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Hedging residual value risk using derivatives

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Abstract

In the leasing industry the lessor faces a risk, at the end of the contract, in not recovering sufficient capital value from resale of the asset. We propose a model to hedge residual value risk using the Gaussian copula methodology. After discussing residual value risk and credit risk modelization, a new derivative product is introduced and analyzed; the Collateralized Residual Values (CRV). The model is applied to an European auto lease portfolio of operating lease contracts pertaining to a major company. Our results indicate that the financial product is easy to customize, and to implement through the contract characteristics and the level of correlation.

Keywords: residual value risk, credit risk, credit derivatives, factor modeling, copula.

JEL Classification: C10, G13.

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1 Introduction

A lease is a contract in which one party transfers the use of an asset to another party for a specific period of time, at a predetermined rate. Leasing equipment is an important means of financing, and consequently represents a significant part in many financial institutional portfolio. In 2006, leasing represented more than one-sixth of the world's annual equipment financing requirement [2]. The value of the entire Global leasing market was estimated to be more than \$633 billion [3]. Academic results suggest that "leasing allows small firms to finance their growth, and/or survival while for large firms, leasing appears to be a financial instrument used by sophisticated financial managers to minimize the after-tax cost of their capital"[4].

In the leasing industry, residual values are the forecasted prices of equipment in the second hand market. A large part of the rent paid by the customer during a life contract is the difference between the list price and the residual value. The leasing company makes money or losses money depending on whether it accurately predicts the value of the asset at the end of the contract (fair market value). If residual values are forecasted to be higher than what the asset is actually worth at lease-end, then there will be a loss. At the opposite, if residual values are forecasted to be lower, then there will be a gain on resale.

In the European auto lease market, most leases are closed-end leases: leasing companies assume the residual value risk. In 2001, car resale's price fell dramatically, as a result; US leasing companies suffered large losses, and some even dropped out of the business[5]. Residual value risk is a key element in the leasing industry, however; there is minimal literature, and few developed models. The few studies were developed in three main areas; Operational Purpose[6]; The operational perspective aims to set the most accurate residual

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[2] Percentage market penetrations are highly significatives in United states (27.7%), Germany (23.6%), and Spain (29.1%).

[3] According to the White Clarke Global Leasing report (2008), "Globally, the industry contnued to growth robustly, with the top 50 countries increasing volume by 8.8%" between 2005-2006. The Percentage of the world market volume was respectively 41.1% and 38% for Europe and North America.

[4] See Lasfer and Levis (1998)

[5] See Gordon (2001) for a description of the 2001 Leasing industry crisis.

[6] Jost and Franke (2005) illustrate the use of a specific tool of statistical modelling to calculate residual value through a wide range of parameters. In Lucko (2003) and Lucko, Anderson-cook, and Vorster (2006), residual values are set using regression methodology. Rode, Paul, and Dean (2002) outline a framework for analysing the uncertainty of residual value for assets, such as power generation facilities, for which few data points exists.

value; Basel 2 requirements[7]; defines how to calculate reserves. Studies evaluate Basel 2 accuracy, and reserve calculation in relation to specific credit risk in the leasing industry, and Leasing Contract Valuation[8]; in the valuation analysis, the residual value risk is included through an American option. It allows a comparison of leasing (financial lease and operating lease) v.s. purchase decision. Unfortunately, it does not aim to hedge the specific Residual value risk, let-alone the correlation issue in a portfolio of equipment.

A lack of development on financial products hedging residual value risk, sent my research to credit risk. The recent important developments in finance modelling and in new financial products were in credit derivatives. This implies a change in credit management involving banks and other financial institutions. A credit derivative is a contract between two parties that allows the use of a derivative instrument to transfer credit risk from one party to another. The risk seller, in form, has to pay a fee to the risk buyer who will take the risk. Over the last ten years, the credit derivative market has faced a substantial increase. A lot of credit risk models have been developed, therefore; increasing investor interest[9].

In 1999, Li's Gaussian copula model [10] facilitated a dramatic success of this derivative sector. He proposed a fairly easy, and intuitive model depicting the payment default of a company like the survival probability of a human life[11]. It was also a new tool to evaluate the ongoing issue of credit risk; i.e. correlation. For instance, in a basket of loans there is an individual risk component. Each loan has a risk to default its payment. The systemic risk is the other component. An economic downturn could also impact the whole portfolio, and the systemic risk implies correlation.

Collateralized Debt Obligation (CDO) turns the correlation problem into a solution. It is a credit

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[7]Schmit produced several articles on Credit risk in leasing industry to analyse Basel 2 requirements accuracy. See Schmit (2003), Schmit (2004), Irotte, Schmit, and Vaessen (2004), Laurent and Schmit (2007).

[8]T. Copeland and J. Weston (1982) apply an American put with a decreasing exercise price and S.E Miller (1995) includes an American Call Option in a net present value formula to estimate the internal rate of return of the deal. S.R Grenadier (1995), focusing on the real estate arena, adds a residual value insurance that is equivalent to a put option on the underlying asset in the pricing of a varriety of leasing contracts.

[9]"At the risky end of finance" The economist (April 21st 2007) gives an up to date on the credit derivatives market: "According to the Bank for International Settlements, the nominal amount of credit-default swaps had reached $20 trillion by June last year. With volumes almost doubling every year since 2000, some reckon the CDS market will soon be worth more than $30 trillion".

[10]See Li (2000).

[11]In "Gaussian copula and credit derivatives" the Wall Street Journal (September 12, 2005) tells the story of David Li discovery and his impact on financial markets.

derivative created from a portfolio of debt instruments[12]. The risk seller transfers the risk, therefore; the risk buyer takes the risk. Of course, the risk seller has to pay a fee to the risk buyer. The CDO became a successful product by allowing the credit risk division among different tranches. Synthetic CDOs[13], in particular, were booming, improving liquidity, and allowing corporate bonds to be sliced and diced on the basis of risk. Investors were able to choose different levels of risk and returns[14]. The growth was so huge that it had a global macroeconomic impact, decreasing the risk of default impact, inflating asset prices and narrowing credit spreads[15]. Prior to summer 2007, there was up to 30% of banking investment profit.

From that time, all products have slumped suddenly in value due to fraud, and low quality loan underwriting. The "credit crunch" allowed identification of several weaknesses in the industry of loan securitization. Severing the link between borrowers and risk takers, it promoted a lack of accountability. In addition, market protagonists contributed to a credit bubble[16]. Investor did not fully understand the products and had an over reliance on ratings provided by specialized agencies. Moreover, some securities were poorly structured. Thereby, on a cleared market with more incentives, some experts are hoping for a recovery. Fortunately, securitization is not confined to consumer or corporate loans.

Residual value risk and credit risk have a clear analogy, constituting of units that are more or less risky. A lease portfolio is similar to a loan portfolio, both could be divided into systematic and idiosyncratic risks. Losses occur when certain events happen, and again, the correlation risk has a huge impact. Hedging a portfolio of leasing equipment using derivative securities is attractive, and the idea to use some of the significant developments in Credit risk modelling is attractive as well.

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[12] "Collateralized debt obligations divide the credit risk among different tranches: First senior tranches (rated AAA), second mezzanine tranches (AA to BB), and finally equity tranches (unrated). Losses are applied in reverse order of seniority. Therefore junior tranches offer higher coupons to compensate for the added risk".

[13] "Synthetic CDOs do not own cash assets like bonds or loans. Using credit default swaps (a derivatives instrument), synthetic CDOs gain credit exposure to a portfolio of fixed income assets without owning those assets".

[14] See Hull (2005)

[15] « La multiplication des émissions de CDO semble avoir contribué au resserrement prononcé de spreads intervenu au cours de ces eux dernières années sur l'ensemble des marché de crédit ». Cousseran and Rahmouni (2006).

«This derivatives "money" is not being used to buy food, clothes or cars, which is why there has been no general pick-up in inflation. But it has been used to inflate asset prices, Mr Roche [Independent Strategy] argues ». "At the risky end of finance" The economist (April 21st 2007).

[16] "Fear and loathing, and a hint of hope" The economist (February 14th 2008).

Therefore the aim of this research is to transfer a model from the Credit risk to the Residual risk. The one factor model is presented and modified. This modification allows the creation of a new product, the Collateralized Residual Value. Pykhtin and Dev (2003), first applied the one factor model to auto lease. They calculated the economic loss associated to Residual risk, leading to an estimate on economic capital. The model was constructed and modified for financial lease with the option to buy out (the lessee has a purchase option at the end of the contract). Moreover, loss distribution was calculated for a fine grained portfolio (specific to large portfolio without significant individual exposure), as a result, the model was only driven by the systematic factor.

Our study differs slightly, as we aim to hedge residual risk using a derivative financial product. This article is intended for people within the leasing industry interested by an innovative financial product, as well as people from the financial market concerned by leasing risk opportunities. More specifically, we aim to hedge risk for a classical European contract. The product should cover operating lease contracts on a defined number of units and defined characteristics equipment parameters. We complete this theoretical development by an empirical analysis in which we confront this new derivative with market reality. Research gathered is organized as follow: Sections 2 and 3 provide some backgrounds on residual value risk and CDO pricing; Section 4 describes the model and the financial product; Section 5 is an empirical analysis and Section 6 concludes.

