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Testing for a nonlinear Fisher relationship

Frederick H Wallace
Universidad de Quintana Roo

Abstract

Empirical evidence regarding the Fisher effect is mixed. One reason may be a nonlinear adjustment process in the real interest rate. The nonlinear unit root test proposed by Sollis, Leybourne, and Newbold (Journal of Money, Credit, and Banking 34: 686-700, 2002) is used to test for stationarity of the U.S. real interest rate over the 1934:01-2011:02 period and selected subperiods. The unit root null in the real rate of interest can be rejected over the full sample, evidence of a Fisher effect. Weaker evidence of a Fisher relation is found in the subsample for 1934:01-1959:12 for which the unit root null can be rejected for one measure of the real interest rate. However, there is no indication of a Fisher effect for subperiods starting in 1960 or later. A conjecture is that temporal aggregation of the interest rate data may explain the different results, but the findings are also consistent with other explanations.

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Contact: Frederick H Wallace - fwalla@uqroo.mx.
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1. Introduction

Does a one-time change in the inflation rate cause an equivalent change in the nominal interest rate? In other words does a Fisher effect hold? If so, then the real interest rate must be stationary. Empirical evidence is inconclusive. Rose (1988) finds that the Fisher relationship does not hold for any of the eighteen OECD countries he studies. His conclusion is based on tests that indicate the presence of a unit root in the nominal interest rate and stationarity of the inflation rate in each country. A necessary condition for the Fisher relation is that the two series are integrated of the same order. In their extension of Rose's study using recently developed unit root tests and updated data Rapach and Weber (2004) look for evidence of a Fisher relationship in sixteen OECD countries. Unlike Rose, they find that both the nominal interest rate and the inflation rate are I(1) processes for ten of the countries. However, their tests reveal little evidence of cointegration between the nominal rate and inflation so that the real rate is not stationary in most cases, thus the Fisher relationship does not hold for most of the countries in their study. Thus, Rapach and Weber reach the same conclusion as Rose although for different reasons.

In contrast, Mishkin (1992) and Wallace and Warner (1993) find empirical evidence for a Fisher effect in U.S. data. Cointegration results presented in Crowder and Hoffman (1996) suggest that a one percent increase in the inflation rate raises the nominal interest rate by more than one percent, an effect they attribute to tax distortions. Using tax-adjusted nominal interest rate data their results indicate a Fisher effect in U.S. data. Recent work by Christopoulos and León-Ledesma (2007), henceforth CL, and Yoon (2010) suggest that the Fisher relationship may be a nonlinear, perhaps due to the presence of thresholds in the conduct of monetary policy.[1] CL posit that the U.S. central bank targeted the inflation rate during the periods in which Paul Volcker and Alan Greenspan were Fed chairs but adjusted monetary policy only when the inflation rate exceeded a threshold level of about three percent.

If a Fisher effect holds, then the macroeconomic impact of a one-time change in the inflation rate is limited as Ahmed and Rogers (2000) show in their study using four variants of a representative agent model.[2] Inflation has a much larger impact on the macroeconomy in the absence of a Fisher relationship. Using U.S. annual data for 1889-1995 they find that a permanent increase in inflation negatively affects the consumption-output ratio and positively affects the investment-output ratio, leading Ahmed and Rogers to conclude that the evidence does not support the Fisher relation.

The objective of this study is to apply the nonlinear unit root test of Sollis, Leybourne, and Newbold (2002) to U.S. real interest rate data. Their nonlinear test can capture behavior of the sort that Christopoulos and León-Ledesma postulate for the Fed, that is adjustments to monetary policy once a policy threshold is breached.

[1] Neely and Rapach (2008) discuss other possible sources of nonlinearities in the Fisher relation and present a thorough review of the empirical evidence.

[2] In the version of their model with a cash-in-advance constraint for consumption only, there is a Fisher effect but superneutrality does not hold because a one-time increase in inflation reduces employment.

