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The unidirectional Hotelling model with spatial price discrimination

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Abstract

The unidirectional Hotelling model where consumers can buy only from firms located on their right (left) is extended to allow for price discriminating firms and a general class of transportation costs. In a two-stage location-price game one firm locates at 1/2 and the other locates at 1 (0). We also study collusion in an infinitely repeated game. The maximum collusive profits sustainable in equilibrium monotonically increase (decrease) with the location of the firm located at the right (left), while initially increase and then decrease with the location of the firm located at the left (right). A higher reservation price of consumers makes perfect collusion less sustainable in equilibrium, but allows firms to agree on higher (albeit imperfect) collusive profits.

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1. Introduction

In a recent paper (Kharbach, 2009), the unidirectional Hotelling model (UHM henceforth) has been introduced. While in the standard bidirectional Hotelling model (BHM henceforth) consumers have a bidirectional purchasing ability, in the UHM a consumer can buy only from firms located at his right or only from firms located at his left. The UHM can be used to describe spatial situations like highways or one way roads, or non-revertible flows in gas and oil pipelines (Kharbach, 2009). In a location-price game with uniform pricing and quadratic transportation costs, Kharbach (2009) shows that when consumers can buy only from firms located on their right (left), one firm locates in position $3/5$ from the left (right) endpoint of the linear market, while the other firm locates at the right (left) endpoint.

The aim of this article is twofold. In the first part, we study the location-price equilibrium emerging within the UHM when spatial price discrimination is introduced.[1] We adopt a general class of transportation cost functions encompassing both linear and quadratic transportation costs and we show that in equilibrium one firm locates in the middle of the market, while the other locates at the right (left) endpoint of the segment when consumers can buy only from firms located on their right (left).

In the second part of the article we develop an infinitely repeated game in order to investigate collusion sustainability within the UHM under spatial price discrimination. In particular, we are interested in the role of firms' location.[2] We show that when consumers can buy only from firms located on their right (left), the maximum collusive profits sustainable in equilibrium monotonically increase (decrease) with the location of the firm located at the right (left), while initially increase and then decrease with the location of the firm located at the left (right).

The article proceeds as follows. In Section 2 we describe the UHM. In Section 3 we analyse the location-price equilibrium. In Section 4 we analyse collusion sustainability in the repeated game framework. Section 5 concludes. Proofs are in the appendix.

2. The model

Assume a linear market of length 1. Consumers are uniformly distributed along the market. Denote by $x \in [0,1]$ the location of each consumer. Each consumer consumes no more than 1 unit of the good. Denote by v the reservation price. There are two firms, A and B, with zero marginal costs, located respectively at a and b. Without loss of generality we assume $0 \le a \le b \le 1$. Both firms can perfectly price discriminate. Denote by p_x^J the price set by firm $J = A, B$ on a consumer located at x. As in Kharbach (2009) we assume that a consumer can buy

[1] Location-price games under price discrimination have received a lot of attention within the BHM. Lederer and Hurter (1986) show that with inelastic demand functions, equilibrium locations minimize transportation costs, thus maximizing welfare. Hamilton *et al*. (1989) assume downward sloping demand functions and show that when firms compete *à la* Bertrand equilibrium locations are strictly between the first and the third quartile, while when firms compete *à la* Cournot in equilibrium firms agglomerate in the middle. Hamilton and Thisse (1992) introduce two-part tariffs, and show that in equilibrium firms locate at the first and the third quartile.

[2] Some authors have analyzed the relationship between firms' location and collusion sustainability within the BHM under price discrimination. Gupta and Venkatu (2002), relying on grim trigger punishment mechanisms, show that in the case of elastic demand functions perfect collusion is easier to sustain the less firms are distant. Miklós-Thal (2008) shows that when optimal punishment is introduced the result is the opposite. See also Colombo (2009) for the case of imperfect price discrimination.

only from a firm located on his right hand side. We maintain the specification of the transportation costs function as general as possible. Namely, let the transportation costs paid by a consumer x buying from firm A and from firm B be given by the functions: $t(a-x)^k$ and $t(b-x)^k$ respectively, with $k \geq 1$.[3] Therefore, the utility of a consumer located at x when he buys from firm A is given by: $u_x^A = v - p_x^A - t(a-x)^k$, while the utility of a consumer located at x when he buys from firm B is given by: $u_x^B = v - p_x^B - t(b-x)^k$. We assume $v \geq t$ to ensure that the market is always covered.

