

E C O N O M I C S B U L L E T I N

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## Ambient environmental monitoring, sequential firm inspections and time–decreasing benefits of inspection

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### *Abstract*

We consider an environmental enforcement agency who uses the measurement of ambient pollution to guide its inspections of individual polluters. We compare two different uses of this information. In a first model, the agency uses a "threshold strategy": if ambient pollution exceeds an endogenous threshold, the agency inspects all individual polluters simultaneously. In a second model, the agency inspects polluters sequentially, and updates its beliefs with respect to the firms' behavior after each firm inspection. If the cost of delaying the inspection of noncompliant firms is low enough, this sequential inspection policy is superior to a simultaneous inspection policy. However, if the cost of delay is high, the agency is better off if it commits itself to ignoring some information embedded in ambient pollution.

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# 1 Introduction

One of the central results in environmental economics is that pollution taxes and marketable permits are more cost-effective than uniform standards. The results obtained by Beavis and Walker (1981), Harford (1978), Keeler (1991), Malik (1990), Martin (1984) and Sandmo (1999) cast some doubt on this conventional result in a context where the environmental agency cannot perfectly enforce environmental standards. However, a casual glance at their models shows that the conclusion depends critically on the relationship between the probability of punishment and the pollution levels.

It is indeed relatively common to assume that the probability of inspecting polluters depends on the relation between pollution and the environmental standard. The underlying idea is that large transgressors will be inspected more frequently than small ones and that the polluter has to take the effect on the monitoring probability into account when he decides how much to pollute. Although this approach is reasonable, none of these papers endogenizes the relationship between ambient pollution levels and inspection probabilities.

On the other hand, several authors have proposed to use ambient-based policies to regulate non-point source pollution - see the surveys in Shortle and Abler (1997) and Xepapadeas (1999). The basic idea, first proposed by Segerson (1988), is to take observed ambient pollution as tax basis, rather than individual emissions of the polluter. There are however some problems with the feasibility of the proposed schemes (see, for instance, Shortle and Abler (1997)).

Ambient levels could however be a useful source of prior information to guide the monitoring efforts of the monitoring agency. For instance, according to Wasserman (1990) the enforcement program in the United States “has placed a high priority on violations of pollution standards in areas exceeding national ambient air quality standards for that pollutant”. In the United Kingdom, the Environment Agency (2000) explicitly recognizes that ambient monitoring “may be carried out (...) for compliance with legislation (...)”.

Franckx (2001, a) has argued that, by considering a setting where the inspection agency inspects ambient environmental pollution before deciding to inspect individual polluters, it should be possible to obtain an explicit relation between the probability of punishment and the level of pollution. It can then be shown that if all firms play the same mixed strategy, the agency will play a “threshold strategy”: it inspects all firms if and only if ambient pollution exceeds an endogenous threshold. Otherwise, no firm is inspected at all. The paper also shows under which circumstances monitoring ambient pollution constitutes an improvement compared to a situation where the enforcement agency does not collect any prior information at all.

In the initial formulation of the problem, it is assumed that, after having observed ambient pollution, the agency inspects all firms *simultaneously* - it does not update its prior beliefs after inspecting individual firms. The purpose of this paper is to extend the basic analysis to a setting where, after having observed ambient pollution, the agency inspects the firms sequentially and updates its beliefs rationally after each firm inspection. Moreover, we consider explicit costs

of postponing firm inspections and show that simultaneous inspections can then sometimes be superior. Finally, we argue that the results obtained by Franckx (2001, a) are also valid for a more general objective function for the enforcement agency.

In Section 2, we first define the general setting of the model. Sequential inspections are treated in Section 3. We compare these results with the results obtained in Franckx (2001, a) in Section 4 and offer concluding remarks in Section 5.

## 2 General setting of the model

Except if stated explicitly, we keep here all the assumptions of Franckx (2001, a).

We consider a game *without* repeated interactions.

For the sake of analytical simplicity, we limit ourselves to an analysis with two polluting firms, who can choose between two levels of abatement expenditure,  $\alpha$  and 0. If a firm spends  $\alpha$ , it is in compliance.

We assume that, for the agency, the cost of the compliant abatement technology is  $D_c$ ; the cost of noncompliance will be represented as  $D_{nc}$ . Note that  $D_{nc}$  and  $D_c$  can be given a wide range of interpretations. For instance, if the agency maximizes social welfare,  $D_{nc}$  and  $D_c$  are the monetary value of environmental damages net of private compliance costs. Or, alternatively, for an agency that narrowly focuses on environmental effects,  $D_{nc}$  and  $D_c$  are just the monetary value of environmental damages. We assume that  $D_{nc} > D_c$ : otherwise, the agency would have no reason to pursue compliance.

