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# Bayesian Ranking and Selection of Fishing Boat Efficiencies

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**Abstract** The steadily accumulating literature on technical efficiency in fisheries attests to the importance of efficiency as an indicator of fleet condition and as an object of management concern. In this paper, we extend previous work by presenting a Bayesian hierarchical approach that yields both efficiency estimates and, as a byproduct of the estimation algorithm, probabilistic rankings of the relative technical efficiencies of fishing boats. The estimation algorithm is based on recent advances in Markov Chain Monte Carlo (MCMC) methods— Gibbs sampling, in particular—which have not been widely used in fisheries economics. We apply the method to a sample of 10,865 boat trips in the US Pacific hake (or whiting) fishery during 1987–2003. We uncover systematic differences between efficiency rankings based on sample mean efficiency estimates and those that exploit the full posterior distributions of boat efficiencies to estimate the probability that a given boat has the highest true mean efficiency.

**Key words** Ranking and selection, hierarchical composed-error model, Markov Chain Monte Carlo, Pacific hake fishery.

JEL Classification Codes Q2, L5, C1.

## Introduction

Ranking production units—firms, employees, or machines, say—by efficiency is useful for a variety of managerial purposes. In retail, a firm may wish to identify and replace its less productive salespersons (Fernandez-Gaucherand *et al.* 1995). In agriculture, more efficient farms may be used as models for less efficient ones (Ahmad and Bravo-Ureta 1996). In fisheries, efficiency rankings may aid firms wishing to compare the relative performances of boats. Such rankings may also assist fisheries managers in gauging the likely impacts of prospective management measures or the effects of measures already enacted. For example, rankings of boat efficiencies before and after the implementation of mesh size limits might indicate a

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differential impact of the regulation on home port groups or boat sizes. Similarly, the effects of a permit buyback on the composition of fleet efficiency might be a concern during formulation of the buyback program, especially if 'high-' or 'low-ranking' boats might be given priority in the buyback. To take another example, efficiency rankings could help assess the likelihood that ostensibly conservation-neutral transfers of days at sea among boats would affect fishing mortality. While such rankings may be useful, they raise a variety of challenging questions related to their derivation and statistical properties. For example, can the most or least efficiency scores, how probable is it that the difference is real and not due to statistical noise? And, can boats be categorized in statistically defensible ways—for example, into high, medium, and low efficiency types?

In this paper, we outline a framework within which these and similar questions can be addressed. One major difference between our approach and those preceding it is that we define 'the best' as that boat that has the highest probability of having the lowest inefficiency score. In this context, the key points we make are three. First, because they are functions of other random variables, technical efficiency scores are themselves random variables with associated probability distribution functions (pdfs). Second, rankings of mean efficiency scores based on the complete distribution of those random variables may produce different results from rankings based on point estimates of mean efficiency. That is, using only sample means in ranking and selection ignores vital information, and a ranking of production units according to point estimates of technical efficiency may lead to error. Third, while frequentist methods have been successfully applied in this context (Horrace 2005), the Bayesian hierarchical approach to ranking and selection has some significant advantages. Its range of applicability is broad, since any function of posterior estimates can be estimated directly. Further, its implementation is attractive, producing the requisite probability calculations as a byproduct of the estimation algorithm.

The basis of our approach is the composed-error stochastic frontier model commonly used for efficiency studies (Kumbhakar and Lovell 2000). While most stochastic production frontiers, both within fisheries and within the wider production economics literature, are estimated using maximum likelihood techniques, we adopt a Bayesian hierarchical approach for three reasons. First, recent advances in Bayesian estimation permit implementation of significantly more complex models than previously thought possible. Second, the hierarchical approach enables us to incorporate, formally and robustly, aspects of intra-sample heterogeneity that are important in the data generating environment in which the technical efficiency scores are established. In the fisheries context, heterogeneity can assume several forms, including variations in performance across seasons, variations in performance across boats within seasons, and variations in performance across trips within seasons and across boats. Such heterogeneity can impact efficiency scores, so it is important to account for it when it is present. Finally, as a byproduct of the estimation algorithm, the Markov Chain Monte Carlo (MCMC) routine used to estimate frontier parameters and unit efficiencies generates estimates of probabilities that a particular vessel or group of vessels is most efficient, as we illustrate below.

We demonstrate the ranking and selection problem with an application to the US Pacific whiting fishery. The data set used consists of a panel of 10,865 Pacific whiting trips taken off the US West Coast<sup>1</sup> by 41 boats between years 1987 and 2003. We suppose that each trip has associated with it a measure of technical efficiency that depends, among other things, on the management of the boat making the trip and the

<sup>&</sup>lt;sup>1</sup> West Coast here refers to California, Oregon, and Washington.

year in which the trip occurs. Importantly, we consider the trip-specific technical efficiency measure to be a draw from an underlying distribution characterized by an unobserved set of parameters. Because these parameters are important determinants of the technical efficiency score associated with each boat, a key objective is to estimate them. In standard hierarchical estimation, the end point of the investigation is a description of the marginal distribution of the parameters in question. Although the marginal distributions that we seek are not available in closed form, an estimation procedure using recent advances in MCMC methods supplies them. Having estimated the parameters in the underlying data-generating process, we then go on to compare the probability that the true values of each boat's mean efficiency score is the maximum among the set of all boats' true mean efficiencies.

In the next section, we describe measures of technical efficiency that have been developed in the econometric literature and consider how they should be modified to evaluate boat efficiency in fisheries. We then describe our estimation techniques and apply them to the US West Coast Pacific hake fishery. The final section of the paper offers an assessment and conclusions.

## **Bayesian Ranking and Selection**

Statistical ranking and selection procedures have developed over the last fifty years, with fundamental contributions from Bechhofer (1954) and Gupta (1956) and discussions of the modifications since then by Gibbons, Olkin, and Sobel (1977); Gupta and Panchapakesan (1979); and Dudewicz and Koo (1982). More recently, a Bayesian procedure for ranking and selection of related means with alternatives to analysis-of-variance methodology is proposed by Berger and Deely (1988). Subsequently, Fong (1992) develops an extension to incorporate covariates. The main ideas underlying ranking and selection are articulated appealingly in Berger and Deely's example of ranking and selecting among baseball scores, which we reproduce as table 1 (Berger and Deely 1988, table 1, p. 365). This table reports the performance levels of twelve leading batters in the National League in 1984 (obtained for all players that had at least 150 at bats), showing each batter's rank, based on his batting average ( $x_i$ , in column 2), his number of at bats ( $n_i$ , column 3), and his

Observed Batting Averages							
i	$x_i$	n <sub>i</sub>	$1,000 \times \sigma_i^2$	$p_i$			
1	0.362	185	1.25	0.159			
2	0.351	606	0.38	0.222			
3	0.351	342	0.67	0.165			
4	0.346	214	1.06	0.125			
5	0.324	262	0.84	0.077			
6	0.321	474	0.46	0.061			
7	0.314	636	0.34	0.038			
8	0.312	600	0.36	0.035			
9	0.311	550	0.39	0.036			
10	0.303	535	0.39	0.024			
11	0.298	181	1.16	0.048			
12	0.290	607	0.34	0.010			

Table 1

Note: Reproduced from Berger and Deely 1988, table 1, p. 365.

associated variance in batting averages  $(1,000 \times \sigma^2, \text{ column 4})$ . Column 5 reports the probabilities that each of the batters is 'best' in the sense of having the highest true probability of a hit,  $\theta_i$ . An important outcome of this problem is that the player with the largest observed sample mean (player one) is judged, by the hierarchical Bayesian method, to have a lower probability of having the highest true mean, because the player with the highest true mean (player two) has a batting average that has associated with it a much smaller variance.

