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Abstract

This paper analyses the impact of the disinflation policy timing on the sign and the magnitude of the sacrifice ratio in a modified price and wage staggered model of Blanchard (1986). When wages are updated every four quarters and prices every two quarters, we show that a "cold-turkey" disinflation is associated to an output boom when the policy is implemented during the last period of life of the wage contract and a recession the other quarters.

Keywords: Disinflation policy; Shock timing; Sacrifice ratio; Price and wage staggered contracts.

JEL classification: E31; E52

1 Introduction

There is a huge literature on the conduct of monetary policy in a framework with optimizing agents facing constraints when they want to adjust their price or wage decisions. Due to these nominal rigidities, monetary shocks have persistent real effects. While some divergences remain concerning the quantitative impact of monetary shocks in such models, a common feature of this literature is to consider that the impact of shocks on economic variables is independent from the date of their occurrence during the calendar year.

This paper shows that the timing of a disinflation policy may have a impact on the size of the sacrifice ratio, and, more importantly, on the sign of

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the output response. We determine the conditions under which an immediate and unexpected disinflation (often called "cold-turkey" disinflation) is associated to an output boom. Some papers already associate output booms to disinflations. However, it is always in the case of gradual or pre-announced disinflations, as in the sticky price models of Ball (1994*a*) or Mankiw and Reis (2002). In these models, the origin of the disinflationary boom is not due to a specific assumption about the kind of nominal rigidities. It comes from the combination of sticky prices with the assumption of an expected disinflation. Taking into account this future disinflation, sticky price setters begin to lower their prices immediately, leading to a rise of real balances and an output boom. However a cold-turkey disinflation in these models is at best associated with a real neutrality.

We derive our result from the 'wage-price' spiral model of Blanchard (1986). It is based on staggered prices and wages contracts *à la* Taylor in a monopolistic competition setup. Our main assumption concerns the distinct duration of the price and wage contracts. We assume that prices are adjusted every two quarters while wages are set annually. This assumption has strong implications concerning the real impact of an unexpected disinflation. We show that the output boom only occurs when the disinflation begins in the last period of the wage contract. If the same policy is implemented in other periods, it implies recessions of various sizes. Our approach shares some common features with Olivei and Teneyro (2007, 2008). They show that the date of a monetary shock may have a different impact on output deviation depending on the percentage of wages which are changed during the quarter of the calendar year. But our work differs since we do not consider a non uniform distribution of staggering contracts *à la* Calvo (1983).

The timing structure of our model is mainly guided by empirical considerations. As Ball (1994*b*) points out, "*in many countries, virtually all wages are set for one year with no time-variation*", and it concerns "*more than 80 percent of US wages*". Taylor (1999) and Dickens *et al.* (2006) also note that most of the wages are negotiated annually. Furthermore, Olivei and Tenreyro (2007, 2008) support the idea that annual wage changes are mostly concentrated in the same quarter of the year in the United States (U.S.) and Japan (but not for major countries of the Euro area). Evidence for a faster adjustment of prices is also well documented. Among others, Bils and Klenow (2004) find that half of goods' prices last 5.5 months or less in the U.S.[1]. Based on these empirical findings, we can plausibly assume that prices

[1] Altissimo *et al.* (2006) highlight that the degree of price rigidity seems to be higher for the Euro area (around one year).

are more often adjusted than wages, leading to a higher duration for wage contracts than price contracts. Moreover this assumption is in line with the "old" Keynesian feature that wages are more sticky than most prices in the U.S. (Friedman, 1999).

The structure of the paper is as follows. Section 2 introduces asymmetric price and wage contracts, and determines analytically the impact of monetary shocks on the dynamics of prices and wages. Section 3 shows that a "cold-turkey" disinflation can have a different impact depending on the date of the shock. Finally, Section 4 presents a brief summary and conclusion.

2 The model

We consider the reduced form of Blanchard (1986) derived from the general equilibrium imperfect competition model of Blanchard and Kiyotaky (1987). Let p, w, m and y be the price level, the nominal wage, the stock of money and the output, respectively.[2] Let p^* and w^* be the price and wage that maximize the profit for a given period. The elasticities a and b, with $0 < a, b < 1$, respectively measure the degree of rigidity of markups and real wage.[3] Following Blanchard and Kiyotaky (1987), the optimal price and the optimal wage at time t are given by:

$$p_t^* = aw_t + (1-a)m_t, \tag{1}$$
$$w_t^* = bp_t + (1-b)m_t. \tag{2}$$

The output depends on the level of real balances:

$$y_t = m_t - p_t. \tag{3}$$

We introduce some nominal rigidities in the economy: prices set by one cohort of firms and wages set by one cohort of workers are not modified each period. The duration of each contract is known by advance and follows the adjustment rule of Taylor (1980): for a contract x_t ($x_t = p_t, w_t$) modified in period t, with a duration of N periods, the probability of changing the contract is equal to 0 during the $N-1$ subsequent periods, and equal to 1

[2] All variables are expressed in logarithm and are measured as deviations from their trend values normalized to zero. The steady state is defined as $w = p = m$ and $y = 0$.

