A Stochastic Linear Programming Model for Asset Liability Management: The Case of an Indian Insurance Company

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Abstract

Asset - Liability management is one of the most critical tasks for any financial institution, determining its cushion against the risk and the net returns. The problem of asset liability management for an insurance company requires matching the cash inflows from premium collections and investment income with the cash outflows due to casualty and maturity claims. Thus, what is required is a prudent investment strategy such that the returns earned on the assets match the liability claims at all points of time in future. Conventionally, the asset allocation has been done using the Mean Variance approach due to Markowitz (1952, 1959). While such a strategy ensures that the asset value always match or are greater than the liability for the next year, it does not maximise the net worth of the firm nor does it take care of all the cash inflows and outflows over a long term period. A stochastic linear programming model (on the lines of Pirbhai, 2004) maximises the net worth of the firm and also takes care of the uncertainties. While there are instances of stochastic linear programming being applied for ALM in financial institutions in developed markets, no such practical application has been reported in this area in Indian context as yet.

In this paper, we describe the development of a multi stage stochastic linear programming model for insurance companies. The multi-stage stochastic linear programming model was developed on the modelling language AMPL (Fourer, 2002).

1. Introduction

Asset Liability Management (ALM) is one of the foremost challenges for any financial institution. It is all the more important for insurance companies as the assets in this case are short term while the liabilities could be long term. Thus the need for an effective investment strategy to minimize this difference and hence the consequent risk on the institution. Different strategies have been tried in the past – most of them have been on static lines. In this paper, we try to look at an asset allocation mechanism based on a stochastic linear programming model for effective Asset Liability Management. While some works on these lines have been done in international context, nothing has been reported in the Indian context. This is particularly important with the liberalisation of the insurance sector in India and the need for the players to manage their own ALM without recourse to public funds.

This paper is organized as follows. Section 2 describes the problem of Asset Liability Management in the Indian context. Section 3 reviews the work done in other parts of the world in the field of Asset Liability Management using stochastic programming techniques. Section 4 gives the exact problem definition while section 5 deals with the details of the model and its formulation. In section 6 we talk about the implementation process for the model and we conclude the paper with the final section, section 7, which looks at possible extensions to the work.

2. The problem of asset liability management for an insurance company

In today's era, insurance firms have started offering a wide variety of products, like term insurance, unit linked insurance, retirement plans, etc. In the Indian context in particular, there have been a number of new product innovations since the liberalisation of insurance sector. The insurance scenario in India is fast changing – an insurance policy is very quickly changing from being a mere 'protection' instrument to an 'investment' instrument. A number of new products are being devised to meet the investment appetite of the consumer. These new products with aggressive assured returns demand a more prudent asset allocation.

For the purpose of this work, we have focussed on one of the most popular and conventional life insurance products in India, namely the fixed maturity endowment policy. The generic features of such a policy are:

1. The individual is supposed to pay a fixed amount, known as premium, at fixed time intervals, usually every month, quarter or year.
2. The premium paid has two components – protection component and savings component.
3. The protection component assures payment of 'sum assured' in case of death of the policy holder before the maturity period.
4. Savings component is paid back at the end of maturity period along with a certain rate of return on the savings component.
5. Hence, if the policyholder dies before the maturity period, the nominee receives the 'sum assured', whereas if the policyholder is alive at the end of maturity period, he/she receives the refund of savings component along with certain rate of return.

The premium received by the insurance company from individuals is invested in different instruments. At any point of time, the cash inflows would be the premium received and the income from investments made in previous years. On the other hand, the cash outflows would be the death claims and maturity refunds along with the operating expenses of the company. In a deterministic world, it would be easy to construct an investment portfolio, which earns returns such that the cash inflows are greater than or equal to cash outflows *(income constraint)*. However, the real world is not deterministic in nature and thus in an uncertain world, the stochastic nature of rate of return on an asset can at best be qualified with the expected rate of return and variance. Thus the return earned from the portfolio of investments would depend on the 'scenario' that exists in future.

