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### Nonlinear Stock Price Adjustment in the G7 Countries

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# Nonlinear Stock Price Adjustment in the G7 Countries

Fredj JAWADI<sup>+</sup> and Georges PRAT<sup>∂</sup>

**Abstract** - This paper seeks to address the stock price adjustment toward fundamentals. Using the class of Switching Transition Error Correction Models (STECMs), we show that two regimes describe the dynamics of stock price deviations from fundamentals in the G7 countries over the period 1969-2005. Deviations appear to follow a quasi random walk in the central regime when prices are near fundamentals (i.e. transaction costs being greater than expected gains, the mean reversion mechanism is inactive), while they approach a white noise in the outer regimes (i.e. transaction costs being lower than expected gains, the mean reversion works). As expected when transaction costs are heterogeneous, the STECM shows that stock price adjustments are smooth, implying that the convergence speed is time-varying according to the size of the deviation. Finally, using appropriate indicators, both the magnitudes of under- and overvaluation of stock price and the speed of the mean reversion are exhibited per date in the G7 countries, showing that the dynamics of stock price adjustment is highly dependent on the date and on the country under consideration.

**JEL:** C22, G15.

**Keywords:** Stock price, heterogeneous transaction costs, STECMs.

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# Nonlinear Stock Price Adjustment in the G7 Countries

## 1- Introduction

Many studies suggest that fundamentals cannot explain the dynamics of stock prices since deviations between price and fundamentals are often large and durable (among others, see Shiller (1981), Campbell and Shiller (2001)). Deviations may be explained in different ways. Shiller (1981) and Summer (1986) suggest that “irrational fads” generate persistent deviations between prices and fundamentals and Daniel *et al.* (1998) explain positive market deviations by investor overconfidence. Barberis and Thaler (2003) suggest that investors under-react to news about fundamentals in the short term, although they gradually incorporate them in the long run. Other studies show that heterogeneity in expectations (i.e. chartists, fundamentalists and noise traders), mimetic behavior and information asymmetry may contribute to a mean-reverting strength leading stock price to converge to fundamentals (see Poterba and Summers (1988), Fama and French (1988), Cecchetti *et al.* (1990), Manzan (2003), De Grauwe and Grimaldi (2006)), and Jawadi (2006)). In particular, Barberis *et al.* (1998) and Boswijk *et al.* (2007) develop two-regime models describing stock price deviations (a trend regime related to “trend follower” investors and a mean-reverting regime related to “fundamentalists”), and show that nonlinearity characterizing the asset price adjustment dynamics can be explained by the heterogeneity in shareholder expectations.

Another approach focuses on transaction costs. In line with Anderson (1997), the transaction costs hypothesis still justifies the nonlinear mechanism describing the stock price adjustment dynamics. This approach appears as a limitation to arbitrage as well as to the market efficiency hypothesis, notably when the expected profit is lower than the assumed costs. As shown hereafter, transaction costs appear to be far from negligible, inducing persistent stock price deviations from their fundamentals. These costs suggest an adjustment process that is mean-reverting, with an adjustment speed increasing with the magnitude of the deviation (i.e. Manzan (2003) and Boswijk *et al.* (2007), Jawadi (2006)).

Taking transaction costs and their heterogeneity into account, this paper aims to measure stock price deviations and to explain stock market adjustments toward their fundamentals. The literature on these issues is relatively scarce, probably because of the difficulty involved in representing the fundamental value and because of the complexity of stock price deviation modeling. On the one hand, we propose an estimation of stock price fundamental value using the Dividend Discount Model (DDM) where expected dividends are represented using a Smooth Transition Autoregressive Model

(STAR). On the other hand, the stock price adjustment process is modeled in a nonlinear framework using a Switching Transition Error Correction Model (STECM). While most previous studies have focused on the American stock market, the present paper extends the field of empirical applications to the G7 countries, including the interdependence or contagion effect between stock markets in the G7 group. Moreover, using indicators proposed by Peel and Taylor (2000) for the foreign exchange market, we identify the periods and the magnitude of under- and overvaluations and compute the time-varying adjustment speeds for the G7 stock markets.

The rest of this article is organized as follows. The nonlinearity characterizing stock price adjustment is formally justified by transaction costs in section 2. Section 3 presents the STECM methodology to model stock price deviations and the empirical results, and we set out our concluding remarks in section 4.

## 2 - Stock price adjustment within transaction costs

### 2.1 - Theoretical framework: why transaction costs cause nonlinearity in stock price adjustment?

Transaction costs represent an institutional reality that is sufficient to generate nonlinear dynamics. For the stock markets, these costs are far from negligible: as shown in Table 1 for 2005-2006, even when only direct transaction costs are taken into consideration, the ratio between the cost and the amount of the transaction generally exceeds the value of interest rates expressed on an annual basis. When implicit costs are included, the total transaction costs more often exceed 20%! Moreover, the transaction costs appear to be largely dependent on the country in question. Between June 2005-July 2006, for example, the USA and Japan showed the lowest transaction costs, while France came in fifth position after Germany and the UK.<sup>1</sup>

**Table 1 - Stock market transaction costs (in % of the amount of the transaction)**  
**Averages 2005-2006**

Transaction costs	Germany	Canada	USA (NYSE)	France	UK	Italy	Japan
Direct cost	5.51	10.23	5.0	6.58	8.8	10.65	5.9
Implicit cost	16.62	13.75	12.51	16.49	15.62	17.13	14.4
Total cost	22.13	23.98	17.51	23.07	24.42	27.78	20.3

Source: Elkins and McSherry reports and Cherbonnier and Vandelanoite (2008, p.89).

<sup>1</sup> Implicit costs cover opportunity costs and market impact. Opportunity costs correspond to the difference between the cost of executing an order and its optimal cost, while market impact measures the effect induced by a financial actor when he or she buys or sells a financial asset. In practice, implicit costs depend on the bid-ask spread and are difficult to define and appreciate (see Cohen, Maier, Schwartz and Whitcomb (1986) for more details). Direct or explicit costs are essentially composed of taxes, regulation costs and other commissions. They generally depend on the nature of the type of the broker, on the nature of the order and on the stock market and they are relatively simple to identify.

With regard to the foreign market, Dumas (1992) shows that transaction costs create two zones. In the first zone, called “the no trade band,” arbitrages and adjustments are not active since the expected returns are lower than the transaction costs, which means that prices can continually deviate from their fundamental values. The deviations are left uncorrected as long as they are low with respect to transaction costs and they follow a near-unit root process in this area. Disequilibrium is only corrected in the second zone, the exchange zone, when price deviations and arbitrage opportunities are large enough to compensate for transaction costs. In this respect, stock price deviations are a white noise and stock price can join their fundamentals with a convergence speed that depends on the size of the deviation. Following Dumas (1992), a more recent study confirms that transaction costs induce some delay and persistence in interest rates (Anderson (1997), foreign exchange rates (Michael *et al.* (1997), Peel and Taylor (2000)) and stock prices (Manzan (2003), Boswijk *et al.* (2007)), and reject the linear, symmetrical, instantaneous and continuous adjustment hypothesis.

Anderson (1997) proposes a model that shows how transaction costs influence the dynamics of the US Treasury Bills rate. The author suggests that the adjustment process can be represented with a nonlinear error-correction model (NLECM), particularly with a STECM that was introduced by Granger and Teräsvirta (1993) and recently developed by Van Dijk *et al.* (2002). Anderson (1997) defines three types of adjustment dynamics depending on transaction costs. Let  $S_{i,t} = P_{i,t} - F_{i,t}$  be the actual deviation between the market price  $P_{i,t}$  of equity  $i$  and its fundamentals  $F_{i,t}$  known by all investors, and let  $\eta_{i,t}$  be the minimal theoretical stock price deviation that is expected by investors when they purchase the asset  $i$ . In the absence of transaction costs, all investors can benefit from a stock price deviation. When  $S_{i,t} = \eta_{i,t}$ , there are no arbitrage opportunities, but when  $S_{i,t} > \eta_{i,t}$  (respectively  $S_{i,t} < \eta_{i,t}$ ), the asset  $i$  is viewed as over-valuated (respectively under-valuated), and the arbitrage is active. In this case, the adjustment process bringing the stock price toward fundamentals is continuous and linear since it is characterized by a constant speed of adjustment:

$$\Delta r_{i,t} = -\rho (S_{i,t-1} - \eta_{i,t-1}) + \Phi(L)\Delta r_{i,t-1} + v_t \quad (1)$$

with:  $r_{i,t} = \sum_{t=1}^T (P_{i,t} - F_{i,t})$ ,  $\forall t = 1, 2, \dots, T$

where  $r_{i,t}$  is a measure of stock price deviations from fundamentals during the period of detention  $T$ , and  $\Phi(L)$  represents the lag operator.  $\Delta$  and  $v_t$  designate the first difference and a white noise respectively.

It can now be seen that the presence of transaction costs reduces arbitrage opportunities. Let  $\tau$  represent the transaction costs supposed in a first instance to be homogeneous according to the operators. When  $(S_{i,t-1} - \eta_{i,t-1}) > \tau$  or when  $(S_{i,t-1} - \eta_{i,t-1}) < -\tau$ , the investor is incited to raise

the detention of asset  $i$ , while when  $-\tau < (S_{i,t-1} - \eta_{i,t-1}) < \tau$ , this arbitrage opportunity disappears. With transaction costs, however, equation (1) is no longer appropriate to reproduce the price adjustment dynamics since it fails to replicate this discontinuity of arbitrages. In this case, Anderson shows that the following nonlinear specification reproduces the adjustment process, taking both the no-trade zone and the arbitrage opportunity zone into account:

$$\Delta r_{i,t} = -\rho [\Omega |S_{i,t-1} - \eta_{i,t-1}| \times (S_{i,t-1} - \eta_{i,t-1})] + \Theta(L) \Delta r_{i,t-1} + \varepsilon_t$$

$$\text{where : } \Omega[|S_{i,t-1} - \eta_{i,t-1}|] = 1 \text{ if } |S_{i,t-1} - \eta_{i,t-1}| > \tau$$

$$= 0 \text{ if } |S_{i,t-1} - \eta_{i,t-1}| \leq \tau.$$
(2)

where  $0 < \Omega(\cdot) < 1$  represents a transition function weighting the two extreme regimes in the adjustment process.