2 Leasing

The initial idea of leasing is that it is the use of equipment in a business which produces benefits, not the ownership. One characteristic of ownership in leasing contract is residual value risk that generates competitiveness or losses.

2.1 Main characteristics

As previously mentioned, a lease[17] is a contract between two parties where a party (the lessor) provides equipment for usage on a specific period of time to another party (the lessee) for specified payment. Three parties are involved in the process; equipment suppliers, lessors and lessees. The lessor is the party that grants the use of the asset to the lessee. The lessee is the party that obtains the use of the asset from the lessor. The lessor purchases the equipment to the supplier. All along the contract, the lessor has the legal ownership of the asset. To use the asset the lessee makes periodic payments to the lessor at an agreed rate of interest.

There are two families of lease contracts. An *operating lease* can be considered as a typical rental allowing the lessee to use an asset without owning it. A *financial lease* aims to transfer all risks and rewards of ownership to the lessee.

A lease is defined as a financial lease if it contains one of the following elements:

- The ownership of the asset is transferred to the lessee by the end of the lease term.

- The lessee has an end of contract option to buy the asset lower than the fair market value.

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[17]Leasing definitions and legislations are quite different from a country to another. As we do not wish to focus on a specific legislation, definitions are made on an international common perspective.

- Whether the asset is transferred or not, the lease period is for a majority of the asset useful life.

- Because of the specialized nature of the asset, the lessee only can use the equipment without major modification.

Otherwise, it is an operating lease[18].

A lease is a financial instrument for the procurement of equipment. Recovery rate on a lease is higher than on a standard loan. But why do enterprises lease ? Regarding large firms, leasing minimizes the after tax cost of their capital. For small asset base companies, leasing increases access to equipment finance. The inherent value of the purchased asset acts as collateral. The lessor is the owner of the equipment, and then is secured by the collateral. Another attractiveness is the leasing companies expertise. Leasing companies are not only intermediaries. Their expertise is a real value added in the leasing process. They have knowledge of the asset. They select the appropriate equipment based on the ability of the asset to contribute to cash flow (through various parameters like equipment characteristics, economic life of the asset, taxes or residual value risk). Leasing companies have also skills in finance, credit, equipment acquisition and dealing. All things considered, they facilitate the flow between equipment suppliers and equipment users.

On lessor side, they are several key elements:

- Asset leased: Used by the lessee for business purpose, it could be any kind of equipment (i.e. printers, trucks...)

- Asset List price: The lessor is usually able to negotiate rebates and the lessee could be part of the acquisition process.

- Lease period: It is a pre requisite agreement between the parties. According to the contract, it could be flexible .

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[18]In next sections, we propose a model to hedge residual value risk on an operating lease that is the most common contract in Europe for Auto Lease. The model can be extended and modified for a financial lease (see Pykhtin and Dev (2003)).

- End of term options: At the end of the contract, they are options allowed to the lessee; Lease period can be extended, lease can be renewed, equipment can be bought or returned.

- Residual value: The lessor forecasts the market value of the asset at the end of the contract.

- Depreciation: It might be seen as the variance between the List price and the Residual value all along the lease period.

- Lease payment: As illustrated by Figure 1, several features are included in payments made by the lessee during the contract; depreciation of the asset (usually the larger component), interests on the lessor investment, servicing charges (including operation cost, insurances, counselling, repairs...).

Lease rental = ((List Price - Residual Value) / Lease period) + interest rate + Servicing charges
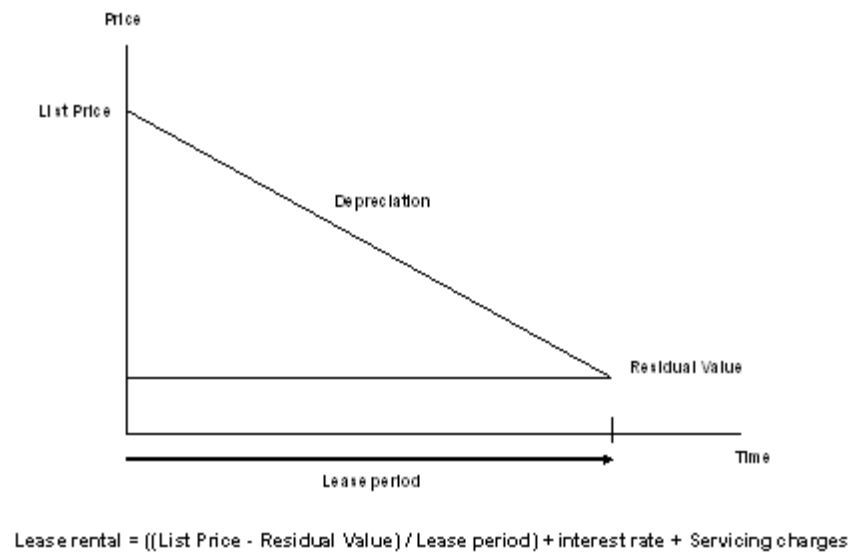
Figure 1: Lease rental calculation

2.2 Residual value risk versus competitiveness

The residual value risk is that the lessor faces the risk to not being able to recover sufficient capital value from the resale or disposal of the asset. As illustrated by Figure 2, the fair market value curve implies a gain on sale or a loss on sale depending on the level of residual value.
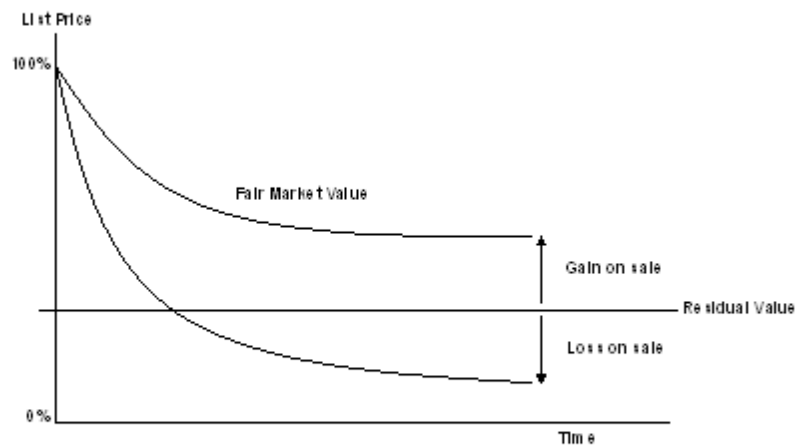

Figure 2: Depreciation curve

Therefore the lessor has a dilemma: The higher the residual value, the lesser the risk of loss on sale. But the higher the residual value, the lower the rental payment. At the same time, the lower the rental payment, the better the competitiveness. And conversely the less the residual value risk, the worse the competitiveness. Figure 3 displays the mechanism of competitiveness and sales results at the end of the contract.

Figure 3: Dynamical benefits

In others words, the lessor has to set a residual value to minimizing residual value risk and maximizing competitiveness. A solution would be the use of a financial product. Hedging residual value risk could be done through a security derivative. Security definition includes financial security (bond, stock) but also capital market securities (mortgage, long-term bonds). It is an investment instrument which offers evidence of debt or equity. A security derivative is a financial security whose value is derived in part from the value and characteristics of another security, the underlying asset[19]. It would allow the lessor to transfer the risk to a fourth party (i.e. insurance company, financial market...).

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[19]Securitization is the process of aggregating similar securities that can be transferred or delivered to another party.

3 Model pre requisites

Because it allows to create a link of two survival functions, Gaussian Copula is a key element in our analysis. CDO pricing, default modeling, and the one factor model are also inherent to the financial product presented in Section 4.

3.1 CDO are a subclass of ABS

Asset Backed Securities are securities backed by a pool of assets. ABS include various subclasses (Commercial Mortgage Backed Securities (CMBS) or credit card ABS...), depending on the underlying asset class. Obligations are usually underlying Collateral Debt Obligation (CDO). The basic idea of CDO is to pool corporate bonds and selling off pieces of the pool. A synthetic CDO replaces pool's bonds by specific credit derivatives, Credit Default Swaps (CDS).

All in all, CDS are triggered by a credit event. A credit event increases the likelihood that the rating of a bond decreases. Consequently, a credit event increases the risk that a bond issuer will default, by failing to repay principal and interest in a timely manner. The events triggering a credit derivative are defined in a bilateral swap confirmation. It is a document that refers to an agreement between the two swap counterparts. There are several standard credit events that could be referred to in credit derivative transactions: Bankruptcy, Failure to Pay, Restructuring, Repudiation, Moratorium.

By selling a CDS, an investor can take exposure to an individual credit. He is receiving periodic payment from his client. At the same time, however, he has to pay contigent payment when default occurs. The client, conversely, can hedge individual credit by buying a CDS. He provides periodic payment to the client and receives contingent when default occurs.

