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Determinacy and Learnability of Monetary Policy Rules in Small Open Economies

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Abstract[1]

This paper evaluates under which conditions different Taylor-type rules lead to determinacy and expectational stability (E-stability) of rational expectations equilibrium in a simple "New Keynesian" small open economy model, developed by Gali and Monacelli (2005). In particular, we extend the Bullard and Mitra (2002) results of determinacy and E-stability in a closed economy to this small open economy framework. Our results highlight an important link between the Taylor principle and both determinacy and learnability of equilibrium in small open economies. More importantly, the degree of openness coupled with the nature of the policy rule adopted by the monetary authorities might change this link in important ways. A key finding is that, contrary to Bullard and Mitra, expectations-based rules that involve the CPI and/or the nominal exchange rate limit the region of E-stability and the *Taylor Principle does not guarantee E-stability*. We also show that some forms of managed exchange rate rules can help to alleviate problems of both indeterminacy and expectational instability, yet these rules might not be desirable since they promote greater volatility in the economy.

Keywords: Learning; Indeterminacy; Monetary Policy Rules; Open Economy
JEL classification: E4; E5; F31: F41

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1. Introduction

The implementation of monetary policy in terms of interest rate feedback rules has been extensively studied in recent research in both closed and open economies contexts. In practice many central banks, most of them in an environment of open economy, have recently adopted inflation-targeting regimes where the policy implementation requires a particular policy feedback-rule.[2]

In open economies the design of the policy rule and, in particular, the choice between consumer and domestic price indexes is key for the implementation of monetary policy. Most "Iters'" open economies define their goals in terms of either consumer price inflation (CPI) or any "adjusted measure" of CPI, implying that the dynamics of the targeted variable not only incorporates the movements of domestic inflation but also responds to changes in the exchange rate and world inflation. An additional concern is whether movements in the exchange rate have to be included in the policy rule besides the standard elements such as inflation and the output gap; see Taylor (2001). For example, Lubik and Schorfheide (2006) have found robust evidence, by using Bayesian structural estimation, that the Bank of Canada and the Bank of England include the nominal exchange rate in their policy rules.[3] Yet, the role of these rules in stabilizing the economy has been criticized recently on many grounds.

One major problem is the issue of whether a policy rule guarantees real determinacy. In a closed economy context, the usual condition for determinacy is the so-called "Taylor Principle" see Woodford (2003b). In particular, this condition suggests that if the nominal interest rate is adjusted positively, and more than one-for-one, in response to inflation movements above its target, and positively to output above target, a determinate Rational Expectation Equilibrium (REE) is attainable. Taylor's intuition is that under such a rule, a rise in inflation brings about an increase in the real interest rate, which reduces demand, and inflationary pressures, bringing the economy back towards the targeted equilibrium. Nevertheless, it has been stressed that some

[2] Nowadays, the list of Inflation Targeters (ITers) covers more than 20 central banks and the number of central banks which could potentially adopt such a policy in the future is non-negligible. See Vega and Winkelried (2005) for a detailed list of both developed and developing countries that are implementing an IT regime.

[3] This is a widespread feature among developing countries as is documented by Calvo and Reinhart (2002). They coined the phrase "Fear of Floating," referring to those Central Banks that systematically tend to defend their exchange rates by increasing interest rates.

kind of Taylor-type rules can induce real indeterminacy with undesirable properties under a rational expectation environment even if the Taylor Principle holds.[4]

A second major issue is that the interest rate feedback rules might not perform satisfactorily if we relax the assumption of rational expectations by assuming that agents follow a learning process. A particular concern to this literature is the notion of *Expectational Stability* (or E-stability) developed by Evans and Honkapohja (1999, 2001): the conditions under which agents are able to learn the reduced form dynamics induced by the model given a monetary policy rule under the assumption of rational expectations. Even when a determinate equilibrium exists, coordination at that equilibrium cannot be assured if the assumption of rational expectations is relaxed. E-stability therefore provides a robustness criterion: if agents make small mistakes in expectations relative to those consistent with the associated REE, then a policy rule that is E-stable ensures such mistakes are corrected over time.

Recently, Evans and Honkapohja (2003) evaluate the issue of E-stability under optimal rules, finding that the optimal rule under discretion is unstable if agents follow adaptive learning.[5] Similarly, Bullard and Mitra (2002, hereafter BM) have shown that if agents follow adaptive learning rules, then the stability of the Taylor-type rules might not be taken for granted. Yet, their results support the Taylor principle based on the learnability criteria. In particular, they find that if the monetary authority is able to commit to a Taylor-type interest rate rule, the REE is E-stable under learning dynamics as long as the Taylor principle is satisfied.

What is clear from the above discussion is that, in open economies, a central bank can implement its policy on a broader variety of instrument rules such as CPI target, domestic inflation target, managed exchange rate targets and, hence, a rational expectation equilibrium can be learnable or not under different conditions. In a closed economy there is no difference between domestic inflation and CPI inflation target, nor is there room for a managed exchange rate rule target. Thus, trade openness modifies the way that any shock is transmitted to domestic variables and therefore the findings concerning determinacy and learnability for the closed economy might be altered.

[4] See Bernanke and Woodford (1997) and Woodford (1999) for a closed economy case, Batini et al. (2004) for a two-country model, and De Fiori and Liu (2005) for the small open economy counterpart.

[5] They also show that this problem can be solved if private expectations are observed and suitably incorporated into the policymaker's optimal rule.

Thus, the goal of this paper is to evaluate the effects of trade openness coupled with a variety of Taylor-rules on the stability of an economy. We study both local determinacy and learnability properties of the Rational Expectations Equilibrium (REE) in the small open economy model proposed by Gali and Monacelli (2005, henceforth GM). The methodology for the learning analysis is the one postulated by Evans and Honkapohja (2001). In particular, we evaluate whether agents in a small open economy can learn the fundamental equilibrium of the system induced by different classes of Taylor-type feedback rules. We use the criterion of expectational stability to calculate whether rational expectations equilibria are stable under real time recursive learning dynamics.[6] In this sense, our work extends BM's (2002) closed-economy results to a small open economy framework.

We perform the analysis of real determinacy and learning under four simple monetary policy feedback rules. The two first rules are typical Taylor rules. For the first one we assume that the central bank adjusts the short-term interest rate by responding systematically to domestic inflation and the output gap. For the second rule it is assumed that the central bank targets CPI inflation instead of domestic inflation. The third and fourth rules modify the two previous rules by adding a reaction to movements in nominal exchange rate. Thus, the third rule combines a reaction to CPI inflation, the output gap and the change in the nominal exchange rate. The fourth rule supplements the first rule with a reaction to movements in the exchange rate as well. Following closely BM (2002) we evaluate the aforementioned rules under two specifications based on the way the central bank and private agents form expectations. In the first one, the monetary authority reacts to current values; this is called *contemporaneous data* specification. Our second specification assumes that policymakers react to forecasts; this is called *forecast-based rule* specification.

In general, our results highlight an important link between the Taylor Principle and both determinacy and learnability of REE. Yet, the degree of openness coupled with the nature of the policy rule adopted by the monetary authorities might change this link in important ways. A key finding is that, contrary to BM (2002), expectations-based rules that involve the CPI and/or the nominal exchange rate limit the region of E-stability, and the Taylor Principle does not guarantee

[6] Evans and Honkapohja (2001) have shown that local convergence of real time recursive algorithms for a variety of models is governed by the expectational stability of the rational expectation equilibrium.

E-stability. We find analytical results for determinacy and E-stability in most of the rules considered.

The main findings of our analysis can be summarized as follows: Under *forecast-based rules*, openness alters the conditions of both determinacy and learnability rules with respect to the closed economy case. The striking result is that unlike BM (2002), forward-looking policy rules that react aggressively to CPI inflation with little or no reaction to the output gap do not necessarily induce both determinate and learnable rational expectation equilibria. For instance, in an open economy environment, it is more likely that the economy lies in an E-unstable region. Openness adds an upper bound to a reaction to inflation. This main finding carries over to the managed exchange rate case as well. Therefore, under both CPI inflation targeting and managed exchange rate rule, there can be important limits on how aggressive a central bank may wish to be with respect to inflation in an open economy setting. Interestingly, domestic inflation targeting with or without a reaction to movement to the nominal exchange rates does not have this flavor. Instead, the analytical results under the previous rules suggest that more aggressive reaction to domestic inflation is all to the good as in the closed economy case suggested by BM. One important implication of these results is that the pure application of the Taylor Principle in open economies could be misleading advice if policymakers target CPI inflation or stabilize exchange rate movements in a forward-looking fashion.

With *contemporaneous rules*, openness affects the determinacy and learnability conditions only quantitatively. Monetary policy rules of this type can easily induce a determinate equilibrium. Moreover, when equilibrium is determinate it is also learnable. The quantitative impact of openness is ambiguous, depending mainly on the degree of elasticity of substitution between foreign and domestic goods.[7] More importantly, conditions for a unique and learnable REE do not depend on whether the central bank responds to domestic or CPI inflation, i.e., the Taylor Principle is a necessary and sufficient condition under both policies. Interestingly, we find that the monetary policy authority in a small open economy can substitute CPI inflation stabilization to some degree for exchange rate smoothing when rules are contemporaneous. That

[7] Central Banks in open economies face a monetary transmission mechanism that differs from the one prevailing in the closed economy counterpart. The standard channel in closed economy models is the intertemporal substitution effect, whereas in open economy the substitutability between goods of different origin becomes relevant, activating the so-called terms of trade effect. In general, the terms of trade effect tends to reinforce the intertemporal substitution effect as long as domestic and foreign goods are substitutes, as GM pointed out.

is, even if the policymakers do not react sufficiently forcefully to CPI inflation they can still induce determinacy and learnability of equilibrium by reacting sufficiently forcefully to the nominal exchange rate. Therefore, policymakers would face a trade-off in the case of contemporaneous rules: a managed exchange rate regime is more suitable than other monetary rules because it increases the areas of determinacy and learnability but at the same time empowers macroeconomic volatility because it might impede the economy's adjustment to fundamental shocks.

Which rule is the more desirable for a small open economy? Under *contemporaneous data* specifications, there is no difference between targeting either current values of domestic or CPI inflation. Indeed, both rules can induce a determinate and learnable equilibrium to the extent that the Taylor Principle holds. However, previous rules augmented with a reaction to movements in the nominal exchange rate might be more suitable since they alleviate problems of indeterminacy and expectational instability. However, these rules, as shown in GM, could generate greater macro volatility. On the other hand, under *forecast-based data specification,* we show that targeting domestic inflation avoids potential expectational unstable problems which arise when the central bank targets either CPI inflation and movements in the exchange rate or both.

Interestingly, when a *forecast-based* domestic inflation Taylor rule is augmented by targeting movements in the nominal exchange rate, the regions of both determinacy and *E-stability* become larger, promoting learnability of the equilibrium. This result can be interpreted as if this type of rule has desirable determinacy and learnability properties, and therefore, it might be an important reason why central banks in small open economies target movements in the exchange rate. Yet, the previous argument should be taken cautiously. In this regard, in Section 4 we show analytically that even though a *forecast-based* domestic inflation Taylor rule supplemented by targeting movements in the nominal exchange rate might have desirable properties in terms of *E-stability*, conditional on the source of shocks, it could also generate larger volatility in the economy (with respect to a rule that does not target changes in the exchange rate). In particular, our analytical results confirm that, if the economy is hit by shocks to the natural interest rate (i.e., productivity), the aforementioned type of rule will be desirable in terms of both *E-stability* properties and macroeconomic volatility (the unconditional volatility of the output gap and domestic inflation are smaller than otherwise). On the contrary, if the economy is hit by a foreign interest rate shock, this type of rule promotes greater volatility in the output gap and domestic inflation and consequently is less desirable compared to rule that does not react to movements in the exchange rate.[8] We conclude that it is worthwhile to recommend not only rules that are desirable in terms of determinacy and learnability properties, but also those that induce benefits in terms of macroeconomic volatility.

The contribution of this paper is twofold. First, we obtain not only *analytical* conditions of determinacy, but also of learnability. Second, our analysis relies on a broad set of policy rules for small open economies, including those supported by the data, e.g., Taylor rules with managed exchange rates.[9] In that sense, our paper contributes to a growing literature that has been studying stability issues in a open economy context. In an small open economy version of the Cooley and Hansen (1989) model, De Fiori and Liu (2005) find that the conditions for determinacy depend crucially on the degree of openness to international trade in both flexible and sticky price specifications. Zanna (2003) in a model with tradable and nontradable goods find similar results regarding to the role of openness. Similarly, focusing on determinacy, Batini et al. (2004) study forward-looking policy rules along with interest rate inertia for different forecast horizons in a two-country model. Their results point out that potential local indeterminacy is exacerbated in the open economy regardless of whether CPI or domestic inflation enters in the policy rule. In a multiple-large-economies model, Bullard and Singh (2006) show *numerically* that the open economy setting puts an important upper bound on the reaction of a central bank with respect to expected CPI inflation deviations in order to guarantee determinacy. Our results with forward looking rules show *analytically* the existence of such a remarkable limit on the aggressiveness of policymakers in open economies. Furthermore, this upper limit arises not only for conditions of determinacy but also for E-stability.

A closer paper to ours is that of Bullard and Schaling (2006) who study determinacy and learnability in a two-country model under both instrument and target rules. Some their results regarding instrument rules parallel ours. For example, they show, as we do, that with *contemporaneous* domestic or CPI inflation targeting, openness alters determinacy and learning conditions at least numerically. Yet, the Taylor Principle is a necessary and sufficient condition

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[8] GM's findings show that a perfect peg rule enhances greater macroeconomic volatility in terms of output and inflation. Yet, they analyze the extreme case of a perfect peg. Our analytical results focus on a flexible managed exchange rate, and we show that this kind of rule does not necessarily induce greater volatility. Volatility will instead depend on the source of shocks.

to be met independently in both home and foreign economies. Still, we study instrument rules more extensively than Bullard and Schaling (2006), including different specifications (contemporaneous and forecast-based) of Taylor-type rules.

The rest of the paper is organized as follows. Section 2 outlines the simple environment for the analysis of determinacy and learning. Here we specify the main equations of the GM (2005) model, emphasizing its differences with respect to the closed economy case. After that, we describe the different specifications of monetary policy rules and methodology. The analysis of determinacy and E-stability is addressed in Section 3. In Section 4 we solve analytically the rational expectations equilibrium of the small open economy in order to establish a link between the conditions that guarantee E-stability and the implied macroeconomic volatility induced by two types of rules, namely, the domestic inflation Taylor rule and the previous rule supplemented with a target to movements in the exchange rate. Finally, Section 5 concludes.

2. The Simple Environment

We study a simple small open economy model developed by GM (2005). The model is built up by assuming a small open economy with staggered prices *à la* Calvo (1983) as one among a continuum of (infinitesimally small) economies making up the world economy. One interesting property of GM's model is that it is isomorphic to the workhorse sticky price model of a closed economy of Woodford (2003b). More specifically, GM's model is identical to the closed economy model if the degree of openness collapses to zero. This feature allows us to isolate the effects of openness and study its interaction with monetary policy. The main purpose is to obtain conditions that are necessary and sufficient to guarantee a determinate and E-stable equilibrium and assess the roles of openness and monetary policy on these grounds.

2.1 The Model

In this section we briefly present and discuss the main equations of the GM (2005) framework. Before proceeding with the exposition of the model, we describe some useful notation used throughout the paper. Subscript H denotes any *domestic* or *home* variable, subscript F denotes

[9] See Lubik and Schorfheide (2006).

foreign or *imported* variables (in domestic currency), superscript `‾' denotes variables in their natural levels, and superscript * denotes *international* or *world* variables.

The small open economy is log-linearized at a steady state and can collapse to the following two equations, (equations 36 and 37 of GM),

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_\alpha x_t$$
$$x_t = E_t x_{t+1} - \frac{1}{\sigma_\alpha}(r_t - E_t \pi_{H,t+1} - \overline{rr}_t)$$
(1) and (2)

where

$\lambda = [(1-\beta\theta)(1-\theta)/\theta], \kappa_\alpha = \lambda(\sigma_\alpha + \varphi), \sigma_\alpha = \frac{\sigma}{[1-\alpha+\alpha\omega]}$,

and $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$.

The variables x_t, $\pi_{H,t}$ and r_t represent the *domestic output gap*, *domestic inflation*, and *domestic interest rate*, respectively. In the model $\overline{rr}_t$ is the small open economy's natural level of real interest rate, and E_t symbolizes the standard expectation operator. We implicitly base our analysis of learning and monetary policy on a "Euler Equation" approach, as suggested in Honkapohja et al. (2003). Therefore, throughout the paper we assume that our systems are valid under both rational expectations and learning. In this sense, the expectation operation is taken to describe aggregate behavior regardless of the precise nature of agents' expectation formation. Recently, Preston (2005) has proposed an interesting reformulation of intertemporal behavior under learning in which agents are assumed to incorporate a "subjective version" of their intertemporal budget constraint into their behavior under learning. In this paper, we abstract from this approach.