However, transaction costs are heterogeneous since investors do not necessarily have the same transaction amounts, stock market costs depending on the total of the transactions.<sup>2</sup> The disparities between individual transaction costs generate different arbitrage thresholds, which mean that the model (2) is no longer appropriate to describe the stock price adjustment. Introducing individual thresholds, Anderson (1997) extends this model and suggests that the adjustment becomes smooth and gradual rather than sudden. Let  $\tau_{ij}$  be the transaction cost associated with the purchase of an asset  $i$  by an investor  $j$ . A rational investor reacts to a price deviation only if  $\tau_{ij}$  is such as  $\tau_{ij} < (S_{i,t-1} - \eta_{i,t-1}) < -\tau_{ij}$ . Let  $H(|S_{i,t-1} - \eta_{i,t-1}|)$  be the cumulative density function of investors' expenses. According to Anderson (1997), this function measures the proportion of assets for which investors expect a gain due to the price deviation. Formally, the introduction of the heterogeneous transaction costs in the equation (2) implies the following adjustment process:

$$\Delta r_{i,t} = -\rho [H(|S_{i,t-1} - \eta_{i,t-1}|) \times (S_{i,t-1} - \eta_{i,t-1})] + \Theta(L) \Delta r_{i,t-1} + \varepsilon_t$$
(3)

where the cumulative density function, ranging between 0 and 1, is represented by an exponential function defined as follows:

$$H_s(\tau) = 1 - \exp[-\beta(\tau)^2], \beta > 0 \text{ and } \forall 0 \leq \tau < \infty$$
(4)

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<sup>2</sup> In particular, spreads between transaction costs supported by individual investors and those supported by institutional investors contribute to heterogeneity.

where  $\beta$  is the transition speed.  $H_s(\tau)$  corresponds to  $H(|S_{i,t-1} - \eta_{i,t-1}|)$  since stock price deviations  $S_{i,t-1} - \eta_{i,t-1}$  just compensate transaction costs  $\tau_{i,j}$  at the equilibrium price, and  $\tau$  represents the average individual transaction costs.

It is worth noting that the structural representation given by equations (3) and (4) can be assimilated with a nonlinear error correction model of STECM specification, where  $H_s(\tau)$  is a smooth exponential transition function to be estimated.

## 2.2 - Empirical evidence of nonlinearity of stock price adjustment in the literature

Studies relating to stock price adjustment are relatively scarce. Using the DDM to estimate the fundamental value for the S&P500 and the Dow Jones indexes, Shiller (1981) identified a “*volatility puzzle*” characterized by the inequality  $\sigma^2(P) < \sigma^2(P^*)$ , where  $P$  and  $P^*$  are the market price and the ex-post rational price respectively. Campbell and Shiller (1987) applied the usual linear cointegration techniques to study the relationships between stock price and dividends and rejected the linear cointegration hypothesis between the two variables. These results suggest that fundamentals fail to explain the stock price dynamics. Interestingly, to explain the S&P deviations, Froot and Obstfeld (1991) compare the bubble hypothesis to the alternative of a threshold dynamic process and conclude with the validity of the last hypothesis. Using a switching model, paper by Driffill and Sola (1998) confirms this conclusion. Allen and Yang (2001) studied British stock price deviations over the period 1986-2000 and showed that a large proportion of them (around 35%) remain unexplained by macroeconomic variables. More recently, Berdin and Hyde (2005) used STAR models to capture nonlinearity in the cyclical character of stock price dynamics for eight countries (Belgium, Canada, France, Germany, Ireland, Japan, the United Kingdom and the United States). The authors showed that the process describing the stock price adjustment toward fundamentals depends on the state of the economy (two regimes are considered: growth and recession).

Manzan (2003) and Boswijk *et al.* (2007) also focus on stock price adjustment in a nonlinear framework, but from another perspective. The authors retain restricted hypotheses to estimate the fundamental value (i.e. a constant risk-free rate and a constant dividend growth). While this value is assumed to be known by all investors, the stock price deviation adjustment processes are individual, depending on the presence of transaction costs and heterogeneity in expectations. They show that the STAR model provides an appropriate tool to represent the mean reversion in the S&P, implying that adjustment is asymmetrical and nonlinear.

Overall, these results suggest that threshold models may be used to describe stock price adjustment dynamics. However, no fundamental value modeling is chosen unanimously. In fact, two key questions arise: how can the expected future cash flows be represented? And which discount rate value is appropriate? In most previous studies, cash flows have been measured by dividends<sup>3</sup> while the expected dividends are estimated using the rational expectation hypothesis (REH) by supposing linear or nonlinear processes to describe the dividend dynamics.<sup>4</sup> It is worth noting that these studies only concern the American stock market (S&P500) and therefore the results cannot be generalized. This paper aims to model stock price adjustment due to heterogeneous transaction costs by using a STECM, which allows us to measure the size of stock price under- and overvaluation at each date and to measure the speed of adjustment. In addition, in line with Driffill and Sola (1998), and Berdin and Hyde (2005), we use a STAR model to estimate the dividend expectations embedded in the fundamental value. Our study investigates the G7 countries over the period 1969-2005 and takes into consideration the interdependences between these stock markets.

### 3 - Stock price adjustment modeling in the G7 countries

We first present the fundamental value estimations (§3.1). We then focus on the stock price adjustment modeling (§3.2 to §3.5).

#### 3.1 – Fundamental value estimation

In a world with perfect foresight and under the transversality condition, the DDM can be expressed by the following recurrent equation defining the fundamental value  $\bar{F}_t$  for a given country, this value corresponding to Shiller's "rational ex-post price":

$$\bar{F}_{t+1} = \bar{F}_t (1 + i_{ot}) - D_{t+1} \quad (5)$$

where  $i_{ot}$  is the one-period to maturity risk-free rate and  $D_{t+1}$  the dividend distributed during the period  $[t, t+1]$ .

Considering the fundamental value under the one-period ahead Rational Expectation Hypothesis (REH), the future dividend  $D_{t+1}$  was replaced by the expected dividends  $E_t(D_{t+1})$ , where  $E_t(\cdot)$  is the expectation conditional to the information available at time  $t$ , the discount rate being defined as the sum of a risk-free rate and a constant risk premium  $\Phi_o$ . The fundamental rational value is then given by the forward resolution of the following relation:

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<sup>3</sup> Among others, see Shiller (1981, 1989, 2000), Manzan (2003) and Boswijk *et al.* (2006). Shefrin and Statman (1984) suggest that dividends should be preferred to earnings for stock price modeling.

<sup>4</sup> For more details on this review, see Jawadi (2009).



$$F_{t+1} = F_t(1 + i_{ot} + \Phi_o) - E_t(D_{t+1}) \quad (6)$$

It can be seen that the generating process of  $F_t$  is based on rational expectations that are revised at each date according to new information, and this is a less restrictive hypothesis than the REH at time  $t$  for all future horizons that is often considered in the literature. The estimation of  $F_t$  according to (6) requires an initial value  $F_o$  at the beginning of the period and the value of the constant risk premium  $\Phi_o$ . To let the fundamental value explain the price as far as possible, these parameters are chosen to reach the minimum sum of squared log-differences between prices and the fundamental values over the period of analysis.

The fundamental values were estimated for the G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States) using monthly data over the period 1969-2005. Stock price and dividend series were found in the *Price Indexes* and the *Gross Indexes* from the Morgan Stanley Capital International database.<sup>5</sup> The monthly free-risk discount rate is given by the one month Monetary Market Rates (MMR), and the industrial production series (CSA) were obtained from the International Monetary Fund's International Financial Statistics. All data are expressed in local currencies. The Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) stationarity tests show that the G7 stock prices in logarithm are I(1). Furthermore, the G7 stock return<sup>6</sup> distributions are found to be asymmetric and leptokurtic. As a result, returns do not follow a normal distribution, and this suggest that nonlinearity characterize the dynamics of stock price.

Depending on the unit root test results applied to the dividend series, the one period expected dividends  $E_t(D_{t+1})$  are represented with a STAR model applied to the level of dividends for Germany, Italy and Japan (equation (7)) and to the dividend growth rates for Canada, the USA, France and the UK (equation (8)):

$$D_t = (\alpha_0 + \alpha_1 D_{t-1} + \dots + \alpha_p D_{t-p}) + (\beta_0 + \beta_1 D_{t-1} + \dots + \beta_p D_{t-p}) \times \Omega(D_{t-d}, \gamma, c) + \varepsilon_t \quad (7)$$

$$\Delta D_t = (\alpha_0 + \alpha_1 \Delta D_{t-1} + \dots + \alpha_p \Delta D_{t-p}) + (\beta_0 + \beta_1 \Delta D_{t-1} + \dots + \beta_p \Delta D_{t-p}) \times \Omega(\Delta D_{t-d}, \gamma, c) + v_t \quad (8)$$

<sup>5</sup> The gross index takes into account the dividend investment while the price index excludes it. All indexes are closing prices.

<sup>6</sup> The stock return is defined as the stock price logarithmic first difference plus the dividends yield.

This dividend modeling implies two regimes for the dividends associated with the extreme values of the transition function ( $\Omega(.)=0$  and  $\Omega(.)=1$ ), but allows for a “continuum” of intermediate regimes when  $0 \leq \Omega(.) \leq 1$ .<sup>7</sup> The results<sup>8</sup> show that the dividend dynamics are nonlinear for all countries, since two significant regimes are identified in the dividend dynamics.<sup>9</sup> This may be due to the coexistence of heterogeneous dividend policies and to changes in management strategies which can induce persistence and discontinuity in dividend dynamics.<sup>10</sup> We find that this latter can be reproduced by an LSTAR process for Germany and the USA and by an ESTAR model for Canada, France, the UK, Italy and Japan (see Appendix 1). The estimated transition speed ( $\hat{\gamma}$ ) is relatively small for most indexes, indicating that the transition between these regimes is slow due to the smooth character of the dividend series. When applying the misspecification tests proposed by Eitrheim and Teräsvirta (1996) to check the specification of the selected STAR model, we find that residual sets have white noise properties, suggesting that representing  $E_t(D_{t+1})$  by a STAR model is in line with the REH.

After replacing  $E_t(D_{t+1})$  in the equation (6) by the values calculated from of the appropriate STAR model  $\hat{D}_{t+1}$ , initial fundamental values  $F_0$  were swept in the interval  $[P_0-50\%, P_0+50\%]$ , while the interval  $[0\%, 8\%]$  is considered for the premium  $\Phi_0$ . Estimates for  $F_0$  and  $\Phi_0$  given in table 2 are those minimizing  $Q = \sum_{t=1}^{n=T} (p_t - f_t)^2$ , where  $p_t$  and  $f_t$  are respectively the log- values of price and fundamental value.

**Table 2 - Initial fundamental values and risk premia estimates**

	Germany	Canada	USA	France	UK	Italy	Japan
$\hat{F}_0$	73.11	80.32	85.12	72.57	86.13	57.25	129.15
$P_0$	100	100	100	103.67	100	80.51	100
$\hat{\Phi}_0$	3.8%	4.8%	5.4%	3.95%	4.29%	6.01%	6.58%

Note:  $P_0$  and  $\hat{F}_0$  are the initial values of price and of the fundamental value respectively, while  $\hat{\Phi}_0$  is the risk premium estimate.

<sup>7</sup> This approach is in line with studies by Driffill and Sola (1998) and Berdin and Hyde (2005). Both the equations (7) and (8) describe the STAR model proposed by Teräsvirta (1994). ( $\alpha_0, \alpha_1, \dots, \alpha_p$ ) and ( $\beta_0, \beta_1, \dots, \beta_p$ ) are respectively the autoregressive coefficients in the first and second regime,  $d$  is the lag parameter defining the transition variable ( $d \geq 1$ ),  $\gamma$  is the transition speed between the regimes, and  $c$  is the threshold parameter.  $\Omega(.)$  is the transition function which is continuous and bounded between 0 and 1.  $\Omega(.)$  is either logistic: ( $\Omega(D_{t-d}, \gamma, c) = (1 + \exp\{-\gamma(D_{t-d} - c)\})^{-1}$ ,  $\gamma > 0$ ) or exponential: ( $\Omega(D_{t-d}, \gamma, c) = 1 - \exp\{-\gamma(D_{t-d} - c)^2\}$ ,  $\gamma > 0$ ). It implies respectively a Logistic STAR (LSTAR) model or an Exponential STAR (ESTAR) model.