3.2 Default, default, default....

Default modeling is about the expected default payment of an obligor in a bank credit portfolio. The obligor (or debtor) is an individual or company that owes debt to another individual or company (the creditor). The obligor borrows or issues bonds.

The following framework defines our model. The model is underlaid by a *probability space*. This probability space is constituted of three parts. $\mathcal{F}$, a $\sigma-$Algebra, is the information available into a sample space called Ω. Elements of $\mathcal{F}$ are the measurable events of the model. Events of default are measurable. For instance, the event that an obligor survives or defaults is a measurable event. The last element is Pr, a probability measure. Pr(default) is the probability of default. Finally, to summarize, the probability space $(\Omega, \mathcal{F}, \mathrm{Pr})$ is underlying our model.

In survival analysis, T is a random variable denoting the time of default and t are others different times. If $T > t$, then the obligor defaults. *The survival function*, usually denoted S is defined as $S(t) = \mathrm{Pr}(T > t)$. This function must be non increasing: $S(t + 1) \leq S(t)$.

We can now define the complement of the survival function. Usually denoted F, it is a *lifetime distribution function*: $F(t) = Pr(T \leq t) = 1 - S(t)$. From this concept a default rate per unit time can be calculated, the *event density*. Usually denoted f, it is the derivative $f(t) = \frac{d}{dt}F(t)$.

All of this allows the creation of an advanced function, the *hazard function*. The hazard function, usually denoted λ, is the event rate at time t conditional until time t or later. It is given by $\lambda(t)dt = \mathrm{Pr}(t \leq T < t + dt \mid T > t) = \frac{f(t)dt}{S(t)} = -\frac{S'(t)dt}{S(t)}$ ($\lambda(t) \geq 0$ and $\int_0^\infty \lambda(t)dt = \infty$ with no continuous or monotonic constraints).

A *cumulative hazard function* is $\Lambda(t) = \int_0^t \lambda(u)du$.

Because $\lambda(t) = -\frac{S'(t)}{S(t)}$, then $\frac{d}{dt}\Lambda(t) = -\frac{S'(t)}{S(t)}$ and $\Lambda(t) = -\log S(t)$.

Several distributions can be used in duration modeling (usually define on $\mathbb{R}^+$), the most common one being the exponential distribution ($S(t) = e^{\frac{t}{\lambda}}$).

3.3   Basic elements on Copulas

Why do we use copula ? In a portfolio, credit risks are non independent. Copulas are a convenient approach to specify a joint distribution of survival times. Using a copula function, we are able to link the survival function of an obligor to the survival function of another obligor in a portfolio .

In our model, we use copula on a three dimensional perspective. For simplification purpose, we will focus on the *bivariate distribution function* and the *two dimensional copula*. The following results, however, can be extended to the multivariate case (see Nelsen (2006) and Vershuere (2006)).

For a "rigorous" copula definition, we first have to define the *unit square* and the concept of *subcopula*.

- The unit square $\mathbf{I}^2$ is the product $\mathbf{I} \times \mathbf{I}$ where $\mathbf{I} = [0, 1]$.

- A two dimensional subcopula is a function C' defined through the four following properties:

    1_ $DomC\prime = S_1 \times S_2$ (with S_1 and S_2 are subsets of $\mathbf{I}$ containing 0 and 1).

    2_ $C\prime$ is grounded[20].

    3_ $C\prime$ is 2-increasing (for every $x_1 \leq x_2$ and $y_1 \leq y_2$, $H(x_1, y_1) \leq H(x_2, y_2)$).

    4_ For every u in S_1 and every v in S_2, $C\prime(u, 1) = u$ and $C\prime(1, v) = v$.

We are now able to define a *two dimensional copula:* It is a two dimensional subcopula C whose domain is $\mathbf{I}^2$.

Figure 4 gives an intuitive notion of a two dimensional copula. The graph of a two dimensional copula is a continuous surface within the unit cube I^3.

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[20] A function H from $S_1 \times S_2$ is grounded if $H(x, a_2) = 0 = H(a_1, y)$ for all (x, y) in $S_1 \times S_2$ with a_1 and a_2 last elements of S_1 and S_2.
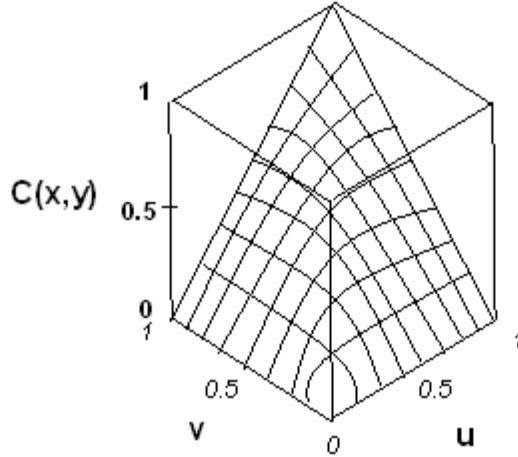
Figure 4: Two dimensional copula

Two others elements are fundamental in our analysis; the *joint bivariate distribution function* and the *Sklar Theorem*.

A *joint bivariate distribution function* is a function H with domain $\mathbb{R}^2$ such that H is 2-increasing:

$H(x, -\infty) = H(-\infty, y) = 0$, and $H(+\infty, +\infty) = 1$. The joint bivariate distribution function is a key element of the *Sklar Theorem*: Let H be a joint distribution function with margins F and G. Then there exists a copula C, such that for all x, y in $\bar{\mathbb{R}}$, $H(x, y) = C(F(x), G(y))$. Furthermore, if F and G are continuous, the copula C is unique. Otherwise C is uniquely determined on $RanF \times RanG$. Conversely, if C is a copula and F and G are distribution functions, then H is a joint distribution function with margins F and G.

We can now include random variables. Let X and Y be random variables with distributions functions F and G, and joint distribution function H. Then there exist a copula C with $H(x, y) = C(F(x), G(y))$. If F and G are continuous, C is unique. Otherwise, C is uniquely determined on $RanF \times RanG$.

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In a few words, a copula function is a function that links univariate marginal to their full multivariate distribution: $C(u, v) = \Pr(u \leq U, v \leq V)$. Therefore, using a copula function, we are able to link the survival function of a credit risk to the survival function of another credit risk in a portfolio.

3.4 Specific pre requisites, the Gaussian copula

In the model presented in this article, we use the Gaussian copula. Let Φ_ρ be the bivariate normal distribution function with correlation coefficient ρ $(0 \leq \rho \leq 1)$. The bivariate normal is a member of the family of elliptically contoured distributions. The density function of Φ_ρ is $\Phi(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{\left(-\frac{1}{2(1-\rho^2)}(x^2+y^2-2\rho xy)\right)}$.

The densities for such distributions have contours that are concentric ellipses with constant eccentricity. Φ^{-1} is the inverse of a normal distribution function.

Finally the Gaussian copula is $C(u, v) = \Phi_2(\Phi^{-1}(u), \Phi^{-1}(v), \rho)$.

Consequently variables are jointly elliptically distributed and we can set ρ using a linear correlation as a measure of dependence: Let X and Y follow, respectively, the distribution F and G. They jointly follow the distribution function H. Then the linear correlation ρ for X and Y is defined, using $u = F(x)$ and $v = G(y)$ as $\rho = (X, Y) = \frac{1}{\sqrt{Var(X)}\sqrt{Var(Y)}} \int_0^1 \int_0^1 [C(u, v) - uv] dF^{-1}(u) dG^{-1}(v)$.

Another property of the bivariate normal distribution is radially symmetry. A bivariate normal distribution with parameters μ_x, μ_y, σ_x^2, σ_y^2 and ρ is radially symmetric about the point (μ_x, μ_y). It means that $H(\mu_x + x, \mu_y + y) = \bar{H}(\mu_x - x, \mu_y - y)$.

Using copula, we are able to work on survival function. Indeed, for a pair of random variable with joint distribution function $H(H(x, y) = P[X < x, Y < y])$, the joint survival function copula is given by $\bar{H}(x, y) = P[X > x, Y > y] = 1 - H(x, y)$. The relationship is $\bar{H}(x, y) = 1 - F(x) - G(y) + H(x, y) = -1 + \bar{F}(x) + \bar{G}(y) + C(1 - \bar{F}(x), 1 - \bar{G}(y))$.

In the next section, we assume that the correlation of default is driven by a common factor through a Gaussian copula.

3.5 The initial one factor model is used for CDO pricing

To resume the model in one sentence, a firm defaults when its "asset value-like" stochastic process X, falls below a barrier. X is commonly identified as the amount of asset and $\overline{X}$ the barrier as the amount of liabilities. The firm defaults when the amount of asset is below the amount of liabilities. The idea was first introduced by Merton (1974). He transferred an option pricing model to the credit risk market. Then he applied the Black and Scholes model to credit risk. We present an alternative model using copula. Value added is in copula flexibility to dependent variables and copula ability to provide scale invariant measure of association between random variables. The intuitive aspect of this model contributed to the growth of credit risk market.

The model described below is the famous standard Gaussian copula developed by Li (2000) and exposed[21] by Gibson(2004).

