EXPLOSIVE HYPERINFLATION, INFLATION TAX LAFFER CURVE AND MODELING THE USE OF MONEY*

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Abstract

This paper analyzes the existence of an inflation tax Laffer curve (ITLC) in the context of two standard optimizing monetary models: a cash-in-advance model and a money in the utility function model. Agents' preferences are characterized in the two models by a constant relative risk aversion utility function. Explosive hyperinflation rules out the presence of an ITLC. In the context of a cash-in-advance economy, this paper shows that explosive hyperinflation is feasible and thus an ITLC is ruled out whenever the relative risk aversion parameter is greater than one. In the context of an optimizing model with money in the utility function, this paper firstly shows that an ITLC is ruled out. Moreover, it is shown that explosive hyperinflations are more likely when the transactions role of money is more important. However, hyperinflationary paths are not feasible in this context unless certain restrictions are imposed.

Key words: Inflation tax, Hyperinflation, Laffer curve
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1 INTRODUCTION

Hyperinflation is usually viewed as the result of an inflationary finance policy. The rationale for an inflationary finance policy is that generating inflation through a persistent rise in the money supply can be understood as a means of raising revenues for the government by using an inflation tax.[1] Most of the inflationary finance models developed in the literature (for instance, Evans and Yarrow, 1981; Kiguel, 1989; Bruno, 1989; Bruno and Fischer 1990) are built on Cagan's demand for money (Cagan, 1956). Under perfect foresight those models imply the possibility of dual equilibria and the existence of a (concave) inflation tax Laffer curve (ITLC), which means that inflation does not explode. Based on this traditional approach, a recurrent theme in the large empirical literature on hyperinflation (Cagan, 1956; Sargent, 1977; Taylor, 1991, Phylaktis and Taylor, 1993; Kiguel and Neumeyer, 1995; among others) is the analysis of whether the economy is on the "efficient" side of the ITLC (that is, whether inflation tax is increasing with inflation during a hyperinflationary episode).[2]

This traditional approach has recently been challenged because it fails to explain many stylized facts observed during hyperinflation. For instance, Buiter (1987), Dornbusch, Sturzenegger and Wolf (1990), Bernholz and Gersbach (1992) and Vázquez (1998) emphasize that hyperinflation is better characterized by an unstable dynamic process where inflation speeds up and real money balances tend to vanish.

This paper uses an analytical approach to analyze whether explosive hyperinflation is feasible and thus to study whether an ITLC is ruled out. We consider two standard optimizing monetary models: a cash-in-advance model and a money-in-the-utility-function model. The two models represent alternative ways of modeling the role of money as a medium of exchange. In both models the representative agent's preferences are represented by a constant relative risk aversion utility function. Interestingly, one can show that the money-in-the-utility-function model collapses in the cash-in-advance model

[1] It must be clear that an inflationary finance policy might not result in an optimal inflation tax raised by a benevolent social planner. Then, why should a government and monetary authorities adhere to a mechanical, presumably grossly inefficient, inflationary finance policy? There are, at least, two complementary reasons, which are well documented in the inflationary finance literature. First, most hyperinflationary episodes are driven mainly by huge government spending after some very destructive war. Second, printing money is usually the easiest way to finance huge government expenditure when it is difficult to increase taxes due to the lack of political consensus and weak governments.

[2] The term ITLC refers to a concave (hump-shaped) function relating inflation tax to inflation. Alternatively, we can also define the ITLC as a hump-shaped function that relates inflation tax to real money balances.

when money is strictly essential for purchasing goods. We argue that a cash-in-advance model (or alternatively, a money-in-the-utility-function model with money being essential) is a sensible approach to analyzing hyperinflationary scenarios because extreme inflation dramatically decreases credit transactions and in general the use of long-term contracts. This implies that money becomes more essential for purchasing goods during hyperinflation than during stable periods. There are two main well-known reasons to avoid the use of long-term contracts during hyperinflation. First, the rapid depreciation of money during hyperinflation that induces agents to spend the money as soon as they have got it (Casella and Feinstein, 1990, report anecdotal evidence on this issue). Second, the instability involving relative price movements during hyperinflation (see Tang and Wang, 1993 for empirical evidence on this issue) that induces a high uncertainty about the outcomes of long-term contracts.

