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An Alternative Nonlinear Perspective on the Consumption, Income and Wealth Relationship

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Abstract

We provide new evidence on the relationship between consumption expenditure and key drivers namely, income and wealth. Using a testing procedure advocated by Bierens applied to US data, we find evidence that all series are in fact stationary around a nonlinear deterministic trend and are co-trended insofar as they share a common nonlinear deterministic trend. This can be seen in the context of cointegration-based studies that have often found against the existence of a long-run relationship. We also contribute to the 'great ratios' debate concerning the time series properties of the average propensity to consume.

We are grateful for the helpful comments provided by the Editor and anonymous referees. Any remaining errors are our own.

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1. Introduction

Personal consumption expenditures account for nearly 70 percent of US GDP. As financial liberalization has taken place, both financial and household wealth have assumed important positions as key drivers of consumption expenditure. However, when it comes to validating a long run equilibrium relationship involving consumption, income and wealth, both theoretical and empirical evidence has not pointed convincingly towards cointegration between these series. Indeed, the possibility of a stable cointegrating relationship has most likely been impacted on by the major changes in financial markets, demographics, productivity growth, tax rates and so on where reference can be made to empirical studies such as Benjamin et al. (2004), Rudd and Whelan (2006), and Carroll et al., (2006) supporting the view of an unstable long run relationship.

In this paper, we address this issue by considering an alternative assessment of the consumption function relationship. This is based on a testing procedure advocated by Bierens (1997a, 1997b, 2000) that considers whether nonlinear trend stationarity is present in consumption, income and wealth and if so, whether they are *co-trended* insofar as sharing the same nonlinear deterministic trend. Rather than focus on a single wealth measure, we further deviate from much of the existing literature by differentiating the impacts from both housing and financial wealth on consumption. Finally, we offer some reflection on the debate concerning the observed non-stationarity of one of the great ratios, namely the average propensity to consume (*APC*). Despite the predictions of theory, research has often found that the *APC* is non-stationary [King et al. (1991), Harvey et al. (2003)]. Within a growth modeling context, the *APC* depends on structural parameters; therefore the *APC* might undergo periodic mean shifts, as the underlying economic structure changes. Accordingly, Attfield and Temple (2010) examine whether the ratios are stationary for the US and UK, allowing for structural breaks that could reflect time-varying parameters. They find stronger evidence for stationarity than previous work. Our perspective on this debate is that the *APC* is stationary, but stationary with respect to a nonlinear deterministic trend.

2. Methodology

For long macroeconomic time-series, it is often implausible to argue that the parameters of the data generation process are unchanged over time. Perron (1997) and others have shown that when a time series has structural breaks in the mean, the unit root hypothesis is often accepted before structural breaks are taken into account. It is conceivable that macroeconomic variables may in fact be stationary around deterministic nonlinear trends. Such trends are meant to capture the evolution of the underlying data generating processes from changes in structural parameters of economies capturing structural instability and the fundamental features of economic systems in the long-run. Therefore, it can be argued that more flexible trend specifications that go beyond the standard linear representations should be entertained.

The Bierens (1997a, 1997b) nonlinear augmented Dickey-Fuller (NLADF) test allows the trend to be an almost arbitrary deterministic function of time. The test is based on an ADF type auxiliary regression model that sees a nonlinear deterministic trend approximated by a linear function of Chebishev polynomials. These offer substantial advantages over regular time polynomials because they are orthogonal (with a closed form) and bounded and allow the researcher to distinguish stationarity around a linear trend from stationarity around a nonlinear deterministic trend under the alternative hypothesis.

Suppose a series is modeled as $\mu_t = \omega + \phi\mu_{t-1} + \nu_t$ where ω is a constant drift parameter and ν_t is a stationary autoregressive process. The usual test for linear adjustment towards mean is based assessing the unit root properties of μ_t through the OLS estimation of ADF regressions such as

$$\Delta\mu_t = \omega + \zeta\mu_{t-1} + \sum_{i=1}^{k}\psi_i\Delta\mu_{t-i} + \nu_t \qquad (1)$$

where $-2 < \zeta < 0$ indicates stationarity of μ_t. The test of the null hypothesis $\phi = 1$ proposed by Bierens is against the alternative of nonlinear trend stationarity:

$$\mu_t = g(t) + \nu_t \qquad (2)$$

where $g(t)$ is a possibly nonlinear trend function. The NLADF regression is written as:

$$\Delta\mu_t = \omega + \zeta\mu_{t-1} + \sum_{i=1}^{p}\psi_i\Delta\mu_{t-i} + \theta^T P_{t,n}^{(m)} + \nu_t \qquad (3)$$

where $P_{t,n}^{(m)} = \left[P_{0,n}^{*}(t), P_{1,n}^{*}(t), \ldots., P_{m,n}^{*}(t)\right]^T$ is a vector of orthogonal Chebishev time polynomials such that $P_{0,n}^{*}(t) = 1$, $P_{1,n}^{*}(t)$ is equivalent to a linear trend and $P_{2,n}^{*}(t)$ through to $P_{m,n}^{*}(t)$ are cosine functions defined as $\sqrt{2}\cos[k\pi(t-0.5)/n]$ for $t = 1,\ldots,n$, and $k = 1,\ldots,m-1$. Under the null hypothesis of a unit root, $\zeta = 0$ and $\theta^T = 0$. The unit root hypothesis can be tested on the basis of the *t*-statistic on ζ or the test statistic $Am = (n-p-1)\zeta\left|1 - \sum_{i=1}^{k}\psi_i\right|$. These are two-tailed tests. If the non-stationary null is rejected, the proper alternative hypothesis will depend on whether there is left- or right-side rejection. A left-rejection favors the alternative of either mean stationarity, linear trend stationarity or nonlinear trend stationarity; whereas a right-rejection favors the alternative of nonlinear trend stationarity alone. The distribution of these tests is non-standard and so p-values are simulated using a wild bootstrap procedure.

Although some macroeconomic time-series are not unit root processes, they might still behave as if they are cointegrated. Similarity in long-run dynamics is normally described as cointegration. However, there might be similar movements that are not to do with unit roots. Nonlinear co-trending is a special case of common feature where the appearance of cointegration could be accounted for by the presence of a common nonlinear deterministic time trend that links several nonlinear trend stationarity series. Bierens (2000) proposes a nonparametric test for nonlinear co-trending based on the eigenvalues of matrices constructed from the partial sums of the variables.[1] The test is nonparametric in the sense that the nonlinear trends and any serial correlation process do not have to be specified. The test is based on the generalized eigenvalues of the matrices M_1 and M_2 defined as:

[1] Bierens (2000) considers nonlinear co-trending in the context of inflation and interest rates in the US. Further applications include Cushman (2002), who analyses the demand for money, and Camarero and Ordonez (2006), who consider European unemployment rates.

$$M_1 = \left(\frac{1}{n}\right)\left[F(1/n)F(1/n)' + ... + F(1)F(1)'\right]$$

$$M_2 = \left(\frac{1}{n}\right)\left[dF(m/n)dF(m/n)' + ... + dF(1)dF(1)'\right] \quad (4)$$

where M_1 and M_2 are estimated from the partial sums of the variables such that $F(t/n) = (1/n)[x_1 + ... + x_t]$, $dF(t/n) = \{F(t/n) - F[(t/n) - (m/n)]\}/(m/n)$, x_t is the de-trended or demeaned μ_t, and $m = n^{\zeta}$ with n equal to the number of usable observations. Solving $\left|\hat{M}_1 - \lambda \hat{M}_2\right| = 0$ for λ, the test statistics are calculated as $n^{1-\zeta}\hat{\lambda}_r$ where r is the number of co-trending vectors under the null. This test has a nonstandard distribution. Bierens (2000) calculates the asymptotic critical values for this test. The critical values that we use in our analysis are based on the simulated data-based sampling distributions used by Bierens. The existence of r co-trending vectors among $r+1$ series indicates the presence of r linear combinations that are stationary around a linear trend where these series share a single $[(r+1)-r]$ common nonlinear deterministic time trend. This is indicative of a strong degree of co-movement across the $r+1$ series.

