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### On the equivalence between labor and consumption taxation

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#### Abstract

This article studies the equivalence between labor and consumption taxes in a stochastic context, where the government can undertake an active portfolio management strategy by investing in both risk-free and risky assets. Using a two-period model we have shown that such taxes let consumers make the same decisions and can finance the same amount of government spending in each period.

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# 1 Introduction

In the absence of risk, a consumption tax is equivalent to a labor tax (see, e.g., Salanié 2003). When however we assume a stochastic context, this equivalence result may fail to hold, unless ad hoc conditions are met.

As shown by Ashan (1989, 1990), equivalence holds under uncertainty only if the wage tax is modified to account for excess returns, i.e., capital gains/losses net of the risk-free rate of return. Applying a risk-free discount rate, he shows indeed that the present value of revenues raised by this modified wage tax (hereafter MWT) is the same as under consumption-based taxation. Zodrow (1995) criticizes Ashan's use of the risk-free rate, by arguing that the government cannot absorb risk costlessly. This implies that the relevant discount rate must incorporate a risk premium. Zodrow (1995) proves that, if the government's discount rate is equal to the private expected return to saving, a consumption tax is equivalent to a standard labor tax (rather than to a MWT). In a more recent article, Ashan and Tsigaris (1998) have responded to Zodrow's (1995) critique by applying a certainty-equivalent approach. They show that if individuals hold efficient portfolios, the relevant discount rate is endogenously determined and, if applied by the government, ensures the equivalence between a consumption tax and a MWT.

It is worth noting that all these articles are based on the calculation of the present value of future tax revenue.<sup>1</sup> Therefore they share the use of an ex-ante approach. However the existence of an ex-ante equivalence does not necessarily imply that such taxes are equivalent from an ex-post perspective. To tackle the ex-post equivalence issue, Kaplow (1994) proposes a new approach, that lets the government invest in risky assets and thus manage resources actively.<sup>2</sup> Using a two-period model, he shows that equivalence holds on both an ex-ante and ex-post basis, provided that the government can manage its portfolio with no limits (i.e., it can take infinite short or long positions). Kaplow's (1994) findings are based on some restrictive assumptions. Indeed, he assumes that all the resources are invested at time

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<sup>1</sup>For a detailed review of the relevant literature on taxation and risk taking see Schindler (2008).

<sup>2</sup>A similar reasoning is used by Fahri (2007). He shows indeed that capital ownership provides the government with a powerful hedging instrument. In such a context, the government can perfectly approximate the complete markets allocation by taking an infinitely long or short position in capital. However, even with smaller positions, a substantial welfare improvement can be achieved

1 and consumed at time 2. Similarly, the government is assumed not to spend at time 1: all the resources raised are indeed spend at time 2. Given these limitations, Kaplow (1994) cannot deal with the fact that consumption taxes postpone tax payments relative to labor taxes. As pointed out by Summers (1981), indeed, under these two alternative systems, tax revenues may differ time by time, and therefore, the equivalence may fail to hold in an intertemporal context.

In this article we aim to generalize Kaplow's (1994) model by letting the individuals consume also at time 1, and the government to spend in each period. We will then show that if the government can manage risky resources, labor and consumption taxes are equivalent in each period, and therefore, the stricter definition of time-by-time equivalence is achieved.

The structure of the article is as follows. Section 2 presents the model and proves our equivalence result. Section 3 summarizes our finding and discusses possible extensions of the model.

## 2 The model

Let us assume a representative individual who lives for two periods, and maximizes an expected utility function  $u(c_t)$ , that is strictly concave in  $c_t$ , i.e., consumption at time  $t = 1, 2$ .<sup>3</sup> By assumption, there are two assets: a risk-free asset with a certain return  $r$ , and a risky asset with an stochastic return equal to  $x$ . At time 1, the individual earns a wage income equal to  $y$ .<sup>4</sup>

The sequence of the model is as follows.

At the beginning of time 1 the government chooses the tax rate and the level of government spending. Then, the individual decides how much to consume at time 1 and how much to invest in the risky and risk-free asset. Given the consumer's choice, the government raises revenues and finances public spending at time 1. Moreover, since the government can undertake an active portfolio strategy, it can invest its surplus in both the risky and risk-free assets.<sup>5</sup>

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<sup>3</sup>See, e.g., Sandmo (1985).

<sup>4</sup>Since income  $y$  is earned only at time 1, we can think of a representative individual who works only when he is young and consumes in both periods. However, the quality of results would not change if we assumed that the individual also works when he is old.

