

THE NEW KEYNESIAN MONETARY MODEL: DOES IT SHOW THE COMOVEMENT BETWEEN OUTPUT AND INFLATION IN THE U.S.?^{*}

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First version: September, 2004. This version: November 2004

Abstract

This paper analyzes the performance of alternative versions of the New Keynesian monetary (NKM) model in order to replicate the comovement observed between output and inflation during the Greenspan era. Following Den Haan (2000), we analyze that comovement by computing the correlations of VAR forecast errors of the two variables at different forecast horizons. The empirical correlations obtained show a weak comovement. A simple NKM model under a standard parametrization provides a high negative comovement at any forecast horizon. However, a generalized version including habit formation and a forward-looking Taylor rule is able to mimic the observed weak comovement. The good performance of this generalized version also extends to the case in which the policymaker is committed to following an optimal contingent plan under certain parametrizations.

Key words: comovement, VAR forecast errors, NKM model, optimal policy
JEL classification numbers: E30, E52

^{*}We are grateful for comments and suggestions from Arantza Gorostiaga, Javier Gómez-Biscarri, Antonio Moreno and participants in seminars at Universidad de Murcia, Universidad de Navarra and Universidad del País Vasco. Financial support from Fundación BBVA, Ministerio de Ciencia y Tecnología and Universidad del País Vasco (Spain) through projects 1/BBVA00044.321-15466, SEJ2004-04811/ECON and 9/UPV00035.321-13511/2001, respectively, is gratefully acknowledged.

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1 INTRODUCTION

For a long time economists widely accepted that output and inflation displayed a positive correlation at least in the short-run. For a large group of economists, the positive short-run correlation between output and inflation (the so-called Phillips curve phenomenon) is still considered a necessary building block of business cycle theory (for instance, Mankiw, 2001). Yet, this view is rather controversial in the literature. For instance, Kydland and Prescott (1990) argue that “*any theory in which procyclical prices figure crucially in accounting for postwar business cycle fluctuations is doomed to failure.*” Moreover, Cooley and Ohanian (1991) find evidence that the U.S. correlation between output and prices is negative during the postwar period.

Den Haan (2000) argues that an important source of disagreement in the literature is the focus on only the unconditional correlation between output and prices. As an alternative, Den Haan proposes using correlations of VAR forecast errors at different horizons. By proceeding in this way one can take into account a full set of statistics characterizing the comovement dynamics in an efficient manner.¹ Using U.S. data from the postwar period, Den Haan (2000) finds that the comovement between output and prices is positive in the short-run (up to two-year horizons) and negative in the long-run (between five- and seven-year horizons).²

In this paper, we argue that another important source of disagreement on the Phillips curve is the choice of variables involved in this relationship.

¹As discussed by Den Haan (2000), this methodology has two main advantages. First, variables need not be stationary for their comovement to be analyzed and then previous filtering is not required. Second, it avoids the type of *ad-hoc* assumptions necessary to compute impulse response functions. There is a shortcoming though. This procedure does not identify the responses to all the different structural shocks. More generally, one could ask why the analysis of comovement between output and inflation is based on VAR's. There are two compelling reasons. First, macroeconomic variables such as output and inflation show a great deal of persistence and VAR's are well suited to deal with persistence. Second, the NKM model leads to a restricted VAR. Then, why do we not test the restrictions imposed on the VAR by the NKM model directly? The reason is simple. Any small-scale business cycle model such as the NKM model is a simple abstraction of a complex world. So, a test with reasonable power will very probably reject the restrictions imposed by the model on the VAR, but the model may still be able to reproduce some stylized facts (for instance, the comovement observed between output and inflation).

²Following Den Haan's methodology, Den Haan and Sumner (2004) analyze data from the G7 countries. They find a negative long-run relationship for all countries. However, the evidence of a positive short-run comovement between output and prices is weaker. Similar to Den Haan and Sumner (2004), Vázquez (2002a) finds evidence of a negative long-run relationship for a large group of EU15 countries, but only few countries France, Italy and Portugal exhibit a type of Phillips-curve effect, that is, a positive comovement between output and prices in the short-run.

Many papers have studied the comovement between output (or another indicator of economic activity) and prices. However, traditional and new Phillips curve proponents claim that there is a positive short-run correlation between output (or another indicator of economic activity) and inflation. As pointed out by Mankiw (2001), the dynamics of prices and inflation (and thus the comovement of output with one of these variables), can be rather different. For instance, in models of staggered price adjustment the price level adjusts slowly, but the rate of inflation can jump instantaneously.

The aim of this paper is twofold. First, it applies the methodology suggested by Den Haan (2000) to study the comovement between the level of economic activity and inflation in the U.S. during the Greenspan period using alternative measures of both economic activity and inflation. Second, it analyzes the ability of alternative versions of the New Keynesian monetary (NKM) model to replicate the dynamic correlations between economic activity and inflation observed in actual data.^{3,4}

Our empirical analysis suggests the presence of a weak comovement between economic activity and inflation whereas the simple NKM model under a standard calibration of model parameters gives a strong negative comovement. However, a generalized version of the NKM model that considers a forward-looking Taylor rule and habit formation à la Fuhrer (2000) is able to replicate the tenuous comovement between economic activity and inflation at medium- and long-run forecast horizons. Similar results are found when the policymaker is committed to following an optimal contingent plan under certain parametrizations. Nevertheless, the generalized model has some difficulties in reproducing the observed weak comovement at short-run (less than one year) forecast horizons.

Using a Bayesian maximum-likelihood estimation procedure, Lubik and Schorfheide (2004) (henceforth LS) have recently shown that in the context of an NKM model the presence or absence of determinacy plays a key role in explaining the different dynamic features displayed by the output gap, inflation and the Fed funds rate before and after the Volcker-Greenspan monetary experience. By contrast with LS, on the one hand we focus on the Greenspan period because a monetary policy characterized by a Taylor rule fits better in this period than in the pre-Greenspan era. Moreover, the choice of parame-

³Early versions of the NKM models include those of Yun (1996), Goodfriend and King (1997), Rotemberg and Woodford (1995, 1997) and McCallum and Nelson (1999). See also Galí (2002) for a detailed analytical derivation of a standard version of the NKM model.

⁴This paper then follows the New Neoclassical Synthesis approach (see Goodfriend and King, 1997) where stochastic general equilibrium models showing short-run price stickiness are confronted with nominal and real data in order to get a better understanding of the effects of macroeconomic policy.

ter values made below is partly based on estimates obtained by Rudebusch (2002) using only data from the Greenspan period. On the other hand, our goal is not to estimate the NKM model, but to analyze the features that the NKM model must exhibit in order to characterize the weak comovement between economic activity and inflation observed in recent U.S. data.

