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Co-movements of international equity markets: a large-scale factor model approach

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Abstract

We analyze the comovements of a set of country-sector indexes from 45 different countries studying their factor decomposition based on a PCA analysis for a large cross section framework. We derive a measure to analyze the comovements over time based on the part of variance explained by the main extracted factors and we apply the method from Bai and Ng to study the relevant number of factors. We conduct rolling estimations for the period 1994-2006 focusing on the set of emerging markets. We show that both, emerging and developed equity markets experienced increasing comovements over the period of study, reflecting the integration of those markets. We have estimated that the main factor accounts for 30\% and 20\% of the whole variation of each data set. We use the comovements to gauge integration in two different ways, both indicating higher integration for developed markets. Finally, we relate the comovements to a measure of diversification and we conclude that it is only possible to reduce 85\% of the average risk of an equity index by diversification at the end of the period compared to 95\% at the beginning for the set of emerging markets.

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1. Introduction

The analysis of comovements is essential in the context of portfolio theory as it has a direct impact on the risk reduction potential by means of diversification. The interest in studying comovements across assets in finance comes from this interest in constructing better portfolios in a mean-variance framework. Here, we analyze the comovements within a set of international country-sector indexes over time and we study their relation to financial integration and diversification. Many documents present results about comovements among developed markets but only a few analyze the comovements among emerging markets or the comovements between emerging and developed markets¹. Our study gives particular attention to those markets pointing out their specific behavior, we also use actual techniques of factor analysis mainly used in macroeconomic studies.

Our work lies within different strands of the literature. First of all, it lies within the studies of comovements, largely explored by academics and practitioners, specially for advanced markets. Longin and Solnik (1995) and Goetzmann et al. (2005) analyze correlations of different developed markets and study their stability across time concluding that correlations are time-varying and are rising during periods of high integration and higher volatility. From our analysis, we find that the level of comovements increases during the whole period of analysis and in a sharper way for the set of developed markets.

Second, this study is based on the factor models literature initiated by Chamberlain and Rothschild (1983) and Stock and Watson (1988). To analyze the structure of comovements, we rely on a factor decomposition of returns. We suggest a dynamic measure capturing the intensity of the comovements based on the part of variance explained by orthogonal factors extracted with principal components analysis (PCA) and estimated with a rolling procedure. The idea, derived from the APT literature², is that markets are ruled by common factors and as comovements become stronger, specific risk and diversification potential get lower. We use a data set with a large number of variables and observations, therefore we estimate the factors as in Connor and Korajczyk (1986) and we can use the inference results developed by Stock and Watson (1998), Bai (2003) and Bai and Ng (2003). Thus, the principal components can estimate consistently the factor space spanned by the true factors. We can therefore use information criteria to estimate the number of factors consistently as in Bai and Ng (2002) and we are able to build confidence intervals around the true factors in order to find if an observed factor lies within them, and consequently give an interpretation to the estimated factors.

Third, we analyze the comovements with the perspective of measuring integration and diversification. The use of comovement measures to capture integration has been largely debated without a definite response³. We argue that in a large data setting, comovements could be used to gauge financial integration. Extending the analysis to the international context, the comovements would be impacted not only by the features of the assets and the markets to which they belong, but also by the barriers existing between different markets. Here, we suggest different approaches to measure integration based on comovements and we conclude that the level of financial integration has increased during the period. The relation between comovements and diversification has been analyzed through the diversification ratio (DR) defined in Solnik (1974). We show graphically that both measures are closely related and therefore we can conclude that the potential of diversification has been reduced over the period.

¹Some examples are: Bekaert and Harvey (1995), Carrieri et al. (2005) or Chambet and Gibson (2008)

²See Ross (1976) or Burmeister and McElroy (1988)

³see for example the work of Bekaert and Harvey (1995) or Errunza et al. (1999).

2. Methodology

2.1. Factor Model and Comovements

We present a random variable X that follows a factor model representation. Let us note X_{it} the excess returns of an index i , $1 \leq i \leq N$, at time t , $1 \leq t \leq T$. The set of these returns obeys a factor model with K (unobserved) factors F_{kt} if:

$$X_{it} = \sum_{k=1}^K \Lambda_{ik} F_{kt} + \varepsilon_{it} \quad (1)$$

where ε_{it} represents the idiosyncratic component, Λ_{ik} the sensitivity (loadings) of return i to factor k and the idiosyncratic term are such that: $E[\varepsilon_t] = 0$ and $V[\varepsilon_t] = \Sigma_\varepsilon$. This relation can be rewritten as $X = F\Lambda' + \varepsilon$. where $X = (X'_1, \dots, X'_T)'$ with $X_t = (X_{1t}, \dots, X_{Nt})'$ denoting the N -dimensional vector of the N random excess returns X_{it} . $\Lambda = (\Lambda_1, \dots, \Lambda_N)'$ is the $N \times K$ matrix of factor loadings (with $\Lambda_i = (\Lambda_{i1}, \dots, \Lambda_{iK})'$) and $F = (F_1, \dots, F_T)'$ (with $F_t = (F_{1t}, \dots, F_{Kt})'$) are the K factors.