3. Data and Results

All data are obtained from the *Bureau of Economic Analysis* where quarterly data for the non-durable consumption (*NDC*), labor income (*LIN*), financial wealth (*FW*) and housing wealth (*HW*) are expressed in real per capita terms for the study period 1952Q1-2010Q3. The data plotted in Figure 1 indicate episodes of sharp swings in wealth and (*NDC-LIN*) making it very likely that structural breaks exists. Table 1A reports ADF unit root tests which are unable to reject non-stationarity at the 5% significance level throughout. In the case of *HW*, there is only a marginal rejection of the null at the 10% level in cases when a deterministic linear trend is included in the more powerful DF-GLS test advocated by Elliott et al. (1996). Table 1B reports results based on the Perron (1997) unit root tests that allow for a single (unknown) structural break. In only one of twelve tests is the non-stationary null rejected at the 5% significance level. This is the case of *HW* where the additive outlier model points towards a structural break at 2009Q2, possibly a reflection of recent events surrounding the global financial crisis. If we consider more powerful tests with unknown breaks such as those proposed by Perron and Yabu (2009), there are mixed findings regarding non-stationarity. Table 1C reports limited evidence of stationarity concerning *NDC* and *LIN*, but this is dependent on the type of model used to conduct the test. While some of these tests point to the possibility of stationarity if a change in the intercept is allowed for, the wealth variables are still found to be non-stationary.

Given the fairly strong evidence that the four series appear as non-stationary series, it is of interest to examine the possibility that they are cointegrated. The single equation tests based on Engle-Granger and Phillips-Ouliaris finds against cointegration. This finding is supported by the multivariate Johansen testing procedure which is unable to reject the null hypothesis of no cointegrating vectors. It is possible that these cointegration tests have low test power on account of structural breaks that are present but not allowed for. Table 3 provides some findings based on Gregory-Hansen. Here there is evidence of cointegration once a full structural break at 1988Q4 is allowed for.

Given that the strongest evidence of cointegration is found where a structural break is present, we now consider the possibility that *NDC*, *LIN*, *FW* and *HW* are in fact co-trended sharing a common non-linear deterministic trend. Our first task is to establish whether the series are stationary around a non-linear deterministic trend. Table 4 presents NLADF test results based on the auxiliary regression in equation (3).[2] The lag length p is chosen using the AIC and the Chebishev time polynomial is set at $m = 10$.[3] This test can potentially present substantial size distortion so relevant critical values are simulated using a wild bootstrap based on 10,000 replications of a Gaussian $AR(m)$ process for $\Delta\mu_t$ with parameters and error variance equal to the estimated $AR(m)$ null model. According to the *t-stat* tests, there is a right hand side rejection of the unit root hypothesis in favor of nonlinear trend stationarity at the 10% significance level or better in all cases.

While time-series studies of consumption have concluded that these variables are first difference stationary, the analysis here looks at this issue in a different light in terms of *NDC*, *LIN*, *FW* and *HW* being stationary around a deterministic nonlinear trend. A key issue for our study is what implication this characterization has for the relationship between these variables. The co-trending test results presented in Table 5 point to the existence of three co-trending vectors ($r = 3$) comprising *NDC*, *LIN*, *FW* and *HW*. Evidence of three linear combinations of *NDC*, *LIN*, *FW* and *HW* that is stationary around a nonlinear trend suggests that these four series share a common nonlinear deterministic time trend where common trending behavior would appear to be a reasonable statistical characterization of the US consumption function. Table 6 reports the three co-trending vectors where each has been standardized by the largest coefficient. Vector 1 most closely resembles a long-run consumption function based on a non-linear trend in *NDC* with positive elasticities of 0.686, 0.188 and 0.200 attached to the non-linear trends for *LIN*, *FW* and *HW* respectively. However, it is possible to solve the three vectors simultaneously to express *NDC* with respect to each one of the explanatory variables in turn. While this should not necessarily be interpreted as causality, using all the information provided by the co-trending vectors the elasticities that relate the non-linear trends in *NDC* with *LIN*, *FW* and *HW* are respectively 1.166, 0.925 and 0.964.

Our findings provide an alternative viewpoint on the debate concerning the non-stationarity of one of the great ratios, namely the *APC*. The balanced growth and neoclassical stochastic growth literatures imply stationarity of the consumption-output ratio. Indeed, the *APC* is usually regarded as bounded between 0 and 1. However, the evidence favoring stationarity is very limited. The *APC* series is plotted in Figure 1. In our analysis, we have established that the non-linear trends in *NDC* and *LIN* move closely in tandem over the long-run with a coefficient of 1.166. In theory, a coefficient that is insignificantly different from unity should mean that (*NDC-LIN*) is not driven by a nonlinear deterministic trend because the (common trends) driving *NDC* and *LIN* are cancelled out. However, a coefficient that is significantly different from unity would mean that (*NDC-LIN*) is driven by a nonlinear deterministic trend. In contrast to *NDC*, *LIN*, *FW* and *HW*, the NLADF tests reported in Table 4 indicate that (*NDC-LIN*) is characterized by a left-rejection which favors the alternative of either mean stationarity, linear trend stationarity or nonlinear trend stationarity. The unit root tests reported in Tables 1A, 1B and 1C provide mixed evidence as to whether or not (*NDC-LIN*) can be characterized as a stationary series even after allowing for a constant, linear trend

[2] Estimation is conducted using the EasyReg International software made available by Herman Bierens.