[3] These elasticities depend on some structural parameters. Following Blanchard (1986), we have $a = 1/\alpha$ and $b = 1 - \alpha(\beta - 1)$, where $\alpha \geq 1$ is the inverse of the degree of returns-to-scale, and $\beta > 1$ the marginal desutility of work. To ensure $b > 0$, we assume $\beta < 1 + (1/\alpha)$.

in period $t+N$. Agents set their price or wage by minimizing the following loss function:

$$\underset{x_t}{Min}\ L_t = \sum_{i=0}^{N-1} E_t \left(x_t - x_{t+i}^{*} \right)^2 , \tag{4}$$

where E_t stands for the mathematical expectation of future variables, given the set of available information at time t (this set of information contains the path of monetary policy announced at time t). The contract is then set as an average of the future optimal values:

$$x_t = \frac{1}{N} \sum_{i=0}^{N-1} x_{t+i}^{*}. \tag{5}$$

Based on the empirical findings developed in the introduction, we assume a one-year duration of wage with wage changes mostly concentrated during the same period. Prices are assumed to be updated every two quarters leading a duration of one semester. It turns out that at time t, the wage cohort sets its nominal wage for four periods, from t to $t+4$, while at time $t+1$ the price cohort sets its price for two periods $t+1$ and $t+2$ and again at time $t+3$ for two periods $t+3$ and $t+4$. Three different price contracts overlap with a wage contract. The first period of the wage contract coexists with the price contract set at time $t-1$, the second and third periods overlap with the price set at time $t+1$ and the last period overlaps with the price set at time $t+3$. We denote by p^1 and p^3 the prices set at the first and the third periods of the wage contract life. The timing of adjustment is summarized by the following Table:[4]

Period t	0	1	2	3	4	5	6	7	8	...
prices	$\longrightarrow$	p_1^1	$\longrightarrow$	p_3^3	$\longrightarrow$	p_5^1	$\longrightarrow$	p_7^3	$\longrightarrow$	...
wage	w_0	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	w_4	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	w_8	...

Table 1: The timing of adjustment

Given Eqs (1), (2) and (5), wages and prices are set according the follow-

[4] A more complex setup with two cohorts of price and one cohort of wage is discussed in section 3.2.

ing rules ($\forall t = 0, 4, 8...$):

$$\begin{aligned}
w_t &= \frac{1}{4}\left[bp^3_{t-1} + (1-b)\, m_t\right] + \frac{1}{4} E_t\left[bp^1_{t+1} + (1-b)\, m_{t+1}\right] \\
&\quad + \frac{1}{4} E_t\left[bp^1_{t+1} + (1-b)\, m_{t+2}\right] + \frac{1}{4} E_t\left[bp^3_{t+3} + (1-b)\, m_{t+3}\right], \quad (6) \\
p^1_{t+1} &= \frac{1}{2}\left[aw_t + (1-a)\, m_{t+1}\right] + \frac{1}{2}\left[aw_t + (1-a)\, E_{t+1} m_{t+2}\right], \quad (7) \\
p^3_{t+3} &= \frac{1}{2}\left[aw_t + (1-a)\, m_{t+3}\right] + \frac{1}{2} E_{t+3}\left[aw_{t+4} + (1-a)\, m_{t+4}\right]. \quad (8)
\end{aligned}$$

The choice of an asymmetric structure has some implications on the weights of other contracts taken into account by price setters. As shown in Table 1, the choice of p^1 is made only with respect to the past value of wages (because the price contract will be renewed before the wage contract), while the determination of p^3 depends also on the future value of wages (because the contract will interact both with the current and following wage contracts).

Eqs (6), (7) and (8) permit to obtain the following dynamic equation for the wage level ($\forall t = 0, 4, 8...$):

$$\begin{aligned}
w_t &= f w_{t-4} + f E_{t-1} w_t + f E_t w_{t+4} + g_1 m_{t-1} + g_1 E_{t-1} m_t \\
&\quad + g_2 m_t + (2g_1 + g_2)\, E_t m_{t+1} + (2g_1 + g_2)\, E_t m_{t+2} \\
&\quad + (g_1 + g_2)\, E_t m_{+3} + g_1 E_t m_{t+4}, \quad (9)
\end{aligned}$$

where

$$f = \frac{ab}{8-5ab},\ g_1 = \frac{b(1-a)}{8-5ab},\ g_2 = \frac{2(1-b)}{8-5ab}.$$

Eq (9) states that the dynamic of w_t under rational expectations depends on both past and future expected values of w_t through the terms $E_{t-1} w_t$ and $E_t w_{t+4}$, as well as expected values of m_t. Solving the model requires the elimination of the expected values of w_t by the method of lag operators (Romer, 2006). As in Blanchard (1986), we distinguish the initial impact of the shock from the subsequent dynamics. We obtain an equation which only depends on the past values of wages (the initial value of wages is known), and the expectations about the money path (which depends on the exogenous and publicly announced monetary policy). The dynamic of w_t is then given by ($\forall t = 0, 4, 8...$):

$$\begin{aligned}
w_t &= \lambda w_{t-4} + \frac{\lambda}{f} \sum_{i=0}^{\infty} \lambda^i E_t \begin{pmatrix} g_2 m_{t+4i} + (2g_1 + g_2)\,(m_{t+1+4i} + m_{t+2+4i}) \\ + (g_1 + g_2)\, m_{t+3+4i} + g_1 m_{t+4+4i} \end{pmatrix} \\
&\quad + \frac{\lambda g_1}{f} \sum_{i=0}^{\infty} \lambda^i E_{t-1}\left(m_{t-1+4i} + m_{t+4i}\right), \quad (10)
\end{aligned}$$

where λ is the stable eigenvalue:

$$\lambda = \frac{\sqrt{2-ab} - \sqrt{2(1-ab)}}{\sqrt{2-ab} + \sqrt{2(1-ab)}}. \tag{11}$$

