MARKOV SWITCHING RISK PREMIUM AND THE TERM STRUCTURE OF INTEREST RATES. Empirical evidence from US post-war interest rates*

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Abstract

This paper considers the basic present value model of interest rates under rational expectations with two additional features. First, following McCallum (1994), the model assumes a policy reaction function where changes in the short-term interest rate are determined by the long-short spread. Second, the short-term interest rate and the risk premium processes are characterized by a Markov regime-switching model. Using US post-war interest rate data, this paper finds evidence that a two-regime switching model fits the data better than the basic model. The estimation results also show the presence of two alternative states displaying quite different features.

Key words: term-structure, risk premium, Markov regime-switching
JEL classification numbers: E43

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1 INTRODUCTION

Blinder (1997) argues that the term structure model is a key element for macroeconomic policy in order to bridge the gap between the nominal short-term interest rate set by monetary policy and the real long-term rates that presumably in‡uence aggregate demand. The expectations theory of the term structure of interest rates postulates that a nominal long-term interest rate is the present value of current and expected future nominal short-term interest rates plus a term premium. There is a great deal of literature showing evidence that the data reject the basic rational expectations term structure model.[1] The reason for this failure is basically that the basic term structure model implies a much smoother long-term interest rate than the one observed.

The aim of this paper is to investigate the role of two (possibly complementary) reasons to explain the failure of the basic model to account for long-term interest rate dynamics. We consider the basic term structure model under rational expectations with two additional features. First, following McCallum (1994), the model assumes a policy reaction function where changes in the short-term interest rate are determined by the long-short spread. Second, the short-term interest rate and the risk premium processes are characterized by a Markov regime-switching model.[2]

As pointed out by Shiller (1979), the term premium is usually described as re‡ecting public attitudes toward and perceptions of risk and those are usually viewed as slow moving. Moreover, we argue that any short-term rate process assumed in empirical studies in order to test the rational expectations hypothesis of the term structure should be viewed as a reduced form that summarizes both behavioral relationships and economic policy rules. In

[1] See, for instance, Shiller (1979), Chow (1989) and Campbell (1995). Recent papers by Hardouvelis (1994), Gerlach and Smets (1997), Hsu and Kugler (1997), and Domínguez and Novales (2000) have found empirical evidence in favor of the rational expectations hypothesis of the term structure using international data. However, the …rst two papers also found empirical evidence that the rational expectations hypothesis of the term structure does not …t well U.S. interest rate data.

[2] This strategy was also followed by Hamilton (1988), although this paper di¤ers in many aspects from his paper. We highlight the following aspects. First, Hamilton considers that the short-term rate is exogenous. Second, Hamilton's paper assumes a constant term premium. Third, the characterization of the alternative regimes is di¤erent. In Hamilton's paper, the constant term around which the process is de…ned and the standard deviation of process innovations are functions of the regime. Given the features of our model, the parameters of the short-term rate process (including the policy reaction parameter) and the standard deviation of the innovations of the short-term rate process are modeled as regime dependent. Fourth, Hamilton (1988) uses quarterly yields on 3-month Treasury bills and 10-year Treasury bonds from 1962 to 1987. We use monthly yields data on di¤erent terms covering the post-war period (from 1950 to 1992).

particular, the short-term rate may react differently to the spread depending on how tight monetary policy is. Therefore, the parameters characterizing the reduced form of the short-term rate are likely to vary over time. These considerations suggest a natural extension of the empirical analysis of the expectational theory of the term structure of interest rates by taking into account the possibility of regime switches in the processes characterizing the short-term interest rate and the risk premium.

Using US post-war interest rate data, this paper finds evidence that a two-regime switching model fits the data better than the basic model. The estimation results show the presence of two quite different regimes.

State 1 mainly characterizes term structure of interest rates in the fifties and the sixties and this coincides with office term of Fed's chairman Martin (1951:4-1970:1). The seventies that cover the terms of office of Fed's chairmen Burns (1970:2-1978:1) and Miller (1978:3-1978:8), term structure of interest rates is characterized by a combination of the two states with state 2 being the dominant state. During the office term of Fed's chairman Volcker (1979:10-1987:8) the term structure of interest rates is determined by state 2. Finally, the first part of Greenspan's office term (1987:8-1992:7), the term structure is mainly characterized by state 2.

The rest of the paper is organized as follows. Section 2 introduces the present value model of interest rates under rational expectations which allows for a Markov regime-switching in the risk premium and the short-term rate processes. Section 3 presents and discusses the empirical evidence. Finally, Section 4 shows the conclusions.