Given that economic models are at best a quantitative parable trying to capture the main aspects of a complex economic environment, we believe that the exercise proposed in this paper is useful because it complements estimation exercises of the NKM model carried out in the literature by looking at an alternative measure of fit chosen independently of the model.⁵ Specifically, our purpose is to study quantitatively whether alternative versions of the NKM model, assuming parameter values in the range of the estimated values found in the literature, are able to replicate the type of comovement patterns observed in actual data.

The rest of the paper is organized as follows. Section 2 presents and discusses the empirical evidence using alternative measures of economic activity and inflation. Section 3 introduces a generalized version of the NKM model that includes habit formation and a Galí and Gertler's (1999) hybrid Phillips curve. These two features derived from optimizing principles induce a certain degree of sluggishness (backward-looking components) as well as stronger forward-looking components on the IS curve and the AS-Phillips curve. Section 4 extends the analysis to an empirical version of the NKM model suggested by Rudebusch (2002) where longer leads and lags are introduced (admittedly) in an *ad-hoc* fashion in order to capture the institutional length of contracts and delays in information flows and processing. Section 5 concludes.

2 THE COMOVEMENT BETWEEN ECONOMIC ACTIVITY AND INFLATION

In this section, we implement Den Haan's methodology to study the comovement between economic activity and inflation in the U.S. during the Greenspan period. Appendix 1 briefly describes this procedure.

As mentioned in the Introduction, we focus on the Greenspan period because a Taylor rule fits better in this period than in the pre-Greenspan era. Therefore the NKM model, which introduces the Taylor rule as a basic

⁵Estimation approaches (for instance, Galí and Gertler (1999) and LS), in principle, consider the model as the true data generating process.

building block, is more likely to perform well in this period than in the pre-Greenspan era. Nevertheless, the weak comovement between economic activity and inflation found in recent data was also present in the pre-Greenspan period.⁶

We study the comovement by considering quarterly and monthly data, and alternative measures of economic activity and inflation.⁷ Using quarterly data, we first study the comovement between GDP and the inflation rate obtained from the implicit GDP deflator. Second, we analyze the comovement between the rate of inflation obtained from the consumer price index (CPI) and the output gap measured as the difference of the logs of GDP and the measure of potential GDP provided by the Congressional Budget Office. Using monthly data, we consider CPI to define inflation and two alternative measures for the level of economic activity. First, we study an index of total industrial production. Since the use of an industrial output index to measure economic activity can be questioned on the grounds that the share of national output represented by industrial output has decreased steadily in all industrial countries over the last 20-30 years, we also consider a second measure of economic activity at the monthly frequency. More specifically, we study the three-month moving average of the *Chicago Fed National Activity* index, which is computed using the methodology suggested by Stock and Watson (1999).⁸

We estimate correlation coefficients based on *VAR*'s that include economic activity, inflation and the Fed funds rate.⁹ The characteristics of these *VAR*'s are described in Table 1. The Akaike information criterion is

⁶Vázquez (2002b) provides additional evidence for the EU15 countries and the U.S.

⁷Appendix 2 describes the alternative measures for the level of economic activity and the rate of inflation as well as the sources and the sample periods considered.

⁸More precisely, the Chicago Fed National Activity index is the first principal component of 85 existing monthly real indicators of economic activity. These 85 monthly indicators can be classified into five groups: production and income (21 series), employment, unemployment and labor hours (24 series), personal consumption and housing (13 series), manufacturing and trade sales (11 series) and inventories and orders (16 series). For more details on this index and demonstrations of how well it works both in forecasting inflation and identifying recessions as defined by the NBER, see also Evans, Liu, and Pham-Kanter (2002) and references therein.

⁹In order to characterize the comovement between economic activity and inflation one may introduce more variables describing monetary policy such as total reserves and the ratio of non-borrowed reserves to total reserves. We decided not to do so in order to facilitate the comparison between the empirical results obtained from actual U.S. data with those derived from synthetic data obtained from the NKM model that includes only the output gap, inflation and the interest rate. Moreover, Vázquez (2002b) finds similar empirical results using the U.S. data set used by Den Haan (2000) that includes those monetary variables.

used to determine the number of lags and whether linear and quadratic trend terms should be included.

Following Den Haan (2000), we estimate the correlation coefficients of *VAR* forecast errors by calculating the forecast errors for each horizon considered (from one quarter to 28 quarters) as the difference between the realizations and the corresponding forecasts and then calculating the correlations of these forecast errors for each horizon.¹⁰ Since the estimated correlation coefficients are subject to sampling variation, confidence bands are constructed using bootstrap methods. More specifically, the bootstrapped errors of each estimated *VAR* are used to generate 2500 simulated data sets. Then, the correlation coefficients at different horizons are estimated for each simulated data set and standard confidence bands are calculated.¹¹

Figures 1-2 display a set of graphs, one for each data set analyzed. Each graph shows the estimated correlation coefficients (solid line) and the 10% – 90% (lines with dots and dashes) and 5% – 95% (lines with dashes) confidence bands constructed using bootstrap methods. Looking at these graphs, we observe that the comovement between economic activity and inflation at medium- and long-run forecast horizons is not significant, except in one case: when the CFNAI-MA3 and the one-year average of CPI inflation are considered. Moreover, the comovement at short-run forecast horizons is significant in most cases, but the sign of the comovement is not always negative (for instance, the comovement between IPI and CPI inflation is significantly positive) and it is rather weak in all cases.

¹⁰Den Haan and Sumner (2004) use an alternative method to estimate the correlation coefficients. This method uses the covariance obtained from the *VAR* coefficients and the variance-covariance matrix of the white noise process, U_t . They argue that using this method leads to efficiency gains especially in estimating the correlation coefficients associated with long-term forecast horizons. However, they also report that bias is larger with this second method. Nevertheless, the empirical weak comovement also shows up when using this second methodology.

¹¹The programs for estimating the correlation coefficients and the confidence bands are adapted versions of programs written in RATS that were download from Den Haan's website. The key RATS instruction for implementing bootstrap procedure is called BOOT. The BOOT instruction is used to draw entry numbers with replacement from the estimated errors of the *VAR*.