In the context of a cash-in-advance economy, we show that an ITLC arises if and only if the relative risk aversion parameter is less than one. This means that an ITLC is feasible whenever the relative risk aversion parameter lies outside the usual range assumed, which implies an unrealistic high intertemporal substitution for consumption.[3] Moreover, explosive hyperinflation can only arise in this framework if the relative risk aversion parameter is greater than one. In the context of an optimizing model with money-in-the-utility-function, this paper shows that explosive hyperinflation is more likely when the transactions role of money becomes more important. Moreover, we show that an ITLC is also ruled out in a money in the utility framework unless other restrictions are imposed, for instance that the utility function would be additive separable in consumption and real money balances and that the utility function assumed for real money balances satisfies certain restrictions.

The main contribution of this paper is to provide a characterization of agent's preferences that are compatible with an ITLC and thus with stable hyperinflation or, alternatively, with explosive hyperinflation. Our characterization shows that stable hyperinflation is only feasible if agents' preferences are described by an unrealistic relative risk aversion parameter. However, explosive hyperinflation is feasible if the relative risk aversion parameter lies inside the usual range, that is, if preferences have curvature higher than the logarithmic utility function. This characterization may help to assess the welfare costs associated with a stabilization policy carried out during hyperinflation since the type of preferences assumed must be consistent with the way the researcher views hyperinflation (as a stable process or, alternatively,

[3] Auerbach and Kotlikoff (1987) consider that reasonable values for the relative risk aversion parameter lie in the range 2 to 10.

as an explosive dynamic process).

The optimizing monetary models considered in this paper are characterized by a unique unstable steady state. A skeptical reader may argue that any model of hyperinflation must be also able to explain the case of low stable inflation. As in Obstfeld and Rogoff (1983),[4] we do not share the view that a model of hyperinflation must explain necessarily the case of low inflation since a model of hyperinflation can be extremely stylized reflecting, for instance, that to a great extent the classical dichotomy between real and nominal variables holds in a hyperinflationary scenario. Moreover, money demand is usually assumed to depend only on expected inflation in order to analyze hyperinflationary episodes, and then it does not depend on real income as occurs in low inflation periods. From our point of view, it is clear that in order to characterize low inflation dynamics one needs to specify a more complex money demand than the one assumed in hyperinflationary models. But for the same reason, it is hard to believe that a simple function characterizing money demand during hyperinflationary periods would be appropriate to characterize the dynamics of low and stable inflation. More generally, hyperinflation distort in many ways economies. Some examples are the following. First, money becomes essential for agents engaging in transactions. Thus, as pointed out by Casella and Feinstein (1990), economic exchange patterns during hyperinflation are dramatically different from those observed in low inflation periods. Second, nominal anchors such as price and wages indexation tend to disappear during hyperinflation affecting inflationary dynamics. Third, expectational dynamics can change substantially in the transition from a low inflation scenario to a high inflation scenario. Thus, adaptive learning can be appropriate to modeling expectations when inflation is low. However, when inflation reaches high levels agents most likely respond rapidly to changes in inflation, and instantaneously during hyperinflation since systematic forecast mistakes on inflation result in prohibitive costs. The analysis of expectational dynamics has been already followed in a recent paper by Nicolini and Marcet (2002) aiming to explain recurrent hyperinflations.

