equilibrium $I_L(g)$ is not monotonic (compare (9) with the investment in the optimal solution, i.e. $I_O(g) = \min\{W, y_O^* - g\}$). The non-monotonicity of the investment function is a consequence of the fact that the incentive compatibility constraint is not always binding and that the value of the status quo is endogenous. The second panel of Figure 2 shows the equilibrium proposed level of the public good, as a function of the state, $y_L(g)$. This curve fully describes the dynamics of public good provision and the steady state. The steady state level of public good $y_L^*$ corresponds to the point where the $45^o$ line intersects the investment curve.

How does the accumulation of public good in the political equilibrium compare to the optimal solution? Do legislatures provide durable public goods efficiently? And how does this depend on the voting rule adopted? The next result addresses these questions.

**Proposition 3.** (a). $0 < y_L^*(1,n) < y_L^*(2,n) < \ldots < y_L^*(n,n) = y_O^*(n)$; (b) $I_L(g;q,n) < I_O(g;n) \; \forall \; n > 1, \; \forall \; q = 1,\ldots,n$, and $\forall \; g \leq y_O^*(n)$.

Proposition 3(a) states that the equilibrium steady state of the legislative game is less than the steady state of the optimal solution for any $q$-voting rule but unanimity, and it is equal to the steady state of the optimal solution for $q = n$ (with efficiency monotonically increasing in $q$). Proposition 3(b) states that for any voting rule, including unanimity, the accumulation of the public good that leads to the steady state is inefficiently slow. This result arises because, in any legislative game (including unanimity), the proposer finds it profitable to divert some resources towards private transfers to his own district and to $q-1$ other districts (as discussed above).

**Example.** Let the utility function for the public good be the power function, $u(y) = \frac{1}{\alpha}y^\alpha$. The unique long run steady state in the optimal solution is $y_O^*(n) = \left(\frac{n}{1-\delta}\right)^{\frac{1}{1-\alpha}}$ and the unique equilibrium steady state of the legislative game is $y_L^*(n,q) = \left(\frac{n}{\frac{n}{q}-\delta}\right)^{\frac{1}{1-\alpha}}$.

## 5.1  Non-Markov Equilibria

We have restricted our attention to symmetric Markov perfect equilibria. However, the legislative game we study is a dynamic game with an infinite horizon with many subgame perfect equilibria. The Markovian assumption of stationary strategies is very restrictive and it is possible that some other equilibria can sustain more efficient outcomes through the use of history-dependent strategies (punishment and rewards for past actions).

This is common in infinite horizon games with perfect information. For instance, the only Markov perfect equilibrium of the infinitely repeated prisoner's dilemma has both players defect in every period. This is because in the prisoner's dilemma the state variable is null (i.e. all histories lead to strategically identical subgames) and the only Markov perfect equilibrium corresponds to the infinite repetition of the unique Nash equilibrium of the stage game. However, it is well known that cooperation can be sustained by history-dependent strategies with punishment (Aumann [1959]). Similarly, as we show below, in our legislative game the public good accumulation in the unique symmetric Markov perfect equilibrium is inefficient but the optimal solution can be supported as the outcome of a subgame perfect equilibrium of the legislative game.

**Proposition 4.** *For any $q$, there is a $\bar{\delta}$ such that for $\delta > \bar{\delta}$ the efficient investment path characterized by the optimal solution is a Subgame Perfect Nash Equilibrium of the legislative game.*

In Appendix A, we derive nonstationary strategies for the legislative game whose outcome is the efficient level of public good (the optimal solution), and show that these strategies are a subgame perfect Nash equilibrium.[10] We analyze separately the cases with $q = 1$, $q = 2, \ldots, n - 1$, and $q = n$, because in the two extreme cases the potential for punishment is reduced.

In all cases, the strategy for the legislator recognized as the proposer is to propose the optimal level of investment, $I_O^*(g)$, and to share equally among all committee members $W - I_O^*(g)$ as private transfers. The voters' strategy entails voting "yes" to a proposal in accordance with equilibrium if no prior deviation has been observed and to switch to a punishment phase after a single deviation by any proposer. Deviation by voters (as well as deviations from punishment strategies) are punished in the same way as deviation by proposers.

For $q = 2, \ldots, n - 1$, a deviation is punished by a) rejecting an inefficient proposal; b) stopping the accumulation of the public good; and c) excluding the deviator from the distribution of the pork whenever someone else is proposing.

For $q = 1$, the proposer does not need the approval of any other member to implement a public policy, and, therefore, there can be no punishment in the event he is recognized as the proposer. In this case, a deviation is punished by reversion to the Markov perfect equilibrium characterized in the previous section.

For $q = n$, everyone's vote (including the deviator's) is needed to pass a punitive proposal and, thus, there is no enforceable harsher punishment than the status quo policy. In this case, a deviation is punished by implementing the status quo policy in all future periods.

The idea of the proof is simple: the required strategy configurations are such that any member who deviates from the prescribed proposals or from the prescribed punishment is certain to be punished. Members expect the punishment to be enforced because they expect that anyone who fails to punish a deviator will in turn be punished, and so on.

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[10]Our goal is to show that the optimal solution is the outcome of some subgame perfect Nash equilibria of the legislative game. We do not claim that the strategies proposed in the proof of Proposition 4 are the best punishment schemes, and there may be different nonstationary strategies that work for lower $\delta$.

# 6 Experimental Design

The experiments were all conducted at the Social Science Experimental Laboratory (SSEL) using students from the California Institute of Technology. Subjects were recruited from a database of volunteer subjects. Six sessions were run, using a total of 90 subjects. No subject participated in more than one session. In all sessions, the committees were composed of five members ($n = 5$), the discount factor was $3/4$ ($\delta = 0.75$), the exogenous amount of resources in each period was 20 ($W = 20$), and the current-round payoff from the public good was proportional to the square root of the stock at the end of that round ($u(y) = \frac{1}{\alpha} y^\alpha$, with $\alpha = 0.5$).[11] Two sessions were run using a simple majority requirement to pass a proposal ($q = 3$, M), two sessions using a unanimity requirement ($q = 5$, U), and two sessions under a dictatorship rule ($q = 1$, D). Table 1 summarizes the theoretical properties of the equilibrium for the three treatments. It is useful to emphasize that, as proven in the previous section, given these parameters the steady state is uniquely defined for all treatments.

| Majority Rule | n | q | $(\mathbf{g}_1, \mathbf{g}_2)$ | $\mathbf{y}_1^*$ | $y_L^*$ | $\mathbf{g}_P$ | $\mathbf{y}_P^*$ |
|---|---|---|---|---|---|---|---|
| Simple Majority (M) | 5 | 3 | (4,18) | 8 | 29.83 | 380 | 400 |
| Dictatorship (D) | 5 | 1 | (1,1) | - | 1.38 | 380 | 400 |
| Unanimity (U) | 5 | 5 | (4.380) | 6 | 400 | 380 | 400 |

Table 1: Experimental Parameters and Equilibrium

Discounted payoffs were induced by a random termination rule by rolling an eight-sided die after each round in front of the room, with the outcome determining whether the game continued to another round (with probability .75) or was terminated (with probability .25). This is a standard technique used in the experimental literature to preserve the incentives of infinite horizon games in the laboratory (Roth and Murnigham [1978], Palfrey and Rosenthal [1994], Dal Bo [2005], Duffy and Ochs [2009]). All sessions were conducted with 15 subjects, divided into 3 committees of 5 members each. Committees stayed the same throughout the rounds of a given match, and subjects were randomly rematched into committees between matches. A match consisted of one multiround play of the game which continued until one of the die rolls eventually ended the match. As a result, different matches lasted for different lengths. Table 2 summarizes the design.

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[11]Payoffs in experimental dollars were calibrated so subjects could trade in fractional amounts.We do this in order to reduce the coarseness of the strategy space and allow subjects to make budget decisions in line with the symmetric Markov perfect equilibrium in pure strategies. This is particularly important for the Dictatorship treatment where the steady state level of the public good is 1.38.

| Majority Rule | n | q | # Committees | # Subjects |
|---|---|---|---|---|
| Simple Majority (M) | 5 | 3 | 60 | 30 |
| Dictatorship (D) | 5 | 1 | 60 | 30 |
| Unanimity (U) | 5 | 5 | 60 | 30 |

Table 2: Experimental Design

Before the first match, instructions were read aloud, followed by a practice match and a comprehension quiz to verify that subjects understood the details of the environment including how to compute payoffs. The experiments were conducted via computers.[12] The current round's payoffs from the public good stock (called *project size* in the experiment) was displayed graphically, with stock of public good on the horizontal axis and the payoff on the vertical axis. Subjects could click anywhere on the curve and the payoff for that level of public good appeared on the screen.

In the M and U treatments, each round had two separate stages, the proposal stage and the voting stage. At the beginning of each match, each member of a committee was randomly assigned a committee member number which stayed the same for all rounds of the match. In the proposal stage, each member of the committee submitted a *provisional budget* for how to divide the budget between the public good, called *project investment*, and private allocations to each member. After everyone had submitted a proposal, one was randomly selected and became the *proposed budget*. Members were also informed of the committee member number of the proposer, but not informed about the unselected provisional budgets. Each member then cast a vote either for the proposed budget or for the *backup budget* with zero public investment and equal private allocations. The proposed budget passed if and only if it received at least $q$ votes. Payoffs for that round were added to each subject's earnings and a die was rolled to determine whether the match continued to the next round. If it did continue, then the end-of-round project size became the next round's beginning-of-round project size. The D treatment followed the same procedure for the proposal stage, but did not involve a voting stage: after everyone had submitted a proposal, one was randomly selected to be the committee decision in that round.

At the end of the last match each subject was paid privately in cash the sum of his or her earnings over all matches plus a showup fee of $10. Earnings ranged from approximately $20 to $50, with sessions lasting between one and two hours. There was considerable range in the earnings and length across sessions because of the random stopping rule.

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[12]The computer program used was an extension to the open source Multistage game software. See http://multistage.ssel.caltech.edu.

# 7 Experimental Results

## 7.1 Public Good Outcomes

### 7.1.1 Median Public Good Stock

We start the analysis of the experimental results by looking at the long-run stock of public good by treatment. We consider as the *long-run* stock of public good, the stock reached by a committee after 10 rounds of play.[13] Table 3 compares the theoretical and observed levels of public good by treatment. In order to aggregate across committees, we use the median level of the public good from all committees in a given treatment at round 10 ($\mathbf{y}_{mdn}^{10}$). Similar results hold if we use the mean or other measures of central tendency.[14] We compare this to the stock predicted by the Markov perfect equilibrium of the legislative game after 10 periods ($y_L^{10}$), and to the stock accumulated in the optimal solution after 10 periods ($y_O^{10}$).

| Majority Rule | q | $\mathbf{y}_{mdn}^{10}$ | $y_L^{10}$ | $y_O^{10}$ |
|---|---|---|---|---|
| Dictatorship (D) | 1 | 12.5 | 1.38 | 200 |
| Simple Majority (M) | 3 | 30.33 | 29.83 | 200 |
| Unanimity (U) | 5 | 63.13 | 72 | 200 |

Table 3: Long-Run Stock of Public Good, Theory vs. Results by Treatment

How do committees get to these stocks of public good? Figure 3 gives us a richer picture, showing the time series of the stock of public good by treatment.[15] The horizontal axis is the time period and the vertical axis is the stock of the public good. As in Table 3, we use the median level of the public good from all committees in a given treatment. Superimposed on the graphs are the theoretical time paths (represented with solid lines), corresponding to the Markov perfect equilibria and to the optimal solution.