Equation (1) is a *new Keynesian Phillips curve* (NKPC) and equation (2) is a dynamic IS-type. Both equations involve several deep parameters. The parameter β denotes the discount factor, σ is the elasticity of intertemporal substitution (or the inverse of risk aversion), φ is the inverse of labor supply elasticity, η is the elasticity of substitution between domestic and foreign goods, γ is the elasticity of substitution between imported goods, α is the inverse of home bias in preferences and can be interpreted as a natural index of trade openness, and θ is the degree of price stickiness.

Notice that the coefficients κ_α and σ_α depend on parameters that are specific to the open economy, i.e., the degree of openness and the substitutability among goods of different origin. On one hand, the degree of openness, α, affects the dynamics of domestic inflation only through its influence on the size of the slope of the Phillips curve, i.e., the size of response to any given variation in the output gap. In the open economy, a change in domestic output has an effect on marginal cost through its impact on employment (captured by φ) and the terms of trade (captured by σ_α). In particular, under the assumption that $\sigma\eta > 1$, an increase in openness dampens the impact of the adjustment on inflation after an output gap shock. On the other hand, the degree of openness influences the sensitivity of the output gap to interest rate changes. In particular, if $\sigma\eta > 1$, an increase in openness raises that sensitivity through the stronger effects of the induced terms if trade changes on demand.

Considering the definitions of κ_α and σ_α given above, a special case arises. When the small open economy is totally autarkic (α is zero), σ_α reduces to σ. In this case, equations (1) and (2) collapse to the standard closed economy model of Woodford (2003b).[10]

Under the assumption of complete international financial markets, GM (2005) obtain a version of the uncovered interest parity condition. Log-linearizing around a perfect foresight steady state,

$$\Delta E_t e_{t+1} = r_t - r_t^* \quad (3)$$

where e_t is the nominal exchange rate and r_t^* is the world interest rate, equation (3) implies that an expected depreciation (appreciation) of the nominal exchange rate is necessary to counterbalance any positive (negative) difference between the domestic interest rate and the world interest rate.

Let us define the log level of terms of trade s_t as,

$$s_t = p_{Ft} - p_{Ht} \quad (4)$$

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[10] Another case discussed by GM (2005) is when $\sigma = \eta = \gamma = 1$, which implies $\omega = 1$. Under this case, there is a balance of trade at all times.

where $p_{F,t}$ and $p_{H,t}$ are the log level of foreign prices and domestic prices, respectively. Given that it is straightforward to obtain an expression for the rate of change in the terms of trade, i.e., $\Delta s_t = s_t - s_{t-1}$,

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} \quad (5)$$

where $\pi_{F,t} = p_{F,t} - p_{F,t-1}$ is foreign inflation and $\pi_{H,t} = p_{H,t} - p_{H,t-1}$. Combining the last equation with equation (14) of GM (2005), it is a matter of a few algebraic operations to obtain the following definition of CPI inflation,

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} \quad (6)$$

where $\pi_t = p_t - p_{t-1}$ is CPI inflation. This makes CPI inflation a weighted average between domestic and foreign inflation in domestic currency, where the weighting factor is the degree of openness.

Assuming that the law of one prices holds for individual goods at all times, GM (2005) shows that

$$p_{F,t} = e_t + p_t^* \quad (7)$$

with p_t^* representing the log level of the world price index. This equation also implies that

$$\pi_{F,t} = \Delta e_t + \pi_t^* \quad (8)$$

From (5) and (8) it follows that the rate of change in the terms of trade, the rate of change of nominal exchange rate, domestic inflation and world inflation are linked according to

$$\Delta s_t = \Delta e_t + \pi_t^* - \pi_{H,t} \quad (9)$$

Let us define the terms of trade gap $\hat{s}_t$ as the deviation of (log) domestic terms of trade s_t from its natural level $\overline{s}_t$, where the latter is in turn defined as the equilibrium level of terms of trade in the absence of nominal rigidities. Formally,

$$\hat{s}_t = s_t - \overline{s}_t \quad (10)$$

Using this definition of terms of trade gap and equation (9) we have

$$\hat{s}_t = \hat{s}_{t-1} + \Delta e_t + \pi_t^* - \pi_{H,t} + \Delta\overline{s}_t. \quad (11)$$

Manipulating equations (29) and (34) from GM (2005), we obtain an equivalence between the output gap and the terms of trade gap,

$$\hat{s}_t = \sigma_\alpha x_t \quad (12)$$

Without loss of generality, it is assumed that world variables (r_t^*, π_t^*) are constant and equal to their steady state level. For the sake of simplicity, we further assume that the world steady state level is centered at zero for both world variables. Additionally, as in BM (2002), domestic variables at their natural levels $(\overline{rr}_t, \Delta\overline{s}_t)$ are driven by exogenous and mutually independent first-order autoregresive processes. We keep this assumption on the basis that $(r_t^*, \pi_t^*, \overline{rr}_t, \Delta\overline{s}_t)$ cannot be affected by the small open economy's policies or aggregate performance around its local equilibrium.

2.2 Simple Taylor Rules

We supplement equations (1) through (12) with a policy rule for the domestic interest rate r_t that represents the behavior of the monetary authority. We consider a handful of possible Taylor-type feedback rules with different sets of target variables. All the feedback rules have two alternative specifications: *contemporaneous data* and *forecast-based data*. In the first type, the interest rate reacts to information observed at time t, that is, current inflation (domestic or CPI), domestic output gap and/or nominal exchange rate changes. In the forecast-based specification, interest rates react to one period ahead expectation of the targeted variables.[11]

1. *Domestic Inflation Taylor Rule (DITR).* We first consider a "simple" rule similar to the one proposed by the seminal work of Taylor (1993). In this rule, it is assumed that the central bank adjusts the domestic interest rate by responding systematically to both (contemporaneous or expected) domestic inflation and the domestic output gap

[11] For a general discussion about this class of policy setting, see Battini and Haldane (1999). Empirical evidence also suggests that central banks indeed set their interest rate in a forward-looking fashion.

$$r_t = \phi_\pi \pi_{H,t} + \phi_x x_t$$
$$r_t = \phi_\pi E_t \pi_{H,t+1} + \phi_x E_t x_{t+1}$$ (13) and (14)

where ϕ_π and ϕ_x are non-negative parameters and measure the aggressiveness of monetary policy response to any deviation of (contemporaneous or expected) domestic inflation and output gap from their target values, respectively.

2. *CPI Inflation Taylor Rule (CPITR).* For the second feedback rule, it is assumed that the central bank targets CPI inflation, rather than domestic inflation, and the domestic output gap. This type of rule seems to be more realistic among actual central banks adopting inflation targeting regimes. Under this specification, domestic inflation in (13) and (14) is replaced by CPI inflation

$$r_t = \phi_\pi \pi_t + \phi_x x_t$$
$$r_t = \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1}$$ (15) and (16)

Notice that the inclusion of CPI inflation in the policy rule implies an indirect response of the interest rate to foreign inflation. The sensitivity of the interest rate to foreign inflation shocks is given by the Central Bank's aggressiveness towards any deviation (contemporaneous or expected) of CPI inflation from its target and the size of openness. Moreover, this rule also implies an indirect reaction to the movements of the nominal exchange rate.

3. *CPI Managed Exchange Rate Taylor Rule (CPI-METR).* Along the same line as Taylor (2001), we focus on an open economy interest rate reaction function where the central bank reacts changes in the exchange rate next to CPI inflation rate and the domestic output gap,

$$r_t = \phi_\pi \pi_t + \phi_x x_t + \phi_e \Delta e_t$$
$$r_t = \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1} + \phi_e E_t \Delta e_{t+1}$$ (17) and (18)

where ϕ_e captures the endogenous response of the Central Bank to (contemporaneous or expected) changes in the nominal exchange rate. As the other policy parameters, we restrict the value of ϕ_e to be non-negative. Lubik and Schorfheide (2006) have found robust evidence that the Bank of Canada and the Bank of England follow similar policy rules.

4. *Domestic Inflation Managed Exchange Rate Taylor Rule (DI-METR)*. The fourth representation adds to the first rule considered a reaction to movements in the nominal exchange rate

$$r_t = \phi_\pi \pi_{H,t} + \phi_x x_t + \phi_e \Delta e_t$$
$$r_t = \phi_\pi E_t \pi_{H,t+1} + \phi_x E_t x_{t+1} + \phi_e E_t \Delta e_{t+1} \quad (19) \text{ and } (20)$$

These two rules are isolated from the effects of nominal exchange rate and openness on the CPI index. Instead, they focus only on those characteristics added by a direct reaction to movements of the nominal exchange rate.

2.3 Methodology

Consider a model given by the general form

$$y_t = \Gamma + \Omega E_t y_{t+1} + \Phi y_{t-1} + \Theta w_t$$
$$w_t = \rho w_{t-1} + \varepsilon_t \quad (21) \text{ and } (22)$$

where y_t is an $nx1$ vector of endogenous variables, Γ is an $nx1$ vector of constants, Ω, Φ, Θ and ρ are n x n matrices of coefficients, and w_t is an $nx1$ vector of exogenous variables which is assumed to follow a stationary VAR, so that ε_t is an $nx1$ vector of white noise errors.

The first issue of concern is under which circumstances a policy rule guarantees a unique or determinate rational expectations equilibrium. The criterion for this purpose is to ask whether a system such as (21) has the right number of eigenvalues inside the unit circle given the number of free and predetermined variables.[12] The second issue is the study of conditions for REE to be learnable under different policy rules. Here, we follow closely the criterion of *Expectational Stability* (or E-stability) developed by Evans and Honkapohja (1999, 2001). Under learning, the agents do not have rational expectations; instead agents form their expected values with adaptive rules which are updated as data is produced by the system.

[12] For details see Blanchard and Kahn (1980).

Consider the Minimal State Variable (MSV) solution (see McCallum, 1983) of (21) and (22) which takes the following form,

$$y_t = a + by_{t-1} + cw_t \quad (23)$$

Taking expectations of (23), $E_t y_{t+1} = a + by_t + c\rho w_t$, and replacing it in (21), we can solve for (a, b, c) by applying the method of undetermined coefficients.

$$(I - \Omega b - \Omega)a = \Gamma$$
$$\Omega b^2 - b + \Phi = 0$$
$$(I - \Omega b)c - \Omega c\rho = \Theta. \quad (24), (25) \text{ and } (26)$$

Under learning, the MSV solution (23) is known as the *Perceived Law of Motion* (PLM) of the agents. Using it to form the next period expectation, $E_t y_{t+1} = a + by_t + c\rho w_t$, we can compute the *Actual Law of Motion* (ALM),

$$y_t = (I - \Omega b)^{-1}(\Gamma + \Omega a) + (I - \Omega b)^{-1}\Phi y_{t-1} + (I - \Omega b)^{-1}(\Omega c\rho + \Theta)w_t. \quad (27)$$

To analyze the E-stability conditions, we have to check the stability of the mapping T from the PLM to ALM,

$$T(a, b, c) = ((I - \Omega b)^{-1}(\Gamma + \Omega a), (I - \Omega b)^{-1}\Phi, (I - \Omega b)^{-1}(\Omega c\rho + \Theta)). \quad (28)$$

The answer of the question of whether the system is stable under learning is given by the principle of E-stability, which comes from analyzing the following matrix differential equation,

$$\frac{\partial T(a, b, c)}{\partial \tau} = T(a, b, c) - (a, b, c) \quad (29)$$

where τ is a notional time. The E-stability conditions are derived in Evans and Honkapohja (2001, p. 238), proposition 10.3,[13]

[13] Those conditions correspond to t dating expectations, which assumes that agents have access to an information set including w_t and y_t. Other information set corresponds to $t-1$ dating expectations. For further details see Chapter 10 of Evans and Honkapohja (2001).

$$DT_a(\bar{a},\bar{b}) = (I-\Omega\bar{b})^{-1}\Omega$$
$$DT_c(\bar{b},\bar{c}) = \rho' \otimes \left[(I-\Omega\bar{b})^{-1}\Omega\right]$$
$$DT_b(\bar{b}) = \left[(I-\Omega\bar{b})^{-1}\Phi\right]' \otimes (I-\Omega\bar{b})^{-1}\Omega. \quad (30), (31) \text{ and } (32)$$

The rational expectation solution $(\bar{a},\bar{b},\bar{c})$ is *E-stable* or *learnable* if all real parts of the eigenvalues of $DT_a(\bar{a},\bar{b}), DT_b(\bar{b}), DT_c(\bar{b},\bar{c})$ are lower than 1. The solution is *E-unstable* if any of them have a real part higher than 1.

2.4 Parametrization

In order to gain an insight into the effects of openness and the alternative policy rules specifications on determinacy and learnability conditions, we illustrate the results by using a calibrated case. In our benchmark calibration most of the parameters are taken from GM (2005). That is, the elasticity substitution between imported goods γ, the probability of not adjusting prices θ, and the discount factor β, are set at 1, 0.75 and 0.99, respectively.

We depart from the GM (2005) calibration in four parameters: α, η, σ and φ. In the first case, we let the degree of openness α take two possible values: 0 or 0.4, where the former characterizes our completely *closed economy*, whereas the latter characterizes our *open economy*.[14] In the second and third case, the elasticity of substitution between foreign and domestic goods η, and the coefficient of risk aversion σ, are set equal to 1.5 and 5, respectively, according to Chari et al. (2002). This allows us to study the effects of openness on both determinacy and learnability through its impacts on the parameters of the system as discussed in Section 2.1. The inverse of the elasticity of labor supply φ takes the value of 0.47 according to Rotemberg and Woodford (1998).

Finally, we consider that all variables in their natural levels ($\overline{rr}_t, \Delta\bar{s}_t$) follow AR(1) processes with persistence less than one and zero cross correlation. As in BM (2002) we calibrate the policy reaction parameters for non-negative values ($0 \leq \phi_\pi \leq 12$, $0 \leq \phi_x \leq 4$ and $\phi_e > 0$)

[14] The value of 0.4 corresponds roughly to the import/GDP ratio for the Canadian economy.

3. Policy Rules, Determinacy and Learning

3.1 Domestic Inflation Taylor Rules

3.1.1 Contemporaneous Specification (DITR)

First we study the case in which the central bank uses a contemporaneous Domestic Inflation Taylor Rule (DITR) of the form of (13). To obtain the determinacy and E-stability conditions under this case, we combine equations (1), (2) and (13), so the model boils down to a two dynamic equation system involving domestic variables x_t and π_{Ht},

$$y_t = \Gamma + \Omega E_t y_{t+1} + \Theta w_t \quad (33)$$

where $y_t = [\pi_{Ht}, x_t]'$, $w_t = \overline{rr}_t$, $\Gamma = 0$ and

$$\Omega = \frac{1}{\sigma_\alpha + \phi_x + \kappa_\alpha \phi_\pi} \begin{bmatrix} \sigma_\alpha \beta + \phi_x \beta + \kappa_\alpha & \kappa_\alpha \sigma_\alpha \\ 1 - \phi_\pi \beta & \sigma_\alpha \end{bmatrix}, \quad (34)$$

We omit Θ since it is not important for either determinacy or E-stability analyses.[15] Determinacy is analyzed by asking under which conditions Ω has both of its eigenvalues inside the unit circle.

Second, we study the stability of the system when agents no longer have rational expectations and instead form expectations using adaptive rules. Under this scenario, we assume that agents utilize the PLM that corresponds to the MSV solution,[16]

$$y_t = \tilde{a} + \tilde{c}\overline{rr}_t \quad (35)$$

with $\tilde{a} = 0$ and $\tilde{c} = (1 - \rho\Omega)^{-1}\Theta$. Then, we question whether or not E-stability conditions hold for different values for the parameters in the policy rule.

Equations (1) and (2) only involve domestic variables, thus the open economy effects come into the model only in the sense that the coefficients are altered relative to the closed economy case. In fact, an important case occurs when α is zero so that the economy is closed and the model is the same as in Woodford (2003b). Furthermore, DITR is in essence the same as the so-called contemporaneous data interest rule of BM (2002). Therefore, it should not be

[15] For the sake of simplicity we henceforth purposely omit matrices that are not relevant for either determinacy or E-stability analyses.

[16] Note that we include an intercept vector although the MSV solution does not have it. However, in practice agents will need to estimate intercept as well as slope parameters.

surprising that determinacy and learnability conditions of the open economy coincide with conditions derived by BM (2002).

Recalling Propositions 1 and 2 of BM (2002) we have that under DITR the necessary and sufficient condition for determinacy and learnability is given by

$$\kappa_{\alpha}(\phi_{\pi}-1)+(1-\beta)\phi_{x}>0. \quad (36)$$

The only difference between (36) and the conditions provided in Propositions 1 and 2 of BM (2002) is that coefficients are now influenced by open economy considerations such as the degree of openness and the substitutability between foreign and domestic goods.[17]

Despite the difference mentioned above, condition (36) still corresponds to the Taylor Principle: facing inflationary pressures, the central bank increases its interest rate by more than the rise in inflation, which raises real interest rates until inflation returns to the target. As emphasized by BM (2002), such a policy succeeds in stabilizing the economy towards its rational expectation equilibrium. When there is no response to the output gap, $\phi_{\pi}>1$ is sufficient for the Taylor Principle to be satisfied. But even for values of $\phi_{\pi}<1$, the policy authority can compensate for a relatively low value of ϕ_{π} by choosing a sufficiently large value of ϕ_{x} in such a way as to still satisfy condition (36).