<sup>8</sup> STAR modeling implies specification and linearity tests. For more details, see Van Dijk *et al.* (2002).

<sup>9</sup> We apply five Lagrange Multiplier (LM) tests that are explicitly detailed in Van Dijk *et al.* (2002).

<sup>10</sup> For more explanations about the nonlinearity characterizing the dividend dynamics, see Jawadi (2009).

We observed that apart from Japan, all the price indexes were over-valuated at the beginning of the period. Otherwise, the risk premium values seem realistic since the G7 premia average is about 5% per year, which is in line with the values obtained in the literature (among others, see Mehra and Prescott (1985), Siegel (1992), Cochrane (1997) and Pastor and Stambaugh (2000)). Figures presented in Appendix 2 show that the fundamental values are smooth in comparison with market prices for the seven countries, and this property is in line with the results proposed by Manzan (2003) and Boswijk *et al.* (2007).<sup>11</sup> This feature leads stock prices to be often last away from their fundamentals for a long time, as underlined by Black *et al.* (2003) and Manzan (2003).

### 3.2 - Modeling stock price deviations with a STECM

In a frictionless market and in particular in the absence of transaction costs, stock price adjustment is symmetrical, continuous and characterized by a constant speed of adjustment (see section 2.1). A linear error correcting model (LECM) is therefore appropriate:

$$\Delta z_t = k + \rho z_{t-1} + \sum_{i=1}^p \phi_i \Delta z_{t-i} + \varepsilon_t \quad (9)$$

where  $\rho$  characterizes the intensity of the stock price mean-reversion mechanism and  $\varepsilon_t$  is a white noise. However, when the stock market is not frictionless, the LECM cannot describe stock price adjustment. In particular, transaction costs induce discontinuities in arbitrages and imply a nonlinear mean reversion phenomena with a time varying speed. Moreover, as shown above, when transaction costs are heterogeneous, the relevant modeling is an STECM. Introduced by Granger and Teräsvirta (1993) and Franses and Van Dijk (2000) (see also Van Dijk *et al.* (2002)), the STECM defines an adjustment process that depends on the sign (LSTECM) or size (ESTECCM) of the deviation. Let  $z_t = p_t - f_t$  be the relative deviation, where  $p_t$  and  $f_t$  are the log-values of price and the fundamentals, respectively. The general expression of an STECM is given by the following relation:

$$\Delta z_t = k + \rho_1 z_{t-1} \times [1 - \Omega(\gamma, z_{t-d}, c)] + \rho_2 z_{t-1} \times \Omega(\gamma, z_{t-d}, c) + \sum_{i=1}^p \phi_i \Delta z_{t-i} + \mu_t \quad (10)$$

where  $\rho_1$  and  $\rho_2$  are the adjustment coefficients in the first and second regime respectively,  $z_{t-1}$  is the lagged error-correction term,  $z_{t-d}$  is the transition variable,  $\phi_i$  are the AR parameters,  $\Omega(\cdot)$  is the transition function and  $\mu_t \rightarrow N(0, \sigma_\mu^2)$  is an error term.

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<sup>11</sup> The smooth character of fundamental values is implied by the DDM, not by the STAR model used to determine the expected dividend. Indeed, according to the DDM, the fundamental value is the sum of discounted future dividends, this sum leading to formally remove the short term movements in dividend and interest rate.

When comparing Anderson's theoretical model (equation (3)) and the STECM representation (10), we can see the accordance between these two specifications under certain conditions. Both  $z_t$  and  $r_{i,t}$  being measures of stock price deviations, the relation (10) corresponds to Anderson's model if the transition function  $\Omega(\cdot)$  is an exponential function and if  $k' = \rho_1 = c = 0$  and  $\phi_i = 0 \quad \forall i = 2, \dots, p$ .<sup>12</sup> For  $\Omega(\cdot) = 0$  or  $\Omega(\cdot) = 1$ , the STECM (10) leads to the LECM (9). For the other values of  $\Omega(\cdot)$  ranging between 0 and 1, the adjustment is gradual rather than abrupt and its speed depends on the size or the sign of the deviation: the larger the deviation, the stronger the tendency to move back to zero. This implies that even though  $\rho_1 \geq 0$ ,  $\rho_2$  and  $(\rho_1 + \rho_2)$  should be strictly negative and the linear adjustment term  $\rho$  must belong to the interval  $[\rho_1, \rho_1 + \rho_2]$  in order to comply with a nonlinear mean-reversion process in stock prices (see Michael *et al.* (1997) among others). In the first regime (i.e. the central regime), when the deviations are small,  $z_t \rightarrow I(1)$  is near a unit root process approaching a random walk, and may also demonstrate explosive behavior (when  $\rho_1 \geq 1$ ). In this regime, the deviations are persistent and stock prices can remain away from their fundamentals for a long time. On the other hand, in the outer regimes, when deviations are large enough to pay for the transaction costs, the process would be mean-reverting with a convergence speed that depends on the size of deviations, and  $z_t$  may approach a white noise. At each date, the adjustment process is described by a combination of the two adjustment patterns weighted by the transition function  $\Omega_t$  and scaled by the coefficients  $\rho_1$  and  $\rho_2$ . The greater the value of  $\rho_2$  relative to  $\rho_1$ , the larger stock price deviations will be. Note that such behavior can escape from the conventional linear cointegration framework in the sense that  $H_0: \rho = 0$  (i.e. LECM) may not be rejected even though stock prices are nonlinearly mean-reverting (i.e.  $(\rho_1 + \rho_2) < 0$  in the STECM). Conventional cointegration tests thus appear to be relatively ineffective in the presence of market frictions (see Taylor *et al.* (2001)). In fact, what appears important is to test the linear adjustment hypothesis against its alternative of nonlinearity.

In line with Peel and Taylor (2000), we now consider three hypotheses leading to a restricted specification of the STECM which have not previously been considered for stock markets:

$$\begin{aligned}
 H_0^a &: k' = c = 0, \\
 H_0^b &: \rho_1 + \rho_2 = -1 / H_0^a, \\
 H_0^c &: \rho_1 = 0 \text{ s.t. } H_0^a \text{ and } H_0^b
 \end{aligned} \tag{11}$$

Under these conditions, the equation (10) simplifies to:

$$\Delta z_t = -z_{t-1} \times \Omega(\gamma, z_{t-d}) + \sum_{i=1}^p \phi_i \Delta z_{t-i} + \mu_t \tag{12}$$

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<sup>12</sup> For more details about these conditions, see equations (11) and (12).

The equation (12) reproduces a relation similar to Anderson's model for stock price deviations, characterized by two regimes, namely, a random walk in the central regime (when transaction costs are larger than expected arbitrage gains) and a white noise in the outer regimes (when transaction costs are smaller than expected arbitrage gains). As we will show below, these hypotheses allow us to calculate two indicators proposed by Peel and Taylor (2000), the first one giving the magnitude of under- and overvaluation of stock prices per date and the second one a measure per date of the speed of convergence between stock prices and fundamentals. In practice, both the unconstrained STECM (10) and the constrained STECM (12) will be estimated independently, so that the restrictive hypotheses ( $H_0^a, H_0^b, H_0^c$ ) will be tested using a likelihood ratio test.

### 3.3 - The STECM specification

The STECM specification requires defining the form of the transition function  $\Omega(\cdot)$  and the basic linear model (LECM) from which regimes can be defined. Concerning the last point, to capture the interdependence or contagion between stock markets, we introduce the current and lagged American stock price deviations in the LECM as an exogenous variable in the adjustment process of the other G7 countries. The German (respectively French) deviations are also introduced in the model for France (respectively Germany) in order to capture the interdependences or contagions between these two markets. In the same manner, the Japanese deviations are introduced in the American stock price adjustment model. Moreover, change in the risk-free interest rate is retained as an exogenous variable in the stock price adjustment model to capture a liquidity effect. In addition, change in the industrial production is also introduced in the stock price adjustment in order to capture the possible influence of the economic activity. Formally, the equation (9) has been extended as follows:

$$\Delta z_t = k + \rho z_{t-1} + \sum_{i=1}^p \phi_i \Delta z_{t-i} + \sum_{j=0}^{p'} \alpha_j \Delta z_{t-j}^{USA} + \sum_{j=0}^{p'} \theta_j \Delta i_{0,t-j} + \sum_{j=0}^{p'} \theta'_j \Delta q_{t-j} + \varepsilon_t \quad (13)$$

where  $z_t^{USA}$  is the American stock price deviations,  $i_0$  is the risk-free interest rate and  $q_t$  is the log-index of industrial production.

In practice, many specifications have been tested to determine the number of lags, using the AIC, BIC, Ljung-Box Statistics and the autocorrelation functions. As a result, we retain  $p = 1$ , for Germany, the USA, France, Italy and Japan;  $p = 2$  for the UK and  $p = 3$  for Canada. The LECM are estimated by the OLS and the results are given in Appendix 3. Since contemporary values of residuals  $\varepsilon_t$  for the seven countries are found to be insignificantly correlated, it was not considered necessary to

estimate the seven equations as a system.<sup>13</sup> Our results show that most of the AR parameters are statistically significant at 5% or 10%. The adjustment coefficient  $\hat{\rho}$  is negative and significant, confirming a mean reversion process in stock prices for all countries, except for Italy. Furthermore, the interdependence or contagion effect is evidenced at 5%, since the American market has a strong positive affect on the other MSCI stock prices. A mutual contagion effect is also shown respectively between German and French stock markets and between American and Japanese markets. Otherwise, as expected with the liquidity effect hypothesis, changes in short term interest rate have a negative influence on the stock price adjustment for all countries, while change in industrial production has a positive, if delayed, affect on stock price adjustment for Canada, the USA, the UK and Japan only.

We will now turn to the relevance of the nonlinear stock price adjustment hypothesis. We applied the LM linearity test where the transition variable is supposed to be the lagged deviation ( $z_{t-d}$ ) for  $1 \leq d \leq 12$  months.<sup>14</sup> With respect to the standard linearity tests generally used in the literature, we applied linearity tests, in preference, that are robust to heteroscedasticity (Van Dijk *et al.* (2002)). According to this test, rejection of linearity implies that nonlinearity is relevant, suggesting the rejection of the one regime hypothesis (Table 3).

**Table 3 – LM<sub>3</sub> linearity test (*p-values*)**

Delay	Germany	Canada	USA	France	UK	Italy	Japan
p	1	3	1	1	2	1	1
$\hat{d}$	10	2	6	2	1	6	10
<i>p-value</i>	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: p is the number of lags in the change of the deviation.  $\hat{d}$  is the optimal number of lags in the transition variable  $z_{t-d}$ .