3. Location-price equilibrium

In this section we study the location-price equilibrium emerging in a two-stage game where in the first stage the firms choose where to locate and in the second stage set the price schedules. The sub-game Nash equilibrium concept is used in solving the game.

In the second stage of the game firms choose simultaneously the price schedules given the locations. Consider the consumers located at $x \in [0, a]$. The consumers located at $x \in [0, a]$ can buy from both firms. In order to avoid ε-equilibria, we assume that if the utility of a consumer is the same when he buys from firm A and when he buys from firm B, he buys from the nearer firm.[4] The equilibrium prices on a consumer located at $x \in [0, a]$ have the following characteristics: the firm which is further from consumer x (firm B) charges a price equal to the marginal costs (which are assumed to be zero in our model), while the nearer firm (firm A) sets a price equal to the difference between the transportation costs.[5] Then, firm A's price schedule is: $p_x^A = t(b-x)^k - t(a-x)^k$. Firm A serves all consumers located at $x \in [0, a]$. Consider now the consumers located at $x \in [a, b]$. The consumers located between the two firms can buy only from firm B. Therefore, firm B sets the price schedule in such a way to extract the whole consumer surplus from the consumers located at $x \in [a, b]$. The equilibrium price schedule set by firm B on the consumers located between a and b is therefore: $p_x^B = v - t(b-x)^k$. The consumers located at $x \in (b, 1]$ cannot buy any product. Therefore, the profits functions of the two firms are: [6]

$$\Pi^{A,N} = \int_0^a p_x^A dx = t[b^{1+k} - (b-a)^{1+k} - a^{1+k}]/(1+k) \tag{1}$$

$$\Pi^{B,N} = \int_a^b p_x^B dx = (b-a)[v(1+k) - t(b-a)^k]/(1+k) \tag{2}$$

Taking the derivative of (1) and (2), we get:[7]

[3] Since a consumer can buy only from a firm located on his right hand side the possibility of "negative transportation costs" is excluded.

[4] This assumption is standard in spatial models. For more details, see among the others Hurter and Lederer (1985), Lederer and Hurter (1986), Thisse and Vives (1988), Hamilton *et al.* (1989), Hamilton and Thisse (1992).

[5] For a formal and general proof, see Lederer and Hurter (1986).

[6] Let us use the superscript N to indicate the equilibrium profits. This will become useful in Section 4 when collusion will be introduced.

[7] It is immediate to see that $\partial^2 \Pi^{A,N} / \partial a^2 < 0$ and $\partial^2 \Pi^{B,N} / \partial b^2 < 0$, so the second order condition is satisfied.