In a reference portfolio of $i = 1, ...N$ credits, for each obligor, default payment occurs when x_i (reference credit normalized asset value) falls below $\overline{x_i}$ (the threshold).

$$x_i = a_i M + \sqrt{(1 - a_i^2)} Z_i \tag{1}$$

x_i has three main components: M, Z_i, and a_i.

M is the common factors affecting all the credits, the systematic risk. Z_i is the factor affecting only credit i. a_i is the correlation parameter $(0 \leqslant a_i \leqslant 1)$ and defines default dependency between companies in the economy. The correlation of asset value between credits i and j is equal to $a_i a_j$. The random variables are assumed to be independently distributed. Therefore unconditionally on the systematic risk, default payments are correlated but conditionally there are independent.

M, Z_i, and x_i are means-zero, unit variance random variables with distribution functions $G(0; 1)$, $H_i(0; 1)$,

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[21] See also Meneguzzo and Vecchiato (2004) for an empirical study of credit derivatives within the copula framework. Cherubini, Luciano, and Vecchiato (2004) gives an overview of copula applications in Finance.

and $F_i(0; 1)$. $q_i(t)$ is a risk neutral probability that credit i default before t. The default threshold $\overline{x_i}$ is equal to $F_i^{-1}(q_i(t))$.

When does a default happen ?

A default happens when x_i falls below $\overline{x_i}$.

But x_i falls below $\overline{x_i}$ if $F(x_i) < q_i(t) \Leftrightarrow x_i < F_i^{-1}(q_i(t)) \Leftrightarrow a_i M + \sqrt{1 - a_i^2} Z_i < F^{-1}(q_i(t))$

and finally $Z_i < \frac{F^{-1}(q_i(t)) - a_i M}{\sqrt{1 - a_i^2}}$

Conditional on the value of the factor M, the probability of default is therefore

$$q_i(t/M) = H^{-1}\left(\frac{F^{-1}(q_i(t)) - a_i M}{\sqrt{1 - a_i^2}}\right) \tag{2}$$

For any number of default in a portfolio of N obligors, we have to estimate the probability of default on time t and conditional on the common factor M.

Therefore, we set the number of default distribution using a binomial function[22].

$$PN(l; t/M) = \binom{l}{N} q_i(t/M) \tag{3}$$

Once we have the conditional default distribution, we estimate the distribution according the distribution of M.

The unconditional default distribution $P^N(l; t)$ can be calculated as

$$P^N(l; t) = \int_{-\infty}^{\infty} P^N(l; t/M) g(M) dM. \tag{4}$$

In a CDO, the investor is responsible for the interval of loss $[L, H]$. The expected loss of a CDO is defined on $[L, H]$. We define the loss for any default as $A(1 - R)$ with A the notional amount of credit and R the

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[22] The number of default distribution is usually computed through a recursion method (Andersen, Sidenius and Basu (2003) or Hull and White (2004)). In the case of homogeneous credits and for simplification purpose, a binomial function is simpler and lead to similar results.

recovery rate.

The expected loss is

$$EL_i = \sum_{l=0}^{N} P^N(l; T_i) * max(min(lA * (1 - R), H) - L, 0) \tag{5}$$

with Ti, $i = 1, ..., n$ the periodic payment.

Now, how to price a CDO ?

A CDO contract specifies two potential cash flow streams: a Contingent leg and Fee leg.

- On the contingent leg side, the protection seller makes one payment only if the references credit default.

  The amount of a contigent payment is the notional amount multiplied by $(1 - R)$.

The contigent leg is

$$contigent = \sum_{i=1}^{n} D_i(EL_i - EL_{i-1}) \tag{6}$$

with D_i the risk free discount factor for payment date i (e^{-rt}, with r the risk free rate). The risk free discount factor is usually derived from the risk free interest rate.

- On the fixed leg side, the buyer of protection makes a series of fixed, periodic payments of CDO premium until the maturity, or until the reference credit defaults.

The expected present value of the Fee leg is

$$Fee = s \sum_{i=1}^{n} D_i \Delta_i \left[(H - L) - EL_i \right] \tag{7}$$

Δ_i is the accrual factor for payment date i and s is the spread per annum paid to the tranche investor ($\Delta_i \approx T_i - T_{i-1}$).

The value of the CDO contract to the tranche investor at any given point of time is the difference between the present value of the contigent leg and the present value of the fixed leg. It is the difference between the protection the buyer expects to pay, and the amount he expects to receive.

- The Mark To Market value of the tranche, from the perspective of the tranche investor is

$$MTM = Fee - Contigent \tag{8}$$

At inception the mark to market is equal to 0, therefore the spread is

$$s = \frac{Contigent}{\sum\limits_{i=1}^{n} D_i \Delta_i \left[(H - L) - EL_i \right]} \tag{9}$$

4 A modified model: The leasing model

From equipment leasing specificities and the one factor model, we create a residual value risk model. A new product called Collateralized Residual Value (CRV) is adjusted through the leasing contract parameters.

4.1 There is a similarity between credit risk and residual value risk. But there are also dissimilarities and specifically in Auto Lease.

The main idea of the leasing model is that a portfolio of leased equipment is comparable to a portfolio of credit. A portfolio with losses on resales is equivalent to a portfolio of credit with companies defaulting.

- As in a CDO, every unit into the lease portfolio, has an idiosyncratic and a systematic risks; asset specific characteristic impact is resale price (model type, obsolescence...). At the same time, resale price of other assets has a significant impact (bid and ask effect, downturn on the resale price market, inflation etc....).

There are also dissimilarities:

- First of all, equipment units are resold only one time at the end of the contract, although for a CDO, there is a risk of default throughout the contract. Therefore, the model presented in next section is set for only one period.

- Another dissimilarity (and not the least) is on correlation estimation. Difference is not on calculation but on data source.

In credit risk, they are four main data sources available:

-Default events that are obviously concrete realization of credit risk. They are rare events and as a result there are usually few data available. Approximations and aggregation have to be made to constitute data bases.

-Companies credit ratings: They are provided by credit agencies and reflect the credit risk of a company according to experts points of views. By and large, they are made through balance sheet and macroeconomic analysis.

-Credit spreads: They reflect market perception of credit risk. A large amount of data is available. But spreads could be impacted by external elements like liquidity

-Equity correlation: The factor model (cf Section 3), assumes a theoretical link between equity and credit risk. Correlations are then more easy to compute.

In residual risk, there is one main data source available: for a residual value calculation, inputs are observations from second hand markets. Correlation estimation of residual value is based on resale market statistics. Resale prices, asset characteristics and price index are used to set modelization variables. A large amount of data is available.

- The last dissimilarity is on standard definitions. As multiple factors define a resale, there are issues to define resale asset classes or homogeneity prices.

Auto lease is an extreme illustration. The high price level in the automotive second hand market involves a high residual value level. Combined to a competitive leasing market, the level of price leads to high risks of loss on sale.

At the same time, automotive is a singular equipment. A car is not only a tool to go from a place to another. It is also a living place and a symbol. Automotive often reflects driver's sociological characteristics. The purchase of a vehicle is a sensitive act, even in business. Therefore, Auto lease is a wide area to analyze. In automotive market multiple factors influence resale price. A second hand vehicle price is impacted by age (time between registration date and resale date), mileage (number of kilometers at the end of the contract), damages (i.e. amount and type), product life cycle (i.e. new model...), make (i.e. Toyota, Renault...), model (i.e. Yaris, Laguna...), version, body type (i.e. break, pick-up...), segment (i.e. small cars...) or external color. Figure 5 gives an overview. Choices have to be made to define similar assets and prices (c.f Section 5).
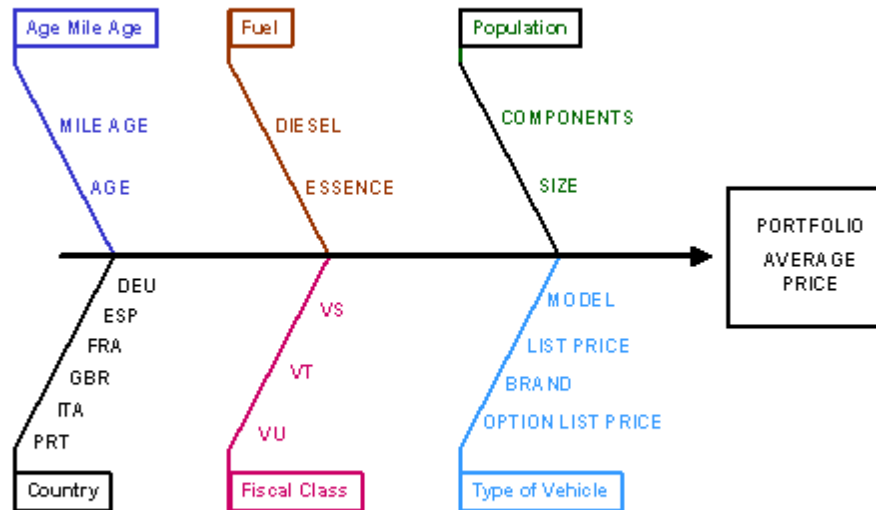

Figure 5: Multiple factors of automotive market

4.2 Homogeneous equipment type model

The initial idea is simple: we use the equipment resale value as the asset value-like x_i and the probability of resale value below residual value ($x_i < \overline{x_i}$) as the probability of default.