At any point of time, the total premium collected would be due to the existing policies as well as new policies registered in the preceding period. The premium inflow in any period would create a liability, which matures at different periods in future. Hence the liability is multi-period in nature. It is a requirement for an insurance company that the total value of assets (in the form of investments) at any point of time should be greater than or equal to liability created due to premiums *(reserve constraint)*.

The above two constraints – income and reserve, form the basis of asset liability management for an insurance company. The asset allocation needs to be such that it satisfies the above two constraints at all periods in future.

It may be worth mentioning that the product chosen here is for illustration purposes only – as will be evident later, any other product could have been chosen. The only difference would be that the constraint equations in the model would need to be appropriately defined.

3. Literature Survey

Historically several attempts have been made to arrive at an effective ALM position by making use of appropriate asset allocation techniques which can help the firm maximise its worth while at the same time hedging it against the risk of shortfall. Conventionally, the asset allocation was done using the Mean – Variance approach described by Markowitz (1952, 1959) which involved determination of the efficient frontier of securities and then selection of a portfolio on the efficient frontier such that the selected portfolio was able to match the liabilities in future. Though such an approach assures asset-liability matching, it fails to maximise the net worth of the company, i.e. though the portfolios are feasible, they are not optimal. Moreover, being a static approach it took care of uncertainty only in the very next period.

The development of ALM models based on stochastic linear programming by Kalber et. al. (1982) and Kusy and Ziemba (1986) was an important breakthrough. A large scale practical application of the same was then developed by Carino et. al. (1994) for a large Japanese insurance firm. The model was dynamic in nature as well as maximised the expected net worth of the firm at the horizon period while fulfilling the income and reserve constraints at all the scenarios likely to occur in the future. This work has since been used as a starting point of a number of such applications for ALM in banks, pension funds and insurance companies in many developed countries.

The most noticeable application of stochastic programming in ALM has been in pension funds. The uncertainties involved in the pension funds are future life expectancy as well as return on investments. Further, the regulations governing the pension funds are different from those for other financial institutions. Thus scenario trees have to be generated for assets as well as liabilities. Consigli and Dempster (1998) developed the computer-aided asset/ liability management (CALM) stochastic programming model for dynamic ALM. Geyer, Herold, Kontriner and Ziemba (2002) describe a financial planning model InnoALM developed by Innovest for the Austrian pension funds. In another such attempt Hilli et al (2004) developed a similar stochastic programming ALM model for a Finnish pension insurance company while Dupacova and Polivka (2004) developed a similar stochastic programming model for ALM of Czech pension funds.

While in the above-cited research, the uncertainty in future is modeled by constructing the scenario trees, Hibki (2003) simulated paths to generate scenarios as Hibki claimed that simulated paths provide higher accuracy of description of uncertainties associated with asset returns.

Grebeck and Rachev (2004) have recently provided a review of the stochastic programming applications in ALM developed so far.

4. The Problem in Indian Context

Our application of linear programming for asset liability management for an insurance firm is based on an endowment type life insurance policy described in section 2. The model has been developed based on the particulars of the policy described earlier. The microstructure of the policy is as follows: In this policy, the issuer does not look at the risk component and savings component separately. Hence, only combined reserves are maintained for risk as well as savings component. The policyholder pays a fixed premium regularly till the maturity. A fixed amount, known as the Sum Assured, is paid to the policyholder either in case of death claim or on the maturity of the policy. This Sum Assured is equal to the sum total of all the premia paid till maturity. The incentive for using it as savings instruments is given by promising the policyholders a share in the profits (return on investments minus operating expenses) earned by the issuer. Every year, the issuer distributes a major portion, say β, (in this work, we take β = 0.9 or 90% as stated later) of its profits as bonus to the policy holders. A typical policy is has a life of anywhere between 10 to 30 years. The policyholder has the option to surrender the policy any time after some lock-in period (say 3 years) and take the surrender value.