$$\partial \Pi^{A,N} / \partial a = t[(b-a)^k - a^k] \quad (3)$$

$$\partial \Pi^{B,N} / \partial b = v - t(b-a)^k \quad (4)$$

Note that (4) is always larger than 0. Therefore, firm *B* locates at the right endpoint of the market. Then: $b^* = 1$. Solving $\partial \Pi^{A,N} / \partial a = 0$ with respect to *a* we get the optimal location of firm *A*: $a = b/2$. In equilibrium we get: $a^* = 1/2$. Therefore, when the consumers can buy only from firms located on the right hand side, one firm locates at the right endpoint of the market, while the other locates in the middle of the market (by symmetry, when the consumers can buy only from firms located on the left hand side, one firm locates at the left endpoint of the market, while the other locates in the middle of the market). The intuition is the following. Consider firm *B*. When both firms price discriminate, firm *B* monopolistically serves the consumers located between the two firms, but it cannot serve the consumers on the left of the rival. Since price discrimination allows firm *B* to extract the whole surplus from the consumers it serves, firm *B* has the incentive to maximize the demand. Therefore, firm *B* tries to expand its demand by locating as far as possible from firm *A*. For any firm *A*'s location firm *B*'s demand is maximized when firm *B* locates at the right endpoint of the segment. Consider now firm *A*. Firm *A* serves only consumers located on its left. Therefore, the higher is *a*, the higher is firm *A*'s demand. This effect coincides with the *demand effect* illustrated by Tirole (1988, p.281) for the location-price game with uniform price and quadratic transportation costs. The *demand effect* pushes firm *A* to locate near to firm *B* in order to increase its own demand. However, firm *A*'s equilibrium price schedule depends also on the distance between the two firms: the higher is product differentiation the higher is the price firm *A* can charge on each consumer buying from it. This coincides with the *strategic effect* discussed by Tirole (1988, p.281). The *strategic effect* pushes firm *A* far from firm *B* in order to soften competition. While in the BHM with uniform pricing firms and quadratic transportation costs the *strategic effect* dominates and determines maximal differentiation in equilibrium (D'Aspremont *et al.*, 1979), in the UHM with price discrimination no effect dominates: the *strategic effect* and the *demand effect* are in equilibrium when firm *A* positions itself in the middle. This result is robust for a general class of transportation costs functions.

4. Collusion

In this section, we consider the conditions of collusion sustainability within the UHM. In order to investigate collusion, we need to depart from the two-stage game analysed in Section 3 and introduce an infinitely repeated game. We are mainly interested in evaluating the impact of the firms' location over collusion sustainability. Locations are kept exogenous, and we analyse how variations in the locations' parameters affect collusion sustainability.

Suppose that firm *A* and firm *B* interact repeatedly in an infinite horizon setting. In supporting collusion, the firms are assumed to use the *grim trigger* strategy of Friedman (1971).[8] The

[8] Clearly, the *grim trigger* strategy is not optimal (Abreu, 1986). However, "*this is one of very realistic punishment strategies because of its simplicity*", as argued by Matsumura and Matsushima (2005, p.263). The most part of the articles which study collusion sustainability through spatial models adopt the *grim trigger* strategy. See for example, Deneckere (1983), Chang (1991), Chang (1992), Friedman and Thisse (1993), Hackner (1994), Hackner (1995) and

market discount factor, $\delta \in (0,1)$, is exogenous and common for each firm. Let us denote by $\Pi^{J,C}$, $\Pi^{J,D}$ and $\Pi^{J,N}$ respectively the collusive profits, the deviation profits and the punishment (or Nash) profits of firm $J = A, B$. It is well known that collusion is sustainable as a sub-game perfect equilibrium when the following incentive-compatibility constraint is satisfied:

$$\delta \geq \delta^* \equiv \max[\frac{\Pi^{A,D} - \Pi^{A,C}}{\Pi^{A,D} - \Pi^{A,N}}, \frac{\Pi^{B,D} - \Pi^{B,C}}{\Pi^{B,D} - \Pi^{B,N}}] \tag{5}$$

Recall that consumers located between *a* and *b* cannot buy from firm *A*, and their surplus is totally extracted by firm *B* in the competitive set-up. Therefore, a collusive agreement cannot generate higher profits than the competitive profits over consumers located between *a* and *b*: the collusive profits over consumers located between the two firms must coincide with the Nash profits obtained by firm *B*. However, a collusive agreement over the consumers located between 0 and *a* may be profitable for both firms. Suppose a collusive agreement of this type: firm *B* agrees to compete less fiercely over the consumers located between 0 and *a*, by setting a price schedule $p_x^{B,C}$ which is strictly positive at least for some $x \in [0, a]$. This allows firm *A* to set a collusive price schedule $p_x^{A,C} = p_x^{B,C} + t(b-x)^k - t(a-x)^k$, which is strictly larger than the equilibrium price schedule arising in competition, at least for some $x \in [0, a]$. Let us denote by $\Lambda \equiv \int_0^a p_x^{A,C} dx$ the *gross* collusive profits obtained by firm *A* under the collusive agreement. In exchange for softening competition firm *B* receives a fraction *s* of the *gross* collusive profits of firm *A*. Therefore, the *net* collusive profits of firm *A* and firm *B* are respectively $\Pi^{A,C} = (1-s)\Lambda$ and $\Pi^{B,C} = s\Lambda + \Pi^{B,N}$, where the second term in $\Pi^{B,C}$ refers to the fact the firm *B* monopolizes the consumers located at $x \in [a, b]$.[9]

