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Social Housing Under Oligopoly

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Abstract

In this paper it is shown that the setting up of a social housing system may decrease the total number of houses built in the market, induce a price of non-social houses greater than the price of houses without that system and increase the profits of housing developers even in situations where they have to sell social houses at a price below production cost. The analysis considers a situation with imperfect competition in the housing market and with a social housing system where housing developers must provide some social houses when they obtain a permit to build non-social houses.

JEL classification codes: L 13, L 50, R 31

Keywords: Social housing, imperfect competition, housing stock.

1 Introduction

Social housing intends to provide affordable housing for poor households. The direct and administrative costs of social housing for the government are, however, very high. This work points to a different possibility for social housing, that may be considered when housing developers have market power in the housing market: a social housing system where housing developers must provide some social houses when they obtain a permit to build non-social houses. The regulator would determine the number of social houses to build, their price and the maximum income level that a household may have to be eligible for a social house.

In the analysis of housing markets it is often implicitly assumed that there is perfect competition in housing supply. However, this is not correct in most local housing markets. These markets can be approximated by districts or cities which are far enough apart for only local supply and demand conditions to affect prices and output, and the number of housing developers in each market is often limited. This work considers that there is imperfect competition in each housing market and, hence, that housing developers have market power.

It is generally accepted that if housing developers are required to sell social houses at a price below cost, they will have to be compensated. However, in this work it is shown that, under imperfect competition, the setting up of a social housing system, combined with permits to build non-social houses, may increase the expected profits of housing developers, even if they have to sell social houses at a price below production cost. Hence, the design of the social housing system may be such that no direct government funding is required.

There has been a long debate on the effects of social housing on house building.[1] In this paper it is shown that the setting up of a social housing system as the one described above may decrease the total number of houses built in the market. Moreover, even if the total number of houses built increases with the social housing system, the price of a non-social house may be greater than the price of a house when there is not social housing.

For the results it is necessary to assume that the households eligible for a social house are not only for those who cannot afford to buy a house when there is not social housing. In this paper it is considered that the regulator

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[1]See, for instance, Sinai and Waldfogel (2005).

requires the allocation of social houses to households with income below some fixed income level. However, there are more households eligible for a social house than the number of social houses available. Then the available social houses are allocated at random among the eligible households.[2] The regulator could eliminate the excess demand of social houses by reducing the maximum income level that qualifies for a social house. But he prefers to maintain that excess demand. A reason for this preference may be that there is asymmetric information on household income between the regulator and each household, as a consequence of some non uniformly distributed fraud in income disclosure among households.

To obtain the results it suffices to use a one period (static) model with linear demand for housing services and Cournot competition among housing developers. As in a one period model there is no difference between renting and selling, the analysis applies to situations where social houses are rented and also to situations where social houses are sold. The presentation will consider this latter case.[3]

The intuition for the results is the following: When there are not social houses the number of houses built is above the monopoly level. Producers would benefit from a reduction in total production towards the monopoly level. But they cannot collude and agree on that lower quantity (commitment to collude may not be feasible and collusion is not allowed). Social housing may make up for the lack of ability of producers to coordinate on a lower production level in the market of non-social houses. In particular, social housing may induce an increase in the profits of housing developers, even if they must sell social houses at a price below production cost.

The paper is organized as follows: Section 2 presents the basic model. Section 3 analyzes the consequences of the setting up of a social housing system. Finally, section 4 concludes.

2   Basic model

To proceed, consider an oligopolistic housing industry with $n \geq 2$ housing developers that compete *à la Cournot*. Entry into the industry is assumed to

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[2]Different ways to allocate social houses among eligible households have been used in practice. See Olsen (2003) and Sinai and Waldfogel (2005).

[3]The simultaneous consideration of renting and selling as differentiated alternatives requires a dynamic model where houses are a durable good.

be unprofitable. Each housing developer builds houses only on one particular site and all sites for housing development are homogeneous.[4]

The presentation considers that substitution effects between new houses and houses built in the past are negligible and, hence, the analysis centers in the market of new houses. It is assumed that all new houses offered in the market are identical from the consumer's point of vue. The inverse demand function for new houses is: $p = a - bQ$, where Q represents the quantity of new houses in the market. Hence, there are $\frac{a-p}{b}$ households that are willing to pay at least p for a new house. The marginal cost of production of each firm, represented by c with $0 < c < a$, is constant. The demand function for new houses and the cost of production are known by households and housing developers. Finally, housing developers are neutral to risk.