In order to examine the effect of openness on the stability of the economy, Figure 1 depicts determinacy and E-stable regions as functions of ϕ_{π} and ϕ_{x} under different degrees of openness. In all cases the rest of the parameters are set at their baseline values.[18].

The numerical results reveal that the line between determinacy and E-stable and Indeterminate and E-unstable regions steps up as the degree of openness approaches to zero. Thus, whenever $\phi_{\pi}<1,$ relatively closed economies need greater responses to the output gap. Therefore, the more closed the economy, the tighter are the constraints faced by policymakers. The explanation behind this outcome relies on the effects of openness on κ_{α}. If $\sigma\eta>1,$ an increase of openness has a positive effect, increasing the area of determinacy and E-stability through the reduction of κ_{α}. This positive effect decays non-monotonically with the degree of

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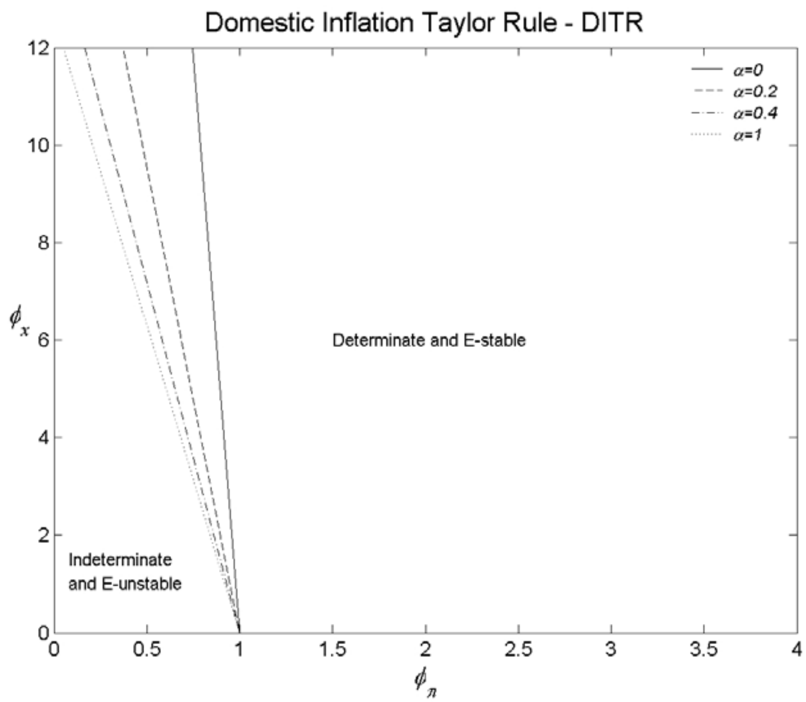
[17] GM have also found the same condition, although in our paper we explore, in addition, the E-stability conditions.

[18] Although one case corresponds to the closed-economy case, the graphic does not coincide with BM (2002, Figure 1) due to differences in calibration.

openness: the area of determinacy and E-stability with a mild degree of openness (α is 0.4) is not greatly different from the corresponding area with a completely open economy (α is 1).

Figure 1. Regions of Determinacy and E-stability for Contemporaneous DITR under different degrees of openness.



The intuition behind the enlargement of the determinate and learnable region stems from the terms of trade effect on inflation dynamics. Specifically, a positive (negative) output gap is offset by an increment (reduction) of the terms of trade, which causes an expenditure switching effect from domestic (foreign) towards foreign (domestic) goods. As a consequence, in relatively more open economies a central bank can be less concerned with the output gap because its fluctuations have lower impacts on domestic inflation. Note that when $\sigma\eta < 1$ the opposite result holds, whereby we would observe a reduction of both the determinate and learnable regions as the degree of openness increases.

3.1.2 Forecast-Based Specification

Under a forecast-based Domestic Inflation Taylor Rule (FB-DITR), we can again collapse the system of equations (1), (2) and (14) to two equations involving the endogenous variables x_t and $\pi_{H,t}$,

$$y_t = \Gamma + \Omega E_t y_{t+1} + \Theta w_t \quad (37)$$

where $y_t = [\pi_{H,t}, x_t]'$, $w_t = \overline{rr}_t$ and Ω is defined by,

$$\Omega = \begin{bmatrix} \frac{1}{\sigma_\alpha}(-\phi_\pi \kappa_\alpha + \sigma_\alpha \beta + \kappa_\alpha) & \frac{\kappa_\alpha}{\sigma_\alpha}(\sigma_\alpha - \phi_x) \\ -\frac{\phi_\pi - 1}{\sigma_\alpha} & \frac{\sigma_\alpha - \phi_x}{\sigma_\alpha} \end{bmatrix}. \quad (38)$$

For t-dating of expectations, the MSV solution takes the form of $y_t = \overline{a} + \overline{c}\,\overline{rr}_t$ with $\overline{a} = 0$, and $\overline{c} = (I - \rho\Omega)^{-1}\Theta$.

Since the feedback policy rule (14) has the same form of the forward expectation rule studied in BM (2002), the same arguments discussed above apply here. Therefore, we use conditions for determinacy and E-stability given by Propositions 4 and 5 of BM (2002), respectively. Proposition 4 states that the necessary and sufficient conditions for a rational expectation equilibrium to be determinate under a forward expectation policy rule are

$$\phi_x < \sigma_\alpha(1 + \beta^{-1})$$
$$\kappa_\alpha(\phi_\pi - 1) + (1 + \beta)\phi_x < 2\sigma_\alpha(1 + \beta)$$
$$\kappa_\alpha(\phi_\pi - 1) + (1 - \beta)\phi_x > 0. \quad (39), (40) \text{ and } (41)$$

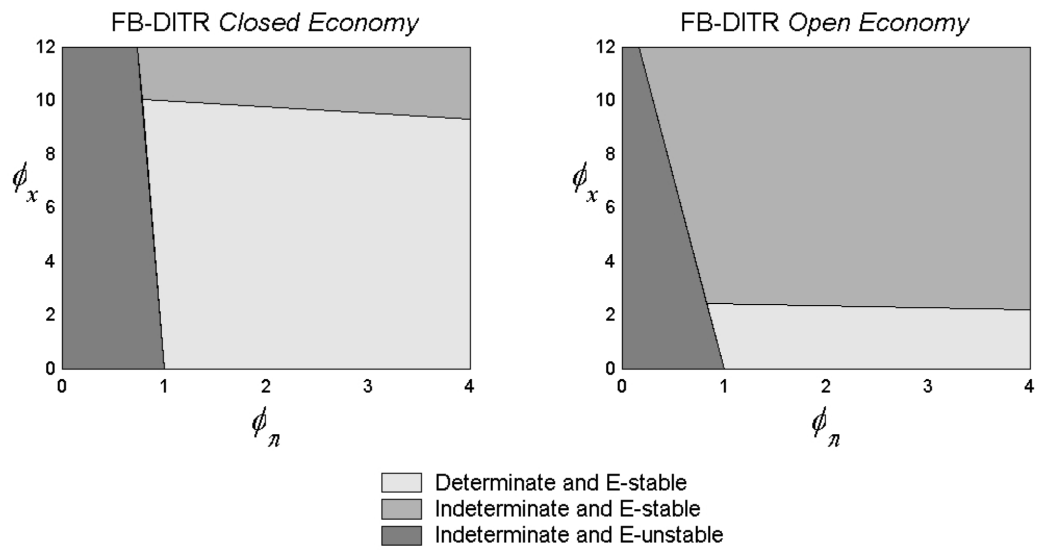
On the other hand, Proposition 5 indicates that a necessary condition of the MSV solution to be *E-stable* is that

$$\kappa_\alpha(\phi_\pi - 1) + (1 - \beta)\phi_x > 0. \quad (42)$$

Figure 2 illustrates the intersections of the regions of determinacy and learnability of the MSV solution at the baseline parametrization under both closed and open economies. Unlike DITR, determinate equilibrium is always expectationally stable, but the opposite does not occur due to restrictions (39) and (40).

In general, for both closed and open economies, a FB-DITR described by $\phi_\pi > 1$ and a relatively small response to output gap guarantees a determinate and learnable equilibrium, while an indeterminate but *E-stable* equilibrium exists for high values of ϕ_x and medium values of ϕ_π.[19] With our baseline parametrization, an increase in the size of openness lowers the determinate and learnable area because restriction (40) tends to bind ϕ_x. This is due to the fact that an increase in openness reduces σ_α and thus reduces the upper bound of ϕ_x. Therefore, under FB-DITR, openness to trade jeopardizes the Central Bank's ability to stabilize the economy.

Figure 2. Regions of Determinacy and E-stability for FB-DITR.



Note: Closed $(\alpha = 0)$ and open $(\alpha = 0.4)$ economies

3.2 CPI Inflation Taylor Rules

3.2.1

In this section we assume that the central bank sets its interest rate according to a contemporaneous CPI Inflation Taylor rule (CPITR), given by (15). In an open economy domestic inflation differs from CPI inflation due to the presence of an additional endogenous variable, the terms of trade. We depart from the earlier analyses by formulating the dynamics of the small open economy in terms of domestic inflation, nominal exchange rate and terms of trade gap. To do that, we combine equations (1), (3), (11) and (15) and use definitions (6) and (12).

Notice that the Taylor rule (15) can be re-expressed as:

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[19] Indeterminacy and instability coexist when ϕ_π is too large. This area is not shown in the graph because the value

$$r_t = \phi_\pi^{CPI}\pi_{H,t} + \phi_x x_t + \phi_e^{CPI}\Delta e_t \quad (43)$$

where $\phi_\pi^{CPI} = (1-\alpha)\phi_\pi$ and $\phi_e^{CPI} = \alpha\phi_\pi$. This rule embeds DITR since, instead of having the reaction to domestic inflation, $\pi_{H,t}$, equal to ϕ_π, under this rule the implied reaction to domestic inflation is smaller and equal to $(1-\alpha)\phi_\pi$. Yet, in addition there is an implicit reaction to contemporaneous changes to the exchange rate that will add some inertia to the rational expectation equilibrium. The model can be re-written as a system of three equations of the form

$$\begin{aligned} \pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa_\alpha \sigma_\alpha^{-1} \widehat{s}_t \\ \Delta E_t e_{t+1} &= \phi_\pi^{CPI}\pi_{H,t} + \phi_x \sigma_\alpha^{-1} \widehat{s}_t + \phi_e^{CPI}\Delta e_t \\ \widehat{s}_t &= \widehat{s}_{t-1} + \Delta e_t - \pi_{H,t} + \Delta \overline{s}_t. \end{aligned} \quad (44), (45) \text{ and } (46)$$

And the exogenous variable $\Delta\overline{s}_t$ follows

$$\Delta\overline{s}_t = \rho\Delta\overline{s}_{t-1} + \epsilon_t \quad (47)$$

The collapsed system of three equations involving the endogenous variables $\pi_{H,t}, \Delta e_t$ and $\widehat{s}_{t-1}$ can be represented as

$$\begin{bmatrix} E_t q_{t+1} \\ z_{t+1} \end{bmatrix} = \Upsilon \begin{bmatrix} q_t \\ z_t \end{bmatrix} + \Lambda w_t \quad (48)$$

where $q_t = [\pi_{H,t}, \Delta e_t]', z_t = \widehat{s}_{t-1}, w_t = \Delta\overline{s}_t$. Variable q_t collects non-predetermined variables, whereas z_t collects states or predetermined variables. Vector w_t denotes the exogenous variables of the system. Matrix Υ is given by

$$\Upsilon = \begin{bmatrix} \frac{1}{\beta} + \frac{\kappa_\alpha}{\beta\sigma_\alpha} & -\frac{\kappa_\alpha}{\beta\sigma_\alpha} & -\frac{\kappa_\alpha}{\beta\sigma_\alpha} \\ -\frac{\phi_x}{\sigma_\alpha} + \phi_\pi^{CPI} & \phi_e^{CPI} + \frac{\phi_x}{\sigma_\alpha} & \frac{\phi_x}{\sigma_\alpha} \\ -1 & 1 & 1 \end{bmatrix} \quad (49)$$

Since there exists one predetermined variable (lag of terms of trade gap), the equilibrium is determinate if and only if the matrix Υ has exactly two eigenvalues outside the unit circle and one

of ϕ_π in this case so far exceeds the limit for this parameter in the calibration.

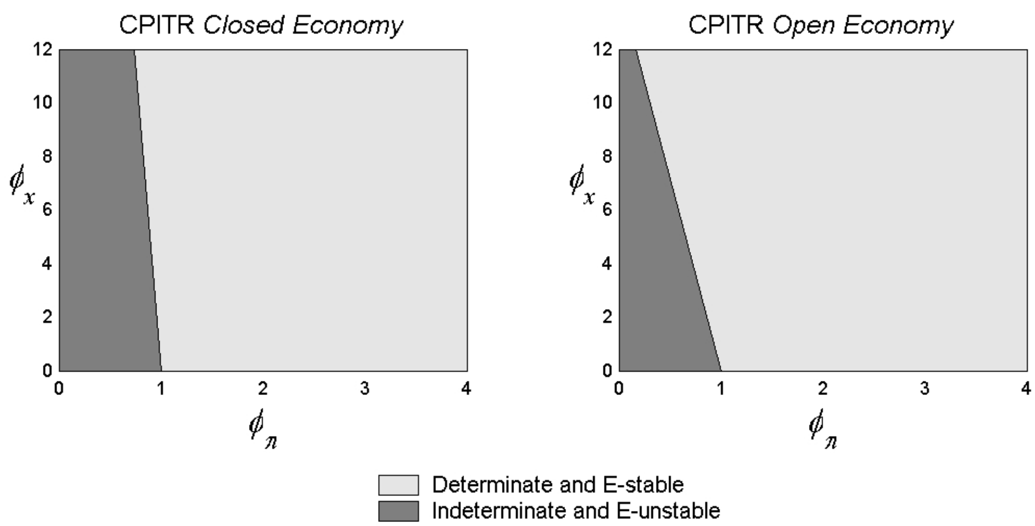
eigenvalue inside the unit circle. Woodford (2003b, Chapter 4) analyzes such a system like (48) and derives the necessary and sufficient conditions for a determinate equilibrium. In fact, the system analyzed here is similar to the one for a closed economy under policy inertia studied by Woodford (2003a). The following proposition summarizes the result.

Proposition 1. *Under CPITR the necessary and sufficient condition for a rational expectations equilibrium to be unique is that*

$$\kappa_{\alpha}(\phi_{\pi}^{CPI} - (1 - \phi_{e}^{CPI})) + (1 - \beta)\phi_{x} > 0 \quad (50)$$

Proof. See Appendix A.

Figure 3. Regions of Determinacy and E-stability for CPITR.



Note: Closed $(\alpha = 0)$ and open $(\alpha = 0.4)$ economies.

Apparently (50) is different from the Taylor principle, but after replacing ϕ_{π}^{CPI} and ϕ_{e}^{CPI}, we can note that (50) becomes $\kappa_{\alpha}(\phi_{\pi} - 1) + (1 - \beta)\phi_{x} > 0$. Therefore, as in the case of contemporaneous DITR, the Taylor Principle completely characterizes determinacy, i.e., any active policy rule ($\phi_{\pi} > 1$) can induce a determinate equilibrium. The reason behind this result relies on the fact that lower reaction to domestic inflation is canceled out by the implicit reaction to nominal exchange movements. The previous finding is an analytically novel result and can also be useful in analyzing determinacy and learnability in a two-sector closed economy model as in Aoki (2001).[20] However, there is a difference between targeting CPI inflation and domestic inflation: as pointed out by GM, the implied macroeconomic volatility of the endogenous variables will be larger under the CPITR.

To analyze the stability under learning, we re-write the system (48) as

$$y_t = \Gamma + \Omega E_t y_{t+1} + \Phi y_{t-1} + \Theta w_t, \quad (51)$$

where $y_t = [\pi_{H,t}, \Delta e_t, \hat{s}_t]', w_t = \Delta \overline{s}_t$. Matrices are $\Gamma = 0$,

$$\Omega = \psi \begin{bmatrix} \beta(\phi_x + \sigma_\alpha \phi_e^{CPI}) & \kappa_\alpha & 0 \\ \beta(\phi_x - \sigma_\alpha \phi_\pi^{CPI}) & \kappa_\alpha + \sigma_\alpha & 0 \\ -\sigma_\alpha \beta(\phi_\pi^{CPI} + \phi_e^{CPI}) & \sigma_\alpha & 0 \end{bmatrix}$$

$$\Phi = \psi \begin{bmatrix} 0 & 0 & \kappa_\alpha \phi_e^{CPI} \\ 0 & 0 & -(\phi_x + \kappa_\alpha \phi_\pi^{CPI}) \\ 0 & 0 & \sigma_\alpha \phi_e^{CPI} \end{bmatrix}$$

$$\Theta = \psi \begin{bmatrix} \kappa_\alpha \phi_e^{CPI} \\ -(\phi_x + \kappa_\alpha \phi_\pi^{CPI}) \\ \sigma_\alpha \phi_e^{CPI} \end{bmatrix} \quad (52), (53) \text{ and } (54)$$

where $\psi = (\phi_x + \kappa_\alpha \phi_\pi^{CPI} + (\kappa_\alpha + \sigma_\alpha)\phi_e^{CPI})^{-1}$.