The use of models with large N and T simplifies the procedure of estimation of the factors and loadings traditionally achieved with maximum likelihood estimation and where normality assumptions for the errors are required. This specification also guarantees the convergence of the estimated factors to the true factors.

The asymptotic estimation within large panel settings have been rewritten in Bai and Ng (2007). From a standardized matrix Z , calculated from X , the factors \hat{F}^k and the loading $\hat{\Lambda}^k$ are estimated with a principal components method such that:

$$N > T : \hat{F}^k \text{ is } \sqrt{T} \text{ times the } k \text{ largest eigenvectors of the } T \times T \text{ matrix } ZZ' \text{ and } \hat{\Lambda}^k = T^{-1} \hat{F}^k' Z \quad (2)$$

$$N < T : \hat{\Lambda}^k \text{ is } \sqrt{N} \text{ times the } k \text{ largest eigenvectors of the } N \times N \text{ matrix } Z'Z \text{ and } \hat{F}^k = N^{-1} Z \hat{\Lambda}^k \quad (3)$$

Another important issue of multivariate factor models is to determine the relevant number of factors. A recent paper from Bai and Ng (2002) presents some panel based information criteria to determine the number of factors in the case of approximate factor models in large N settings.

Based on a k -factor model, they have studied the overall sum of the squared residuals

$$V_k = \frac{1}{NT} \sum_{t=1}^T e_t e_t'$$

where $e_t = Z_t - \Lambda F_t$ and suggested several information criteria $IC(k)$, of the form:

$$IC(k) = Ln(V(k)) + kg(N, T)$$

where $g(N, T)$ is a penalty function. The estimated number of common factors \hat{k} will be the number k that minimize the information criterium. We use two information criteria $IC_1(k)$ and $IC_2(k)$ presented by Bai and Ng:

$$IC_1(k) = \ln V(k) + k \log\left(\frac{NT}{N+T}\right) \frac{N+T}{NT} \quad (4)$$

$$IC_2(k) = \ln V(k) + k \log\left(\frac{\min(N, T)}{N+T}\right) \frac{N+T}{NT} \quad (5)$$

We evaluate the number of factors with different sizes of data sets, and we combine these results with the measures of comovements. We measure the comovements based on the main k factors. The comovements at time t , are calculated for a matrix with W observations⁴, as:

$$C_t^{1:k} = \frac{\sum_{i=1}^k l_i}{\sum_{i=1}^N l_i} \quad (6)$$

where l_i corresponds to the i^{th} eigenvalue of the sample correlation matrix. This measure indicates the intensity of markets comovements and represents the part of variance explained by the k first common factors over the set of standardized variables. We can also define a measure accounting only for the comovements explained by one of those factors as:

$$C_t^k = \frac{l_k}{\sum_{i=1}^N l_i} \quad (7)$$

After analyzing the contribution of the different number of factors, we conclude that the main factor is responsible for fundamental comovements and therefore we focus on it for the rest of the analysis. We compute the measure C_t^1 of comovements that we note C_t for simplicity.

$$C_t = \frac{l_1}{\sum_{i=1}^N l_i} \quad (8)$$

C_t is equivalent to the average squared correlation of each of the N variables with the estimated factor \widehat{F}_{1t} . We have,

$$C_t = \frac{1}{N} \sum_{i=1}^N \widehat{r}^2(Z_{it}, \widehat{F}_{1t})$$

with

$$\widehat{r}(Z_{it}, \widehat{F}_{1t}) = \frac{\sum_{s=t-W}^t Z_{is} \widehat{F}_{1s}}{\sqrt{\sum_{s=t-W}^t Z_{is}^2 \sum_{s=t-W}^t \widehat{F}_{1s}^2}}$$

where W denotes the number of observations in the window and \widehat{F}_{1t} is the main factor.

We construct confidence intervals for the eigenvalues of the correlation matrix to be able to compare different levels of C_t based on the asymptotic results Saporta (2003) for PCA with standardized variables. He has defined a confidence interval for the eigenvalues of the correlation matrix using the limiting distribution of $\sqrt{N}(\log(\widehat{\lambda}_i) - \log(\lambda_i))$ defined in Anderson (1963). The interval applied to the measure C_t^k is:

$$\widehat{C}_t^k \exp(-1.96 \sqrt{\frac{2}{N-1}}) < C_t^k < \widehat{C}_t^k \exp(1.96 \sqrt{\frac{2}{N-1}}) \quad (9)$$

Many authors have reported similar measures of comovements without analyzing them over time or including confidence intervals allowing to compare the levels from different periods.

2.2. Market Integration

A disadvantage of factor analysis is the difficulty to interpret the factors. The first factor has been interpreted by Heston et al. (1995) as a market factor closely related to an

⁴from the date $(t - W)$ to the date t

equally weighted index (EWI) composed of all N assets. We pushed further this analysis using the method of Bai and Ng (2006) to build a confidence interval around the true factor and to evaluate if an observed factor lies within its bounds⁵. The resulting interval for an observed variable G has the following form:

$$\widehat{G} + \widehat{\varepsilon}_{jt} - 1.96\widehat{s}_{jt} < G < \widehat{G} + \widehat{\varepsilon}_{jt} + 1.96\widehat{s}_{jt} \quad (10)$$

Where $\widehat{\varepsilon}_{jt}$ is the error term from the regression of the observed variable G on the estimated factors F and \widehat{s}_{jt} is the asymptotic variance of this error term.