From table 3, the LM<sub>3</sub> test suggests a strong rejection of the linearity hypothesis at 5% for all the MSCI indexes. This result is in line with Manzan (2003) and Boswijk *et al.* (2007).<sup>15</sup> Although the optimal value of  $d$  varies across the different countries ( $d = 10$  for Germany and Japan,  $d = 2$  for Canada and France,  $d = 6$  for the USA and Italy, and  $d = 1$  for the UK), the validity of the STECM to describe stock price adjustment suggests that the expected effects of heterogeneous transaction costs are not rejected.<sup>16</sup>

<sup>13</sup> We nevertheless applied an SUR system estimation: estimates were insignificantly different from those obtained with the OLS. This result confirms that the seven equations can be estimated independently.

<sup>14</sup> In line with Teräsvirta (1994) and recently Van Dijk *et al.* (2002), we applied several LM tests (LM<sub>1</sub>, LM<sub>2</sub>, LM<sub>3</sub>, LM<sub>3</sub><sup>e</sup> and LM<sub>4</sub>) for all possible values of  $d$ :  $1 \leq d \leq 12$ . The optimal value of the delay parameter  $\hat{d}$  is such that the linearity is rejected the most strongly. Thus,  $\hat{d}$  should maximize the LM statistics and minimize the p-values of the linearity tests.

<sup>15</sup> These authors only apply the standard linearity test.

<sup>16</sup> We briefly describe the STECM methodology and LM tests. More details can be found in Van Dijk *et al.* (2002) and Jawadi (2006).

The last step in the STECM specification is the choice of transition function  $\Omega(\cdot)$ . Even though several previous studies retained *a priori* an exponential function which is in keeping with the transaction cost hypothesis (i.e. Michael *et al.* (1997), Manzan (2003) and Boswijk *et al.* (2007)), we tested the ESTECM against the LSTECM on the basis of tests developed by Teräsvirta (1994) and Escribano and Jordá (1999). Table 4 gives the results for the unrestricted STECM.

**Table 4 - Selecting the transition function  $\Omega(\cdot)$**

Countries	Delay parameter	<i>p-values</i> (Teräsvirta tests)			<i>p-values</i> (Escribano and Jordá tests)		Conclusion
		$H_{03}$	$H_{02}$	$H_{01}$	$H_{0L}$	$H_{0E}$	
	$\hat{d}$						<i>Model</i>
<b>Germany</b>	10	0.09	0.01	0.00	0.00	0.001	ESTECM
<b>Canada</b>	2	0.01	0.00	0.00	0.008	0.00	ESTECM
<b>The USA</b>	6	0.0009	0.00	0.001	0.003	0.00	ESTECM
<b>France</b>	2	0.15	0.008	0.04	0.002	0.00	ESTECM
<b>The UK</b>	1	0.00	0.00	0.01	0.00	0.00	ESTECM or LSTECM
<b>Italy</b>	6	0.21	0.002	0.54	0.007	0.00	ESTECM
<b>Japan</b>	10	0.24	0.004	0.001	0.00	0.00	ESTECM or LSTECM

Note: Teräsvirta tests and Escribano and Jordá tests are useful for specifying the transition function while testing whether it is exponential ( $\Omega(z_{t-d}, \gamma) = 1 - \exp\{-\gamma (z_{t-d})^2\}$ ) or logistic.  $\Omega(z_{t-d}, \gamma) = (1 + \exp\{-\gamma (z_{t-d})^2\})^{-1}$ .  $H_{01}$ ,  $H_{02}$  and  $H_{03}$  are the null hypotheses in Teräsvirta tests which are based on Fisher tests.  $H_{0L}$  and  $H_{0E}$  are null hypotheses tested by Escribano and Jordá and correspond to the auxiliary regression of the linearity tests (LM<sub>3</sub> and LM<sub>4</sub>).<sup>17</sup>

From table 4, the ESTECM can be retained to describe the stock price adjustment for most of the countries since the  $H_{02}$  hypothesis is rejected more strongly than the  $H_{01}$  and  $H_{03}$  hypotheses. This result is as predicted by the theoretical effects expected from heterogeneous transaction costs. Moreover, both models may be retained for the UK and Japan. However, while estimating these two models, the information criteria appear to conclude in favor of the ESTECM. The ESTECM is therefore retained for all the G7-MSCI indexes.

### 3.4 – Working with the ESTECM in the G7 countries

The no-restricted ESTECM (10) and the restricted ESTECM (12)) are estimated by the NLS method, both models being augmented with exogenous variables as indicated in (13). We tested the  $H_0^a, H_0^b, H_0^c$  restrictions using the likelihood ratio  $LR=2 [L(\theta_1) - L(\theta_0)]$ , where  $L(\theta_0)$  and  $L(\theta_1)$  are

<sup>17</sup> More details about these tests and the  $H_{01}$ ,  $H_{02}$  and  $H_{03}$  null hypotheses can be found in Van Dijk *et al.* (2002) and Jawadi (2006).

respectively the log-likelihood of the restricted and non-restricted STECM. The LR ratio follows a  $\chi^2(q)$  distribution where  $q$  is the number of constraints. The results in Table 5 show that, for the seven MSCI indexes, the  $H_0^a$ ,  $H_0^b$  and  $H_0^c$  restrictions are statistically accepted at 5%.

**Table 5 - Testing  $H_0^a$ ,  $H_0^b$  and  $H_0^c$  restrictions with the Likelihood Ratio**

Countries	Germany	Canada	USA	France	UK	Italy	Japan
LR <sup>a</sup>	0.8	0.79	0.85	0.58	0.12	0.79	0.28
LR <sup>b</sup>	0.89	0.93	0.98	0.82	0.09	0.77	0.11
LR <sup>c</sup>	0.93	0.74	0.97	0.90	0.08	0.67	0.80

Note: the table gives the  $p$ -values issued from the LR test.

It is worth noting that, according to the restricted specification of the ESTECM, transaction costs are implicitly captured at each date. Indeed, since the calculated value of the endogenous variable at time  $t$  is a weighted average of the values corresponding to the outer and central regimes, the first regime (white noise) will appear to be dominant when transaction costs are smaller than expected gains while the second regime will appear to be dominant (random walk) when transaction costs are higher than expected gains. This property of the model is far more interesting than it appears at first sight since transaction costs are not constant per date and have tended to decrease in recent years.

The ESTECM estimates under  $H_0^a$ ,  $H_0^b$  and  $H_0^c$  are reported in Table 6.<sup>18</sup> The AR parameters are statistically significant at 5%. There is strong evidence of contagion or interdependence between the MSCI stock indexes. In particular, the current and lagged US stock price deviations significantly affect the stock price adjustment of the other countries. There is also significant interdependence between the French and German markets and between the American and Japanese markets. Furthermore, interest rate variations negatively affect the stock market deviations, while changes in industrial production have a significant positive effect only for Japan at 5% and for the USA at 10%.

The transition speed  $\gamma$  is statistically significant at 5% (only 10% for the UK). The values of  $\gamma$  are relatively low, hence confirming the hypothesis of a smooth transition. This implies that stock prices are nonlinearly mean-reverting with an adjustment speed that depends at each date on the size of deviations from the fundamentals. For small deviations, stock prices last away from their

<sup>18</sup> The estimated restricted ESTECM augmented with exogenous variables as indicated in (13) is defined as follows:

$$\Delta z_t = -z_{t-1} \times \Omega(\gamma, z_{t-d}) + \sum_{i=1}^p \phi_i \Delta z_{t-i} + \sum_{j=0}^{p'} \alpha_j \Delta z_{t-j}^{USA} + \sum_{j=0}^{p'} \theta_j \Delta i_{0,t-j} + \sum_{j=0}^{p'} \theta'_j \Delta q_{t-j} + \mu_t$$



fundamentals for a long time, but for large deviations - when they exceed the transaction costs - arbitrage becomes active and the prices quickly revert back. Such results are in line with those of Black *et al.* (2003) and Bohl (2003) who suggest strong evidence of nonlinear mean-reversion in the S&P. To illustrate how the G7-MSCI indexes adjust toward fundamentals, we calculate the transition functions and plot them (on the vertical axis) against the lagged values of the stock price deviations (see figures in Appendix 4). We can see that the observations are distributed around the equilibrium on the left and the right side, hence confirming the choice of the exponential function and the relevance of the regimes. Moreover, these functions slope are more sharp for France, Italy and Japan (i.e. the functions increase quickly with deviations), implying that the transition is faster in these countries compared with others.

Finally, to check the validity of the ESTECM estimations under  $H_0^a$ ,  $H_0^b$  and  $H_0^c$ , three misspecification tests are applied: a test of residual autocorrelation, a test of parameter stability and a test of omitted linearity (Appendix 5). First, the results show that the residuals are independent for all the MSCI indexes. Second, the hypothesis of parameter stability is accepted at 5% except for the UK. Third, applying the robust linearity tests to the ESTECM residuals for different values of  $d$ ,  $1 \leq d \leq 12$ , we find that the nonlinearity is well captured by the ESTECM except for the UK. Consequently, these results confirm the ESTECM specification.

**Table 6 - Restricted ESTECM estimations**

	Germany	Canada	USA	France	UK	Italy	Japan
p	1	3	1	1	2	1	1
$\hat{d}$	10	2	6	2	1	6	10
$\hat{\gamma}$	0.62 (3.8)*	0.10 (4.4)*	0.57 (3.6)*	8.53 (3.29)*	0.64 (1.63)**	9.94 (2.7)*	7.65 (2.18)*
$\hat{\phi}_1$	-0.06 (-1.75)**	-0.08 (-1.63)**	-0.03 (-1.69)**	0.06 (2.1)*	-0.02 (-0.44)	0.14 (2.9)*	-0.02 (-1.63)**
$\hat{\phi}_2$	-	-0.02 (-1.1)	-	-	-0.46 (-9.7)*	-	-
$\hat{\phi}_3$	-	0.17 (5.4)*	-	-	-	-	-
$\hat{\alpha}_0$	0.16 (3.07)*	0.68 (16.1)*	-	0.44 (7.9)*	1.08 (21.7)*	0.98 (13.1)*	0.06 (1.2)
$\hat{\alpha}_1$	0.12 (2.4)*	0.16 (2.9)*	-	-	-0.05 (-0.9)	0.38 (5.08)*	0.35 (6.07)*
$\hat{\alpha}_2$	-	-	-	-	0.37 (6.2)*	0.42 (5.8)*	-
$\hat{\alpha}'_0$	0.19 (3.6)*	-	-	-	-	-	-
$\hat{\alpha}''_0$	-	-	-	0.9 (20.4)*	-	-	-
$\hat{\beta}_0$	-	-	0.18 (3.9)*	-	-	-	-
$\hat{\theta}_0$	-0.007 (-1.73)**	-0.01 (-4.2)*	-0.03 (-6.06)	-0.02 (-5.8)*	-0.005 (-1.8)**	-0.06 (-10.3)*	-0.01 (-2.3)*
$\hat{\theta}'_0$	-	-	-	-	-	-	0.34 (1.98)*
$\hat{\theta}'_1$	-	-	0.41 (1.8)**	-	-	-	-
$\hat{\gamma} \times \sigma_z^2$	0.07	0.006	0.08	1.2	0.04	1.3	1.1
ADF (p)	-13.9* (p=0)	-14.3* (p=0)	-14.8* (p=0)	-14.6* (p=0)	-20.3* (p=0)	-14.6* (p=0)	-14.07* (p=0)
DW	1.97	2.04	2.02	2.03	2.01	2.0	2.02
Q(4)	0.12	0.6	2.07	1.5	0.95	4.6	2.2
Q(12)	5.31	29.2	9.34	13.07	14.2	15.5	6.7
ARCH (q)	5.06* (q=1)	10.8* (q=1)	14.3* (q=1)	0.55* (q=1)	17.7* (q=1)	7.9* (q=1)	18.8* (q=2)
Nb. of iterations	18	47	30	45	27	25	28

*Note:* The restricted ESTECM augmented with exogenous variables as indicated in (13) is defined as follows:

$$\Delta z_t = -z_{t-1} \times \Omega(\gamma, z_{t-d}) + \sum_{i=1}^p \phi_i \Delta z_{t-i} + \sum_{j=0}^{p'} \alpha_j \Delta z_{t-j}^{USA} + \sum_{j=0}^{p'} \theta_j \Delta i_{0, t-j} + \sum_{j=0}^{p'} \theta'_j \Delta q_{t-j} + \mu_t$$

The values under the estimates are the t-ratios. Q(4) and Q(12) are the Ljung-Box statistics. (\*) and (\*\*) indicate respectively the significance at 5% and 10%. ADF and ARCH are the statistics of the ADF and ARCH tests.