First, we perform a numerical evaluation for the conditions of E-stability, then we will explain the analytics. We calculate the MSV solution using the method of undetermined coefficients. We assume the following form of the MSV solution

$$y_t = \tilde{a} + \tilde{b} y_{t-1} + \tilde{c} w_t \quad (55)$$

where $\tilde{a} = 0$ and matrices $\tilde{b}$ and $\tilde{c}$ satisfy

$$\tilde{b} = (I - \Omega \tilde{b})^{-1} \Phi$$
$$\tilde{c} = (I - \Omega \tilde{b})^{-1} (\Theta + \Omega \tilde{c} \rho) \quad (56) \text{ and } (57)$$

[20] As emphasized by Aoki (2001), there is a parallel between a small open economy model like the one we use and his two-sector closed economy model. In a small open economy, the domestic sector is analogous to a sector showing prices stickiness, whereas the foreign sector is analogous to the one showing flexible prices. Therefore, our result suggests that the Taylor principle could be a necessary and sufficient condition for determinacy in a two-sector closed economy model.

provided the matrix $(I - \Omega\tilde{b})^{-1}$ is invertible. Restriction (56) could lead to multiple stationary solutions for $\tilde{b}$. A determinate equilibrium requires a unique solution for $\tilde{b}$ with all eigenvalues inside the unit circle. After solving numerically (56) and (57), E-stability conditions given by (30) through (32) are evaluated.

Figure 3 plots the determinate and learnable areas for both closed-economy and open-economy. The numerical results suggest that determinate equilibrium is always learnable. Therefore, with contemporaneous data in the policy rule there is no difference between targeting domestic inflation or consumer price inflation.[21] Numerical results also show that the area of E-stability augments when openness increases if and only if $\sigma\eta > 1$. It is surprisingly that, when the policymakers include CPI inflation in the contemporaneous specification nothing changes with respect to DITR, i.e., Figure 3 is unchanged from Figure 1. In Appendix G we provide some intuition and a sketch of the analytical results for coincidences of the areas of determinacy and learnability under a CPITR and DITR specifications.

3.2.2 Forecast-Based Specification (FB-CPITR)

Under a forecast-based CPI Inflation Taylor Rule (FB-CPITR), the Central Bank follows a policy rule of the form of (16). Plugging (3) and (6) into the rule, the domestic interest rate can be rewritten as

$$r_t = \phi'_\pi E_t \pi_{H,t+1} + \phi'_x E_t x_{t+1} \quad (58)$$

where $\phi'_\pi = \left(\frac{\phi_\pi(1-\alpha)}{1-\phi_\pi\alpha}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_\pi\alpha}\right)$.

By combining (1) and (2) with (58), we can reduce the system to two equations involving the endogenous variables x_t and $\pi_{H,t}$. The reduced system is then given by

$$y_t = \Gamma + \Omega E_t y_{t+1} + \Theta w_t \quad (59)$$

where $y_t = [\pi_{H,t}, x_t]'$, $w_t = \overline{rr}_t$, $\Gamma = 0$ and Ω is defined by

[21] Bullard and Schaling (2005) study a similar environment. Their results coincides with ours in the case of a small open economy. The authors also found that when small open economy assumption is dropped, interaction with the rest of the world is important in the sense that it modifies the conditions for a determinate and learnable equilibrium in the domestic economy.

$$\Omega = \begin{bmatrix} \frac{-\phi'_\pi \kappa_\alpha + \sigma_\alpha \beta + \kappa_\alpha}{\sigma_\alpha} & \frac{\kappa_\alpha}{\sigma_\alpha}(\sigma_\alpha - \phi'_x) \\ -\frac{\phi'_\pi - 1}{\sigma_\alpha} & \frac{\sigma_\alpha - \phi'_x}{\sigma_\alpha} \end{bmatrix} \quad (60)$$

Since both x_t and π_{Ht} are free variables, determinacy requires both the eigenvalues of Ω to be inside the unit circle. The following proposition summarizes the necessary and sufficient conditions for a rational expectations equilibrium to be unique.

Proposition 2. *Under FB-CPITR, the necessary and sufficient conditions for a rational expectations equilibrium to be unique are that*

$$\begin{aligned} \phi_\pi &< \frac{1}{\alpha} \\ \phi_x &< \sigma_\alpha(1+\beta^{-1})(1-\phi_\pi\alpha) \\ \kappa_\alpha(\phi_\pi - 1) + (1+\beta)\phi_x &< 2\sigma_\alpha(1+\beta)(1-\phi_\pi\alpha) \\ \kappa_\alpha(\phi_\pi - 1) + (1-\beta)\phi_x &> 0 \end{aligned}$$

(61), (62), (63) and (64)

Proof. See Appendix B.

For t-dating of expectations, the MSV solution takes the form of $y_t = \bar{a} + \bar{c}w_t$ with $\bar{a} = 0,$ and $\bar{c} = (I - \rho\Omega)^{-1}$ and the T-mapping from the PLM to the ALM is given by, $T(a,c) = (\Omega a, \Omega c\rho + \Theta)$ The following proposition provides the conditions for E-stability of the MSV solution.

Proposition 3. *Suppose the time t information set is* $(1, w_t)'$. *Under FB-CPITR, the necessary and sufficient conditions for an MSV solution* $(0, \bar{c})$ *to be E-stable are that*

$$\begin{aligned} \phi_\pi &< \frac{1}{\alpha} \\ \kappa_\alpha(\phi_\pi - 1) + (1-\beta)\phi_x &> 0 \end{aligned}$$

(65) and (66)

Proof. See Appendix C.

It is noticeable that FB-CPITR modifies both determinacy and learnability conditions respect to FB-DITR. The main effect of openness is given by conditions (63) and (65), which clearly constrain the higher permissible values for ϕ_π. On the opposite, the lower bound for ϕ_π is still dictated by conditions (64) and (66). For example, in the case of determinacy, if there is a null response to the expected output gap (i.e., ϕ_x is zero), the limits for ϕ_π are

$$1 < \phi_\pi < \frac{\kappa_\alpha + 2\sigma_\alpha(1+\beta)}{\kappa_\alpha + 2\sigma_\alpha\alpha(1+\beta)} \lessapprox \frac{1}{\alpha}.$$

whereas in the case of E-stability the limits are,

$$1 < \phi_\pi < \frac{1}{\alpha}.$$

Thus, there exists a determinate and learnable equilibrium as long as the sensitivity of the interest rate to expected CPI inflation is approximately lower than the inverse of openness. Consequently, as the degree of openness increases, the scope of values for ϕ_π that guarantees determinacy and E-stability shrinks significantly. Remarkably, the Taylor Principle should be viewed as a necessary but not as a sufficient condition for learnability. This result contrasts with those of a closed economy, which suggest that the Taylor Principle guarantees E-stability; see BM (2002). The idea that the Taylor principle or "active" policy is a matter of changing nominal interest rates more than one-for-one with inflation is a celebrated result in the literature. It is almost always thought of as a pure inequality. The idea that open economy considerations create an upper bound on how aggressive policymakers can be with respect to inflation is striking and simple in this framework

To clarify these results, Figure 4. depicts determinacy and learnability conditions at the baseline parameter values for closed and open economies. Because in a closed economy domestic and CPI inflation are the same concept,[22] the plot corresponding to the closed economy case coincides with the left panel of Figure 2.

As in the case of FB-DITR, determinate equilibrium is always expectationally stable, but the reverse does not occur. However, as discussed above, activism against future CPI inflation deviations from its target is remarkably bound not only for determinacy but also for E-stability. For example, in our benchmark calibration ϕ_π must lie between 1 and (around) 2.5 in order to achieve a determinate and learnable equilibrium in the open-economy case.[23] Unlike previous

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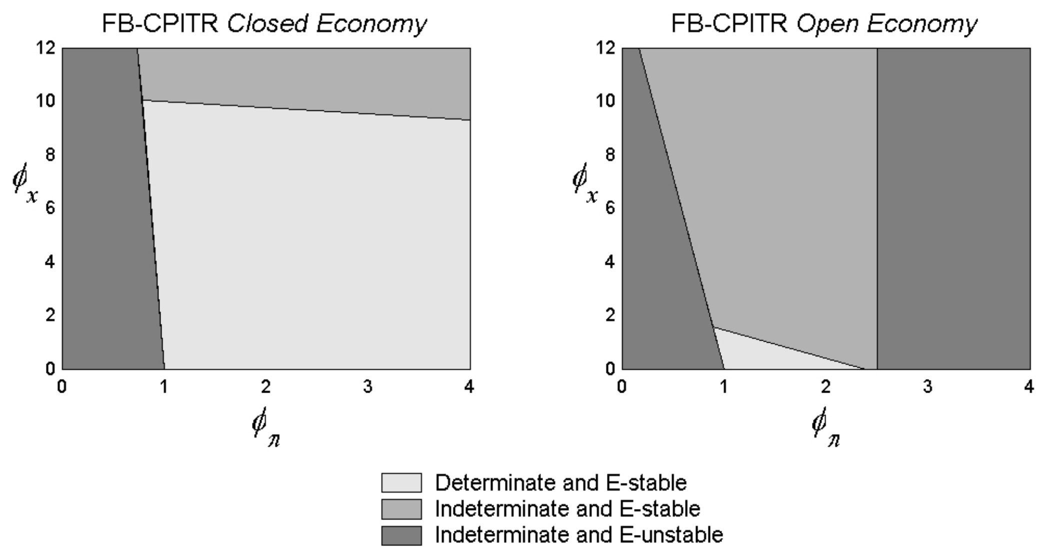
[22] Notice that as $\alpha \to 0$, when CPI inflation coincides with domestic inflation, determinacy and E-stability conditions for FB-CPITR converge to the conditions found by BM (2002) for the closed economy counterpart.

[23] Moreover, the parametrization of Taylor (1993), $\phi_\pi = 1.5$ and $\phi_x = 0.5$, implies that a degree of openness of roughly more than 0.66 could easily induce both indeterminacy and E-instability.

feedback rules, it is certain that the degree of openness together with the presence of expected CPI inflation in the policy rule *unambiguously* reduces both determinate and E-stable areas.

Figure 4. Regions of Determinacy and E-stability for FB-CPITR



Note: Closed $(\alpha = 0)$ and open $(\alpha = 0.4)$ economies.

Our interpretation is that the reduction of determinate and E-stable areas comes from the interaction between activism in the policy rule and openness. Any increase (decrease) in the interest rate due to inflationary (deflationary) expectations triggered by an expected depreciation (appreciation) of nominal exchange rate reinforces the expectation of higher (lower) CPI inflation. In this context, the likelihood of a consequent movement in the interest rate relies on the preferences of the central bank, given by ϕ_π, and the degree of openness.

Therefore, if either the degree of openness or the aggressiveness of the monetary policy with respect to expected CPI inflation is high, the economy is likely to be stuck in an indeterminate equilibria that private agents would not be able to learn. The final consequence is that the central bank would face excessive volatility of the macroeconomic aggregates due to self-fulfilling expectations. In this circumstance, it would not be possible to anchor private agents' expectations to their target values.

3.3 CPI Managed Exchange Rate Taylor Rule

3.3.1 Contemporaneous Specification (CPI-METR)

Under the contemporaneous Managed Exchange Rate Taylor rule (CPI-METR), we follow the same criteria used in the case of CPITR. Combining equations (1), (3), (11) and (17) and using definitions (6) and (12), we obtain a collapsed system of three equations involving the endogenous variables $\pi_{H,t}, \Delta e_t$ and $\hat{s}_{t-1}$ that can be represented as

$$\begin{bmatrix} E_t q_{t+1} \\ z_{t+1} \end{bmatrix} = \Upsilon \begin{bmatrix} q_t \\ z_t \end{bmatrix} + \Lambda w_t \qquad (67)$$

where $q_t = [\pi_{H,t}, \Delta e_t]', z_t = \hat{s}_{t-1}, w_t = \Delta \overline{s}_t$. Variable q_t collects non-predetermined variables, whereas z_t collects states or predetermined variables. Vector w_t denotes the exogenous variables of the system. Matrix Υ is given by

$$\Upsilon = \begin{bmatrix} \frac{1}{\beta} + \frac{\kappa_\alpha}{\beta\sigma_\alpha} & -\frac{\kappa_\alpha}{\beta\sigma_\alpha} & -\frac{\kappa_\alpha}{\beta\sigma_\alpha} \\ -\frac{\phi_x}{\sigma_\alpha} + \phi_\pi(1-\alpha) & \phi_e + \alpha\phi_\pi + \frac{\phi_x}{\sigma_\alpha} & \frac{\phi_x}{\sigma_\alpha} \\ -1 & 1 & 1 \end{bmatrix} \qquad (68)$$

The equilibrium is determinate if and only if the 3x3 matrix Υ has exactly two eigenvalues outside the unit circle and one eigenvalue inside the unit circle. The proof is similar to the CPITR case.

Proposition 4. *Under CPI-METR the necessary and sufficient condition for a rational expectations equilibrium to be unique is that*

$$\kappa_\alpha(\phi_\pi - (1-\phi_e)) + (1-\beta)\phi_x > 0 \qquad (69)$$

Proof. Appendix D.

Condition (69) is slightly different from condition (50), given in Proposition 1. Any additional reaction of the interest rate to movements in the nominal exchange rate, measured by ϕ_e, shrinks the lower limits of both ϕ_π and ϕ_x. Notice that (69) can be re-written as $\kappa_\alpha(\phi_\pi + \phi_e - 1) + (1-\beta)\phi_x > 0$ Therefore, *ceteris paribus*, the determinacy region increases with the degree of reaction of the interest rate to the nominal exchange rate, which is clear from the $(\phi_\pi + \phi_e - 1)$ component of the "new" Taylor Principle. Moreover, when interest rate reacts one-for-one to nominal exchange rate movements (i.e., ϕ_e is 1), monetary policy can induce determinacy even with a negligible response to CPI inflation and/or the output gap. Analogous to DITR and CPITR, the degree of openness modifies determinacy conditional on whether $\sigma\eta > 1$ holds or not.

We emphasize that managed exchange rate promotes both determinacy and learnability of equilibrium in open economies in the same way as the lagged interest rate in the policy rule (so-called policy inertia) does it in the closed economy counterpart, see Woodford (2003a) and Bullard and Mitra (2006). In fact, since the current nominal exchange rate varies one-for-one with the lagged of domestic interest rate, the inclusion of the former in the policy rule works as if there actually were inertia in the domestic interest rate.

E-stability analysis is performed by re-writing (67) in a matrix system in the form of $y_t = \Gamma + \Omega E_t y_{t+1} + \Phi y_{t-1} + \Theta w_t$ where $y_t = [\pi_{H,t}, \Delta e_t, \hat{s}_t]', w_t = \Delta \bar{s}_t$. Matrices are $\Gamma = 0$,

$$\Omega = \psi \begin{bmatrix} \beta(\phi_x + \alpha\sigma_\alpha\phi_\pi + \sigma_\alpha\phi_e) & \kappa_\alpha & 0 \\ \beta(\phi_x - \sigma_\alpha(1-\alpha)\phi_\pi) & \kappa_\alpha + \sigma_\alpha & 0 \\ -\sigma_\alpha\beta(\phi_e + \phi_\pi) & \sigma_\alpha & 0 \end{bmatrix}$$

$$\Phi = \psi \begin{bmatrix} 0 & 0 & \kappa_\alpha(\alpha\phi_\pi + \phi_e) \\ 0 & 0 & -(\phi_x + \kappa_\alpha(1-\alpha)\phi_\pi) \\ 0 & 0 & \sigma_\alpha(\alpha\phi_\pi + \phi_e) \end{bmatrix}$$

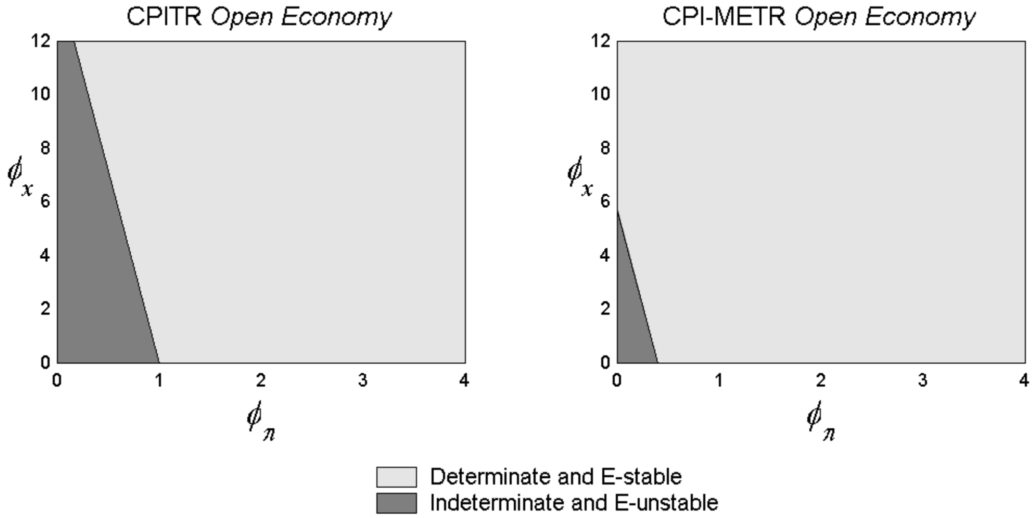
$$\Theta = \psi \begin{bmatrix} \kappa_\alpha(\alpha\phi_\pi + \phi_e) \\ -(\phi_x + \kappa_\alpha(1-\alpha)\phi_\pi) \\ \sigma_\alpha(\alpha\phi_\pi + \phi_e) \end{bmatrix}$$

(70), (71) and (72)

where $\psi = (\phi_x + (\kappa_\alpha + \alpha\sigma_\alpha)\phi_\pi + (\kappa_\alpha + \sigma_\alpha)\phi_e)^{-1}$. In the same way as CPITR, we study the E-stability of the MSV quantitatively. The sketch of the proof for E-stability can be derived specularly to the sketch of the proof for CPITR case.[24]

[24] See Section 3.2.1. A detailed proof can be obtained from the authors upon request.