If the EWI lies within the confidence interval for the first true factor, we can conclude that the first factor represents a market factor which is close to the EWI.

With the first factor representing a market factor it makes sense to interpret C_t as a measure of integration. We used two different approaches to measure integration. First, a region would be considered more integrated if C_t is higher, we analyze different regions and compare the results. Second, we measure integration to global markets⁶. We calculate the measure of comovements explained by the main factor of the region, noted C_t^{F1R} . We compare it with the variance explained by the first factor extracted from the whole set of variables noted C_t^{F1G} on the set of variables from each group (emerging - developed). We compare the importance of a global factor compared to a regional factor.

2.3. Diversification

As the level of comovements arises, less opportunities of diversification are left for the investor. Comovements are therefore inversely related to diversification. Solnik (1974) has introduced a measure capturing the part of risk that cannot be reduced by diversification, defined as the ratio of the average sample variance of a portfolio of n assets over the sample variance of a typical security⁷. The diversification ratio DR_t , for a set of n variables x_{it} representing stocks or indexes returns, is defined as:

$$DR_t(n) = \frac{\overline{Var(\sum_{i=1}^n \frac{x_{it}}{n})}}{\frac{1}{n} \sum_{i=1}^n Var(x_{it})} \quad (11)$$

We show that C_t is closely related to this measure DR_t in the sense that it lies within the confidence interval of C_t .

3. Data Description and Empirical Results

3.1. Data Description

We used Datastream⁸ global country-sector indexes in US Dollars for 10 sectors and 46 countries. The period of study is 1994:07-2006:06. We decomposed the set of countries into developed and emerging markets and distinguished different regions within emerging markets, we present the details of the different countries, regions and sectors included in the data set in Tables ?? and ??.

Log returns were calculated from the price series at weekly horizons. We have analyzed some descriptive statistics for the set of country-sector indexes. The results are summarized by regions in Table ??. For the set of developed countries, the average an-

⁵The details of the method are found in the appendix.

⁶This approach used in other integration studies is similar to international asset pricing models (i.e. CAPM) based on an observed global factor.

⁷The average variance of a single stock.

⁸Datastream International Ltd. 09/07. We used their total return market indexes as Datastream use the same methodology, include at least 80% of each market and has large coverage on emerging markets.

nualized return is 8.2% and the average annualized volatility is 19.3%. The figures for the set of Emerging markets present lower levels of returns for the considered period and higher volatility. These statistics for this set of markets are 6.1% and 27.5%.

We implemented a rolling estimation procedure from the set of normalized returns, as we have series with very different levels of variance. We have used a rolling window of size W equal to one year.

3.2. Empirical results and analysis

3.2.1. Factor Model and Comovements

We analyzed the number of common factors within each data set with Bai and Ng's method. We considered rolling estimations because the inclusion of very distant past information could be misleading⁹. We have calculated the two criteria presented in section 2.1. but we only report the results for IC2¹⁰. We observed that the number of factors is different for each zone, it ranges from 1 to 4 for the rolling estimations with a window length of one year. The criteria from Bai and Ng are based on the minimization of the percentage of variance explained by a given number of factors, therefore it would be sound to analyze jointly the number of factors and measure of comovements. We analyze it for different number of factors. We perform this analysis for emerging and developed data sets separately. Figure ?? presents the combined analysis.

For the set of developed markets, the first factor ($\widehat{F1}$) explains on average nearly 30% of the variance of the whole data set. We observe a positive trend in the measure of comovements explained by $\widehat{F1}$, from a level of 20% at the beginning of the period to a level of 40% at the end. For emerging markets, $\widehat{F1}$ explains on average 16% varying from 12% to 24%. Regressing the measures C_t for emerging and developed countries over a trend confirms this observation, both trend coefficients are significant and positive but the coefficient for developed markets is higher and is more significant¹¹.

Looking closer to Figure ??, we observe different behavior of the variance explained by the first and second factors in emerging or developed markets. The cumulative variance explained by the factors should explain the whole variability of the data set when all the factors are included. In some way, when C_t^1 increases, the part explained by the rest of factors should decrease. This explains why for developed markets, during some short periods, the measure C_t^1 drops when C_t^2 (and even C_t^3) rises. Another explanation for the presence of the second or third factors should be the temporary different behavior of a subset of markets. This second argument seems to be plausible to explain the behavior of the second factor for the emerging markets data set during the years 1998 and the set of developed markets during 2001. We relate the peaks of C_t^1 with downturns of the market and, in the same way, comovements' drops should correspond to bullish phases of the market¹². For the set of developed markets, we observe three peaks (in 1999, 2002-2003 and 2005) and three minimums (in 1996, 2000 and 2004). The years 1997-1998 correspond to the presence of a relevant second factor certainly related to the asian crisis, the same as the year 2001 for developed countries where the second factor is undoubtedly associated to specific factors related to the technological bubble.