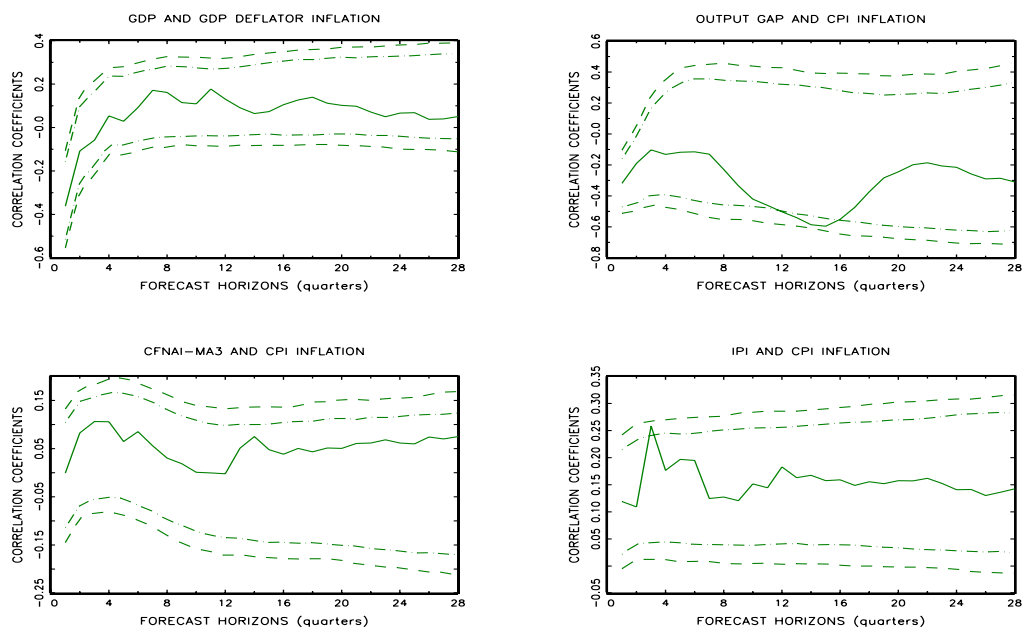


Figure 1: Comovement between economic activity and current inflation

Table 1. VAR characteristics

Variables	Number of lags	Linear and Quad. Trends
CFNAI-MA3 vs CPI inflation	4	No
GDP vs GDP deflator inflation	4	Yes
Output gap vs CPI inflation	3	No
IPI vs CPI inflation	4	Yes
CFNAI-MA3 vs 1-year average of CPI inflat.	4	No
GDP vs 1-year aver. of GDP deflator inflat.	4	Yes
Output gap vs 1-year average of CPI inflat.	4	No
IPI vs 1-year average of CPI inflation	5	Yes

3 A NEW KEYNESIAN MONETARY MODEL

The model analyzed in this paper is a generalized version of the now-standard NKM model that includes habit formation and Galí and Gertler's hybrid

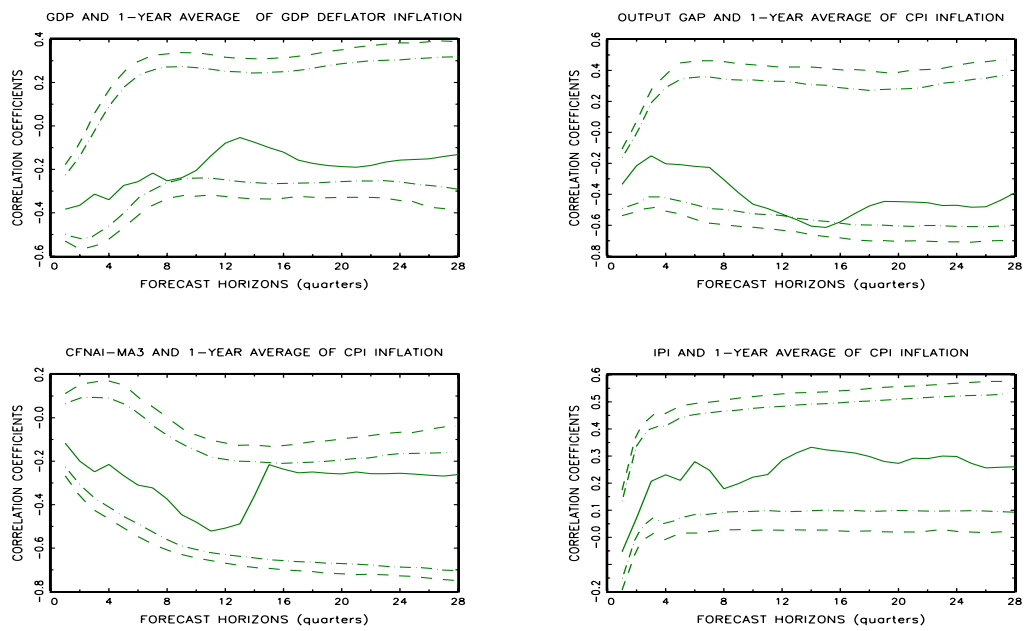


Figure 2: Comovement between economic activity and 1-year average inflation

Phillips Curve, which is given by the following set of equations:

$$\left[\tau + \frac{1 + \beta\gamma^2 + \gamma}{1 - \beta\gamma}(1 - \tau) \right] y_t = \frac{\gamma(1 - \tau)}{1 - \beta\gamma} y_{t-1} + \left[\tau + \frac{1 + \beta\gamma(1 + \gamma)}{1 - \beta\gamma}(1 - \tau) \right] E_t y_{t+1} + \frac{\beta\gamma(1 - \tau)}{1 - \beta\gamma} E_t y_{t+2} - \tau(i_t - E_t \pi_{t+1}) + g_t, \quad (1)$$

$$\pi_t = \frac{\beta}{1 + \beta\omega} E_t \pi_{t+1} + \frac{\kappa}{1 + \beta\omega} y_t + \frac{\omega}{1 + \beta\omega} \pi_{t-1} + z_t, \quad (2)$$

$$i_t = \rho i_{t-1} + (1 - \rho)(\psi_1 \pi_t + \psi_2 y_t) + \epsilon_{it}. \quad (3)$$

where y , π and i denote the log-deviations from the steady states of output, inflation and nominal interest rate, respectively. E_t denotes the conditional expectation based on the agents' information set at time t . g and z denote aggregate demand and aggregate supply shocks, respectively. These two shocks are further assumed to follow first-order autoregressive process

$$g_t = \rho_g g_{t-1} + \epsilon_{gt}, \quad (4)$$

$$z_t = \rho_z z_{t-1} + \epsilon_{zt}, \quad (5)$$

where ϵ_{gt} and ϵ_{zt} denote i.i.d. random shocks.