Figure 5. Regions of Determinacy and E-stability for CPITR and CPI-METR



Note: Both graphics correspond to open economies $(\alpha = 0.4)$. The graphic on the left shows the case of CPITR or no managed exchange rate $(\phi_e = 0)$ and the graphic on the right shows the case of CPI-METR $(\phi_e = 0.6)$.

Figure 5 shows our results under two different values of ϕ_e for a given degree of openness (α equals 0.4). The picture on the left plots the results when there is no response to the nominal exchange rate, ϕ_e is zero, i.e., policymakers follow a CPITR. In the picture on the right, we assume that monetary authority reacts to the nominal exchange rate as well as CPI inflation and the output gap. In the latter, we calibrate the value of ϕ_e to be 0.6. In both cases, determinacy and E-stable areas perfectly coincide and multiple equilibria are not learnable. More interesting is that a central bank reacting passively to inflation $(\phi_\pi < 1)$ and simultaneously targeting movements in the exchange rate in the policy rule $(\phi_e > 0)$ can induce a determinate and E-stable equilibrium even with a null response to the output gap. For instance, when ϕ_x is zero, the lower bound of ϕ_π under CPI-METR is around 0.4. Even more important, if ϕ_e is larger than one, any positive values for ϕ_π and ϕ_x would imply both determinate and E-stable equilibrium. Therefore, additional reaction to nominal exchange rate increases the determinate and learnable regions.[25] A central bank that reacts to exchange rates movements is implicitly taking into

[25] In contrast to the previous analyses, in this section we have focused on managed exchange rate rather than openness. Nevertheless, it is worth to emphasize that our numerical results, not shown, confirm that the impact of the size of openness ambiguously alters both determinacy and E-stability. Again, the impact on determinacy and

account reactions to exogenous shocks, which makes the determinacy region larger. However, we should be aware that even though it is easier to induce determinacy, it is also easier to generate greater volatility in the endogenous variables, as was pointed out by GM (2005). In a nutshell, even if policymakers do not react sufficiently strongly to CPI inflation they can still induce determinacy and learnability of equilibrium by reacting sufficiently strongly to the nominal exchange rate, although this policy could enhance macroeconomic volatility.

3.3.2 Forecast-Based Specification (FB-CPI-METR)

In this section we suppose that the monetary authority follows a forecast-based Managed Exchange Rate (FB-CPI-METR). First, with the same procedure used for FB-CPITR, the interest rate feedback rule (18) can be rewritten as

$$r_t = \phi'_{\pi} E_t \pi_{H,t+1} + \phi'_x E_t x_{t+1} \quad (73)$$

where $\phi'_{\pi} = \left(\frac{\phi_{\pi}(1-\alpha)}{1-\phi_{\pi}\alpha-\phi_e}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_{\pi}\alpha-\phi_e}\right)$. Notice that the parameter of nominal exchange rate reaction in the rule, ϕ_e, has modified ϕ'_{π} and ϕ'_x with respect to FB-CPITR case.

The system is reduced to two equations involving the endogenous variables x_t and $\pi_{H,t}$. The reduced system is then given by

$$y_t = \Gamma + \Omega E_t y_{t+1} + \Theta w_t \quad (74)$$

where $y_t = [\pi_{H,t}, x_t]'$, $w_t = \overline{rr}_t$, $\Gamma = 0$, and Ω is defined by

$$\Omega = \begin{bmatrix} \frac{-\phi'_{\pi}\kappa_{\alpha}+\sigma_{\alpha}\beta+\kappa_{\alpha}}{\sigma_{\alpha}} & \frac{\kappa_{\alpha}}{\sigma_{\alpha}}(\sigma_{\alpha} - \phi'_x) \\ -\frac{\phi'_{\pi}-1}{\sigma_{\alpha}} & \frac{\sigma_{\alpha}-\phi'_x}{\sigma_{\alpha}} \end{bmatrix}. \quad (75)$$

Since both x_t and $\pi_{H,t}$ are free variables, determinacy requires both the eigenvalues of Ω to be inside the unit circle. The following proposition summarizes the necessary and sufficient conditions for a rational expectations equilibrium to be unique. The proof is straighforward, and we can follow the steps of the proof for the FB-CPITR case.

expectational stability mostly depends on the degree of substitutability between foreign and domestic produced goods and the coefficient of risk aversion.

Proposition 5. *Under FB-CPI-METR, the necessary and sufficient conditions for a rational expectations equilibrium to be unique are that*

$$\phi_\pi < \frac{(1-\phi_e)}{\alpha}$$
$$\phi_x < \sigma_\alpha(1+\beta^{-1})(1-\phi_e-\phi_\pi\alpha)$$
$$\kappa_\alpha(\phi_\pi-(1-\phi_e))+(1+\beta)\phi_x < 2\sigma_\alpha(1+\beta)(1-\phi_e-\phi_\pi\alpha)$$
$$\kappa_\alpha(\phi_\pi-(1-\phi_e))+(1-\beta)\phi_x > 0$$

(76), (77), (78) and (79)

Proof. See Appendix E.

For t-dating of expectations, the MSV solution takes the form of $y_t = \bar{a} + \bar{c}w_t$ with $\bar{a} = 0$, and $\bar{c} = (I-\rho\Omega)^{-1}\Theta$. The following proposition provides the condition for E-stability of the MSV solution.

Proposition 6. *Suppose the time t information set is* $(1, w_t)'$. *Under FB-CPI-METR interest rate rules, the necessary and sufficient conditions for an MSV solution* $(0, \bar{c})$ *to be E-stable are that*

$$\phi_\pi < \frac{(1-\phi_e)}{\alpha}$$
$$\kappa_\alpha(\phi_\pi-(1-\phi_e))+(1-\beta)\phi_x > 0$$

(80) and (81).

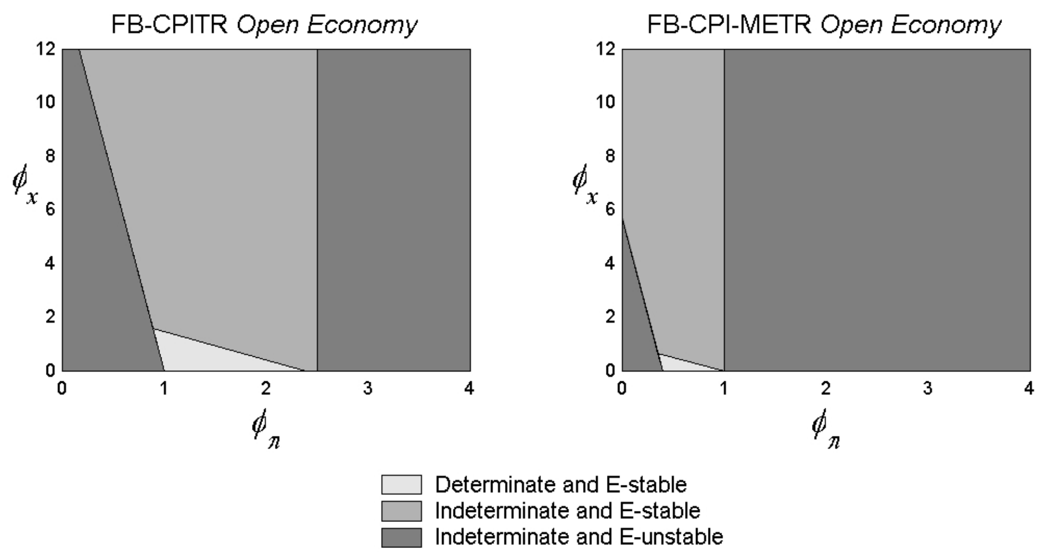
Proof. Appendix F.

Figure 6 plots the numerical results under two possible values for ϕ_e. The graph on the left depicts the case of FB-CPITR, i.e., when there is a null response to the expected nominal exchange rate (ϕ_e is zero), whereas the graph on the right depicts the case of FB-CPI-METR (ϕ_e is 0.6).

Comparing the conditions under FB-CPI-METR with the conditions obtained under FB-CPITR, we can note that the degree of managed exchange rate, measured by ϕ_e, has affected both determinacy and learnability conditions. There are two major effects through which ϕ_e impact on the stability of the system. First, conditions (79) and (81) imply that an increase in ϕ_e

reduces the lower-bound for ϕ_π. For instance those conditions can be rewritten as $\kappa_\alpha(\phi_\pi + \phi_e - 1) + (1-\beta)\phi_x > 0$. The overall reaction to inflation is now captured by the terms $\phi_\pi + \phi_e$. Like in the case of METR rule, ϕ_π less than one can guarantee a determinate and learnable equilibrium; i.e. under $\phi_e = 0.6$ and $\phi_x = 0$, the lower bound of ϕ_π is 0.4. Second, coupled with the degree of openness, any positive reaction to expected nominal exchange rate movements reduces the area of determinacy and learnability through (78) and (80), respectively. For example, when ϕ_e is 0.6, those conditions imply that the upper limit for ϕ_π is around 1. Different from CPI-METR, if ϕ_e is constraint to be lower than one, given the fact that both ϕ_π and ϕ_x are assumed to be non-negative.

Figure 6. Regions of Determinacy and E-stability for FB-CPITR and FB-CPI-METR.



Note: Both graphics correspond to open economies $(\alpha = 0.4)$. The graphic of the left shows the case of FB-CPITR or no managed exchange rate $(\phi_e = 0)$ and the graphic of the right shows the case of FB-CPI-METR $(\phi_e = 0.6)$.

Consequently, highly open economies joint with a central bank reacting too strongly to either future CPI inflation or expected nominal exchange rate movements are more prone to indeterminacy and instability. Yet, if the degree of openness and activism towards CPI and exchange rate are moderate, the monetary authority is able to push the economy towards the determinate and E-stable region, even with no response to the output gap. More important, a passive reaction to expected CPI inflation could success in generating a determinate and E-stable path. Overall, it is relevant to analyze this type of rule and its properties since there exists robust evidence that Bank of Canada and the Bank of England have been following a similar policy rule; see Lubik and Schorfheide (2006).

3.4. Domestic Inflation Managed Exchange Rate Taylor Rule

3.4.1 Contemporaneous Specification (DI-METR)

Under contemporaneous domestic inflation Managed Exchange Rate Taylor rule (DI-METR), we follow the same criteria used in the case of CPITR. Combining equations (1), (3), (11) and (19), and using definition (12), we obtain a collapsed system of three equations involving the endogenous variables $\pi_{H,t}, \Delta e_t$ and $\hat{s}_{t-1}$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_\alpha \sigma_\alpha^{-1} \hat{s}_t$$

$$\hat{s}_t = \hat{s}_{t-1} + \Delta e_t + \pi_t^* - \pi_{H,t} + \Delta \overline{s}_t$$

$$\Delta E_t e_{t+1} = \phi_\pi \pi_{H,t} + \phi_x \sigma_\alpha^{-1} \hat{s}_t + \phi_e \Delta e_t$$

(82), (83) and (84)

And the exogenous variable $\Delta \overline{s}_t$ follows,

$$\Delta \overline{s}_t = \rho \Delta \overline{s}_{t-1} + \epsilon_t \quad (85)$$

This system of equations can be represented as,

$$\begin{bmatrix} E_t q_{t+1} \\ z_{t+1} \end{bmatrix} = \Upsilon \begin{bmatrix} q_t \\ z_t \end{bmatrix} + \Lambda w_t \quad (86)$$

where $q_t = [\pi_{H,t}, \Delta e_t]', z_t = \hat{s}_{t-1}, w_t = \Delta \overline{s}_t$. Variable q_t collects non-predetermined variables, whereas z_t collects states or predetermined variables. Vector w_t denotes the exogenous variables of the system. Matrix Υ is given by

$$\Upsilon = \begin{bmatrix} \frac{1}{\beta} + \frac{1}{\beta}\kappa_{\Gamma} & -\frac{1}{\beta}\kappa_{\Gamma} & -\frac{1}{\beta}\kappa_{\Gamma} \\ -\frac{\phi_x}{\rho_{\gamma}} + \phi_{\pi} & \phi_e + \frac{\phi_x}{\rho_{\gamma}} & \frac{\phi_x}{\rho_{\gamma}} \\ -1 & 1 & 1 \end{bmatrix} \quad (87)$$

The equilibrium is determinate if and only if the 3x3 matrix Υ has exactly two eigenvalues outside the unit circle and one eigenvalue inside the unit circle.

Proposition 7. *Under DI-METR the necessary and sufficient condition for a rational expectations equilibrium to be unique is that*

$$\kappa_{\alpha}(\phi_{\pi} - (1 - \phi_e)) + (1 - \beta)\phi_x > 0 \quad (88)$$

Proof. The proof is straightforward and can be obtained from the authors upon request.

Condition (88) is exactly the same as we found under CPI-METR. Therefore, the determinacy region increases with the degree of reaction of interest rate to nominal exchange rate regardless of which index of inflation is in the rule.

E-stability analysis is performed by re-writing (86) in a matrix system in the form of

$$y_t = \Gamma + \Omega E_t y_{t+1} + \Phi y_{t-1} + \Theta w_t \quad (89)$$

where $y_t = [\pi_{H,t}, \Delta e_t, \hat{s}_t]', w_t = \Delta \overline{s}_t$. Matrices are $\Gamma = 0$,

$$\Omega = \psi \begin{bmatrix} \beta(\phi_x + \sigma_{\alpha}\phi_e) & \kappa_{\alpha} & 0 \\ \beta(\phi_x - \sigma_{\alpha}\phi_{\pi}) & \kappa_{\alpha} + \sigma_{\alpha} & 0 \\ -\sigma_{\alpha}\beta(\phi_e + \phi_{\pi}) & \sigma_{\alpha} & 0 \end{bmatrix}$$

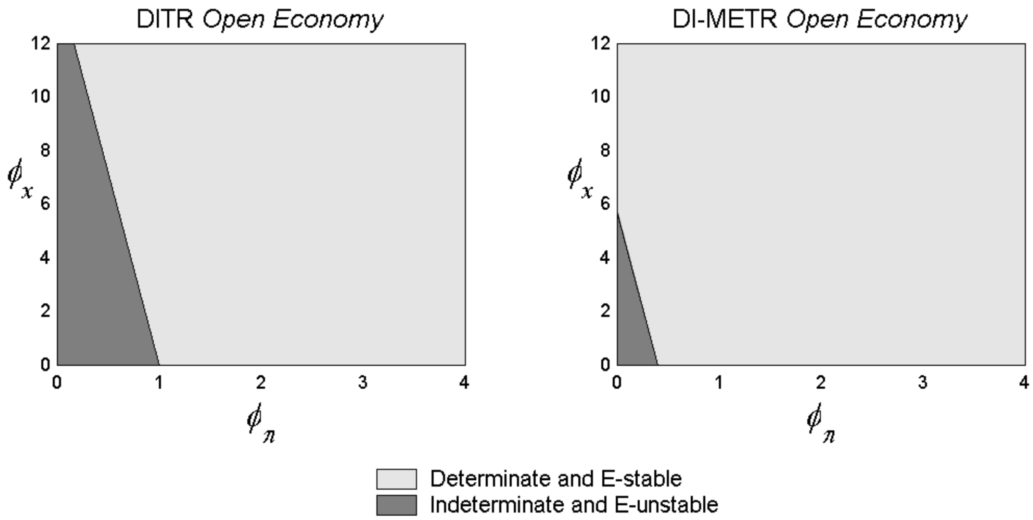
$$\Phi = \psi \begin{bmatrix} 0 & 0 & \kappa_{\alpha}\phi_e \\ 0 & 0 & -(\phi_x + \kappa_{\alpha}\phi_{\pi}) \\ 0 & 0 & \sigma_{\alpha}\phi_e \end{bmatrix}$$

$$\Theta = \psi \begin{bmatrix} \kappa_{\alpha}\phi_e \\ -(\phi_x + \kappa_{\alpha}\phi_{\pi}) \\ \sigma_{\alpha}\phi_e \end{bmatrix}$$

(90), (91) and (92)

where $\psi = (\phi_x + \kappa_\alpha \phi_\pi + (\kappa_\alpha + \sigma_\alpha)\phi_e)^{-1}$. We study the E-stability of the MSV quantitatively.[26]

Figure 7. Regions of Determinacy and E-stability for DITR and DI-METR



Note: Both graphics correspond to open economies $(\alpha = 0.4)$. The graphic on the left shows the case of DITR or no managed exchange rate $(\phi_e = 0)$ and the graphic on the right shows the case of DI-METR $(\phi_e = 0.6)$.