To analyze the impact of a shock on the growth rate of the money, we have to determine both the growth path of the wage and the initial response (see Appendix). We denote by $\Delta w_t = w_t - w_{t-4}$ the growth rate of wage ($\forall t = 0, 4, 8..$), and $\Delta m_t = m_t - m_{t-1}$ the growth rate of money ($\forall t = 0, 1, 2..$). We have:

$$\begin{aligned} f(1-\lambda)\lambda^{-1}\Delta w_t &= f(1-\lambda)\Delta w_{t-4} - g_1 E_{t-5}(M_1) - E_{t-4}(M_2) \\ &\quad + g_1 E_{t-1}(M_3) + E_t(M_4) \end{aligned} \tag{12}$$

where

$$\begin{aligned} M_1 &= \Delta m_{t-4} + \sum_{i=1}^{\infty} \lambda^i \left(\Delta m_{t-4+4i} + \Delta m_{t-5+4i}\right), \\ M_2 &= (2g_1+g_2)\sum_{i=2}^{3} \Delta m_{t-i} + (2g_1+g_2)\Delta m_{t-3} + (g_1+g_2)\sum_{i=1}^{3}\Delta m_{t-i} \\ &\quad + g_1 \sum_{i=0}^{3} \Delta m_{t-i} + \sum_{i=1}^{\infty} \lambda^i \left(\begin{array}{c} g_2 \Delta m_{t-4+4i} + (2g_1+g_2)(\Delta m_{t-3+4i} + \Delta m_{t-2+4i}) \\ + (g_1+g_2)\Delta m_{t-1+4i} + g_1 \Delta m_{t+4i} \end{array} \right), \\ M_3 &= \sum_{i=1}^{4} \Delta m_{t-i} + \sum_{i=0}^{4} \Delta m_{t-i} + \sum_{i=1}^{\infty} \lambda^i \left[\Delta m_{t-1+4i} + \Delta m_{t+4i}\right], \\ M_4 &= g_2 \sum_{i=0}^{3} \Delta m_{t-i} + (2g_1+g_2)\left[\sum_{i=0}^{4} \Delta m_{t+1-i} + \sum_{i=0}^{5} \Delta m_{t+2-i}\right] \\ &\quad + (g_1+g_2)\sum_{i=0}^{6} \Delta m_{t+3-i} + g_1 \sum_{i=0}^{7} \Delta m_{t+4-i} \\ &\quad + \sum_{i=1}^{\infty} \lambda^i \left[\begin{array}{c} g_2 \Delta m_{t+4i} + (2g_1+g_2)(\Delta m_{t+1+4i} + \Delta m_{t+2+4i}) \\ + (g_1+g_2)\Delta m_{t+3+4i} + g_1 \Delta m_{t+4+4i} \end{array} \right], \end{aligned}$$

The terms $M_1 - M_4$ contain the current and past values of money growth, as well as many past expected terms on money growth. Eq (12) shows that the wage dynamic is driven by current and past expectations of the growth of the money stock, with different weights for each date of expectation (reflecting the position in the cycle of adjustment presented in Table 1). Let us remark that the lags of the expectation terms depends on the longer contract and go up to $t-4$ and $t-5$.

The initial impact of a change in the growth rate of money is given by:

$$
\begin{aligned}
(1-f)\,\Delta w_t \;=\;& f\left(-E_{t-5}\Delta w_{t-4}+2\Delta w_{t-4}-E_{t-4}\Delta w_t+E_{t-1}\Delta w_t+E_t\Delta w_{t+4}\right)\\
&-g_1E_{t-5}\Delta m_{t-4}-E_{t-4}\left[(2g_1+g_2)\left(2\Delta m_{t-1}+2\Delta m_{t-2}+3\Delta m_{t-3}\right)\right]\\
&+2g_1\Delta m_{t-4}+4\left(2g_1+g_2\right)\left[\Delta m_{t-1}+\Delta m_{t-2}+\Delta m_{t-3}\right]+g_1E_{t-1}\Delta m_t\\
&+2\left(3g_1+g_2\right)\Delta m_t+\left(2g_1+g_2\right)E_t\left[\Delta m_{t+3}+2\Delta m_{t+2}+3\Delta m_{t+1}\right]\\
&+g_1E_t\Delta m_{t+4}. \qquad (13)
\end{aligned}
$$