[3] Bierens (1997a) reports results for m=10 and argues there is no definitive method for choosing m. If m is too low, it may be insufficient to approximate the nonlinearity under the alternative. If m is too high, it may cause the test to lack power.

or structural breaks. Depending on the model used, the relatively more powerful Perron and Yabu tests offer some limited support of stationarity. It is therefore quite likely that the left-side rejection of the *NLADF* test occurs because (*NDC-LIN*) is stationary around a deterministic nonlinear trend.

4. Conclusion

Given the limited evidence based on cointegration, this paper has provided an alternative perspective on understanding the behavior of the consumption function. While consumption expenditure and its income, financial wealth and housing wealth determinants appear to be stationarity around nonlinear trends, they can be regarded as co-trended insofar as they share a common nonlinear deterministic time trend. While several studies have found that the average propensity to consume is non-stationary, our perspective is that it could be stationary, but around a deterministic non-linear trend.

References

Attfield, C. and J.R.W. Temple (2010) "Balanced Growth and the Great Ratios: New Evidence for the US and UK" *Journal of Macroeconomics* **32**, 937-56.

Benjamin, J., Chinloy, P. and D. Jud (2004) "Real Estate Versus Financial Wealth in Consumption" *Journal of Real Estate Finance and Economics* **29**, 341-354.

Bierens, H.J. (1997a) "Testing the Unit Root with Drift Hypothesis Against Nonlinear Trend Stationarity, with an Application to the U.S. Price Level and Interest Rate" *Journal of Econometrics* **81**, 29–64.

Bierens, H.J. (1997b) "Nonparametric Cointegration Analysis" *Journal of Econometrics* **77**, 379-404.

Bierens, H.J. (2000) "Nonparametric Nonlinear Co-trending Analysis, With an Application to Inflation and Interest in the U.S." *Journal of Business and Economic Statistics* **18**, 323–337.

Camarero, M. and J. Ordonez (2006) "Is There a Nonlinear Co-movement in the EU Countries' Unemployment?" *Economics Letters* **93**, 157-62.

Carroll, C., Otsuka, M. and J. Slacalek (2006) "How Large is the Housing Wealth Effect? A New Approach" NBER Working Paper No. 12746.

Cushman, D. (2002) "Nonlinear Trends and Co-trending in Canadian Money Demand" *Studies in Nonlinear Dynamics and Econometrics* **6**, No. 1, Article 4, 1-27.

Dufrénot, G. and V. Mignon (2002) *Recent Developments in Nonlinear Cointegration with Applications to Macroeconomics and Finance*. Springer. Kluwer Academic Publishers, Netherlands.

Elliott, G., Rothenberg, T.J. and J.H. Stock (1996) "Efficient Tests for an Autoregressive Unit Root" *Econometrica* **64**, 813-836.

Engle, R.F. and C.W.J. Granger (1987) "Co-integration and Error Correction: Representation, Estimation, and Testing" *Econometrica* **55**, 251-276.

Gregory A.W. and B.E. Hansen (1996) "Residual-based Tests for Cointegration in Models with Regime Shifts" *Journal of Econometrics* **70**, 99–126.

Harvey, D. I., Leybourne, S. J. and P. Newbold (2003) "How Great Are the Great Ratios?" *Applied Economics* **35**, 163-77.

Johansen, S. (1991) "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models" *Econometrica* **59**, 1551-1580.

King, R.G., Plosser, C.I., Stock, J.H. and M.W. Watson (1991) "Stochastic Trends and Economic Fluctuations" *American Economic Review* **81**, 819–840.

Perron, P. (1997) "Further Evidence on Breaking. Trend Functions in Macroeconomic Variables" *Journal of Econometrics* **80**, 355-385.

Perron, P. and T. Yabu (2009) "Testing for Shifts in Trend With an Integrated or Stationary Noise Component" *Journal of Business and Economic Statistics* **27**, 369-396.

Phillips, P.C.B. and S. Ouliaris (1990) "Asymptotic Properties of Residual Based Tests for Cointegration" *Econometrica* **58**, 165–193.