Figure 7 shows our results under two different values of ϕ_e for a given degree of openness (α equals 0.4). The picture on the left plots the results when there is no response to nominal exchange rate (i.e., ϕ_e is zero): policymakers follow either a CPITR or DITR. In the picture on the right, we assume that monetary authority reacts to the nominal exchange rate besides domestic inflation and output gap. In the latter, we calibrate the value of ϕ_e to be 0.6. In both cases, determinacy and E-stable areas perfectly coincide and multiple equilibria are not learnable. Interestingly a central bank reacting passively to inflation $(\phi_\pi < 1)$ and simultaneously targeting movements in the exchange rate in the policy rule $(\phi_e > 0)$ can induce a determinate and E-stable equilibrium even with null response to the output gap. Moreover, if ϕ_e is larger than one, any positive values for ϕ_π and ϕ_x would imply both determinate and E-stable equilibrium.

This class of rule elicits some interesting aspects of both determinacy and E-stability in small open economies. Compared with CPI -METR, this type rule delivers the same result.

[26] The sketch of E-stability analysis follows Section 3.2.1.

Therefore, regardless of the inflation index targeted by the Central Bank, a certain degree of exchange rate management helps to avoid both indeterminacy and instability under learning. Furthermore, this implies that the direct reaction towards movements in the exchange rate is the factor that relaxes both determinacy and E-stability conditions. Instead, contemporaneous reaction to CPI inflation does not add anything in terms of determinacy and E-stability even if it implies an indirect reaction to nominal exchange rate changes. As noted above, such implicit reaction cancels out with the lower reaction to domestic inflation.[27]

3.4.2 Forecast-Based Specification

The central bank follows a policy rule of the form of (20). Plugging (3) into the rule, the domestic interest rate can be rewritten as

$$r_t = \phi'_\pi E_t \pi_{H,t+1} + \phi'_x E_t x_{t+1} \quad (93)$$

where $\phi'_\pi = \left(\frac{\phi_\pi}{1-\phi_e}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_e}\right)$. Notice that ϕ_e modifies ϕ'_π and ϕ'_x.

The system is reduced to two equations involving the endogenous variables x_t and $\pi_{H,t}$. The reduced system is then given by

$$y_t = \Gamma + \Omega E_t y_{t+1} + \Theta w_t \quad (94)$$

where $y_t = [\pi_{H,t}, x_t]', w_t = \overline{rr}_t, \Gamma = 0,$ and Ω is defined by

$$\Omega = \begin{bmatrix} \frac{-\phi'_\pi \kappa_\alpha + \sigma_\alpha \beta + \kappa_\alpha}{\sigma_\alpha} & \frac{\kappa_\alpha}{\sigma_\alpha}(\sigma_\alpha - \phi'_x) \\ -\frac{\phi'_\pi - 1}{\sigma_\alpha} & \frac{\sigma_\alpha - \phi'_x}{\sigma_\alpha} \end{bmatrix}. \quad (95)$$

Since both x_t and $\pi_{H,t}$ are free variables, determinacy requires both the eigenvalues of Ω to be inside the unit circle. The following proposition summarizes the necessary and sufficient conditions for a rational expectations equilibrium to be unique.

Proposition 8. *Under FB-DI-METR, the necessary and sufficient conditions for a rational expectations equilibrium to be unique are that*

[27] Contrary to this, Bullard and Schaling (2006) found that the interaction with the rest of the world is important in the sense that it modifies the conditions for a determinate and learnable equilibrium in the domestic economy.

$$\phi_e < 1$$
$$\phi_x < \sigma_\alpha(1+\beta^{-1})(1-\phi_e)$$
$$\kappa_\alpha(\phi_\pi - (1-\phi_e)) + (1+\beta)\phi_x < 2\sigma_\alpha(1+\beta)(1-\phi_e)$$
$$\kappa_\alpha(\phi_\pi - (1-\phi_e)) + (1-\beta)\phi_x > 0$$

(96), (97), (98) and (99)

Proof. The proof is straightforward and can be obtained from the authors upon request.

For t-dating of expectations, the MSV solution takes the form of $y_t = \bar{a} + \bar{c}w_t$ with $\bar{a} = 0,$ and $\bar{c} = (I - \rho\Omega)^{-1}$. The following proposition provides the conditions for E-stability of the MSV solution.

Proposition 9. *Suppose the time t information set is* $(1, w_t)'$. *Under FB-DI-METR, the necessary and sufficient conditions for an MSV solution* $(0, \bar{c})$ *to be E-stable are that*

$$\phi_e < 1$$
$$\kappa_\alpha(\phi_\pi - (1-\phi_e)) + (1-\beta)\phi_x > 0$$

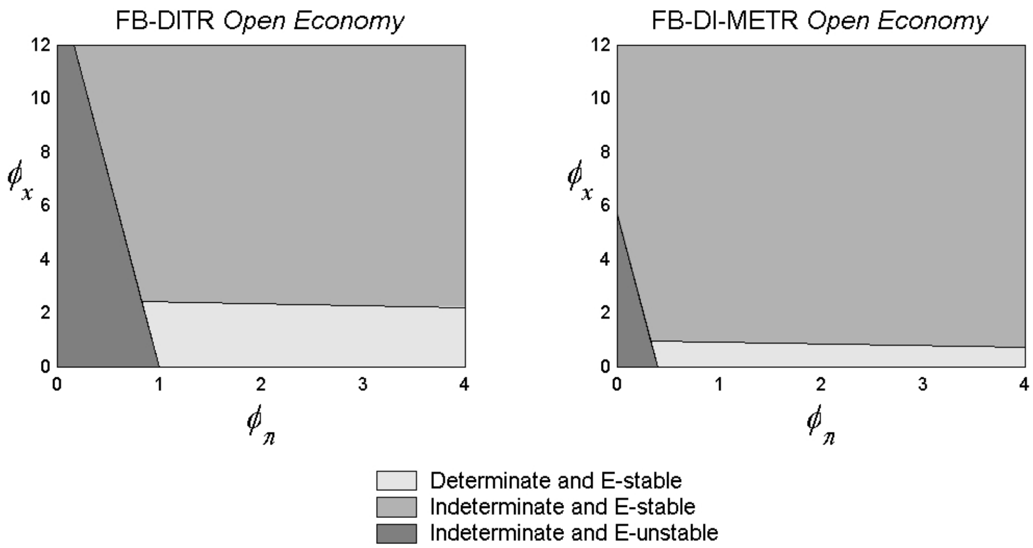
(100) and (101)

Proof. The proof is straightforward and can be obtained from the authors upon request.

First, note that the degree of managed exchange rate ϕ_e affects both determinacy and learnability conditions. On one side, ϕ_e restricts the determinacy region through conditions (97) and (98). On the other side, a positive ϕ_e relaxes both determinacy and E-stability conditions through conditions (99) and (101), respectively. However, although ϕ_e helps, reacting excessively to expected exchange rate movements causes indeterminacy and expectational instability.

Figure 8 illustrates the intersections of the regions of determinacy and learnability of the MSV solution at the baseline parametrization assuming the open economy case. The graph on the left shows the case of FB-DITR or no managed exchange rate whereas the graph on the right shows the case of FB-DI-METR. We can note that a managed exchange rate is detrimental in terms of determinacy because shrinks the upper limit to ϕ_x. However, as in other rules with managed exchange rate, FB-DI-METR guarantees stability even if a central bank reacts passively to domestic inflation $(\phi_\pi < 1)$.

Figure 8. Regions of Determinacy and E-stability for FB-DITR and FB-DI-METR



Note: Both graphics correspond to open economies $(\alpha = 0.4)$. The graphic on the left shows the case of FB-DITR or no managed exchange rate $(\phi_e = 0)$ and the graphic on the right shows the case of FB-DI-METR $(\phi_e = 0.6)$.

Analyzing this type of rule helps us to disentangle some key features observed under forward-looking CPI-based rules. As we stressed earlier, reacting to expected CPI inflation imposes an upper bound to ϕ_π approximately equal to the inverse of openness. Thus, we concluded that as the degree of openness increases, the scope of values for ϕ_π that guarantees determinacy and E-stability shrinks significantly. Our interpretation was that the reduction of the determinate and E-stable area comes from the interaction between activism against CPI inflation and openness: any increase (decrease) of the interest rate due to inflationary (deflationary) expectations triggered by an expected depreciation (appreciation) of nominal exchange rate, reinforces the expectation of higher (lower) CPI inflation. In this context, we claimed, the likelihood of a consequent movement in the interest rate relies on the preferences of the central bank, given by ϕ_π, and the degree of openness. Our results with FB-DI-METR confirm that interpretation: reacting to the expected changes in nominal exchange rate does not threaten E-stability as long as the central bank is not targeting future CPI inflation.

4. Learnability and Volatility

As shown in previous sections, some forms of managed exchange rate rules make the conditions of determinacy and learnability less stringent in small open economies. For example, in the particular case of the FB-DI-METR rule (equation (20)), to the extent that ϕ_e lies between zero and one, the region of both *E-stability* and determinacy gets larger. In fact, the larger ϕ_e the less likely the economy will fall in a indeterminate or expectational instable region. The above result suggests that a FB-DI-METR rule might be desirable based on the criteria of both determinacy and learnability compared to a FB-DITR rule (equation (14)).

Yet, there is another dimension to consider in order conclude whether managed exchange rate rules (i.e., FB-DI-METR) are desirable. In particular, it is important to quantify the volatility that these types of rules induce to the endogenous macro variables, such as the output gap and inflation. We illustrate this issue by obtaining analytically the unconditional volatility of domestic inflation and output gap under different Taylor rules. In particular, in this section we establish a link between the implied volatility that a particular rule generates vis-à-vis the conditions of *E-stability* implied by the rule. We do so by obtaining the analytical solutions of the rational expectations of two specifications: FB-DITR specification $(r_t = \phi_\pi E_t \pi_{H,t+1} + \phi_x E_t x_{t+1})$ and FB-DI-METR $(r_t = \phi_\pi E_t \pi_{H,t+1} + \phi_x E_t x_{t+1} + \phi_e E_t \Delta e_{t+1})$.

We argue that if the managed exchange rate rule (FB-DI-METR) generates larger volatility in the economy, the rule is less desirable in this dimension. As will become clear in the next two sub-sections, the benefits of each rule will depend on the source of shocks. Under a natural interest rate shock, if the managed exchange rate rule $(\phi_e > 0)$ induces smaller volatility with respect to the domestic inflation Taylor rule $(\phi_e = 0)$, then the FB-DI-METR will be more desirable. On the other hand, when the economy is hit by a foreign nominal interest rate shock, the FB-DITR induces smaller volatility compared to that generated by a FB-DI-METR rule, hence the latter rule is less desirable.

We follow Gali and Monacelli in this discussion and we find reasonable to stay closer to that analysis given that we are focusing in understanding variants on standard policy prescriptions that would apply in small open economy settings. An alternative would be to follow Evans and Honkapohja (2003, RES) and find optimal policy rules in the linear class that will also be consistent with determinacy and learnability.

4.1 Volatility and Natural Interest Rate Shock

We solve the rational expectations of the economy by using the undetermined coefficient method. We first assume that the natural interest rate shock is the only driving force of dynamics in this economy. We combine the aggregate supply equation (1), the aggregate demand equation (2) and the FB-DITR equation (14) to solve the system. We guess the solutions for domestic inflation and the output gap

$$\pi_{H,t} = \eta_{\pi r}\overline{rr}_t$$
$$x_t = \eta_{xr}\overline{rr}_t$$

and

$$\overline{rr}_t = \rho\overline{rr}_{t-1} + \varepsilon_t$$

where $\eta_{\pi r}$ and η_{xr} denote the partial elasticity of domestic inflation and the output gap with respect to the natural interest rate shock, respectively. Rewriting the AS equation

$$\pi_{H,t} = \beta\rho\pi_{H,t} + \eta_{xr}\kappa_\alpha\overline{rr}_t$$
$$\pi_{H,t} = \frac{\eta_{xr}\kappa_\alpha}{1-\beta\rho}\overline{rr}_t$$

re-writing the IS equation

$$x_t = \frac{[-\frac{1}{\sigma_\alpha}(\phi_\pi - 1)\rho\eta_{\pi r} + \frac{1}{\sigma_\alpha}]}{1 - \left(1 - \frac{\phi_x}{\sigma_\alpha}\right)\rho}\overline{rr}_t$$

and combining the above equations with our guessed solution we obtain

$$\eta_{\pi r} = \frac{\eta_{xr}\kappa_\alpha}{1-\beta\rho}$$
$$\eta_{xr} = \frac{[-\frac{1}{\sigma_\alpha}(\phi_\pi - 1)\rho\eta_{\pi r} + \frac{1}{\sigma_\alpha}]}{1 - \left(1 - \frac{\phi_x}{\sigma_\alpha}\right)\rho}$$

After some algebra we can obtain the solutions for $\eta_{r\pi}$ and η_{rx}

$$\eta_{\pi r} = \frac{\kappa_\alpha}{(1-\beta\rho)[\sigma_\alpha(1-\rho)+\rho\phi_x]+(\phi_\pi-1)\kappa_\alpha\rho}$$

$$\eta_{xr} = \frac{(1-\beta\rho)}{(1-\beta\rho)[\sigma_\alpha(1-\rho)+\rho\phi_x]+(\phi_\pi-1)\kappa_\alpha\rho}$$

(102) and (103)

From the above analytical solutions it is straightforward to see that the volatility of both domestic inflation and the output gap are decreasing in ϕ_x and ϕ_π (to the extent that $\phi_x > 0$ and $\phi_\pi > 1$). Remember that the Taylor principle is a necessary condition for *E-stability*. *Therefore, to the extent that the FB-DITR satisfies the Taylor Principle, the greater the reaction to inflation the more stable the system is.*

Now we obtain the analytical solutions of the system based on FB-DI-METR. Combining rule (20) with the UIP condition (3) we can re-write the rule as

$$r_t = \phi_\pi^{'} E_t \pi_{H,t+1} + \phi_x^{'} E_t x_{t+1} \quad (104)$$

where $\phi_\pi^{'} = \frac{\phi_\pi}{1-\phi_e}$ and $\phi_x^{'} = \frac{\phi_x}{1-\phi_e}$.

Notice that the natural interest rate shock is the only source of volatility $(\overline{rr}_t)$, so that $r_t^* = 0 \; \forall t$.

From the above implied Taylor rule, the analytical solutions of the rational expectations equilibrium are the following

$$\eta_{\pi r}^{e} = \frac{\kappa_\alpha}{(1-\beta\rho)\left[\sigma_\alpha(1-\rho)+\rho\frac{\phi_x}{1-\phi_e}\right]+\left(\frac{\phi_\pi}{1-\phi_e}-1\right)\kappa_\alpha\rho}$$

$$\eta_{xr}^{e} = \frac{(1-\beta\rho)}{(1-\beta\rho)\left[\sigma_\alpha(1-\rho)+\rho\frac{\phi_x}{1-\phi_e}\right]+\left(\frac{\phi_\pi}{1-\phi_e}-1\right)\kappa_\alpha\rho}$$

(105) and (106)

where $\eta_{\pi r}^{e}$ and η_{xr}^{e} represent the partial elasticity of domestic inflation and the output gap with respect to the natural interest rate shock induced by a FB-DI-METR rule.