The second row of graphics in Figure ?? presents the optimal number of factors

⁹We reported the results for window sizes of one year length.

¹⁰Both criteria give similar results, we present the results for the IC2 criterium which gives a more stable number of factors.

¹¹The results are presented in table ??

¹²When investors are more confident and there is less variability in the equity market.

calculated for the same windows with the method of Bai and Ng. During periods where the second or third factors account for an important part of the variance, the criteria suggest to use two or three factors, otherwise only one factor is relevant. The second and third factors reveal specific phases of the market and are therefore important to be included but only during some short periods. We observe a lower level in the comovements of emerging markets but also a lower number of factors according to the applied criteria. For developed markets, the number of factors is very unstable and difficult to interpret economically, probably the IT bubble has an effect on the larger number of factors.

We decide to focus on the first factor for many reasons. First, we observed that only the first factor drives the major dynamics in comovements; second, the number of factors is very unstable for developed markets making difficult to interpret them economically and finally, the results for emerging markets suggest to use only one factor for the later period.

3.2.2. Market Integration

We used two approaches to quantify how integrated are the different sets of markets based on the measure C_t . Before that, we test empirically if the first factor could be interpreted as an EWI composed of the indexes in the data set calculating a 95% confidence interval for the true factor according to equation 10. The results of this analysis are presented in Figure ?? for the two sets of markets. We observe that the EWI lies within the confidence interval for the whole period of study for rolling windows of two years. We conclude that $F1$ can be interpreted as the EWI of the data set.

The first approach is to test if there is an increase of C_t for the different data sets. This would reflect a process of integration for those markets. In Figure ??, we present the measure C_t for the two main sets of markets: developed and emerging. In order to compare these two measures and their changes over time, we have built confidence intervals around C_t as presented in equation 9. We observe in Figure ?? that at the beginning of the period, these two measures are not significantly different but after 2000 the difference is significantly high. It seems possible that the introduction of the Euro was responsible for this increase in comovements¹³. Also, we observe that only the set of developed countries presents a significant increase in integration over the period.

Second, we compare if a set of variables is better explained by a global factor or a regional factor. If the part of variance explained by the global factor is small compared to the part of variance explained by the regional factor, then the set of markets are segmented from the global market. We compare C_t^{F1R} and C_t^{F1G} for the sets of developed and emerging markets, and we also look within emerging markets if the group of Latin American (LA) markets or the group of Asian (EAS) countries present some increase in their comovements with global or regional factors that could be related to a higher level of integration. The results are presented in Figure ?. For each data set, the regional factor accounts for a larger part of the variance than the global factor. Only for the set of developed markets, we observe that the two measures are very similar. This result suggests that developed markets are integrated to the global factors but this is not the case for the group of emerging markets. Focusing on emerging regions, only the group of asian countries presents an upward trend in the comovements, suggesting that they have become more integrated to the global factors. At the beginning of the period, the part of variance explained by C_t^{F1G} for the set of LA markets is insignificant compared to C_t^{F1R} , this is a sign of segmentation before 1997. We have also reported correlation

¹³The European (euro zone) countries represent an important part of the developed world, and as we do not take into account size effects, their importance is even higher.

coefficients between the factors C_t^{F1G} and C_t^{F1R} for each figure. For the set of developed markets, the two factors are highly correlated. This coefficient combined with the analysis of C_t^{F1R} and C_t^{F1G} over time give important information for asset pricing applications.

3.2.3. Diversification

The last important feature about comovements is their relation to diversification. We have computed the ratio DR_t described in equation 11 and we have compared it to the measure C_t . Figure ?? shows that both measures are closely related. Both measures are defined between zero and one, where one indicates the highest level of comovements and no diversification opportunities; and zero indicates the opposite. The diversification ratio captures the part of variance that can not be diversified away, which should correspond to the part of variance explained by the common factors or the market factor. The measure of comovements then captures the dynamics of the potential diversification¹⁴.

The diversification ratio in Solnik (1974) was calculated for a limited size of portfolios, here we calculate this ratio over a portfolio of the largest possible size, so we can interpret this measure of diversification as the risk reduction limit available from our data set. We observe that for emerging countries the DR_t value is 0.06 at the beginning of the period and 0.14 at the end, and it takes a value close to 0.18 at the beginning compared to 0.3 at the end for developed ones. This means that there are fewer diversification opportunities within developed than within emerging markets and that for both kinds of markets the diversification potential has been reduced with time. We concluded that for the recent period 70% of the average risk of a security within the set of developed countries could be reduced by diversification and 86% within emerging ones. We observe that the diversification ratio lies within the confidence intervals calculated for C_t for the set of developed markets and it is close to the lower confidence band for the emerging markets data set.