The inflation rates $\Delta p^1_{t+1}=p^1_{t+1}-p^3_{t-1}$ and $\Delta p^3_{t+3}=p^3_{t+3}-p^1_{t+1}$ are given by ($\forall t=0,2,4..$):

$$
\begin{aligned}
\Delta p^1_{t+1} \;=\;& \frac{a}{2}\left[2\Delta w_t-E_{t-1}\Delta w_t\right] \qquad (14)\\
&+\left(\frac{1-a}{2}\right)\left[-E_{t-1}\Delta m_t+2\Delta m_t+2\Delta m_{t+1}+E_{t+1}\Delta m_{t+2}\right],\\
\Delta p^3_{t+3} \;=\;& \frac{a}{2}\left[E_{t+3}\Delta w_{t+4}\right] \qquad (15)\\
&+\left(\frac{1-a}{2}\right)\left[-E_{t+1}\Delta m_{t+2}+2\Delta m_{t+2}+2\Delta m_{t+3}+E_{t+3}\Delta m_{t+4}\right].
\end{aligned}
$$

The inflation rate in Eq (14) contains past expectations of the current wage growth, while Eq (15) contains contemporary expectations of future wage growth. This feature is important when considering the impact of monetary shocks.

3 The impact of a disinflation policy

Many papers focuse on disinflation because it is often associated with high real costs that are difficult to reproduce in standard models (Ball, 1994*a*, Mankiw and Reis, 2002). The estimated costs of disinflation policies are also quite heterogenous (Ball, 1994*b*). Explanations for the empirical variations of these costs often rely on the speed of disinflation, some labor market features (Ball, 1994*b*) or imperfect credibility (Ball, 1995, Ireland, 1995). To illustrate the properties of our model, we consider a credible disinflation policy[5] occurring at time s. For $t<s$, we set $\Delta m_t=\mu$ and for $t\geq s$, $\Delta m_t=\mu'$, with $\mu>\mu'$. Since the shock is unique, permanent and totally unexpected, we have $E_t\Delta m_{t+i}=\Delta m_t$. In our numerical application, we pass from an inflation rate of $\mu=1\%$ to a long term objective of price stability

[5] The derivation of the general price dynamics allows us to study other types of monetary shocks, since it just modifies the value of expectations on money growth.

$\mu' = 0$. The output cost obtained after the implementation of the disinflation policy is measured by the sacrifice ratio. This is defined as the output loss for one percent of inflation reduction. Before the shock, we assume that, at the regular state, prices and wage are set as an average of the expected levels of money during the contract life, such that $x_t = (1/N) \sum_{i=0}^{n} E_t m_{t+i}$ for $x_t = p_t, w_t$. This implies that on average the output y equals its equilibrium value set to zero. The long run equilibrium values of wages and prices are equal to the value of the money stock.

3.1 Dynamics

The monetary shock can occur during each of the different phases of the price/wage cycle of adjustment. A shock in $t = 0$ coincides with a date of wage adjustment, while shocks in $t = 1$ and $t = 3$ coincide with periods of price revision. In $t = 2$, neither wages nor prices are adjusted. We show that, depending on the date of the shock, the disinflation can be associated either to an output loss or to an output boom.[6]