Table 3 and Figure 3 exhibit several systematic regularities, which we discuss below in comparison with the theoretical time paths.

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[13]In the experiment, the length of a match is stochastic and determined by the roll of a die. No match lasted longer than 13 rounds and we have very few observations for rounds 11-13.

[14]In the D and M treatments, the medians are somewhat higher than the means in early periods and lower in later periods, but the differences are small. In the appendix, we report averages, medians and standard errors of the stock of the public good by round for each treatment. The statistical tests in the remainder of this section compare average stocks between different treatments using t-tests.

[15]These and subsequent figures show data from the first ten rounds. Data from later rounds (11-13) are excluded from the graphs because there were so few observations. The data from later rounds are included in all the statistical analyses.
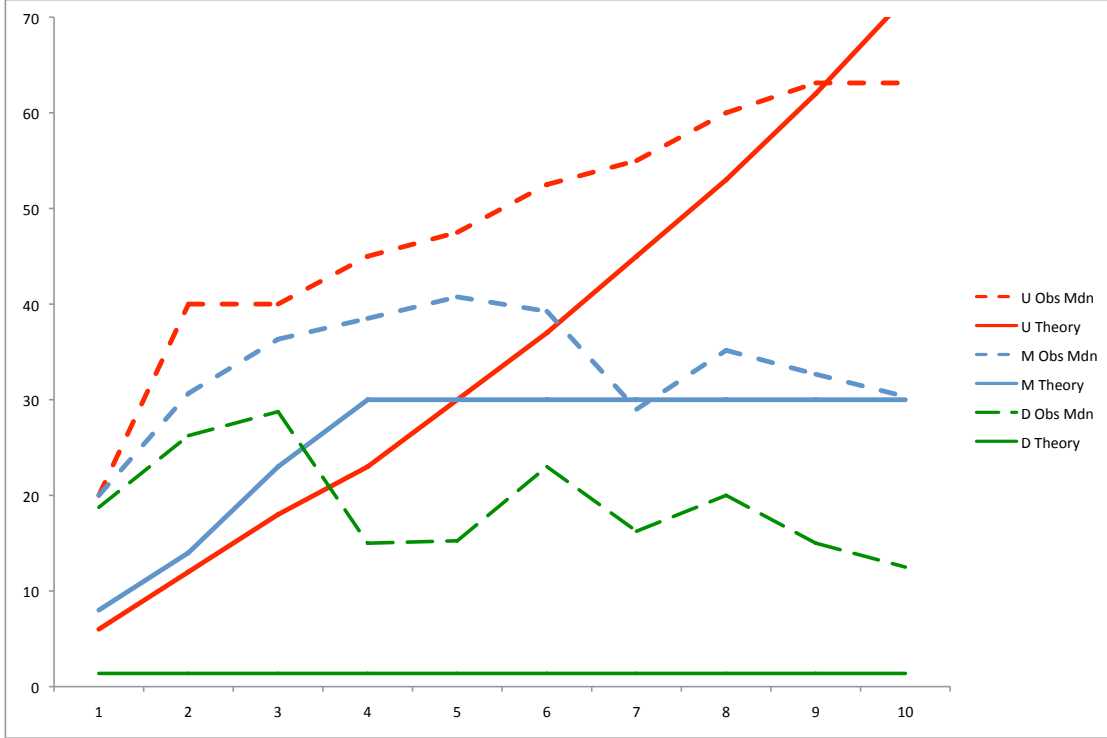
Figure 3: Median Time Paths of the Stock of g, All Treatments.

**FINDING 1. Higher $q$ leads to higher public good production: Dictatorship leads to lower public good production than Simple Majority and Unanimity; Unanimity leads to higher public good production than Simple Majority.** According to t-tests[16], the average stock of public good is significantly lower in D than in U in every single period. This difference is statistically significant at the 1% level (p-value<0.01) in every period. The stock of public good is larger in M than in D and larger in U than in M in every single period. These differences are statistically significant for periods 1 through 6.[17] The lack of statistical significance for the later rounds is due to the small sample size for the M treatment.[18]

Not only are the differences statistically significant, but they are large in magnitude. The median stock of public good is two times greater in the U treatment than in the D treatment, averaged across all 13 rounds for which we have data (20.1 in D vs. 34.7 in M vs. 39.8 in

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[16]The p-values associated with these tests are reported in the appendix. The null hypothesis of a t-test is that the averages in the two samples are the same. We are treating as unit of observation a single committee.

[17]The difference between the average in D and the average in M is significant at the 5% level (p-value<0.05) in periods 1,2,3,4, and 6 and significant at the 1% level (p-value<0.01) in period 5. The difference between the average in U and the average in M is significant at the 1% level (p-value<0.01) in periods 2,3, and 4, and significant at the 5% level (p-value<0.05) in periods 1, 5, and 6.

[18]Due to the stochastic length of each match, only 6 committees reached round 7 or above, and only 3 committees reached round 10 in the M treatment.

U). The differences between the three voting rules are relatively small in the initial round, but they increase sharply as more rounds are played. By round 10, the differences are very large (12.5 vs. 30.3 vs. 63.1).

**FINDING 2. All voting rules lead to significantly inefficient long-run public good levels.** The optimal steady state is $y^*=400$ and the optimal investment policy is the fastest approach: invest W in every period until $y^*$ is achieved. After 10 rounds, the median stock of public good achieved with the optimal investment trajectory is 200. With a legislature, the median stock of public good levels out at about 15 under dictatorship, at about 30 under simple majority, and at about 60 under unanimity. The median stock averages 14.5 in rounds 7-10 in D, 31.8 in rounds 7-10 in M, and 57.9 in rounds 7-10 in U. These very inefficient long run public good levels occur in spite of initial round median investment that is fully efficient (I=W) in M and U[19] and very close to efficiency (I=0.94W) in D. In all treatments the average stock of public good in the last rounds (rounds 8 on) is significantly smaller than the level predicted by the optimal solution (the level attainable investing W each round) according to the results of a t-test on the equality of means (p-value<0.01).

**FINDING 3. In all voting rules, there is overinvestment relative to the equilibrium in the early rounds. This is followed by either negative investment (in the D and the M treatment) or by zero investment (in the U treatment) approaching the theoretical predictions.** The median investment in the first two rounds are (18.8, 11.3) in D, (20, 10.7) in M, and (20, 20) in U. As a result the median public good stock by the end of round 2 equals, respectively, 26.3, 30.7 and 40. This compares with equilibrium investment policies in the first two rounds equal to (1.38, 0) for D, (8, 6) for M, and (6, 6) for U, and a predicted stock equal to 1.38 for D, 14 for M, and 12 for U.[20] Thus, in all treatments, committees overshoot the equilibrium in early rounds by a factor of twenty (D), two (M) and three (U). This overshooting is largely corrected in later rounds, either via disinvestment (in D and M) or an arrest in investment (in U). In the M treatment, convergence is especially close to equilibrium, with the difference between the median public good levels and the equilibrium public good levels in the last 4 rounds of data measuring less than 2 units of the public good (31.79 vs. 29.83). A similar pattern of overshooting in the D mechanisms is also evident. Beginning in round 4, the stock of public good in D declines sharply, with the median public good stock averaging 16.6 in rounds 4-10. In U,

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[19]In M efficient investment occurs only in the first round, while in U it occurs in the first two rounds.

[20]The difference between the average investment in the early rounds and the predicted investment in these same rounds is statistically significant at the 1% level.

the investment slows down considerably after the second round: the median investment in rounds 3-10 is 0. Given the nature of the equilibrium investment function for this voting rule (a convex function), the investment cessation following the initial overshooting, brings the level of the stock closer to the predicted one and, eventually, below it (the median stock in round 10 is 63.1 vs. a predicted stock of 72).

### 7.1.2 Variation Across Committees

Because of the possibility of nonstationary equilibria it is natural to expect a fair amount of variation across committees. Figure 3, by showing the median time path of the stock of public good, masks some of this heterogeneity. Do some committees reach full efficiency? Are some committees at or below the equilibrium? We turn next to these questions.

Figure 4 illustrates the variation across committees by representing, for each round, the first, second and third quartile of investment levels for the D (panel (a)), M (panel (b)), and U game (panel (c)).
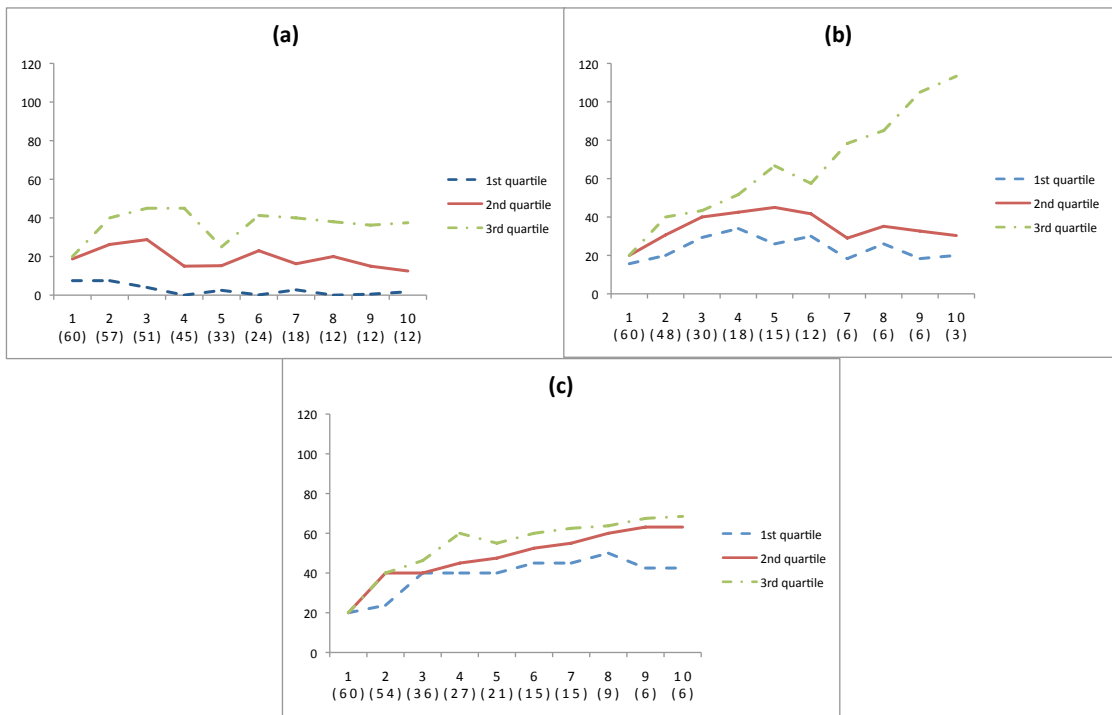


Figure 4: Quartiles of time paths of g, (a) Dictatorship, (b) Simple Majority, (c) Unanimity. The number of observations (committees) per round is reported on the x axis below the round number.

There was remarkable consistency across committees, especially considering this was a complicated infinitely repeated game with many non-Markov equilibria.[21] There were a few

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[21]In periods 6-10 of the $M$ treatment the top quartile continues to increase. However, this is due to a

committees who invested significantly more heavily than predicted by the Markov perfect equilibrium, but this only happened rarely, and nearly always such cooperation fell apart in later rounds. The most efficient committee in $M$ invested $W$ in each of the first 7 rounds, resulting in a public good level of 140. That committee did not invest anything for the remaining 2 rounds. Even this very successful committee stopped investing well short of the efficient level (400). The most efficient committee in $D$ invested $W$ in each of the first 4 rounds, resulting in a public good level of 80. That committee disinvested the entire stock of public good in the following round. In $U$ only two committees reached levels above 80 and not a single committee invested $W$ for more than 4 consecutive rounds.