4. Discussion and Conclusions

We have analyzed the comovements of a global set of equity markets for a 12 years period based on a factor decomposition in a large data setting. We have observed an upward trend in the global comovements and also that this trend is stronger for developed than emerging countries. We have analyzed the measures of comovements considering them as proxies for the levels of integration. First, we have shown that the level of integration is significantly higher for the set of developed markets and that integration has increased for these markets during the period. Second, we have analyzed the effects of global and regional factors as indications of integration concluding that only the set of developed markets are integrated to global markets. Finally, we have shown that our measure of comovements is closely related to a measure of diversification which is very important in the context of portfolio construction. The measure of comovements can therefore be interpreted as a indicator of potential diversification. Many extensions of this analysis are possible. First, the analysis of comovements and the number of factors is a descriptive analysis of the past and for any portfolio application it would be relevant to estimate the future level of comovements and the future number of factors to use in a model. Second, we have analyzed the comovements for a global set of indexes and we have identified periods of high comovements and fewer diversification opportunities and periods of low comovements and higher opportunities of diversification. It would be interesting for future analysis to see how to take this information into account for the management of equity portfolios, to build portfolio strategies based on the different levels of global comovements.

¹⁴Regressing DR on C_t and a constant results in coefficients close to 1 significant at 0.1%

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Appendix

1 Confidence Intervals calculation

(From Bai and Ng (2006))

Based on a factor representation for the data X_{it} , such that:

$$X_{it} = \sum_{k=1}^K \Lambda_{ik} F_{kt} + e_{it}$$

where F_{kt} is an unobserved factor, Λ_{ik} is the sensitivity of asset i to factor k (or factor loading) and e_{it} is an error term.

Factors and loadings are estimated with the method of principal components. The estimated variables are denoted \tilde{F}_{kt} and $\tilde{\Lambda}_{ik}$. We can write $\tilde{\Lambda} = (\tilde{\Lambda}_1, \dots, \tilde{\Lambda}_N)'$ as the matrix of factor loadings and $\tilde{F} = (\tilde{F}_1, \dots, \tilde{F}_T)$ the matrix of estimated factors. Then:

$$\tilde{e}_{it} = X_{it} - \tilde{\Lambda}'_i \tilde{F}_t$$

Letting $\lambda = (\lambda_1, \dots, \lambda_N)$ be the vector of eigenvalues of the matrix $\frac{X'X}{NT}$, $\tilde{V} = \text{diag}(\lambda^k)$ is defined as the diagonal matrix containing the k largest eigenvalues.

The matrix $\tilde{\Gamma}_t$ is a covariance estimator defined as:

$$\tilde{\Gamma}_t = \frac{1}{N} \sum_{i=1}^N \tilde{e}_{it}^2 \tilde{\Lambda}_i \tilde{\Lambda}'_i$$

when e_{it} is cross-sectionally uncorrelated. If $E[e_{it}^2] = \sigma_e^2, \forall t$ and $\forall i$, then $\tilde{\Gamma}_t = \hat{\sigma}_e^2 \frac{\tilde{\Lambda}'\tilde{\Lambda}}{N}$ where $\hat{\sigma}_e^2 = \frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T \tilde{e}_{it}^2$

The observed variable is denoted G_t . And the OLS estimate from the regression of G_{jt} on F_t is:

$$\hat{G}_{jt} = \hat{\gamma}'_j \tilde{F}_t$$

Letting the measurement error be $\hat{\varepsilon}_{jt} = G_{jt} - \hat{G}_{jt}$, can be defined the term s_{jt}^2 estimated as:

$$\hat{s}_{jt}^2 = \frac{1}{T} \tilde{F}_t' \tilde{F}_t \hat{\sigma}_\varepsilon^2(j) + \frac{1}{N} \widehat{Avar}(\hat{G}_{jt})$$

when ε_{jt} is conditionally homoscedastic.

The term $\widehat{Avar}(\hat{G}_{jt})$ can be estimated as:

$$\widehat{Avar}(\hat{G}_{jt}) = \hat{\gamma}'_j \tilde{V}^{-1} \tilde{\Gamma}_t \tilde{V}^{-1} \hat{\gamma}'_j$$

And finally, the confidence intervals at a 95% confidence level are defined as:

$$CI : [\hat{\varepsilon}_{jt} - 1.96\hat{s}_{jt}, \hat{\varepsilon}_{jt} + 1.96\hat{s}_{jt}]$$

Tables and Figures

Table I: Countries and Sectors

Note: AM: Americas; AS: Asia; EUP: Europe; EUR: Euro Zone; EAS: Emerging Asia; EEMEA: Emerging Europe Middle East and Africa; LA: Latin America; Others: Other Emerging countries.
 NB: Number of series per country; NB-R: Number of series per region; NB-T: Number of series per type of market.

