A heuristic procedure for one dimensional bin packing problem with additional constraints

-- Kumar Satyendra*, Rao V. Venkata, Tirupati Devanath

Abstract

We proposed a heuristic algorithm to solve the one-dimensional bin-packing problem with additional constraints. The proposed algorithm has been applied to solve a practical vehicle-allocation problem. The experimental results show that our proposed heuristic provides optimal or near-optimal results, and performs better than the first fit decreasing algorithm modified to incorporate additional constraints.

Keywords: Bin-packing algorithm, Vehicle-allocation, Heuristic procedure

---

*Corresponding author: Email: **satya@iimahd.ernet.in**

The authors' name is in alphabetical order of last name.

1. Introduction

A well-known one-dimensional bin-packing problem solves for minimum number of bins to pack 'N' items subject to various constraint viz. an item should be accommodated in a single bin, etc. This is an NP-hard problem [Coffman et al. (1978)] and many authors have developed algorithms to solve the classical (single-dimensional items and with no additional constraints) bin-packing problem. Eilon and Christofides (1971) had developed a heuristic procedure to solve the problem with different objective viz. minimize the number of bins / slack; minimize the un-accommodated number / value of items; and a combination of both. Later Johnson et al. (1974) had developed first fit decreasing (FFD) and best fit decreasing (BFD) algorithms to solve the one-dimensional bin-packing problem. They have also calculated the upper bound as $\{(11/9) * N + 4\}$ vehicles, where 'N' is the optimal number of bins. Recently, Gupta et al. (1999) had developed minimum bin slackness algorithm to solve the bin-packing problem. They have also utilized local search method and showed that their algorithm provides better results than FFD algorithm.

There are many extensions in the classical bin packing algorithm to model real-world situations. Some of the extension areas are packing two-dimensional [Martello and Vigo (1998)] and three-dimensional [Martello et al. (2002)] items, calculating bounds of various bin-packing problems [Fekete et al. (2001), Fleszar et al. (2002), Labbe et al. (2003), etc.], and incorporating additional constraints [Robb and Trietsch (1999), Ralphs et al. (2003), etc.].

Other extension areas of bin packing problems deal with constraints like grouping of items for packing and maximum number of items allowed in a bin. Anily and Federgruen (1991) had solved the problem where items are clubbed into various groups for services i.e. packing. They had utilized the concept in vehicle routing problem, partitioning problems etc. Rhee (1993) had considered the maximum number of items per bin. He showed that the difference between the expected numbers of bins is of the order of square root of ‘N’ when the maximum numbers of items is changed from two to three. The benefit diminishes when they considered higher values of number of items.

In this research paper, we have the bin packing algorithm to a vehicle allocation problem as one common objective of vehicle allocation problems is to minimize the number of vehicles used for delivering required quantity at ‘N’ different locations with additional constraints like zonal constraint, maximum number of visits per vehicle. The following section discusses the problem conceptualization and formulation. Section 3 discusses a heuristic approach to solve the problem, while section 4 deals with some experimental results. Finally, we conclude our paper in section 5.

2. Problem formulation

We came across a practical vehicle allocation problem while dealing with a dairy in a large city. The dairy has to deliver pouch milk to various retail outlets as per their demand. The demand of the retail outlets may vary on day-to-day basis based on the excess stock left on the previous day and the demand assessment of the present day. The dispatch section of the dairy has to decide the number of vehicles to be used as every vehicle used for supply purpose has fixed transportation cost. Identical insulated vehicles make the supplies and there is no vehicle constraint. Thus, the objective is to minimize the number of vehicles used for delivery purposes such that vehicle norms are satisfied. The vehicle movement is restricted to certain geographical zones. There is an upper limit to the number of retail outlets being serviced by the vehicles. In addition, the retail outlets can be serviced only by single vehicle.

The conceptual diagram showing the vehicle allocation is presented in figure 1. Each closed path starting and ending at the central warehouse corresponds to the route of a vehicle, and each point it visits is a retailer.