<sup>5</sup>If public spending is higher than tax revenue at time 1, the government can issue a bond, that will be repaid at time 2.

At time 2, the government earns the returns of its investment and, under consumption taxation, also raises revenues. All these resources are aimed at financing public spending at time 2.

Given these assumptions we want to find under what conditions:

1. consumers make the same decisions;
2. government spending in each period is the same under both regimes in each period.

If both conditions are met we can therefore say that these taxes are equivalent. Denoting  $\tau_w$  and  $\tau_c$  as the wage and consumption tax rate, respectively,  $s$  as saving at time 1, and  $a$  as the portion of resources invested in the risky asset, we can write the consumer's budget constraints at time 1 and 2, respectively:

$$(1 - \tau_w) y = (1 + \tau_c) c_1 + s \tag{1}$$

$$(1 + \tau_c) c_2 = [1 + r + a(x - r)] s. \tag{2}$$

It is worth noting that (1) and (2) are the consumer's budget constraints in a general setting (i.e., with  $\tau_w > 0$  and  $\tau_c > 0$ ). In what follows however, we will focus on two alternative regimes: a pure labor-tax system, with  $\tau_w > \tau_c = 0$ , and a pure consumption-based one, i.e., with  $\tau_c > \tau_w = 0$ . Contrary to labor taxation, that ensures revenue raising only at time 1, consumption taxation is levied in both periods.

Assuming for simplicity that a representative individual's utility function is additively separable,<sup>6</sup> and defining  $\beta$  as the relevant discount factor, his/her problem will be:

$$V(c_1, c_2, a) = \max_{c_1, c_2, a} E[u(c_1) + \beta u(c_2)], \tag{3}$$

*s.t.* (1) and (2).

Before solving (3) we can rewrite it as follows. Let us solve (1) for  $s$  and substitute the result into (2) so as to obtain:

$$\begin{aligned} (1 + \tau_c) c_2 &= [1 + r + a(x - r)] s \\ &= [1 + r + a(x - r)] [(1 - \tau_w) y - (1 + \tau_c) c_1], \end{aligned}$$

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<sup>6</sup>Notice that the quality of results would not change if, following Zodrow (1995), we assumed that the utility function is not additively separable.

which gives:

$$c_2 = [1 + r + a(x - r)] \left[ \frac{(1 - \tau_w)}{(1 + \tau_c)} y - c_1 \right]. \quad (4)$$

Let us next substitute (4) into (3), so as to rewrite a consumer's problem as follows:

$$V^*(c_1, a) = \max_{c_1, a} E \left\{ u(c_1) + \beta u \left( [1 + r + a(x - r)] \left[ \frac{(1 - \tau_w)}{(1 + \tau_c)} y - c_1 \right] \right) \right\} \quad (5)$$

The first order conditions of problem (5) are well known:<sup>7</sup>

$$(c_1) : \quad \frac{\partial u(c_1)}{\partial c_1} - E \left[ \beta \frac{\partial u(c_2)}{\partial c_2} [1 + r + a(x - r)] \right] = 0, \quad (6)$$

$$(a) : \quad E \left[ \beta \frac{\partial u(c_2)}{\partial c_2} [(x - r)] \left( \frac{(1 - \tau_w)}{(1 + \tau_c)} y - czXs \right) \right] = 0. \quad (7)$$

As shown in (6), neither regime has a direct impact on consumption. On the other hand, Eq. (7) shows that the impact of these tax regimes on asset allocation is the same only if tax rates are such that the equality  $(1 - \tau_w) = \frac{1}{1 + \tau_c}$  (or equivalently,  $\tau_w = \frac{\tau_c}{1 + \tau_c}$ ) holds.

Let us next analyze the government's policy, that consists of both tax rate setting and portfolio decisions. Given (6) and (7), we will calculate under what conditions, the government can make consumption and labor taxation equivalent.

**Labor income tax** Let us denote  $g_t$  as public spending at time  $t = 1, 2$ . Under labor taxation, the government's public budget constraint at time 1 is:

$$\tau_w y - g_1 = \alpha_w^s + \alpha_w^r, \quad (8)$$

where  $\alpha_w^s$  and  $\alpha_w^r$  are the amount of resources invested in the safe ( $s$ ) and risky ( $r$ ) activity.<sup>8</sup> Solving (8) for  $\alpha_w^s$  gives  $\alpha_w^s = \tau_w y - g_1 - \alpha_w^r$ . At time 2,

<sup>7</sup>For a detailed discussion of these conditions in a more general setting, see e.g. Sandmo (1985).