When there is a significant substitution effect between used and new houses the results in the paper still follow. If there exist U used houses and a used house is equivalent to a proportion β of a new house with $0 < \beta \leq 1$ (the value of a used house for a buyer is a proportion β of the value of a new house for that buyer), the analysis below will remain valid using as the inverse housing demand function for new houses

$$a - bQ - b\beta U = \bar{a} - bQ.$$

where $\bar{a} = a - b\beta U$. In this case a used house might be sold at a price equal to the price of the new house multiplied by β.

When there is not a social housing system each active housing developer i, $i = 1, ..., n$, will solve the following problem:

$$\max_{q_i}(a - bQ - c)q_i,$$

where $Q = \sum_{i=1}^{n} q_i$. The first order condition of this problem is:

$$a - bQ - c - bq_i = 0.$$

Adding up the n first order conditions we get:

$$q_i^* = \frac{a - c}{b(n + 1)}.$$

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[4]Assume that the extension of a site is big enough to allow several-story buildings to be built on it. Hence, a site is bigger than a lot, and it may be seen as closer to a subdivision.

Hence,

$$p^* = a - bQ^* = a - \frac{n(a-c)}{n+1} = \frac{a+nc}{n+1}.$$

3 Social housing

Consider now a situation where the regulator establishes that housing developers must sell D social houses ($\frac{D}{n}$ houses each housing developer) at a price equal to r, with $D < \frac{a-r}{b}$. Moreover, each housing developer is free to decide the number of non-social houses to build on his site, besides the $\frac{D}{n}$ social houses required by the regulator. We may consider that there is a zone in each site where social houses are built or assume that all social houses are built in some particular sites and each housing developer has a site for building only non-social houses.

In this work the cost of production of social houses is the same as the cost of production of non-social houses. However, if the quality of social houses were smaller than the quality of non-social houses, the former would be cheaper to produce. It is immediate to prove that the results obtained below will hold if the marginal cost of each social house is smaller than c.

The regulator requires the allocation of social houses to households with incomes below some fixed income level. Assuming that the valuation of housing services increases with income, consider that the limit in the income level established to be eligible for a social house implies that social houses are assigned to households with willingness to pay for the services of a house smaller than m, with $r < m < a$.[5] Assume also that it is $r < p^* = \frac{a+nc}{n+1} < m$. This implies that some households that would buy a house without social housing are eligible for social houses.

Consider, finally, that there are more than D households eligible for a social house ($\frac{m-r}{b} > D$).[6] The regulator prefers not to eliminate the excess demand of social houses by reducing the maximum income level that qualifies for a social house. The reason may be that there is asymmetric information on household income between the regulator and each household,

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[5]Hence, the social housing policy considered in this work is a policy of the kind considered in Nichols and Zeckhauser (1982): a targeting policy. However, sometimes the social housing policy is more a policy of the type suggested by Akerlof (1978): a tagging policy, that classifies households according to characteristics over which they have no control (disability or age, for instance).

[6]Households with willingness to pay for a house smaller than r will not ask for a social house.

as a consequence of a non uniformly distributed fraud in income disclosure among households. The available social houses are then allocated at random among the eligible households. The probability of obtaining a social house for an eligible household is equal to $\frac{bD}{m-r}$. As expected, this probability decreases with m and increases with D and with r.

Let us denote by Q^n the quantity of non-social houses built. Note that $Q^n < \frac{a-r}{b} - D$ is required to have non-social houses that are more expensive than social houses. If $0 < Q^n \leq \frac{a-m}{b}$ the price of non-social houses will be $a - bQ^n$ (the same as without social housing). The reason is that, in this case, there are not households eligible for a social house who are willing to pay the price of non-social houses $(a - bQ^n \geq a - b\frac{a-m}{b} = m)$.

Let us consider now that $\frac{a-m}{b} < Q^n < \frac{a-r}{b} - D$. In this case the price of non-social houses will be, in general, smaller than $a - bQ^n$. The reason is that the households with willingness to pay for a house between $a - bQ^n$ and m are eligible for a social house and some of them may obtain a social house. The price of non-social houses will, thus, depend on the result of the allocation process of social houses. As these houses are allocated at random among the eligible households the price of non-social houses for Q^n between $\frac{a-m}{b}$ and $\frac{a-r}{b} - D$ might take many different values. Hence, that price will be a random variable: $\tilde{p}^n$.[7] However, as housing developers are neutral to risk they will take into account only the expected price of non-social houses, $E(\tilde{p}^n)$.