Only two types of asset class – debt and equity, have been considered here for the purpose of model. The number of asset class can be easily extended in our generalised model described below. For the purpose of simplicity, the returns on equity and debt market indices under different market scenarios have been used as benchmarks for the returns on investment of the Company in the two asset classes under same scenarios.

5. Model Formulation

Based on the above description of the characteristics of the particular life insurance policy of the Company, a multi-stage stochastic linear programming model was developed as described below.

Since a common reserve is kept for risk as well as savings component, a common liability account has been taken in the model for keeping track of total reserve. The company does not promise any return on the savings component. Instead at the maturity it simply refunds the total premium deposited. Hence there was no need to model separately an account for interest earned by policy holders on their premiums. Only the principal account has to be maintained which carries the total premium deposited till date. As assumed earlier in section 4, β ($0 \leq \beta \leq 1$; in this case we shall take β = 0.9) proportion of the net profit earned by the Company in a year is declared as bonus to the shareholders but it is paid only at the time of maturity. Hence, every year the bonus given to the policyholders is added to the principal reserve which would have to be repaid at the time of maturity. Here we are assuming that the bonus earned by the policyholders in a year would also earn them income in subsequent years since the bonus declared today would be paid only at the time of maturity. Lastly, since no differentiation is made between the income return and the price return on an asset, we model only the total return on an asset.

The challenge for any company is to make prudent investments of its premium in the two asset classes such that at all points in time in future, the total cash inflows are able to meet the expected outflows due to maturity, death claims, commission and other expenses. The cash inflows would be due to the premium and income earned from the investments made in the previous years in the two asset classes. The return on assets would depend on the possible 'scenarios' that exist in future. While theoretically, there can be infinite 'scenarios', a finite number of scenarios, along with probability for each scenario, can be defined based on past trends. While even the liabilities and other parameters like premium income are stochastic in nature and can be assumed to be 'scenario dependent', for the purpose of keeping the model within prudent limits of complexity, we model only the return on assets to be 'scenario dependent'. Figure 1 gives an illustration of a typical scenario tree.

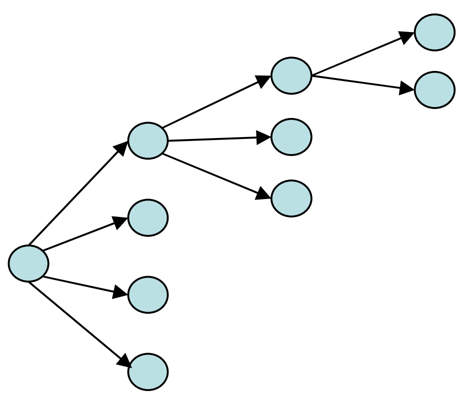

Figure 1: Illustration of a Scenario Tree

The objective is to maximise the expected net worth (policyholders' and shareholders' reserves) of the firm at the horizon period while matching the cash inflows with the cash outflows at all nodes in the scenario tree.

We define the following notations used in the model:

A node in the scenario tree is defined by the year 'i' and the scenario 'α', $p_{i\alpha}$ as the probability of the scenario α for a given year i, such that $\sum_{\alpha} p_{i\alpha} = 1$.

$L_{i\alpha}$ is defined as the total (principal & interest liability of policyholders' accounts) reserve at the end of year i and scenario α, $G_{i\alpha}$ as the total value of the shareholders' account at the end of year i and scenario α, $D_{i\alpha}$ as the total income earned in the year i under scenario α and F_i as the premium inflow in the year i.

Also, M_i is defined as the maturity outgo in the year i (*it may be noted that maturity claims denote only the refund of the principal savings component without including the share in the bonus returned to the policy holder at the time of maturity*).

Further, Y_i is the death claims in the year i, S_i is the surrender outgo in the year i, C_i is the commission expense in the year i and E_i is other expenses (operating, etc.) incurred in the year i.