In a reference portfolio of $i = 1, ...N$ units (vehicles, equipment...), for each obligor, losses occur when x_i (reference unit normalized asset value) falls below $\overline{x_i}$ (reference unit normalized residual value).

$$x_i = a_i M + \sqrt{(1 - a_i^2)} Z_i \tag{10}$$

-The correlation of resale's prices between unit i and j is equal to $a_i a_j$.

-M is the sectorial factor affecting equipment units on resale's market and Z_i is the risk of loss on resales on unit i

-Xi, M, and Zi are means-zero, unit variance random variables with distribution functions $F_i(0;1)$, $G(0;1)$, and $H_i(0;1)$.The random variables are assumed to be independently distributed.

At that point, the construction is similar to the credit model, but we include residual risk. Resale's value can be lower than residual value. There is a risk of loss on sale.

Three new elements will have an impact on the leasing adjustment of the model.

V_i is the residual value or in other words the expected fair market value. $mFMV_i$ is the historical average fair market value, $eFMV_i$ is the historical standard deviation.

$mFMV_i$, $eFMV_i$, and V_i are set on a percentage of Lp, List price by unit. As an example, an asset bought € 10000 and leased for a Residual value of € 5000 has V_i= 50%.

Then residual risk is added: Probability of loss at the end of the contract is $q_i(t)$. $q_i(t)$ is a variable with mean $mFMV_i$, variance $eFMV_i$, and distribution function $E_i(mFMV_i; eFMV_i)$ The probability of loss depends on residual value: $q_i = E_i(V_i)$. So default threshold $\overline{x_i}$ is equal to $F_i^{-1}(q_i)$.

Conditional on the value of the sectorial factor M, the probability of default is therefore

$$q_i(M) = H^{-1}\left(\frac{F^{-1}(q_i) - a_i M}{\sqrt{1 - a_i^2}}\right) \tag{11}$$

Again, conditional probability is $P^N(l; M) = \binom{l}{N} q_i(M)$ and the unconditional probability can be calculated as $P^N(l) = \int_{-\infty}^{\infty} P^N(l) g(M) dM$.

As a result the recovery rate is equal to the probability of loss. By construction the recovery rate is $R = qi$. The loss on sale for any unit is $(1 - R)Lp$. Finally, resale price becomes RLp.

Previous elements allow the creation of a financial product, inspired by Collateral debts obligations:

The Expected loss is

$$EL = \sum_{l=0}^{N} PN(l) * max(min(Lpl * (1 - R), H) - L, 0) \tag{12}$$

The contigent leg is

$$contigent = \sum_{i=1}^{n} D_i(EL) \tag{13}$$

The premium leg is

$$Fee = s \sum_{i=1}^{n} D_i \Delta_i \left[(H - L) - EL_i\right] \tag{7}$$

The spread is

$$s = \frac{Contigent}{\sum_{i=1}^{n} D_i \Delta_i \left[(H - L) - EL_i\right]} \tag{9}$$

4.3 Heterogeneous equipment type model: a portfolio of three different assets

The model is extended to a portfolio with non similar units. A company fleet is commonly constituted of various car model. In an European leasing contract for medium size European company, lessee usually request different categories of cars for an auto lease contract. The fleet is usually divided into three groups: Executives' cars (usually high brand car), Employee cars (medium level cars) and Small cars.

Basically the construction is similar to the homogeneous equipment type model (cf Section 4.2).

Three representative's vehicles constitute the model: Ex, Em and Sm.

Now there are different types of asset residual values, number of units, List price etc....

$V1_i$, $V2_i$, $V3_i$ are residual values for group 1, 2, 3

$mFMV1_i$, $mFMV2_i$, $mFMV3_i$, are fair market values historical averages,

$eFMV1_i$, $eFMV2_i$, $eFMV3_i$ are historical standard deviation historical averages.

$mFMV1_i$, $eFMV1_i$, and $V1_i$ are set on a percentage of List price. The recovery rate for group 1 is $R1 = q1i$, the loss on sale for any unit is $R1(1 - Lp1)$ with $Lp1$ unit list price. Indeed, resale price is $R1Lp1$. The principle is the same for others groups.

For each vehicle, asset value is still $x_i = a_i M + \sqrt{(1 - a_i^2)} Z_i$.

For group 1, default threshold $\overline{x_i}$ is equal to $F_i^{-1}(q_i)$ with $q1_i = E_i(V1_i)$ and $E_i(mFMV1_i; eFMV1_i)$. The principle is the same for other groups.

The distribution of the number of default, conditional on the common factor M, is computed for each group as $P^{N_1}(u; M) = \binom{u}{N_1} q1_i(M)$, $P^{N_2}(v; M) = \binom{v}{N_2} q2_i(M)$, $P^{N_3}(w; M) = \binom{w}{N_3} q3_i(M)$ with N_1, N_2, N_3 number of units, $P^{N_1}, P^{N_2}, P^{N_3}$ the conditional probabilities, and u, v, w number of defaults for group 1, 2, 3.

The probability of default is computed on the whole portfolio.

$P^N(u; v; w; M) = P^{N_1}(u; M) P^{N_2}(v; M) P^{N_3}(w; M)$

The Expected loss is:

$$EL = \sum_{l=0}^{N} P^N(u; v; w; M) * max(min((Lp1u*(1-R1)) + (Lp2u*(1-R2)) + (Lp3u*(1-R3)), H) - L, 0) \quad (14)$$

Premium leg, contigent leg and spread are calculated like in 4.2.

It is straightforward to generalize this approach to more than three vehicle type.

4.4 Collateralized Residual Values

We propose a financial product, the Collateralized Residual Values, that covers residual value risk. We display a sensitivity analysis of the CRV to the main characteristics of the leased asset.

4.4.1 A CRV is a new class of ABS

The Collateralized Residual Values (CRV) is a new class of Asset Backed Securities (ABS). The CRV is inspired by synthetic Collateralized Debt Obligations (CDO) structure. Like a CDO, CRV can be sliced and diced, and tranches can be sold. But CRV is not about credit risk. The purpose is to hedge residual value risk on a portfolio of leases. A credit derivatives, obviously, is more accurate to hedge credit risk in a portfolio of contract.

4.4.2 Sensitivity Analysis on a CRV

What is the sensitivity of a CRV to size, residual value, and fair market variance ?

In the following sensibility analysis, all underlying reference assets are cars. The portfolio is homogeneous. List price (€ 15000), Fair market value (€ 4500) and Correlation ($\sqrt{0.3}$) by car are equal. Cars are leased on a three years contract.

We value four tranches of the CRV. The first tranche absorbs all losses until the first 25% of the portfolio, the second tranche until 50%, the third tranche until 75% and the fourth until 100%.

Impact of Fleet Size   Table 1 shows that the buyer of protection on a fleet of 600 units should be willing to pay 125,21 basis points to hedge the first 50% losses using a CRV.

Spread bp

Population	Tranche			
	25%	50%	75%	100%
10	285,54	136,90	90,04	67,07
100	285,53	136,90	90,04	67,07
200	285,54	136,90	90,04	67,07
500	285,54	136,90	90,04	67,07
550	281,11	134,87	88,71	66,10
600	260,18	125,21	82,44	61,45
700	192,67	93,63	61,84	46,17
1000	54,67	27,11	18,02	13,50

List price= €15000,
Residual value= €4500,
Fair Market Value Average = €4500,
Fair Market Value Variance= €1500.

Table 1: Sensitivity to fleet size

According to results, the spread is stable until 500 units. Then the increasing size reduces the cost of protection. An increase in size reduces idiosyncratic effect. There is a diversification of risk .

Impact of Residual Value level

Spread bp

		Tranche			
		25%	50%	75%	100%
Residual Value	1500	0,21	0,10	0,07	0,05
	3000	17,06	8,51	5,67	4,25
	4500	285,53	136,90	90,04	67,07
	7500	2459,82	898,42	549,57	395,86

List price= € 15000
Population= 100 Units
Fair Market Value Average = € 4500
Fair Market Value Variance= € 1500

Table 2: Sensitivity to residual value

The higher the residual value, the higher the pricing of CRV (Table 2). As illustrated in Section 2.2, decreasing residual value reduces the risk of loss on sales.

Impact of Fair Market Value variance

Spread bp

Fair Market Value Variance		Tranche			
		25%	50%	75%	100%
	750	138,50	67,84	44,92	33,58
	1500	285,53	136,90	90,04	67,07
	2250	384,76	181,88	119,09	88,53
	3000	402,61	189,84	124,20	92,29

List price= € 15000,
Population= 100 Units
Residual value= € 4500,
Fair Market Value Average = € 4500,

Table 3: Sensitivity to fair market value variance

Fair market value distribution tails depends of FMV variance (Table 3). For an higher variance, tail are

larger. As a result, the spread is an increasing function of the variance.