The expected price of non-social houses when $\frac{a-m}{b} < Q^n < \frac{a-r}{b} - D$ is given by the price of non-social houses that is obtained when social houses happen to be distributed among the eligible households in a particular way. This particular distribution corresponds to the case where social houses are distributed among the eligible households with different willingness to pay for a house in proportion to their relative numbers. That is, for any x and y such that $m > y > x \geq r$, the proportion of households with willingness to pay for a house between x and y that obtain a social house is equal to the probability that any eligible household has of obtaining a social house $(\frac{bD}{m-r})$.

With that particular distribution of social houses among the eligible

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[7]If the eligible households with higher willingness to pay are the ones that obtain the social houses, the price of non-social houses required to sell the Q^n non-social houses will have to be smaller than for other distributions of social houses among the eligible households. The contrary will occur if the eligible households with lower willingness to pay are the ones that obtain the social houses.

households, the number of non-social houses sold at a price p of non-social houses such that $m > p > r$ will be $\frac{a-p}{b} - \frac{m-p}{b}(\frac{bD}{m-r})$.[8] Hence, if Q^n non-social houses are built, with $\frac{a-m}{b} < Q^n < \frac{a-r}{b} - D$, the expected price of non-social houses, $E(\tilde{p}^n(Q^n))$, will be such that

$$Q^n = \frac{a - E(\tilde{p}^n(Q^n))}{b} - \frac{m - E(\tilde{p}^n(Q^n))}{b}(\frac{bD}{m-r}),$$

that is,

$$E(\tilde{p}^n(Q^n)) = \frac{a(m-r) - mbD}{m-r-bD} - \frac{b(m-r)}{m-r-bD}Q^n.$$

Therefore, the expected inverse demand function for non-social houses when a social housing system has been established is

$$E(\tilde{p}^n(Q^n)) = \begin{cases} \frac{a(m-r)-mbD}{m-r-bD} - \frac{b(m-r)}{m-r-bD}Q^n & \text{for } \frac{a-m}{b} \le Q^n < \frac{a-r}{b} - D, \\ a - bQ^n & \text{for } Q^n \le \frac{a-m}{b}. \end{cases}$$

As $r < p^* = \frac{a+nc}{n+1} < m$, we have from the analysis in section 2 that the equilibrium price of non-social houses, when there is social housing, will be at most m. The equilibrium in the market of non-social houses may correspond to a corner solution, with the expected price of non-social houses equal to m, or to an interior solution, with the expected price of non-social houses in the interval (r, m). To obtain this interior solution we proceed as in the previous section using $\frac{a(m-r)-mbD}{m-r-bD}$ instead of a, and $\frac{b(m-r)}{m-r-bD}$ instead of b, and assuming that the situation is such that the n housing developers remain active.[9] Then, we get[10]

$$q_i^{n*} = \left[\frac{a-c}{b} - \frac{(m-c)D}{m-r}\right]\frac{1}{n+1} < q_i^* \tag{1}$$

and the expected price of non-social houses will be[11]

$$E(\tilde{p}^n)^* = \left[a + nc + \frac{(a-m)bD}{m-r-bD}\right]\frac{1}{n+1} > p^*. \tag{2}$$

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[8]Note that there are $\frac{a-p}{b} - \frac{a-m}{b} = \frac{m-p}{b}$ households that are willing to pay between p and m for a new house.

[9]We have to add the term $(r-c)\frac{D}{n}$ to the total profits of each firm, but the (interior) solution in the market of non-social houses does not depend on this additional term, as long as the n firms remain active.

[10]Note that $r+bD < m < a$ implies $(a-c)(m-r) - (m-c)bD \ge (a-c)(m-r-bD) > 0$ and $q_i^{n*} > 0$.

[11]Note that the price of non-social houses for a particular allocation at random of social houses may be below p^*. Consider, for instance, that the eligible households with greater willingness to pay for a house are the ones that obtain a social house. Then if m is well above p^*, the price of non-social houses might be $\frac{a-bD+nc}{n+1} < p^*$.

The interior solution given by (1) and (2) requires $E(\tilde{p}^n)^* \leq m$. If $\left[a + nc + \frac{(a-m)bD}{m-r-bD}\right]\frac{1}{n+1} > m$ the corner solution $p^{nc} = m$ and $q_i^{nc} = \frac{a-m}{bn}$ will be obtained. From (1) we have that the quantity of non social houses increases with m and decreases with D and with r. From (2) we have that the expected price of non social houses decreases with m and increases with D and with r.