Also note that $X_{1_{i,\alpha}}$ is the allocation made from the policyholders' account to asset 1 (here, asset 1 is assumed to be equity) at the end of year i and scenario α, $X_{2_{i,\alpha}}$ is the allocation made from the policyholders' account to asset 2 (here, asset 2 is assumed to be debt) at the end of year i and scenario α, $X^G_{1_{i,\alpha}}$ is the allocation made from the shareholders' account to asset 1 (equity) at the end of year i and scenario α and $X^G_{2_{i,\alpha}}$ is the allocation made from the shareholders' account to asset 2 (debt) at the end of year i and scenario α.

It may also be noted that $r_{1\alpha}$ is the return earned on asset 1 under scenario α, $r_{2\alpha}$ is the return earned on asset 2 under scenario α, u_{i_α} is the shortfall of income (from investments made in previous year) at the end of year i and scenario α over commission and other expenses and v_{i_α} as the surplus of income (from investments made in previous year) at the end of year i and scenario α over commission and other expenses.

β, as defined earlier, is the proportion of the profits passed on to the policyholders as bonus (we assume β = 0.9) and T is the horizon year at which the expected net worth of the firm is to be maximized. Also, r is taken as the cost of capital of shareholders.

The liabilities and other parameters have been modelled using their expected values as estimated by the company. Based on market trends, certain standards norms in insurance business (like mortality tables) and statistical analysis, a company can have a prior estimate of the premium inflows (F), maturity claims (M), death claims (Y), surrender outgo (S), commission expenses (C) and other expenses (E) for the next few years (life of the policy).

For a particular scenario α, the total income earned in policyholders' account in year i is

$D_{i\alpha} = r_{1\alpha} X_{1_{i-1,\alpha'}} + r_{2\alpha} X_{2_{i-1,\alpha'}}$ *(where α' is the scenario that occurred in the year i-1).* **(1)**

The Income constraint is defined as:

$$D_{i\alpha} + u_{i_\alpha} - v_{i_\alpha} = C_i + E_i \qquad \textbf{(2)}$$

If any shortfall u_{i_α} of income over commission and other expenses occurs, it is funded from the shareholders' account G. On the other hand, if the surplus v_{i_α} occurs, then it is shared between the policy holders and the shareholders in the ratio β and 1- β respectively. This surplus declared as bonus to policyholders is not paid in the current year but at the maturity. Therefore, this surplus should be added to the total reserve.

Hence the total reserve at the end of year i is given by,

$$L_{i_\alpha} = L_{(i-1)_{\alpha'}} + F_i - (1 + \beta \times \frac{\sum_{k=i-10}^{i} v_k}{\sum_{K=i-10}^{i} F_k}) M_i - Y_i - S_i + \beta \times v_{i_\alpha} \qquad \textbf{(3)}$$

Equation (3) takes care of the reserve constraint, that is, at any period in future for any possible scenario, the total value of the reserve should be greater than the payouts due to maturity, death claims and surrender. Here, M signifies only the principal maturity amount. The income surplus $(\beta \times v_{i\alpha})$ during the tenure of policy is added to policyholders account and is repaid at the time of maturity. Here we assumed that the policy is for 10 years and hence the policyholder receives not just the total premium deposited (M), but also the average return on the premium in ratio of total income surplus to the total premium collected in last 10 years (life of the policy). Hence, in equation (3), we have the term

$$(1+\beta\times\frac{\sum_{k=i-10}^{i} v_k}{\sum_{K=i-10}^{i} F_k})M_i .$$

1-β of surplus income v_{i_α} is the net gain of the shareholders. On the other hand, if an income shortfall (u_{i_α}) occurs in the policyholders' account, that shortfall in policyholders' account should be met by withdrawing the equivalent amount from shareholders' account. Hence the total value of shareholders account, at any cost, must be greater than the income shortfall; thus the shareholder reserve constraint is defined as:

$$G_{i-1,\alpha'} + r_{1_\alpha} X^G_{1_{i-1,\alpha'}} + r_{2_\alpha} X^G_{2_{i-1,\alpha'}} \geq u_{i_\alpha} . \quad \textbf{(4)}$$