2 THE PRESENT VALUE MODEL OF INTEREST RATES

As shown by Shiller's (1979) seminal paper, the rational expectations theory of the term structure of interest rates postulates the following relation between a long-term rate and a short-term rate

$$R_t = (1 - \delta) \sum_{i=0}^{\infty} \delta^i E_t r_{t+i} + c_{s_t}; \qquad (1)$$

where R_t denotes a long-term rate at time t, r_t is a short-term rate at time t, E_t denotes the conditional expectation operator given the information set, I_t, available to the economic agents at the beginning of time t. I_t includes current and past values of all random variables included in the model. δ denotes the discount factor and c_{s_t} is the risk premium and it is usually assumed constant. In this paper, we assume that the risk premium follows a first-order two-state Markov process with $p(s_t = 1/s_{t-1} = 1) = p$ and $p(s_t = 2/s_{t-1} = 2) = q$. The important point is that the inclusion of a time-varying risk premium in (1) keeps the essence of the expectations theory of the term structure, that is, the long-term rate differs from a weighted average sum of expected future short-term rates only randomly.

We further assume that the short-term interest rate r_t is characterized by the following process

$$r_t - r_{t-1} = \rho_{0s_t} + \rho_{1s_t}(R_{t-1} - r_{t-1}) + v_t, \qquad (2)$$

where $\rho_1^{s_t}$ is a positive policy reaction parameter reflecting how changes in the short-term interest rate try to narrow the long-short spread. v_t is an i.i.d. random variable with mean zero and variance $\sigma^2_{vs_t}$. As the risk premium, parameters ρ_{0s_t}, ρ_{1s_t} and σ_{vs_t} are assumed to follow a two-state Markov process. v_t is included in I_t since r_t and s_t are also included.

Taking into account equation (1) to evaluate $E_t R_{t+1}$ and subtracting $\delta E_t R_{t+1}$ from (1) we obtain

$$R_t = (1 - \delta) r_t + \delta E_t R_{t+1} + c_{s_t} - \delta E_t c_{s_{t+1}}. \qquad (3)$$

Equations (2) and (3) form a bivariate system of difference equations. Using the undetermined coefficient method we begin by writing R_t as a linear function of a minimal set of state variables: r_t, and a constant that is state dependant,

$$R_t = \pi_{0s_t} + \pi_{1s_t} r_t. \qquad (4)$$

In this paper, we focus our attention on the unique fundamental solution satisfying McCallum's (1983) criterion. This solution is given by[3] (see mathematical workings at the end of the manuscript)

$$\begin{aligned} \pi_{11} &= \pi_{12} = 1, \\ \pi_{01} &= \frac{A_2(c_1 + \delta B) + \delta(1 - p)(c_2 + \delta D)}{A_1 A_2 - \delta^2(1 - p)(1 - q)}, \\ \pi_{02} &= \frac{c_2 + \delta[(1 - q)\pi_{01} + D]}{A_2}, \end{aligned} \qquad (5)$$

[3] McCallum (1983) suggest the minimum state variable criterion to single out a unique rational expectations equilibrium solution in a context of multiple equilibria with the additional requirement that the solution must be valid for any admissible parameter value of the forcing variable process. In particular, it can be shown that the equilibrium solution analyzed in this paper is the only solution that remains valid for any admissible parameter value of the forcing variable process when there is a single state (that is, if $p = 1$).

where

$$\begin{aligned} A_1 &= 1 - \delta[p(1+\rho_{11}) + (1-p)\rho_{12}]; \\ A_2 &= 1 - \delta[q(1+\rho_{12}) + (1-q)\rho_{11}]; \\ B &= p(\rho_{01} - c_1) + (1-p)(\rho_{02} - c_2); \\ D &= q(\rho_{02} - c_2) + (1-q)(\rho_{01} - c_1). \end{aligned}$$

We then estimate the following bivariate system:

$$\begin{aligned} R_t - r_t &= \pi_{0s_t} + \theta_{s_t} u_t; \\ r_t - r_{t-1} &= \rho_{0s_t} + \rho_{1s_t}(R_{t-1} - r_{t-1}) + v_t; \end{aligned} \qquad (6)$$

where u_t is an i.i.d. standard normal variable and θ_1 and θ_2 are positive constants.[4]

3 EMPIRICAL EVIDENCE

We estimate our model using two monthly U.S. Treasury yield series (1-month U.S. Treasury bill rate and the U.S. Treasury 20-year yields) available from 1950 to 1992 from Salomon Brothers' Analytical Record of Yields and Yield Spreads (1992).[5]

3.1 Estimation results for the basic model

We start by estimating the basic (one-state) term structure model of interest rates. In this case, the bivariate system is

$$\begin{aligned} R_t - r_t &= \pi_0 + \theta u_t; \\ r_t - r_{t-1} &= \rho_0 + \rho_1(R_{t-1} - r_{t-1}) + v_t; \end{aligned} \qquad (7)$$

[4] As Driffill and Sola (1998) in a related context, we have augmented the model with a random disturbance, u_t, which may be interpreted as a measurement error.