In the remainder of this paper, we develop a similar analysis of technical efficiency in the Pacific hake fishery. Using a notation that we later justify, given a set of unobserved mean inefficiency scores,  $\{z_A, z_B, ..., z_N\}$  for N boats, we wish to select the boat that is most efficient, which is the boat with the smallest  $z_i$ . Suppose that Boat A has a higher estimated mean inefficiency than does boat B (that is,  $\hat{z}_A > \hat{z}_B$ ). Suppose also that boat A, with only a few trips, has a considerably higher variance in inefficiency score than Boat B (that is,  $\hat{\omega}_A^2 > \hat{\omega}_B^2$ ). Given these differences in the mean and variance of inefficiency scores, which of the two boats can correctly be adjudged more efficient in the sense that it has higher probability of having true mean inefficiency ( $z_A$  or  $z_B$ , as the case may be) that is lower than the one associated with the other boat? By exploiting Gibbs sampling techniques, which were not available to Berger and Deely (1988) at the time of their investigation, we demonstrate how this question can be resolved simply and intuitively as a byproduct of the estimation algorithm, without the need to resort to complex integration such as that in Berger and Deely (1988) and in the literature preceding it.

The main attraction of the procedure is its simplicity. Although we observe neither the true values  $\{z_A, z_B, ..., z_N\}$  nor the marginal probability density functions that generate them, say  $\{f^A(z_A), f^B(z_B), ..., f^N(z_N)\}$ , we are able to estimate the corresponding pdfs and, as a consequence, draw samples from the underlying distributions. Consequently, we can compute the numerical analogue of the integral associated with the probability  $\mathcal{O}^A(z_A \leq z_J, \forall J \neq A)$ . This estimate is:

 $\hat{\wp}^{A}(z_{A} \leq z_{J} \forall J \neq A) \equiv G^{-1} \times [\text{the total number of occurences in } G \text{ draws from} (1)$   $\{f^{A}(z_{A}), f^{B}(z_{B}), \dots, f^{N}(z_{N})\} \text{ in which the condition } z_{A} \leq z_{B}, \dots z_{A} \leq z_{N} \text{ holds}].$ 

Similarly, we are able to repeat this calculation for each of the remaining boats in the sample. If we are able to draw samples of sufficient size, we can appeal to the asymptotic rule that, in the sequence of draws, g = 1, 2, ..., G, the limit of the expression on the right converges to the true probability that we seek. However, in order for the procedure to work, it is necessary that we simulate draws from the appropriate pdfs. Although these pdfs are unavailable to us, we do have available the corresponding fully conditional distributions, as required by the Gibbs sampling procedure. As a consequence, we can estimate the marginal pdfs by the process of Rao-Blackwellization (see Casella and George (1992) for discussion) in the sequence of iterations in the Gibbs sample, simulating as a byproduct of the estimation algorithm the true probabilities that we seek.

As we have seen, the ideas of statistical ranking and selection date at least to 1954, yet there have been few applications in the resource and environmental economics literature. Holloway and Ehui (2002) show how the Gibbs output can be further exploited in order to estimate the probability that a particular policy is best for achieving stated objectives. In assessing alternative measures for effecting market participation among households in the Ethiopian highlands, they compute the probability that a particular policy dominates in the sense that it effects outcomes at least cost. Atkinson and Dorfman (2005) pursue similar objectives in an application

that is contextually similar to the one in this paper. Using data from a sample of electric generating plants in the US, they focus on identifying preferred sets of efficient plants, where efficient means those plants that produce closest to the frontier of a minimum distance function. Finally, related ideas, particularly those pertaining to the truncated-Normal composed-error setting of this paper, appear in Horrace (2005). His method is sample-theoretic and is focused on estimating sets of sample units that are 'most efficient,' in the sense that with a preselected level of confidence, that set of firms lies closest to the production frontier.

Several features of our econometric approach merit emphasis. First, by virtue of the fact that the Gibbs sampler iterates over a large number of draws, the estimate in expression (1) is a *marginal probability* measure even though the distributions from which it is obtained are fully conditional. Second, with reference to the probability calculations and previous work (Newey and West 1987), we can place numerical standard errors on the probabilities computed, and thereby place confidence bounds on the estimates derived. Lastly, the procedure is attractive for its generality as well as its simplicity, as the probability calculations are equally applicable to arbitrary functions of the estimated model parameters. This makes the procedure applicable to a wide class of statistics that the analyst or the policymaker may wish to rank.

#### **Bayesian Composed-error Model Estimation**

Our statistical framework is the composed-error stochastic production frontier, with heterogeneous production inefficiency incorporated using truncated normal distributions. The composed-error model has origins in the deterministic-frontier works of Aigner and Chu (1968) and Afriat (1972) and the subsequent extension by Aigner, Lovell, and Schmidt (1977) to make the frontier stochastic. Since the appearance of these seminal works, composed-error modelling has experienced routine application due, in part, to the availability of specialized software, particularly the freely available *FRONTIER* software package (Coelli 1996). In addition, a growing literature emphasizes the versatility and value of applying the composed-error model in a broad set of circumstances. Some of the more important developments since its inception include extensions of the distributional assumptions on the inefficiencies (Jondrow *et al.* 1982); incorporating explicit linkages between the mean inefficiencies and covariates to model efficiency change over time (Battese and Coelli 1992); and extensions to panel settings in which production units are observed repeatedly over time (see Battese and Coelli (1995) for a review).

Application of the composed-error model to fisheries came relatively late, but the number of studies has grown in recent years. The first analysis of fisheries efficiency appears to be Hannesson (1983). Recent applications include Kirkley, Squires, and Strand (1995); Squires and Kirkley (1999); Grafton, Squires, and Fox (2000); Bjorndal, Kondouri, and Pascoe (2002); Pascoe and Coglan (2002); Herrero and Pascoe (2003); and Pascoe *et al.* (2003).<sup>2</sup> From these recent studies, several general themes emerge. First, the null hypothesis of zero inefficiency is generally refuted; non-negligible inefficiencies appear to be the rule in fisheries. Second, the overall distribution of technical performance in fisheries appears to be highly contextual, with some studies deriving widely dispersed estimates of technical efficiency, whereas others produce estimates that are concentrated. Third, the effects

<sup>&</sup>lt;sup>2</sup> Overviews of technical efficiency and productivity in fisheries are available in Alvarez (2001); Felthoven and Morrison Paul (2004); and Holloway, Tomberlin, and Irz (2005). A recent review of the more general economic efficiency literature is Murillo-Zamorano (2004).

of regulation are an over-arching theme, and the analysis of how regulation affects technical efficiency is a matter of considerable interest. Generally speaking, measures aimed at solving the open-access nature of fisheries tend to increase the overall level of technical efficiency, and this positive force appears greater when the fishery intervention is more flexible, although exceptions exist (see Grafton, Squires, and Fox 2000; Felthoven 2002; and Pascoe *et al.* 2003).

While most studies of technical efficiency in fisheries are conducted with panel data, they seem in most cases not to fully exploit the panel structure. This is partly due to the econometric difficulties that arise in panel estimation when some regressors are time-invariant. Our approach circumvents such issues by employing a Bayesian hierarchical methodology, as has previously been done by Fernandez, Ley, and Steel (2002) and Holloway, Tomberlin, and Irz (2005). Bayesian applications of the composed-error model began with a series of seminal papers (Koop, Osiewalski, and Steel 1994, 1997; Koop, Steel, and Osiewalski 1995) which were made possible by the advent of MCMC techniques, Gibbs sampling in particular (Gelfand and Smith 1990). These papers form the basis for our hierarchical investigation of technical efficiency in the Pacific hake fishery.