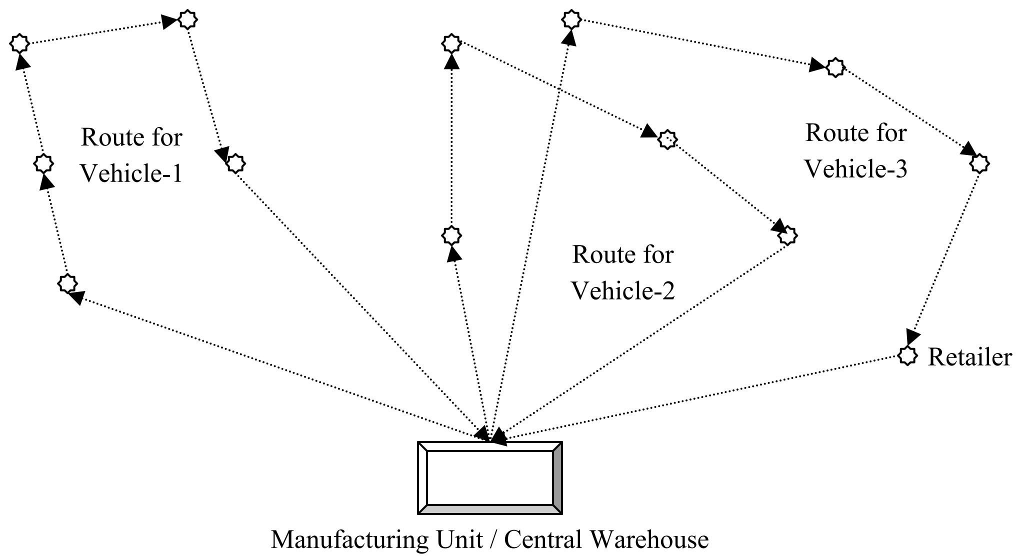

Figure 1: Conceptual diagram

2.1 Notation

The notation used in the model formulation is as follows:

- r: Set of retailers (1 to R)
- v: Indices of vehicles (1 to V)
- Q_r: Quantity to be delivered at retailer 'r' (Units).
- TC: Transportation cost per vehicle per trip (Rs.)
- VL: Vehicle load capacity (Units)
- M: Maximum number of retail outlets a vehicle can visit (Number)
- N_r: Set of retailers that can not be serviced along with retailer ‘r’

- Y_v: Is 1 if the vehicle 'v' is used for delivery purposes, otherwise 0.
- D_{vr}: Is 1 if the vehicle 'v' is used to deliver at retailer 'r', otherwise 0.

2.2 Objective

The objective is to minimize the number of vehicles for the delivering Q_r quantities at retailer 'r' subject to various vehicle norms.

$$Z = \text{Minimize } \sum_{v} Y_v \qquad \ldots 1$$

Subject to

The constraint sets are as follows:

$$\sum_{v} D_{vr} * Q_r \leq Y_v * VL \qquad v \in \{1\ldots V\} \qquad \ldots 2$$

$$\sum_{v} D_{vr} \leq M \qquad v \in \{1\ldots V\} \qquad \ldots 3$$

$$\sum_{v} D_{vr} = 1 \qquad r \in \{1\ldots R\} \qquad \ldots 4$$

$$D_{vr} + D_{vr*} \leq 1 \qquad r \in \{1\ldots R\},\ r^* \in N_r \qquad \ldots 5$$

Constraint 2 ensures that vehicle load capacity is not violated. Constraints 3 and 4 ensure that a single vehicle can service each retail outlet and each vehicle can visit maximum of 'M' retail outlets. Since, the delivery quantity at each retail outlet is less than 20% of the vehicle load capacity; it is always desirable to service the retail outlets by a single vehicle only. The constraint on the maximum number of retail outlets is to restrict the vehicle movement. It also takes care of the time window constraint. Constraint 5 deals with the situation that the same vehicle cannot service extreme retail outlets. For each retailer 'r' there is a set of retailers N_r that cannot be serviced by the same vehicle along with retailer ‘r’. For some values of N_r (which depends on the location), we have grouped the retail outlets in three zones [A, B, and C]. Further, a vehicle could not service the retail outlets of zone A and zone B, but can service the retail outlets of zone A and zone C, and zone B and zone C together.