| COUNTRY | CODE | TYPE | REGION | BASIC | CYCGD | CYSER | GENIN | ITECH | NCYCG | NCYSR | RESOR | TOTLF | UTLS | NB | NB-R | NB-T |
|----------------|------|------|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------|----|------|------|
| CANADA | CN | DM | AM | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | 20 | 204 |
| U.S.A | US | DM | AM | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | | |
| AUSTRALIA | AU | DM | AS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | 47 | |
| HONG.KONG | HK | DM | AS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | | |
| JAPAN | JP | DM | AS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | | |
| NEW.ZEALAND | NZ | DM | AS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| SINGAPORE | SG | DM | AS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| DENMARK | DK | DM | EUP | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | 44 | |
| NORWAY | NW | DM | EUP | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| SWEDEN | SD | DM | EUP | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | | |
| SWITZERLAND | SW | DM | EUP | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| UNITED.KINGDOM | UK | DM | EUP | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | | |
| AUSTRIA | OE | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | 93 | |
| BELGIUM | BG | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | | |
| FINLAND | FN | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| FRANCE | FR | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| GERMANY | BD | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| GREECE | GR | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | | |
| IRELAND | IR | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | | |
| ITALY | IT | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | | |
| NETHERLANDS | NL | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| PORTUGAL | PT | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | | |
| SPAIN | ES | DM | EUR | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| CHINA | CH | EM | EAS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | 72 | 172 |
| INDIA | IN | EM | EAS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | | |
| INDONESIA | ID | EM | EAS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 4 | | |
| KOREA | KO | EM | EAS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| MALAYSIA | MY | EM | EAS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| PAKISTAN | PK | EM | EAS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| PHILIPPINES | PH | EM | EAS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | | |
| TAIWAN | TA | EM | EAS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | | |
| THAILAND | TH | EM | EAS | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | | |
| CZECH.REPUBLIC | CZ | EM | EEMEA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | 51 | |
| HUNGARY | HN | EM | EEMEA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | | |
| ISRAEL | IS | EM | EEMEA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | | |
| POLAND | PO | EM | EEMEA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | | |
| RUSSIA | RS | EM | EEMEA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 6 | | |
| SOUTH.AFRICA | SA | EM | EEMEA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | | |
| TURKEY | TK | EM | EEMEA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 10 | | |
| ARGENTINA | AR | EM | LA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 9 | 49 | |
| BRAZIL | BR | EM | LA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7 | | |
| CHILE | CL | EM | LA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | | |
| COLOMBIA | CB | EM | LA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 6 | | |
| MEXICO | MX | EM | LA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | | |
| PERU | PE | EM | LA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 8 | | |
| VENEZUELA | VE | EM | LA | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 3 | | |
| TOTAL | | | | | | | | | | | | | | | 376 | |

Table II: Sector codes and names

| Sector Code | Sector Name | |
|-------------|-----------------------------|----------------------------------------|
| BASIC | Basic Industries | (Chemicals, construction) |
| CYCGD | Cyclical Consumer Goods | (Automobiles, Household, Textiles etc) |
| CYSER | Cyclical Service | (Leisure, media, transportation) |
| GENIN | General Industrials | (Aerospace, Industrials, machinery) |
| ITECH | Information Technology | (Hardware, Software) |
| NCYCG | Non Cyclical Consumer Goods | (Food, Beverage etc) |
| NCYSR | Non Cyclical Services | (Food retailers, telecom services) |
| RESOR | Resources | (Mining, Oil and Gas) |
| TOTLF | Financials | (Banks, Insurance, Investments) |
| UTILS | Utilities | (Electricity, Gas, Water) |

Table III: Descriptive Statistics

Note: AM: Americas; AS: Asia; EUP: Europe; EAS: Emerging Asia; EEU: Emerging Europe; LA: Latin America; Others: Other Emerging countries.

Mean: Average annualized return; Std: Average annualized volatility; Skew: Skewness coefficient; Kurt: Kurtosis coefficient; AC1-AC4: Autocorrelation coefficient for lags 1 to 4; ADF: Augmented Dickey-Fuller Test of Stationarity.

| | Mean | Std | Skew | Kurt | AC1 | AC2 | AC3 | AC4 | ADF |
|------------------|--------|--------|---------|-------|---------|-------|-------|---------|----------|
| AM | 10.52% | 16.11% | - 0.340 | 1.532 | - 0.026 | 0.003 | 0.051 | 0.028 | - 15.087 |
| AS | 3.46% | 19.42% | - 0.290 | 1.301 | 0.040 | 0.044 | 0.038 | - 0.012 | - 15.364 |
| EU | 9.36% | 19.70% | - 0.520 | 2.353 | - 0.059 | 0.033 | 0.042 | 0.004 | - 17.135 |
| <i>DEVELOPED</i> | 8.18% | 19.32% | - 0.454 | 2.052 | - 0.035 | 0.033 | 0.042 | 0.003 | - 16.572 |
| EAS | 0.56% | 32.06% | - 0.278 | 3.189 | 0.027 | 0.076 | 0.068 | - 0.006 | - 12.393 |
| EEU | 13.09% | 35.67% | - 0.286 | 1.673 | 0.025 | 0.065 | 0.094 | - 0.018 | - 15.424 |
| LA | 2.67% | 28.89% | - 0.355 | 3.942 | 0.030 | 0.064 | 0.056 | 0.018 | - 14.135 |
| Others | 7.85% | 23.61% | - 0.632 | 2.068 | 0.005 | 0.065 | 0.050 | 0.003 | - 18.236 |
| <i>EMERGING</i> | 6.12% | 27.49% | - 0.390 | 2.817 | 0.000 | 0.058 | 0.061 | 0.000 | - 14.773 |

