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Dependence in Private Values and Efficiency in Bilateral Trade.

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Abstract

In a bargaining model with dependent private information, I extend the result on existence of efficient mechanisms allowing any distribution of the surplus among traders. This generalization introduces flexibility into the model since full surplus extraction by one trader is incompatible with ex post individual rationality.

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1 Introduction.

Most of the existing literature on trading problems analyzes the private value model. In this model, the distribution of values can be viewed as expressing the beliefs of agents. When the beliefs are independent of any privately observed signal, the agents' strategic behavior leads typically to losses in efficiency. With dependent beliefs, however, Crémer and McLean (1985, 1988) and McAfee and Reny (1992) showed that it is possible to eliminate the inefficiencies associated with private information. In their setting, the conditional distribution of private information varies, creating a diversity of beliefs across types. This diversity of beliefs gives an opportunity to construct arbitrarily large penalties and rewards (positive and negative transfer payments) to deter the agents from misrepresenting their preferences.

These results were established in an auction framework, by imposing the restriction that the seller extracts all benefits from trade on average. Even though this restriction may be desirable in auctions, it is artificial in situations where the achievement of efficiency is the major concern. As an example consider the case of two divisions of a business unit involved in internal transactions. The company's main interest is not how the surplus is distributed between the divisions but rather whether the outcome is efficient or not. In fact, I show here, that the requirement of full surplus extraction by the seller is not only restrictive but inconsistent with ex post individual rationality. This finding is all the more important since many trading mechanisms including auctions are effectively ex post individually rational.

In light of this result, I proceed to analyze the problem of achieving efficiency in ex ante budget balanced trading mechanisms with greater generality without incorporating the restriction of full surplus extraction by the seller. In the bilateral trading framework, the result by Crémer and McLean (1988) at the ex ante stage is easily generalized to show that efficient mechanisms based on penalties and rewards, mechanisms that induce any distribution of the ex ante surplus can still be constructed. These type of mechanisms, however, do not satisfy the stricter and more realistic constraint of ex post budget balance (the transfers cannot balance for every sample of valuations). In order to induce truthtelling, in ex post budget balanced mechanisms, the designer can no longer resort to penalties in every occasion of misreporting.

2 The Model.

A buyer and a seller consider trading an object. The seller's privately known cost s_i, takes values in the set $S = \{s_1, s_2, \ldots, s_m\}$. Similarly, the buyer's value t_j, is drawn from the set $T = \{t_1, t_2, \ldots, t_n\}$. Both agents are risk neutral. Before any form of uncertainty about their valuations, what I will call their types, is resolved, the agents have a probability distribution π over the elements of $T \times S$. This paper considers a private value model with dependent beliefs and restricts attention to efficient direct revelation mechanisms.[1]

Let $x_b(s_i, t_j)$ be the payment of the buyer, $x_s(s_i, t_j)$ the receipt of the seller and let $p(s_i, t_j)$ denote the probability that the object is transferred to the buyer when the reported valuation of the buyer and the seller are s_i and t_j respectively (efficiency requires $p(s_i, t_j) = 1$ if $t_j > s_i$, $p(s_i, t_j) = 0$ if $t_j < s_i$ and $p(s_i, t_j) \in [0, 1]$ if $t_j = s_i$).

The expected payoff of a buyer of type t_j who announces t_j^* is:

$U_b(t_j^*|t_j) = \sum_{i=1}^{m} \pi(s_i|t_j)[p(s_i, t_j^*)t_j - x_b(s_i, t_j^*)]$.

Likewise the expected utility of a seller with type s_i who reports s_i^* is:

$U_s(s_i^*|s_i) = \sum_{j=1}^{n} \pi(t_j|s_i)[x_s(s_i^*, t_j) - p(s_i^*, t_j)s_i]$.

The constraints imposed on a mechanism can be classified according to the three temporal stages of a Bayesian game: the ex ante, interim and ex post stages. Based on this distinction, I express below the constraints of the problem at the different stages they appear in the analysis.