The problem being an NP-Hard problem, we have developed a heuristic procedure to solve the problem. We have focused our heuristic procedure on following the two points:

- We have first utilized the benefit of sorting the retail outlets as per decreasing value of quantities [Johnson et al. (1974)]. We have further analyzed that the allocation of retail outlets from top and bottom of the array will fulfill the balance between number of retailers served and the quantity allocated.
- We have also utilized the Tabu search to minimize the slackness of the vehicles used. This will ensure the usage of a new vehicle only when further allocation cannot be done in the presently used vehicles.

3. Heuristic procedure

Johnson et al. (1974) have discussed the benefits of sorting the items based on their weights. In our heuristic procedure also, we have first sorted the retail outlets based on their quantities. We have further picked the retail outlets from top and bottom of the array alternatively to balance the number of retail outlets as well as the load allocated to vehicles. Finally, we have also applied local improvement by minimizing the slackness of vehicles similar to Gupta et al. (1999), which have attained the maximum number of retail outlets. The heuristic steps are provided as below:

1. Sort the retailers as per decreasing value of Q_r.

2. Set sum of quantities (S_v) and count of retailers (C_v) = 0. Put the vehicles in the allocation set.
3. For j = 1 to R, do steps 4 to 10.
4. If j is odd, select the retailer from the top of the sorted array; otherwise select the retailer from the bottom of the sorted array.
5. Allot the selected retailer to the best-fit vehicle of corresponding zone in allocation set (vehicle with maximum S_v value accommodating this retailer without capacity violation). Assign the zone to the vehicle as and when required.
6. $S_v \leftarrow S_v + Q_r$, $C_v \leftarrow C_v + 1$, $MQ \leftarrow$ minimum Q_r value of the array.
7. If $C_v = M$, keep the vehicle in the Full_Item set.
8. Put the vehicles of allocation set in Full_Load set if $S_v + MQ > VL$.
9. Readjust the retailers between the vehicles of Full_item and Full_load set.
    a. Sort the vehicles as per the decreasing value of S_v.
    b. Pick up the top vehicle, identify its zone, calculate the slackness ($VL - S_v$) and identify the bottom vehicle from Full_load set of corresponding zone.
    c. Within the two vehicles, identify the retailers whose interchange will minimize the slackness of the top vehicle considered.
    d. Repeat the above steps till reallocation is possible.
    e. After reallocation, if any vehicle of Full_load set satisfies for allocation set ($S_v + MQ <= VL$ and $C_v < M$), then put the vehicle in the allocation set.
10. Sort the vehicles of allocation set as per decreasing value of S_v.

4. Performance measures

We have designed and performed various experiments to measure the quality of heuristic results. The parameters for measuring the performance are as follows:

- Average and maximum deviation from the optimal solution (for small problems). We have obtained the optimal solution by solving the problem as mixed integer program in GAMS (Generalized algorithm modeling Software) 19.8 by CPLEX solver.
- Average and maximum deviation from the lower bound solution (for large problems).
- Percentage of instances in which heuristic provides optimal solution.
- Comparison with First Fit Decreasing [Johnson et al. (1974)] algorithm modified to incorporate maximum number of retailers per vehicle. We have started with the lower bound on the number of vehicles and try to allocate the retail outlets in those vehicles. The number of the vehicles has been increased by one, when the present number of vehicles does not fit all the retail outlets.

4.1 Experiment-1

In this experiment, we have considered

- 50 / 75 / 100 retailers in single zone.
- Quantity is randomly generated between 0.1 and 0.5 (of vehicle load) for every retailer.
- Maximum number of retailers per vehicle is considered 3 / 4 / 5 / 6 / 7 / 8.

We have generated 18 test problems and for each test problem, we have run 50 instances. The performance measure parameters are average deviation from the lower bound solution and comparison with modified FFD algorithm. The results of these test problems are discussed in the following table:

Table 1: Results showing performance of heuristic solution

Test	No. of retailers	Max ret. / vehicle	% deviation (lower bound - heuristic)	% deviation (lower bound –mFFD)	% deviation (heuristic -mFFD)
1	50	3	0.00%	5.00%	5.00%
2	50	4	1.36%	9.04%	7.58%
3	50	5	1.36%	4.75%	3.34%
4	50	6	1.36%	2.71%	1.34%
5	50	7	1.81%	2.37%	0.55%
6	50	8	1.69%	2.37%	0.67%
7	75	3	0.28%	5.60%	5.31%
8	75	4	0.45%	8.49%	8.00%
9	75	5	0.68%	4.02%	3.31%
10	75	6	0.76%	2.58%	1.81%
11	75	7	0.45%	2.12%	1.66%
12	75	8	0.83%	2.05%	1.20%
13	100	3	0.00%	3.04%	3.04%
14	100	4	0.63%	8.59%	7.91%
15	100	5	0.80%	4.58%	3.75%
16	100	6	0.86%	2.81%	1.93%
17	100	7	0.63%	2.41%	1.77%
18	100	8	0.86%	2.12%	1.25%