These findings are perhaps surprising since, from Proposition 4, we know that, for the parameters of the experiment, the optimal solution can indeed be supported as the outcome of the game in $M$ and $U$ using nonstationary strategies (at least for the first 10 rounds). In $D$, even if the parameters of the experiment do not allow an efficient level of the public good to be attainable in equilibrium, using nonstationary strategies can sustain a level of the public good much higher than the one predicted by the Markov perfect equilibrium, namely around 50 (with respect to 1.38). Figure 3 and 4, therefore, make clear that the predictions of the Markov perfect equilibrium are substantially more accurate than the prediction of the "best" subgame perfect equilibrium (that is the Pareto superior equilibrium from the point of view of the agents), even when this best equilibrium is unique and reasonably focal (being the efficient solution). This observation may undermine the rationale for using the "best" equilibrium" as a solution concept.

## 7.2    Strategies and Behavior

### 7.2.1    Proposals and Coalitions

We now turn to a descriptive analysis of the proposed allocation of pork, as a function of $g$ and $q$. For this analysis we focus primarily on the number of members receiving significant amounts of pork in the proposed allocation, and whether the proposals had negative investment in the public good. We break down the proposed allocations into 4 canonical types. These types are: (1) *Invest W*: 100% allocation to the public investment; (2) *Proposer only*: The allocation divided between public investment and private consumption of the proposer only; (3) *Proposer + 2*: The allocation divided between public investment and a coalition that includes the proposer and two other members of the committee (notice that this is a minimal winning coalition in M); (4) *Universal*: Positive private allocations to all

small sample size: only 2 committees are in the third quartile for periods 6-9, and only one for period 10.

five members.[22] The last three categories are further broken down by whether investment in the public good is positive, zero, or negative.

| Proposal Type | Dictatorship % Prop. | Simple Maj % Prop. | Simple Maj % Acc. | Unanimity % Prop. | Unanimity % Acc. |
|---|---|---|---|---|---|
| INVEST W | 27% | 43% | 92% | 58% | 80% |
| PROPOSER ONLY | | | | | |
| * with positive inv | 16% | 3% | 100% | 3% | 0% |
| * with no inv | 7% | 1% | 0% | - | - |
| * with negative inv | 14% | - | - | - | - |
| PROPOSER + 2 | | | | | |
| * with positive inv | 1% | 7% | 85% | - | - |
| * with no inv | - | 5% | 77% | - | - |
| * with negative inv | - | 6% | 72% | - | - |
| UNIVERSAL | | | | | |
| * with positive inv | 19% | 21% | 84% | 29% | 67% |
| * with no inv | 2% | 2% | 46% | 4% | 6% |
| * with negative inv | 3% | 5% | 56% | 6% | 19% |

Table 4: Proposal Types, # Observations: 330 for D, 204 for M and 330 for U.

Table 4 shows the breakdown of proposals for the three treatments. In each table, the first column lists the various proposal types. The second column lists the proportion of proposals of each type that were proposed at the provisional stage (i.e., before a proposal was randomly selected to be voted on). The final column gives the proportion of proposals of each type that passed when they were voted on.

**FINDING 4. In all treatments, most proposals are either (i) invest the entire budget; or (ii) universal private allocations with positive investment. The proportion of proposals that belong to these two categories increases with the majority requirement adopted.** With all voting rules, most proposals were to either invest

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[22]There are two residual categories, not shown in the table, where pork is offered to exactly 2 or 4 members. In M and U, pork was never offered to exactly 2 members; there were only 21 provisional proposals with pork offered exactly to 4 members and, when these were observed, they were always accepted. In D, around 11% of provisional proposals belong to these two categories: 32 provisional proposals offer pork to exactly 2 members and 154 to exactly 4 members.

W or universal allocations with a positive amount of investment. In D, these two proposal types account for 46% of all budget proposals (including provisional budget proposals); in M and U, these two types account, respectively, for 64% and 86%. Of the remaining proposals, approximately half were MWC proposals (17% of all provisional budgets in L5 committees and 12% in L3 committees). Proposals that offered private allocation to the proposer only were quite rare in both treatments. Proposals with zero or negative investment occurred 21% of the time in L5 committees and 16% of the time in L3 committees. In contrast to the data, the Markov perfect equilibrium proposals should have been concentrated in the two categories: "proposer only" and MWC. However, it should be noted that even when pork is provided to more than a minimum winning coalition, most of the pork is concentrated on a minimum winning coalition.

Since one of the most common proposal type is one where some pork is offered to all members of the committee, it is interesting to check whether transfers are egalitarian or whether they favors a minimal winning coalition of voters or the proposer. Figure 5 shows the cumulative distribution of transfers in provisional proposals when committee members are ordered in decreasing amount of pork.
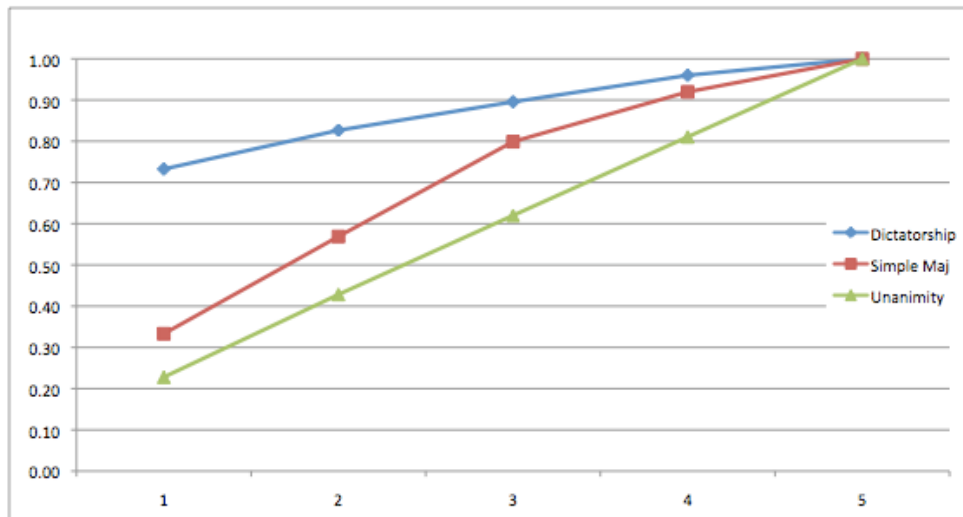


Figure 5: Cumulative Distribution of Pork, All Treatments.

**FINDING 5. In D and M, a minimal winning coalitions of players receives a more than proportional share of transfers. In U, when pork is proposed, transfers are egalitarian.** In the D treatment, when positive pork is allocated, 75% of the pork goes to the proposer. In the M treatment, when positive pork is allocated, 80% of it goes to the proposer and two other coalition partners and over 90% goes to the proposer and

three other committee members. Thus, universal allocations are not equitable in the sense of giving non-proposers the same amount of pork. In U, instead, everyone gets a very similar amount of pork: the proposer gets 23% of pork, and the member who get the least amount of pork still gets 19% of the total resources allocated to private transfers.

### 7.2.2  Voting Behavior

### 6.2.2.1 Testing for Forward Looking Behavior

Table 5 displays the results from logit regressions[23] where the dependent variable is vote (0=no;1=yes). An observation is a single voter's vote decision on a single proposal. The proposer's vote is excluded.[24] The data is broken down according to the treatment (M or U). The independent variables are: *EU(status quo)*, the expected value to the voter of a "no" outcome (including the discounted theoretical continuation value); *EU(proposal)*, the expected value to the voter of a "yes' outcome; and *pork*, the amount of private allocation offered to the voter under the current proposal. Theoretically, a voter should vote yes if and only if the expected utility of the proposal passing is greater than or equal to the expected utility of the status quo. This would imply a negative coefficient on *EU(status quo)* and a positive coefficient on *EU(proposal)*, with the magnitudes of these coefficients being approximately equal. The effect of pork should be fully captured by *EU(proposal)* and therefore, we do not expect a significant coefficient on *pork*.

|  | (1) | (2) | (3) | (5) |
|---|---|---|---|---|
| *Treatment* | M | M | U | U |
| EU(proposal)-EU(sq) | 0.12*** (0.02) |  | 0.11*** (0.01) |  |
| EU(proposal) |  | 0.13*** (0.02) |  | 0.11*** (0.01) |
| EU(status quo) |  | -0.12*** (0.02) |  | -0.11*** (0.01) |
| Constant | 0.29 (0.10) | -1.20 (0.81) | 1.05 (0.79) | -0.45 (0.37) |
| Pseudo-R2 | 0.2899 | 0.2943 | 0.2049 | 0.2104 |
| Observations | 816 | 816 | 1032 | 1032 |

Table 5: Logit estimates. Dependent var: Pr {vote=yes}. SE in parentheses; * significant at 10% level; ** significant at 5% level; *** significant at 1% level

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[23]The results presented in Tables 5 and 7 are robust to the use of a Probit specification.

[24]Proposers vote for their own proposals nearly 100% of the time (203 times out of 204 in M, and 254 times out of 258 in U).

**FINDING 6. Voters are forward looking.** The results from Tables 5 are clear. The main effect on voting is through the difference between the expected utility of the status quo and the proposal. The signs of the coefficients are highly significant, large in magnitude, and not significantly different from each other in absolute value. The constant term is not significantly different from zero, suggesting that voters are not a priori inclined to favor or disfavor proposals.

### 6.2.2.2 Proposals Acceptance Rates

The theory predicts that all proposals should pass. Is this consistent with the data? Table 4 displays the probability the proposal passes for each type of proposal (for M and U).

**FINDING 7. The vast majority of proposals pass.** Overall, 84% of the M proposals and 69% of the U proposals receive committee support. Many of the M proposals are unanimously supported, especially the "invest W" proposals and the universal proposals with positive investment.[25]

Acceptance rates differ by type of proposal. Some kinds of proposals are rejected somewhat frequently. This is particularly true for proposals with negative investment. In M committees, only 59% of proposals with negative investment pass and in U committees, only 19% pass. Proposals that give private allocation only to the proposer also fare relatively poorly, passing 71% of the time in M committees and 0% of the time in U committees. The most common proposal types, "invest W" and universal with positive investment, nearly always pass. The acceptance rates for proposals to invest everything are 95% and 68% for the M and U treatments, respectively. The corresponding acceptance rates for universal proposals with positive investment are 97% and 74%. One surprise in the data is the relatively low acceptance rates for MWC proposals in M.

Even if our legislative game is different from the standard Baron-Ferejohn setting, it is interesting to note that these numbers are in line with the acceptance rates for first round proposals in experiments testing that bargaining protocol (with simple majority): in Frechette et al. [2003] 96.4% of first round proposals are accepted (closed rule treatment), while in Kagel et al. [2010] 89% of first round proposals pass when the cost of delay is high and 72% pass when the cost of delay is low (both numbers refer to the control treatment without a veto player).