Also, the value of shareholders' account at the end of year i at scenario α would be given by,

$$G_{i_\alpha} = G_{i-1,\alpha'} + r_{1_\alpha} X^G_{1_{i-1,\alpha'}} + r_{2_\alpha} X^G_{2_{i-1,\alpha'}} + (1-\beta)v_{i_\alpha} - u_{i_\alpha} . \quad \textbf{(5)}$$

At the end of year i under the scenario α, the allocations of the amount in policyholders' account and shareholders' account to assets 1 and 2 are made as,

$$L_{i_\alpha} = X_{1_{i,\alpha}} + X_{2_{i,\alpha}} \quad \textbf{(6)}$$

$$G_{i_\alpha} = X^G{}_{1_{i,\alpha}} + X^G{}_{2_{i,\alpha}} \quad \textbf{(7)}$$

This way the value of the policyholders' account and shareholders account are derived for each of the scenarios for every year till horizon period and subsequently the allocation amounts in various asset classes are decided. Finally, determining the probability of each scenario at the horizon period, the expected value of the firm (sum of value of the policyholders' account and shareholders account) can be calculated. The objective is to maximise the expected total worth of the firm (policyholders' plus shareholders' account) at the horizon period while penalising for every shortfall (u_{i_α}) that occurs in all the intermediate periods.

Now, the formulation of the linear programming for the above described model is as follows:

Decision variables: $X_{1i\alpha}$, $X_{2i\alpha}$, $X^{G}_{1i\alpha}$, $X^{G}_{2i\alpha}$ for all years 'i' and for all the scenarios α in each of the year i,

Parameters: M_i, Y_i, S_i, C_i, E_i for each year 'i'

Objective Function: **Maximise** $\sum_{\alpha} p_{T\alpha}(L_{T_\alpha} + G_{T_\alpha}) - \sum_{i=1}^{T}\sum_{\alpha} u_{i_\alpha}(1+r)^{T-i}$

Subject to: The constraints defined by equations (1), (2), (3), (4), (5), (6) and (7) as well as the *non-negativity constraint defined as:*

F_i, M_i, Y_i, S_i, C_i, E_i, D_i, u_i, v_i, L_i, G_i, X_{1i}, X_{2i}, $X^{G}_{1_i}$, $X^{G}_{2_i} \geq 0$

6. Model Implementation

Data and Assumptions

The horizon period for which the model was built for illustration purposes was 15 years – the range of the summations defined earlier is suitably adjusted. As already mentioned, the investment is only in two assets – equity and debt. Also, the cost of capital r of the shareholders was taken to be 20 %, he proportion (β) of the profits passed on to the policyholders was taken to be 0.9.

The next step, which is the most critical step, in the implementation was building of scenarios. In order to keep the complexity of the model within prudent limit, it was assumed that only two future scenarios – favourable and unfavourable (U and D) could exist for any present state. The returns (%) that equity and debt would give for every favourable (U) scenario and for every unfavourable (D) scenario were taken to be constant for all periods in future. Based on these assumptions, the following 'binary tree' depicts all the possible scenarios that will exist till the 15th year horizon period.