In this experiment, our heuristic procedure provides optimal results in 2 out of 18 tests. The maximum average percentage deviation of the heuristic solution from the lower bound solution is less than 2%. Our heuristic procedure provides better results than the modified FFD algorithm in almost all the tests.

4.2 Experiment-2

In this experiment, we have considered

- 100 retailers in 3 zones (30 in zone A, 30 in zone B, and 40 in zone C)

- Quantity is randomly generated between 0.1 and 0.5 / 0.3 and 0.8 / 0.1 and 0.9 (of vehicle load) for every retailer.
- Maximum number of retailers per vehicle is considered 4 / 5 / 6.

We have generated 9 test problems and for each test problem we have run 50 instances. The results of these test problems are discussed in the following table:

Table 2: Heuristic solution results for three zone problems

Test	Quantity range	Max ret. / vehicle	Average % deviation	Minimum % deviation	Maximum % deviation
1	0.1 – 0.5	4	3.38%	0.00%	7.41%
2	0.1 – 0.5	5	3.63%	1.28%	9.33%
3	0.1 – 0.5	6	5.41%	1.72%	7.41%
4	0.3 – 0.8	4	6.96%	1.12%	8.33%
5	0.3 – 0.8	5	7.89%	2.96%	11.11%
6	0.3 – 0.8	6	9.65%	4.40%	14.33%
7	0.1 – 0.9	4	3.58%	0.76%	6.58%
8	0.1 – 0.9	5	4.07%	1.21%	7.17%
9	0.1 – 0.9	6	4.91%	2.00%	8.33%

In this experiment, the minimum and the maximum deviation from the lower bound solution are 0.00% and 14.33% respectively. The maximum average deviation from the lower bound solution is less than 10%.

4.3 Experiment-3

In this experiment, we have considered

- 15 / 24 retailers equally distributed in 3 zones.

- Quantity is randomly generated between 0.0 / 0.1 and 0.5 / 0.9 (of vehicle load) for every retailer.
- Maximum number of retailers per vehicle is considered 4 / 6.

We have generated 8 test problems with 10 instances each. We have compared our heuristic solution with the lower bound and optimal solution [refer appendix-E]. The optimal result has been obtained by solving the problem as MIP (by CPLEX solver in GAMS). The results of these test problems are discussed in the following table:

Table 3: Comparing heuristic solution with optimal solution

Test	R	Quantity range	M	Av L.B. solution	Av optimal solution	Av heuristic solution	Instances (Heu - Opt)
1	15	0.0 – 0.5	4	4.1	4.2	4.7	5
2	15	0.0 – 0.5	6	3.9	4.2	4.9	3
3	15	0.1 – 0.9	4	8.1	8.9	9.3	6
4	15	0.1 – 0.9	6	7.5	8.2	8.9	3
5	24	0.0 – 0.5	4	6.0	6.3	6.9	4
6	24	0.0 – 0.5	6	5.8	6.0	6.7	3
7	24	0.1 – 0.9	4	12.4	13.5	14.0	5
8	24	0.1 – 0.9	6	11.9	13.2	13.9	4

In this experiment, there are 33 instances out of 80 instances where the heuristic procedure provides optimal solution. There was only one instance where the deviation of heuristic solution from the optimal solution was two vehicles. The average percentage deviation of heuristic solution from the optimal solution is less than 2%.

4.4 Summary of the Experimental Results

The summary of the experimental results of vehicle allocation problem is as follows:

- Our heuristic procedure provides better results than the FFD algorithm modified to incorporate maximum retailer a vehicle can visit.
- The percentage deviation increases when the randomly generated quantity range approaches towards the full vehicle load capacity.
- The percentage deviation increases with the increase in the maximum number of retail outlets a vehicle can visit.
- We have also compared our heuristic solution with FFD algorithm and optimal solution for some bin packing problems defined as hard problem by Eilon and Christofides (1971). Our heuristic solution provides results equal to optimal solution.
- We have not registered the time as our heuristic procedure takes less than 5 seconds even for 100 retailers' problem.