Figure 1: Percentage of Variance explained by global and regional F1

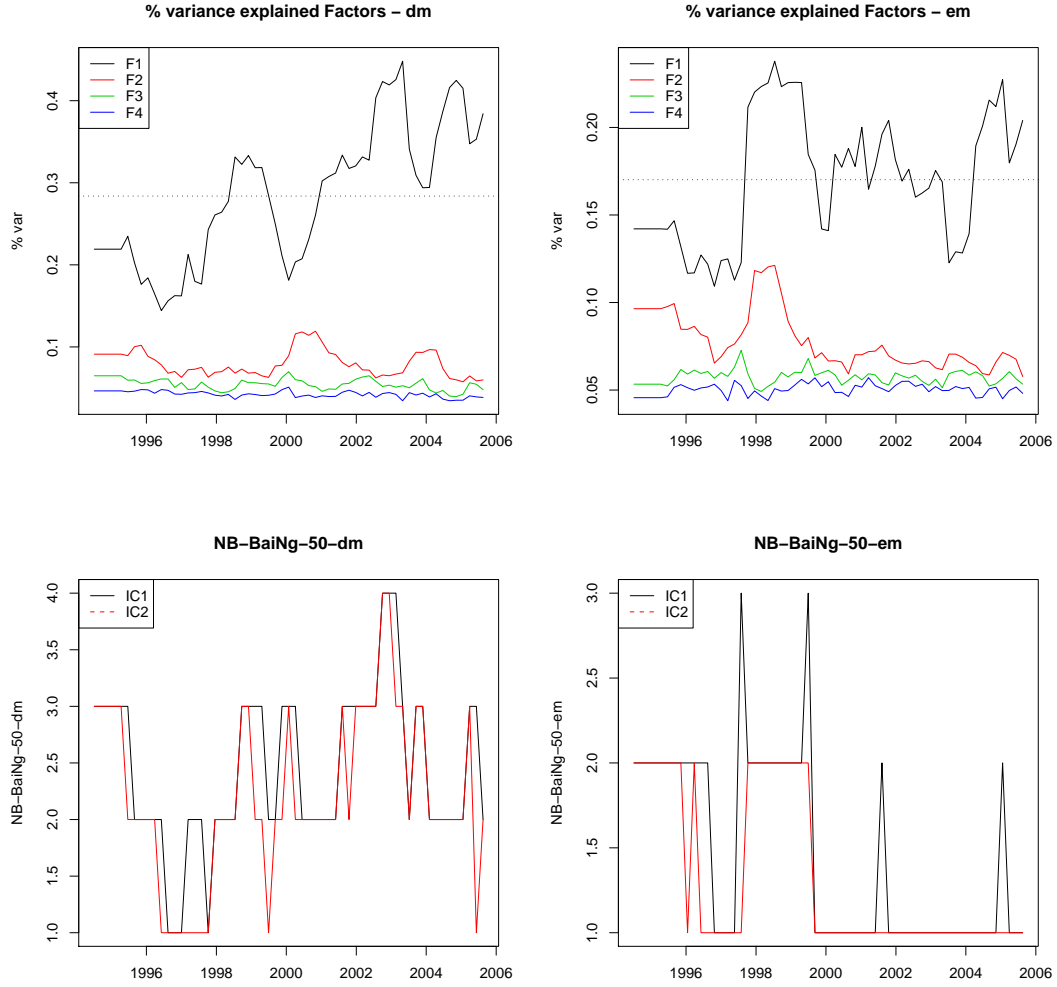


Table IV: Time trend regressions

Note: Standard errors in parentheses
 *** and ** indicate 0.1% and 1% significance

| | I_t -DM | I_t -EM |
|-------------|-------------------|-------------------|
| (Intercept) | 0.17*** (0.01) | 0.15*** (0.01) |
| Trend | 0.39*** (0.04) | 0.08** (0.03) |
| N | 59 | 59 |
| R^2 | 0.64 | 0.14 |
| adj. R^2 | 0.63 | 0.13 |
| Resid. sd | 0.05 | 0.03 |