5 Empirical Analysis

The model is applied to a six years historic resale's portfolio. The observations, between 2000 and 2008, are from a major European leasing company (General Electric Capital Solutions). We first estimate the correlation of assets to a common factor. Then fair market value parameters and residual value are estimated

5.1 Correlation to the one sector factor

According to section 4.2, we have to set the linear correlation (a_i) between the portfolio and a sectorial factor affecting equipment units on resale's market (M). The sectorial factor is assessed using Eurostat Harmonized Consumer Price index (HCPI[23]). The index Purchase of vehicles price allows comparison within European markets. Additionally, a portfolio index has to be created. The portfolio index provides a non-biased historical trend analysis and exposes portfolio sale price at different time.

5.1.1 Automotive Price Index

To set a common factor that would affect the whole portfolio, several HCPIs are available ; HCPI all items, HCPI Energy, HCPI Petroleum products, HCPI Road transport equipment.

The Index "Purchase of vehicle" (Figure 6) appears to be the most relevant. It covers purchases of new vehicles and purchases of second-hand vehicles from other institutional sectors. It is available by country and on European level. This index is included in the modelization as the sectorial factor indicator (M). Vehicle customers have to choose between resale market and new market. As a consequence, resale market is strongly impacted by new vehicle market. Therefore, a positive or a negative correlation of the sectorial index with the portfolio can be expected.

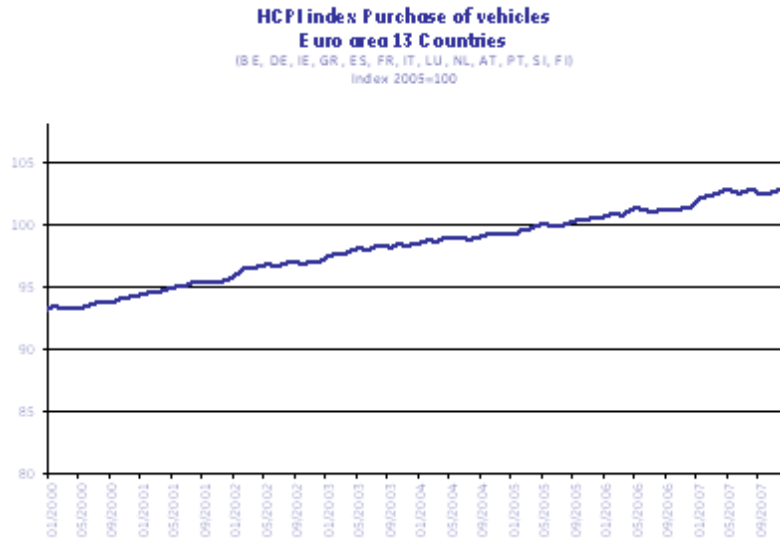[23]Graphics of HCPI series are reported in Appendix.

Figure 6: European HCPI

5.1.2 Portfolio Index Creation and Computation

A large amount of parameters impacts resale price and there is a non homogeneity of the portfolio mix from one month to another. As a consequence, average price never reflects accurately portfolio sales price variance through time. Therefore a consistent price variable is created through an index replicating a same artificial portfolio. The idea is to replicate a portfolio mix to allow time series analysis.

Portfolio Index Creation The information comes from resale vehicles statistics from 01/2000 to 01/2008 including France, Germany, Italy, Portugal, Spain and Sweden. We only include normal terminations sales (sales types like wrecks or litigations are excluded). Observations with extreme and incorrect values are cleaned. High damages (95th decile by country) that would alter resale's price and therefore are filtered.

Calculation is a five-steps process:

1   Creation of buckets for Age and Mileage (details in appendix).

2.   Keys are created including the following components:

30

$age/mileage/model/fiscalclass/fueltype/country$

3.   Keys with population of less than 100 units on the whole history are excluded

e.g.:$GBR/Toyota/previa/private/lightgoods/petrol/\_6.33, 39month/\_3.75000, 105000km$ is excluded.
During the last 8 years, less than 100 units in this bucket have been sold.

Representative and similar samples are created all along the history using the historical key frequency:

4.   A Random selection is processed by month through the following criteria:

-      1% of units by bucket are selected:

(e.g. for a bucket of 200 units, then 2 units are selected)

-      Restricted random sampling with replacement (SAS proc survey)

-      Priority levels: The sample is replicated on a monthly basis according to key frequency and by order
of priority; selected month, semester of the selected month, whole history (e.g. If data are not available in
the current month, then data are selected in the current quarter etc...).

5.   A monthly resale percentage is computed from the sample. The percentage of resale is $\frac{resaleprice}{Listprice+OptionListprice}$.
It allows a comparison of resale performance between vehicles with different levels of price and option price.

The process is replicated several time to create several sample. 1000 random samples are created by
month.

Portfolio Index Computation and results   565 representative buckets are selected from an initial pool
of 38257 units. 97000 samples are calculated (97 period).

Among other perspective, it provides a graph distribution by month. Results are available by country
and on European level. For instance, Figure 7 displays the simulation result for Portugal on January 2001.
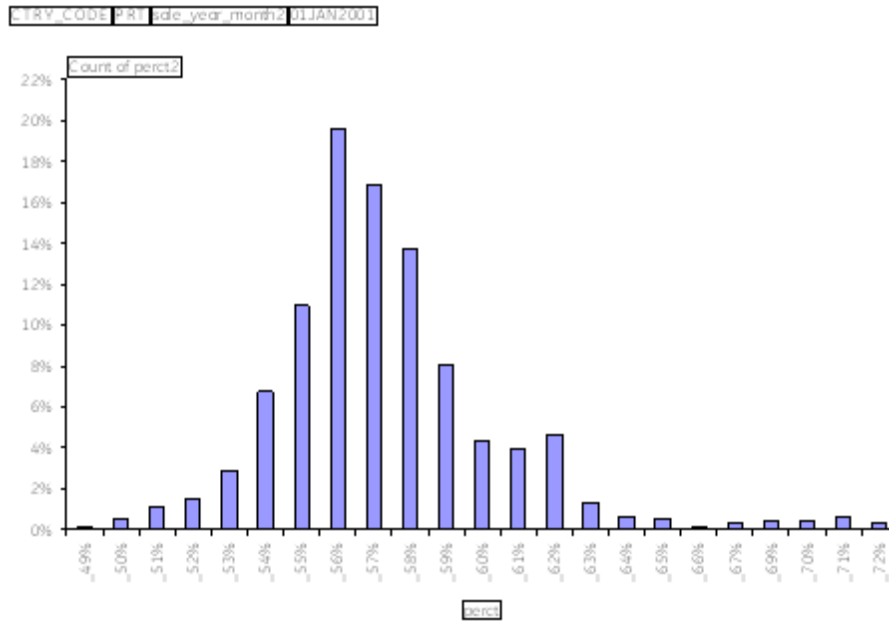The percentage of resale distribution is on a range of [49%-72%].

Figure 7: Portugal depreciation distribution on January 2001

5.1.3 Estimation of the correlation between the sectorial factor and the portfolio price

Time series are seasonally adjusted using the TRAMO-SEATS methodology[24]. Graphical results by countries

are displayed in Appendix.

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[24] TRAMO-SEATS: They consist of new versions of programs TRAMO, "Time series Regression with ARIMA noise, Missing values and Outliers", and SEATS, "Signal Extraction in ARIMA Time Series", created by Gómez and Maravall in 1996, of program TERROR, "TRAMO for Errors", and program TSW, a Windows version of TRAMO-SEATS with some modifications and additions, developed by G. Caporello and A. Maravall at the Banco de España.
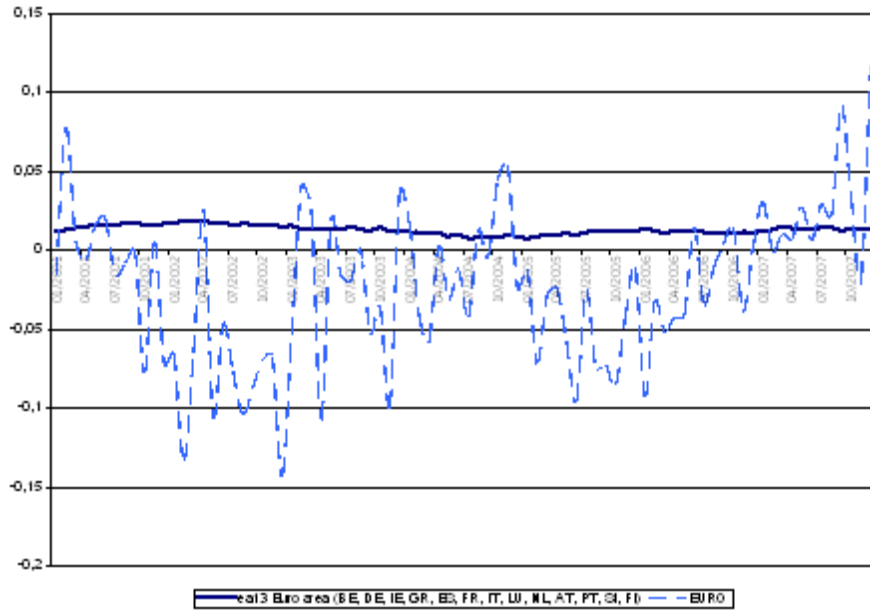
Figure 8: European HCPI and European portfolio YoY variance

Figure 8 illustrates results for Europe. The European HCPI is more stable than the European Portfolio YoY variance value.