We assume that the data are generated from the observational equation:

$$y_{iik} = \mathbf{x}'_{iik}\mathbf{\beta} + u_{iik} - z_{ii}, \quad i = 1, 2, \dots, N; \quad j = 1, 2, \dots, T_i; \quad k = 1, 2, \dots, S_{ii},$$
(2)

where the subscript i = 1, 2, ..., N, denotes the boats in the sample;  $j = 1, 2, ..., T_i$ , denotes the years in the sample in which boat *i* operates; and  $k = 1, 2, ..., S_{ij}$ , denotes the trips undertaken by boat i in year j;  $y_{ijk}$  denotes (the natural logarithm of) catch by boat *i* in year *j* on trip *k*;  $\mathbf{x}'_{ijk} \equiv (x_{ijk1}, x_{ijk2}, ..., x_{ijkM})$  denotes an M-vector of (the natural logarithm of) relevant covariates affecting catch (inputs to production or exogenous factors such as weather);  $\boldsymbol{\beta} \equiv (\boldsymbol{\beta}_1, \, \boldsymbol{\beta}_2, ..., \, \boldsymbol{\beta}_M)'$  denotes an M-vector of corresponding coefficients;  $u_{ijk}$  denotes a random error affecting catch by boat i in season j on trip k; and  $z_{ij}$  denotes the level of technical inefficiency corresponding to the  $i^{th}$  boat in the  $j^{th}$  year. Implicit is the assumption that, for a given boat in a given year, each trip has the same level of technical inefficiency associated with it, but that this  $z_{ii}$  may vary across boats and across years. We will make assumptions about the  $u_{ijk}$  and the  $z_{ij}$  for which some additional notation will prove useful.<sup>3</sup> Let  $f^a(b|c,$  $d, \dots, e$ ) denote a probability density function (pdf) of type a for random variable b, fully described by the (unknown) parameters c, d,..., e. Consequently, our assumption that the sampling errors are *iid* Normal can be formalized by writing  $u_{iik} \sim f^N(u_{iik}|0,\sigma)$ . We need also to make assumptions about the data-generating process for the inefficiency terms. Many distributions have been proposed for the inefficiency terms  $z_{ii}$ , in both the Bayesian and classical settings. These forms include the exponential, the Gamma, the half-normal, and the truncated normal pdf (see Dorfman and Koop 2005, and the literature cited there, for a review of this work). While offering considerable flexibility, the truncated-normal distribution is, arguably, among the easiest to implement in hierarchical settings. Hence, we assume that each  $z_{ii}$  is

<sup>&</sup>lt;sup>3</sup> It remains to explain our choice of notation in the presentation of the rankings  $\{z_A, z_B, ..., z_N\}$ . The observed data are denoted **x** and **y** and the unknown parameters are denoted by  $\theta$ . We use **z** to denote latent or missing data which, here, refers to the boat-and-year specific inefficiencies. However, the boat-specific inefficiencies  $\{z_1, z_2, ..., z_N\}$  are also latent data in the hierarchy, thus the notation  $\{z_A, z_B, ..., z_N\}$ . An alternative is to assume that the z's are parameters contained in  $\theta$ , as advocated, for example, in Koop (2003, see, in particular, the discussion on pages 169–70). For most of the estimation, the choice of interpretation is moot. However, the choice is significant in the context of marginal-likelihood estimation (see Chib (1995) for discussion), and we prefer the latent-data interpretation of the inefficiencies, finding it to be somewhat more intuitive.

drawn from a normal distribution, truncated at zero, with unknown mean,  $z_i$ , and variance,  $\omega_i^2$ . We assume, in the hierarchical manner, that the boat-specific means (*i.e.*, the  $z_i$ ) are *iid* draws from another truncated-normal distribution with mean  $\mu$  and variance  $\lambda^2$ . As an alternative, one could assume that the  $z_{ij}s$  are *iid* across years with year-specific means,  $z_j$ , and that the year-specific means are *iid* from the upper-level distribution. However, in previous use of the data (Tomberlin, Irz, and Holloway 2006) we find that the former specification is preferred.<sup>4</sup> The form of the hierarchical structure is presented diagrammatically in figure 1 and can be defined formally by the relations  $z_{ij} \sim f^{iN}(z_{ij}|z_i,\omega_i)$  and  $z_i \sim f^{iN}(z_i|\mu,\lambda)$ . Note that each boat-level distribution—each  $f^{iN}(.lz_i,\omega_i)$ —is defined by a separate mean and a separate variance, which is important in the context of calculating probabilities of 'the best.' This problem, which is the main motivation of our inquiry, would degenerate if the means and variances were constrained *a priori* to be the same. Yet the validity of such a constraint can only be established *a posteriori* using our Bayesian hierarchical methodology. In short, what makes it interesting and motivates our investigation



Figure 1. A Three-layer Hierarchy of Boat Efficiency

<sup>&</sup>lt;sup>4</sup> In our context of Bayesian model comparison, 'preferred' means the model that has higher posterior probability over all alternatives under consideration. Marginal likelihoods are the essential inputs in Bayes factors which are, in turn, essential in calculating posterior probabilities in favour of a particular model (see Zellner (1996), pp. 291–318, for a comprehensive discussion). Until quite recently, computation of marginal likelihoods had proven extremely troublesome (see for alternative attempts at computation Carlin and Polson 1991; Gelfand and Dey 1994; Newton and Raftery 1994; Carlin and Chib 1995). The MCMC approach to estimation leads, by a simple extension (see Chib 1995), to estimates of marginal likelihoods, hence to diagnostics for model selection.

is the notion that different boats have different  $f^{iN}(.|z_i,\omega_i)$ . For later reference it will prove useful to group the boat inefficiency means into a vector  $\mathbf{z}^i \equiv (z_1, z_2, ..., z_N)'$ and group also the boat inefficiency standard errors into  $\mathbf{\omega}^i \equiv (\omega_1, \omega_2, ..., \omega_N)'$ . Note the use of superscripts in these definitions in order to distinguish between the collection of boat-level inefficiencies  $(\mathbf{z}^i)$  and the collection of year-specific inefficiencies pertaining to boat  $i(\mathbf{z}_i)$ .

Suitably rearranging the system in matrix terms will enable us to develop the estimation procedure more transparently. Stacking observations across the trips yields:

$$\mathbf{y}_{ij} = \mathbf{x}_{ij} \mathbf{\beta} + \mathbf{u}_{ij} - \mathbf{\iota}_{ij} z_{ij}, \ i = 1, 2, \dots, N; \ j = 1, 2, \dots, T_i,$$
(3)

where  $\mathbf{y}_{ij} \equiv (y_{ij1}, y_{ij2}, ..., y_{ijSij})'$  denotes a vector of  $S_{ij}$  observations on observed catch;  $\mathbf{x}_{ij} \equiv (\mathbf{x}_{ij1}, \mathbf{x}_{ij2}, ..., \mathbf{x}_{ijSij})'$  denotes a matrix of observed covariates of dimension  $S_{ij} \times \mathbf{M}$ ;  $\mathbf{u}_{ij} \equiv (u_{ij1}, u_{ij2}, ..., u_{ijSij})'$  denotes a vector of random disturbances; and  $\mathbf{u}_{ij}$  denotes an  $S_{ij}$ -unit vector. It will prove useful to write the system in a boat-specific form. Stacking observations over the respective years yields:

$$\mathbf{y}_i = \mathbf{x}_i \boldsymbol{\beta} + \mathbf{u}_i - \mathbf{v}_i \mathbf{z}_i, \quad i = 1, 2, \dots, N,$$
(4)

where  $\mathbf{y}_i \equiv (\mathbf{y}'_{i1}, \mathbf{y}'_{i2}, ..., \mathbf{y}'_{iTi})'$  denotes a vector of  $\Sigma_j S_{ij}$  observations on boat *i*'s catch;  $\mathbf{x}_i \equiv (\mathbf{x}'_{i1}, \mathbf{x}'_{i2}, ..., \mathbf{x}'_{iTi})'$  denotes a corresponding matrix of covariates;  $\mathbf{u}_i \equiv (\mathbf{u}'_{i1}, \mathbf{u}'_{i2}, ..., \mathbf{u}'_{iTi})'$  denotes a corresponding vector of disturbances;  $\mathbf{v}_i$  denotes a binary matrix of dimension  $\Sigma_j S_{ij} \times T_i$  containing the unit vectors  $\mathbf{t}_{i1}, \mathbf{t}_{i2}, ..., \mathbf{t}_{Ti}$ ; and  $\mathbf{z}_i \equiv (\mathbf{z}_{i1}, \mathbf{z}_{i2}, ..., \mathbf{z}_{iTi})'$  denotes the technical inefficiency of boat *i* across the  $T_i$  seasons in which it operates. Finally, by stacking observations over the corresponding boats, the system can be written compactly as:

$$\mathbf{y} = \mathbf{x}\mathbf{\beta} + \mathbf{u} - \mathbf{w}\mathbf{z},\tag{5}$$

where vector  $\mathbf{y} \equiv (\mathbf{y}'_1, \mathbf{y}'_2, ..., \mathbf{y}'_N)'$  has dimension S;  $\mathbf{x} \equiv (\mathbf{x}'_1, \mathbf{x}'_2, ..., \mathbf{x}'_N)'$  has dimension S×M;  $\mathbf{u} \equiv (\mathbf{u}'_1, \mathbf{u}'_2, ..., \mathbf{u}'_N)'$  is an S-vector of disturbances; and  $\mathbf{w}$  denotes a binary matrix of dimension S× $\Sigma_i T_i$  containing an appropriate arrangement of the unit matrices  $\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_N$ .