A direct bargaining mechanism characterized by the functions p and x is <u>Bayesian incentive compatible, (IC)</u> if for every t_j and t_k in T:

$\sum_{i=1}^{m} \pi(s_i|t_j)[p(s_i, t_j)t_j - x_b(s_i, t_j)] \geq \sum_{i=1}^{m} \pi(s_i|t_j)[p(s_i, t_k)t_j - x_b(s_i, t_k)]$,

and for every s_i and s_l in S:

$\sum_{j=1}^{n} \pi(t_j|s_i)[x_s(s_i, t_j) - p(s_i, t_j)s_i] \geq \sum_{j=1}^{n} \pi(t_j|s_i)[x_s(s_l, t_j) - p(s_l, t_j)s_i]$.

This set of inequalities asserts that both individuals, at the interim stage, expect to do at least as well by reporting truthfully as they would do by misreporting.

The mechanism is <u>interim individually rational, (IIR)</u> if for every t_j in T:

$\sum_{i=1}^{m} \pi(s_i|t_j)[p(s_i, t_j)t_j - x_b(s_i, t_j)] \geq 0$,

and for every s_i in S:

[1] This is the discrete type version of the model considered by Myerson and Satterthwaite (1983) with an additional element, namely the dependence among the agent's types.

$\sum_{j=1}^{n} \pi(t_j|s_i)[x_s(s_i,t_j) - p(s_i,t_j)s_i] \geq 0.$

This condition ensures that every individual will voluntarily participate in the mechanism at the time he knows his private information.

The mechanism is ex ante budget balanced, (EABB) if on average the payments of the buyer equal the receipts of the seller, i.e.,

$\sum_{j=1}^{n} \pi(t_j) \sum_{i=1}^{m} \pi(s_i|t_j) x_b(s_i,t_j) = \sum_{i=1}^{m} \pi(s_i) \sum_{j=1}^{n} \pi(t_j|s_i) x_s(s_i,t_j).$

Finally the mechanism is ex post budget balanced, (EPBB) if the transfers of the agents are equal for every profile of announcement, i.e., if for every s_i in S and t_j in T:

$x_b(s_i,t_j) = x_s(s_i,t_j).$

In a closely related model, describing an auction environment, Crémer and McLean (1988) considered the possibility of full surplus extraction by the seller and proved that there exists an efficient, incentive compatible, interim individually rational and ex ante budget balanced mechanism as long as the matrix of conditional probabilities is nonsingular. Theorem 1 below shows that, in the bilateral trading framework, full surplus extraction by the seller is incompatible with ex post individual rationality since the payments required in some occasions are higher than the buyer's true value. This result can be easily extended to the setting of an open auction.

Theorem 1 : No ex post individually rational mechanism exists that would allow the seller to extract all gains from trade on average.

Proof: A mechanism satisfies ex post individual rationality if: $p(s_i,t_j)t_j - x_b(s_i,t_j) \geq 0 \quad \forall\ s_i,\ t_j$. Incorporating the assumption of full surplus extraction into the buyer's incentive compatibility condition results in:

$\sum_{s_i} \pi(s_i|t_j)[x_b(s_i,t_k) - p(s_i,t_k)t_j] \geq 0 \quad \forall t_j,\ t_k \neq t_j.$

The ex post individual rationality condition implies that:

$p(s_i,t_k)t_k \geq x_b(s_i,t_k) \quad \forall s_i.$

Finally, replacing x_b in the incentive compatibility condition leads to the following condition:

$\sum_{s_i} \pi(s_i|t_j)[p(s_i,t_k)t_k - p(s_i,t_k)t_j] \geq 0.$ which is not satisfied for any $t_k < t_j$.

Since there is no ex post individually rational mechanism that allows the seller to extract the gains from trade on average (EABB mechanism), there isn't any that could be balanced at every instance (EPBB mechanism). *Q.E.D.*

Theorem 2 provides the sufficient conditions on the agents' beliefs for existence of an efficient, IC, IIR and EABB mechanism allowing the possibility that the gains from trade might be shared by the agents. These conditions require that the vector of conditional probabilities corresponding to any value of a trader's type can not be expressed as a linear combination of the vectors corresponding to his other types, with nonnegative weight placed upon them. They are sufficient conditions to create enough room for mechanisms based on penalties and rewards to deter the traders from misreporting their values. The proof of theorem 2 is based on Farkas' lemma. It generalizes the result on existence of ex ante budget balanced mechanisms in Crémer and McLean (1988), by allowing any distribution of the ex ante surplus among traders[2]. When achievement of efficiency is the primary goal in the design of mechanisms, any distribution of the surplus that leads to efficient outcomes should be considered.