Table 6 instead displays the percentage of accepted proposals that are predicted by theory to be accepted and the percentage of refused proposals that predicted by the theory to be

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[25]In M, 67% of the "invest all" proposals pass unanimously, and 40% of universal proposals with positive investment pass unanimously.

refused, divided by proposal type. Theoretically, a legislator should support a proposal if the expected value to him of a "no" outcome (including the discounted theoretical continuation value) is smaller than the expected value to him of a "yes" outcome. We say that theory predicts a proposal to pass in M (U) if the expected utility of the proposal passing is greater than or equal to the expected utility of the status quo for at least three (for all five) legislators.

| Proposal Type | Simple Maj | | Unanimity | |
|---|---|---|---|---|
| | % Acc as Pr | % Rej as Pr | % Acc as Pr | % Rej as Pr |
| INVEST W | 100% (86) | 0% (2) | 100% | 0% (28) |
| PROPOSER ONLY | | | | |
| * with positive inv | 100% (5) | - | - | 0% (7) |
| * with no inv | - | 0% (2) | - | - |
| * with negative inv | - | - | - | - |
| PROPOSER + 2 | | | | |
| * with positive inv | 100% (11) | 0% (5) | - | - |
| * with no inv | 100% (7) | 0% (6) | - | - |
| * with negative inv | 60% (5) | 100% (4) | - | - |
| UNIVERSAL | | | | |
| * with positive inv | 100% (37) | 0% (3) | 100% (48) | 0% (23) |
| * with no inv | 0% (1) | 80% (5) | 100% (3) | 0% (17) |
| * with negative inv | 0% (4) | 100% (5) | 0% (1) | 77% (13) |

Table 6: Proposal Acceptance Rates, Theory vs. Experiments. "% Acc as Pr" is the percentage of accepted proposal that theory predicts to pass and "% Ref as Pr" is the percentage of refused proposals that theory predicts to be refused. The number of observations is in parentheses.

The theory does remarkably well. Overall, the voting outcome is correctly predicted by the theory around 85% of the times for M and around 70% of the time for U. The cases the theory fails to predict are proposals with positive investment that should be rejected but instead pass and proposals with negative or no investment that should be accepted but instead fail.

One possible explanation for this discrepancy between voting behavior and theoretical prediction is that, rather than playing a stationary equilibrium, some committees are supporting more efficient allocations by using non-stationary strategies. This possibility is in

line with recent experimental results on repeated games: Dal Bo [2005] and Dal Bo and Frechette [2011] suggest the likelihood that in this games the infinite horizon dynamics allow efficient or nearly efficient public goods provision.

**6.2.2.3 Behavioral Factors Affecting Voting**

We next explore the extent to which voting behavior depends on factors other than just the expected utility from the status quo and the expected utility from the current policy proposal (this is the basic assumption from the Markov perfect equilibrium). For instance, the voting behavior could be affected by non-stationary strategies or other-regarding preferences. Table 7 reports the results of a logit regression of voting behavior on the same variables in Table 5, but includes three additional regressors that could indicate some degree of punishment or reward behavior being used to affect proposals and support equilibrium outcomes that differ from the theoretical stationary solutions: the proposed investment level $I$; a Herfindahl index, $h$, that captures how unequal the proposed division of pork is across committee members; and the amount of own-private allocation by the proposer (that we call "greed"). In case of non-stationary behavior, the sign on $I$ should be positive (in the sense that voting strategies punish proposals that do not offer sufficient public good), while the sign on $h$ and *greed* should be negative (in the sense that greedier or less egalitarian proposals are punished with more negative votes).

|  | (1) | (2) |
| --- | --- | --- |
| *Treatment* | *M* | *U* |
| EU(status quo) | -0.15*** (0.03) | -0.15*** (0.01) |
| EU(proposal) | 0.14*** (0.03) | 0.14*** (0.01) |
| pork | 0.17*** (0.04) | 0.17** (0.07) |
| I | 0.03*** (0.01) | 0.03** (0.01) |
| h | -1.20 (1.07) | 0.83 (0.94) |
| greed | -2.05*** (0.71) | -0.31 (0.28) |
| constant | 0.62 (0.99) | -2.043* (1.22) |
| Pseudo-R2 | 0.3230 | 0.2340 |
| Observations | 576 | 1032 |

Table 7: Logit estimates. Dependent var: Pr {vote=yes}. Including i, h, and greed. SE in parentheses; * significant at 10% level; ** significant at 5% level; *** significant at 1% level.

The results are presented in Table 7. First of all, we notice that adding these new variables does not change the main result from Table 5: the coefficients on $EU(statusquo)$ and

$EU(proposal)$ still have the correct (opposite) signs, are not significantly different from each other, and they are highly significant. Some of the new factors we introduced in our analysis in Table 7 are statistically significant but have a smaller impact. For the $M$ treatment, all three of the "non-stationary" variables have the expected sign, and two are highly significant ($I$ and $greed$). More efficient proposals receive greater support, as do proposals that are less greedy. The size of the positive sign of the effect of $I$, however, seems too small to provide evidence in favor of an equilibrium in which non Markovian strategies reward efficient behavior, especially since a positive sign is consistent with equilibrium behavior.[26] Because of this, we are reluctant to conclude that the significant coefficient on $I$ is indicative of nonstationary behavior. On the other hand the significance of the coefficient of $greed$ demonstrates the existence of voting behavior that rewards exactly the types of proposals we see more of relative to the equilibrium predictions (invest W and universal). Results for the $U$ treatment are similar, but the new variables have less importance in explaining the variance in voting behavior: $I$, $h$, and $greed$ all the have the right sign but here only $I$ is significant.

We conclude from this analysis that there is some evidence of the use of non-stationary strategies but that subjects are forward looking, the main determinant of voting being the difference between the expected utility of the status quo and the proposal.

# 8    Discussion and Conclusions

This paper investigated the dynamic provision of durable public goods by a legislature, operating with procedures that entail bargaining and voting. Despite the fact that most, if not all, public goods provided by governments are durable, very little is known on this subject, both from a theoretical and empirical point of view. We attempt to provide a first answer to some basic questions that can be helpful as a starting point for further research on the politics of dynamic public good provision.

The main questions we ask are: Do legislatures provide public goods efficiently in a dynamic setting? To what extent does this depend on the voting rule adopted by the floor? The theoretical properties of the Markov perfect equilibrium in our legislative bargaining game imply that the steady state level of public investment approved by a legislature is inefficiently low for any voting rule but unanimity: the inefficiency of the long-run steady state decreases with the majority requirement adopted and disappears completely when a proposal

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[26]In the M treatment, the significant coefficient on $I$ is probably due to spurious correlation. When $g$ is small, the equilibrium predicts a high investment level and a *unanimous* yes vote; when $g$ is high, investment is predicted to be smaller, and proposals are predicted to pass by a bare majority.

on resources allocation passes only with a unanimous vote. However, even with unanimity, convergence to the steady state is inefficiently slow, as the proposer will appropriate rents along the path.

The experimental analysis on three alternative voting rules (Simple Majority and the two polar extremes, Dictatorship and Unanimity) supports these key predictions. A higher majority requirement leads unambiguously to significantly higher public good production. This result confirms, from an experimental point of view, the importance of institutions in public good provision, and the fact that incentives matter in a way predicted by complex theoretical models. In all cases, investment is generally below the Pareto efficient levels, regardless of the voting rule used. Although we often observe more investment than predicted in the early stage of the game, overinvestment does not persist: the long run public good levels approximate the Markov perfect equilibrium steady state.

The final questions we attempt to address are: To what extent are the models we use adequate to study this problem? What equilibrium concepts should be used? This is a particularly important question since, depending on the equilibrium concept, we can have very different predictions for the same model. It is clearly difficult to identify the equilibrium adopted by players, but the analysis of proposal and voting behavior provides some interesting insights. First, as discussed in Section 6.1.2, we observe a consistent pattern of behavior across committees, despite the fact that we have multiplicity of potential equilibria. The Markov perfect equilibrium that we have adopted as benchmark does not fully capture the complexity of the agents' strategies, which also depend to a limited extent on additive history-dependent variables as the distribution of pork in previous periods. However, these nonstationary effects are small and the Markovian equilibrium benchmark is far closer to the data than the prediction of the best subgame equilibrium sustainable with nonstationary strategies, the alternative benchmark that is routinely applied when studying cooperation in repeated games. In our setting, this alternative would predict efficient outcomes for any $q > 1$, which is far off the mark.

There are many possible directions for the next steps in this research. On the experimental side, our design was intentionally very simple and used a limited set of treatments. The theory has interesting comparative static predictions about the effect of other parameters of the model that we have not explored in this work, such as: the discount factor; the production technology; preferences; and endowments. Any of these would be useful extensions of our experimental design. We have also limited the analysis to legislatures that differ on the q-rule adopted and use a specific procedure. It would be interesting to consider the impact of different proposal and voting procedures (e.g. alternating offer bargaining without a status quo alternative, à la Baron and Ferejohn [1989]). Moreover, our political process does not

have elections and parties, and there is no executive branch or "president" to oversee the general interest common to all districts. Elections, parties, and non-legislative branches are all important components of democratic political systems, and incorporating such institutions into our framework would be a useful and challenging direction to pursue. Finally, it would be interesting to allow for a richer set of preferences and feasible allocations, such as allowing for diversity of preferences or multiple public goods.

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# Appendix A - Proofs of Propositions

## The Optimal Public Policy

In the steady state $y(y_O^*) = y_O^*$ and $x(y_O^*) > 0$. Since $y(g)$ is constant for $g \geq \max\{y_O^* - W, 0\}$, it is straightforward to show that the derivative of the value function in this region is

$v'(g) = \frac{\partial}{\partial g} [W + g - y_O^* + nu(y_O^*) + \delta v_O(y_O^*)] = 1$. Using the first order condition for optimality, we must have $nu'(y_O^*) + \delta = 1$, so:

$$y_O^* = [u']^{-1} \left( \frac{1 - \delta}{n} \right) \tag{10}$$

## Proof of Proposition 1

Define a function

$$
\begin{aligned}
v_L^1(g) &= \frac{W - (y_L^* - g)}{n} + u(y_L^*) + \frac{\delta}{1 - \delta} \left[ \frac{W}{n} + u(y_L^*) \right] \\
&= \frac{1}{1 - \delta} \left[ \frac{W}{n} + u(y_L^*) \right] + \frac{g - y_L^*}{n}.
\end{aligned}
$$

where $y_L^* = [u^{-1}]' \left( \frac{n/q - \delta}{n} \right)$. Note that this function is continuous, increasing, concave, and differentiable with respect to $g$, with $\frac{\partial}{\partial g} v_L^1(g) = \frac{1}{n}$. Now define $\widetilde{y}(g)$ implicitly by the equation:

$$u(\widetilde{y}(g)) + \delta v_L^1(\widetilde{y}(g)) = W/n + u(g) + \delta \left[ \frac{W - \widetilde{y}(g) + g}{n} + u(\widetilde{y}(g)) + \delta v_L^1(\widetilde{y}(g)) \right]$$

This equation can be rewritten as:

$$u(\widetilde{y}(g))(1 - \delta) + \frac{\delta}{n}\widetilde{y}(g) = u(g) + \frac{\delta^2}{n}g + \frac{W}{n} - \delta \left( u(y_L^*) - (1 - \delta)\frac{y_L^*}{n} \right) \tag{11}$$

Note that (11) implicitly defines a differentiable and increasing function of $g$. To see this note that differentiating (11) with respect to $\widetilde{y}$ and $g$ we have:

$$\widetilde{y}'(g) = \frac{u'(g) + \frac{\delta^2}{n}}{(1 - \delta) u'(\widetilde{y}(g)) + \frac{\delta}{n}} > 0 \tag{12}$$