Ambient environmental inspections costs  $a$  per time period. Inspecting the firm costs  $b$ . If a firm is inspected and is found in noncompliance it will have to pay a fine  $\Psi > 0$  with certainty. We assume that this fine is set by a higher authority in government, say the legislator, and is thus exogenous in this model.

We also assume that the agency derives some benefit  $\Delta$  from inspecting a *noncompliant* firm. For instance, the career perspectives of the agency's staff may depend on the number of detected noncompliant firms, or the staff may derive some moral satisfaction from fining noncompliant firms. Alternatively, the agency might have the authority to put a noncompliant in compliance during an inspection;  $\Delta$  then represents the environmental benefit (net of private compliance costs) of inspecting a noncompliant firm. In order to allow for this latter interpretation, we shall from now on assume that a firm that is found in noncompliance has to incur a fraction  $\sigma$  of the costs of purchasing the new abatement technology, where  $\sigma \in \{0, 1\}$  - note that Franckx (2001, a and b) only considers  $\sigma = 1$ . However, we shall assume that there is no redistribution of fines to the agency, so that  $\Delta$  is completely independent from  $\Psi$ .

If a firm complies, its expected costs are always  $\alpha$ . If a noncompliant firm is not inspected, its expected costs are zero. If a noncompliant firm is inspected, its expected costs are  $\Psi + \sigma\alpha$ . This implies immediately that if  $(1 - \sigma)\alpha > \Psi$ , then the firm will never comply, even if it is inspected with certainty. Therefore,

we shall from now on assume that  $\Psi > (1 - \sigma)\alpha$ .

The firms can choose between complying and not complying;  $p_i^\alpha$  is the probability that firm  $i$  complies. The agency commits itself to a permanent monitoring of ambient pollution, and can choose between inspecting and not inspecting an individual firm.  $p(i|k)$  is the probability that the agency inspects firm  $i$  if ambient inspections show that  $k$  firms comply.

We shall use the perfect Bayesian equilibrium (PBE) as solution concept. This means that each firm's strategy must be optimal, *given the agency's and the other firm's strategy*, but the agency's strategy must also be optimal, *given the firms' strategies*. Moreover, the agency's beliefs with respect to the actions the firms have undertaken must be obtained from the firms' equilibrium strategies and from the observation of ambient pollution levels, using Bayes' rule<sup>1</sup> - thus, the agency's strategies must be sequentially rational. We assume that the agency perfectly observes ambient pollution. This implies that it faces two singleton information sets (both firms comply, no firms comply) and one non-singleton information set (one firm complies).  $\mu_i$  is the agency's belief that firm  $i$  does not comply, given that ambient inspections show that one and only one firm complies and that the agency has not yet inspected any individual firm.

We shall only consider equilibria where the firms all play the same strategy.

Finally, if  $b > \Delta$ , the cost of inspecting one firm is higher than the maximal possible environmental benefit of inspecting that firm. The agency will then never inspect any individual firm. We shall from now on ignore this possibility.

### 3 Sequential inspections

The timing of the game is the most important change compared to Franckx (2001, a):

- Step 0 - The agency commits itself to a permanent observation of ambient pollution, and to the sequential inspection policy of step 3
- Step 1 - The firms simultaneously choose their abatement technology
- Step 2 - The inspection agency observes ambient levels
- Step 3 - As with simultaneous inspections, sequential rationality requires the agency to inspect all firms immediately if ambient inspections show that none complies<sup>2</sup> and not to inspect the firms if ambient inspections show that they all comply. If one and only one firm complies, the inspection agency decides whether or not it will inspect an individual firm. If it does not, then the game ends. If the agency inspects an individual firm and finds that the firm is not in compliance, then it levies the fine (and,

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<sup>1</sup>For a formal treatment of this concept, see for instance Fudenberg and Tirole (1995).

<sup>2</sup>If ambient inspections show that no firm complies, the inspection agency does not need to inspect the firms individually to identify the noncompliant ones. However, it can be doubted that a court would find this a sufficient proof to impose a fine, certainly if pollution is stochastic.

depending on the setting, it imposes the purchase of the compliant technology). After this firm inspection, the agency updates its beliefs that the other firm complies and finally decides whether it will inspect the second firm.