Theorem 2: An efficient, Bayesian incentive compatible, interim individually rational and ex ante budget balanced mechanism exists if:

(1) there does not exist a $t_k \in T$ and a family $\{\lambda(t_j)\}\, t_j \in T$ such that:

(a) $\lambda(t_j) \geq 0 \;\forall\; t_j \in T$ and

(b) $\pi(s_i|t_k) = \sum_{t_j \neq t_k} \lambda(t_j)\pi(s_i|t_j)$ for all $s_i \in S$ and

(2) there does not exist an $s_l \in S$ and a family $\{\mu(s_i)\}\, s_i \in S$ such that:

(a) $\mu(s_i) \geq 0 \;\forall\; s_i \in S$ and

(b) $\pi(t_j|s_l) = \sum_{s_i \neq s_l} \mu(s_i)\pi(t_j|s_i)$ for all $t_j \in T$.

Proof: If the conditions of Theorem 2 hold, one can consider a fixed sharing rule that satisfies individual rationality and introduce deviations from that rule that have zero expected value when the agents report truthfully and penalize them with arbitrarily large negative expected transfers when they misreport.

According to Farkas' lemma[3], when these conditions hold, there exists for each i and s_i a vector with elements $g_i(s_i, t_j) \in R^n$ where $t_j \in T$, for which:

$\sum_{t_j} \pi(t_j|s_i)g_i(s_i,t_j) > 0$ and $\sum_{t_j} \pi(t_j|s_l)g_i(s_i,t_j) \leq 0, \quad \forall\; s_l \neq s_i$.

[2]In fact, the author has found both necessary and sufficient conditions for existence of efficient, IC, IIR and EABB mechanisms using a different approach, namely Tucker's theorem of the alternative. Since one of the conditions involves beliefs and values and its form does not provide any general characterization about beliefs, I chose to formulated the theorem only in terms of the sufficient conditions involving beliefs and provide an alternative shorter proof. The sufficient conditions of this alternative method are the conditions presented above.

[3]See Mangasarian (1969).

Following Crémer and McLean (1988), let $\epsilon_i = \sum_{t_j} \pi(t_j|s_i)g_i(s_i,t_j)$ and $h_i(t_j,s_i) = g_i(s_i,t_j) - \epsilon_i$. Then: $\sum_{t_j} \pi(t_j|s_i)h_i(s_i,t_j) = 0$ *and* $\sum_{t_j} \pi(t_j|s_l)h_i(s_i,t_j) < 0 \quad \forall\, s_l \neq s_i$.

Considering another version of the same lemma, there exists for each j and t_j a vector with elements $g_j(s_i,t_j) \in R^m$ with $s_i \in S$, for which $\sum_{s_i} \pi(s_i|t_j)g_j(s_i,t_j) < 0$ and $\sum_{s_i} \pi(s_i|t_k)g_j(s_i,t_j) \geq 0$, $\forall\, t_k \neq t_j$. Let $\sum_{s_i} \pi(s_i|t_j)g_j(s_i,t_j) = -\epsilon_j$ and $h_j(s_i,t_j) = g_j(s_i,t_j) + \epsilon_j$. It follows that:
$\sum_{s_i} \pi(s_i|t_j)h_j(s_i,t_j) = 0$ *and* $\sum_{s_i} \pi(s_i|t_k)h_j(s_i,t_j) > 0 \quad \forall\, t_j \neq t_k$.

Now consider the following payment schemes for the buyer and the seller for $0 \leq w \leq 1$:
$x_j(s_i,t_j) = p(s_i,t_j)[ws_i + (1-w)t_j] + \gamma_j(t_j)h_j(s_i,t_j)$
$x_i(s_i,t_j) = p(s_i,t_j)[ws_i + (1-w)t_j] + \gamma_i(s_i)h_i(s_i,t_j)$.