With this notation at hand, the likelihood is easy to formalize. Given the assumptions about the stochastic disturbance in equation (2), a Jacobian transformation from  $\mathbf{u}$  to  $\mathbf{y}$  allows us to write the *complete-data likelihood*:

$$f(\mathbf{y}|\boldsymbol{\theta}, \mathbf{z}) \equiv \prod_{i=1}^{N} \prod_{j=1}^{T_{i}} f^{N}(\mathbf{y}_{ij} | \mathbf{x}_{ij} \boldsymbol{\beta} - \boldsymbol{\iota}_{ij} \boldsymbol{z}_{ij}, \boldsymbol{\sigma}) \times f^{TN}(\boldsymbol{z}_{ij} | \mathbf{z}_{i}, \boldsymbol{\omega}_{i}) \times f^{TN}(\mathbf{z}_{i} | \boldsymbol{\mu}, \boldsymbol{\lambda}).$$
(6)

At this point, we distinguish between the parameters  $\boldsymbol{\theta} \equiv (\boldsymbol{\beta}', \boldsymbol{\sigma}, \boldsymbol{\omega}', \boldsymbol{\mu}, \boldsymbol{\lambda})'$ , which are unobserved; the observed data, **x** and **y**; and the unobserved or latent data, **z**, which are the boat-year technical inefficiencies. A natural-conjugate prior over the elements of  $\boldsymbol{\theta}$  consists of independent normal and inverted-Gamma components, namely:

$$\pi(\boldsymbol{\theta}) \equiv f^{iG}(\boldsymbol{\sigma}|\boldsymbol{s}_{\sigma\sigma}, \boldsymbol{v}_{\sigma\sigma}) \times f^{mvN}(\boldsymbol{\beta}|\boldsymbol{\beta}_{\sigma}, \mathbf{C}_{\beta\sigma})$$

$$\times f^{iG}(\boldsymbol{\lambda}|\boldsymbol{s}_{\lambda\sigma}, \boldsymbol{v}_{\lambda\sigma}) \times f^{N}(\boldsymbol{\mu}|\boldsymbol{\mu}_{\sigma}, \boldsymbol{v}_{\mu\sigma}) \times \Pi_{i} f^{iG}(\boldsymbol{\omega}_{i}|\boldsymbol{s}_{\omegai\sigma}, \boldsymbol{v}_{\omegai\sigma}).$$

$$(7)$$

It follows that the complete-data posterior is proportional to the product of equations (6) and (7), or:

$$\pi(\mathbf{\theta}|\mathbf{y},\mathbf{z}) \propto f(\mathbf{y}|\mathbf{\theta},\mathbf{z}) \times \pi(\mathbf{\theta}).$$
(8)

We also distinguish between the observed data likelihood  $f(\mathbf{y}|\mathbf{\theta})$  and the complete data likelihood  $f(\mathbf{y}|\mathbf{\theta}, \mathbf{z})$ , noting that  $f(\mathbf{y}|\mathbf{\theta}) = \int f(\mathbf{y}|\mathbf{\theta}, \mathbf{z}) dz$ . An artifact of this distinction is that we must obtain estimates of the latent  $\mathbf{z}$  in the process of Gibbs sampling the posterior in equation (8) and, hence, there are seven blocks of parameters for which the fully conditional distributions must be derived. Some algebra (available from the authors upon request) reveals that the fully conditional distributions comprising the posterior consist of inverted-Gamma, univariate-Normal, multivariate-Normal, and truncated-Normal forms, namely:

$$\pi(\boldsymbol{\sigma}|\mathbf{y}, \mathbf{z}, \mathbf{z}^{i}, \boldsymbol{\beta}, \boldsymbol{\omega}^{i}, \boldsymbol{\mu}, \boldsymbol{\lambda}) \propto f^{iG}(\boldsymbol{\sigma}|_{\boldsymbol{v}_{\sigma}}, s_{\sigma}^{2}),$$
(9)

where  $v_{\sigma} \equiv S$ ,  $v_{\sigma}s_{\sigma}^2 \equiv (\mathbf{y} - \mathbf{x}\mathbf{\beta} + \mathbf{w}\mathbf{z})'(\mathbf{y} - \mathbf{x}\mathbf{\beta} + \mathbf{w}\mathbf{z}) + v_{\sigma\sigma}s_{\sigma\sigma}^2$ ;

$$\pi(\boldsymbol{\beta}|\mathbf{y}, \mathbf{z}, \mathbf{z}^{i}, \boldsymbol{\sigma}, \boldsymbol{\omega}^{i}, \boldsymbol{\mu}, \boldsymbol{\lambda}) \propto f^{mvN}(\boldsymbol{\beta}|\hat{\boldsymbol{\beta}}, \mathbf{C}_{\hat{\boldsymbol{\beta}}}),$$
(10)

where 
$$\hat{\boldsymbol{\beta}} \equiv (\sigma^{-2} \mathbf{x}' \mathbf{x} + \mathbf{C}_{\boldsymbol{\beta}}^{-1})^{-1} \Big[ \sigma^{-2} \mathbf{x}' (\mathbf{y} - \mathbf{x} \boldsymbol{\beta}) + \mathbf{C}_{\boldsymbol{\beta}_{o}}^{-1} \boldsymbol{\beta}_{0} \Big]$$
 and  $\mathbf{C}_{\boldsymbol{\beta}} \equiv (\sigma^{-2} \mathbf{x}' \mathbf{x} + \mathbf{C}_{\boldsymbol{\beta}_{o}}^{-1})^{-1};$ 

$$\pi(z_{ij}|\mathbf{y}, \mathbf{z}^i, \sigma, \boldsymbol{\beta}, \boldsymbol{\omega}^i, \boldsymbol{\mu}, \boldsymbol{\lambda}) \propto f^{tN}(z_{ij}|\hat{z}_{ij}, c_{\hat{z}_{ij}}), \quad j = 1, 2, \dots, T_i; \quad i = 1, 2, \dots, N, \quad (11)$$

where 
$$\hat{z}_{ij} \equiv (\sigma^{-2} \mathbf{\iota}'_{ij} \mathbf{\iota}_{ij} + \omega_i^{-2})^{-1} \Big[ \sigma^{-2} \mathbf{\iota}'_{ij} (\mathbf{y}_{ij} - \mathbf{x}'_{ij} \boldsymbol{\beta}) + \omega_i^{-2} z_i \Big]^{-1}$$
 and  $c_{z_{ij}} \equiv (\sigma^{-2} \mathbf{\iota}'_{ij} \mathbf{\iota}_{ij} + \omega_i^{-2})^{-1}$ ;

$$\pi(\boldsymbol{\omega}_i | \mathbf{y}, \mathbf{z}, \mathbf{z}^i, \boldsymbol{\sigma}, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda}) \propto f^{iG}(\boldsymbol{\omega}_i | \boldsymbol{v}_{\boldsymbol{\omega} i}, \boldsymbol{s}_{\boldsymbol{\omega} i}^2), \ i = 1, 2, \dots, N,$$
(12)