### 3.5 - Gauging under- and overvaluation phases and mean reversion strengths

To gauge the degree of the under- and overvaluation of stock prices and the mean reversion strength at a particular point in time, we estimate two indicators  $\Pi(z_t)$  and  $\Psi(z_t)$  proposed by Peel and Taylor (2000) for the foreign exchange market, but which has not yet been applied to stock markets. The first indicator is defined as follows:

$$\Pi(z_t) = 100 \times \Omega(z_t) \times \text{sign}(z_t), \text{sign}(z_t) \equiv \frac{z_t}{|z_t|}, \quad -100 \leq \Pi(z_t) \leq 100 \quad (14)$$

The use of  $\Pi(z_t)$  is based on the property that the transition function  $\Omega(\cdot)$  measures the magnitude of the deviation from equilibrium since it implies a low degree of mean reversion for small deviations and a high degree of mean reversion for large deviations. This is why substituting  $z_t$  to  $z_{t-d}$  in the exponential function  $\Omega(\cdot)$  and affecting the sign of  $z_t$  to the latter enables us to determine the stock price under- and overvaluation phases. The condition  $\Pi(z_t) \rightarrow 0$  means that stock prices approach their fundamental values, while  $\Pi(z_t) > 0$  (respectively  $\Pi(z_t) < 0$ ) implies that stock prices are over-valued (respectively under-valued).

The second indicator depends directly on the importance of the autoregressive component of the STECM, and it can be shown that it just equals one minus the transition function:

$$\Psi(z_t) = 1 - \Omega(z_{t-d}), \quad 0 \leq \Psi(z_t) \leq 1 \quad (15)$$

When  $\Psi(z_t)$  moves toward 1, the speed of adjustment decreases and  $z_t$  converges toward a random walk. Conversely, when  $\Psi(z_t)$  moves toward 0, the speed of adjustment increases and  $z_t$  converges toward a white noise.

Calculating these two indicators for the stock markets is a new empirical contribution and leads to interesting results. The values of  $\Pi(z_t)$  per date for the G7 indexes are reported in the figures given in Appendix 6. These figures exhibit long durations of strong under- and overvaluation of the MSCI stock indexes over the period. The values per date of  $\Psi(z_t)$  are reported on figures given in Appendix 7. The average adjustment delay from prices to fundamentals is about 5 months for the seven countries.<sup>19</sup> Overall, the convergence speeds appear to be strongly time varying, asymmetrical and nonlinear. The adjustment speeds often appear to be greater when the stock price deviations are strong. Adjustment speeds tend to be higher during periods of crises (i.e. 1973, 1979, 1987). For the USA, our results are in line with those of Manzan (2003) who shows that the S&P500 index was not

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<sup>19</sup> This average is given by the sum of the optimal values of  $d$  for the G7-MSCI indexes divided by 7.

mean-reverting during the period 1990-95. Overall, the dynamics of  $\Pi(z_t)$  and  $\Psi(z_t)$  indicators show that at each date stock price adjustment is highly dependant on the country in question. However, during the last years of the period, it can be seen that, for almost all the countries, stock price is near the fundamental value, suggesting low expected profits. Although the fall in transaction costs has been a recognized fact in recent years, it is not surprising to observe that the speed of adjustment tends to be slow or decreasing at the end of the period.

#### **4 - Concluding remarks**

This paper analyses the stock price adjustments toward fundamentals as an “on/off” threshold error-correction model which works only when deviations exceed a threshold defined by the investors’ transaction costs. We found strong evidence of such a nonlinear mean reversion process in the G7 stock markets, the adjustment speeds rising with the magnitude of the deviations from fundamentals. According to the restricted ESTECM proposed, stock price deviations appear to follow a process close to a random walk in the central regime when prices are close to fundamentals (i.e. transaction costs are higher than expected gains) while deviations approach a white noise process in the outer regimes (i.e. transaction costs are lower than expected gains). This model shows that the transition from one regime to the other is smooth, a result which is in accordance with the expected effects due to heterogeneous transaction costs. Finally, transaction costs cannot be neglected since the results presented in this paper suggest that they can significantly affect stock price dynamics.

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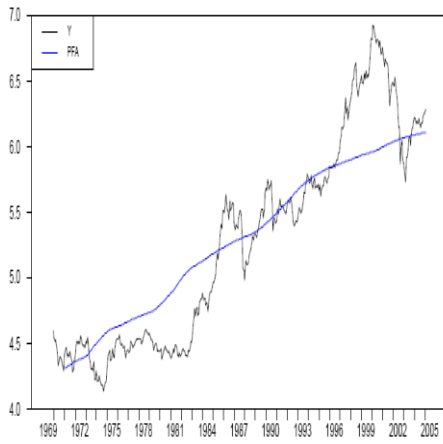
## Appendix 1: STAR estimations of dividends

	Germany	Canada	USA	France	UK	Italy	Japan
$\alpha_0$	0.06 (0.9)	-0.26 <sup>a</sup> (-2.8)	0.28 <sup>a</sup> (2.2)	-0.006 (-0.1)	-0.008 (-0.27)	0.01 (0.1)	5.9 <sup>a</sup> (12.7)
$\alpha_1$	0.08 (1.1)	0.9 <sup>a</sup> (2.1)	-0.17 <sup>a</sup> (-10.8)	-0.73 <sup>a</sup> (-7.9)	-0.62 <sup>a</sup> (-4.6)	0.08 <sup>a</sup> (2.3)	-1.02 <sup>a</sup> (-5.7)
$\alpha_2$	-0.002 (-1.04)	-0.27 (-0.4)	-3.5 <sup>a</sup> (-6.3)	-0.71 <sup>a</sup> (-7.5)	-0.78 <sup>a</sup> (-7.3)	-0.06 <sup>a</sup> (-2.3)	-2.3 <sup>a</sup> (-6.8)
$\alpha_3$	0.2 <sup>b</sup> (1.9)	0.75 (1.1)	-2.0 <sup>a</sup> (-3.6)	-0.82 <sup>a</sup> (-9.7)	-0.49 <sup>a</sup> (-4.3)	0.03 (0.7)	-1.62 <sup>a</sup> (-13.9)
$\alpha_4$	0.01 <sup>a</sup> (2.1)	-0.17 (-0.28)	-1.7 <sup>a</sup> (-3.2)	-0.64 <sup>a</sup> (-5.6)	-0.52 <sup>a</sup> (-4.7)	0.01 (0.5)	-1.6 <sup>a</sup> (-15.1)
$\alpha_5$	-0.004 (-1.2)	1.5 <sup>a</sup> (2.2)	-0.33 (-0.7)	-0.68 <sup>a</sup> (-6.9)	-0.13 (-1.2)	-0.1 <sup>a</sup> (-3.3)	0.41 (1.3)
$\alpha_6$	0.08 <sup>a</sup> (2.2)	3.1 <sup>a</sup> (4.0)	0.13 (0.2)	-0.95 <sup>a</sup> (-8.1)	0.57 <sup>a</sup> (3.8)	0.25 <sup>a</sup> (8.6)	-0.69 <sup>a</sup> (-5.6)
$\alpha_7$	-0.04 (-0.7)	-3.09 <sup>a</sup> (-4.1)	0.39 (0.8)	-0.86 <sup>a</sup> (-7.3)	0.44 <sup>a</sup> (2.8)	-0.03 (-0.5)	-0.66 <sup>a</sup> (-3.7)
$\alpha_8$	0.07 (1.6)	1.02 <sup>a</sup> (3.5)	1.07 <sup>a</sup> (2.2)	-0.84 <sup>a</sup> (-6.7)	0.34 <sup>a</sup> (2.1)	0.04 <sup>b</sup> (1.8)	1.7 <sup>a</sup> (6.0)
$\alpha_9$	0.11 <sup>b</sup> (1.7)	0.15 <sup>b</sup> (1.7)	1.2 <sup>a</sup> (2.7)	1.02 <sup>b</sup> (1.6)	0.004 (0.03)	-0.11 (-1.0)	- -
$\alpha_{10}$	0.09 (1.3)	- -	1.6 <sup>a</sup> (2.9)	-2.2 <sup>a</sup> (-10.7)	-0.09 (-0.7)	0.02 (0.4)	- -
$\alpha_{11}$	0.06 <sup>b</sup> (1.9)	- -	0.59 (1.5)	-0.35 <sup>a</sup> (-2.1)	-0.32 <sup>a</sup> (-3.2)	0.06 (0.6)	- -
$\alpha_{12}$	0.05 <sup>a</sup> (7.5)	- -	- -	0.27 <sup>a</sup> (2.8)	-0.16 <sup>b</sup> (-1.9)	-2.1 <sup>a</sup> (-2.0)	- -
$\beta_0$	1.9 <sup>a</sup> (5.5)	0.3 <sup>a</sup> (3.1)	-0.26 <sup>a</sup> (-2.0)	5.1 <sup>a</sup> (2.2)	2.36 <sup>a</sup> (5.5)	4.1 <sup>a</sup> (7.1)	-5.8 <sup>a</sup> (-12.6)
$\beta_1$	0.42 <sup>a</sup> (4.2)	-2.0 <sup>a</sup> (-4.8)	0.7 <sup>a</sup> (3.8)	3.3 <sup>a</sup> (2.5)	-1.06 <sup>a</sup> (-5.2)	0.01 <sup>a</sup> (0.1)	0.96 <sup>a</sup> (4.3)
$\beta_2$	-0.31 <sup>a</sup> (-3.7)	-0.99 <sup>b</sup> (-1.7)	2.5 <sup>a</sup> (-4.4)	-1.9 <sup>a</sup> (-0.9)	-1.4 (-0.6)	0.04 (0.2)	2.4 <sup>a</sup> (6.8)
$\beta_3$	0.1 (1.1)	-1.3 <sup>b</sup> (-1.9)	1.3 <sup>a</sup> (2.3)	4.6 <sup>a</sup> (2.4)	-1.7 <sup>a</sup> (-5.5)	-2.3 <sup>a</sup> (-5.3)	1.6 <sup>a</sup> (13.8)
$\beta_4$	-0.43 <sup>a</sup> (-5.2)	-0.3 (-0.5)	1.0 <sup>b</sup> (1.7)	-4.1 <sup>a</sup> (-1.6)	-1.6 <sup>a</sup> (-4.4)	-1.1 <sup>a</sup> (-5.8)	1.7 <sup>a</sup> (15.1)
$\beta_5$	-0.12 (-1.3)	-2.0 <sup>a</sup> (-2.7)	-0.34 (-0.7)	0.21 (0.22)	-2.03 <sup>a</sup> (-5.1)	0.06 (0.2)	-0.4 (-1.2)
$\beta_6$	-0.25 <sup>a</sup> (-2.4)	-3.7 <sup>a</sup> (-4.6)	-0.59 (-1.1)	1.5 <sup>b</sup> (1.8)	-2.4 <sup>a</sup> (-7.4)	0.1 (0.4)	1.4 <sup>a</sup> (10.2)
$\beta_7$	-0.09 (-1.0)	-3.4 <sup>a</sup> (-4.4)	-0.84 (-1.5)	1.8 <sup>a</sup> (2.1)	-2.5 <sup>a</sup> (-7.9)	-3.8 <sup>a</sup> (-8.7)	0.86 <sup>a</sup> (3.8)
$\beta_8$	-0.33 <sup>a</sup> (2.6)	-1.3 <sup>a</sup> (-4.2)	-1.7 <sup>a</sup> (-3.3)	1.1 (1.3)	-2.1 <sup>a</sup> (-8.1)	-0.03 (-0.1)	-1.6 <sup>a</sup> (-5.8)
$\beta_9$	0.46 <sup>a</sup> (4.9)	-0.08 (-0.3)	-1.8 <sup>a</sup> (-3.6)	-0.7 (-0.8)	-1.6 <sup>a</sup> (-6.6)	7.3 <sup>a</sup> (3.8)	- -
$\beta_{10}$	-0.26 (-0.9)	- -	-1.9 <sup>a</sup> (-3.3)	2.1 <sup>a</sup> (2.4)	-1.9 <sup>a</sup> (-7.1)	-1.2 <sup>a</sup> (-12.8)	- -
$\beta_{11}$	-0.09 (-0.3)	- -	-0.8 <sup>a</sup> (-2.0)	0.3 (0.4)	-1.1 <sup>a</sup> (-6.4)	-0.17 <sup>a</sup> (-1.0)	- -
$\beta_{12}$	0.29 <sup>a</sup> (2.0)	- -	- -	-0.5 <sup>b</sup> (-1.7)	-0.08 <sup>b</sup> (-1.8)	0.32 <sup>b</sup> (1.8)	- -
$\gamma$	5.3 <sup>a</sup> (2.8)	1.43 <sup>a</sup> (6.9)	0.24 <sup>a</sup> (2.5)	5.2 <sup>a</sup> (2.8)	0.17 <sup>a</sup> (4.9)	0.16 <sup>a</sup> (3.8)	66.4 <sup>a</sup> (5.8)
c	0.78 <sup>a</sup> (14.8)	-0.34 <sup>a</sup> (-23.1)	-0.27 <sup>a</sup> (-6.9)	0.05 <sup>a</sup> (4.7)	-0.31 <sup>a</sup> (-2.0)	0.45 <sup>b</sup> (1.8)	0.04 <sup>a</sup> (22.1)
R <sup>2</sup>	0.78	0.87	0.85	0.81	0.91	0.91	0.92
N	32	26	71	53	40	50	51