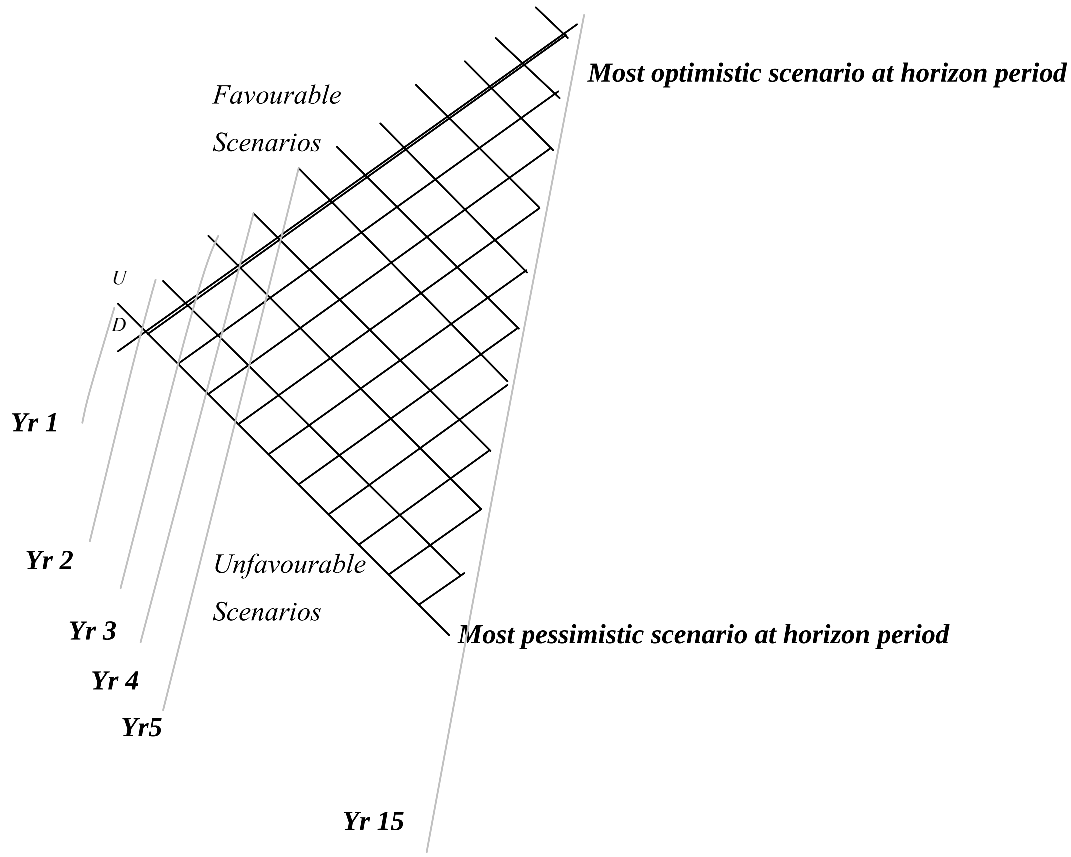

Figure 2. Binary tree for scenarios till horizon period

The returns of equity and debt for two scenarios were found as described below. Nifty was taken as proxy for equity returns while T-bill Index was taken as proxy for debt returns. Monthly geometric returns were calculated from 1997 till 2004 (92 months) for Nifty and T-bill Index followed by calculation of 92-month average return on the two indices. Now all the months for which the returns were greater than average return were taken and average return of such months was found. Similarly, all the months for which the returns were less than monthly mean return were taken and average returns of such months was found. It was found that out of 92 months considered, 46 months had equity returns greater than 92-month equity average return while 45 months had T-bill returns greater than 92-month T-bill average return. Moreover, a majority of the months in which equity returns were more than 92-month equity average were the same as the months in which T-bill returns were higher than 92-month T-bill average. Thus, overall two types of scenarios – favourable (U) and unfavourable (D), were defined. A favourable scenario would have the assets giving returns higher than their 92-month average while an unfavourable scenario would have assets giving returns lesser than their 92-month average. Since 46 and 45 months out of 92 months gave favourable returns for equity and debt respectively with lot of months as common, we took the probability of each scenario as 0.5. The average return on equity and debt for the two scenarios were found to be as shown in Table 2.

Table 2: Average equity and debt returns under the different scenarios

Scenario	Probability	Average Equity monthly return	Avg. equity yearly return	Avg. debt monthly return	Avg. yearly debt. Return
U	0.5	6.9173%	123.14%	1.0732%	13.67%
D	0.5	-5.407%	-48.68%	0.3555%	4.35%

It is to be noted that all the returns found were geometric returns and all the means found were geometric means.