We believe that introducing some of these features in a model would be useful to characterize dynamics from a low inflation steady state to a high inflation steady state. However, the aim of this paper is different and rather modest. We try to characterize explosive hyperinflation where an economy is just in a high inflation scenario, where there is perfect foresight,

[4]Obstfeld and Rogoff (1983) characterize explosive hyperinflation in the context of a money-in-the-utility function model where the utility function is additive separable in consumption and real money balances. In their model, there is a unique unstable steady state as in the models studied in this paper.

money demand depends only on inflation and money is essential. The point raised in this paper is that hyperinflationary dynamics derived from standard maximizing models are consistent with a characterization of hyperinflation as an explosive process. However, these models are not consistent with a characterization of the economy being in the inefficient side of the inflation tax Laffer curve as suggested in traditional inflationary finance models.

The rest of the paper is organized as follows. Section 2 characterizes the existence of an ITLC (that implies the existence of an inflation tax maximization rule) in terms of a money demand restriction. Moreover, this section reviews the approach followed in the empirical literature in order to analyze whether the economy is on the "efficient" side of the ITLC where ITLC is increasing (decreasing) with inflation (real money balances). Section 3 studies a cash-in-advance economy. Section 4 studies an optimizing model with money-in-the-utility-function. Section 5 concludes.

2 INFLATION TAX LAFFER CURVE

Inflationary finance models usually assume that a constant per-capita share of government's deficit d is financed by issuing high-powered money. Formally,

$$\frac{\dot{M}(t)}{P(t)} = d,$$

where $M(t)$ is per-capita nominal money supply at time t and $P(t)$ is the price level. As usual a dot on a variable denotes its time derivative.

Using the latter expression, denoting per-capita real money balances by $m(t)$ and omitting the time index for the sake of simplicity, we can write the law of motion for real money balances as follows

$$\dot{m} = d - m\,\pi, \tag{1}$$

where π is the rate of inflation (that is, $\dot{P}/P$). The inflation tax, T^i, is defined as the loss of purchasing power of a given amount of nominal money balances due to the existence of a positive inflation rate. Formally,

$$T^i = m\,\pi. \tag{2}$$

According to (1) real money balances decrease whenever inflation tax is greater than d (that is, the share of government's deficit that is financed by issuing high-powered money). Since d is a constant, we can conclude from (1) and (2) that T^i is strongly concave (convex) if and only if $\dot{m}$ is a strongly convex (concave) function of real money balances.

The traditional approach builds upon Cagan's demand for money

$$ln(m) = \kappa - \mu\pi, \tag{3}$$

where κ is a positive constant and μ is the (positive and constant) semi-elasticity of money demand with respect to the nominal interest rate. Based on (3), Figures 1 and 2 summarize the two well known key aspects of the traditional view on hyperinflationary dynamics. First, dual steady states may co-exist for a given level of government deficit d whenever this level is not too high and κ, μ are determined accordingly. Second, the existence of a high-inflation trap since the high-inflation steady state, m_1, is stable and the low-inflation steady state, m_2, is unstable. Notice that the high-inflation steady state, m_1, is on the "wrong" side of the ITLC, that is, an increase in real money balances raises inflation tax (or alternatively, a decrease in inflation raises inflation tax).

A hump-shaped ITLC is characterized by the existence of a level of real money balances for which the ITLC is maximized. Let us assume that the demand for money is a general decreasing function of the inflation rate, the inverse of money demand function exists, and the money market instantaneously clears. The inverse of money demand function, f, is given by $\pi = f(m)$ with $f'(.) < 0$. The following proposition relates the inflation tax maximization rule to the demand for money.