where  $v_{\omega i} \equiv \Sigma T_i$ ,  $v_{\omega i} s_{\omega i}^2 \equiv \Sigma_j (z_{ij} - z_i)^2 + v_{\omega io} s_{\omega io}^2$ ;

$$\pi(z_i|\mathbf{y}, \mathbf{z}, \sigma, \boldsymbol{\beta}, \boldsymbol{\omega}^i, \boldsymbol{\mu}, \boldsymbol{\lambda}) \propto f^N(z_i|\hat{z}_i, c_{\hat{z}_i}), \ i = 1, 2, \dots, N,$$
(13)

where  $\hat{z}_i \equiv (\omega_i^{-2} \mathbf{t}'_i \mathbf{t}_i + \lambda^{-2})^{-1} (\omega_i^{-2} \mathbf{t}'_i z_{ij} + \lambda^{-2} \mu)$  and  $c_{\hat{z}_i} \equiv (\omega_i^{-2} \mathbf{t}'_i \mathbf{t}_i + \lambda^{-2})^{-1}$ ;

$$\pi(\lambda | \mathbf{y}, \mathbf{z}, \mathbf{z}^i, \sigma, \boldsymbol{\beta}, \boldsymbol{\omega}^i, \mu) \propto f^{iG}(\sigma | v_{\lambda}, s_{\lambda}^2),$$
(14)

where  $v_{\lambda} \equiv N, v_{\lambda}s_{\lambda}^2 \equiv (\mathbf{z}^i - \mathbf{\iota}_N \mu)'(\mathbf{z}^i - \mathbf{\iota}_N \mu) + v_{\lambda o}s_{\lambda o}^2$ ; and

$$\pi(\boldsymbol{\mu}|\mathbf{y}, \mathbf{z}, \mathbf{z}^{i}, \boldsymbol{\sigma}, \boldsymbol{\beta}, \boldsymbol{\omega}^{i}, \boldsymbol{\lambda}) \propto f^{N}(\boldsymbol{\mu}|\hat{\boldsymbol{\mu}}, c_{\hat{\boldsymbol{\mu}}}),$$
(15)

where  $\hat{\boldsymbol{\mu}} \equiv (\lambda^{-2} \boldsymbol{\iota}'_N \boldsymbol{\iota}_N + c_{\mu_o}^{-1})^{-1} (\lambda^{-2} \boldsymbol{\iota}'_N \mathbf{z}^i + c_{\mu_o}^{-1} \mu_o)^{-1}$  and  $c_{\mu} \equiv (\lambda^{-2} \boldsymbol{\iota}'_N \boldsymbol{\iota}_N + c_{\mu_o}^{-1})^{-1}$ .

Each of these probability distribution functions is easy to sample from. Note that all but one of the unknown random variables is required to initiate calculations, and we select the draw for  $\sigma$  to initiate the chain. It follows that, given arbitrary starting values { $\beta^{(0)'}$ ,  $\mathbf{z}^{(0)'}$ ,  $\mathbf{\omega}^{(0)'}$ ,  $\mathbf{z}^{i(0)'}$ ,  $\mu^{(0)}$ ,  $\lambda^{(0)}$ }, an efficient algorithm for estimating the parameters of the composed-error model consists of the steps:

A<sub>1</sub>: Draw, respectively,  $\sigma^{(g)}$  from (9);  $\boldsymbol{\beta}^{(g)}$  from (10);  $\mathbf{z}^{(g)}$  from (11);  $\omega_i^{(g)}$  from (12);  $\mathbf{z}_i^{(g)}$  from (13);  $\lambda^{(g)}$  from (14); and  $\mu^{(g)}$  from (15). Iterate a sufficient number of times until the draws are independent of the starting values and then repeat for iterations g = 1, 2, ..., G, collecting output  $\{\sigma^{(g)}, \boldsymbol{\beta}^{(g)'}, \boldsymbol{\omega}^{i(g)'}, \mu^{(g)}, \lambda^{(g)}\}_{g=1}^{G}$  and  $\{\mathbf{z}^{(g)}, \mathbf{z}^{i(g)}\}_{g=1}^{G}$ .

From the outputs collected we can conduct posterior inference about the locations and scales of the marginal distributions of interest and, indeed, any function of the parameters which is of interest. Concerns about how long to iterate algorithm  $A_1$  are answered by assessing convergence of the Markov chain, using appropriately chosen convergence statistics. A large and growing literature on convergence exists (see Gelman and Rubin (1992), Geweke (1992), Brooks and Gelman (1998), Mengersen and Guihenneuc-Joyaux (1999), and Brooks and Giudici (2000) for comprehensive coverage). In our case, convergence is assessed by using the convergence diagnostic:

$$CD = \frac{\hat{g}_{S_A} - \hat{g}_{S_C}}{\frac{\hat{\sigma}_A}{\sqrt{S_A}} + \frac{\hat{\sigma}_C}{\sqrt{S_C}}},$$
(16)

which is advocated, for example, by Koop (2003, p. 66). The convergence diagnostic, *CD*, is obtained by iterating for a sequence of draws  $g = 1, 2, ..., S_0$ ; then, another sequence,  $g = 1, 2, ..., S_A$ ; then, a third sequence,  $g = 1, 2, ..., S_B$ ; and finally, a fourth sequence  $g = 1, 2, ..., S_C$ ; and collecting outputs from the second and fourth sequences which, if the third sequence is sufficiently large, are likely to be independently distributed. Then, with  $\hat{g}_{S_A}$  and  $\hat{g}_{S_C}$  estimates of two functions of interest, derived with respect to the samples  $S_A$  and  $S_C$ , and with  $\hat{\sigma}_{S_A} / \sqrt{S_A}$  and  $\hat{\sigma}_{S_C} / \sqrt{S_C}$ their respective numerical standard errors, the statistic *CD* is asymptotically standard-normally distributed, making possible inference about the number of iterations deemed to be sufficient for convergence. We estimate that convergence is achieved after about G = 2,000 iterations, but we iterate conservatively using G = 25,000. The empirical reports that follow are derived from this G = 25,000 sample, after a 'burnin' phase of G = 25,000.

#### **Study Fishery, Data, and Prior Information**

Pacific hake (*Merluccius productus*), also known as Pacific whiting, is a migratory species found from Baja California to the Gulf of Alaska. Most of the commercial catch is taken with mid-water trawl gear in the northern half of this range. The hake fishery is a high-volume, low-margin fishery that accounted for 64% of the catch (by weight) in the shore-based, limited-entry trawl fleet during 1993–2003. Because of a tendency to spoil rapidly, hake were long regarded as undesirable by US fisher-

men. During the 1990s, growth in the surimi market and improved processing techniques led to huge increases in domestic catch, with most boats having refrigerated seawater storage to reduce spoilage, making them a fairly distinct segment.

Along the West Coast of the United States, the hake fishery is managed by the Pacific Fisheries Management Council, which allocates the total allowable catch among a tribal fishery and three non-tribal sectors (a shore-based fleet, a catcher-processor fleet, and a factory trawler fleet). In this paper, we consider only the shore-based fleet; specifically, the 41 boats that caught hake on at least 100 trips during our study period (1987–2003). While trip limits apply before and after the main season (to allow for incidental catch), during the main season there are no trip limits.

Pacific whiting were declared overfished in 2002—meaning that estimated biomass had fallen below 25% of estimated unfished biomass—but have rebounded recently. In 2004, the National Marine Fisheries Service declared the stock rebuilt, which means that current biomass is over 40% of unfished biomass. Although there are bycatch concerns (salmon and rockfish, in particular), trawlers operating off the coastal shelf can often net hake with little or no bycatch. Hence, we adopt a singleoutput stochastic production function in our analysis.