Technical Details of the Model

The binary scenario tree till 15 years would entail 105 nodes. For each of the nodes four decision variables ($X_{1i}, X_{2i}, X_{1_i}^G, X_{2_i}^G$) needs to be defined. Thus, the model would have a total of 420 decision variables. Every node would have to satisfy equations (1) to (7). The model of such a scale and complexity is clearly out of the realms of a simpler solver like Microsoft Excel. Thus the model had to be developed either on a matrix generator or a modelling language. Since the modelling language provides the ease of verifiability, modifiability and solver independence, the modelling language AMPL (for details see Fourer, 2002) was chosen for developing the model.

The opening value of shareholders' account at year 1 is taken to be Rs 500,000. This is to be able to meet the cash flow requirements in case the unfavourable scenarios take place for all the next 15 years. All shortfalls in the income (denoted by 'u' in the formulation) over the 15 years are penalised in the objective function with the terminal value at the end of 15 years of all the shortfalls being calculated using 20% cost of capital (r).

Results

On solving, the programme gives the optimal allocation between equity and debt in the year 1 as Equity = Rs 27935 and Debt = Rs 708000. This allocation is out of net inflow of Rs 987353 which takes place in year 1 due to the premium inflow and outflows occurring due to maturity and other sources. Besides this, the optimum value of objective function, that is, the expected total worth of the firm (Policyholders' and shareholders' account) at the end of 15 years is found to be Rs 36,19,65,967.10.

Figures 3 and 4 give the range in which the policyholders' and shareholders' account can vary at the horizon period (depending on the scenario) by following the optimal allocation policy provided by the model output.

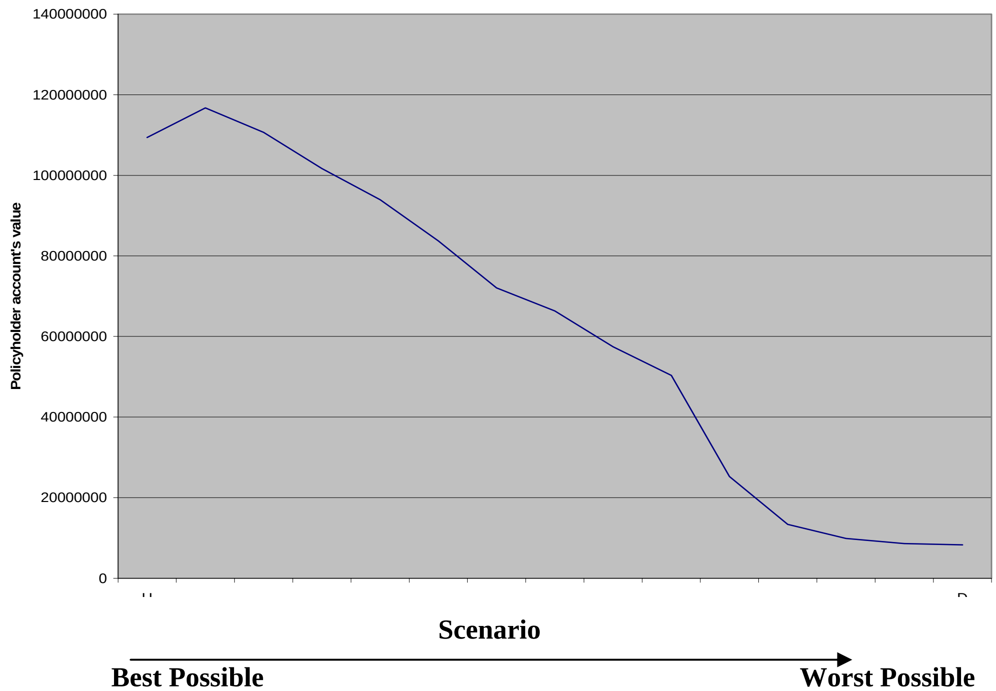

Figure 3: Value of Policyholders' account under different scenarios at T = 15years