The appropriate choice of the functions γ_i and γ_j, satisfies the incentive compatibility conditions. This mechanism is also interim individually rational and balanced on average since
$\sum_{j=1}^{n} \pi(t_j) \sum_{i=1}^{m} \pi(s_i|t_j)x_b(s_i,t_j) = \sum_{i=1}^{m} \pi(s_i) \sum_{j=1}^{n} \pi(t_j|s_i)x_s(s_i,t_j) =$
$\sum_{j=1}^{n} \sum_{i=1}^{m} \pi(s_i,t_j)p(s_i,t_j)[ws_i + (1-w)t_j]$.
This completes the proof of the result. *Q.E.D.*

Notice that if no efficient Bayesian incentive compatible, IIR and EABB mechanism exists $\sum_j \lambda(t_j) = 1$ and $\sum_i \mu(s_i) = 1$. Otherwise, the beliefs will not be well defined.

In the case of the independent private value model, Myerson and Satterthwaite (1982) proved that it is impossible to construct efficient mechanisms in the bilateral trading framework. In this extreme case, the inefficiencies arise because every agent must be paid on the margin the surplus generated by his private information in order to induce him to reveal what he knows privately. The updated distribution of the opponents' types is the same conditional on any type of private information. As a result, no payment scheme can help differentiate the expected payments made by agents who misreport and those who don't. Once the updated distribution differs depending on the private information of each agent, one can introduce payments that impose large penalties when someone is misrepresenting his preferences and rewards when he is telling the truth. The designer can then adjust the size of positive and negative transfer payments to restrict the surplus that is appropriated by each agent.

As is demonstrated below, however, the mechanisms proposed in theorem 2 cannot be ex post budget balanced. In an ex post budget balanced mechanism the deviations from the fixed sharing rule should be the same for both traders, for any pair of types, i.e., $\gamma_j(t_j)h_j(s_i,t_j) = \gamma_i(s_i)h_i(s_i,t_j)$

$= \phi(s_i, t_j)$. In this case $\phi(s_i, t_j)$ should satisfy, for all s_i, $s_l \neq s_i$, t_j, and $t_k \neq t_j$, the following set of conditions:

$$\sum_i \pi(s_i|t_k)\phi(s_i, t_j) > 0, \quad \sum_j \pi(t_j|s_i)\phi(s_i, t_j) = 0, \tag{1}$$

$$\sum_i \pi(s_i|t_j)\phi(s_i, t_j) = 0, \quad \sum_j \pi(t_j|s_i)\phi(s_l, t_j) < 0. \tag{2}$$

The conditions in (2) imply:

$\sum_j \pi(t_j, s_i)\phi(s_l, t_j) < 0 \Rightarrow \sum_j \pi(s_i|t_j)\pi(t_j)\phi(s_l, t_j) < 0 \Rightarrow$

$\sum_i \sum_j \pi(s_i|t_j)\pi(t_j)\phi(s_l, t_j) < 0 \Rightarrow \sum_j \pi(t_j) \sum_i \pi(s_i|t_j)\phi(s_l, t_j) < 0 \Rightarrow \sum_j \pi(t_j)\phi(s_l, t_j) < 0.$

Moreover, combining the conditions in (1) results in:

$\sum_i \pi(s_i|t_k)\phi(s_i, t_j) > 0 \Rightarrow \sum_j \pi(t_j) \sum_i \pi(s_i|t_k)\phi(s_i, t_j) > 0 \Rightarrow \sum_i \pi(s_i|t_k) \sum_j \pi(t_j)\phi(s_i, t_j) > 0.$

Since $\sum_j \pi(t_j)\phi(s_i, t_j) < 0 \ \forall s_i$, the last inequality produces a contradiction.

The deviations from the fixed sharing rule impose penalties at the interim stage every time an agent misreports his type. This, however, is not a characteristic that is necessary of mechanisms that induce truthtelling. It is possible that, for some realizations of his type, an agent has no incentive to misreport when the payments are determined by a fixed sharing rule. For these types, the deviations from the fixed rule could have a positive expected value on occasions of misreporting as long as it is small enough to ensure that the agents' incentives are not altered. Since these conditions are not necessary for the construction of ex ante budget balanced mechanisms leading to efficient outcomes, the previous argument does not prove nonexistence of ex post budget balanced mechanisms, in general. It merely points to the fact that it is not possible to construct ex post budget balanced mechanisms that penalize the agents for every possible deviation from truthtelling.

3 References.

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