4.5 Performance Analysis

The performance of the vehicle allocation algorithm deteriorates with the increase in retailer requirement as a ratio of vehicle load capacity, and the maximum number of retailers allowed per vehicle. Compared to the lower bound solution, the heuristic will provide worst solution when every retailer requirement is slightly more than 50% of vehicle load capacity. In this case, the percentage deviation of heuristic solution from the lower bound solution will be approximately 100%. When the retailer requirement varies from 1% to 100% of vehicle load capacity, and the number of maximum retailer per vehicle is more than 3, the heuristic solution will deviate from the optimal and lower bound solution to the maximum extent of approximately 25%.

5. Conclusion

In this paper, we have proposed a new algorithm to solve the one-dimensional bin-packing problem with additional constraints. We have applied the heuristic procedure to a vehicle-allocation problem where the objective is to minimize the number of vehicle used for delivery purposes. The algorithm can be utilized for other applications and can be easily modified to incorporate some additional constraints also. We have also identified extension areas for future research. First, the development of the worst-case performance bounds for the proposed algorithm will be useful. Second, the incorporation of non-identical vehicles i.e. bins will be desirable to capture the real-world situations.

Acknowledgements

I am also thankful to Profs. Pankaj Chandra, Diptesh Ghosh, Girija Sharan, and Sanjay Verma for going through the previous draft of this paper (as a part of my thesis) and providing valuable suggestions for improving it.

References

1. Anily, S., and A. Federgruen, (1991). Structured partitioning problem, *Operations Research,* Volume 39, Issue 1, pp. 130-149.
2. Coffman, E. G., M. R. Garey, and D. S. Johnson, (1978). An application of bin-packing to multi-machine scheduling, *SIAM Journal of Computing,* Volume 7, Issue 1, pp. 1-17.
3. Eilon, S., and N. Christofides, (1971). The loading problem, *Management Science,* Volume 17, Issue 5, pp. 259-268.

4. Fekete, S. P., and S. Jorg, (2001). New classes of fast lower bounds for bin packing problems, *Mathematical programming,* Volume 91, Issue 1, pp. 11-31.

5. Fleszar, K., and K. S. Hindi, (2002). New heuristics for one-dimensional bin packing, *Computers and Operations Research,* Volume 29, Issue 7, pp. 821-839.

6. Gupta, J. N. D., and J. C. Ho. (1999). A new heuristic algorithm for the one-dimensional bin-packing problem, *Production Planning and Control*, Volume 10, Issue 6, pp. 598-603.

7. Johnson, D. S., A. Demers, and J. D. Ullman, (1974). Worst case performance bounds for simple one-dimensional packing algorithms, *SIAM Journal of Computing,* Volume 3, Issue, 4, pp. 299-325.

8. Labbe, M., G. Laporte, and M. Silvano, (2003). Upper bounds and algorithms for the maximum cardinality bin packing problem, *European Journal of Operational Research,* Volume 149, Issue 3, pp. 490-498.

9. Martello, S, and D. Vigo, (1998). Exact solution of the two dimensional finite bin packing problem, *Management Science*, Volume 44, Issue 3, pp. 388-399.

10. Martello, S., A. Lodi, and D. Vigo, (2002). Heuristic algorithm for the three dimensional bin packing problem, *Management Science,* Volume 141, Issue 2, pp. 410-420.

11. Ralphs, T. K., L. Kopman, W. R. Pulleyblank, and L. E. Trotter, (2003). On the capacitated vehicle routing problem, *Mathematical Programming,* Volume 94, Issue 2 / 3, pp. 343-359.

12. Rhee, W. T., (1993). Optimal bin packing of items of sizes uniformly distributed over [0,1], *Mathematics of Operations Research,* Volume 18, Issue 3, pp. 694-704.

13. Robb, D. J., and D. Triestsch, (1999). Maximizing the number of mixed packets subject to variety constraints, *Computers and Operations Research*, Volume 26, Issue 13, pp. 1323-1333.