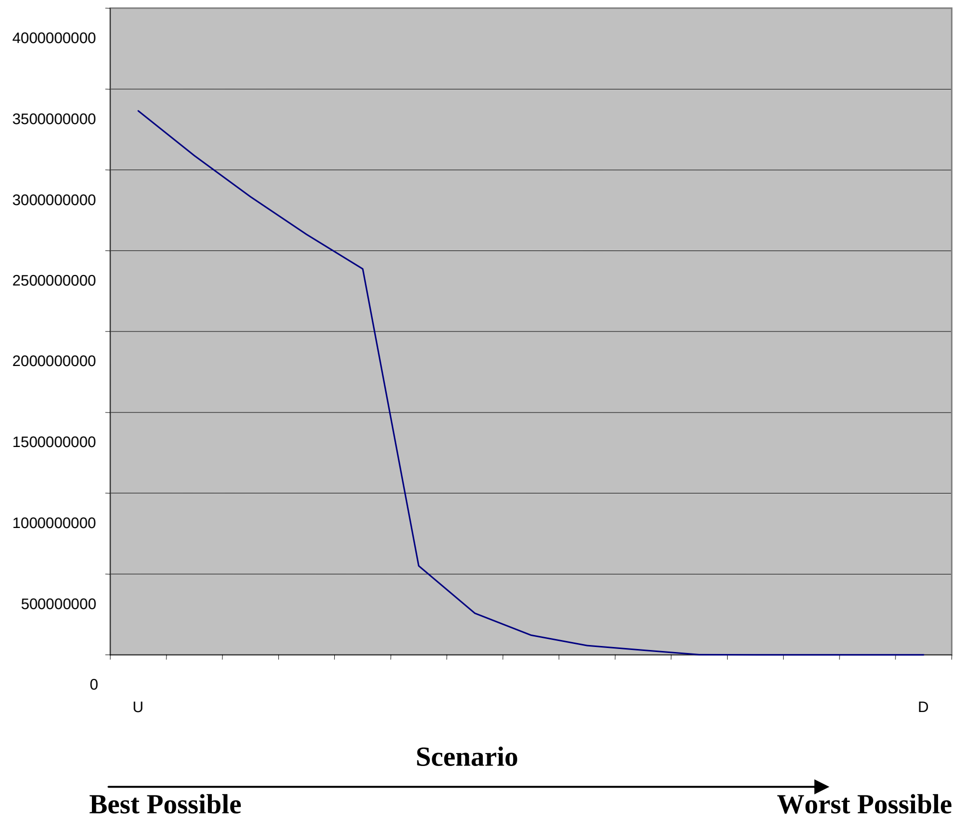

Figure 4: Value of Shareholders' account under different scenarios at T = 15years

Moreover, the allocation of shareholders' account is also achieved as the output of the programme. Initially, the model allocates all of Rs 5,00,000 of shareholders account in the equity and none in debt in the first year. Optimal allocations in subsequent years are provided by the model based on the scenarios that exist. Similarly, the model also provides the optimal allocation between equity and debt for the shareholder's account for all possible future scenarios.

7. Extensions

One simple step in future work can be more detailing of the model, i.e. with more scenarios per period and for longer period. Right now the model works with lot of assumptions like only two scenarios in future, rate of return constant for all future years for same scenario and the penalty for shortfalls remaining same for all amounts. These assumptions can be further refined to make the model more accurate. Moreover, even liabilities and other parameters can be assumed to stochastic in nature and 'scenario dependent'. However, such a sophisticated model would require events probability distributions and entail higher design complexity and more solution time.

Another important future scope of this model is extending it to other financial institutions like banks, mutual funds which also have to tread the tight rope of asset liability management. The difference between the asset liability model of insurance company and a bank would be – in an insurance company we have the expected values of death and maturity claims based on the standard mortality table provided by IRDA. On the other hand, in case of bank, the withdrawals by the customers would be stochastic in nature and a statistical analysis needs to be carried out for defining the probabilities. Hence, a major uphill task would be calculation of the probability distribution under different scenarios for different types of customers.

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