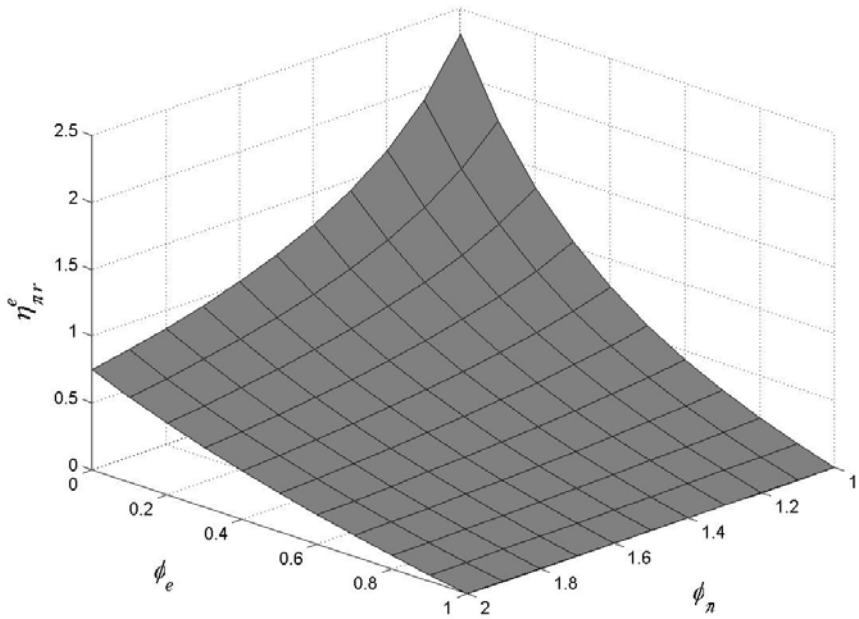
First, notice that in the limiting case, when $\phi_e \to 1$, $V(\pi_{H,t}) \to 0, V(x_t) \to 0$. However, this is not a relevant case since from the *E-stability* condition (100) we know that $0 < \phi_e < 1$. The interesting case is the one in which $0 < \phi_e < 1$. Under this scenario the unconditional variances of both $\pi_{H,t}$ and x_t are decreasing in ϕ_e. Therefore, conditional on a natural interest rate shock, reacting to the expected changes in the exchange rate is beneficial in terms of volatility since the analytics show smaller partial elasticities; that is $\eta^e_{\pi r} < \eta_{\pi r}$ and $\eta^e_{xr} < \eta_{xr}$. The economic intuition for previous findings can be explained as follows. Suppose that there is an increase in the natural interest rate $(\overline{rr}_t \uparrow)$. Following the shock we should observe increases in both the output gap and inflation. The central bank reacts by increasing the nominal interest rate generating an expected depreciation (through the UIP condition), which in turn induces a further increase in inflation. On the other hand, if the central bank puts some weight, in its reaction function, on expected movements in the exchange rate, it will partially offset the expected depreciation generated by the increase in the domestic nominal interest rate, therefore making domestic inflation to increase by less.

Interestingly, under perfect peg, that is $\phi_e = \infty, \eta^e_{\pi r} \rightarrow \infty, \eta^e_{xr} \rightarrow \infty$, we obtain that $V(\pi_{Ht}) \rightarrow \infty, V(x_t) \rightarrow \infty$. Thus, a perfect peg will generate instability in a small open economy, a result that is consistent with GM's findings. Instead, if $0 < \phi_e < 1$ the economy can become more stable; yet this case has not been analyzed by GM.

To gather more insight of the previous result, Figure 9 depicts $\eta^e_{\pi r}$ under different values of ϕ_π and ϕ_e. We set the rest of parameters at their baseline parametrization and assume that the degree of openness is 0.4 and ϕ_x is 0.5. The figure confirms the analytics: as the degree of managed exchange rate increases, $\eta^e_{\pi r}$ (and volatility) decreases. A similar pattern is observed for ϕ_π. Nevertheless, the reduction in volatility is more notorious under ϕ_e than ϕ_π. This result highlights an interesting trade-off in open economies: compared with ϕ_π, increasing ϕ_e might be more beneficial in terms of macroeconomic volatility, but at the same time increases the likelihood of indeterminacy and expectational instability. Obviously, the same discussion applies to η^e_{xr}.

Yet, notice that if we allow simultaneously for foreign nominal interest rate shocks, FB-DI-METR might generate larger volatility in the endogenous variables, mitigating its beneficial effects. We develop this result in the next sub-section.

Figure 9. Sensitivity of $\eta^{e}_{\pi r}$ with respecto to ϕ_{π} and ϕ_{e}.



Note: The rest of parameters are set at their baseline parametrization. The degree of openness is 0.4 and ϕ_x is 0.5.

4.2. Volatility and Foreign Interest Rate Shock

Notice that the implied managed exchange rate rule (20), once the foreign interest rate shock is taken into account, can be re-written as

$$r_t = \phi^{'}_{\pi} E_t \pi_{H,t+1} + \phi^{'}_{x} E_t x_{t+1} - \phi^{'}_{e} r^{*}_{t} \quad (107)$$

where $\phi^{'}_{e} = \frac{\phi_e}{1-\phi_e}$ and $\phi^{'}_{\pi}$ and $\phi^{'}_{x}$ have been defined previously. The term $\phi^{'}_{e}$ was absent in the previous case, so it is clear from the above rule that the central bank has a direct reaction to the foreign interest rate shock. Let us assume the foreign interest rate shock that hits the economy $\left(\overline{rr}_t = 0 \text{ for all } t\right)$ has the following AR(1) process

$$r^{*}_{t} = \rho^{*} r^{*}_{t-1} + \epsilon^{*}_{t} \quad (108)$$

where ρ^{*} is the autorregresive coefficient. If we assume the solutions for the endogenous variables are

$$\pi_{Ht} = \eta_{\pi r^*} r_t^*$$
$$x_t = \eta_{x r^*} r_t^*$$

the analytical solutions collapse to

$$\eta^e_{\pi r^*} = \frac{\phi_e \kappa_\alpha}{(1-\beta\rho^*)\left[\sigma_\alpha(1-\rho^*) + \rho^* \frac{\phi_x}{1-\phi_e}\right] + \left(\frac{\phi_\pi}{1-\phi_e} - 1\right)\kappa_\alpha \rho^*}$$
$$\eta^e_{x r^*} = \frac{\phi_e(1-\beta\rho^*)}{(1-\beta\rho^*)\left[\sigma_\alpha(1-\rho^*) + \rho^* \frac{\phi_x}{1-\phi_e}\right] + \left(\frac{\phi_\pi}{1-\phi_e} - 1\right)\kappa_\alpha \rho^*}$$

which can be re-expressed as

$$\eta^e_{\pi r^*} = \frac{\phi_e \kappa_\alpha}{\Lambda_0 + \frac{\Lambda_1}{(1-\phi_e)}}$$
$$\eta^e_{x r^*} = \frac{\phi_e(1-\beta\rho^*)}{\Lambda_0 + \frac{\Lambda_1}{(1-\phi_e)}}$$ (109) and (110)

where $\Lambda_0 = [(1-\beta\rho^*)\sigma_\alpha(1-\rho^*) - \kappa_\alpha\rho^*]$ and $\Lambda_1 = \phi_\pi\kappa_\alpha\rho^* + (1-\beta\rho^*)\rho^*\phi_x$.

The variances of both domestic inflation and the output gap are increasing in ϕ_e. Given the *E-stability* conditions, the only relevant case is the one when $0 < \phi_e < 1$. Notice that under reasonable parametrization $\Lambda_1 \rightarrow 0$. Therefore, $\eta^e_{\pi r^*}$ and $\eta^e_{x r^*}$ will be increasing in ϕ_e.

To sum up, a FB-DI-METR can be beneficial in terms of volatility if only shocks to the natural interest rate are present. But, as long as foreign interest rate shocks are considered, a FB-DI-METR could be less desirable in terms of macroeconomic volatility. Therefore, we argue that, in addition to the *E-stability* criterion that a Taylor rule has to meet, it is important to evaluate which are the implications in terms of volatility of a Taylor-type rule in order to conclude whether this rule is desirable.

5. Conclusions

Using the GM (2005) small open economy model, we have studied the determinacy and learnability conditions the of rational expectations equilibrium. In particular, we have extended BM (2002) results to a small open economy framework under a handful of possible Taylor-type instrument rules. Our analytical results highlight an important link between the Taylor Principle and both determinacy and learnability of REE in small open economies. The degree of openness coupled with the nature of the policy rule adopted by the monetary authorities might change this link in important ways. Perhaps the main conclusion is that a pure, naive application of the Taylor principle in open economy settings could be misleading.

With *contemporaneous rules*, we show that openness affects stability conditions quantitatively. The final impact of openness, in terms of enlargement of the determinacy region, is ambiguous and depends on the degree of the elasticity of substitution between tradable goods. More importantly, conditions for unique and learnable REE do not depend on whether the central bank responds to domestic or CPI inflation, i.e., the Taylor Principle is a necessary and sufficient condition under both policies. Yet, we have shown that a managed exchange rate regime relaxes the constraint on the degree of response to inflation and alleviates problems of indeterminacy and expectational instability.

We have stressed that in the case of *forecast-based* monetary rules, openness imposes an additional constraint, making it more difficult to induce a determinate and learnable solution. Indeed, the *Taylor Principle does not guarantees E-stability*, as it is the case in a closed economy (BM 2002). When the central bank follows either CPI inflation targeting or a managed exchange rate, the determinacy and learnability region shrinks significantly. Domestic inflation targeting does not suffer from this problem, instead suggesting that more aggressive reaction towards inflation is all to the good as in the closed economy case. Therefore, in order to avoid indeterminacy and expectational instability problems forward-looking central banks in open economies should adopt some kind of “inward-looking” policy by focusing on domestic inflation.

In this paper we emphasize the crucial role of openness along with alternative policy rules for the analysis of E-stability in open economies. The analysis of stability under learning in open economies provides new insights regarding the desirability of the policy rule. We find that conditions of learnability are more stringent in open economies with respect to closed economies. Therefore, it is more likely that a small open economy will to fall into an E-unstable region, so policymakers should be quite cautions about the instrument rule employed.

However, some managed Taylor rules exhibit desirable determinacy and learnability properties. In particular, a domestic inflation Taylor rule augmented by an exchange rate target allows the monetary authority to mitigate the threats of indeterminacy and expectational instability, although in terms of macroeconomic volatility these rules might not be desirable. We conclude that it is worthwhile to recommend not only rules that are desirable in terms of determinacy and learnability properties but also those that induce benefits in terms of macroeconomic volatility.

Finally, one important question our paper raises but does not answer is the following: If a rule is desirable in terms of both macroeconomic stability and E-stability, how fast do private agents learn this rule? Analyzing the speed of learning under the broad set of rules analyzed in this paper will add another dimension through which the desirability of a rule should be evaluated, and we think this would be a highly useful undertaking.

6. Appendices: Proofs

6.1 Appendix A: Proof of Proposition 1

Here we closely follow Woodford's proof of determinacy of a Taylor rule with some form of partial adjustment of the short term interest rate (Woodford 2003b, Chapter 4). Let the characteristic equation of the matrix Υ (defined in (49)) be written in the form

$$P(\mu) = \mu^3 + A_2\mu^2 + A_1\mu + A_0 = 0$$

where

$$A_2 = -\beta^{-1}(1+\kappa_\alpha\sigma_\alpha^{-1}) - 1 - \sigma_\alpha^{-1}\phi_x - \phi_e^{CPI} < 0 \quad \text{(A1)}$$

$$A_1 = \beta^{-1} + \beta^{-1}\sigma_\alpha^{-1}(\kappa_\alpha\phi_\pi^{CPI} + \phi_x) + (1 + \beta^{-1} + \beta^{-1}\sigma_\alpha^{-1}\kappa_\alpha)\phi_e^{CPI} > 0 \quad \text{(A2)}$$

$$A_0 = -\phi_e^{CPI}\beta^{-1} < 0 \quad \text{(A3)}$$

and where $\phi_e^{CPI} = \alpha\phi_\pi$

The above equation has one root inside the unit circle and two roots outside if and only if:

either (Case I)

$$1 + A_2 + A_1 + A_0 < 0 \quad \text{(A4)}$$

and

$$-1 + A_2 - A_1 + A_0 > 0 \quad \text{(A5)}$$

We can rule out this first case because coefficients A_i contradict $-1 + A_2 - A_1 + A_0 > 0$.

Now we have to analyze other two cases,

(Case II):

$$1 + A_2 + A_1 + A_0 > 0$$
$$-1 + A_2 - A_1 + A_0 < 0$$
$$A_0^2 - A_0A_2 + A_1 - 1 > 0$$

and (Case III):

$$1 + A_2 + A_1 + A_0 > 0$$
$$-1 + A_2 - A_1 + A_0 < 0$$
$$A_0^2 - A_0 A_2 + A_1 - 1 < 0$$
$$|A_2| > 3$$

Notice that both cases share the first condition $(1 + A_2 + A_1 + A_0 > 0)$, which can be reduced to

$$\kappa_\alpha(\phi_\pi^{CPI} - (1 - \phi_e^{CPI})) + (1 - \beta)\phi_x > 0$$

By replacing ϕ_π^{CPI} and ϕ_e^{CPI} we obtain

$$\kappa_\alpha(\phi_\pi - 1) + (1 - \beta)\phi_x > 0 \quad \text{(A6)}$$

which is a necessary condition for determinacy. By considering the signs of coefficients A_i , $-1 + A_2 - A_1 + A_0 < 0$ holds.

The additional condition required for Case II $(A_0^2 - A_0 A_2 + A_1 - 1 > 0)$ can be written after some manipulation as

$$(\phi_\pi - \beta^{-1}\phi_e^{CPI}) + \frac{(1 - \phi_e^{CPI})}{\kappa_\alpha}\phi_x + (\beta^{-1} - 1)[\kappa_\alpha^{-1}\sigma_\alpha(1 - \phi_e^{CPI})(\beta - \phi_e^{CPI})] > 0 \quad \text{(A7)}$$

and the remaining condition needed for Case III $(|A_2| > 3)$ can be written as

$$\beta^{-1}(1 + \kappa_\alpha \sigma_\alpha^{-1}) + \phi_e^{CPI} + \sigma_\alpha^{-1}\phi_x > 2. \quad \text{(A8)}$$

Equilibrium is determinate if and only if the coefficients of the policy rule (15) satisfy (A6) and either (A7) or (A8) . We will show that under the sign assumption, (A6) is both necessary and sufficient for determinacy.

We prove this by showing that any parameter values that satisfy (A6) and not (A8) *must* necessarily satisfy (A7).

First let's write (A8) as,

$$\beta^{-1}\kappa_\alpha \sigma_\alpha^{-1} + (\phi_e^{CPI} - \beta) + (\beta^{-1} + \beta) + \sigma_\alpha^{-1}\phi_x > 2. \quad \text{(A9)}$$

Note that under the sign assumption, the above equation can fail to hold only if $\phi_e^{CPI} = \alpha\phi_\pi < \beta$. (here we use the fact that $\beta^{-1} + \beta > 2$). Note that $\phi_e^{CPI} = \alpha\phi_\pi < \beta$ necessarily implies that $\phi_e^{CPI} = \alpha\phi_\pi < 1$ since $0 < \beta < 1$.

Now we need to show that under these circumstances (A7) holds given $\phi_e^{CPI} = \alpha\phi_\pi < \beta$. Notice that (A7) can be expressed as

$$\kappa_\alpha(\phi_\pi - 1) + (1-\beta)\phi_x + \kappa_\alpha(1 - \beta^{-1}\phi_e^{CPI}) + (\beta - \phi_e^{CPI})\phi_x$$
$$+ (\beta^{-1} - 1)\sigma_\alpha(1 - \phi_e^{CPI})(\beta - \phi_e^{CPI}) > 0. \quad \text{(A10)}$$

The first two terms (A10) corresponds to condition (A6) which, along with $\phi_e^{CPI} = \alpha\phi_\pi < \beta$, guarantees that (A10) will hold. Therefore, (A6) or (50) in the main text, is a necessary and sufficient condition for determinacy.

6.2 Appendix B: Proof of Proposition 2

The characteristic polynomial of Ω (given by (60)) is $\rho(\mu) = \mu^2 + A_1\mu + A_0$ where

$$A_0 = \frac{\beta(\sigma_\alpha - \phi'_x)}{\sigma_\alpha} \quad \text{(B1)}$$

$$A_1 = \frac{\phi'_x + \kappa_\alpha(\phi'_\pi - 1) - \sigma_\alpha(\beta + 1)}{\sigma_\alpha} \quad \text{(B2)}$$

with $\phi'_\pi = \left(\frac{\phi_\pi(1-\alpha)}{1-\phi_\pi\alpha}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_\pi\alpha}\right)$. Both eigenvalues of Ω are inside the unit circle if and only if both of the following conditions hold

$$|A_0| < 1 \quad \text{(B3)}$$

$$|A_1| < 1 + A_0. \quad \text{(B4)}$$

After replacing the definitions of ϕ'_π and ϕ'_x, we can note that condition (B3) implies (62), whereas condition (B4) implies (63) and (64). The only relevant case is $\phi_\pi < 1/\alpha$, given by (61). The other case, $\phi_\pi > 1/\alpha$, can be ruled out by showing that it contradicts condition (B4).

6.3 Appendix C: Proof of Proposition 3

Using results of Evans and Honkapohja (2001), E-stability requires that the eigenvalues of $\rho\Omega$ (Ω is given by equation. (60) to have real parts less than one. The eigenvalues of $\rho\Omega$ are given by the product of the eigenvalues of Ω and ρ, and since $0 < \rho < 1$, it suffices that eigenvalues of Ω have parts less than 1. On the other hand, the MSV solution will not be E-stable if any eigenvalue of Ω has a real part greater than 1. The characteristic polynomial of $\Omega - I$ is given by $p(\mu) = \mu^2 + A_1\mu + A_0$ where

$$A_1 = \frac{\kappa_\alpha(\phi'_\pi - 1) + \phi'_x + \sigma_\alpha(1-\beta)}{\sigma_\alpha} \quad \text{(C1)}$$

$$A_0 = \frac{\phi'_x(1-\beta) + \kappa_\alpha(\phi'_\pi - 1)}{\sigma_\alpha} \quad \text{(C2)}$$

where $\phi'_\pi = \left(\frac{\phi_\pi(1-\alpha)}{1-\phi_\pi\alpha}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_\pi\alpha}\right)$.