A Pearson's product moment is computed on a year on year annual variance, and results are given in table 4.

	EURO	DEU	ESP	FRA	ITA	PRT	GBR
Pearson's product-moment coefficient	0,156298	-0,022541	0,050449	-0,008072	-0,032967	0,027337	0,014147
coefficient of determination	0,024429	0,000508	0,002545	0,000065	0,001087	0,000747	0,000200

Table 4: Pearson Product moment on year on year annual variance

As expected, results are different by country. Correlation are negative or positive with different level of intensity. Impacts are negative for Germany, France and Italy. If "Purchase of vehicle" HCPI increases, then the resale portfolio performance decreases. Therefore unlike the initial credit model, the correlation parameter could be negative $(-1 \leqslant a_i \leqslant 1)$.

5.2 Fair Market Value and Residual Value setting

$mFMV_i$, average fair market value, $eFMV_i$, standard deviation and residual value (V_i) are parameters to include in the model.

Fair Market Value estimation is complex   We assess the Fair Market value at the end of the contract. In others words, we estimate the depreciation of the asset for the next years.

Resale percentage   Mean and Variance of resale percentage are calculated from historical statistics. For simplification purpose, resale price is computed through a percentage of List Price ($\frac{resaleprice}{Listprice+OptionListprice}$).

FMV subtlety   To illustrate our presentation we focus on a specific Key: PEUGEOT 307 Tourisme Diesel _6.]33,39]month _4.]105000,145000]km _FRA.
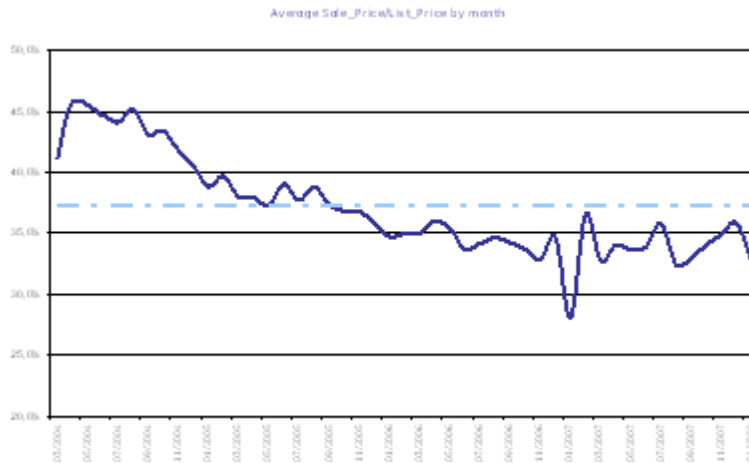
Figure 9: 36 months contracts / Peugeot 307 depreciation and historical average

Figure 9 shows the time series depreciation of the key. It also shows the historical average depreciation. Since January 2004, the key average depreciation is 37.26% and the standard deviation is 46.76%.
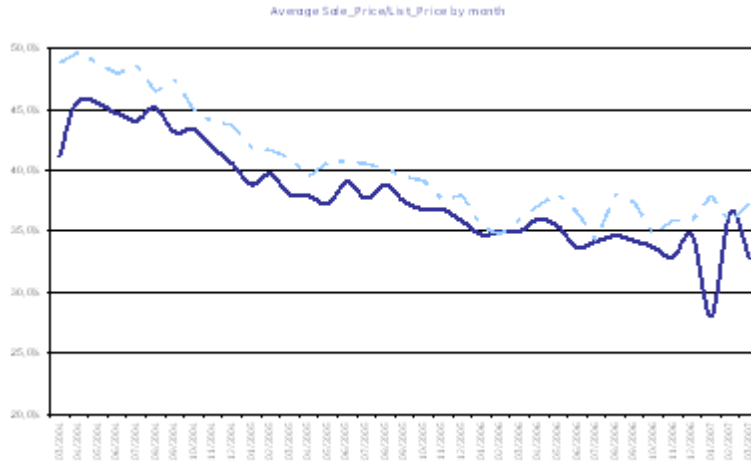
Figure 10: 24 months contracts and 36 months contracts depreciations

Figure 10 compares the depreciation with a 24 months Key: PEUGEOT 307 Tourisme Diesel _ 6.]21,27]month _ 4.]105000,145000]km _FRA .

Figure 11: Peugeot 306 and Peugeot 307

Figure 11 is a graphic of Peugeot 306[25] and Peugeot 307 depreciation: PEUGEOT 306 Tourisme Diesel _ 6.]33,39]month _ 4.]105000,145000]km _FRA.

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[25] Peugeot 306 is the previous model version of Peugeot 307.

Previous graphics illustrate the fact that FMV is not a constant value. There are trends and cycles that are not straightforward to identify. Depreciation in the value of a car occurs based on a range of factors. The factors include cars condition, kilometers traveled and brand reputation. Moreover, brand reputation contains mechanics and popularity. Consequently, different methodologies are possible to forecast average fair market value.

Leasing industry usually works with internal modelization. Standard models have inside a model life cycle and a segment analysis. New legislations or macroeconomic impacts also are sometime included. Additionally, External companies (Eurotax, X-ray, Cap) provides forecasted FMV. Forecasts are based on market data, modelization and expertise.

5.2.1   RV and FMV, a new perspective

In an operating lease contract, Residual value is defined as the forecasted fair market value. It is an input in rental calculation. It also drives the risk of loss on sales at the end of the contract. What is the impact of a CRV ?

Elements of the contract become different. Using securitization product, elements of the contract have to be redefined.

The fair market value still has to be forecasted. But residual value is now a threshold. As illustrated in Figure 12, the threshold is a level of risk chosen by the lessor and the lessee. In the model, $mFMV$ is a forecasted average of fair market value at the end of the contract. And $eFMV_i$ is the estimated standard deviation of fair market value. So considered residual value is now an adjustment variable. Therefore the securitization product allows several choices within different levels of risk, different levels of rents, different market spreads, and different fair market value variance. Additionally, hedging can be made on specific tranches.
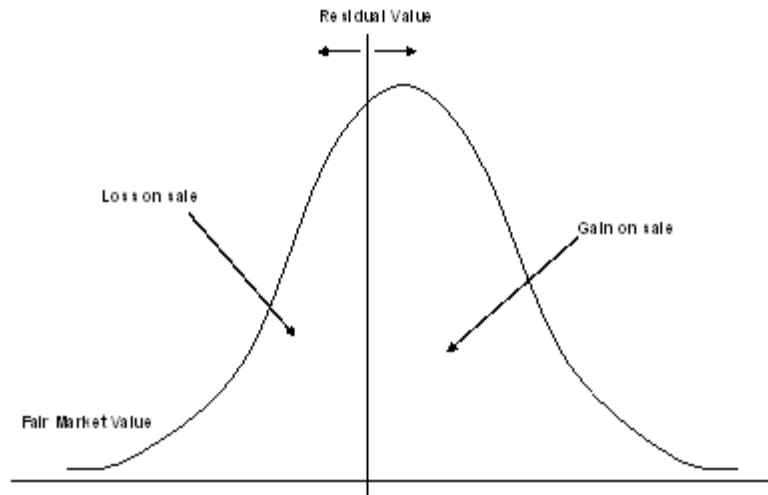
Figure 12: Sale results through fair market value and residual value level

For simplification purpose, the threshold is set at $mFMV$ value in the next illustration. It means that the contract position is neither conservative or risk taking.

5.2.2 Six CRV

Spread bp

	Bucket					
	1	2	3	4	5	6
Population	200 units	200 units	200 units	200 units	200 units	200 units
List price	30 000 €	30 000 €	25 000 €	20 000 €	20 000 €	20 000 €
Residual value	10 533 €	14 745 €	4 468 €	8 414 €	4 262 €	10 628 €
Fair Market Value Average	10 533 €	14 745 €	4 468 €	8 414 €	4 262 €	10 628 €
Fair Market Value Variance	31	37	27	57	48	168
Correlation	-0,008072	-0,022541	-0,032967	0,050449	0,014147	0,027337
Tranche 25%	261,04	200,37	571,03	282,84	751,42	399,49
Tranche 50%	125,60	97,26	262,99	133,85	337,65	184,97
Tranche 75%	82,69	64,22	170,83	87,67	217,75	120,34
Tranche 100%	61,64	47,93	126,50	65,18	160,69	89,18

Bucket 1	PEUGEOT 307 Véhicule de Tourisme Diesel _6.]33,39]month _4.]105000,145000]km _FRANCE
Bucket 2	Audi A4 Personenkraftwagen D 6.]33,39]month 4.]105000,145000]km GERMANY
Bucket 3	FORD MONDEO Vetture D 6.]33,39]month 4.]105000,145000]km ITALY
Bucket 4	RENAULT MÉGANE Turismos D _8.]45,51]month _4.]105000,145000]km _SPAIN
Bucket 5	VAUXHALL VECTRA PRIVATE/ LIGHT GOODS P 6.]33,39]month 2.]45000,75000]km UK
Bucket 6	RENAULT MEGANE Ligeiro de Passageiros S95 _8.]45,51]month _4.]105000,145000]km _PORTUGAL

Table 5: Pricing of six CRV

A CRV is built accordingly to leasing contract characteristic. As illustrated in Table 5 for six CRV, inputs in pricing are population size, list price, fair market value mean, fair market value variance and residual value. Through the selected residual value level and tranches limits, lessor and lessee can choose a level of risk. Moreover, in case of negative correlation parameter, CRV could go against a downturn in the sectorial market and create opportunities for risk diversification. Like standards derivatives, CRV allows insurance or hedging for the lessor and, for the buyer taking opposite position in the financial market, speculation or arbitrary.