Proposition 1: *Assume that an ITLC exists (that is, T^i is strongly concave). The inflation tax is maximized when the amount of real money balances is such that the elasticity of the inverse of the demand for money function η_f is one.*

Proof: The inflation tax is given by $mf(m)$. The first-order condition for inflation tax maximization is

$$m^* f'(m^*) + f(m^*) = 0,$$

where m^* denotes the level of real money balances which maximizes the inflation tax. Denoting the elasticity of the inverse of money demand function with respect to real money balances by η_f, the former first-order condition can be written as follows

$$\eta_f = -f'(m^*)\frac{m^*}{f(m^*)} = 1. \tag{4}$$

The second-order condition given by

$$2f'(m^*) + m^* f''(m^*) < 0,$$

establishes the condition for a maximum (that is, the condition characterizing a strongly concave ITLC). The proof is completed.■

Notice that equation (4), in general, characterizes the inflation tax maximization rule because, as we shall show below, an inverse money demand function, such as $f(.)$, typically arises from an agent's optimization problem. Even if $f(.)$ include more arguments than m (for instance, the real interest rate and the level of income), condition (4) still characterizes the inflation tax maximization policy with $f'(.)$ being replaced by $f_m(.)$ (i.e., the partial derivative of f with respect to m).

The empirical literature on hyperinflation analyzes whether huge inflation rates are consistent with the policy to maximize the inflation tax by monetary authorities. Notice that an inflation tax maximization rule only makes sense when the inflation tax function, T^i, has a maximum. By using Proposition 1 (equation (4)), we can easily show the well-known result that when the demand for money is characterized by Cagan's demand for money (3), the rate of inflation that maximizes the inflation tax is given by the inverse of the semi-elasticity of demand for money with respect to the nominal interest rate, $1/\mu$. Thus, the empirical literature on hyperinflation built on Cagan's demand for money usually tests whether inflation dynamics are consistent with an inflation tax maximization rule by, first, averaging the rate of inflation over the sample period and, then, by testing whether or not averaging inflation is significantly different from $1/\hat{\mu}$, where $\hat{\mu}$ is the estimated value of μ.

We think that this way of proceeding has two caveats. First, it relies on an ad-hoc form of characterizing money demand. Second, since the inflation process seems to be nonstationary during hyperinflation, averaging the rate of inflation over the sample period might not be appropriate. The first caveat (arguably) seems to be a mild one since, on the one hand, Cagan's money demand fits data reasonably well in most hyperinflationary episodes studied in the literature. On the other hand, as shown by Calvo and Leiderman (1992), Cagan's demand for money can be derived from first principles by assuming that money enters directly in the utility function and by considering a particular functional form of the utility function for money. We think, however, that the second caveat is more serious because inflation is always rising during hyperinflationary periods. The rise in inflation could be due, firstly, to the fact that inflation really follows an explosive (nonstationary) process or, alternatively, to the fact that the inflation rate is along the convergence path leading to a steady state rate of inflation. In either case, the consideration of the averaging rates of inflation over the sample period is, at least, problematic because, in the former case, the average is specific to

the sample period (that is, the first moment of the inflation process does not exist). In the later case, averaging the rates of inflation over the sample period does not approximate well the steady state of the inflation rate.

Due to these limitations of the empirical evidence found following the traditional approach, we henceforth follow an analytical strategy by studying in the context of two optimizing monetary models whether the huge rates of inflation observed during hyperinflationary episodes are consistent with an ITLC, which implies a stable dynamic process for real money balances and inflation as described by the high-inflation steady state represented in Figures 1 and 2 or, alternatively, those huge rates of inflation are the result of an unstable dynamic process that leads to explosive hyperinflation.

3 A CASH-IN-ADVANCE ECONOMY

The two optimizing monetary models considered in this paper assume a continuous time model where the economy consists of a large number of identical infinitely-lived households. There is no uncertainty. Each household has a non-produced endowment $y(t) > 0$ of the non-storable consumption good per unit of time. In the cash-in-advance model, the representative household's utility at time 0 is

$$\int_0^\infty e^{-rt} U[c(t)]dt, \tag{5}$$

where $c(t)$ is per-capita consumption, $U(.)$ is the increasing and concave utility function and $0 < r < 1$ denotes the constant subjective discount rate which, following Calvo (1987), is assumed to be equal to the real rate of interest. As shown below, this assumption implies that the demand for money function is time-invariant as the demand for money function assumed in the traditional inflationary finance literature.[5] The utility function is further assumed to be of the constant relative risk aversion class,[6] given by

$$U[c(t)] = \frac{[c(t)]^{1-\alpha} - 1}{1-\alpha}, \tag{6}$$

where $0 < \alpha < \infty$ denotes the constant relative risk aversion parameter that measures the curvature of the utility function.