Figure 1: Shock in time 0

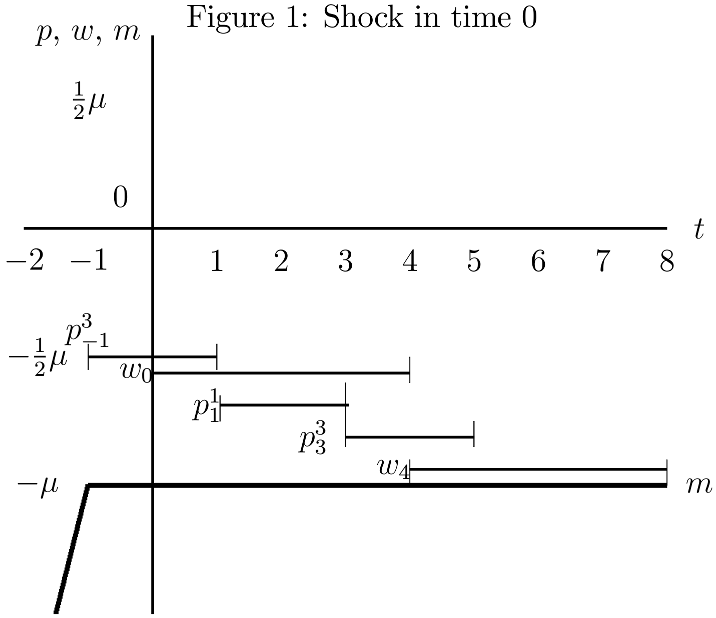


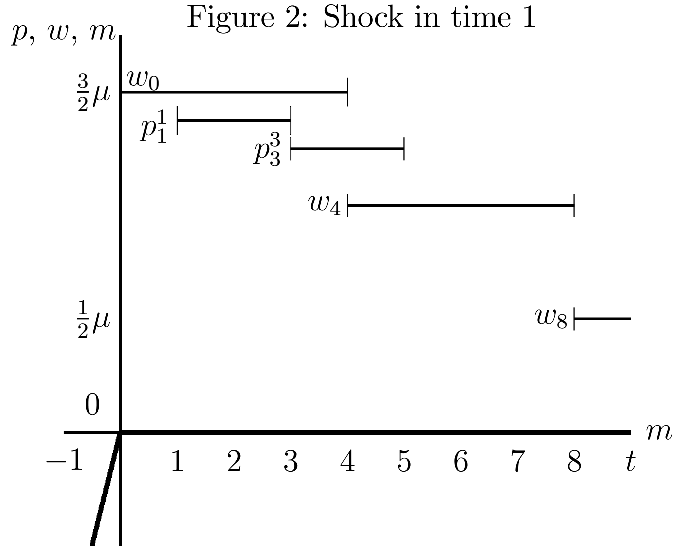
Figure (1) displays the response of prices and wages when the disinflation policy occurs at time 0. At this time, the money growth is stopped ($\mu' = 0$), implying a value of the money stock equal to its past value $m_0 = m_{-1} =$

[6] The dynamics of both the price and wage inflation and the initial responses are displayed in Appendix.

$-\mu$. Since the shock was not expected, the price p_{-1}^3 set before the shock is based on the previous growth of the money stock (equal to μ) implying $p_{-1}^3 = (1/2)(m_{-1} + m_0) = -(1/2)\mu$. When the disinflation occurs, the stop of the money growth yields a reduction of the real balances ($p_0 > m_0$) and a recession ($y_0 = m_0 - p_0 < 0$). To explain the slow convergence of the variables to their long run equilibrium values and the associated persistent output cost, note that the wage cohort sets optimally in $t = 0$ its wage using Eq (2) as a weighted average between the price level and the value of the money stock (the weight depends on the parameter b). It turns out that the wage level also overshoots the money stock. The same kind of adjustment holds for prices p_1^1 and p_3^3 according to the rule (1). Since the price level always overshoots the money stock, the disinflation policy is associated to output costs.

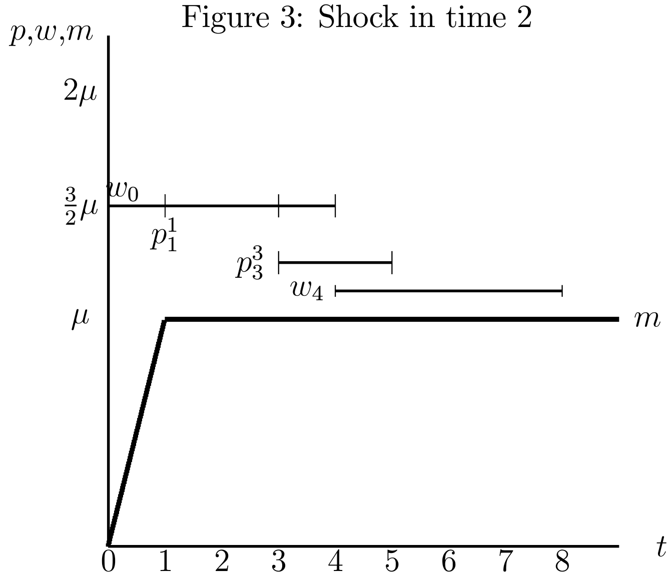
The same mechanisms are at stake when the disinflation policy is implemented in time 1 (Figure 2) and in time 2 (Figure 3). The main differences concern the magnitude of the nominal variables adjustment.

Figure 2: Shock in time 1



When the shock occurs in $t = 1$, wages have been set in $t = 0$ with respect to the constant expected rate of the monetary growth. $w_0 = (1/4)\left(\sum_{t=0}^{4} m_t\right)$, so we obtain $w_0 = (1/4)(0 + \mu + 2\mu + 3\mu) = (3/2)\mu$. This wage level remains unchanged until period 4. The price p_1^1 is updated at the date of the shock with respect to the new value of the money stock ($m_1 = m_0 = 0$) but also on the basis of the past value of the wage w_0. Since wages are too high for the new monetary path, and because of the interactions between prices