We can therefore define a point $g_L^2 = \min [g \geq 0 \,|\, \widetilde{y}(g) \geq y_L^*]$. This point has the property that for any $g \geq g_L^2$, we have $\widetilde{y}(g) \geq y_L^*$; moreover, $g_L^2 < y_L^*$. Now define the function:

$$v_L^2(g) = \begin{cases} v_L^1(g) & g > g_L^2 \\ \frac{W - \widetilde{y}(g) + g}{n} + u(\widetilde{y}(g)) + \delta v_L^1(\widetilde{y}(g)) & else \end{cases} \tag{13}$$

Let $\overline{g} > 0$ be defined by $\overline{g} = u'^{-1}(1)$. We have:

**Lemma A.1.** *There is a $\overline{\delta}$ such that for $\delta > \overline{\delta}$, $\widetilde{y}(g)$ and $v_L^2(g)$ are increasing and continuous and concave respectively in $g \in [\overline{g}, g_L^2]$, and in $g \geq g_L^2$.*

**Proof.** We showed above that $\widetilde{y}'(g) > 0$. Furthermore, differentiating (12) with respect to $g$, we have:

$$\widetilde{y}''(g) = \frac{u''(g)\left[(1-\delta)\,u'(\widetilde{y}(g)) + \frac{\delta}{n}\right] - \left[u'(g) + \frac{\delta^2}{n}\right](1-\delta)\,u''(\widetilde{y}(g))\widetilde{y}'(g)}{\left[(1-\delta)\,u'(\widetilde{y}(g)) + \frac{\delta}{n}\right]^2} \tag{14}$$

It is clear that there is a $\overline{\delta}$ such that for $\delta > \overline{\delta}$, $\widetilde{y}''(g) < 0$ for any $g \in [\overline{g}, g_L^2]$. To see this note that for $\delta = 1$ we have $\widetilde{y}''(g) < 0$, as the numerator of (14) is smaller than 0 and its denominator greater than 0, and recall that $\widetilde{y}(g)$ is continuous. For $v_L^2(g)$, note that for $g \geq g_L^2$ the function is linear. For $g \leq g_L^2$

$$\begin{aligned}
v_L^2(g) &= \frac{W - \widetilde{y}(g) + g}{n} + u(\widetilde{y}(g)) + \delta v_L^1(\widetilde{y}(g)) \\
&= \frac{W - \widetilde{y}(g) + g}{n} + u(\widetilde{y}(g)) + \delta\left[\frac{W - (y_L^* - \widetilde{y}(g))}{n} + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u\left(y_L^*\right)\right]\right] \\
&= \frac{W + g}{n} + u(\widetilde{y}(g)) + (\delta - 1)\frac{\widetilde{y}(g)}{n} + \delta\left[\frac{W - y_L^*}{n} + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u\left(y_L^*\right)\right]\right]
\end{aligned}$$

so concavity in $[\overline{g}, g_L^2]$ follows from the concavity of $\widetilde{y}(g)$ for $\delta$ sufficiently large. Concavity in $g \geq g_L^2$ follows from the fact that in this range $v_L^2(g)$ is differentiable everywhere except at most at $g_L^2$, and its derivative is non increasing in $g$. ∎

Define $y_1^* = \arg\max_{y'}\{u(y') - y' + \delta v_L^2(y')\}$ and $g_L^1 = \min\left[g \geq 0\,|\widetilde{y}(g) \geq y_1^*\right]$. Note that $y_1^* < y_L^* = [u']^{-1}\left(\frac{n/q - \delta}{n}\right)$, an upper bound that is independent of $W$, and $g_L^1 \leq g_L^2$; moreover $g \leq g_L^1$, implies $\widetilde{y}(g) \leq y_1^*$. We can now construct the following value function:

$$v_L^*(g) = \begin{cases} v_L^2(g) & g \geq g_L^1 \\ \frac{W - y_1^* + g}{n} + u(y_1^*) + \delta v_L^1(y_1^*) & else \end{cases}$$

which is a continuous and non decreasing function of $g$. We can also construct the strategies:

$$y_L^*(g) = \begin{cases} y_1^* & g \leq g_L^1 \\ \widetilde{y}(g) & g \in (g_L^1, g_L^2] \\ y_L^* & else \end{cases}$$

and $x_L^*(g) = W - y_L(g) + g - (q-1)s_L(g)$. We now show that the value function $v_L^*(g)$ and the strategies $y_L^*(g)$ and $x_L^*(g)$ are an equilibrium for a sufficiently large $W$ and $\delta$. Consider the Proposer's problem (5). One of two cases is possible. First, the incentive compatibility constraint is not binding, so the proposer can effectively ignore the other legislators. Second, the incentive compatibility constraint binds and so the proposer has either to modify the

level of public good, or provide pork transfers to a minimal winning coalition or both.

**Case 1: non binding IC**. Assume first that we can ignore the incentive compatibility constraint and set $s = 0$. The problem becomes:

$$\max_{y} \left\{ \begin{array}{c} W - [y - g] + u(y) + \delta v_L^*(y) \\ s.t. \ \ W - y + g \geq 0 \end{array} \right\} \tag{15}$$

If we ignore the constraint in (15), then it is optimal (without loss of generality) to choose $y$ such that:

$$y \in \arg\max_{y'} \{u(y') - y' + \delta v_L^*(y')\} \tag{16}$$

It is useful to have the following result:

**Lemma A.2.** *The threshold $g_L^1$ is a non increasing continuous function of $W$ and for any $\varepsilon$ there is a $W_\varepsilon$ such that for $W > W_\varepsilon$, then $g_L^1 < \varepsilon$.*

**Proof.** Let $k$ be defined as before by $u'(k) = 1$. Then since $v_L^2(y)$ is non decreasing in $y$, $y_1^* \geq k > 0$. Let $f(W)$ be defined by

$$u(y_1^*)(1 - \delta) + \frac{\delta}{n} y_1^* = u(f(W)) + \frac{\delta^2}{n} f(W) - \frac{W}{n} + \delta \left( u(y_L^*) - (1 - \delta) \frac{y_L^*}{n} \right)$$

So $g_L^1 = \max\{0, f(W)\}$. Since $f(W)$ is a continuous decreasing function of $W$, it is then immediate that $g_L^1$ is a continuous and monotonically non increasing function of $W$. It is also immediate to verify that for any $\varepsilon > 0$ there is a $W_\varepsilon$ such that $g_L^1 < \varepsilon$ for $W > W_\varepsilon$. ∎

By Lemma A.2 we can find a $W_1$ such that for $W > W_1$, $g_L^1$ is sufficiently small to guarantee that $u'(y) + \delta v_L^{*\prime}(y) > 1$ for any $g \leq g_L^1$, so

$$y \in \arg\max_{y'} \{u(y') - y' + \delta v_L^*(y')\}$$

implies $y > g_L^1$. Lemma A.1 then guarantees that for $\delta \geq \bar{\delta}$ the unique solution to (15) is $y_1^*$. It is easy to see that in correspondence to $y_1^*$ we have $x_L(g) \geq 0$ if and only if $g$ is greater than or equal to $\max\{y_1^* - W, 0\}$. Since $y_1^*$ is bounded, this is verified for any $g \geq 0$ when $W > W_1$, and $W_1$ is chosen to be sufficiently large. The incentive compatibility constraint is satisfied if and only if $\widetilde{y}(g) \leq y_1^*$ that is if $g \leq g_L^1$. We can therefore conclude that, for $\delta > \bar{\delta}$ and $W > W_1$, when $g \leq g_L^1$ the optimal policy is $y_L^*(g)$ and $x_L^*(g)$.

**Case 2: binding IC constraint.** When $g > g_L^1$ the incentive compatibility constraint can

not be ignored. In this case, the problem solved by the proposer is:

$$\max_{y,s} \left\{ \begin{array}{c} [W - [y - g] - (q-1)s] + u(y) + \delta v_L^*(y) \\ s.t.s + u(y) + \delta v_L^*(y) \geq \frac{W}{n} + u(g) + \delta v_L^*(g) \\ s \geq 0 \end{array} \right\} \tag{17}$$

Note that we can assume without loss of generality that the solution to this problem is larger or equal than $y_1^*$ (if this were not the case, by increasing $y$ the proposer would increase his utility and relax the constraint, a contradiction). By Lemma A.1, it follows that we can treat (17) as a concave maximization problem when $\delta \geq \bar{\delta}$. There are two possibilities. First, the proposer continues to provide no consumption to the districts of other legislators, but he increases the provision of the public good $y_L(g)$ in order to satisfy the incentive compatibility constraint (no transfer case). Second, he provides consumption to the districts of $q-1$ other legislators and to his own district (transfers case).

Consider the second case first, assuming $s > 0$. We can write (17) as:

$$\max_{y} \left\{ \begin{array}{c} W - [y - g] \\ -(q-1)\left[\frac{W}{n} + \Psi(g) - \Psi(y)\right] + \Psi(y) \end{array} \right\} \tag{18}$$

where $\Psi(\cdot) = u(\cdot) + \delta v_L(\cdot)$. Choosing an optimum in problem (18) is equivalent to choosing an optimum in problem: $\max_y \{q\Psi(y) - y\}$. So an optimal choice for the proposer is to propose $y_L(g) = y_L^*$   This case is feasible only if $s = \frac{W}{n} + \Psi(g) - \Psi(y_L^*) \geq 0$, that is if and only if $g \geq g_L^2$. In the case in which $g \in [g_1, g_2]$ then we must have $u(y) + \delta v_L(y) = \frac{W}{n} + u[g] + \delta v_L(g)$, so the chosen $y$ is $\widetilde{y}(g)$. It follows that in this range the optimal proposal is $y_L^*(g)$ and $x_L^*(g)$.

Finally, we need to show that $v_L^*(g)$ is the expected utility of a player when the strategies are $y_L^*(g)$, $x_L^*(g)$. This is immediate for $g > g_L^2$. For $g \in [0, g_L^2]$, observe that, by a similar argument as in Lemma A.2, for for any $\gamma$ there is a $W_\gamma$ such that for $W > W_\gamma$, then $\widetilde{y}(y_1^*) > \gamma$. It follow that when $W > W_{g_L^2}$, $y_L^*(g) = \widetilde{y}(g) > g_L^2$ for any $g \in (g_L^1, g_L^2]$, so in this range the value function is given by (13). Finally it is easy to see that for $W > W_{g_L^2}$, $\widetilde{y}(\widetilde{y}(g)) \geq \widetilde{y}(y_1^*) > g_L^2$, so the value function is $v_L^*(g)$ in $[0, g_L^2]$. We conclude that there is a $\bar{\delta}, \overline{W}$ such that for $\delta > \bar{\delta}$ and $W > \overline{W}$ the value function $v_L^*(g)$ and the strategies $y_L^*(g)$ and $x_L^*(g)$ are an equilibrium.