Note that, given the level of *gross* collusive profits Λ, there exist many different collusive discriminatory price schedules yielding the same level of profits.[10] In general, finding the optimal collusive price schedule for any given level of collusive profits would be very difficult. Nevertheless, we are able to characterize a condition for collusion sustainability which is based only on collusive profits and not on collusive prices. This allows us to perform the analysis by using directly profits functions instead of price schedules.

First, note that firm *A* has never the incentive to deviate from the collusive agreement. In fact, the consumers located at $x \in [a, b]$ are always monopolized by firm *B* and cannot be stolen by firm *A* even if it deviates from the collusive agreement. On the contrary, the consumers located at $x \in [0, a]$ are served by firm *A*, and firm *A* cannot set a price higher than $p_x^{A,C}$ when firm *B* is setting the collusive price $p_x^{B,C}$. Since the deviation profits are equal to the collusive profits, firm *A* has never the incentive to deviate from the collusive agreement. Let us consider now firm *B*. The deviation profits of firm *B* are defined in the following Lemma:

Matsumura and Matsushima (2005). Exceptions are Hackner (1996) and Miklós-Thal (2008), where optimal punishments are assumed.

[9] Note that for collusion to be profitable for firm *A*, we have to impose $s \leq 1 - \Pi^{A,N} / \Lambda$: this guarantees that firm *A*'s *net* collusive profits are higher than firm *A*'s Nash profits.

[10] The only exception is represented by perfect collusion, because in this case there is only *one* collusive price schedule generating perfect collusive profits (see later). For a similar problem, see Liu and Serfes (2007, Section 5).

Lemma 1. The deviation profits of firm *B* are: $\Pi^{B,D} = \Lambda - \Pi^{A,N} + \Pi^{B,N}$.

Substituting the collusive profits, the deviation profits and the punishment profits of firm *B* into (5) we get that collusion is sustainable if and only if:

$$\delta \geq \delta^* = 1 - s\Lambda / (\Lambda - \Pi^{A,N}) \tag{6}$$

We state the following Lemma:

Lemma 2. If $s \geq 1-\delta$ any collusive profits Λ is sustainable in equilibrium.

The intuition of Lemma 2 is straightforward. The higher is *s*, the higher are firm *B*'s collusive profits. When firm *B*'s participation in the *gross* collusive profits of firm *A* is sufficiently high (i.e. $s \geq 1-\delta$), firm *B* has never the incentive to deviate from the collusive agreement, whatever are the collusive profits Λ firms agree upon and whatever are firms' locations. Therefore, in order to investigate the effect of firms' location on collusion sustainability, in the rest of this section we assume $s < 1-\delta$.

Consider perfect collusion. Under perfect collusion, firm *B* completely renounces to compete over the consumers located at $x \in [0, a]$: in this case, firm *A* sets the (unique) discriminatory price schedule which extracts the whole surplus from the consumers located at $x \in [0, a]$, that is: $p_x^{A,C*} = v - t(a-x)^k$. Perfect *gross* collusive profits are the following:

$$\Lambda^* = \int_0^a p_x^{A,C*} dx = va - ta^{1+k}/(1+k) \tag{7}$$

It is immediate to see that Λ^* is strictly increasing with *a*, while it does not depend on *b*. In fact, a higher *a* implies a larger subset of consumers over which the collusive agreement is profitable for both firms, while a higher *b* only increases the number of consumers monopolized by firm *B*. Suppose now that perfect collusion is not sustainable as a sub-game perfect equilibrium because the market discount factor is too low. Note from (6) that the critical discount factor is strictly increasing with the *gross* collusive profits of firm *A*. Therefore, when perfect collusion is not sustainable, firms rationally agree on the maximum imperfect firm *A*'s *gross* collusive profits, $\overline{\Lambda}$, that can be sustained in equilibrium given the market discount factor. This profits level is simply obtained by solving (6) with respect to Λ. We get:

$$\overline{\Lambda} = (1-\delta)\Pi^{A,N} / (1-s-\delta) \tag{8}$$

From (3) we have that $\partial\Pi^{A,N}/\partial b > 0$ and $\partial\Pi^{A,N}/\partial a \geq (\leq) 0$ when $a \leq (\geq) b/2$. Therefore, we can immediately state the following proposition:

Proposition 1. The maximum imperfect collusive profits sustainable in equilibrium increase (decrease) with *a* when $a \leq (\geq) b/2$, and monotonically increase with *b*.[11]

The intuition for Proposition 1 is the following. From (8) it follows that the higher is $\Pi^{A,N}$ the higher are the maximum collusive profits that can be sustained in equilibrium. This is due to the fact the punishment profits of firm *A* coincide with the transportation costs firm *B* pays to steal firm *A*'s consumers (see the proof of Lemma 1). Therefore, an higher $\Pi^{A,N}$ implies lower firm *B*'s deviation profits, which in turn reduce the incentive for firm *B* to deviate from the collusive agreement. Consider now the impact of firms' locations over $\Pi^{A,N}$. First, for given *b*, note that $\Pi^{A,N}$ increases with *a* (thus increasing the maximum collusive profits sustainable in equilibrium) when $a \leq b/2$, but decreases with *a* (thus decreasing the maximum collusive profits sustainable in equilibrium) when $a \geq b/2$. In fact, when *a* increases, the individual transportation costs decrease as firms are nearer (this is the analogous of the *strategic effect* analysed in Section 3), but at the same time firm *B* steals more consumers, as the market of firm *A* increases with *a* (this is the analogous of the *demand effect* analysed in Section 3). As we showed in Section 3, when $a \leq b/2$ the *demand effect* prevails: as a consequence the overall transportation costs of firm *B* increase with *a*; when $a \geq b/2$ the *strategic effect* prevails: as a consequence the overall transportation costs of firm *B* decrease with *a*. Consider now the effect of an increase of *b* for given *a*. $\Pi^{A,N}$ increases with *b*, thus increasing the maximum collusive profits sustainable in equilibrium. The intuition is straightforward. A change in *b* does not modify firm *A*'s demand. Therefore, the *demand effect* does not arise. On the contrary, the distance between the firms increases with *b*. Therefore, the *strategic effect* arises: firm *A*'s punishment profits increase, and deviation profits of firm *B* decrease as firm *B* has to pay higher transportation costs in order to serve firm *A*'s consumers.

Figure 1 and 2 illustrate the impact of firms' location over the maximum collusive profits sustainable in equilibrium. Figure 1 considers the impact of *a* over collusion sustainability. First, consider the curve Λ^* and the curve $\overline{\Lambda}$. The bold lines represent the collusive profits sustained in equilibrium as a function of *a*. When *a* is low enough ($a \leq a'$), perfect collusion is sustainable in equilibrium, as $\Lambda^* < \overline{\Lambda}$. Moreover, a higher *a* allows higher perfect collusive profits. However, when *a* is high enough ($a \geq a'$), perfect collusion is no more sustainable. Therefore, firms agree on imperfect collusive profits, $\overline{\Lambda}$, and the impact of *a* over the maximum collusive profits sustainable in equilibrium is negative.[12] Moreover, note from (7) that the slope of Λ^* depends positively on the reservation price, *v*, while the slope of $\overline{\Lambda}$ does not depend on *v*. In Figure 1, three different perfect collusive profits levels, Λ^*, $\dot{\Lambda}^*$ and $\ddot{\Lambda}^*$, are represented. The higher is *v*, the steeper is the perfect collusive profits curve. It is immediate to see that the higher is *v* the lower is the intercept between the perfect collusive profits curve and $\overline{\Lambda}$. It follows that the higher is *v*, the narrower is the subset of *a*'s on which perfect collusion is sustainable. When *v* is sufficiently high (i.e. when the perfect collusive profits curve is $\ddot{\Lambda}^*$), perfect collusion is