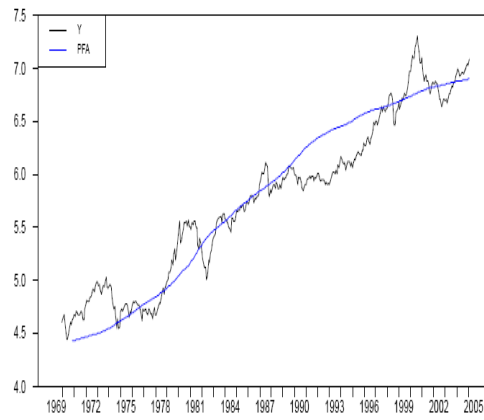
Notes: The values between brackets are the t-ratio of the estimators. (a) and (b) designate respectively the significativity at 5% and 10%. Canada: 1969:12-2005:02, France: 1970:01-2004:10, Germany: 1969:12-2005:02, Italy: 1971:01-2005:02, Japan: 1969:12-2005:02, the UK: 1969:12-2005:01 and the USA: 1969:12-2005:02.

## Appendix 2: Stock prices and fundamental values <sup>20</sup>

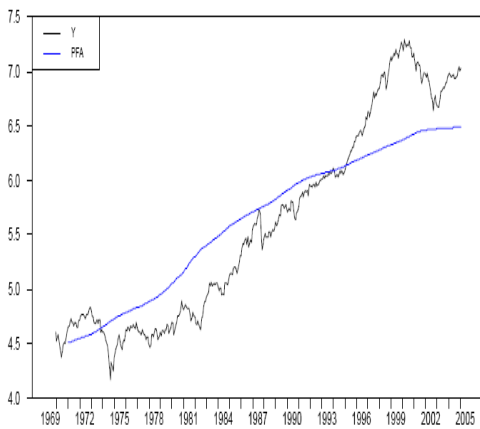
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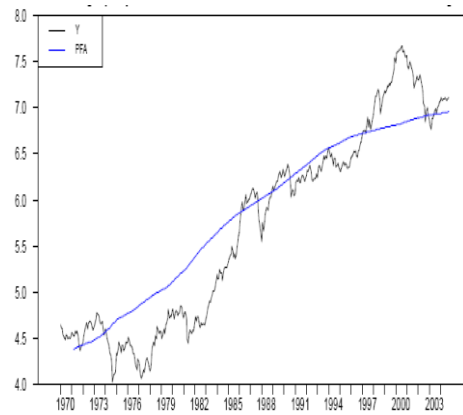
*Canada*



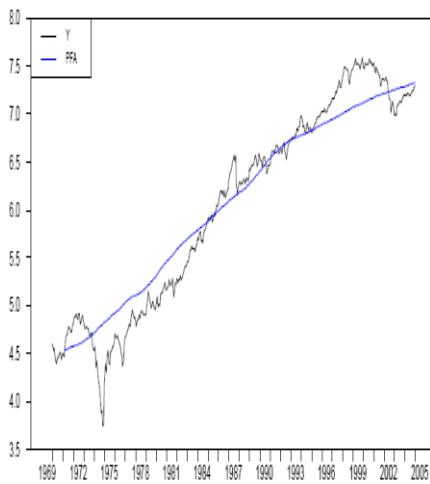
*USA*



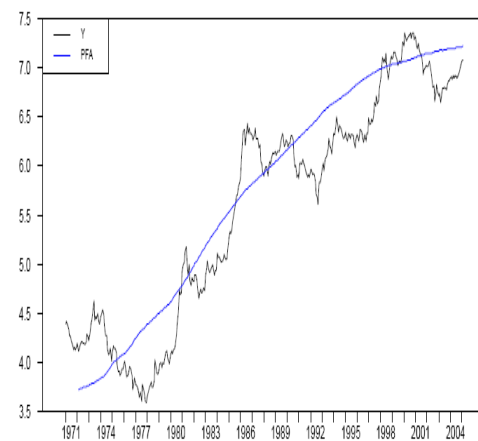
*France*



*UK*

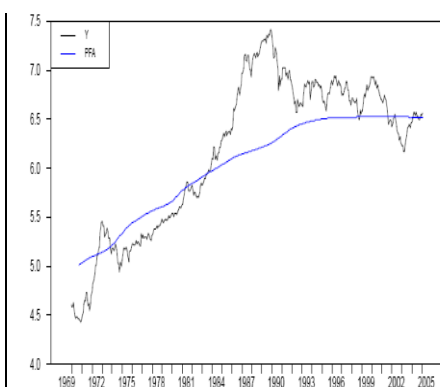


*Italy*





## Japan



**Note:** Y and PFA are respectively the observed price and its estimated fundamental value in logarithm.

### Appendix 3: Stock price deviations: LECM estimations

	Germany	Canada	USA	France	UK	Italy	Japan
p	1	3	1	1	2	1	1
$\hat{\rho}$	-0.015 (-2.31)*	-0.011 (-1.68)**	0.06 (0.9)*	-0.0001 (-1.74)**	-0.025 (-2.95)*	-0.005 (-0.63)	-0.012 (-2.05)*
$\hat{\phi}_1$	-0.013 (-1.63)**	0.017 (0.4)	-0.04 (-1.8)**	0.029 (1.71)**	-0.007 (-1.15)	-0.016 (-1.81)**	0.012 (1.83)**
$\hat{\phi}_2$	-	-0.019 (-1.74)**	-	-	-0.14 (-2.92)*	-	-
$\hat{\phi}_3$	-	0.102 (3.18)*	-	-	-	-	-
$\hat{\alpha}_0$	0.293 (5.14)*	0.83 (22.6)	-	0.4 (7.05)*	0.79 (16.2)*	0.52 (7.06)*	0.43 (7.91)*
$\hat{\alpha}_1$	0.131 (2.35)*	0.09 (1.65)**	-	-	0.09 (1.65)**	0.16 (2.0)	0.2 (3.37)*
$\hat{\alpha}_2$	-	-	-	-	0.13 (2.09)*	0.14 (1.96)*	-
$\hat{\alpha}_3$	-	-	-	-	0.15 (3.06)*	-	-
$\hat{\alpha}_0'$	0.49 (11.7)*	-	-	-	-	-	-
$\hat{\alpha}_0''$	-	-	-	0.51 (11.67)*	-	-	-
$\hat{\beta}_0$	-	-	0.15 (4.4)*	-	-	-	-
$\hat{\theta}_0$	-0.0007 (1.65)**	-0.011 (-3.8)*	-0.008 (2.57)	-0.011 (-2.4)*	-0.022 (-5.42)*	-0.011 (-2.16)*	-0.001 (-1.99)*
$\hat{\theta}_0'$	-	-	-	-	-	-	0.29 (1.64)**
$\hat{\theta}_1$	-	0.22 (1.7)**	0.29 (1.69)**	-	-	-	-
$\hat{\theta}_2$	-	-	-	-	0.25 (1.76)**	-	-
$R^2$	0.49	0.60	0.44	0.53	0.46	0.17	0.21
$\sigma_L$	0.04	0.03	0.03	0.04	0.04	0.06	0.04
Q(4)	0.09	0.46	2.37	1.77	1.25	3.18	1.84
Q(12)	3.56	31.01	10.06	13.1	14.9	17.56	5.8
J-B	31.95*	23.58*	7.66**	27.54*	372.2*	20.3*	24.55*