2. Data and Unit Root Test

U.S. monthly data on the not seasonally adjusted consumer price index for all urban consumers and 3 month Treasury bill rate are from the FRED database of the Federal Reserve Bank of St. Louis. The data span nearly seventy-eight years, 1934:01-2011:05. The Fisher relation expressed in terms of the expected inflation and real interest rates, π_t^e, r_t^e, respectively, and the known nominal interest rate, i_t, is given by equation (1).

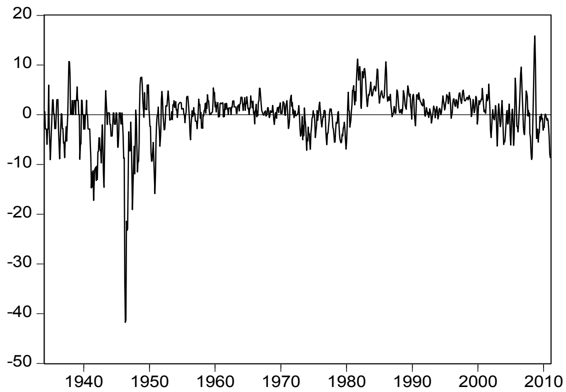
$$i_t - \pi_t^e \approx r_t^e \tag{1}$$

Since the Fisher relationship is an approximate one in discrete time an alternative measure of the real interest rate based on equation (2) is used as a robustness check on the empirical results.

$$\frac{1+i_t}{1+\pi_t^e} = 1 + r_t^e \tag{2}$$

As is common in the literature, actual inflation for period t is employed as a proxy for expected inflation. The actual rate of period t inflation is the annualized change in the monthly CPI from period t to period t+3. If inflation expectations are rational, then actual and expected inflation differ only by a white noise error term and will have identical integration properties, thus the actual inflation rate will be a suitable proxy.[3] The use of actual inflation in the calculations means that the real interest rate measures used in the empirical estimations are ex post real rates. Figure 1 shows the real interest rate derived from the Fisher relation over the entire study period. In particular, observe the extreme volatility occurring during the initial twenty years and the final five years of the study, an issue addressed later in the paper.

Figure 1-U.S. Real Interest Rate, 1934:01-2011:02



The Sollis, Leybourne, and Newbold (henceforth SLN) unit root test given by equation (3) is based on a modified logistic function, the term in braces.

$$\Delta r_t = \alpha\left\{-0.5 + \left[1+\exp\left(-\gamma^2 r_{t-d}^2\right)\right]^{-1}\right\} r_{t-1} + \sum_{i=1}^{k} \beta_i \Delta r_{t-i} + \varepsilon_t \quad (3$$

[3] Neely and Rapach elaborate upon this point and note that the assumption of rational expectations is not a necessary condition. If inflation expectation errors are stationary the integration properties of actual and expected inflation are the same.

Δ is the change operator, r_t is the actual, demeaned real interest rate, d is a delay parameter, α, β, and γ are coefficients, and ε_t is a white noise error.[4] To eliminate autocorrelation k lags of the month-to-month change in the real interest rate are included. The basic idea of the test is that mean reversion is an increasing function of the squared deviations from the mean. The null hypothesis is $\alpha = 0$, meaning a unit root is present in the demeaned variable; thus indicating that the real interest rate is not stationary and the Fisher effect does not hold. Therefore, rejection of the null is evidence of a stationary real rate and the Fisher effect. Two features of the test are noteworthy. First, if the period t-d demeaned real interest rate is zero, the term in braces is zero and the real rate behaves as a unit root. Second, the test reduces to the augmented Dickey-Fuller test if the term in braces is one.

3. Results

Equation (3) is estimated using nonlinear least squares with the real interest rate calculated from the Fisher relationship [equation (1)] over the full sample 1934:01-2011:02 and various restricted samples; 1934:01-1959:12, 1960:01-2004:12, 1934:01-1979:07, and 1979:08-2006:01.[5] The subperiod beginning in 1960:01 is selected because it corresponds to the full study period considered by Christopoulos and León-Ledesma. The restricted sample starting in 1979:08 is chosen since it spans the Volcker and Greenspan terms as Federal Reserve chair. As noted above, CL suggest that the Fed used an inflation target during the terms of these two chairs.