Moreover, from (1) we obtain

$$Q^{n*} + D = \left[\frac{a-c}{b} - \frac{(m-c)D}{m-r}\right]\frac{n}{n+1} + D = \frac{(a-c)n}{b(n+1)} + \frac{m+nc-(n+1)r}{(m-r)(n+1)}D.$$

Hence, it cannot be guaranteed that the setting up of a social housing system as the one considered in this paper will increase the total number of houses built, even considering a linear demand function for houses (the number of houses built will increase in that case if and only if $m + nc > (n+1)r$). We have:

Proposition 1 *The setting up of a social housing system may reduce the total number of houses built in the market.*

The variation in expected profits of housing developers with the setting up of the social housing system is

$$E(\pi^n)^* - \pi^* = ((a + nc + \frac{(a-m)bD}{m-r-bD})\frac{1}{n+1} - c)(\frac{a-c}{b} - \frac{(m-c)D}{m-r})\frac{n}{n+1}$$

$$+ (r-c)D - (\frac{a+nc}{n+1} - c)\frac{(a-c)n}{b(n+1)}$$

$$= nD\frac{a(m-r)(a-2m) + c(m-r-bD)(2m-c) + bDm^2}{(m-r-bD)(m-r)(n+1)^2} + (r-c)D. \quad (3)$$

Therefore, the setting up of a social housing system may increase the expected profits of housing developers (for instance, when $r \geq c$ and $a \geq 2m$ it is $\hat{\pi}^* - \pi^* > 0$). Moreover, from (3) we have that the expected profits of housing developers may increase even if $r < c$, that is, even in situations where they have to sell social houses at a price below production cost. A social housing system will be neutral with respect to the profits of housing developers if the expression in (3) is equal to 0. We conclude:

Proposition 2 *The setting up of a social housing system may increase the expected profits of housing developers, even if social houses have to be sold at a price below production cost.*

The intuition behind Propositions 1 and 2 is the following: When there are not social houses the number of houses built is above the monopoly level as there is Cournot competition among housing developers. Producers would benefit from a reduction in total production towards the monopoly level. But they cannot collude and agree on that lower quantity. With the setting up of a social housing system the expected profits of housing developers may increase. This increase in profits of housing developers may occur even if they must sell social houses at a price below production cost and although, at the relevant prices for interior solutions, the expected demand curve for non-social houses is on the left of the demand curve for houses without social housing. Hence, social housing may make up for the lack of ability of producers to coordinate on a lower production level in the market of non-social houses and, thus, housing developers may favor the setting up of a social housing system of the type discussed in this work.

4 Conclusion

This work has considered a housing market where there is imperfect competition in the supply of houses and a social housing system where housing developers must provide some social houses when they obtain a permit to build non-social houses. The regulator determines the number of social houses to build, their price and the maximum income level that a household may have to be eligible for a social house. The number of housing developers is given to the regulator.

The results obtained explain why these social housing systems must be designed carefully in this context. First, the total number of houses built may decrease with the social housing system. Even if the total number of houses built increases with that system, the price of a non-social house may be greater than the price of a house without social housing. A price of a non-social house greater than the price of a house without social housing implies that some eligible households that do not obtain a social house, and were able to buy a house when there was not social housing, will not be able to afford the purchase of a non-social house when the social housing system is established.

The setting up of a social housing system may also increase the expected profits of housing developers and this may occur even if social houses are

sold at a price below production cost. In this latter case, there would be a cross subsidy from the buyers of non-social houses to households that obtain a social house.

In this context housing developers may try to convince the regulator to design a social housing system profitable for them.[12] As imperfect competition allows for designs of the social housing system that are profitable for housing developers, no direct government funding is required.[13] However, in that case the buyers of non-social houses would pay for the increase in profits of housing developers.

If the regulator has enough information, he may design a social housing system neutral with respect to the profits of housing developers. The design of the social housing system may even be used as a peculiar competition policy. As without that system housing developers have positive profits, there may be social housing systems of the type considered in this paper that may serve to reduce the profits of housing developers. The reason is that social houses are an alternative to non-social houses for some (or many) eligible households that could afford to buy a non-social house and, thus, the situation becomes analogous to one where housing developers face more competition in the housing market. There may even exist social housing systems that limit the profits of housing developers and assign social houses in a way that implies a price of non-social houses smaller than the price of houses without social housing.

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[12]This possibility depends on the ability of housing developers to capture the regulator of the social housing system. See Laffont and Tirole (1993) for a general analysis of the problem of capture of the regulator by economic agents.

[13]Of course, some government expenses will be required to control the correct implementation of the social housing system.

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