[11] By symmetry, when consumers can buy only from firms located on the left hand side, the maximum imperfect collusive profits sustainable in equilibrium increase (decrease) with *b* for $b \leq (\geq)(1+a)/2$ and monotonically decrease with *a*. Details are omitted.

[12] In Figure 1 this depends on the fact that $a' \geq b/2$. In general, the impact of *a* on $\overline{\Lambda}$ is positive as long as $a \leq b/2$ and is negative when $a \geq b/2$, as shown in Proposition 1.

never sustainable in equilibrium whatever is firm A's location. However, even if a higher v reduces the possibility to sustain perfect collusion, a higher reservation price is beneficial for the colluding firms. In fact, as a consequence of a higher v, the level of the collusive profits that can be sustained in equilibrium increases: when v is sufficiently high, the maximum collusive profits constraint is binding everywhere, and the firms are able to collude on higher collusive profits, even if perfect collusion is impossible. To clarify this point, suppose for example that $a = b/2$. Moreover, suppose that initially v is such that the perfect collusive profits are represented by the curve Λ^*. The maximum collusive profits constraint is not binding, and firms collude on perfect collusive profits $\Lambda^*(a = b/2)$. Now, suppose that v increases: the perfect collusive profits curve is now $\dot{\Lambda}^*$. It is easy to see that when $a = b/2$, the perfect collusive profits, $\dot{\Lambda}^*(a = b/2)$, are not sustainable. The maximum collusive profits constraint is binding, and firms collude on the imperfect collusive profits $\overline{\Lambda}(a = b/2)$. However, firms are benefited by the increase of v: the imperfect collusive profits they now agree upon, $\overline{\Lambda}(a = b/2)$, are higher than the perfect collusive profits, $\Lambda^*(a = b/2)$, which were sustained in equilibrium when v was lower.

Figure 2 considers the impact of b over collusion sustainability. First, consider the curve Λ^* and the curve $\overline{\Lambda}$. Proposition 1 shows that the maximum collusive profits, $\overline{\Lambda}$, are strictly increasing with firm B's location, while perfect collusive profits do not depend on b. The bold line represents the collusive profits sustained in equilibrium as a function of b. Perfect collusive profits are sustainable in equilibrium as long as b is sufficiently high ($b \geq b'$). When b is sufficiently low ($b \leq b'$), only imperfect collusive profits are sustainable in equilibrium, and the effect of firm B's location over the level of maximum collusive profits sustainable in equilibrium is strictly positive. Finally, note that perfect collusive profits are a positive function of v. In Figure 2, three different perfect collusive profits levels, Λ^*, $\dot{\Lambda}^*$ and $\ddot{\Lambda}^*$, are represented. The higher is v, the higher is the intercept between the perfect collusive profits curve and $\overline{\Lambda}$. When v is sufficiently high and perfect collusive profits are represented by curve $\ddot{\Lambda}^*$, perfect collusive profits cannot be sustained in equilibrium: the maximum collusive profits constraint is binding everywhere, but firms are able to agree on higher (albeit imperfect) collusive profits.

5. Conclusions

In this article we extended the unidirectional Hotelling model introduced by Kharbach (2009) to allow for price discriminating firms. We also adopted a more general class of transportation cost functions encompassing both linear and quadratic transportation costs. In the first part of the article, we investigated the location-price equilibrium emerging in a two-stage game where firms first choose locations and then set the prices. We showed that when consumers can buy only from firms located on their right (left), in equilibrium one firm locates in the middle, while the other firm locates at the right (left) endpoint. In the second part of the article, we adopted an infinitely repeated game in order to focus on the impact of firms' location over collusion sustainability. We showed that when consumers can buy only from firms located on their right (left), the maximum imperfect collusive profits sustainable in equilibrium monotonically increase (decrease) with the location of the firm located at the right (left), while initially increase and then decrease with the location of the firm located at the left (right). A higher reservation price of consumers makes perfect collusion less sustainable in equilibrium *ceteris paribus*, but allows firms to agree on higher (albeit imperfect) collusive profits.