Our primary data source is the logbook information required of all vessels with federal limited-entry groundfish permits. Logbooks record information on each trip and tow, including species and estimated catch weight, gear used, location of fishing, and duration of tow. Supplementary data on boat characteristics were obtained from the Pacific Fisheries Information Network. In order to keep the analysis focused on the boats that form the core of the whiting fleet, we do not include boats that had fewer than 100 whiting trips during 1987–2003. We further limit attention to trips made during May-September, when trip limits are not in effect. This leaves a data set consisting of 10,865 whiting trips taken by an unbalanced panel of 41 boats. These data account for 82% of the shore-based fleet's whiting catch during 1987–2003.

In order to implement the marginal likelihood calculation, a proper prior must be chosen. Although the frontier estimates are in the form of input elasticities, we have decidedly diffuse information about the remaining quantities of interest. Hence, we adopt a weakly informative specification of the prior in equation (7), consisting of values  $s_{\sigma_0} = 0.1$ ,  $v_{\sigma_0} = 1$ ,  $\mathbf{\beta}_0 = \mathbf{0}_M \mathbf{C}_{\mathbf{\beta}_0} = 100 \times \mathbf{I}_M$ ,  $s_{\lambda_0} = 0.1$ ,  $v_{\lambda_0} = 1$ ,  $\mu_0 = 0$ ,  $v_{\mu_0} = 100$ , and for i = 1, 2, ..., N,  $s_{\omega_{i0}} = 0.1$  and  $v_{\omega_{i0}} = 1$ . Experiments with alternative priors suggest that inferences from the 10,865 observation sample are little affected by the choice of prior.

### **Empirical Model and Results**

The empirical specialization of equation (2) that we use in estimation is to transform the observed catch and covariates levels into natural logarithms. This yields a Cobb-Douglas form for the frontier. We also considered a production frontier in the Translog form, but found little difference in the marginal-likelihood values of the two models. It is important to recognize that any specialization of the frontier and the hierarchy will affect the magnitudes of the technical inefficiencies that are recorded, and the only way to investigate whether such alterations make inferences derived from a chosen specification fragile is to re-estimate the model and compute associated model diagnostics. A fairly detailed investigation prior to the current exercise suggests that the chosen specification performs better than other conventional specifications that a typical investigation might consider (Tomberlin, Irz, and Holloway 2006). Moreover, for our purposes, the focus is on the ranking of inefficiencies, so we opt for the parsimonious Cobb-Douglas form. Accordingly, coefficient estimates represent input elasticities. Table 2 presents summary statistics for the variables included in our preferred model. Table 3 presents point estimates (posterior means) of parameters for the model in which boats occupy the higher position in the hierarchy and years the lower. Horsepower and tow duration per trip have a clearly positive effect on catch, while crew size does not. Ratios of posterior means to standard deviations in the Gibbs sample are reported in parentheses. As we report in Tomberlin, Irz, and Holloway (2006), when we included exploitable biomass as a covariate, its coefficient is negative and significant, the opposite of the generally expected effect of biomass on catch. Two factors help to explain this result. The first is that Pacific hake school, and a boat's ability to catch them has much more to do with the avail-

 Table 2

 Summary Statistics for Model Data

	Mean	Std. Deviation	Min.	Max.
Catch (at-sea, in lbs.)	129,195	57,840	3	472,657
Horsepower	642	249	335	1800
Tow Hours (trip total)	4.0	2.9	0.0	29.5
Crew Size	3	0.3	1	6

Table 3 Results

Parameter	Posterior Mean Estimate (Mean/Standard Deviation)	
$\beta_1$ (Horsepower) <sup>1</sup>	0.53 (10.16)	
$\beta_2$ (Tow hours)	0.05 (5.78)	
$\beta_3$ (Crew size)	0.06 (0.67)	
β <sub>4</sub> 1987 Dummy Coefficient	8.30 (20.23)	
$\beta_5$ 1988 Dummy Coefficient	11.64 (19.93)	
$\beta_6$ 1989 Dummy Coefficient	12.47 (9.57)	
β <sub>7</sub> 1990 Dummy Coefficient	13.11 (34.14)	
β <sub>8</sub> 1991 Dummy Coefficient	12.52 (35.37)	
β <sub>9</sub> 1992 Dummy Coefficient	12.82 (38.03)	
$\beta_{10}$ 1993 Dummy Coefficient	13.20 (39.44)	
β <sub>11</sub> 1994 Dummy Coefficient	13.02 (38.86)	
$\beta_{12}$ 1995 Dummy Coefficient	13.22 (39.97)	
β <sub>13</sub> 1996 Dummy Coefficient	13.35 (38.93)	
β <sub>14</sub> 1997 Dummy Coefficient	13.34 (39.01)	
β <sub>15</sub> 1998 Dummy Coefficient	13.36 (38.69)	
$\beta_{16}$ 1999 Dummy Coefficient	13.39 (39.20)	
β <sub>17</sub> 2000 Dummy Coefficient	13.62 (40.63)	
β <sub>18</sub> 2001 Dummy Coefficient	13.58 (40.87)	
β <sub>19</sub> 2002 Dummy Coefficient	13.67 (42.02)	
$\beta_{20}$ 2003 Dummy Coefficient	13.74 (42.08)	
$\sigma$ (Sampling Error Std. Dev.)	0.52 (17.11)	
$\mu$ (Mean of $f^{iN}(z_i   \mu, \lambda)$ )	5.2 (169.47)	
$\lambda$ (Std. dev. of $f^{N}(z_i   \mu, \lambda)$ )	0.13 (5.93)	

 $^{1}\beta$  terms are elasticities (a Cobb-Douglas production function is assumed).

ability of a single school than with more regional measures of biomass. The second is that the biomass was declining quite steadily throughout our study period, when catch per trip was also increasing steadily, so that the biomass term essentially functioned as a time trend in our regression. As an alternative, we allow for the possibility of time-varying movement in the production frontier by including dummy variables for the years. Coefficient estimates for these dummy variables are also reported in table 3, and they show a significant upward trend. An alternative model with both biomass and year-specific dummy variables was deemed inadmissible due to extremely high levels of multicollinearity. We refer readers interested in our model selection and diagnostics to Tomberlin, Irz, and Holloway (2006).

Table 4 presents the probabilities that each of the boats in our study fleet is the most efficient. Column one reports the ranking of probabilities, *i*, across the 41 boats in the sample. Column two reports  $\wp_i$ , the computed probability that boat i has lowest inefficiency. Column three reports the mean inefficiency score,  $\mathbf{z}_i$ , derived from the Gibbs samples, and column four reports the associated standard deviation of the distribution,  $\omega_i$ . Column five reports the number of years that each boat was in operation, and column six reports the total number of trips,  $n_i$ , that the boat took. Comparing the second and third columns shows that in many cases boats with lower mean inefficiency scores (*i.e.*, more efficient boats by that measure) also have lower probabilities of being the most efficient. This effect can be seen, for example, by comparing Boats 4 and 5. Boat 4 has a higher estimated mean inefficiency score than does Boat 5 (implying that Boat 4 is less efficient by the usual comparison of means); but Boat 4 also has a significantly higher inefficiency standard deviation,  $\omega_i$ , resulting in a higher probability that it is the more efficient of the two. In all, 30 such cases exist in table 4. Hence, we conclude that rankings based on point estimates of mean inefficiency scores and rankings based on computed probabilities of being most efficient, which account for the standard deviation as well as the mean of the efficiency estimates, lead to distinctly different conclusions.

We include information on years of active fishing and number of trips because these are both factors that might be thought to have a positive effect on efficiency as managers learn from experience (see Coelli, Rao, and Battese (1998) for references to this line of research). In the context of fisheries, this idea is related to the 'goodcaptain hypothesis' that differences in catch between vessels are largely explained by differences in skipper skill (see Alvarez, Perez, and Schmidt (2003) for a review). While we have not carried out a formal analysis of possible relationships between these variables and the rankings reported in the second and third columns, we include these numbers to demonstrate that these relationships, if any, are not obvious. The most active and experienced boats can be found throughout the entire distribution of rankings, and it is definitely not the case that the boat with the most or the fewest trips is the one that is most or the least efficient. These observations suggest the potential for additional inquiry about the distribution of performance measures and the importance of learning by doing in the Pacific hake fishery.