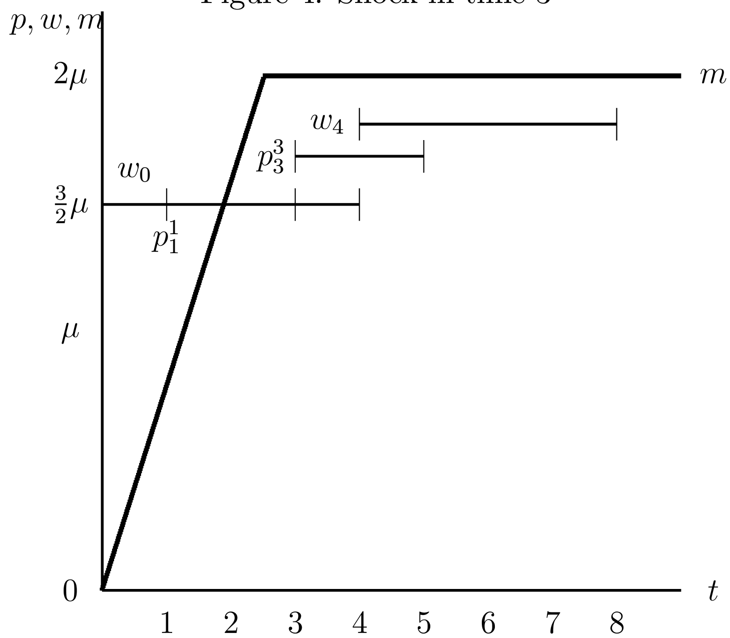
Figure 3: Shock in time 2



and wages, the price level continues to rise in order to catch the value of wages. It occurs despite of the stability of the money stock (which should optimally imply a stability of prices). In period $t = 3$, prices are revised for the second time after the implementation of the disinflation. p_3^3 is set relatively to the future value of wages according to Eq (15). Hence, expecting an adjustment of wages in the following period, prices begin to fall and converge towards their long-run value. Since prices are higher than the money stock during all the process, there is a persistent recession in the economy. A similar mechanism is at work for recessions occurring when the disinflation shock is in $t = 2$. At this time we have $m_2 = m_1 = \mu$, $w_0 = p_1^1 = (3/2)\,\mu$.

Figure 4 presents prices and wages dynamics for a disinflation implemented in period $t = 3$ at the last period of life of the wage contract. As indicated previously, before the shock, wages are set equal to the mean of the money stock expected to hold during the whole contract life (Eq 5). Then, wages at time $t = 0$ are fixed expecting a continuous growth of money at rate μ such that $w_0 = (3/2)\,\mu$. When the disinflation occurs in period 3, the money stock is given by $m_3 = m_2 = 2\mu$. It turns out that the money stock is higher than the wage level in period 3. Since prices are defined as a weighted average of the wage level and the money stock, it also implies that in period 3, they are set at a value located between these two variables (see Eq 1). The price level is then set below the money stock yielding an initial output boom. Subsequent adjustments of wages and prices are gradual to their equilibrium values but they remain below the money stock yielding a persistent output boom. Remark that this output boom does not depend on particular values

Figure 4: Shock in time 3



of a and b; these two parameters only affect the magnitude of the boom.[7]

In Figure 5, we present the comparative response of output for each scenario, for values $a = b = 0.99$.

[7] In absolute value, the size of the boom and the recessions decreases if a or b decreases, but even for very small values of a and b, the qualitative features presented remain valid. Decreases in a imply more important reductions in the sacrifice ratio measured in absolute value.

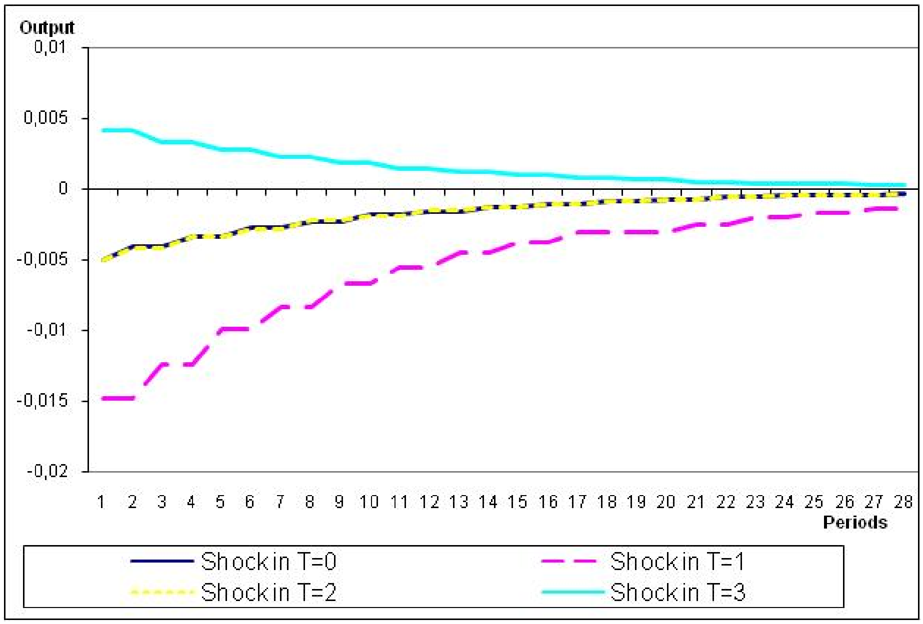


Table 2 shows the robustness of our qualitative results. We display the sacrifice ratios for several values[8] of a and b with respect to the dates of the disinflation policy. In absolute value, the size of the boom and the recessions decreases if a or b decreases, but even for lower values of a and b, our qualitative results remain valid. The highest sacrifice ratio occurs for a shock in $t = 1$ when prices are revised. Interestingly, it means that a date of price adjustment can imply a higher sacrifice ratio than a date which is not associated to any adjustment of nominal variables (this is the case for a shock occurring in $t = 2$). This result may seem to be paradoxical. However it comes from the asymmetric structure which implies that prices set during the first period are backward-looking with respect to the wages. Hence, prices continue to rise despite the stability of the money stock, increasing the output cost of the disinflation. The adjustment of prices indeed rises the initial output cost, and convergence towards the new path of money only begins in period 3. Then, for a shock in period 2, there is no initial rise of price. It results into a lower real cost.