To address autocorrelation, thirty-six lags of the change in the demeaned real rate are included in the initial estimation. The longest lags are eliminated three at a time as long as the marginal significance level (msl) of the Breusch-Godfrey Lagrange multiplier (henceforth, BG-LM) statistic remains greater than .5. In cases where the significance level of the BG-LM test statistic falls below .5 after eliminating three lags, between one and three lags are added until an msl of .5 is restored. This approach eliminates autocorrelation in the full sample and one restricted sample, 1934:01-1959:12. Autocorrelation is more problematic in estimations over the other restricted samples as it appears even with thirty-six lags of the dependent variable. For these other subperiods forty-eight lags are included in the estimation with individual lags added or subtracted until the msl of the BG-LM test statistic exceeds .5. For the 1960:01-2004:12 period no specification can be found satisfying the .5 msl criterion so an msl of .2 is used instead. The same steps are followed in applying the SLN test to the alternative real interest rate measure derived from equation (2). Estimation results for both measures of the real rate over the full and restricted samples are shown in Table I.[6] The figures reported in the table are based on a delay parameter of one in the SLN unit root tests, that is d = 1 in equation (3).

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[4] The sample mean is used to demean the real interest rate. See SLN for the justification.
[5] Since actual inflation for periods t to t+3 proxies for the expected inflation rate in period t, the final three observations in the data sample are lost. Thus the empirical estimations are through 2011:02 rather than 2011:05.
[6] Critical values are from Table 1 of SLN. Critical values shown are for a sample size of 300, close to the sample size for the shortest subperiod considered. The critical values reported by SLN vary only slightly with the number of observations. For example the 5% critical values are -3.19 and -3.11 for sample sizes of 100 and 500 respectively.

Table I-SLN Unit Root Tests on the U.S. Real Interest Rate, Selected Periods

Real Interest Rate from Equation (1), Fisher Relation			
Period	Lags of Δr_t	Estimated α	t-statistic on estimated α
1934:01-2011:02	27	-0.168	-4.145**
1934:01-1959:12	22	-0.334	-3.269*
1960:01-2004:12	48	-0.137	-2.326
1934:01-1979:07	48	-0.131	-1.781
1979:08-2006:01	48	-0.211	-2.587
Real Interest Rate from Equation (2)			
Period	Lags of Δr_t	Estimated α	t-statistic on estimated α
1934:01-2011:02	27	-0.167	-4.098**
1934:01-1959:12	18	-0.269	-2.653
1960:01-2004:12	51	-0.154	-1.052
1934:01-1979:07	49	-0.121	-1.689
1979:08-2006:01	48	-0.208	-2.542

* significant at the 5% level, critical value = -3.12
** significant at the 1% level, critical value = -3.70

The empirical results indicate a rejection of the unit root null at the 1% significance level over the full sample, 1934:01-2011:02, regardless of the interest rate measure used. The estimates of the coefficient α change only slightly and the marginal significance level of the t-statistic remains less than 1% if the delay parameter is set at two or three.[7] The same conclusion of stationarity over the full sample, regardless of interest rate measure, is also obtained from the ADF test with lags selected by the Schwarz criterion.

The null of a unit root in the real interest rate measure derived from the Fisher relation, equation (1), can also be rejected for the restricted sample 1934:01-1959:12 at the 5% significance level. Interestingly, the choice of real rate measure does affect results for this subperiod. High annual rates of inflation, at times exceeding 10% during the first two years of World War II, combined with a constant nominal T-bill rate (0.38%) from 1942:07 to 1947:06 frequently produced highly negative real rates during the war and its immediate aftermath. The sensitivity of the results to the real rate measure may arise because the Fisher relation poorly approximates the real rate when inflation is high provided the real rate is different from zero. By contrast, there is no ambiguity with the ADF tests, which indicate rejection of the unit root null for both real interest rate measures in this subsample.