Rudd, J. and K. Whelan (2006) "Empirical Proxies for the Consumption-Wealth Ratio" *Review of Economic Dynamics* **9**, 34-51.

Figure 1. US Non-durable Consumption, Labor Income, Wealth and the APC

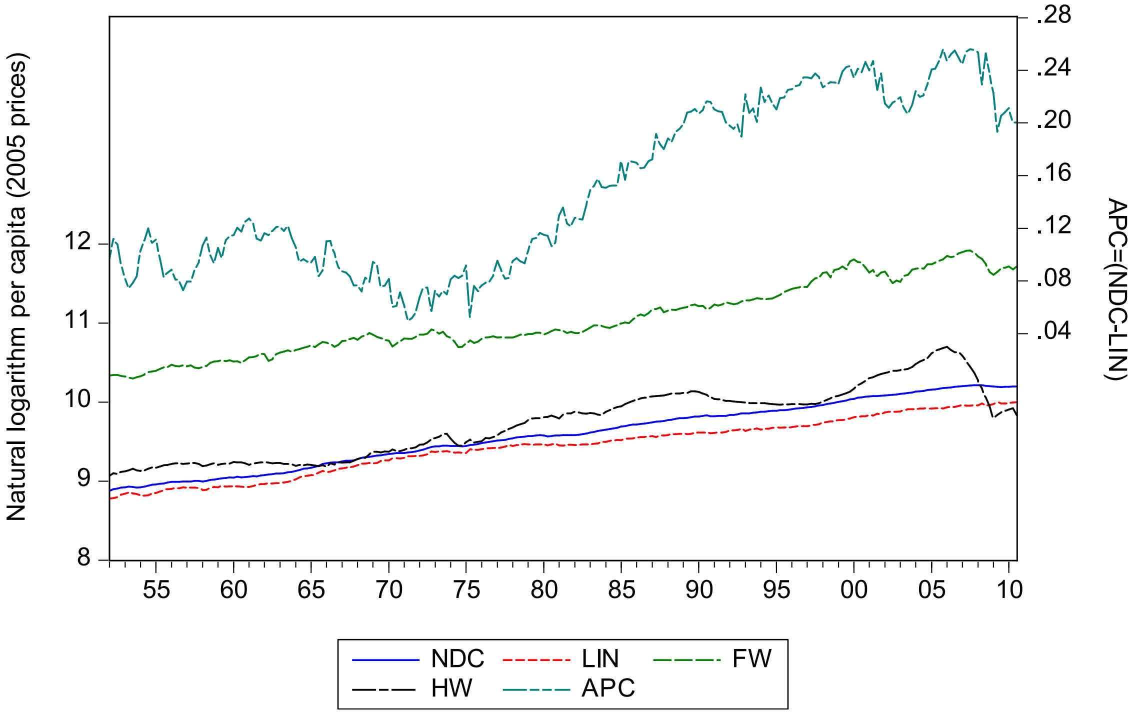

Table 1A. Unit root tests

	ADF (no trend)	ADF (trend)	DF-GLS (no trend)	DF-GLS (trend)
NDC	-1.695	-1.017	0.822	-1.817
LIN	-0.709	-1.524	1.203	-1.406
FW	-0.623	-2.519	1.625	-2.470
HW	-1.594	-2.696	-0.745	-2.821[c]
(*NDC-LIN*)	-0.964	-1.581	-0.240	-1.430

Notes: in all cases, the lag length is selected according to the Akaike Information Criterion (AIC). The superscript *c* denotes rejection of the null hypothesis at the 10% significance level.

Table 1B. Perron (1997) unit root tests

Model:	IO1		IO2		AO	
	T_b	$t_{\hat{\alpha}}$	T_b	$t_{\hat{\alpha}}$	T_b	$t_{\hat{\alpha}}$
NDC	1967Q3	-4.074	1971Q3	-3.712	1977Q3	-3.567
LIN	1963Q2	-4.085	1963Q3	-4.287	1973Q2	-3.005
FW	1972Q3	-3.851	1972Q3	-3.921	1980Q4	-2.901
HW	1975Q2	-3.637	1974Q2	-3.580	2009Q2	-4.886[b]
(*NDC-LIN*)	1963Q2	-3.341	1963Q2	-3.340	1972Q3	-2.488

Notes: the models are the *Innovational Outlier model* (IO1) incorporating a change in the intercept, *the Innovational Outlier model* (IO2) incorporating a change in the intercept and the slope, and the *Additive Outlier* (AO) model incorporating a change in the slope only, but both segments of the trend function are joined at the time break. T_b denotes the time of the break and $t_{\hat{\alpha}}$ denotes the test statistic for a unit root. b denotes rejection of the null at the 5% significance level based on a critical value of -4.65.