Figure 2: EWI vs F1

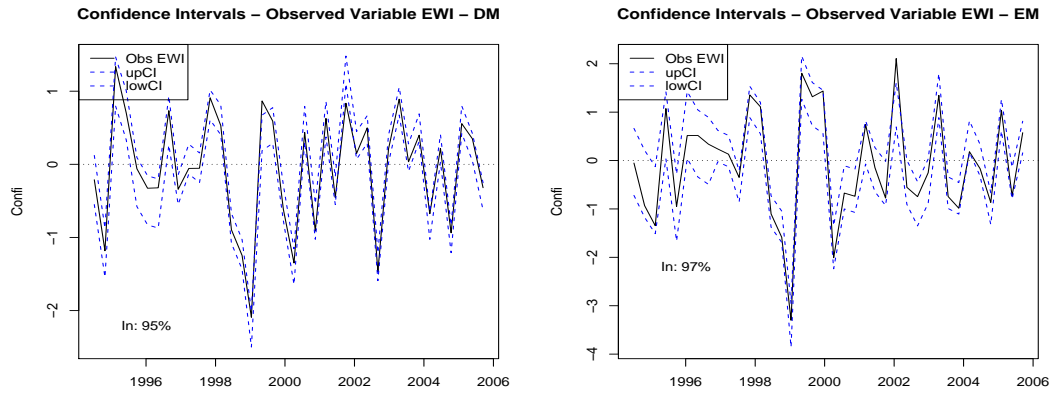


Figure 3: Comovements and Confidence intervals DM-EM

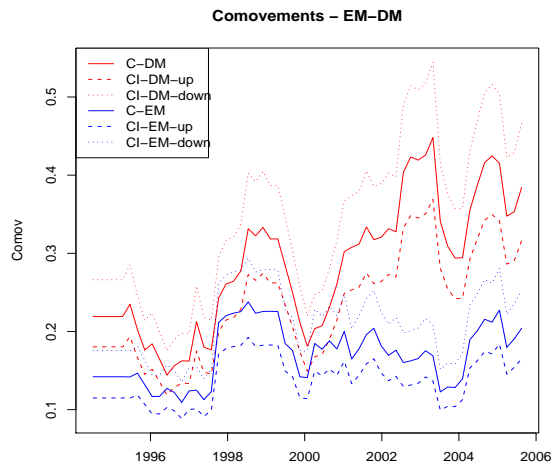


Figure 4: Percentage of Variance explained

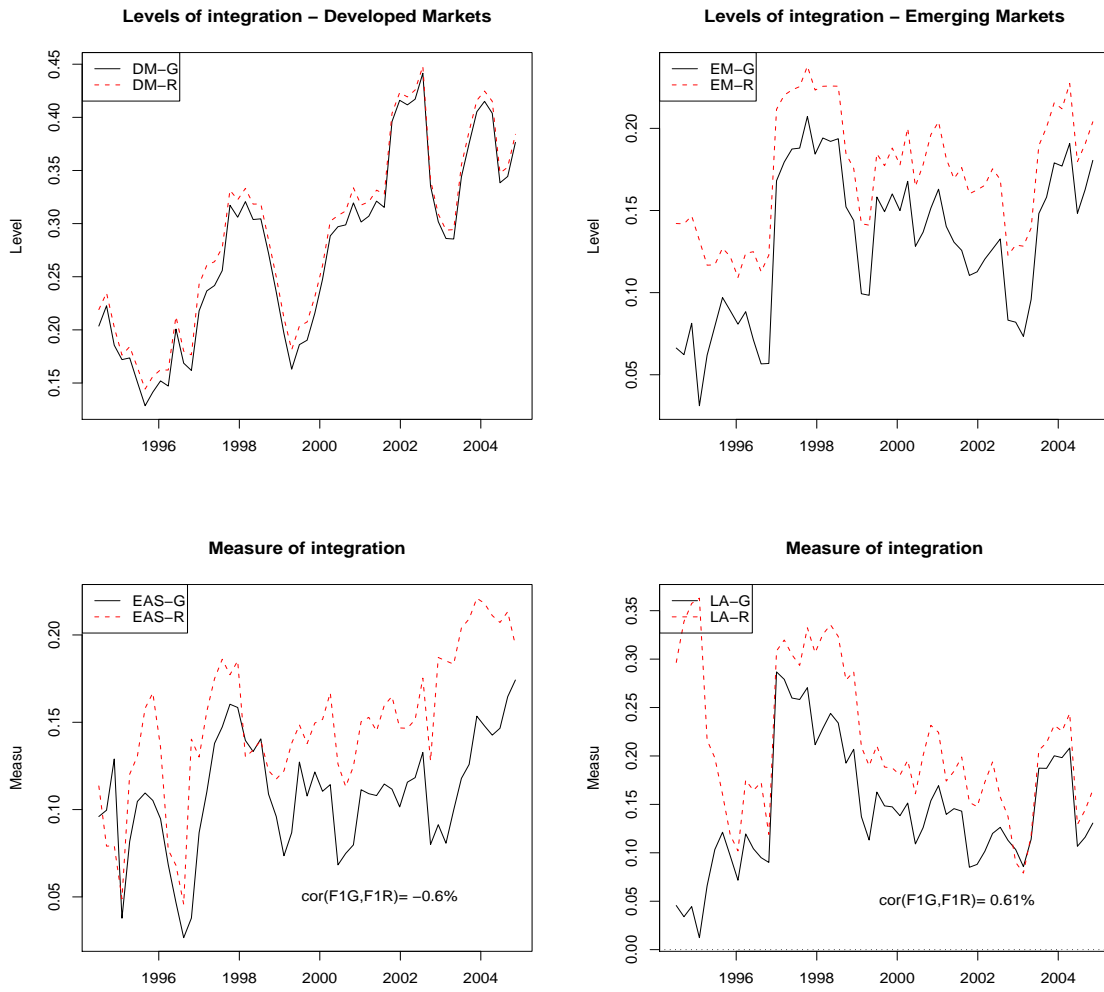


Figure 5: Percentage of Variance explained