[5] Moreover, empirical papers on hyperinflation aim at uncover a stable (time-invariant) demand for real money balances in such unstable scenarios. By working with a time-invariant demand for real money balances, we aim to convince the reader that our results are not driven by an unstable demand for real money balances.

[6] We focus our attention on this class of utility functions because they play a central role in modern intertemporal macroeconomics.

We define financial wealth and nominal interest rate as

$$w(t) = m(t) + b(t),$$

$$i(t) = r + \pi(t),$$

respectively, where $b(t)$ denotes real per capita government debt. The household's budget constraint is

$$\dot{w} = rw(t) + y(t) - \tau(t) - [c(t) + i(t)m(t)], \tag{7}$$

where τ is a lump-sum tax (government transfer if negative) and the term in brackets denotes full consumption, that is, the sum of consumption and costs of holding money.

In addition, as in the cash-in-advance literature, we assume that money is strictly essential to buy the consumption good. In particular, in order to consume c units of the consumption good at time t, the household must hold a stock of real money balances, m, greater or equal than c. Formally,

$$m(t) \geq c(t). \tag{8}$$

At time 0 the household chooses the paths of c and m in order to maximize (5) subject to (7) and (8). Assuming the existence of an interior solution for c, and that money is return-dominated by the bond (that is, $i(t) = r + \pi(t) > 0$, and this implies that (8) holds with equality), the following first-order condition is obtained from the household's optimization problem:

$$[m(t)]^{-\alpha} = \lambda[1 + i(t)], \tag{9}$$

where λ is the associated Lagrange multiplier, which is time-invariant since the instantaneous rate of discount is assumed to be equal to the real rate of interest. Then, equation (9) characterizes the time-invariant demand for real money balances. The transversality condition implies that

$$\lim_{t\to\infty} e^{-rt}\lambda w(t) = 0. \tag{10}$$

By using the definition of the nominal interest rate, and after some small algebra, equation (9) can be written as follows

$$\pi(t) = \frac{[m(t)]^{-\alpha} - k}{\lambda}, \tag{11}$$

where $k = \lambda(1 + r)$ is a constant. Notice that equation (11) is truly the inverse of a demand for money function, thus relating the rate of inflation

to real money balances. The following proposition characterizes the rate of inflation and real money balances, which are consistent with the inflation tax maximization rule and thus with an ITLC, as a function of the relative risk aversion parameter, α, and the rate of discount, r.

Proposition 2: *Assume a cash-in-advance economy and that the utility function is characterized by a constant relative risk aversion parameter, α. There is a (strongly concave) ITLC if and only if $\alpha < 1$. Moreover, the rate of inflation that maximizes the inflation tax is given by*

$$\pi^* = \frac{(1+r)\alpha}{1-\alpha}.$$

Proof: The first part of the proposition is proven by showing that T^i is a strongly concave function if and only if $\alpha < 1$. After some calculation, we can show that

$$\frac{\partial^2[T^i]}{\partial m^2} = \frac{\alpha(\alpha-1)}{\lambda[m(t)]^{1+\alpha}},$$

which implies that there is an ITLC (that is, $\frac{\partial^2[T^i]}{\partial m^2} < 0$) if and only if $\alpha < 1$.