## Proof of Proposition 2

Fix $q, W, \delta$, and $n$, such that $\delta > \bar{\delta}$ and $W > \overline{W}$ as defined in Proposition 1. Consider any regular equilibrium $v(g), y(g), x(g)$, with steady state $y^*$. We need to prove that $y^* =$

$[u']^{-1}\left(\frac{n/q-\delta}{n}\right)$. The incentive compatibility constraint in state $g$ if policy $y(g)$ is chosen is:

$$s(y(g)) \geq \frac{W}{n} + \Psi(g) - \Psi(y(g))$$

where $\Psi(\cdot) = u(\cdot) + \delta v(\cdot)$. In the steady state, this condition becomes: $s(y^*) \geq \frac{W}{n} + \Psi(y^*) - \Psi(y^*) = W/n > 0$. In equilibrium, this constraint is satisfied with equality (if not, the proposer could decrease $s$ and be better off). The proposer's policy must therefore solve:

$$\max_y \left\{ \begin{array}{c} W - [y - y^*] \\ -(q-1)\left[\frac{W}{n} + \Psi(y^*) - \Psi(y)\right] + \Psi(y) \end{array} \right\} \tag{19}$$

By continuity of $\Psi(\cdot)$, for any $g$ in a neighborhood of $y^*$, $s(g) > 0$. By continuity of the value function, this implies $v(g) = \frac{1}{1-\delta}\left[\frac{W}{n} + u(y^*)\right] + \frac{g-y^*}{n}$ in a neighborhood of $y^*$, implying $v'(g) = \frac{1}{n}$. This fact, together with the first order necessary condition of (19) $(qu'(y) + q\delta v'_L(y) = 1)$, implies $y^* = [u']^{-1}\left(\frac{n/q-\delta}{n}\right)$. ∎

## Proof of Proposition 3

We first prove the claims regarding the steady state. The unique equilibrium steady state in the optimal solution is $y_O^* = [u']^{-1}\left(\frac{1-\delta}{n}\right)$ and the unique equilibrium steady state in the legislative game is $y_L^* = [u']^{-1}\left(\frac{n/q-\delta}{n}\right)$. Notice that $\frac{1-\delta}{n} \leq \frac{n/q-\delta}{n}$ for any $q = 1, \ldots n$, $n > 0$, and $\delta > 0$. Since $u(\cdot)$ is, by assumption, increasing and concave (i.e. $u'(g) > 0$, $u''(g) < 0$), this implies that $y_O^* \geq y_L^*$ for any $q = 1, \ldots n$, $n > 0$, and $\delta > 0$. This inequality is strict when $q = 1, \ldots n - 1$ (i.e. when $n/q > 1$) and $y_O^* = y_L^*$ when $q = n$ (i.e. when $n/q = 1$). Moreover, notice that $\frac{n/q-\delta}{n}$ is monotonically decreasing in $q$. Therefore, $y_L^*$ is monotonically increasing in $q$. Since $y_O^*$ is independent of $q$, this also mean that the efficiency of the equilibrium steady state in the legislative game is monotonically increasing in $q$.

We now want to show that - with $n > 1$ and $q = n$ - even if $y_L^* = y_O^*$, the convergence to $y_L^*$ is slower than the convergence to $y_O^*$. In order to do this, we will compare the two investment functions, $I_O(g)$ and $I_L(g)$:

$$\begin{aligned} I_O(g) &= \min\{W, y_O^* - g\} \\ I_L(g) &= \left\{ \begin{array}{ll} y_1^* - g & g \leq g_1 \\ \widetilde{y}(g) - g & g \in (g_1, g_2] \\ y_L^* - g & else \end{array} \right. \end{aligned}$$

First notice that, when $n > 1$, $y_L^* = y_O^* > y_1^*$ and $y_L^* = y_O^* > \widetilde{y}(g)$ for any $g < g_2$. There

are two cases: 1) $g_1 = 0$, 2) $g_1 > 0$. If $g_1 = 0$, for any $g < g_2$, $I_L(g) = \widetilde{y}(g) - g$ which is smaller than $I_O(g) = \min\{W, y_O^* - g\}$. If $g_1 > 0$, for any $g \leq g_1$, $I_L(g) = y_1^* - g < W$ for any $g \leq g_1$ and $I_L(g) = y_1^* - g < y_O^* - g$ for any $g \leq g_1$ (since $y_O^* > y_1^*$). This implies $I_L(g) < I_O(g) = \min\{W, y_O^* - g\}$ for any $g \leq g_1$.■

## Proof of Proposition 4

**A)** $q = 2, \ldots n - 1$

To support the optimal stock of the public good as the outcome of a subgame perfect Nash equilibrium, employ the following strategy configuration:

1. whenever a member is recognized, he proposes a public policy $x$, that entails a level of investment equal to $I_O^*(g)$ and an even share of $W - I_O^*(g)$ as private transfer to all committee members; everyone votes in favor of $x$ (i.e. the proposal is implemented);

2. if a member $j$ is recognized and deviates by proposing $y \neq x$, every $i \neq j$ votes against the proposal; from the following period on, whenever $j$ is the proposer, every $i \neq j$ votes against any proposal, and whenever $i \neq j$ is the proposer, he proposes to divide $W$ equally among all members but $j$ as private transfers; every $i \neq j$ votes for this proposal (i.e. a punishment is carried on in all future periods in which the deviator is the proposer; when the deviator is the proposer, the status quo is implemented);

3. if a member $k$ deviates by voting contrary to the strategies above or if member $k$ deviates from the prescribed proposal in the punishment phase, implement the punishment in strategy 2 with $k$ replacing $j$;

We first show that the proposer has no profitable deviation from the equilibrium strategy on the equilibrium path. The proposer's payoff if she follows the equilibrium strategy is:

$$
\begin{cases}
u(g + W) + \delta V_{EQ}(g + W) & \text{if } g < g_O \\
\frac{W - (y_O^* - g)}{n} + u(y_O^*) + \delta V_{EQ}(y_O^*) & \text{if } g > g_O
\end{cases}
$$

where:

$$
\begin{aligned}
V_{EQ}(y_O^*) &= \frac{W}{n} + u(y_O^*) + \delta V_{EQ}(y_O^*) \\
&\implies V_{EQ}(y_O^*) = \frac{1}{1 - \delta}\left[\frac{W}{n} + u(y_O^*)\right]
\end{aligned}
$$

According to the proposed equilibrium voting strategies, whenever the proposal is different from the equilibrium one, a punishment will be carried out in every future period in which the proposer is not the deviator and the status quo will be implemented in all the other periods. Thus, all deviations are payoff equivalent. The proposer's payoff if she deviates is:

$$\frac{W}{n} + u(g) + \delta V_{DEV}(g) \quad \forall g$$

where:

$$
\begin{aligned}
V_{DEV}(g) &= \frac{n-1}{n}\left(u(g) + \delta V_{DEV}(g)\right) + \frac{1}{n}\left(\frac{W}{n} + u(g) + \delta V_{DEV}(g)\right) \\
&\implies V_{DEV}(g) = \frac{1}{1-\delta}\left[\frac{W}{n^2} + u(g)\right] < \frac{1}{1-\delta}\left[\frac{W}{n} + u(g)\right]
\end{aligned}
$$

To check that the proposer's strategy is an equilibrium it is sufficient to check that the proposer has no profitable deviation. Notice that the expected payoff from a deviation is lower than the payoff that would derive from implementing the status quo for the current and all future periods. We can, thus, check that the expected payoff from the equilibrium strategy is higher than the payoff from implementing the status quo forever. We have three cases, depending on what region of the state space we are in:

**Case 1:** $g \in [g_O, y_O^*)$   In this case we have:

$$\frac{W - (y_O^* - g)}{n} + u(y_O^*) + \delta V_{EQ}(y_O^*) \geq \frac{W}{n} + u(g) + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u(g)\right]$$

$$
\begin{aligned}
\iff & \frac{W - (y_O^* - g)}{n} + u(y_O^*) + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u(y_O^*)\right] \geq \frac{W}{n} + u(g) + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u(g)\right] \\
\iff & \left[\frac{u(y_O^*) - u(g)}{(y_O^* - g)}\right] \geq \frac{1-\delta}{n} \\
\iff & \frac{1}{(y_O^* - g)}\int_g^{y_O^*} u'(x)dx \geq \frac{1-\delta}{n}
\end{aligned}
$$

The inequality above holds for any $\delta \in [0,1]$, To see this note that by concavity of $u(\cdot)$ and the optimality condition in the efficient solution we have $u'(x) > \frac{1-\delta}{n}$ for any $x < y_O^*$.

39

**Case 2:** $g \in [0, g_O)$ **&** $(g + W) > g_O$ **:**   In this case we have:

$$u(g + W) + \delta V_{EQ}(g + W) \geq \frac{W}{n} + u(g) + \frac{\delta}{1 - \delta}\left[\frac{W}{n} + u(g)\right]$$

where:

$$V_{EQ}(g+W) = \frac{W - (y_O^* - g)}{n} + u(y_O^*) + \delta V_{EQ}(y_O^*) = \frac{W - (y_O^* - g)}{n} + u(y_O^*) + \frac{\delta}{1 - \delta}\left(\frac{W}{n} + u(y_O^*)\right)$$

And therefore the inequality we want to check becomes:

$$u(g + W) + \delta V_{EQ}(g + W) \geq \frac{W}{n} + u(g) + \frac{\delta}{1 - \delta}\left[\frac{W}{n} + u(g)\right]$$

$$\iff u(g + W) + \delta\left(\frac{W - (y_O^* - g)}{n} + u(y_O^*) + \frac{\delta}{1 - \delta}\left(\frac{W}{n} + u(y_O^*)\right)\right)$$

$$\geq \frac{W}{n} + u(g) + \frac{\delta}{1 - \delta}\left(\frac{W}{n} + u(g)\right)$$

$$\iff u(g + W) + \frac{\delta u(y_O^*)}{(1 - \delta)} - \frac{u(g)}{(1 - \delta)} \geq \frac{W}{n} + \delta\frac{y_O^*}{n} - \delta\frac{g}{n}$$

Using the fact that $u(g + W) \geq u(g) + u'(y_O^*)W = u(g) + \frac{(1 - \delta)}{n}W$ we have a lower bound on the LHS and we can plug it in to have:

$$\iff u(g) + (1 - \delta)\frac{W}{n} + \frac{\delta u(y_O^*)}{(1 - \delta)} - \frac{u(g)}{(1 - \delta)} \geq \frac{W}{n} + \delta\frac{y_O^*}{n} - \delta\frac{g}{n}$$

$$\iff \frac{\delta}{(1 - \delta)}[u(y_O^*) - u(g)] \geq \delta\frac{W + y_O^* - g}{n}$$

$$\iff [u(y_O^*) - u(g)] \geq (1 - \delta)\frac{W + y_O^* - g}{n}$$

The inequality above holds for $\delta \geq \bar{\delta} \in [0, 1]$.

We can find a lower bound on $\delta$ using the actual $V_{DEV}(g)$ rather than the expression we used above (given by the expected utility of implementing the status quo for any period following a deviation which is greater than $V_{DEV}(g)$ for any $g$). In this case, the inequality

40

we want to prove is:

$$u(g+W) + \frac{\delta\left[W - (y_O^* - g)\right]}{n} + \delta u(y_O^*) + \frac{\delta^2}{1-\delta}\left(\frac{W}{n} + u(y_O^*)\right)$$

$$\geq \quad \frac{W}{n} + u(g) + \frac{\delta}{1-\delta}\left[\frac{W}{n^2} + u(g)\right]$$

$$\iff \quad u(g+W) + \frac{\delta u(y_O^*)}{(1-\delta)} - \frac{u(g)}{(1-\delta)} \geq \frac{(1-2\delta)n + \delta}{(1-\delta)n}\frac{W}{n} + \delta\frac{y_O^*}{n} - \delta\frac{g}{n}$$

Using the fact that $u(g+W) \geq u(g) + u'(y_O^*)W = u(g) + \frac{(1-\delta)}{n}W$ we have a lower bound on the LHS and we can plug it in to have:

$$\iff \quad u(g) + (1-\delta)\frac{W}{n} + \frac{\delta u(y_O^*)}{(1-\delta)} - \frac{u(g)}{(1-\delta)} \geq \frac{(1-2\delta)n + \delta}{(1-\delta)n}\frac{W}{n} + \delta\frac{y_O^*}{n} - \delta\frac{g}{n}$$

$$\iff \quad \frac{\delta}{(1-\delta)}\left[u(y_O^*) - u(g)\right] \geq \frac{(1-\delta n)\delta}{(1-\delta)n}\frac{W}{n} + \delta\frac{y_O^*}{n} - \delta\frac{g}{n}$$

$$\iff \quad \left[u(y_O^*) - u(g)\right] \geq \frac{(1-\delta n)}{n}\frac{W}{n} + (1-\delta)\frac{y_O^* - g}{n}$$

The inequality above holds for $\delta \geq \bar{\delta} \in [0,1]$. To see this notice that the LHS is always positive while the RHS is non-positive as long as $\delta \geq \frac{W(2n+1)-n.}{W(3n+1)-n} \in (0,1)$ (because $n > 1$ and $W > 0$ and in this region $g \in (y_O^* - 2W, y_O^* - W))$ .).