[5] The 1-month Treasury bill rates are shown on a discount basis whereas the Treasury 20-year yields are shown on a bond yield basis. In order to get the appropriate bond yield associated with the 1-month Treasury bill rate we use the Conversion Table for issues Quoted on a Discount Basis, displayed in Salomon Brothers' Analytical Record of Yields and Yield Spreads. Thus, by adding the appropriate percentage shown in the Conversion Table to the discount yield, we obtain the 1-month Treasury bill rate on a bond yield basis.

where $\pi_0 = [\delta\rho_0 + c(1 - \delta)]/[1 - \delta(1 + \rho_1)]$. Table 1 shows the estimation results of the basic model. The estimated value of the policy reaction parameter ρ_1 is rather small but statistically significant at standard critical values. The estimated values of ρ_0 and the term premium parameter, c, are not statistically significant. Moreover, the standard deviation of the measurement error term, θ, is twice larger than the standard deviation of the innovation entering the short-term rate process, σ_v.

Table 1. Estimation results of the basic model

	Estimated Value	Standard Error
ρ_0	-0.1167	0.0735
ρ_1	0.0857	0.0405
σ_v	0.6988	0.0563
δ	0.9969	0.0161
c	-0.2340	0.8796
θ	1.3458	0.0457
Mean log − likelihood	-2.77105	
AIC	2838.471	
SIC	2901.284	
HQ	2848.432	
T	510	

Notes for Tables 1-2: Heteroskedastic-consistent stadard errors are shown. The Akaike, Schwarz and Hannan-Quinn model selection criteria are computed as $AIC = -2L + 2n$, $SIC = -2L + 2n\ln(T)$ and $HQ = -2L + 2n\ln(\ln(T))$, respectively, where L is the maximum value of the Gaussian log-likelihood function, n is the number of estimated parameters and T is the number of observations.

3.2 Estimation results for the Markov regime-switching model

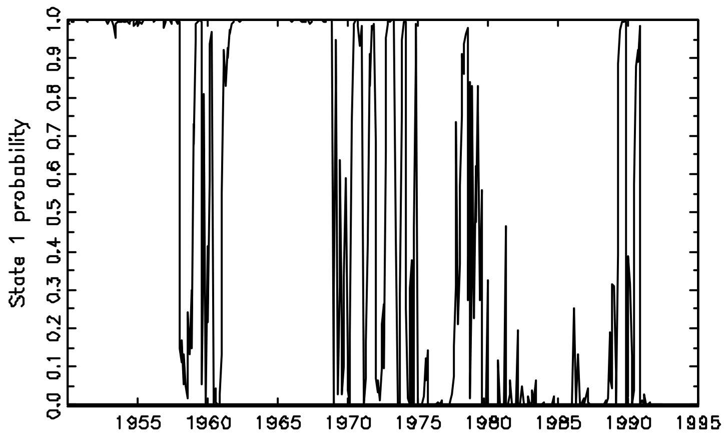
The estimation of the regime-switching model follows the procedures suggested by Hamilton (1994, ch. 22). Table 2 shows the estimation results for the bivariate Markov regime-switching model given by the system (6). The estimation results show the presence of two alternative states with quite different features. The different features between the two states can be summarized as follows. First, a positive policy reaction parameter that is statistically significant at any standard critical value characterized state 1 whereas this policy reaction parameter is not significant in state 2. Second, the risk premium is not significantly different from zero in state 1 whereas it is positive and significant in state 2. Third, state 1 is more persistent that state 2, that is, p is statistically larger than q. Fourth, the variance of the innovation entering the short-term process is four times smaller in state 1 than in state 2. Finally, the standard deviation of the measurement error is much larger in state 2 than in state 1.

Figure 1 shows the allocation of time periods to the two states. The fifties and sixties are allocated mostly to state 1, with brief departures in the late fifties. The first half of the seventies are characterized by frequents jumps from one state to another and the second half is attributed with high probability to state 2 with a brief departure around 1978. The eighties and the early nineties are also allocated with high probability to state 2, with brief departures around 1990. Figure 2 provides a clearer picture of these results by plotting the most likely state period by period.

Table 2. Estimation results of the bivariate Markov regime-switching model

	Estimated Value	Standard Error
ρ_{01}	-0.0847	0.0336
ρ_{02}	-0.2672	0.2085
ρ_{11}	0.1208	0.0440
ρ_{12}	0.1169	0.0736
σ_{v1}	0.2516	0.0165
σ_{v2}	1.0191	0.1065
p	0.9931	0.0033
q	0.9429	0.0132
δ	0.9824	0.0162
c_1	-0.1068	0.4005
c_2	1.4665	0.7062
μ_1	0.6278	0.0508
μ_2	1.6623	0.1369
Mean log-likelihood	-2.10993	
AIC	2178.129	
SIC	2314.223	
HQ	2199.711	
T	510	

Figure 1. U.S. INTEREST RATES (PVM – 2 STATES)



4 CONCLUSIONS

This paper considers the basic term structure model under rational expectations with two additional features. First, following McCallum (1994), the model assumes a policy reaction function where changes in the short-term interest rate are determined by the long-short spread. Second, the short-term interest rate and the risk premium processes are characterized by a Markov regime-switching model.