<sup>8</sup>Notice that  $\alpha_w^s$  and  $\alpha_w^r$  can be either positive or negative, because we have assumed that the government can take both long or short positions. If for instance,  $g_1$  is larger than the amount of resources collected at time 1, the government can finance its deficit by issuing a risk-free bond. Resources raised by means of the debt issue can be used to buy the risky asset too.

government spending will be equal to the amount of resources earned by the government. Therefore the budget constraint will be:

$$\begin{aligned} g_2 &= (1+r)\alpha_w^s + (1+x)\alpha_w^r \\ &= (1+r)(\tau_w y - g_1) + \alpha_w^r(x-r). \end{aligned} \tag{9}$$

**Consumption tax** Under a consumption tax, the amount of resources collected at time 1 is equal to  $\tau_c c_1$ . Given  $g_1$ , the public budget constraint is equal to:

$$\tau_c c_1 - g_1 = \alpha_c^s + \alpha_c^r, \tag{10}$$

where  $\alpha_c^s$  and  $\alpha_c^r$  are the amount of resources invested in the safe ( $s$ ) and risky ( $r$ ) asset.<sup>9</sup> Solving (10) for  $\alpha_c^s$  gives  $\alpha_c^s = \tau_c c_1 - g_1 - \alpha_c^r$ . Therefore, public spending at time 2 will be equal to:

$$g_2 = (1+r)\alpha_c^s + (1+x)\alpha_c^r + \tau_c c_2, \tag{11}$$

Substituting (4) into (11) we therefore obtain:

$$\begin{aligned} g_2 &= (1+r)\alpha_c^s + (1+x)\alpha_c^r + \tau_c c_2 \\ &= (1+r)\left(\frac{\tau_c}{1+\tau_c}y - g_1\right) + \left[\alpha_c^r + \tau_c a\left(\frac{1}{1+\tau_c}y - c_1\right)\right](x-r). \end{aligned} \tag{12}$$

Let us next compare (9) with (12). We can show that, given  $g_1$  and condition  $\tau_w = \frac{\tau_c}{1+\tau_c}$ , both regimes can finance the same amount public spending at time 2 if:

$$\alpha_w^r - \alpha_c^r = \tau_c a\left(\frac{1}{1+\tau_c}y - c_1\right) > 0. \tag{13}$$

Condition (13) states that, given  $a$ ,  $y$ , and  $c_1$  (that are known to the government), the government can adjust the amount of resources invested in the risky asset so as to ensure the same value of public spending in both regimes. Since  $g_1$  is the same and condition  $\tau_w = \frac{\tau_c}{1+\tau_c}$  holds, if condition (13) is met, equivalence holds.

It is worth noting that the differential  $(\alpha_w^r - \alpha_c^r)$  is equal to the product between  $\tau_c$  and the amount of resources invested in risky assets under consumption taxation. Since the RHS of (13) is positive, the amount of resources

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<sup>9</sup>Notice that  $\alpha_c^s$  and  $\alpha_c^r$  can be both positive and negative. Their sign depends on the government's portfolio strategy.

invested in risky assets is larger under a labor tax than under a consumption tax. The intuition behind this result is as follows: under a consumption tax the amount collected at time 2 is stochastic. Therefore, consumption taxation has two sources of risk: that related to tax revenue collection and that due to investment in the risky asset. On the other hand, labor taxation is subject only to the latter source of risk. This means that in order for equivalence to hold, the amount of resources invested in the risky asset under a consumption tax must be less than that invested under a labor tax. If the difference ( $\alpha_w^r - \alpha_c^r$ ) is such that equality (13) holds, then the two sources of risk under consumption taxation entail the same riskiness as under labor taxation.

### 3 Concluding remarks

In this article we have shown that labor and consumption taxes are equivalent in each period. This result is guaranteed by the fact that the government has two objectives (i.e.,  $g_1$  and  $g_2$ ) and two policy tools, i.e., tax rates and the portfolio management strategy.

It is worth noting that equivalence result has not been achieved by assuming the existence of complete markets. Rather, it can be obtained if the government can buy and sell, with no specific limitation, the same assets managed by private investors.<sup>10</sup>

The equivalence conditions obtained in this article can be studied in a more general setting, where, for instance, there are more than two periods, and, like Summers (1981), overlapping generations exist. We leave these extensions to future research.

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<sup>10</sup>I wish to thank Enrico Minelli for this comment.

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