Equation (1) is the consumption first-order condition obtained by introducing multiplicative habit formation à la Fuhrer (2000) where the period utility function at time t is given by

$$U(C_t) = \frac{1}{1 - 1/\tau} \left(\frac{C_t}{C_{t-1}^\gamma} \right)^{1-1/\tau}.$$

The term C_{t-1}^γ can be understood as the habit stock. This term vanishes when $\gamma = 0$, which leads to the standard constant relative risk aversion utility function.¹²

Galí-Gertler's hybrid new Phillips curve, equation (2), can be obtained in a sticky price à la Calvo (1983) model under the assumption that among the fraction of monopolistically competitive firms unable of re-optimizing their prices in response to shocks in any given period, a fraction ω revise their prices according to the lagged inflation whereas a fraction $1 - \omega$ increase

¹²The analytical derivation of Euler equation (1) is straightforward and it can be found in a Technical Appendix to LS in Frank Schorfheide's website.

their prices at the steady state rate of inflation.¹³ For $\omega = 0$ equation (2) becomes the standard New Phillips curve.

Equation (3) is a standard Taylor-type monetary rule where the nominal interest rate exhibits smoothing behavior, captured by parameter ρ , for which there are several motivating arguments in the literature. These arguments range from the traditional concern of central banks for the stability of financial markets (see Goodfriend, 1991 and Sacks, 1997) to the more psychological one posed by Lowe and Ellis (1997), who argue that there might be a political incentive for smoothing whenever policymakers are likely to be embarrassed by reversals in the direction of interest-rate changes if they believe that the public may interpret them as repudiations of previous actions. By contrast, a series of interest-rate changes in the same direction looks like a well-designed programme, and that may give rise to the sluggish behavior of the intervention interest rate. Moreover, Taylor (3) assumes that the nominal interest rate responds to current deviations of output and inflation from their respective steady state values. Later on, we shall discuss the effects of considering backward-looking and forward-looking Taylor rules.

The system of equations (1)-(5) can be written in matrix form as follows

$$\Gamma_0 \xi_t = \Gamma_1 \xi_{t-1} + \Psi \epsilon_t + \Pi \eta_t, \quad (6)$$

where

$$\begin{aligned} \xi_t &= (y_t, \pi_t, i_t, E_t y_{t+2}, E_t y_{t+1}, E_t \pi_{t+1}, g_t, z_t)', \\ \epsilon_t &= (\epsilon_{it}, \epsilon_{gt}, \epsilon_{zt})', \\ \eta_t &= (E_t y_{t+1} - E_{t-1} y_{t+1}, y_t - E_{t-1} y_t, \pi_t - E_{t-1} \pi_t)', \end{aligned}$$

$$\Gamma_0 = \begin{pmatrix} -a_0 & 0 & -\tau & a_3 & a_2 & \tau & 1 & 0 \\ \frac{\kappa}{1+\beta\omega} & -1 & 0 & 0 & 0 & \frac{\beta}{1+\beta\omega} & 0 & 1 \\ -(1-\rho)\psi_2 & -(1-\rho)\psi_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

¹³See Galí and Gertler (1999) for a detailed derivation of this hybrid Phillips curve.

$$\Gamma_1 = \begin{pmatrix} -a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\omega}{1+\beta\omega} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_g & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_z \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\Pi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\Psi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$a_0 = \left[\tau + \frac{1 + \beta\gamma^2 + \gamma}{1 - \beta\gamma}(1 - \tau) \right],$$

$$a_1 = \frac{\gamma(1 - \tau)}{1 - \beta\gamma},$$

$$a_2 = \left[\tau + \frac{1 + \beta\gamma(1 + \gamma)}{1 - \beta\gamma}(1 - \tau) \right],$$

$$a_3 = \frac{\beta\gamma(1 - \tau)}{1 - \beta\gamma}.$$

Equation (6) is a linear rational expectations (LRE) system. It is well known that LRE systems deliver multiple stable equilibrium solutions for certain parameter values. Following LS, we deal with multiple equilibria by

assuming that agents observe an exogenous sunspot shock ζ_t , in addition to the fundamental shocks, ϵ_t . Since system (6) is linear, the forecast errors can be expressed as a linear function of ϵ_t and ζ_t

$$\eta_t = A_1\epsilon_t + A_2\zeta_t,$$

where A_1 is 3×3 and A_2 is 3×1 .

Lubik and Schorfheide (2003) characterize the complete set of LRE models with indeterminacies and provide a method for computing them that builds on Sims' (2002) approach.¹⁴ There are three possible scenarios: (i) No stable equilibrium. (ii) Existence of a unique stable equilibrium in which A_1 is completely determined by the structural parameters of the model and $A_2 = 0$. (iii) Multiple stable equilibria in which A_1 is not uniquely determined by the structural parameters of the model and A_2 can be non-zero.

We start the analysis of the comovement between output and inflation in the NKM model by considering the estimates of the standard NKM model obtained by LS for the Volcker-Greenspan period as a benchmark parameterization. Table 2 displays the benchmark parameter values. The value assumed for β is consistent with a value for the steady state real interest rate of 3.01.

Table 2. Benchmark parameter values

$\tau = 0.54$	$\beta = 0.99$	$\rho = 0.84$	$\kappa = 0.58$	$\psi_1 = 2.19$	$\psi_2 = 0.30$	$\rho_g = 0.83$
$\rho_z = 0.85$	$\gamma = 0.0$	$\omega = 0.0$	$\sigma_g = 0.18$	$\sigma_z = 0.64$	$\sigma_i = 0.18$	$\pi^* = 3.43$

For the benchmark parameter values, the NKM model exhibits a unique stable equilibrium.¹⁵ Figure 3 displays the comovement between output and inflation derived from the NKM model under the benchmark parameter values (line with dots and dashes), the corresponding 5%–95% confidence bands (lines with short dashes) together with the comovement between GDP and GDP deflator inflation (solid line) and the corresponding 5% – 95% confidence bands (lines with dashes).¹⁶ This figure shows that the NKM model

¹⁴Sims' method generalizes the methods developed by Blanchard and Khan (1980), King and Watson (1998) and Klein (2000). The GAUSS code for computing the equilibria of LRE models can be found in Frank Schorfheide's website.

¹⁵The GAUSS programs for computing the solutions and the comovement between output and inflation corresponding to the alternative versions of the NKM model considered in this paper are available from the authors upon request.

¹⁶The confidence bands for the comovement derived from synthetic data are computed simulating the NKM model 2500 times.