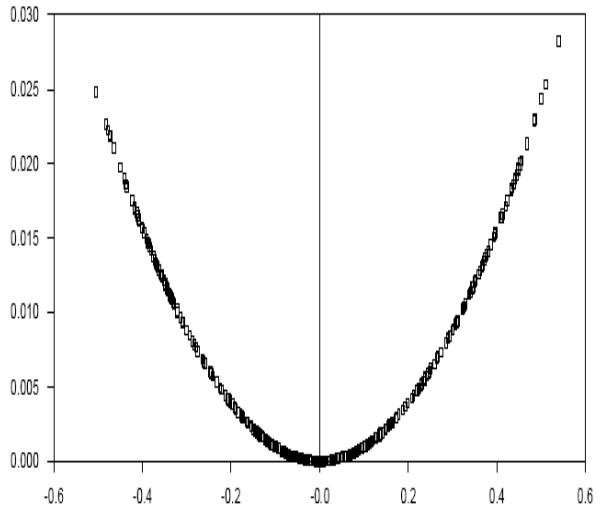
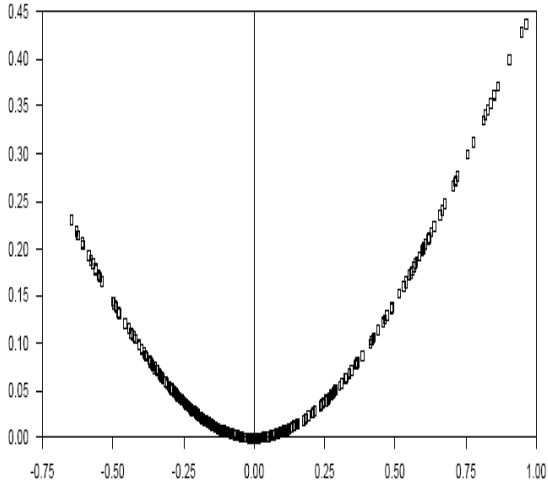
Note: Values under regression coefficients are the t-ratios of estimators.  $R^2$  is the coefficient of determination, J-B is statistic of Jarque-Berra test and  $\sigma_L$  is standard deviation of linear model. Q(4) and Q(12) are Ljung-Box statistics. (\*) and (\*\*) designate respectively the significativity at 5% and 10%.

### Appendix 4: Estimating the transition functions $\Omega(\cdot)$

*ESTECM are estimated under  $H_0^a, H_0^b$  and  $H_0^c$*

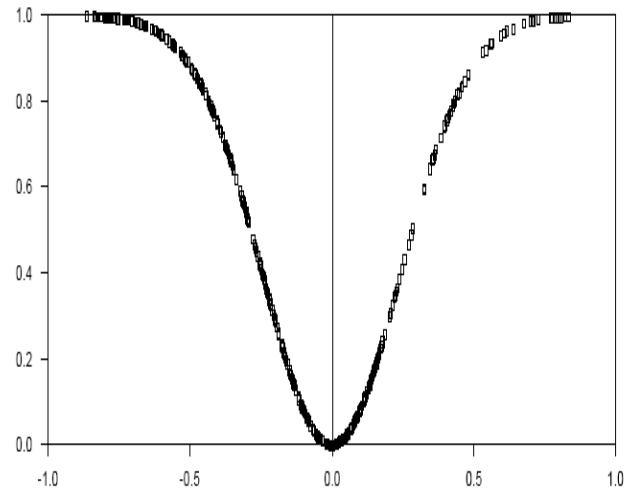
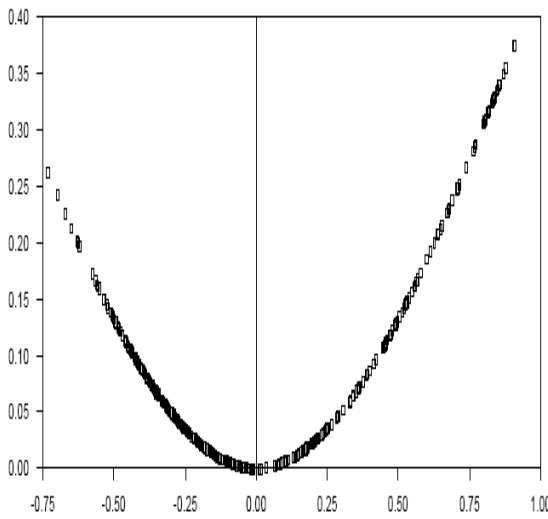
**Germany**

**Canada**



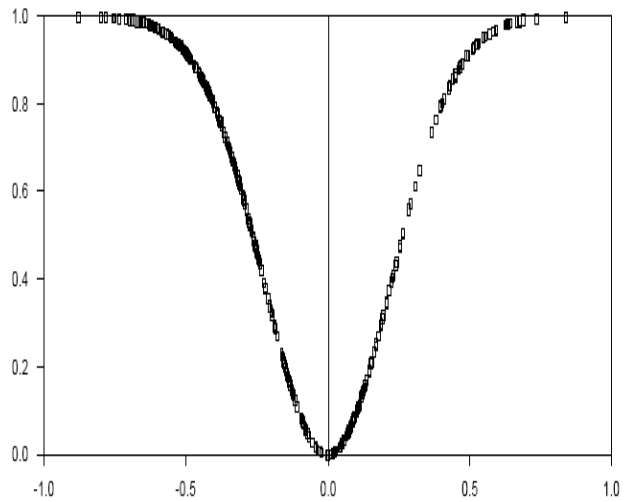
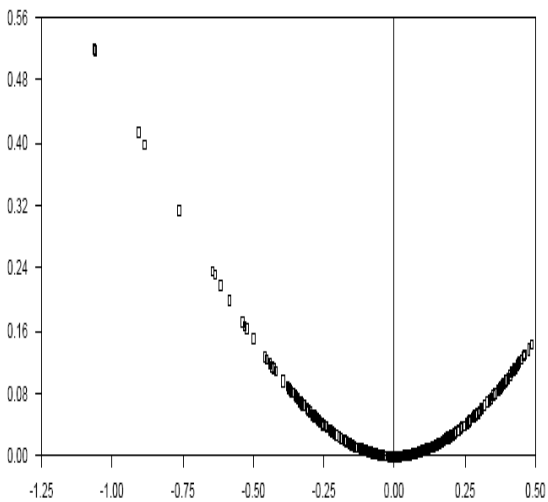
**USA**

**France**

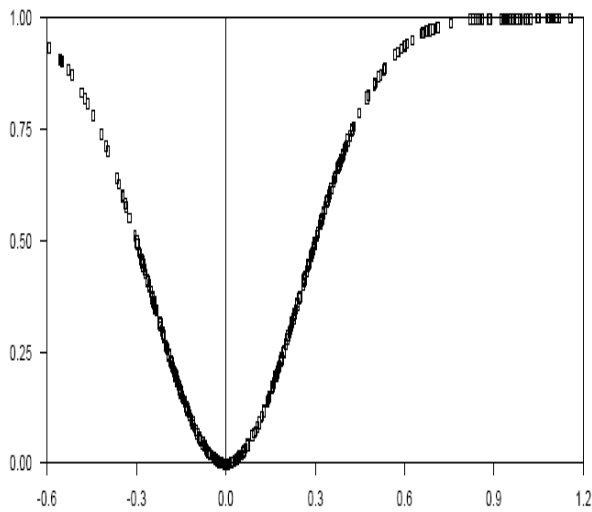


**UK**

**Italy**



## Japan



### Appendix 5: Misspecification tests

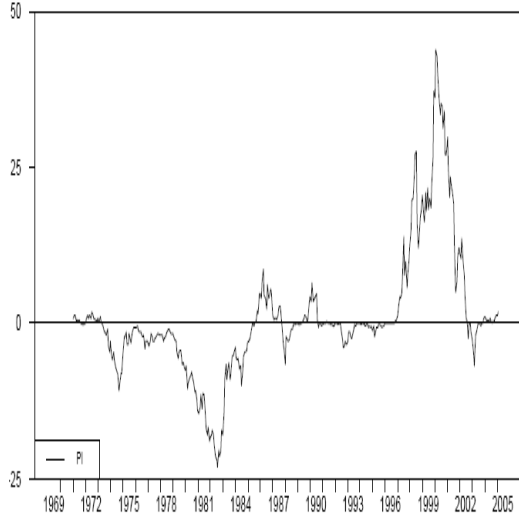
*ESTEEM estimations under  $H_0^a$ ,  $H_0^b$  and  $H_0^c$*

Tests of no error autocorrelation (p-values of $LM_{SI}$ )							
q / serie	Germany	Canada	USA	France	UK	Italy	Japan
q = 1	0.35	0.11	0.17	0.24	0.20	0.55	0.13
q = 2	0.62	0.13	0.24	0.44	0.22	0.46	0.28
q = 3	0.80	0.12	0.42	0.51	0.43	0.53	0.31
q = 4	0.90	0.23	0.53	0.63	0.57	0.33	0.29
q = 8	0.69	0.20	0.73	0.28	0.39	0.16	0.23
q = 12	0.89	0.35	0.75	0.27	0.10	0.17	0.40
Test of parameter stability (p-values of $LM_{c,i}$ , $\forall i = 1, 2, 3$ )							
$LM_{c,1}$	0.48	0.22	0.18	0.17	0.02	0.34	0.23
$LM_{c,2}$	0.67	0.23	0.44	0.10	0.01	0.55	0.38
$LM_{c,3}$	0.88	0.55	0.68	0.30	0.03	0.75	0.63
Test of no remaining nonlinearity (p-values of $LM_{AMR}$ )							
$d = 1$	0.84	0.63	0.97	0.19	0.11	0.11	0.11
$d = 2$	0.92	0.49	0.94	0.27	0.01	0.59	0.07
$d = 3$	0.94	0.57	0.87	0.46	0.13	0.11	0.06
$d = 4$	0.95	0.64	0.79	0.62	0.05	0.15	0.16
$d = 5$	0.98	0.54	0.92	0.74	0.11	0.18	0.39
$d = 6$	0.98	0.47	0.92	0.63	0.14	0.13	0.07
$d = 7$	0.92	0.45	0.80	0.40	0.29	0.48	0.15
$d = 8$	0.92	0.29	0.93	0.37	0.04	0.16	0.23
$d = 9$	0.87	0.53	0.86	0.39	0.11	0.87	0.30
$d = 10$	0.68	0.43	0.80	0.52	0.03	0.30	0.13
$d = 11$	0.80	0.41	0.69	0.64	0.03	0.57	0.52
$d = 12$	0.66	0.32	0.66	0.68	0.07	0.74	0.29

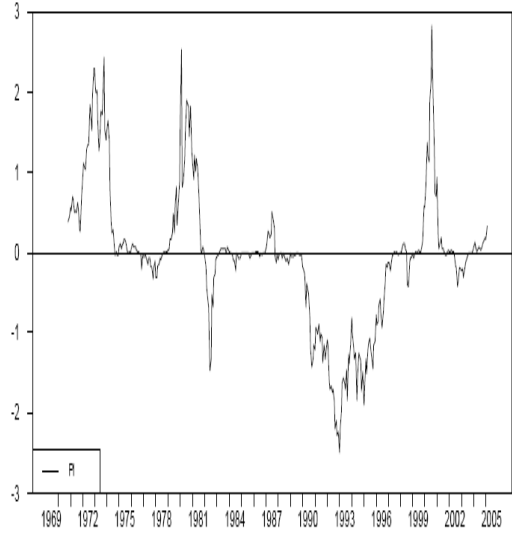
## Appendix 6: Under- and overvaluation of stock price $\Pi(z_t)$