Table 1C. Perron and Yabu (2009) unit root tests

Model:	Model 1		Model 2		Model 3	
	T_b	$t_{\hat{\alpha}}$	T_b	$t_{\hat{\alpha}}$	T_b	$t_{\hat{\alpha}}$
NDC	1965Q3	1.9214[b]	1976Q4	0.2554	1965Q3	2.1515
LIN	1966Q2	5.1999[b]	1974Q3	1.4957	1967Q4	7.8893[b]
FW	1973Q3	1.4592	1982Q4	-0.1852	1974Q1	2.1372
HW	1977Q4	0.7681	2001Q3	-0.0926	2001Q3	1.1951
(*NDC-LIN*)	1966Q3	3.0636[b]	1974Q1	0.6000	1979Q1	4.6678[b]

Notes: the models are *Model 1* which incorporates a change in the intercept with a corresponding 5% critical value of 1.74; *Model 2* which incorporates a change in the slope with corresponding 5% critical value of 1.67; and *Model 3* which incorporates a change in the intercept and the slope with corresponding 5% critical value of 3.12. T_b denotes the time of the break, $t_{\hat{\alpha}}$ denotes the test statistic for a unit root and *b* denotes rejection of the null at the 5% significance level. In cases, the lag length is chosen by the AIC and the sample is trimmed at 15%.

Table 2. Non-cointegration tests on *NDC*, *LIN*, *FW* and *HW*

τ (Engle-Granger)	τ (Phillips-Ouliaris)	*Trace* (Johansen)
-3.033 (0.365)	-3.159 (0.324)	34.823 (0.457)

Notes: τ (Engle-Granger) and τ (Phillips-Ouliaris) refer to the non-cointegration tests advocated by Engle and Granger (1987) and Phillips and Ouliaris (1990). *Trace* refers to the Trace statistic advocated by Johansen (1991) for the null hypothesis of no cointegrating vectors involving *NDC*, *LIN*, *FW* and *HW*. In each case, lag length selection was based on the AIC and *p*-values are reported in parentheses.

Table 3. Gregory and Hansen (1996) cointegration tests

Level break, no trend		Level break, trend		Full structural break	
T_b	$t_{\hat{\alpha}}$	T_b	$t_{\hat{\alpha}}$	T_b	$t_{\hat{\alpha}}$
1981Q3	-4.995	1984Q2	-5.361	1988Q4	-6.091*

Notes: the 1 and 5% critical values are respectively -5.77 and -5.28 for the level break model with no trend. -6.05 and -5.57 for the level break model with trend, and -6.51 and -6.00 for the full structural break model. * denotes rejection of the non-cointegration null at the 5% significance level. T_b denotes the time of the break and $t_{\hat{\alpha}}$ denotes the minimum test statistic for a unit root. In each case, the lag length is determined by the AIC.

Table 4. NLADF tests

	NDC	*LIN*	*FW*	*HW*	(*NDC-LIN*)
t-stat	0.923	0.951	0.982	0.949	0.029

Notes: these are p-values based on bootstrapped critical values.

Table 5. Nonlinear co-trending analysis

Null	Alternative	Test statistic	10% crit. value	5% crit. value	Outcome
$r = 1$	$r = 0$	0.048	0.352	0.466	Accept
$r = 2$	$r = 1$	0.176	0.536	0.674	Accept
$r = 3$	$r = 2$	0.296	0.704	0.860	Accept
$r = 4$	$r = 3$	1.899	0.862	1.035	Reject

Table 6. Co-trending vectors

Vector 1	Vector 2	Vector 3	
1	-0.437	-0.159	← NLT in NDC
-0.686	-0.992	-0.324	← NLT in LIN
-0.188	0.231	1	← NLT in FW
-0.200	1	-0.622	← NLT in HW
Nonlinear trend in NDC = 1.166 x nonlinear trend in LIN Nonlinear trend in NDC = 0.925 x nonlinear trend in FW Nonlinear trend in NDC = 0.964 x nonlinear trend in HW			

Notes: NLT denotes nonlinear trend. Standardized co-trending vectors are reported.