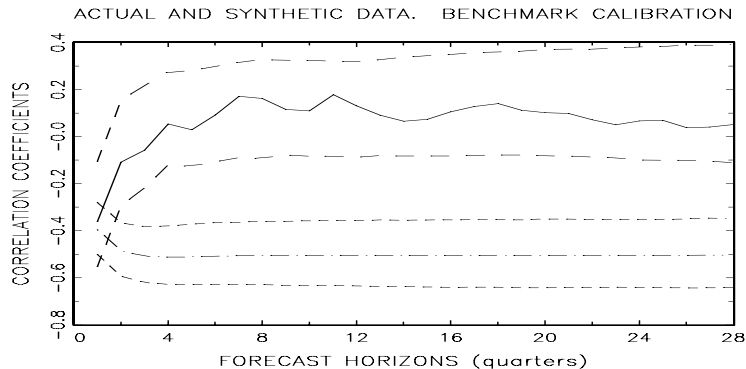


Figure 3: Comovement under benchmark parametrization

under the benchmark parametrization provides high negative correlation coefficients between the forecast errors of output and inflation, and these correlation coefficients are significantly different from those obtained from actual data. Moreover, a similar conclusion is reached when comparing the comovement implied by the model with those observed in actual data depicted in Figures 1-2 whenever the confidence bands are relatively narrow.¹⁷

A sensitivity analysis (not shown here but available from the authors upon request) choosing parameter values belonging to the confidence intervals displayed in the last column of Table 3 in LS shows no relevant improvement in replicating the comovement between economic activity and inflation observed in U.S. data. Moreover, nor does introducing habit formation and a hybrid Phillips curve (i.e., assuming positive values for γ and ω , respectively) help in replicating the observed comovement.

Several authors (for instance, Benhabib, Schmitt-Grohé and Uribe, 2003; Giannoni and Woodford, 2003; and Woodford, 2003) have paid attention to the features of the NKM model when the smoothing parameter ρ is greater than unity. The NKM model studied in this paper displays multiple stable equilibria when $\rho > 1$. More interestingly, once $\rho > 1$ all the equilibria are

¹⁷In this paper, we always evaluate the performance of the NKM model based on the observed comovement between GDP and inflation derived from the GDP deflator. This is done for two reasons. First, the parameter values considered come from estimation results obtained using quarterly data where output is measured by GDP. Second, GDP is (arguably) the best measure of economic activity.

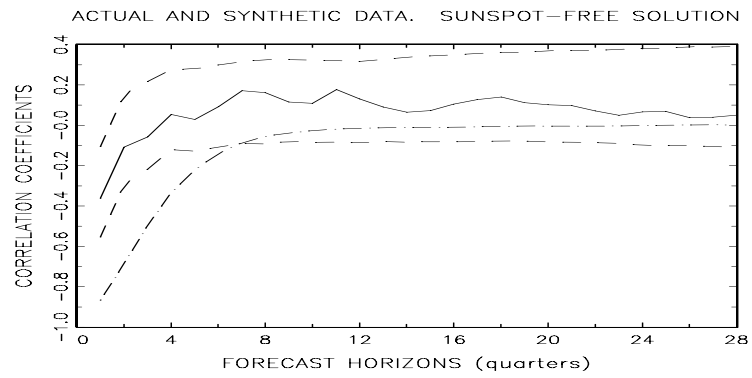


Figure 4: Comovement under sunspot-free equilibrium with $\rho = 1.05$

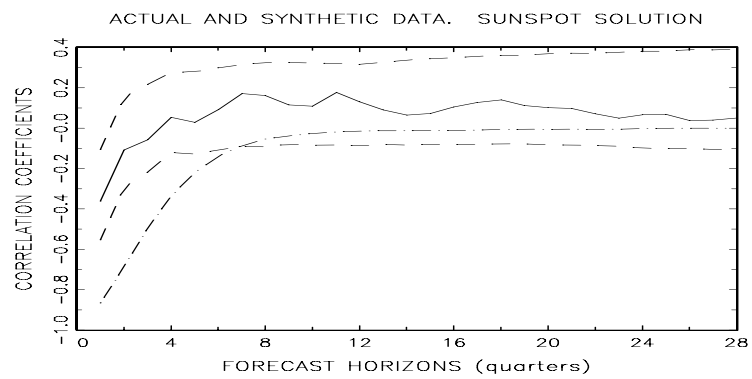


Figure 5: Comovement under sunspot equilibrium with $\rho = 1.05$

much closer to featuring the type of weak comovement between output and inflation, at least at medium- and long-run forecast horizons, observed in actual data.¹⁸ Figures 4-5 show the comovement exhibited by the sunspot and sunspot-free equilibrium solutions assuming $\rho = 1.05$ and assuming values for the remaining parameters as displayed in Table 2. In particular, these figures show that the correlation coefficients for all forecast horizons derived from synthetic data, except for those from the first eight quarters, are within the confidence bands estimated using actual data.

Notice that the Taylor rule with $\rho > 1$ can be solved *forward* to obtain a stable solution of the interest rate:

$$i_t = (1 - \rho^{-1}) \sum_{j=0}^{\infty} \rho^{-j} [\psi_1 E_t \pi_{t+j+1} + \psi_2 E_t y_{t+j+1}],$$

which is indeed a forward-looking Taylor rule. As discussed below in more detail, the importance of forward-looking components in the Taylor rule is a crucial feature for replicating the observed comovement.

A remarkable feature of the NKM model with a smoothing parameter in the Taylor rule larger than one is that in equilibrium the comovement between output and inflation vanishes for long-run forecast horizons even though a strong correlation is imposed by assuming a Phillips curve.

The fact that the comovement features displayed by sunspot and sunspot-free equilibria are almost identical for any version of the NKM displaying indeterminacy is surprising on the one hand, because forecast errors are determined by the presence of sunspots. On the other hand, the robustness of the comovement features obtained from Den Haan's method to the presence of sunspots provides additional support for using those features as relevant statistics for evaluating model performance.

By imposing $\rho = 1.05$, we next analyze whether introducing habit formation or a hybrid Phillips curve is useful in replicating the comovement observed between output and inflation. As shown in Figure 6, assuming habit formation with $\gamma = 0.57$ does not help to replicate the observed comovement: in fact the fit of the model in this dimension gets worse!¹⁹

Allowing for a hybrid Phillips curve with $\omega = 0.30$ (a reasonable value according to the estimates found by Galí and Gertler, 1999) does not help to replicate the observed comovement either. As shown in Figure 7, the introduction of a hybrid Phillips curve has no significant effect on the comovement

¹⁸Instrumental variable estimation of equation (3) using data from the Greenspan period provides a point estimated value of ρ equal to 0.96, close to the value of 1.05 considered.

¹⁹ $\gamma = 0.57$ is the point estimate reported by LS for the pre-Greenspan era. This is smaller than the 0.8 obtained by Fuhrer (2000).