Christopoulos and León-Ledesma find that the nominal interest rate and inflation are cointegrated in a nonlinear fashion over the full 1960:01-2004:12 period and various subperiods. They conclude that the nonlinear relationship is well approximated by either the logistic or exponential smooth transition model depending on the sample period. Contrary to the CL findings the SLN test reveals no evidence of a nonlinear Fisher effect as the null hypothesis of a unit root in the U.S. real interest rate cannot be rejected for the 1960:01-2004:12 sample for either interest rate measure. ADF tests also fail to reject the null for either rate measure.

SLN test results reveal no evidence of stationarity for the other subperiods, 1934:01-1979:07 and 1979:08-2006:01, although the ADF test indicates stationarity of the real rate

[7] As changes in the delay parameter have very little effect on results for any subperiod, they are not reported in the paper but may be obtained from the author.

over the earlier restricted sample.[8] Figure 1 reveals that the real interest rate tended to be negative from late 1972 through 1979 during the period of relatively high U.S. inflation. Starting in 1980 real rates tended to be positive, indeed were uniformly positive from 1980:10 to 1986:11 with rates exceeding 9% in some months. Real rates tended to remain positive until the 2001 recession. The extended periods of negative real rates and large positive real rates suggest some sluggishness in the adjustment of inflation expectations, given that the nominal rate on short term T-bills might be expected to incorporate quickly new information about inflation.

4. Conclusions

Results using the full sample, spanning more than seventy years, indicate that the real interest rate is stationary in the United States. The conclusion occurs for both interest rate measures and is robust to changes in the delay parameter as well as modest modifications to the lag length. Curiously, evidence of a stationary real rate is weaker for the 1934:01-1959:12 restricted sample and absent over the other subperiods.

A possible explanation for the seemingly conflicting results may be that the Fed has chosen to target variables like the inflation rate or money supply rather than the real interest rate so that the real interest rate thresholds that trigger responses from the Fed are quite wide. This would be consistent with central bank perceptions that periods of very high or low real rates are necessary to change expectations. If so, the SLN test results will reveal evidence of a nonlinear, stationary Fisher effect only when the real interest rate varies widely over the sample, thus lies outside the thresholds, as it does between 1934 and 1951 and after 2006. Smaller real rate changes after 1960 but prior to 2007 may mean that the real rate generally stayed within the threshold values hence behaved as a unit root during most of the subperiods considered. Rejection of the unit root null over the full sample, then, could be the result of the inclusion of both periods of high real rate volatility in the sample.

The monthly T-bill rate data are business day averages. More than fifty years ago Working (1960) recognized that averaging "can introduce correlations not present in the original series (p. 916)." Taylor (2001) generalizes Working's results and shows that the temporal averaging of exchange rate data can bias upward the estimated speed at which the real exchange rate reverts to its purchasing power level. Taylor concludes that employing long spans of data in empirical estimations can reduce the consequences of temporal aggregation bias. Thus an alternative explanation for the differing results is that temporal aggregation of the interest rate data affects the SLN test statistics in a form analogous to the impact of such averaging in exchange rate studies. If so the strong evidence of a stationary real rate may be due to the long span of data used in the full sample with the much weaker evidence over the subsamples attributable to the shorter spans rather than absence of a Fisher effect.

Finally, Crowder and Hoffman find that the Fisher relation holds if U.S. tax distortions are taken into account. Thus tax rate changes over the sample period may have resulted in mean shifts in the Fisher relationship making it difficult to reject the unit root null. Determining the contribution of each of these alternative explanations to the empirical results is a challenge for future work.

[8] Excluding the period of the Fed's monetarist experiment and extending the period to include the most recent data (1982:10-2011:02) yields results qualitatively similar to those shown in the table for 1979:08-2006:01 hence are not reported.

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