[8] These elasticities depend on the structural parameters $\alpha = 1/a$ and $\beta = 1 - a(b-1)$ of the model (see footnote 3). We consider two values for α (1.01 and 1.42) and four values for β (1.007, 1.009, 1.105 and 1.297).

a	b	shock in 0	shock in 1	shock in 2	shock in 3
0.99	0.99	4.8	16.47	4.95	-4.4
0.7	0.99	1.11	4.1	1.17	-0.67
0.7	0.85	0.9	3.8	1.07	-0.57
0.99	0.7	1.12	5.82	1.44	-0.94

Table 2: Sacrifice ratios (in %)

3.2 Discussion about the price-wage structure

Our asymmetric temporal structure is highly stylized. In a more realistic framework, at least a fraction of prices should be modified each quarter, implying changes in the aggregate price index every quarter and not only in quarters 1 and 3 as in the previous structure. It does not alter the possibility of an output boom. Nevertheless, the boom will not be systematic as in our model. One can imagine a standard staggered prices structure *à la* Taylor (with two cohorts of equal size, setting 2 periods contracts, as in Romer, 2006), combined with an annual wage modification (which still represents the U.S. case or the Japanese *Shunto*). Denoting the prices set by the first cohort of firms by p_i and the prices set by the second cohort of firms by p_j, the general price level is given by $p = (1/2)(p_i + p_j)$. In this case, the optimal wage and prices of the model of Blanchard and Kiyotaki (1987) are:

$$\begin{aligned} w^* &= \frac{a_1}{2}(p_i + p_j) + (1 - a_1)\, m \\ p_i^* &= b_1 w + b_2 p_j + (1 - b_1 - b_2)\, m \\ p_j^* &= b_1 w + b_2 p_i + (1 - b_1 - b_2)\, m \end{aligned}$$

with $a_1 = 1 - \alpha(\beta - 1)$, $b_1 = \left(\frac{2}{2+(\alpha-1)(\theta+1)}\right)$, and $b_2 = \left(\frac{(\alpha-1)(\theta+1)}{2+(\alpha-1)(\theta+1)}\right)$.

Assume that cohort i changes its price at even periods for two periods, cohort j at odd periods for two periods, and the wage cohort at even periods for four periods. Based on the timing procedure presented in section 2, we can derive a more complicated dynamic structure which is much less tractable. However, an output boom is still possible. However our result will depend on two contradictory effects. The first effect relies on the interactions between price setters in the staggered structure of Taylor. This generates output costs of disinflation (Musy, 2006). The second effect is due to the duration of wage contracts. It can generate an output gain when the disinflation occurs just before its updating. When the two effects are active, the net impact depends on the relative weights of the parameters in the previous equations. Mechanisms are the same than those presented, excepted that, in addition, price setters have to take into account the prices set by their competitors. If the weight relative to competitors is not relatively high, the output boom is still present.

4 Conclusion

Most current models of output and inflation dynamics, despite their size and complexity, are built on very simple uniform staggering structures. They predict no relationship between the impact of a monetary shock and its date of occurrence. Olivei and Tenreyro (2007, 2008) challenge this prediction, giving some evidence that shocks have asymmetric effects depending on their date of occurrence during the process of wage adjustment. This paper presents an original price and wage structure based on Blanchard (1986), assuming that wages have a longer duration than prices. We show that the strategic interactions between staggered price and wage give a combination of multiple expectations terms that differ for each period of the cycle of wage adjustments. Indeed, the date of the shock appears to be important because each date corresponds to a particular combination of price and wage expectations; and each combination determines a specific initial impact of the shock. Then this initial response is transmitted to the following periods due to the overlapping structure of the contracts. In our example, a disinflation causes a boom when the shock occurs during the last quarter of wage rigidity and recessions of different magnitudes occur during the other quarters. The highest sacrifice ratio occurs during a period of price change. It challenges the idea that the sacrifice ratio would always be lower when the nominal rigidity is also the lower (as it is the case in simple models building on Calvo, 1983).

One reader could object that our results are model specific. Indeed, the quantitative results obtained are model dependent, and are merely an illustration of the unexpected results that can be obtained when we combine price and wage rigidities using finite length contracts *à la* Taylor. However, the mechanisms presented rest only on the expected terms inherent to the structure of Taylor. Similar results could be obtained with alternative structures based on contracts *à* la Taylor. The Calvo structure is more used in the literature but its very specific nature erases all the expectations terms in the dynamics. This gives more tractable models but at the expense of less interesting dynamics as we have shown in this paper.

APPENDIX: Disinflation dynamics with asymmetric contracts

We present the general dynamics of the price and wage inflation for a specific money supply process and the initial responses of the monetary shock.

Wage and price inflation dynamics

Eq (12) can be rewritten as for all $t = 0, 4, 8...$:

$$\Delta w_t = \lambda \Delta w_{t-4} + \frac{\lambda}{f(1-\lambda)^2} \begin{pmatrix} -h_5 \Delta m_{t-5} - h_4 \Delta m_{t-4} + h_3 \Delta m_{t-3} \\ +h_3 \Delta m_{t-2} + h_2 \Delta m_{t-1} + h_1 \Delta m_t \end{pmatrix}$$

with

$$\begin{aligned} h_1 &= (19-13\lambda) g_1 + 2(5-3\lambda) g_2; \; h_2 = (9-7\lambda) g_1 + 4(1-\lambda) g_2 \\ h_3 &= 4(1+\lambda)(2g_1+g_2); \; h_4 = (11-7\lambda) g_1 + 3(2-\lambda) g_2 \\ h_5 &= (1+\lambda) g_1 \end{aligned}$$

Price dynamics are derived from eqs (14) and (15).