## Conclusions

The Bayesian hierarchical approach to estimating technical efficiency in fisheries affords great flexibility in model specification and imposes only modest computational demands on the analyst. This paper exploits a further advantage of the estimation framework and the computational method used to implement it, which is that important additional information about industry performance can be derived as a byproduct of the estimation exercise. In our case, this information is the probability that a particular observational unit (here, a boat) has the highest likelihood of

	Ineffic	iency Kanking a	ind Boat-specific	mormation	
i	$\wp_i^{1}$	$z_i^2$	$\omega_i^{3}$	$T_i^4$	$n_i^5$
1	0.4987	4.96	0.2689	5	177
2	0.1489	5.01	0.2419	9	462
3	0.0776	5.09	0.3723	9	431
4	0.0508	5.10	0.2973	6	159
5	0.0452	5.08	0.1972	7	187
6	0.0362	5.07	0.2014	5	87
7	0.0329	5.17	1.7745	9	95
8	0.0197	5.21	2.1757	7	51
9	0.0140	5.21	1.8225	8	121
10	0.0136	5.18	0.3322	11	414
11	0.0132	5.21	0.8916	5	134
12	0.0085	5.15	0.2765	6	142
13	0.0070	5.17	0.2912	6	272
14	0.0044	5.17	0.2188	7	349
15	0.0032	5.19	0.2779	11	501
16	0.0027	5.19	0.2845	16	652
17	0.0025	5.21	0.2307	13	536
18	0.0022	5.27	0.5585	6	93
19	0.0020	5.23	0.2197	3	54
20	0.0020	5.24	0.4206	5	121
21	0.0018	5.29	0.3747	9	358
22	0.0015	5.31	0.5076	4	159
23	0.0013	5.23	0.2246	4	101
24	0.0011	5.26	0.2865	11	500
25	0.0010	5.25	0.2140	4	109
26	0.0010	5.18	0.1974	7	186
27	0.0010	5.22	0.3568	7	232
28	0.0009	5.26	0.2236	7	403
29	0.0008	5.3	0.3186	12	617
30	0.0007	5.23	0.2644	10	331
31	0.0007	5.18	0.2027	10	269
32	0.0006	5.21	0.2858	5	108
33	0.0006	5.26	0.3048	11	92
34	0.0006	5.25	0.2735	9	508
35	0.0005	5.28	0.2445	7	226
36	0.0004	5.25	0.1876	10	359
37	0.0002	5.26	0.1812	8	272
38	0.0002	5.27	0.2224	5	66
39	0.0001	5.21	0.1477	12	502
40	0.0000	5.21	0.1656	5	162
41	0.0000	5.29	0.2651	7	277

Table 4 Inefficiency Ranking and Roat-specific Information

<sup>1</sup> The probability that boat *i* is most efficient in the fleet,  $p(z_i \le z_J \forall j \ne i)$ . <sup>2</sup> Boat *i*'s mean (over years) inefficiency score. <sup>3</sup> The standard deviation of boat *i*'s yearly efficiency scores. <sup>4</sup> Number of years in which boat *i* is active. <sup>5</sup> Total trips made by boat *i* during 1987–2003.

being 'best' in the sense that it produces at minimum distance from a frontier which, although unobservable, can be estimated simultaneously along with the technical efficiency scores. We argue that efficiency rankings based on these probabilities may be more useful to industry observers than rankings based on point estimates of mean inefficiency estimates alone.

As Atkinson and Dorfman (2005) detail, a modest set of additional steps in the Gibbs sampling algorithm can be incorporated to derive probabilistic statements about many statistics of interest. For example, researchers may want to rank percent efficiency scores to test the sensitivity of rankings to vessel omission and to assess the probability that boats with different estimated efficiency scores are, in fact, different. Here, we have focused on the probability that a given boat is the least inefficient, which we believe provides a ranking with intuitive appeal and an instructive contrast to ranking by point estimates of mean inefficiency scores. Conducting this analysis within the context of a model that is hierarchical in the inefficiency scores has enabled us to examine the importance of heterogeneous inefficiency per se. However, our results are conditional on a model that ignores another likely source of heterogeneity, namely differences in production frontiers among boats. Because the location of the frontier affects the derivation of the inefficiency scores and the corresponding probabilities, we should be concerned about sample heterogeneity leading to different frontiers. In this regard, recent work by Tsionas (2003) makes compelling reading, and provides a platform for further developing Bayesian efficiency-in-fisheries analyses where the common-frontier assumption can be tested, and, if necessary, relaxed.

## References

- Afriat, S.N. 1972. Efficiency Estimation of Production Functions. *International Economic Review* 13:568–98.
- Ahmad, M., and B.E. Bravo-Ureta. 1996. Technical Efficiency Measures for Dairy Farms Using Panel Data: A Comparison of Alternative Model Specifications. *Journal of Productivity Analysis* 7(4):399–415.
- Aigner, D.J., and S.F. Chu. 1968. On Estimating the Industry Production Function. *American Economic Review* 58:826–39.
- Aigner D., K. Lovell, and P. Schmidt. 1977. Formulation and Estimation of Stochastic Frontier Models. *Journal of Econometrics* 6:21–37.
- Alvarez, A., 2001. Some Issues in the Estimation of Technical Efficiency in a Fishery. Efficiency paper series 2/2001, Department of Economics, University of Oviedo, Spain.
- Alvarez, A., L. Perez, and P. Schmidt. 2003. The Relative Importance of Luck and Technical Efficiency in a Fishery. Efficiency paper series 3/2003, Department of Economics, University of Oviedo, Spain.
- Atkinson, S.E., and J.H. Dorfman. 2005. Multiple Comparisons with the Best: Bayesian Precision Measures of Efficiency Rankings. *Journal of Productivity Analysis* 23:359–82.
- Battese, G., and T. Coelli. 1992. Frontier Production Functions, Technical Efficiency and Panel Data: with Application to Paddy Farmers in India. *Journal of Productivity Analysis* 3:153–69.
- \_\_\_\_. 1995. A Model for Technical Inefficiency Effects in a Stochastic Frontier Production Function for Panel Data. *Empirical Economics* 20:325–32.
- Bechhofer, R.E. 1954. A Single-Sample Multiple Decision Procedure for Ranking Means of Normal Populations With Known Variances. Annals of Mathematical Statistics 23:16–39.