By using equation (11), the inflation tax maximization condition, equation (4), can be written as follows

$$1 = \eta_f = \alpha(m^*)^{-(1+\alpha)}\frac{m^*}{(m^*)^{-\alpha} - k}, \tag{12}$$

which implies

$$(m^*)^{-\alpha} = \frac{k}{1-\alpha}. \tag{13}$$

Substituting equation (13) in equation (11), we have that

$$\pi^* = \frac{(1+r)\alpha}{1-\alpha}.$$

Thus, the proof is completed.■

Proposition 2 establishes that huge inflation rates under the inflation tax maximization rule occur when α is smaller than (but close to) one. Furthermore, when α is equal to one (that is, the utility function is logarithmic), there is no (finite) inflation rate maximizing the inflation tax.

As shown in Section 1, a strongly convex $\dot{m}$ (or alternatively, a strongly concave T^i) is consistent with the traditional view of describing hyperinflation as a stable dynamic process. Therefore, Proposition 2 shows that in the

context of a cash-in-advance economy the condition $\alpha < 1$ is a necessary and sufficient condition to characterize hyperinflation as a stable dynamic process.

We now turn on to describe hyperinflation as an unstable dynamic process. In particular, the following proposition shows that if $\alpha > 1$ hyperinflation can be characterized as an explosive dynamic process.

Proposition 3: *Assume a cash-in-advance economy and that the utility function is characterized by a constant relative risk aversion parameter, α. The law of motion of real money balances is a strongly concave function (i.e. $\frac{\partial^2 \dot{m}}{\partial m^2} < 0$) and $\lim_{m \longrightarrow 0} \dot{m} = -\infty$ if and only if $\alpha > 1$.*

Proof: The first part of Proposition 3 is a direct corollary of Proposition 2 since T^i is strongly concave (convex) if and only if $\dot{m}$ is strongly convex (concave). The second part is proven by taking into account (1) and then showing that

$$\lim_{m \longrightarrow 0} \dot{m} = \lim_{m \longrightarrow 0} (d - \frac{1}{\lambda m^{\alpha - 1}}) = \begin{cases} -\infty & if \quad \alpha > 1, \\ d - \lambda^{-1} & if \quad \alpha = 1, \\ d & otherwise. \end{cases}$$

Thus, the proof is completed.■

Figure 3 describes hyperinflation as an unstable dynamic process. Notice that to the left of the steady state m^*, the economy will be moving along an unstable hyperinflationary path where real money balances tend to vanish and inflation explodes. Along these hyperinflationary paths the transversality condition (10) always holds since real money balances tend to vanish and financial wealth $w(t)$ tends to be only determined by $b(t)$, and it is reasonable to assume that $b(t)$ has an upper bound. Moreover, notice that the hyperinflationary paths generated by a cash-in-advance economy do not cross the vertical axis. The rationale is that a cash-in-advance constraint means that money is essential for purchasing and consuming goods. Therefore, if money is essential the marginal utility of consumption goes to infinity as real money balances goes to zero since the Inada condition $U'(0) = \infty$ is satisfied by (6). The fact that the marginal utility of consumption goes to infinity implies that inflation tax T^i goes to infinite (and $\dot{m}$ goes to minus infinity) whenever $\alpha > 1$ since inflation (that is an increasing function of the marginal utility of consumption) in this case grows more rapidly than real money balances decreases when the later goes to zero.

In sum, we have shown in this section that the huge rates of inflation observed during hyperinflation are consistent with explosive inflation in a

cash-in-advance economy if and only if the relative risk aversion parameter is greater than one. However, an ITLC in a cash-in-advance economy is not consistent with huge inflation rates unless an unrealistic high intertemporal substitution for consumption (that is, $\alpha < 1$) is assumed.

Next section shows that money-in-the-utility-function models neither generate feasible hyperinflationary paths nor an ITLC arises unless some particular restrictions are imposed.