6   Conclusion

Li Gaussian copula model, initially used for credit risk, is transposed in residual value risk of the leasing industry. The Collateralized Residual Value (CRV), a new derivative product, is proposed. Pooling together a large portfolio of equipment that has been leased, the derivative converts end of contract risks into an instrument that may be sold in the capital market. As a standard derivative, it is a tool that transfers risk, and can be used for hedging or speculation. Moreover, it allows the lessor and lessee to select their degree of exposure to residual value risk and to improve competitiveness. As a result, the model is a contribution geared for people from the leasing industry interested by an innovative financial product, as well as people from the financial market concerned by leasing risk opportunities.

The present analysis could be extended in various ways. The accuracy of the correlation parameter can be improved by a complete macroeconomic analysis, and the fair market value parameter can also be improved. Finally, other families of copula could be tested.
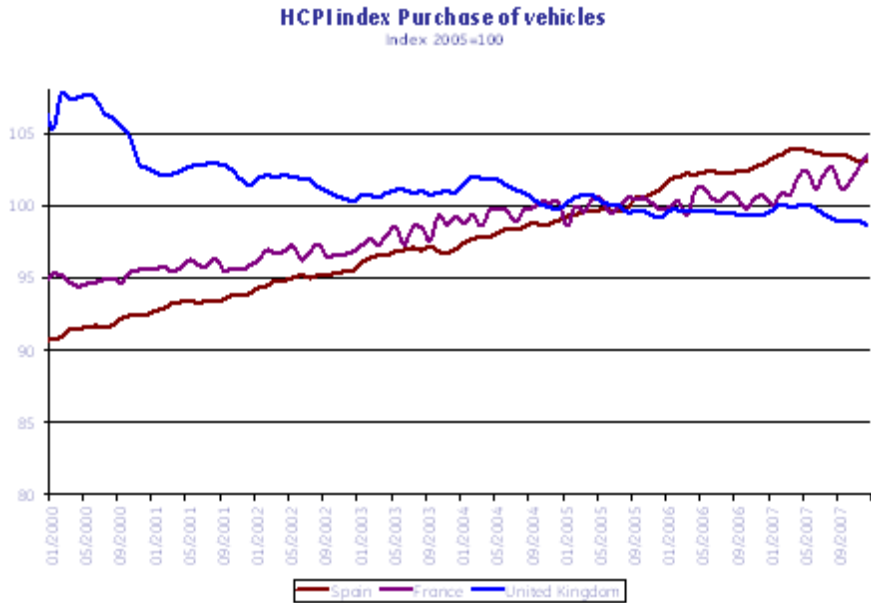
APPENDIX:

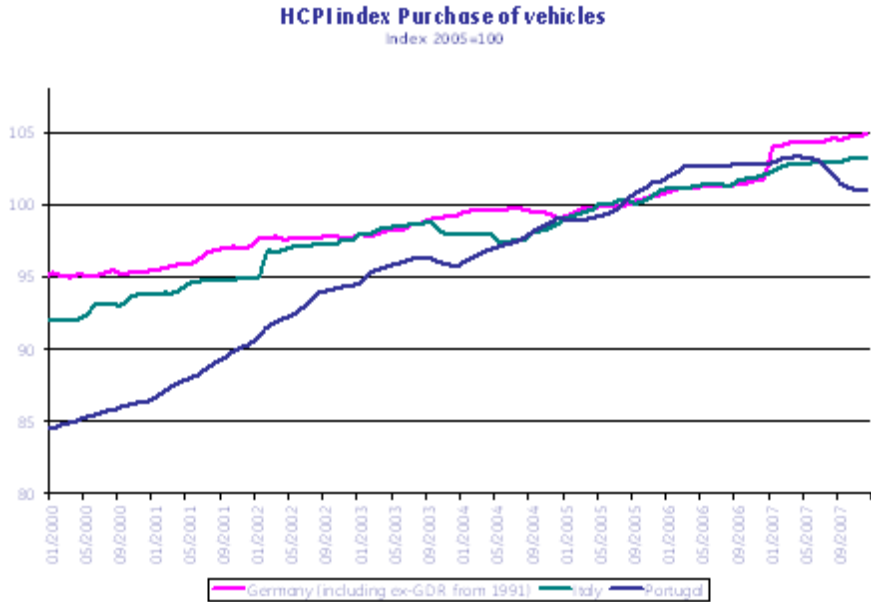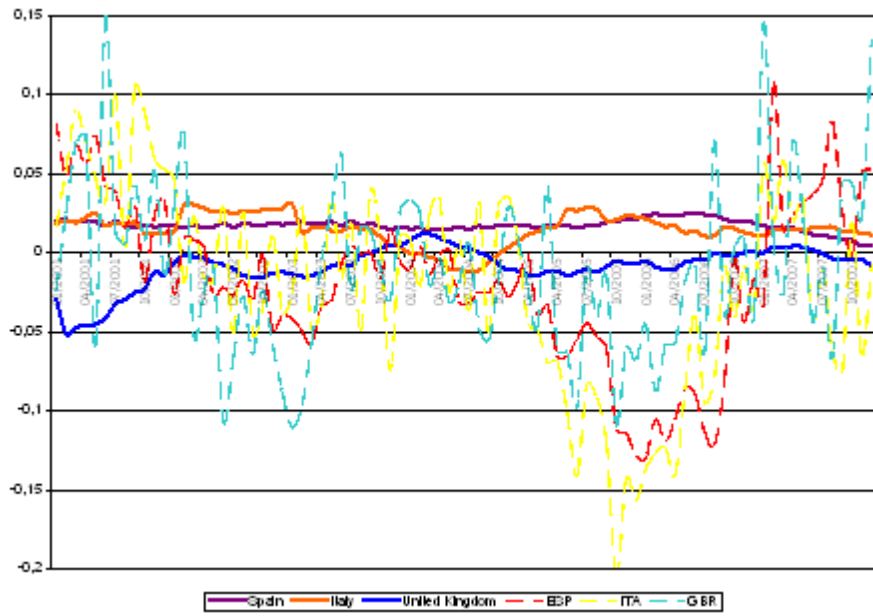**HCPI index Purchase of vehicles**
Index 2005=100

Figure 13

HCPI index Purchase of vehicles
Index 2005=100

Figure 14

Figure 15: HCPI YoY variance
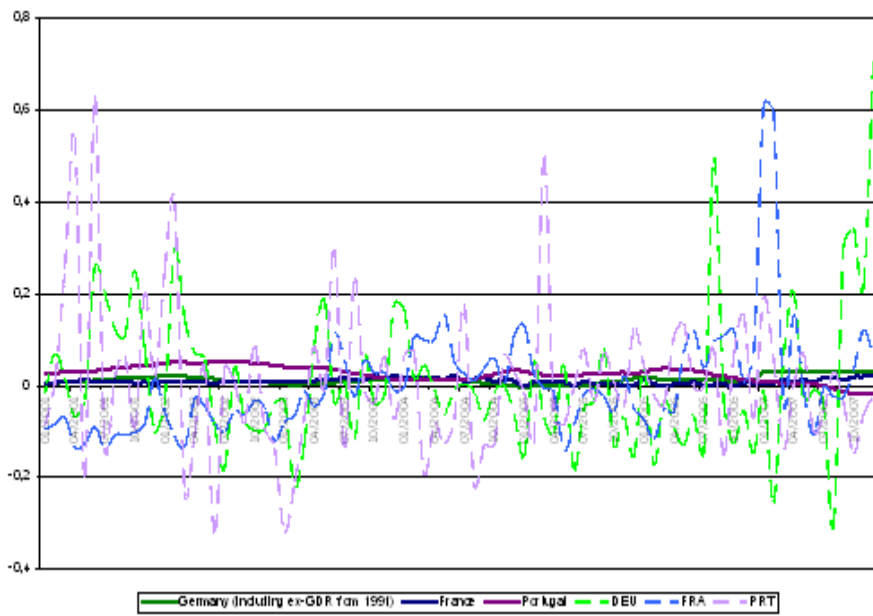
Figure 16: HCPI YoY variance

Figure 17: Age and Mileage buckets

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