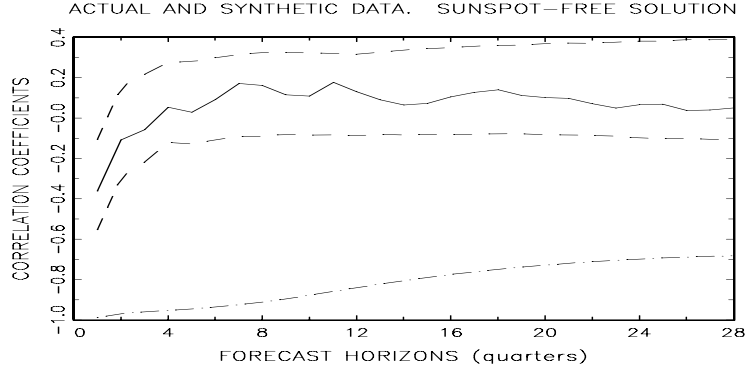


Figure 6: Comovement under habit formation ($\gamma = 0.57$)

implied by the model.

We now extend the analysis to consider alternative Taylor rules studied in the literature. First, we consider a Taylor rule where the nominal interest rate responds to expected deviations of inflation and output from their respective steady state levels, which describes how the central bank may react to anticipated movements in output and inflation. Formally, the forward-looking Taylor rule is given by

$$i_t = \rho i_{t-1} + (1 - \rho)(\psi_1 E_t \pi_{t+1} + \psi_2 E_t y_{t+1}) + \epsilon_{it}. \quad (7)$$

Second, a backward-looking Taylor rule is considered where the nominal interest rate responds to lagged deviations of output and inflation from their respective steady state values as a way of capturing delays in information flows. Formally,

$$i_t = \rho i_{t-1} + (1 - \rho)(\psi_1 \pi_{t-1} + \psi_2 y_{t-1}) + \epsilon_{it}. \quad (8)$$

Finally, we consider a Taylor rule where the nominal interest rate responds to deviations of current output and 1-year average inflation from their respective steady state values. Formally,

$$i_t = \rho i_{t-1} + (1 - \rho) \left[\frac{\psi_1}{4} \left(\sum_{i=0}^3 \pi_{t-i} \right) + \psi_2 y_t \right] + \epsilon_{it}. \quad (9)$$

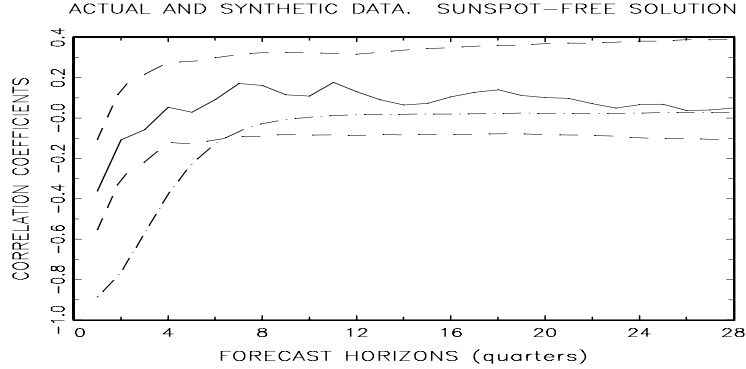


Figure 7: Comovement under hybrid Phillips Curve

Once the benchmark NKM model has been solved, solving the NKM model with any of these three alternative Taylor rules only requires replacing equation (3) by (7), (8) or (9), which amounts only to slight modifications of matrices Γ_0 and Γ_1 .

The version of the NKM model that includes the Taylor rule (7) also exhibits a unique equilibrium solution under the benchmark parametrization described in Table 2. Figure 8 shows a poor fit of the comovement obtained from the forward-looking Taylor rule (7) under the benchmark parametrization. In fact, as shown in Figure 9, model performance is improved by introducing a moderate degree of habit formation ($\gamma = 0.25$) and a hybrid Phillips curve ($\omega = 0.20$) whereas the uniqueness property still remains. We further explore two other Taylor rules that lie half-way between Taylor rules (3) and (7). The Fed rate in one of them is determined by expected inflation and current output whereas in the other it is determined by current inflation and expected output. For these Taylor rules the best fit in the comovement dimension is obtained with a moderate degree of habit formation ($\gamma \approx 0.25$) but imposing a standard New Phillips curve ($\omega \approx 0$). The rationale for these results can be understood if one believes that the Fed has actually followed a (near) optimal forward-looking Taylor rule. Intuitively, the optimal Taylor rule, derived from a central bank optimization problem where an IS curve and a Phillips curve appear jointly with a quadratic loss function, has to show an

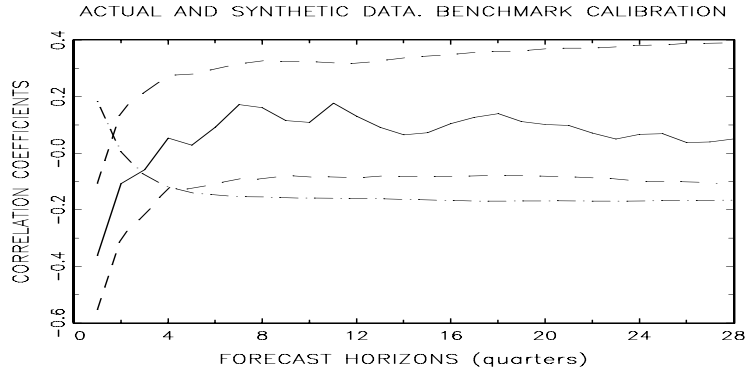


Figure 8: Comovement under forward-looking Taylor rule and benchmark parametrization

optimal balance between backward- and forward-looking components. Thus, if there is strong forward-looking behavior in the IS curve due to habit formation, then the optimal Taylor rule obtained by solving this problem must be forward-looking, taking into account the forward-looking behavior of the private sector. Moreover, the effects of a hybrid Phillips curve are harder to analyze since as ω goes to zero the coefficient associated with expected inflation in the hybrid Phillips curve increases (that is, $\beta/(1 + \beta\omega)$ increases) and the one associated with lagged inflation tends to vanish. But at the same time, the coefficient associated with current output increases (that is, $\kappa/(1 + \beta\omega)$ increases) which explains why fewer forward-looking components in the Taylor rule are required when $\omega = 0$.²⁰ In sum, if one believes that the Fed acts optimally, one should expect better fit results when habit formation is combined with a forward-looking Taylor rule. This conjecture is analyzed in more depth in the following subsection.