In this model, there is an explicit time dimension and we shall introduce a cost of delaying the inspection of noncompliant firms: if the agency inspects firm  $i$  first, then the benefit of inspecting firm  $j$  if it is noncompliant decreases to  $\gamma\Delta$  (for instance, because the environment has further deteriorated while the agency inspected firm  $i$ ).

We can now immediately turn to the three central results of this paper:

**Proposition 3.1** *If  $\gamma\Delta > b$ , then the following strategy-belief profile is a PBE:  $p(i|2) = p(j|2) = 0$ ;  $p(i|0) = p(j|0) = p(i|1) = 1$ ;  $p(j|1) = 1$  if firm  $i$  is inspected and complies and  $p(j|1) = 0$  if firm  $i$  is inspected and does not comply;  $p_i^\alpha = p_j^\alpha = 1$ . If  $2b > \gamma\Delta$ , then  $\mu_i > \frac{2b-\gamma\Delta}{b+(1-\gamma)\Delta}$ . If  $\gamma\Delta > 2b$ , then  $1 \geq \mu_i \geq 0$ .*

**Proof** We first show that the each firm's strategy is an optimal response to the agency's and the other firm's equilibrium strategy.

The agency's equilibrium strategy implies that it inspects firm  $i$  with certainty except if all firms comply. Firm  $i$  is then inspected every time it does not comply, which implies that firm  $i$ 's optimal reaction is to comply. If both the agency and firm  $i$  play their equilibrium strategy, then firm  $j$  will always be inspected if it does not comply. Indeed, if the non-singleton information set is reached, then the agency will find firm  $i$  in compliance and will also inspect firm  $j$ . But then firm  $j$  also optimally complies!

Thus, each firm's strategy is an optimal response to the agency's and to the other firm's equilibrium strategy.

Let us now turn to the agency's sequentially rational strategies in the non-singleton information set.

If the agency does not inspect the firms, then its expected costs in this information set are:

$$a + D_{nc} + D_c \tag{1}$$

Suppose that in the non-singleton information set, the agency inspects firm  $i$  first. With probability  $\mu_i$ , firm  $i$  does not comply, the agency puts it in compliance (and thus obtains benefit  $\Delta$ ) and levies the fine. The agency knows now that firm  $j$  complies and it is sequentially rational to stop the inspection game. If firm  $i$  complies (which happens with probability  $1-\mu_i$ ), then the agency knows that firm  $j$  does not comply. Because  $\gamma\Delta > b$ , it is also sequentially rational to inspect firm  $j$ .

Thus, if the agency inspects firm  $i$  first in the non-singleton information set, then its expected costs in this set are:

$$a + D_{nc} + D_c + b - \mu_i \Delta + (1 - \mu_i)(b - \gamma \Delta) \quad (2)$$

From Expressions 1 and 2, the agency will inspect firm  $i$  in the non-singleton information set if and only if:

$$b + (1 - \mu_i)b < \mu_i \Delta + (1 - \mu_i)\gamma \Delta \quad (3)$$

Suppose first that  $\gamma \Delta > 2b$ . Because  $\Delta > b$ , this implies immediately that Inequality 3 is fulfilled for any  $\mu_i$ .

If  $2b > \gamma \Delta$ , then  $1 > \gamma$  implies that Inequality 3 is fulfilled if and only if  $\mu_i > \frac{2b - \gamma \Delta}{b + (1 - \gamma)\Delta}$ . Because the non-singleton information set is only reached if one of the two firms deviates from its equilibrium strategy, we cannot impose any restriction on  $\mu_i$ , and it is always possible to find a  $\mu_i < 1$  that satisfies these restrictions (because  $\Delta > b$  implies  $1 > \frac{2b - \gamma \Delta}{b + (1 - \gamma)\Delta} > 0$ ).  $\square$  QED  $\square$

**Proposition 3.2** *If  $b > \gamma \Delta$  and  $\Delta > 2b$ , then the following strategy-belief profile is a PBE: If one firm complies: with probability  $q$ , the agency inspects firm  $i$  and does not inspect firm  $j$ ; with probability  $1 - q$ , the agency inspects firm  $j$  and does not inspect firm  $i$ ;  $q = \frac{1}{2}$ ;  $p(i|2) = p(j|2) = 0$ ;  $p(i|0) = p(j|0) = 1$ ;  $p_i^\alpha = p_j^\alpha = \min\{1, 2\frac{(\sigma-1)\alpha + \Psi}{\sigma\alpha + \Psi}\}$ ; if  $\Psi > (2 - \sigma)\alpha$ , then  $1 \geq \mu_i \geq 0$ ; if  $(2 - \sigma)\alpha > \Psi$ , then  $\mu_i = \frac{1}{2}$ .*

**Proof** We first show that the each firm's strategy is an optimal response to the agency's and the other firm's equilibrium strategy.