4 MONEY-IN-THE-UTILITY- FUNCTION

The model considered in this section only differs from the model studied in the previous section in three aspects. First, we assume that the representative household's utility at time 0 is

$$\int_0^\infty e^{-rt} U[c(t), m(t)]dt. \tag{14}$$

Following Bental and Eckstein (1997) we assume a constant relative risk aversion utility function. In particular, we assume that

$$U[c(t), m(t)] = \frac{\{[c(t)]^{1-\omega}[m(t)]^{\omega}\}^{1-\alpha} - 1}{1-\alpha}, \tag{15}$$

where ω is a parameter measuring the transaction requirement of money. A value of ω close to zero implies that money is not essential in buying consumption. The higher ω is, the more important the transactions role of money becomes. Second, it is assumed that there is a lower bound for real money balances, $\widehat{m} \geq 0$. Formally,

$$m(t) \geq \widehat{m}, \tag{16}$$

for all t. As discussed below, the restriction (16) guarantees the existence of feasible hyperinflationary paths. Finally, there is no a cash-in-advance constraint as (8).

This is an optimization problem with a bounded control. The solution to this problem is obtained (see Kamien and Schwartz, 1991) by setting the Lagrangian which is built by appending the constraint (16) to the corresponding Hamiltonian. Let us denote by $\widehat{t}$ the period where real money balances reaches $\widehat{m}$. The first-order conditions obtained from the household's optimization problem (that is, maximize (14) subject to (7) and (16)) imply that for any $t < \widehat{t}$ the marginal rate of substitution between real money balances and consumption is equal to the nominal interest rate. Assuming that $U[c(t), m(t)]$ is given by (15), this condition can be written as follows

$$\frac{\omega c(t)}{(1-\omega)m(t)} = i(t),$$

or, alternatively,

$$\pi(t) = \frac{\omega c(t)}{(1-\omega)m(t)} - r. \tag{17}$$

Notice that, as in the cash-in-advance model, we also assume that the discount rate is equal to the real rate of interest. Taking into account (17), the law of motion for real money balances is

$$\dot{m} = d - \frac{\omega c(t)}{(1-\omega)} + rm(t).$$

This expression shows that the law of motion for real money balances is a linear and increasing function on m. Moreover, there is only a steady state whenever $d \leq \frac{\omega c(t)}{(1-\omega)}$ for all t. This unique steady state is unstable. Notice that the closer is ω to one (that is, the more important the transactions role of money is) the more likely the existence of unstable hyperinflationary path is. Figure 4 displays the real money balances dynamics in this framework. Notice that to the left of the steady state m^*, the economy will be moving along an unstable hyperinflationary path where real money balances go to $\widehat{m}$ and inflation speeds up. Imposing a lower bound for real money balances, $\widehat{m}$, prevents that the $\dot{m}$ schedule will eventually cross the vertical axis at some finite point, which implies negative real balances. This type of restriction is relevant because some authors (for instance, Obstfeld and Rogoff, 1983) have argued that paths implying negative real money balances are infeasible in a perfect foresight framework and therefore must be excluded from the class of possible paths. However, one must notice that (16) is an *ad-hoc* restriction and that the existence of a lower bound for real money balances is not easy to motivate in the context of a money-in-the-utility-function model.[7]

>From the previous analysis one can draw the conclusion that feasible hyperinflationary paths (those hyperinflationary paths satisfying first-order conditions and implying always non-negative real money balances) arise naturally in the basic cash-in-advance model studied in Section 3. However,

[7] In the context of an optimizing monetary model with transaction costs, Vázquez (1998) introduces a lower bound for consumption that, in the transaction cost monetary model considered in his paper, also implies a lower bound for real money balances. Vázquez (1998) interprets the lower bound for consumption as the threshold at which agents repudiate domestic money and decide to start engaging in barter trading, complete currency substitution or home production for their own consumption.

money-in-the-utility-function models have certain limitations to generate feasible hyperinflationary paths. In particular, these limitations appear in the money-in-the-utility-function model when real money balances and consumption are substitutes. As shown by Obstfeld and Rogoff (1983), if the utility function is additive separable in consumption and real money balances, feasible hyperinflationary paths cannot be ruled out unless other severe restrictions are imposed on individual preferences. This discussion points to the conclusion that the basic cash-in-advance model is a *natural* framework to analyze explosive hyperinflation since there is no need to impose additional restrictions in this framework to generate feasible hyperinflationary paths.