*ESTECM are estimated under  $H_0^a, H_0^b$  and  $H_0^c$*

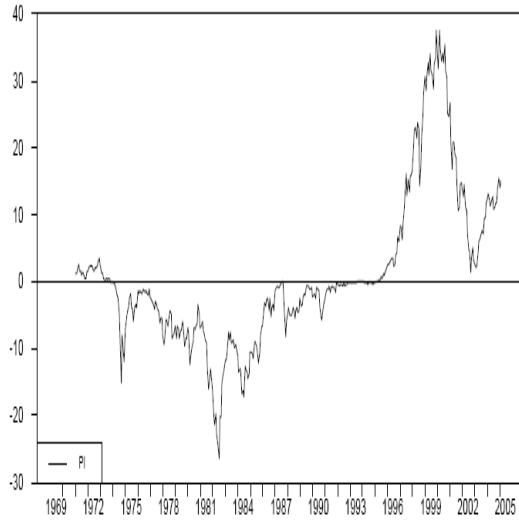
**Germany**



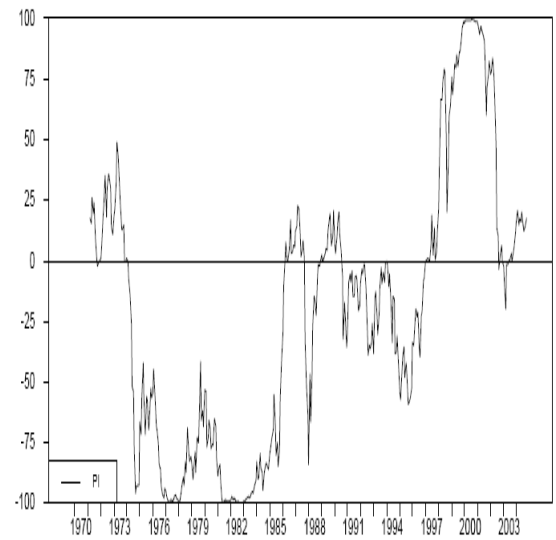
**Canada**



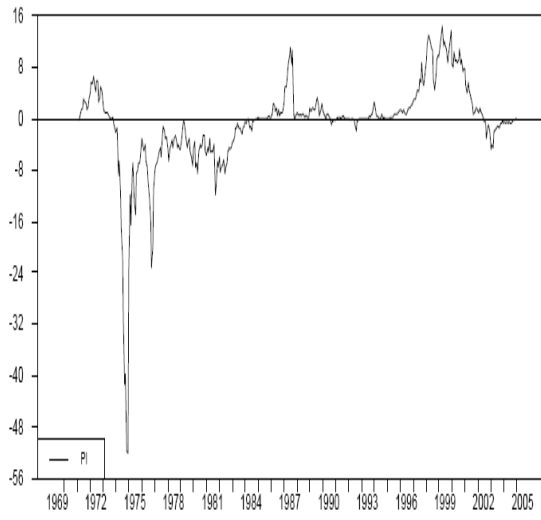
**USA**



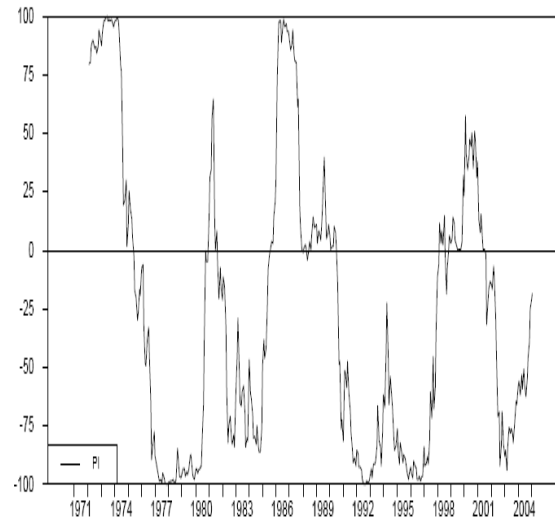
**France**



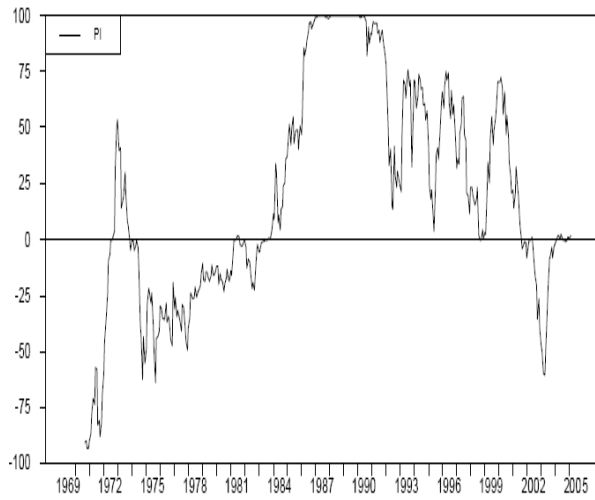
### UK



### Italy



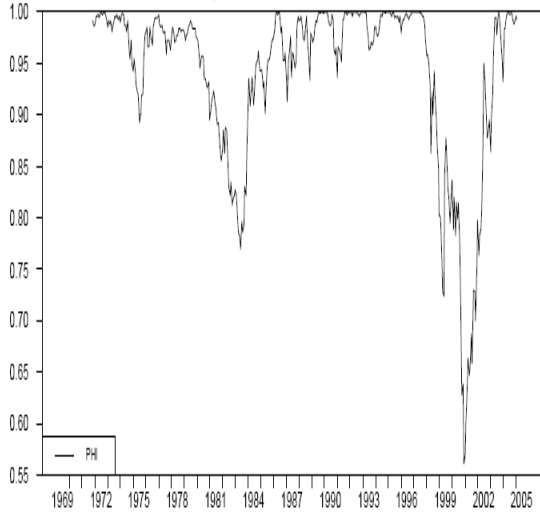
### Japan



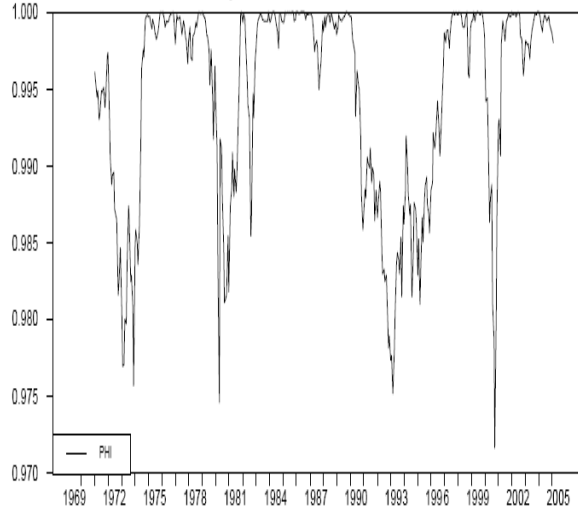
## Appendix 7: Stock price adjustment speeds $\Psi(z_t)$

*ESTECM are estimated under  $H_0^a, H_0^b$  and  $H_0^c$*

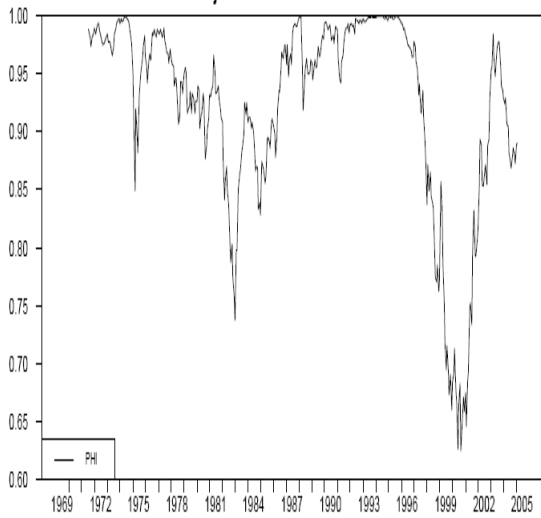
**Germany**



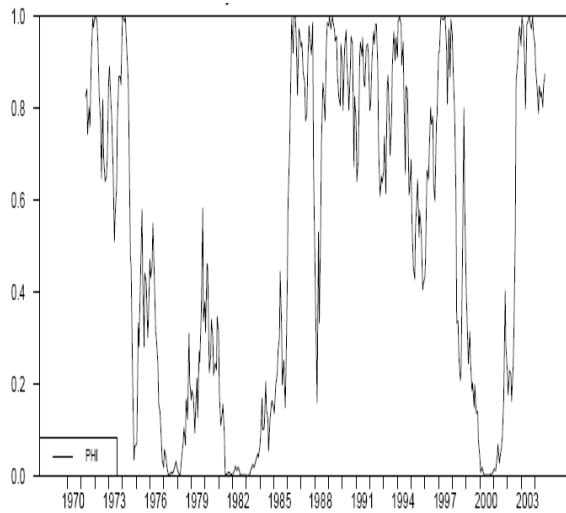
**Canada**



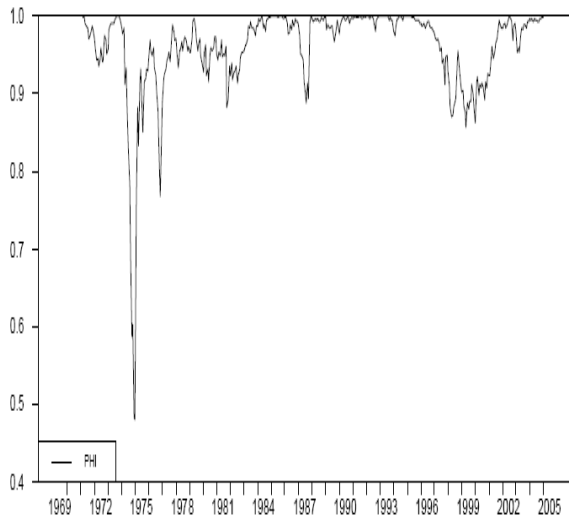
**USA**



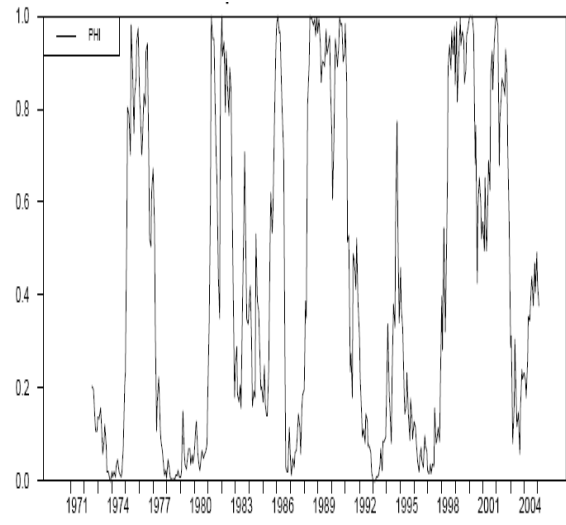
**France**



### UK



### Italy



### Japan