Given the agency's equilibrium strategy, firm  $i$ 's expected costs are:

$$p_i^\alpha \alpha + (1 - p_i^\alpha)[qp_j^\alpha + (1 - p_j^\alpha)](\sigma\alpha + \Psi)$$

Indeed, if firm  $i$  does not comply, it is inspected with probability  $q$  if firm  $j$  complies and it is inspected with certainty if firm  $j$  does not comply. Thus, given the agency's strategy, firm  $i$  will be indifferent with respect to the choice of  $p_i^\alpha$  if and only if  $qp_j^\alpha + (1 - p_j^\alpha) = \frac{\alpha}{\sigma\alpha + \Psi}$ .

Similarly, given the agency's strategy, firm  $j$  will be indifferent with respect to the choice of  $p_j^\alpha$  if and only if  $(1 - q)p_i^\alpha + (1 - p_i^\alpha) = \frac{\alpha}{\sigma\alpha + \Psi}$ .

If the firms play the same strategy,  $(1 - q)p_i^\alpha + (1 - p_i^\alpha) = qp_j^\alpha + (1 - p_j^\alpha)$  is only possible if  $q = \frac{1}{2}$ .

This implies immediately:  $p_i^\alpha = p_j^\alpha = 2\frac{(\sigma-1)\alpha + \Psi}{\sigma\alpha + \Psi}$ . Of course, if  $\Psi > (2 - \sigma)\alpha$ , then  $p_i^\alpha = p_j^\alpha = 1$ .

Let us now turn to the agency's sequentially rational strategies in the non-singleton information set.

Suppose that the agency has inspected a first firm after observing that one and only one firm complies.  $b > \gamma \Delta$  implies that it is not sequentially rational to inspect a second firm whether or not the first firm was found in compliance.

Thus, if the agency plays its equilibrium strategy in the non-singleton information set, then its expected costs in this set are:

$$\begin{aligned} a + D_{nc} + D_c + q[b - \mu_i \Delta] + (1 - q)[b - (1 - \mu_i) \Delta] &= \\ a + D_{nc} + D_c + b - q\mu_i \Delta - (1 - q)(1 - \mu_i) \Delta & \end{aligned} \quad (4)$$

From Expressions 4 and 1, the agency will play its equilibrium strategy in the non-singleton information set if and only if:

$$b < q\mu_i \Delta + (1 - q)(1 - \mu_i) \Delta$$

$q = \frac{1}{2}$  implies that this condition is fulfilled if and only if  $\Delta > 2b$ , independently from  $\mu_i$ .  $\square$  QED  $\square$

**Proposition 3.3** *If  $2b > \Delta > b > \gamma\Delta$ , then the following strategy-belief profile is a PBE:  $p(i|2) = p(j|2) = p(i|1) = p(j|1) = 0$ ,  $p(i|0) = p(j|0) = 1$ ,  $p_i^\alpha = p_j^\alpha = \frac{(\sigma-1)\alpha+\Psi}{\sigma\alpha+\Psi}$  and  $\mu_i = \frac{1}{2}$ .*

**Proof** Given the agency's and firm  $j$ 's equilibrium strategy, firm  $i$ 's expected costs are:

$$p_i^\alpha \alpha + (1 - p_i^\alpha)(1 - p_j^\alpha)(\sigma\alpha + \Psi)$$

Indeed, if firm  $i$  does not comply, then it is only inspected if firm  $j$  does not comply either. Thus, given the agency's strategy, firm  $i$  will be indifferent with respect to the choice of  $p_i^\alpha$  if and only if  $p_j^\alpha = \frac{(\sigma-1)\alpha+\Psi}{\sigma\alpha+\Psi}$ .

Similarly, given the agency's strategy, firm  $j$  will be indifferent with respect to the choice of  $p_j^\alpha$  if and only if  $p_i^\alpha = \frac{(\sigma-1)\alpha+\Psi}{\sigma\alpha+\Psi}$ .