It is interesting to notice that the money-in-the-utility-function model collapses in the cash-in advance model in the extreme case that $\omega = 1$ (that is, money is strictly essential) since equation (15) then becomes (6) with the cash-in-advance constraint (8) already substituted into the utility function (6). Therefore, the cash-in-advance model can be viewed as the limiting case of a money-in-the-utility-function model when money is considered strictly essential (that is, when $\omega = 1$). As argued in the Introduction, a cash-in-advance model (or, alternatively, a money-in-the-utility-function model assuming that money is strictly essential) is a sensible approach to study hyperinflation since huge inflation rates causes that credit transactions vanish and money becomes strictly essential for purchasing goods.

Finally, we consider the case where the utility function is additive separable in consumption and real money balances

$$U[c(t), m(t)] = U[c(t)] + V[m(t)], \tag{18}$$

where $U(.)$ and $V(.)$ are increasing and concave functions. Both utility functions are further assumed to be of the constant relative risk aversion class. In this context, the first-order condition obtained from the household's optimization problem (that is, maximize (14) with $U[c(t), m(t)]$ given by (18) subject to (7)) establishes that the marginal utility of real money balances is proportional to the nominal interest rate. Thus, the inverse of the money demand function is given by

$$\pi(t) = f(m(t)) = \frac{V'(m(t))}{\lambda} - r. \tag{19}$$

After doing the same type of calculation carried out in the proof of Proposition 2, we can show that there is an ITLC whenever the risk aversion parameter characterizing $V'(m(t))$ is lower than one. Alternatively, if we specialize

$$V(m(t)) = m(t)[\kappa + 1 - \ln(m(t))],$$

as in Calvo and Leiderman (1992), we have that equation (19) becomes Cagan money demand (3) and the associated ITLC is the one characterized in Section 1.

5 SUMMARY AND CONCLUSIONS

Two competing approaches have been followed to characterize hyperinflationary dynamics in the literature. The traditional approach posits the presence of an ITLC where the huge rates of inflation observed during hyperinflationary episodes are viewed as the result of the economy being in a stable, but "inefficient", high-inflation steady state. By contrast, another group of researchers view hyperinflationary dynamics as the result of an unstable dynamic process where real money balances vanish and inflation explodes.

This paper studies two prominent optimizing monetary models to characterize hyperinflationary dynamics. In the context of a cash-in-advance economy, we show that an ITLC is ruled out and explosive hyperinflation is feasible whenever the relative risk aversion parameter is greater than one.

In an optimizing model with money-in-the-utility-function an ITLC is also ruled out. Moreover, we show that explosive hyperinflationary dynamics are more likely in this type of model when the transactions role of money becomes more important. However, hyperinflationary paths are not feasible in this model unless additional restrictions are considered. Therefore, the money-in-the-utility-function model presents more limitations than the simple cash-in-advance model to characterize hyperinflation as an explosive process.

Another important conclusion of this paper is that hyperinflationary dynamics derived from standard optimizing monetary models are consistent with a characterization of hyperinflation as an explosive process. However, they are not consistent with a characterization of the economy being in the "inefficient" side of the ITLC (where inflation may be huge, but stable) unless some restrictive assumptions are imposed on agents' preferences.

It seems that the empirical literature on hyperinflation has been focussed on the traditional approach because the econometrics of explosive roots has not been fully developed yet. We believe that pioneer papers on explosive roots (for instance, Juselius and Mladenovic, 1999) together with theoretical papers on the alternative view of hyperinflation suggest promising avenues for future research on hyperinflationary dynamics.

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