Figure 1

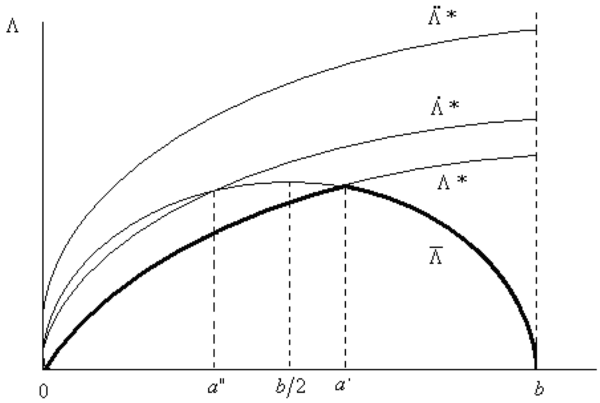
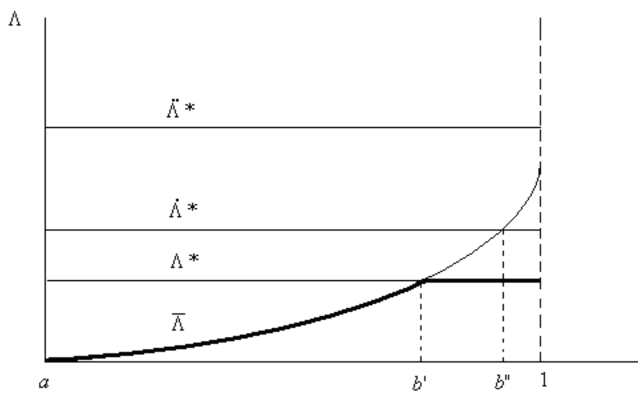


Figure 2



Appendix

Proof of Lemma 1. Consider first the consumers located at $x \in [a,b]$. Firm *B*, both under collusion and under punishment, extract the whole consumer surplus from these consumers. Therefore, firm *B* has never the incentive to deviate from the collusive price schedule and obtains $\Pi^{B,N}$. Consider now a consumer located at $x \in [0,a]$. We assume that when the consumer is indifferent between the deviating firm and the colluding firm, he buys from the deviating firm.[13] If firm *B* deviates, the deviation price on *x* satisfies the indifference condition: $v - p_x^{A,C} - t(a-x)^k = v - p_x^{B,D} - t(b-x)^k$. It follows that: $p_x^{B,D} = p_x^{A,C} + t(a-x)^k - t(b-x)^k$. Aggregating the deviation prices over the consumers stolen by firm *B*, we get: $\int_0^a p_x^{B,D} = \int_0^a p_x^{A,C} - \int_0^a [t(b-x)^k - t(a-x)^k]dx = \Lambda - \Pi^{A,N}$. It follows that the overall firm *B*'s deviation profits are $\Pi^{B,D} = \Lambda - \Pi^{A,N} + \Pi^{B,N}$. ■

Proof of Lemma 2. Define $w \equiv \Lambda/(\Lambda - \Pi^{A,N}) > 1$. Suppose $s = 1 - \delta + \gamma$, where $\gamma \geq 0$. Condition (6) can be rewritten as: $0 \geq (1-\delta)(1-w) - \gamma w$ which is always satisfied. ■

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[13] This assumption – which avoids the technicality of ε-equilibria – can be rationalized noting that the deviating firm can always offer to a consumer *x* a utility which is strictly larger than the utility he receives from the colluding firm by setting a price equal to $\hat{p}_x^B - \zeta$, where $\hat{p}_x^B$ is the price that gives to consumer *x* the same utility he receives from firm *A* at the collusive price, and ζ is a positive small number.

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