The agency's sequentially rational strategies follow from the Proof of Proposition 3.2.  $\square$  QED  $\square$

## 4 Comparison with simultaneous firm inspections

How do the results compare to Franckx (2001, a)? It can easily be verified that all the main results of that paper can be extended to a context where  $\sigma \neq 1$ . The comparison with this analysis is then straightforward - see Table I. In the left column, we summarize the results for sequential firm inspections. In the right column, we summarize the results obtained in Franckx (2001, a).

We see that if  $2b > \Delta > b$  and  $\gamma\Delta > b$ , then sequential inspections induce perfect compliance, while simultaneous inspections induce the firms to play mixed strategies. Moreover, with a sequential inspection policy, there will never be firm inspections in equilibrium.

Also, for some parameter values, the probabilities of compliance do not change (for instance, if  $b > \gamma\Delta$ ,  $\Delta > 2b$  and  $\Psi > (2 - \sigma)\alpha$ , or if and  $2b > \Delta > b > \gamma\Delta$ ). It can easily be verified that the equilibrium probability that the agency will conduct firm inspections stays unchanged as well.

However, if  $b > \gamma\Delta$ ,  $\Delta > 2b$  and  $\Psi < (2 - \sigma)\alpha$ , then sequential inspections lead to a decrease in the probability of compliance. Indeed,  $\Delta > 2b$  implies that the agency is better off if it inspects both firms simultaneously in the non-singleton information set than if it does not inspect them at all. Thus, if the firms know that the agency conducts simultaneous firm inspections, they will both comply. However, with sequential firm inspections, the cost of waiting to put firms in compliance is so high ( $b > \gamma\Delta$  and  $\Delta > 2b$  is only possible if  $\gamma < \frac{1}{2}$ ) that the agency never inspects a second firm in the non-singleton information set. Thus, in equilibrium, the probability that a firm gets inspected (and thus also the expected cost of noncompliance) decreases, for any given probability that the other firm complies. If the fine is low enough compared to the cost of compliance ( $\Psi < (2 - \sigma)\alpha$ ), then firm  $i$  will never comply if firm  $j$  always complies - this explains why, in equilibrium, the firms play mixed strategies.

These mixed strategies can be given a very natural interpretation in the context of this game. Harsanyi's (1973) purification theorem implies that the PBE in the inspection game with complete information can be interpreted as a pure-strategy PBE in a game with the same structure where the environmental effect of noncompliance is deterministic but where a firm found in noncompliance faces costs that are only observed by this firm (for instance, administrative costs linked to the payment of the fines) - the random changes in these costs imply that each firm will comply with the frequency that makes the other firm indifferent between complying and not complying.

## 5 Conclusion

This paper shows that, except if the cost of waiting to put firms in compliance is too high, conducting sequential firm inspections (rather than simultaneous ones) reinforces the argument in favor of ambient inspections. However, if the cost of delaying the inspection of noncompliant firms is too high, then the agency is better off if it inspects both firms simultaneously, and deliberately decides not to gather better information. Thus, the agency can be better off if it commits itself to ignoring some of the information embedded in ambient pollution.

It may seem very restrictive to consider only two polluting firms. However, the analysis with simultaneous inspection has been extended to a setting with an arbitrary number of firms - detailed results are available from the author on request. Franckx (2001, b) has conducted the analysis with sequential firm inspections and three polluting firms, but without cost of waiting. Not surprisingly, in that setting, sequential firm inspections always dominate simultaneous firm inspections. Preliminary results suggest that introducing explicit time considerations in a model with three firms adds a lot in analytic complexity but does not provide new insights compared to the analysis in the present paper.



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Table I: Simultaneous versus sequential firm inspections

	Sequential firm inspections	Simultaneous firm inspections
Perfect compliance	$\gamma\Delta > b$ $b > \gamma\Delta, \Delta > 2b$ and $\Psi > (2 - \sigma)\alpha$	$\Delta > 2b$
$p_i^\alpha = p_j^\alpha = 2 \frac{(\sigma-1)\alpha + \Psi}{\sigma\alpha + \Psi}$	$b > \gamma\Delta, \Delta > 2b$ and $\Psi < (2 - \sigma)\alpha$	never
$p_i^\alpha = p_j^\alpha = \frac{(\sigma-1)\alpha + \Psi}{\sigma\alpha + \Psi}$	$2b > \Delta > b > \gamma\Delta$	$2b > \Delta > b$