Using US post-war interest rate data, this paper …nds evidence that a two-regime switching model …ts the data better than the basic model. The estimation results show the presence of two alternative states, that we call state 1 and state 2. The estimation results show a connection between the allocation of periods to the two states and the Federal Reserve chairman. Thus, state 1 mainly characterizes the term structure of interest rates during the …fties and the sixties and this coincides with the o¢ce term of Fed's chairman Martin (1951:4-1970:1). The seventies that cover the terms of o¢ce of Burns (1970:2-1978:1) and Miller (1978:3-1979:8), the term structure is characterized by a combination of the two states with state 2 being the dominant state. During the o¢ce term of Fed's chairman Volcker (1979:10-1987:8), the term structure is determined by state 2. Finally, the …rst part of Greenspan's o¢ce term (1987:8-1992:7) the term structure of interest rates is mainly characterized by state 2.

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Equations (2) and (3) form a bivariate system of difference equations. Using the undetermined coefficient method (Muth (1961), McCallum (1983) among others) we begin by writing R_t as a linear function of a minimal set of state variables: r_t and a constant that is state dependant,

$$R_t = \pi_{0s_t} + \pi_{1s_t} r_t. \tag{4}$$

For appropriate real values of π_{0s_t} and π_{1s_t}, the expectational variable $E_t R_{t+1}$ will then be given by

$$\begin{aligned} E_t R_{t+1} &= E_t(\pi_{0s_{t+1}}) + E_t(\pi_{1s_{t+1}} r_{t+1}) = E_t(\pi_{0s_{t+1}}) + E_t(\pi_{1s_{t+1}}\rho_{0s_{t+1}}) + & (8)\\ & E_t[\pi_{1s_{t+1}}\rho_{1s_{t+1}}(\pi_{0s_t} + \pi_{1s_t} r_t)] + E_t[\pi_{1s_{t+1}}(1 - \rho_{1s_{t+1}})] r_t. & (9) \end{aligned}$$

To evaluate the π's, we substitute (2), (4) and (8) into (3), which gives

$$\begin{aligned} \pi_{0s_t} + \pi_{1s_t} r_t = {} & \delta E_t(\pi_{0s_{t+1}} + \pi_{1s_{t+1}}\rho_{0s_{t+1}} + \pi_{1s_{t+1}}\rho_{1s_{t+1}}\pi_{0s_t}) + \\ & \delta E_t\left[\pi_{1s_{t+1}}\left(\pi_{1s_t}\rho_{1s_{t+1}} + 1 - \rho_{1s_{t+1}}\right)\right] r_t + (1 - \delta) r_t + c_{s_t} - \delta E_t c_{s_{t+1}}. \end{aligned}$$

Recalling that s_t belongs to the information set at time t, this equation implies identities in the constant term and r_t for each state $s_t = 1, 2$ as follows:

$$\begin{aligned} \pi_{01} &= \delta[p\pi_{01} + (1 - p)\pi_{02} + p\pi_{11}\rho_{01} + (1 - p)\pi_{12}\rho_{02} + & (10)\\ & (p\pi_{11}\rho_{11} + (1 - p)\pi_{12}\rho_{12})\pi_{01}] + c_1 - \delta(pc_1 + (1 - p)c_2); \\ \pi_{02} &= \delta[q\pi_{02} + (1 - q)\pi_{01} + q\pi_{12}\rho_{02} + (1 - q)\pi_{11}\rho_{01} + \\ & (q\pi_{12}\rho_{12} + (1 - q)\pi_{11}\rho_{11})\pi_{02}] + c_2 - \delta(qc_2 + (1 - q)c_1); \\ \pi_{11} &= \delta[(p\pi_{11}\rho_{11} + (1 - p)\pi_{12}\rho_{12})\pi_{11} + p\pi_{11}(1 - \rho_{11}) + & (11)\\ & (1 - p)\pi_{12}(1 - \rho_{12})] + 1 - \delta; \\ \pi_{12} &= \delta[(q\pi_{12}\rho_{12} + (1 - q)\pi_{11}\rho_{11})\pi_{12} + q\pi_{12}(1 - \rho_{12}) + \\ & (1 - q)\pi_{11}(1 - \rho_{11})] + 1 - \delta; \end{aligned}$$

After some algebra, we can show that there are four solutions to the system of equations (10).