The comovements between output and inflation under Taylor rules (8) and (9) are similar to those obtained under equation (3). Namely, (i) a unique stable equilibrium arises under the benchmark parametrization, (ii)

²⁰Svensson (1997) derives an optimal Taylor rule for monetary policy assuming that private sector behaviour is taken as given and is represented by a backward-looking Phillips curve and a backward-looking IS curve. He assumes that Central Bank preferences are quadratic and obtains that the nominal interest rate responds to actual inflation and the output gap.

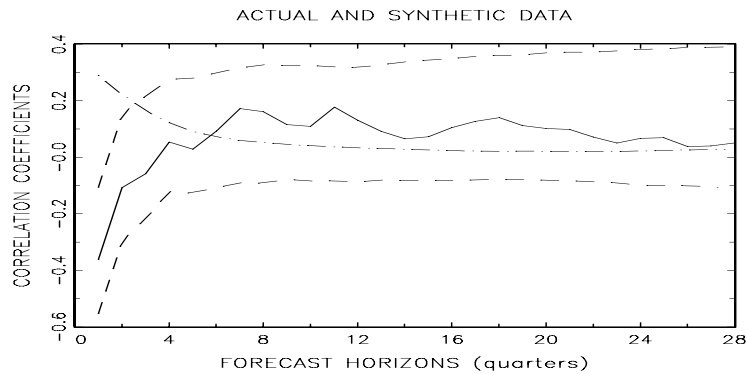


Figure 9: Comovement under forward-looking Taylor rule with habit formation ($\gamma = 0.25$) and hybrid Phillips curve ($\omega = 0.20$)

imposing $\rho > 1$ and keeping the remaining parameters at their benchmark values improves matters at medium- and long-run forecast horizons as shown in Figures 10-11 and (iii) introducing habit formation results in a worse fit whereas introducing a hybrid Phillips curve fails to improve the performance of the model. Result (iii) reinforces the intuitive arguments on the optimal balance between forward- and backward-looking components in the Taylor rule driven by the degree of forward-looking behavior attached to the private sector.

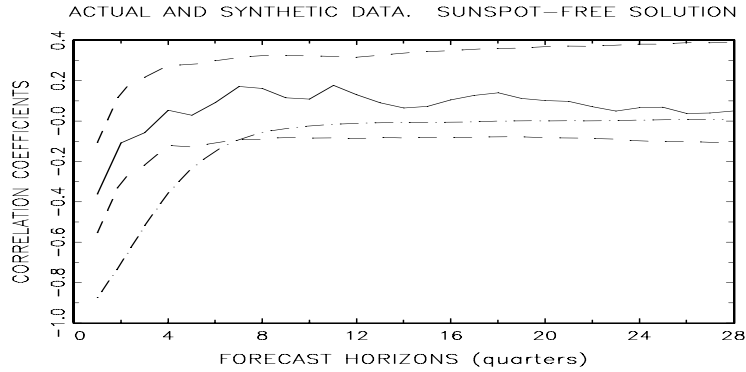


Figure 10: Comovement under backward-looking Taylor rule (8) with $\rho = 1.05$

3.1 Comovement dynamics under optimal monetary policy

In order to analyze the conjecture stated above, we next derive the optimal plan for the Central Bank and study the comovement between output and inflation under optimal monetary policy.

Following Woodford (2003), let us assume that the Central Bank minimizes the expected value of a loss criterion of the form

$$W = E_0 \left[\sum_{t=0}^{\infty} \beta^t L_t \right],$$

where the loss in each period is given by

$$L_t = \frac{1}{2}(\pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2).$$

In order to characterize the optimal plan it is useful to write the Lagrangian

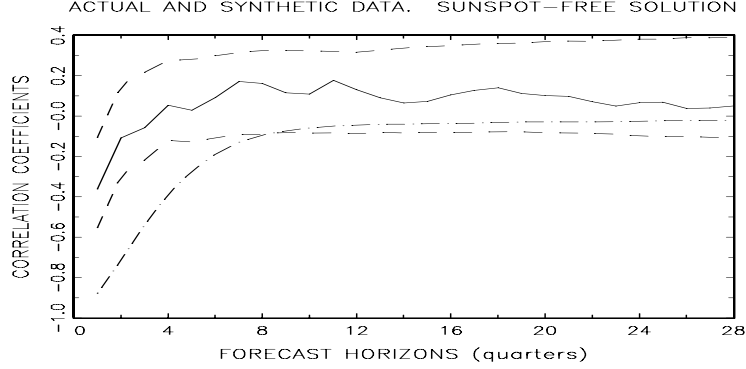


Figure 11: Comovement under Taylor rule (9) with $\rho = 1.05$

associated with the optimal control problem for the Central Bank²¹

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ L_t + \mu_{1t} [a_0 y_t - a_1 y_{t-1} - a_2 y_{t+1} - a_3 y_{t+2} + \tau(i_t - \pi_{t+1}) - g_t] \right. \right. \\ \left. \left. + \mu_{2t} \left[\pi_t - \frac{\beta}{1 + \beta\omega} \pi_{t+1} - \frac{\kappa}{1 + \beta\omega} y_t - \frac{\omega}{1 + \beta\omega} \pi_{t-1} - z_t \right] \right\} \right\}$$

An optimal plan must satisfy the following F.O.C.:²²

$$E_t(\pi_t - \tau\beta^{-1}\mu_{1t-1} + \mu_{2t} - \frac{1}{1 + \beta\omega}\mu_{2t-1} - \frac{\omega}{1 + \beta\omega}\beta\mu_{2t+1}) = 0, \quad (10)$$

$$E_t(\lambda_y y_t + a_0 \mu_{1t} - a_1 \beta \mu_{1t+1} - a_2 \beta^{-1} \mu_{1t-1} - a_3 \beta^{-2} \mu_{1t-2} - \frac{\kappa}{1 + \beta\omega} \mu_{2t}) = 0, \quad (11)$$

$$\lambda_i i_t + \mu_{1t} \tau = 0, \quad (12)$$

obtained by differentiating the Lagrangian with respect to π_t , y_t and i_t , respectively. Under the optimal plan these conditions must hold at each $t \geq 0$ together with initial conditions

$$\mu_{1,-1} = \mu_{1,-2} = \mu_{2,-1} = 0.$$

²¹By the law of iterated expectations, the conditional expectation operators inside the restrictions are removed.

²²As is well known, the optimal plan obtained from these conditions will, in general, not be time consistent as discussed by Kydland and Prescott (1977).