**Case 3:** $g \in [0, g_O)$ **&** $(g+W) < g_O$    In this case we want to prove the following inequality:

$$u(g+W) + \delta V_{EQ}(g+W) \quad \geq \quad \frac{W}{n} + u(g) + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u(g)\right]$$

where $V_{EQ}(g+W) = u(g+2W) + \delta V_{EQ}(g+2W)$.

Note that, since $u'(x) > \frac{1-\delta}{n}$ for any $x < y_O^*$., we have $u(g+x) > u(g) + \frac{(1-\delta)}{n}x$ if $(g+x) < y_O^*$. Using this inequality we have a lower bound on the RHS and it is sufficient to prove that:

$$u(g) + \frac{(1-\delta)}{n}W + \delta V_{EQ}(g+W) \quad \geq \quad \frac{W}{n} + u(g) + \delta V_{DEV}(g)$$

$$\iff \quad V_{EQ}(g+W) \geq \frac{W}{n} + \frac{1}{1-\delta}\left[\frac{W}{n} + u(g)\right]$$

Note that $V'_{EQ}(x) > 1/n$ for any $x < y_O^*$ (as it is clear from the equation for $V_{EQ}(y_O^*)$ and the condition for optimality in the efficient solution) and, thus, $V_{EQ}(g+x) > V_{EQ}(g) + \frac{1}{n}x$. This gives us a lower bound on $V_{EQ}(g+2W)$ $(> V_{EQ}(g+W) + \frac{W}{n})$ that we can use to get a

41

lower bound on $V_{EQ}(g+W)$ :

$$
\begin{aligned}
V_{EQ}(g+W) &= u(g+2W) + \delta V_{EQ}(g+2W) \\
V_{EQ}(g+W) &> \frac{1}{1-\delta}u(g+2W) + \frac{\delta}{1-\delta}\frac{W}{n} \\
&> \frac{1}{1-\delta}u(g) + 2\frac{W}{n} + \frac{\delta}{1-\delta}\frac{W}{n}
\end{aligned}
$$

It is therefore sufficient to show that:

$$
\begin{aligned}
\frac{1}{1-\delta}u(g) + 2\frac{W}{n} + \frac{\delta}{1-\delta}\frac{W}{n} &\geq \frac{W}{n} + \frac{1}{1-\delta}\left[\frac{W}{n} + u(g)\right] \\
&\Longleftrightarrow \left(\frac{2-\delta}{1-\delta}\right) \geq \left(\frac{2-\delta}{1-\delta}\right)
\end{aligned}
$$

This inequality holds for any $\delta \in (0,1)$.

Next, we need to prove that there is no profitable deviation from the prescribed voting strategy on the equilibrium path (i.e. that the expected utility from voting "yes" to an equilibrium proposal is weakly higher than the expected utility from voting "no"). A unilateral deviation in the voting stage does not change the outcome (i.e. the efficient proposal will be implemented) and the only difference in the two expected utilities (voting "yes" vs. voting "no") is in the continuation values, $V_{EQ}(g)$ and $V_{DEV}(g)$, which are the same as the ones specified for the proposer (since the punishment takes the same form). We have three cases:

**Case 1:** $g \in [g_O, y_O^*)$

$$
\begin{aligned}
\frac{y_O^* - g}{n} + u(y_O^*) + \delta V_{EQ}(y_O^*) &\geq \frac{y_O^* - g}{n} + u(y_O^*) + \delta V_{DEV}(y_O^*) \\
V_{EQ}(y_O^*) &\geq V_{DEV}(y_O^*) \\
\frac{1}{1-\delta}\left[\frac{W}{n} + u(y_O^*)\right] &\geq \frac{1}{1-\delta}\left[\frac{W}{n^2} + u(y_O^*)\right]
\end{aligned}
$$

which clearly holds for any $\delta$.

**Case 2:** $g \in [0, g_O)$ **&** $(g + W) > g_O$ :

$$u(g + W) + \delta V_{EQ}(g + W) \geq u(g + W) + \delta V_{DEV}(g + W)$$

$$V_{EQ}(g + W) \geq V_{DEV}(g + W)$$

$$\frac{W - (y_O^* - g)}{n} + u(y_O^*) + \frac{\delta}{1 - \delta}\left(\frac{W}{n} + u(y_O^*)\right) \geq \frac{1}{1 - \delta}\left[\frac{W}{n^2} + u(g + W)\right]$$

$$\frac{1}{1 - \delta}[u(y_O^*) - u(g + W)] \geq \frac{1}{1 - \delta}\left[\frac{W(1 - n)}{n^2}\right] + \frac{y_O^*}{n} - \frac{g}{n}$$

Notice that $u(y_O^*) \geq u(g + W) + u'(y_O^*)(y_O^* - g - W) = u(g + W) + \frac{1 - \delta}{n}(y_O^* - g - W)$. This gives us a lower bound on the LHS, $y_O^*/n - g/n - W/n$. Therefore we can check the following inequality:

$$\Longleftrightarrow \frac{y_O^*}{n} - \frac{g}{n} - \frac{W}{n} \geq \frac{1}{1 - \delta}\left[\frac{W(1 - n)}{n^2}\right] + \frac{y_O^*}{n} - \frac{g}{n}$$

$$\Longleftrightarrow -\frac{W}{n} \geq \frac{1}{1 - \delta}\left[\frac{W(1 - n)}{n^2}\right]$$

$$\Longleftrightarrow \delta \geq \frac{1}{n}$$

**Case 3:** $g \in [0, g_O)$ **&** $(g + W) < g_O$

$$u(g + W) + \delta V_{EQ}(g + W) \geq u(g + W) + \delta V_{DEV}(g + W)$$

$$V_{EQ}(g + W) \geq V_{DEV}(g + W)$$

$$V_{EQ}(g + W) \geq \frac{1}{1 - \delta}\left[\frac{W}{n^2} + u(g + W)\right]$$

As noted above, $V_{EQ}(g + W) > \frac{1}{1-\delta}u(g + 2W) + \frac{\delta}{1-\delta}\frac{W}{n}$. This means that we can prove the following inequality instead:

$$\frac{1}{1 - \delta}u(g + 2W) + \frac{\delta}{1 - \delta}\frac{W}{n} \geq \frac{1}{1 - \delta}\left[\frac{W}{n^2} + u(g + W)\right]$$

$$\frac{1}{1 - \delta}[u(g + 2W) - u(g + W)] \geq \frac{1}{1 - \delta}\left[\frac{W}{n^2} - \delta\frac{W}{n}\right]$$

$$u(g + 2W) - u(g + W) \geq \frac{W}{n^2} - \delta\frac{W}{n}$$

For $\delta \geq 1/n$, the RHS is negative, while the LHS is always positive, which gives the desired result.

Next, we need to prove that there is no profitable deviation from the prescribed voting

strategy in the punishment phase (i.e. that the expected utility from voting "yes" to the punishment proposal is weakly higher than the expected utility from voting "no"):

$$\frac{W}{n-1} + u(g) + \frac{1}{1-\delta}\left[\frac{W}{n-1} + u(g)\right] \geq \frac{W}{n-1} + u(g) + \frac{1}{1-\delta}\left[\frac{W}{n^2} + u(g)\right]$$

It is clear that the above inequality holds for any $\delta \in [0,1]$ .

Finally, we need to prove that there is no profitable deviation from the prescribed proposal strategy in the punishment phase. Any deviation will bring to rejection of the proposal and, thus, implementation of the status quo. This means that a proposer different than the deviator will stick to the punishment proposal as long as:

$$\frac{W}{n-1} + u(g) + \frac{1}{1-\delta}\left[\frac{W}{n-1} + u(g)\right] \geq \frac{W}{n} + u(g) + \frac{1}{1-\delta}\left[\frac{W}{n^2} + u(g)\right]$$

which clearly holds for any $\delta \in [0,1]$ and we're done.∎

**B)** $q = n$

In this case, following a deviation, the status quo will be implemented every period. This is because - in the punishment phase - the deviator never accepts a proposals with an harsher punishment and his vote is needed to pass any proposal. To support the optimal stock of the public good as the outcome of a subgame perfect Nash equilibrium, employ the following strategy configuration:

1. whenever a member is recognized, he proposes a public policy $x$, that entails a level of investment equal to $I_O^*(g)$ and an even share of $W - I_O^*(g)$ as private transfer to all committee members;

2. if a member $j$ is recognized and deviates by proposing $y \neq x$, all future proposers implement the status quo policy.

We first show that the proposer has no profitable deviation from the equilibrium strategy on the equilibrium path. The proposer's payoff if she follows the equilibrium strategy is:

$$\begin{cases} u(g + W) + \delta V_{EQ}(g + W) & \text{if } g < g_O \\ \frac{W - (y_O^* - g)}{n} + u(y_O^*) + \delta V_{EQ}(y_O^*) & \text{if } g > g_O \end{cases}$$

44

where:

$$V_{EQ}(y_O^*) = \frac{W}{n} + u(y_O^*) + \delta V_{EQ}(y_O^*)$$

$$\implies V_{EQ}(y_O^*) = \frac{1}{1-\delta}\left[\frac{W}{n} + u(y_O^*)\right]$$

According to the proposed equilibrium voting strategies, whenever the proposal is different from the equilibrium one, a punishment will be carried out in every future period in which the proposer is not the deviator and the status quo will be implemented in all the other periods. Thus, all deviations are payoff equivalent. The proposer's payoff if she deviates is:

$$\frac{W}{n} + u(g) + \delta V_{DEV}(g) \quad \forall g$$

where:

$$V_{DEV}(g) = \frac{1}{1-\delta}\left[\frac{W}{n} + u(g)\right]$$

The expected payoff from a deviation is the payoff from implementing the status quo for the current and all future periods. To check that the proposer's strategy is an equilibrium it is sufficient to check that the proposer has no profitable deviation. As before, we have three cases, depending on what region of the state space we are in:

**Case 1:** $g \in [g_O, y_O^*)$   In this case we have:

$$\frac{W - (y_O^* - g)}{n} + u(y_O^*) + \delta V_{EQ}(y_O^*) \geq \frac{W}{n} + u(g) + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u(g)\right]$$

$$\iff \frac{W - (y_O^* - g)}{n} + u(y_O^*) + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u(y_O^*)\right] \geq \frac{W}{n} + u(g) + \frac{\delta}{1-\delta}\left[\frac{W}{n} + u(g)\right]$$

$$\iff \left[\frac{u(y_O^*) - u(g)}{(y_O^* - g)}\right] \geq \frac{1-\delta}{n}$$

$$\iff \frac{1}{(y_O^* - g)}\int_g^{y_O^*} u'(x)dx \geq \frac{1-\delta}{n}$$

The inequality above holds for any $\delta \in [0,1]$, To see this note that by concavity of $u(\cdot)$ and the optimality condition in the efficient solution we have $u'(x) > \frac{1-\delta}{n}$ for any $x < y_O^*$.