- Berger, J.O., and J. Deely. 1988. A Bayesian Approach to Ranking and Selection of Related Means with Alternatives to Analysis-of-Variance Methodology. *Journal of the American Statistical Association* 83(402):364–73.
- Bjorndal, T., P. Kondouri, and S. Pascoe. 2002. Multi-output Distance Function for the North Sea Beam Trawl Fishery. Working paper, Department of Economics, University of Reading, England.
- Brooks, S.P., and A.E. Gelman. 1998. General Methods for Monitoring Convergence of Iterative Simulations. *Journal of Computational and Graphical Statistics* 7:434–55.
- Brooks, S.P., and P. Giudici. 2000. Markov Chain Monte Carlo Convergence Assessment via Two-Way Analysis of Variance. *Journal of Computational and Graphical Statistics* 9:266–85.
- Carlin, B., and S. Chib. 1995. Bayesian Model Choice via Markov Chain Monte Carlo. *Journal of the Royal Statistical Society: Series B* 57:473–84.
- Carlin, B., and N. Polson. 1991. Inference for Nonconjugate Bayesian Models Using Gibbs Sampling. *Canadian Journal of Statistics* 19:399–405.
- Casella, G., and E. George. 1992. Explaining the Gibbs Sampler. American Statistician 46(2):167–74.
- Chib, S. 1995. Marginal Likelihood from the Gibbs Output. *Journal of the American Statistical Association* 90:1313–21.
- Chib, S., and I. Jeliazkov. 2001. Marginal Likelihood from the Metropolis-Hastings Output. *Journal of the American Statistical Association* 96:270–81.
- Coelli, T. 1996. A Guide to Frontier 4.1: A Computer Program for Stochastic Frontier Production and Cost Function Estimation. Working paper 96/07, Center for Efficiency and Productivity Analysis, University of New England, Australia.
- Coelli, T., D.S.P. Rao, and G.E. Battese. 1998. An Introduction to Efficiency and Productivity Analysis. Boston, MA: Kluwer Academic Publishers.
- Dorfman, J.H., and G. Koop. 2005. Current Developments in Productivity and Efficiency Measurement. *Journal of Econometrics* (Special Issue on Current Developments in Productivity and Efficiency Measurement) 126(2):233–40.
- Dudewicz, E.J., and J.O. Koo. 1982. *The Complete Categorized Guide to Statistical Selection and Ranking Procedures*. Columbus, OH: American Science Press.
- Felthoven, R.G. 2002. Effects of the American Fisheries Act on Capacity Utilization and Technical Efficiency. *Marine Resource Economics* 17(3):181–205.
- Felthoven, R.G., and C.J. Morrison Paul. 2004. Directions for Productivity Measurement in Fisheries. *Marine Policy* 28:161–69.
- Fernandez, C., E. Ley, and M. Steel. 2002. Bayesian Modelling of Catch in a Northwest Atlantic Fishery. *Journal of the Royal Statistical Society: Series C (Applied Statistics)* 51:257–80.
- Fernandez, C., J. Osiewalski, and M.F.J. Steel. 1997. On the Use of Panel Data in Stochastic Frontier Models with Improper Priors. *Journal of Econometrics* 79:169–93.
- Fernandez-Gaucherand, E., S. Jain, H. Lee, A. Rao, and M. Rao. 1995. Improving Productivity by Periodic Performance Evaluation: A Bayesian Stochastic Model. *Management Science* 41(10):1669–78.
- Fong, D.K.H. 1992. Ranking and Estimation of Related Means in the Presence of a Covariate – A Bayesian Approach. Journal of the American Statistical Association 87(420):1128–36.
- Gelfand, A.E., and D.K. Dey. 1994. Bayesian Model Choice: Asymptotics and Exact Calculations. *Journal of the Royal Statistical Society: Series B* 56:501–14.
- Gelfand, A., and A. Smith. 1990. Sampling-Based Approaches to Calculating Marginal Densities. *Journal of the American Statistical Association* 85(3):972-85.

- Gelman, A., and D.B. Rubin. 1992. Inference from Iterative Simulations Using Multiple Sequences. *Statistical Science* 7:457–511.
- Geweke, J. 1992. Evaluating the Accuracy of Sampling-Based Approaches to the Calculation of Posterior Moments. Proceedings of the Fourth Valencia International Conference on Bayesian Statistics, J.M. Bernardo, J.O. Berger, A.P. Dawid, and A.F.M. Smith, eds., pp. 169–93. New York, NY: Oxford University Press.
- Gibbons, J.D., I. Olkin, and M. Sobel. 1977. Selecting and Ordering Populations. New York, NY: John Wiley.
- Grafton, R.Q., D. Squires, and K.J. Fox. 2000. Private Property and Economic Efficiency: A Study of a Common-Pool Resource. *Journal of Law and Economics* 43(2):679–713.
- Gupta, S.S. 1956. On a Decision Rule for a Problem in Ranking Means. University of North Carolina at Chapel Hill, Institute of Statistics. Unpublished PhD Thesis.
- Gupta, S.S., and S. Panchapakesan. 1979. *Multiple Decision Procedures*. New York, NY: John Wiley.
- Hannesson, R. 1983. Bioeconomic Production Function in Fisheries: Theoretical and Empirical Analysis. *Canadian Journal of Aquatic Sciences* 40:968–82.
- Herrero, I., and S. Pascoe. 2003. Value versus Volume in the Catch of the Spanish South-Atlantic Trawl Industry. *Journal of Agricultural Economics* 54(2):325–41.
- Holloway, G.J., and S.K. Ehui. 2002. The Simple Econometrics of Impact Assessment: Theory with an Application to Milk-Market Development in the Ethiopian Highlands. Paper presented at International Conference on Impacts of Agricultural Research and Development, San Jose, CA, February 4-7.
- Holloway, G.J., D. Tomberlin, and X. Irz. 2005. Hierarchical Analysis of Production Efficiency in a Coastal Trawl Fishery. *Simulation Methods in Environmental* and Natural Resource Economics, R. Scarpa and A. Alberini, eds., pp. 159–85. Boston/Dordrecht/London: Kluwer Academic Publisher.
- Horrace, W.C. 2005. On Ranking and Selection from Independent Truncated Normal Distributions. *Journal of Econometrics* 126(2):335–54.
- Jondrow, J., K. Lovell, I. Matterov, and P. Schmidt. 1982. On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Model. *Journal of Econometrics* August:233–38.
- Kirkley, J.E., D. Squires, and I.E. Strand. 1995. Assessing Technical Efficiency in Commercial Fisheries: The Mid-Atlantic Sea Scallop Fishery. *American Journal* of Agricultural Economics 77:686–97.
- Koop, G. 2003. Bayesian Econometrics. Chichester: Wiley.
- Koop, G., J. Osiewalski, and M.F.J. Steel. 1994. Bayesian Efficiency Analysis with a Flexible Cost Function. *Journal of Business and Economic Statistics* 12:93-106.
- \_\_\_\_. 1997. Bayesian Efficiency Analysis through Individual Effects: Hospital Cost Frontiers. *Journal of Econometrics* 76:77–105.
- Koop, G., M.F.J. Steel, and J. Osiewalski. 1995. Posterior Analysis of Stochastic Frontier Models Using Gibbs Sampling. *Journal of Computational Statistics*. 10:353–73.
- Kumbhakar, S.C., and C.A.K. Lovell. 2000. Stochastic Frontier Analysis. Cambridge: Cambridge University Press.
- Mengersen, R., and C. Guihenneuc-Jouyaux. 1999. MCMC Convergence Diagnostics: A "Review." *Bayesian Statistics* 6, J.O. Berger, J.M. Bernardo, A.P. Dawid, D.V. Lindley, and A.F.M. Smith, eds., 880 pp. Oxford: Oxford University Press.
- Murillo-Zamorano, L.R. 2004. Economic Efficiency and Frontier Techniques. Journal of Economic Surveys 18:33-78.

- Newey, W.K., and K.D. West. 1987. A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55:703–08.
- Newton, M.A., and A.E. Raftery. 1994. Approximate Bayesian Inference by the Weighted Likelihood Bootstrap (with discussion). *Journal of the Royal Statistical Society Series B* 56:3–48.
- Pascoe, S., and L. Coglan. 2002. The Contribution of Unmeasurable Inputs to Fisheries Production: An Analysis of Technical Efficiency of Fishing Vessels in the English Channel. American Journal of Agricultural Economics. 84(3):585–97.
- Pascoe, S., P. Hassaszahed, J. Anderson, and K. Korsbrekkes. 2003. Economic Versus Physical Input Measures in the Analysis of Technical Efficiency in Fisheries. *Applied Economics* 35:1699–710.
- Squires, D., and J. Kirkley. 1999. Skipper Skill and Panel Data in Fishing Industries. *Canadian Journal of Fisheries and Aquatic Sciences* 56(11):2011–18.
- Tomberlin, D., X. Irz, and G. Holloway. 2006. Bayesian Estimation of Technical Efficiency in the Pacific Hake Fishery. Fisheries Centre Working Paper #2006-22. Vancouver, BC, Canada: The University of British Columbia.
- Tsionas, E.G. 2003. Stochastic Frontier Models with Random Coefficients. *Journal* of Applied Econometrics 17:127–47.
- Zellner, A. 1996. An Introduction to Bayesian Inference in Econometrics. Wiley Classics Library Edition. New York, NY: John Wiley and Sons.