Initial Impact

The initial responses are given by eq (13). They depend on the date of the shock.

Shock at time 0

When the monetary policy is implemented at time 0 during a period of wages revision. The wage growth path is given by (with $\digamma = (1-f)[1-f(1+\lambda)] - 2f^2 > 0$):

$$\begin{aligned} \digamma \Delta w_0 &= \begin{bmatrix} [1-f(1+\lambda)](4f+13g_1+6g_2) \\ -f(4f+g_1) \end{bmatrix} \mu \\ &+ \begin{bmatrix} [1-f(1+\lambda)](19g_1+10g_2) \\ +f[4f(1-\lambda)+33g_1+16g_2] \end{bmatrix} \mu' \\ \digamma \Delta w_4 &= [2f[4f+13g_1+6g_2] - (4f+g_1)(1-f)] \mu \\ &+ \begin{bmatrix} (1-f)[4f(1-\lambda)+33g_1+16g_2] \\ +2f(19g_1+10g_2) \end{bmatrix} \mu' \\ \Delta w_t &= \lambda \Delta w_{t-4} + 4(1-\lambda)\mu' \text{ for } t = 8, 12, ... \end{aligned}$$

Concerning the price inflation path, we obtain:

$$\begin{aligned} \Delta p_{-1}^3 &= 2\mu \\ \Delta p_1^1 &= a\Delta w_0 + 5\left(\frac{1-a}{2}\right)\mu' - \left(\frac{1+3a}{2}\right)\mu \\ \Delta p_t^3 &= \Delta p_{t+2}^1 = \frac{a}{2}\Delta w_{t+1} + 2(1-a)\mu' \text{ for } t = 3, 7, 11... \end{aligned}$$

Shock at time 1

The shock occurs when prices p^1 are set, leading to a wage growth path:

$$\begin{aligned}
\Delta w_0 &= 4\mu \\
\Delta w_4 &= -6\left[\frac{(2g_1+g_2)}{1-f(2+\lambda)}\right]\mu + \left[\frac{4f(1-\lambda)+22(2g_1+g_2)}{1-f(2+\lambda)}\right]\mu' \\
\Delta w_t &= \lambda \Delta w_{t-4} + 4(1-\lambda)\mu' \text{ for } t=8,12
\end{aligned}$$

and a price inflation

$$\begin{aligned}
\Delta p_1^1 &= \left(\frac{1+3a}{2}\right)\mu + \frac{3(1-a)}{2}\mu' \\
\Delta p_t^3 &= \Delta p_{t+2}^1 = \frac{a}{2}\Delta w_{t+1} + 2(1-a)\mu' \text{ for } t=3,7,11...
\end{aligned}$$

Shock at time 2

Since no prices are set at this period, the wage growth path is:

$$\begin{aligned}
\Delta w_0 &= 4\mu \\
\Delta w_4 &= -2\left[\frac{2g_1+g_2}{1-f(2+\lambda)}\right]\mu + \left[\frac{4f(1-\lambda)+18(2g_1+g_2)}{1-f(2+\lambda)}\right]\mu' \\
\Delta w_t &= \lambda \Delta w_{t-4} + 4(1-\lambda)\mu' \text{ for for } t=8,12
\end{aligned}$$

and the price inflation is given by:

$$\begin{aligned}
\Delta p_1^1 &= 2\mu \\
\Delta p_3^3 &= \frac{a}{2}\Delta w_4 - \left(\frac{1-a}{2}\right)(\mu - 5\mu') \\
\Delta p_5^1 &= \Delta p_t^3 = \Delta p_{t+2}^1 = \frac{a}{2}\Delta w_{t+1} + 2(1-a)\mu' \text{ for } t=3,7,11...
\end{aligned}$$

Shock at time 3

The shock occurs when prices p^3 are set, we obtain the wage growth path:

$$\begin{aligned}
\Delta w_0 &= 4\mu \\
\Delta w_4 &= 2\left[\frac{2g_1+g_2}{1-f(2+\lambda)}\right]\mu + \left[\frac{4f(1-\lambda)+14(2g_1+g_2)}{1-f(2+\lambda)}\right]\mu' \\
\Delta w_t &= \lambda \Delta w_{t-4} + 4(1-\lambda)\mu' \text{ for } t=8,12
\end{aligned}$$

and the price inflation:

$$\begin{aligned}
\Delta p_1^1 &= 2\mu \\
\Delta p_3^3 &= \frac{a}{2}\Delta w_4 + \left(\frac{1-a}{2}\right)(\mu + 3\mu') \\
\Delta p_5^1 &= \Delta p_t^3 = \Delta p_{t+2}^1 = \frac{a}{2}\Delta w_{t+1} + 2(1-a)\mu' \text{ for } t=7,11,..
\end{aligned}$$

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