45

**Case 2:** $g \in [0, g_O)$ **&** $(g + W) > g_O$ **:** In this case we have:

$$u(g + W) + \delta V_{EQ}(g + W) \geq \frac{W}{n} + u(g) + \frac{\delta}{1 - \delta}\left[\frac{W}{n} + u(g)\right]$$

where:

$$V_{EQ}(g+W) = \frac{W - (y_O^* - g)}{n} + u(y_O^*) + \delta V_{EQ}(y_O^*) = \frac{W - (y_O^* - g)}{n} + u(y_O^*) + \frac{\delta}{1 - \delta}\left(\frac{W}{n} + u(y_O^*)\right)$$

And therefore the inequality we want to check becomes:

$$u(g + W) + \delta V_{EQ}(g + W) \geq \frac{W}{n} + u(g) + \frac{\delta}{1 - \delta}\left[\frac{W}{n} + u(g)\right]$$

$$\Longleftrightarrow \quad u(g + W) + \delta\left(\frac{W - (y_O^* - g)}{n} + u(y_O^*) + \frac{\delta}{1 - \delta}\left(\frac{W}{n} + u(y_O^*)\right)\right)$$

$$\geq \quad \frac{W}{n} + u(g) + \frac{\delta}{1 - \delta}\left(\frac{W}{n} + u(g)\right)$$

$$\Longleftrightarrow \quad u(g + W) + \frac{\delta u(y_O^*)}{(1 - \delta)} - \frac{u(g)}{(1 - \delta)} \geq \frac{W}{n} + \delta\frac{y_O^*}{n} - \delta\frac{g}{n}$$

Using the fact that $u(g + W) \geq u(g) + u'(y_O^*)W = u(g) + \frac{(1 - \delta)}{n}W$ we have a lower bound on the LHS and we can plug it in to have:

$$\Longleftrightarrow \quad u(g) + (1 - \delta)\frac{W}{n} + \frac{\delta u(y_O^*)}{(1 - \delta)} - \frac{u(g)}{(1 - \delta)} \geq \frac{W}{n} + \delta\frac{y_O^*}{n} - \delta\frac{g}{n}$$

$$\Longleftrightarrow \quad \frac{\delta}{(1 - \delta)}[u(y_O^*) - u(g)] \geq \delta\frac{W + y_O^* - g}{n}$$

$$\Longleftrightarrow \quad [u(y_O^*) - u(g)] \geq (1 - \delta)\frac{W + y_O^* - g}{n}$$

The inequality above holds for $\delta \geq \bar{\delta} \in [0, 1]$. To see this notice that the LHS is strictly positive while the RHS converges to zero as $\delta$ goes to 1.

**Case 3:** $g \in [0, g_O)$ **&** $(g + W) < g_O$ In this case we want to prove the following inequality:

$$u(g + W) + \delta V_{EQ}(g + W) \geq \frac{W}{n} + u(g) + \frac{\delta}{1 - \delta}\left[\frac{W}{n} + u(g)\right]$$

where $V_{EQ}(g + W) = u(g + 2W) + \delta V_{EQ}(g + 2W)$.

Note that, since $u'(x) > \frac{1-\delta}{n}$ for any $x < y_O^*$., we have $u(g + x) > u(g) + \frac{(1-\delta)}{n}x$ if $(g+x) < y_O^*$. Using this inequality we have a lower bound on the RHS and it is sufficient to prove that:

$$u(g) + \frac{(1-\delta)}{n}W + \delta V_{EQ}(g+W) \geq \frac{W}{n} + u(g) + \delta V_{DEV}(g)$$

$$\iff V_{EQ}(g+W) \geq \frac{W}{n} + \frac{1}{1-\delta}\left[\frac{W}{n} + u(g)\right]$$

Note that $V_{EQ}'(x) > 1/n$ for any $x < y_O^*$ (as it is clear from the equation for $V_{EQ}(y_O^*)$ and the condition for optimality in the efficient solution) and, thus, $V_{EQ}(g+x) > V_{EQ}(g) + \frac{1}{n}x$. This gives us a lower bound on $V_{EQ}(g+2W)$ $(> V_{EQ}(g+W) + \frac{W}{n})$ that we can use to get a lower bound on $V_{EQ}(g+W)$ :

$$V_{EQ}(g+W) = u(g+2W) + \delta V_{EQ}(g+2W)$$
$$V_{EQ}(g+W) > \frac{1}{1-\delta}u(g+2W) + \frac{\delta}{1-\delta}\frac{W}{n}$$
$$> \frac{1}{1-\delta}u(g) + 2\frac{W}{n} + \frac{\delta}{1-\delta}\frac{W}{n}$$

It is therefore sufficient to show that:

$$\frac{1}{1-\delta}u(g) + 2\frac{W}{n} + \frac{\delta}{1-\delta}\frac{W}{n} \geq \frac{W}{n} + \frac{1}{1-\delta}\left[\frac{W}{n} + u(g)\right]$$

$$\iff \left(\frac{2-\delta}{1-\delta}\right) \geq \left(\frac{2-\delta}{1-\delta}\right)$$

This inequality holds for any $\delta \in (0, 1)$.

Next, we need to prove that there is no profitable deviation from the prescribed voting strategy on the equilibrium path (i.e. that the expected utility from voting "yes" to an equilibrium proposal is weakly higher than the expected utility from voting "no"). If $q = n$, voting "no" to an efficient proposal leads to the implementation of the status quo in the current period and all following periods. This means that the comparison between the two expected utilities (from the equilibrium strategy and from a deviation) is exactly the same as for the proposer and the result showed above holds.

Next, we need to prove that there is no profitable deviation from the prescribed voting strategy in the punishment phase (i.e. that the expected utility from voting "yes" to the punishment proposal is weakly higher than the expected utility from voting "no"). In this

case the punishment proposal is the same as the status quo and therefore the expected utility from voting "yes" and "no" are the same. Finally, we need to prove that there is no profitable deviation from the prescribed proposal strategy in the punishment phase. Any deviation will bring to rejection of the proposal and, thus, implementation of the status quo. This means that a proposer is indifferent between the punishment proposal and any deviation and we're done. ∎

**C)** $q = 1$

When $q = 1$ the proposer does not need the approval of any other member to implement a public policy. To support the optimal stock of the public good as the outcome of a subgame perfect Nash equilibrium, employ the following strategy configuration:

1. whenever a member is recognized, he proposes a public policy $x$, that entails a level of investment equal to $I_O^*(g)$ and an even share of $W - I_O^*(g)$ as private transfer to all committee members;

2. if a member $j$ is recognized and deviates by proposing $y \neq x$, all future proposers implement the Markov perfect equilibrium policy, i.e. the policy that maximizes the individual problem, i.e. $I_L^*(q = 1)$ and $W - I_L^*$ to the proposer as private transfer.

Denote $y_L^*(q = 1)$ as $y_D^*$. The gains from deviation are greater the closer $g$ is to $y_O^*$. Therefore, we will check whether the proposer has an incentive to deviate when $g \in [g, y_O^*]$, or whether:

$$W - (y_O^* - g) + u(y_O^*) + \delta V_{EQ}(y_O^*) \geq W - (y_D^* - g) + u(y_D^*) + \delta V_{DEV}(y_D^*)$$

where $V_{EQ}(y_O^*)$ is defined as before and $V_{DEV}(y_D^*) = \frac{1}{1-\delta}\left[\frac{W}{n} + u(y_D^*)\right]$. Plugging in $V_{EQ}(y_O^*)$ and $V_{DEV}(y_D^*)$ we have:

$$W - (y_O^* - g) + u(y_O^*) + \frac{\delta}{1 - \delta}\left[\frac{W}{n} + u(y_O^*)\right] \geq W - (y_D^* - g) + u(y_D^*) + \frac{\delta}{1 - \delta}\left[\frac{W}{n} + u(y_D^*)\right]$$
$$u(y_O^*) - u(y_D^*) \geq (1 - \delta)(y_O^* - y_D^*)$$

There is $\bar{\delta}$ such that $\forall \delta > \bar{\delta}$ the inequality above holds. To see this note that the LHS is greater than zero (since $u(\cdot)$ is increasing and $y_O^* > y_D^*$) and that as $\delta$ approaches 1, the RHS approaches zero. ∎

# Appendix B - Additional Tables

| Round | Dictatorship (D) | | | | Simple Majority (M) | | | | Unanimity (U) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n | Avg | Mdn | SE | n | Avg | Mdn | SE | n | Avg | Mdn | SE |
| 1 | 60 | 14.00 | 18.75 | 0.91 | 60 | 16.39 | 20.00 | 0.59 | 60 | 18.50 | 20.00 | 0.66 |
| 2 | 57 | 22.50 | 26.25 | 2.04 | 48 | 27.12 | 30.67 | 1.17 | 54 | 33.17 | 40.00 | 1.43 |
| 3 | 51 | 26.08 | 28.75 | 2.91 | 30 | 32.2 | 36.33 | 1.57 | 36 | 41.24 | 40.00 | 1.92 |
| 4 | 45 | 23.84 | 15.00 | 3.83 | 18 | 36.3 | 38.50 | 3.05 | 27 | 46.76 | 45.00 | 2.40 |
| 5 | 33 | 19.92 | 15.25 | 3.81 | 15 | 39.45 | 40.75 | 4.78 | 21 | 50.64 | 47.50 | 2.78 |
| 6 | 24 | 25.48 | 23.00 | 4.67 | 12 | 38.67 | 39.25 | 5.87 | 15 | 54.90 | 52.50 | 4.60 |
| 7 | 18 | 25.92 | 16.25 | 6.20 | 6 | 42.17 | 29.00 | 13.80 | 15 | 57.32 | 55.00 | 5.73 |
| 8 | 12 | 22.15 | 20.00 | 6.98 | 6 | 46.72 | 35.17 | 13.11 | 12 | 57.92 | 60.00 | 4.52 |
| 9 | 12 | 21.21 | 15.00 | 6.93 | 6 | 42.77 | 32.67 | 15.65 | 6 | 58.54 | 63.13 | 6.82 |
| 10 | 12 | 22.15 | 12.50 | 7.34 | 3 | 42.56 | 30.33 | 18.72 | 6 | 58.75 | 63.13 | 6.88 |

Table 8: Summary statistics of public good stocks per round, all treatments.

| Round | D vs. U | M vs. U | D vs. M |
|---|---|---|---|
| 1 | 0.0001 | 0.0184 | 0.0290 |
| 2 | 0.0000 | 0.0013 | 0.0500 |
| 3 | 0.0002 | 0.0005 | 0.0332 |
| 4 | 0.0000 | 0.0096 | 0.0126 |
| 5 | 0.0000 | 0.0488 | 0.0023 |
| 6 | 0.0001 | 0.0370 | 0.0850 |
| 7 | 0.0009 | 0.3199 | 0.2882 |
| 8 | 0.0008 | 0.4236 | 0.1085 |
| 9 | 0.0037 | 0.3723 | 0.1106 |
| 10 | 0.0059 | 0.4359 | 0.1610 |

Table 9: P-values of t-tests on the equality of public good stock averages.

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