It is necessary for both eigenvalues of $\Omega - I$ to have negative real parts. According to the Routh Theorem, that condition holds if and only if $A_1 > 0$ and $A_0 > 0$. We can note that

$$A_1 = A_0 + \frac{\sigma_\alpha(1-\beta) + \beta\phi'_x}{\sigma_\alpha} \quad \text{(C3)}$$

After replacing the definitions of ϕ'_π and ϕ'_x, under the case of $\phi_\pi < 1/\alpha$, $A_0 > 0$ implies $A_1 > 0$. In this case, the second E-stability condition, given by (66), is derived from $A_0 > 0$. As in determinacy analysis, there is a second case, $\phi_\pi > 1/\alpha$. However, this case is not relevant, since it contradicts $A_0 > 0$.

6.4 Appendix D: Proof of Proposition 4

Here we closely follow proof 1 for CPITR. Let the characteristic equation of the matrix Υ (defined in equation (68) be written in the form

$$P(\mu) = \mu^3 + A_2\mu^2 + A_1\mu + A_0 = 0$$ where

$$A_2 = -\beta^{-1}(1+\kappa_\alpha\sigma_\alpha^{-1}) - 1 - \alpha\phi_\pi - \sigma_\alpha^{-1}\phi_x - \phi_e < 0 \quad \text{(D1)}$$

$$A_1 = \alpha\phi_\pi(1+\beta^{-1}) + \beta^{-1} + \beta^{-1}\sigma_\alpha^{-1}(\kappa_\alpha\phi_\pi + \phi_x) + (1+\beta^{-1}+\beta^{-1}\sigma_\alpha^{-1}\kappa_\alpha)\phi_e > 0 \quad \text{(D2)}$$

$$A_0 = -(\alpha\phi_\pi + \phi_e)\beta^{-1} < 0. \quad \text{(D3)}$$

The above equation has one root inside the unit circle and two roots outside if and only if *either* Case I, II or III holds (see proof of Proposition 1). Since coefficients A_i contradict $-1+A_2-A_1+A_0 > 0$, we can rule out this first case.

Condition $1+A_2+A_1+A_0 > 0$(shared by Case I and II) can be reduced to

$$\kappa_\alpha(\phi_\pi - (1-\phi_e)) + (1-\beta)\phi_x > 0 \quad \text{(D4)}$$

Therefore, condition (D4) is a necessary condition for determinacy. Given the signs of the coefficients A_i ,$-1+A_2-A_1+A_0 < 0$(shared by Case I and II) holds.

The additional condition required for Case II $(A_0^2 - A_0A_2 + A_1 - 1 > 0)$can be written, after some manipulation, as

$$(\phi_\pi - \beta^{-1}(\alpha\phi_\pi + \phi_e)) + \phi_e + \frac{(1-\alpha\phi_\pi-\phi_e)}{\kappa_\alpha}\phi_x$$
$$+ (\beta^{-1}-1)[\kappa_\alpha^{-1}\sigma_\alpha(1-\alpha\phi_\pi-\phi_e)(\beta-\alpha\phi_\pi-\phi_e)] > 0 \quad \text{(D5)}$$

and the remaining condition needed for Case III $(|A_2| > 3)$can be written as

$$\beta^{-1}(1+\kappa_\alpha\sigma_\alpha^{-1}) + (\alpha\phi_\pi + \phi_e) + \sigma_\alpha^{-1}\phi_x > 2. \quad \text{(D6)}$$

Equilibrium is determinate if and only if the coefficients of the policy rule (17) satisfy (D4) and either (D5) or (D6). We will show that under the sign assumption, (D4) is both necessary and sufficient for determinacy.

We prove this by showing that any parameter values that satisfy (D4) and not(D6) *must* necessarily satisfy (D5).

First we will write (D6) as

$$\beta^{-1}\kappa_\alpha\sigma_\alpha^{-1} + (\alpha\phi_\pi + \phi_e - \beta) + (\beta^{-1}+\beta) + \sigma_\alpha^{-1}\phi_x > 2 \quad \text{(D7)}$$

Note that under the sign assumption, the above equation can fail to hold only if $\alpha\phi_\pi + \phi_e < \beta$. (here we use the fact that $\beta^{-1} + \beta > 2$). Note that $\alpha\phi_\pi + \phi_e < \beta$, necessarily implies that $\alpha\phi_\pi + \phi_e < 1$ since $0 < \beta < 1$.

Now we need to show that, under these circumstances, (D5) holds given $\alpha\phi_\pi + \phi_e < \beta$. Notice that (D5) can be expressed as

$$\kappa_\alpha(\phi_\pi - (1-\phi_e)) + (1-\beta)\phi_x + \kappa_\alpha(1 - \beta^{-1}(\alpha\phi_\pi + \phi_e)) + (\beta - \alpha\phi_\pi - \phi_e)\phi_x$$
$$+ (\beta^{-1} - 1)\sigma_\alpha(1 - \alpha\phi_\pi - \phi_e)(\beta - \alpha\phi_\pi - \phi_e) > 0 \quad \text{(D8)}$$

The first two terms (D8) corresponds to condition (D4) which along with $\alpha\phi_\pi + \phi_e < \beta$, guarantees that (D8) will hold. Therefore, (D4), or (69) in the main text, is a necessary and sufficient condition for determinacy.

6.5 Appendix E: Proof of Proposition 5

The characteristic polynomial of Ω (given by (75)) is $p(\mu) = \mu^2 + A_1\mu + A_0$ where

$$A_0 = \frac{\beta(\sigma_\alpha - \phi'_x)}{\sigma_\alpha} \quad \text{(E1)}$$

$$A_1 = \frac{\phi'_x + \kappa_\alpha(\phi'_\pi - 1) - \sigma_\alpha(\beta + 1)}{\sigma_\alpha} \quad \text{(E2)}$$

where $\phi'_\pi = \left(\frac{\phi_\pi(1-\alpha)}{1-\phi_\pi\alpha-\phi_e}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_\pi\alpha-\phi_e}\right)$.

Both eigenvalues of Ω are inside the unit circle if and only if conditions (B3) and (B4) hold. After replacing the definitions of ϕ'_π and ϕ'_x, we can note that condition (B3) implies (77), whereas condition (B4) implies (78) and (79). The only relevant case is $\phi_\pi < (1-\phi_e)/\alpha$, given by (76). The other case is $\phi_\pi > (1-\phi_e)/\alpha$, which implies $\phi_\pi > 1-\phi_e$, given that α lies between 0 and 1. However, this case contradicts condition (B4).

6.6 Appendix F: Proof of Proposition 6

As in the previous cases, E-stability conditions are given by analyzing the characteristic polynomial of $\Omega - I$ (where Ω is given by 75) given by $\rho(\lambda) = \mu^2 + A_1\mu + A_0$ where

$$A_1 = \frac{\kappa_\alpha(\phi'_\pi - 1) + \phi'_x + \sigma_\alpha(1-\beta)}{\sigma_\alpha} \quad \text{(F1)}$$

$$A_0 = \frac{\phi'_x(1-\beta) + \kappa_\alpha(\phi_\pi - 1)}{\sigma_\alpha} \quad \text{(F2)}$$

where $\phi'_\pi = \left(\frac{\phi_\pi(1-\alpha)}{1-\phi_\pi\alpha-\phi_e}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_\pi\alpha-\phi_e}\right)$

It is necessary for both eigenvalues of $\Omega - I$ to have negative real parts. According to the Routh Theorem, that condition holds if and only if $A_1 > 0$ and $A_0 > 0$. We can note

$$A_1 = A_0 + \frac{\sigma_\alpha(1-\beta) + \beta\phi'_x}{\sigma_\alpha} \quad \text{(F3)}$$

After replacing the definitions of ϕ'_π and ϕ'_x, we can note that under the case of $\phi_\pi < (1-\phi_e)/\alpha$, $A_0 > 0$ implies $A_1 > 0$. In this case, the second E-stability condition, given by (81), is derived from $A_0 > 0$. As in determinacy analysis, there is a second case which implies $\phi_\pi > (1-\phi_e)/\alpha$. However, this case is not relevant, since it contradicts $A_0 > 0$.

6.7 Appendix G: Coincidence of Determinacy and Learnability under CPITR and DITR

6.7.1 MSV Solution, Intuition and Analytics under Learning

In this appendix we provide some intuition and a sketch of the analytical results for coincidences of the areas of determinacy and learnability under a CPITR and DITR specifications. In order to do so, we first obtain the stationary MSV solution of the system. Notice that given that the Taylor principle guarantees that the system is determinate, the MSV solution of the system has to be stationary. Under the contemporaneous CPITR rule the system takes the form of (55).

Given that there is just one predetermined variable, i.e., $\hat{s}_{t-1}$, we know that $\tilde{b}$ has its two first columns filled with zeros. Thus,

$$\widetilde{b} = \begin{bmatrix} 0 & 0 & \eta_\pi \\ 0 & 0 & \eta_e \\ 0 & 0 & \eta_s \end{bmatrix} \quad \text{(G1)}$$

where η_π, η_e and η_s are coefficients. Note that the MSV solution requires $|\eta_s| < 1$ to be stationary. Explicitly, the MSV solution takes the form

$$\begin{aligned} \pi_{H,t} &= \eta_\pi \hat{s}_{t-1} + \varsigma_\pi \Delta \bar{s}_t \\ \Delta e_t &= \eta_e \hat{s}_{t-1} + \varsigma_e \Delta \bar{s}_t \\ \hat{s}_t &= \eta_s \hat{s}_{t-1} + \varsigma_s \Delta \bar{s}_t \end{aligned} \quad \text{(G2)}$$

By applying the method of undetermined coefficients we obtain,

$$\begin{aligned} \beta \eta_\pi \eta_s + \kappa_\alpha \sigma_\alpha^{-1} \eta_s &= \eta_\pi \\ \eta_e \eta_s - \phi_\pi^{CPI} \eta_\pi - \phi_x \sigma_\alpha^{-1} \eta_s &= \phi_e^{CPI} \eta_e \\ 1 + \eta_e - \eta_\pi &= \eta_s \end{aligned} \quad \text{(G3)}$$

Now we express the coefficients η_π and η_e as functions of η_s

$$\begin{aligned} \eta_\pi &= \frac{\kappa_\alpha \eta_s}{\sigma_\alpha (1 - \eta_s \beta)} \\ \eta_e &= \frac{\kappa_\alpha \eta_s - \sigma_\alpha (1 - \eta_s \beta)(1 - \eta_s)}{\sigma_\alpha (1 - \eta_s \beta)} \end{aligned} \quad \text{(G4)}$$

The solutions for η_s are given by the roots of a cubic polynomial of the form

$$p(\eta_s) = \eta_s^3 + a_2 \eta_s^2 + a_1 \eta_s + a_0 \quad \text{(G5)}$$

where

$$\begin{aligned} a_2 &= -\frac{\beta \phi_x + \beta \sigma_\alpha + \sigma_\alpha + \beta \phi_e^{CPI} \sigma_\alpha + \kappa_\alpha}{\beta \sigma_\alpha} < 0 \\ a_1 &= \frac{\sigma_\alpha + \beta \phi_e^{CPI} \sigma_\alpha + \phi_e^{CPI} \sigma_\alpha + \phi_e^{CPI} \kappa_\alpha + \phi_x + \kappa_\alpha \phi_\pi^{CPI}}{\beta \sigma_\alpha} > 0 \\ a_0 &= -\frac{\phi_e^{CPI}}{\beta} < 0 \end{aligned} \quad \text{(G6)}$$

By applying the Descartes Rule of signs we know that there are three positive roots or one positive root and a pair of complex conjugates. Notice that when the policy rule corresponds to domestic inflation targeting that is $\phi_\pi^{CPI} = \phi_\pi$ and $\phi_e^{CPI} = 0,$ we obtain $\eta_s = 0, \eta_\pi = 0$ and $\eta_e = -1$. It follows that

$$\Delta e_t = -\hat{s}_{t-1} \quad \text{(G7)}$$

Similarly, this solution is also obtained when the degree of openness is zero. In this case, the inertia displayed by the system only comes from the inertia of exogenous variables.

Yet, under the CPITR, the MSV solution needs $|\eta_s| < 1$ to be stationary. We evaluate the above polynomial in -1, 1 and 0. It is straightforward to show that $p(-1)$ and $p(0)$ are both negative. Solving for $p(1)$ we have

$$p(1) = \kappa_\alpha(\phi_\pi^{CPI} - (1 - \phi_e^{CPI})) + (1-\beta)\phi_x \quad \text{(G8)}$$

which could be positive or negative. In order to have a positive root between 0 and 1, we need $p(1) > 0.$ After replacing ϕ_π^{CPI} and ϕ_e^{CPI}, one can note that $p(1) > 0$ if and only if the condition for determinacy holds; see Proposition 1. Hence, the previous results confirm that there exists a unique and stationary solution given that $0 < \eta_s < 1$. This is an alternative way to show that the Taylor Principle is a necessary and sufficient condition for determinacy.

6.7.2 E-Stability: Analytics

Now we show that the Taylor Principle is a necessary condition for stability under learning dynamics. First we analyze the E-stability condition for DT_a, given by (31). The MSV solution will be E-stable if all eigenvalues of DT_a have a real part less than 1. Therefore, it is necessary for both eigenvalues of $DT_a - I$ to have negative real parts.

It can be shown that $DT_a - I$ has one eigenvalue equal to -1. The rest of eigenvalues can be obtained from the following characteristics polynomial[28]

$$p(\mu) = \mu^2 + A_1\mu + A_0 \quad \text{(G9)}$$

where

$$A_1 = \delta^{-1} a_1$$
$$A_0 = \delta^{-1} a_0 \quad \text{(G10)}$$

with (for sake of exposition, let us assume that $\phi_x = 0$):

$$\delta = \kappa_\alpha(\phi_e^{CPI} + \phi_\pi^{CPI} - \eta_s) + \sigma_\alpha(1-\eta_s)(1-\beta\eta_s) + (1-\beta\eta_s)\sigma_\alpha\phi_e^{CPI}$$
$$a_1 = \kappa_\alpha(2\phi_e^{CPI} + 2\phi_\pi^{CPI} - 1 - \eta_s(2-\beta)) + \sigma_\alpha(1-\eta_s\beta)((1-\beta) + (2-\beta)(\phi_e^{CPI} - \eta_s))$$
$$a_0 = \kappa_\alpha(\phi_e^{CPI} + \phi_\pi^{CPI} - 1 - \eta_s(1-\beta)) + \sigma_\alpha(1-\beta)(1-\eta_s\beta)(\phi_e^{CPI} - \eta_s)$$
(G11)

To be E-stable, we need $A_1 > 0$ and $A_0 > 0$ (Routh-Hurwitz Theorem). It is straightforward to see that A_1 and A_0 are positive as long as the equilibrium is determinate and $\phi_\pi^{CPI} = (1-\alpha)\phi_\pi > (1-\beta)$ and $\phi_e^{CPI} = \alpha\phi_\pi > \eta_s$. Suppose that $\beta = 1$, then $\phi_\pi^{CPI} = (1-\alpha)\phi_\pi > 0$ will always be the case. Hence, to the extent that $\phi_e^{CPI} = \alpha\phi_\pi > \eta_s$ the Taylor Principle guarantees the first *E-stability* condition. Our quantitative results confirm this analysis.

In the rest of the proof we use the fact that the eigenvalues of the Kronecker product of two matrices are equal to the cross product of the eigenvalues of each matrix.[29] Using this property we can note that the second *E-stability* condition (32) needs ρ to hold given that DT_a has eigenvalues with real parts less than 1. Similarly, the *E-stability* condition DT_b depends on whether or not $\left[(I-\Omega\tilde{b})^{-1}\Phi\right]'$ has eigenvalues with real parts less than 1 provided that *E-stability* condition for DT_a holds. Using (56), we can re-write DT_b as

$$\tilde{b}' \otimes (I-\Omega\tilde{b})^{-1}\Omega. \quad \text{(G12)}$$

Since the eigenvalues of any matrix are exactly the same as that of its transpose, the eigenvalues of $\tilde{b}$ must have its real part less than one. From the MSV solution we know that $\tilde{b}$ has two eigenvalues equal to -1 and one eigenvalue equal to η_s. As shown, the Taylor Principle

[28] To obtain these results we adapted some Mathematica programs used by Bullard and Mitra (2006).

[29] Let A and B square matrices of dimensions n and m. If $\{\lambda_i | i = 1,\ldots,n\}$ are the eigenvalues of A and $\{\mu_j | j = 1,\ldots,m\}$ are the eigenvalues of B, then $\{\lambda_i\mu_j | i = 1,\ldots,n, i = 1,\ldots,m\}$ are the eigenvalues of $A \otimes B$.

guarantees that η_s lies between 0 and 1. This result, coupled with our proposition for DT_a, implies that the eigenvalues of DT_b have real parts less than 1 if the equilibrium is determinate. Therefore, the Taylor Principle is a necessary condition for *E-stability* for a CPITR.

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