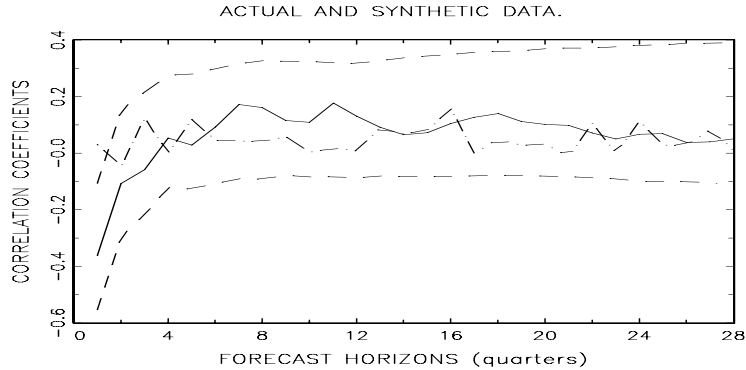


Figure 12: Comovement under optimal policy with benchmark parametrization

To solve the NKM model under optimal monetary policy the solution must be found for the system formed by equations (1), (2), (4), (5), (10), (11) and (12). This solution is derived in Appendix 3. When solving the model under the optimal monetary plan, we consider as benchmark parameter values those displayed in Table 3, where instead of the Taylor rule parameters those associated with the Central Bank loss function (λ_y and λ_i) appear. The parameter values assumed for λ_y and λ_i are those considered by Woodford and coauthors (see, for instance, Giannoni and Woodford, 2003, and Woodford, 2003). Moreover, a certain degree of habit formation and a hybrid Phillips curve are considered.

Table 3. Benchmark parameter values

$\tau = 0.54$	$\beta = 0.99$	$\kappa = 0.58$	$\lambda_y = 0.048$	$\lambda_i = 0.236$	$\rho_g = 0.83$
$\rho_z = 0.85$	$\gamma = 0.25$	$\omega = 0.20$	$\sigma_g = 0.18$	$\sigma_z = 0.64$	$\pi^* = 3.43$

For these benchmark parameter values, the NKM model under the optimal contingent plan exhibits a unique stable equilibrium. Comparing Figures 9 and 12, we observe that the comovement between output and inflation under the optimal plan reproduces both the pattern displayed by the NKM model under the forward-looking rule, except in the short-run, and that displayed by actual U.S. data.

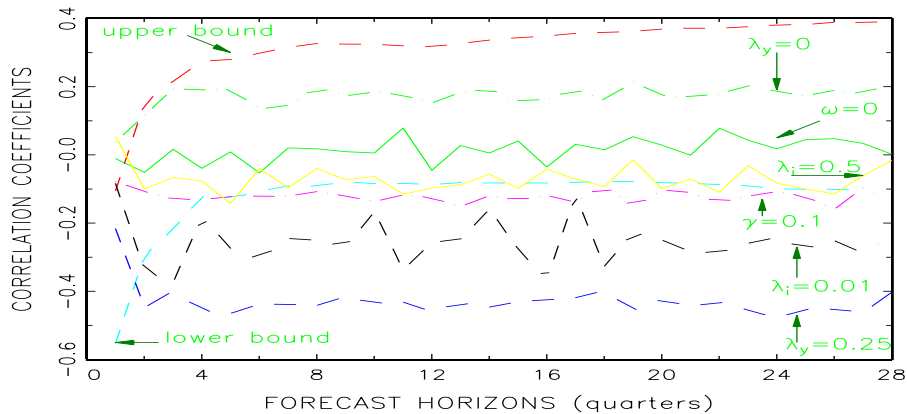


Figure 13: Comovement features under alternative parametrizations

Several articles (Amato and Laubach, 2004; Giannoni and Woodford, 2003; Rotemberg and Woodford, 1997; and Woodford, 2003) have studied the links between Central Bank preference parameters (λ_y , λ_i) and structural parameters in order to obtain an implicit monetary plan that is optimal from a social-welfare perspective. Two conclusions emerge from this literature. First, the links are complex functions of structural parameters (numerical solutions are often required). Second, the values of Central Bank preference parameters are highly sensitive to the model's assumptions. We next perform a sensitivity analysis by computing comovement under alternative parametrizations, changing only one parameter value with respect to the benchmark parametrization described in Table 3 in each case. Figure 13 shows that the comovement features are sensitive to different parametrizations characterizing Central Bank preferences and the degree of habit formation. However, the performance of the NKM model is good when $\omega = 0$, (that is, when the standard New Phillips curve is considered). These results are consistent with the results obtained above when considering other Taylor rules that lie half-way between Taylor rules (3) and (7).

4 AN EMPIRICAL NKM MODEL

The analysis in the previous section shows, on the one hand, that an NKM model that includes habit formation, a hybrid Phillips curve and a forward-looking Taylor rule exhibits a unique equilibrium and does a good job in replicating the comovement between output and inflation exhibited by U.S. data. On the other hand, analyzing the NKM model with either standard or backward-looking Taylor rules shows that by imposing a smoothing parameter, ρ , greater than one, the fit of the model to the observed comovement between output and inflation improves, but then multiple equilibria arise. Recall that assuming $\rho > 1$ in a standard or backward-looking rule imposes the existence of forward-looking components in the Taylor rule. Therefore, a tentative conclusion emerges from this analysis: a “right” balance between forward- and backward-looking components characterizing the three main blocks of the model is crucial for improving the model’s performance in replicating the comovement between economic activity and inflation exhibited by U.S. data.

In this section we consider an empirical version of the NKM model analyzed by Rudebusch (2002) where longer leads and lags seem appropriate given the institutional length of contracts and delays in information flows and processing. By studying this empirical version we can further assess the importance of forward- and backward-looking components of the IS and Phillips curves in order to characterize the observed comovement. Formally, the empirical NKM model is given by the following four equations:

$$\pi_t = \frac{\mu_\pi}{4} E_{t-1} \left(\sum_{i=0}^3 \pi_{t+i} \right) + (1 - \mu_\pi) \sum_{i=1}^4 \alpha_{\pi i} \pi_{t-i} + \alpha_y y_{t-1} + z_t, \quad (13)$$

$$y_t = \mu_y E_{t-1} y_{t+1} + (1 - \mu_y) (\beta_{y1} y_{t-1} + \beta_{y2} y_{t-2}) - \beta_r (r_{t-1} - r^*) + g_t, \quad (14)$$

$$r_{t-1} = \frac{\mu_r}{4} E_{t-1} \left(\sum_{i=0}^3 i_{t+i} \right) - \frac{\mu_r}{4} E_{t-1} \left(\sum_{i=1}^4 \pi_{t+i} \right) + \frac{(1 - \mu_r)}{4} \left(\sum_{i=1}^4 i_{t-i} - \pi_{t-i} \right), \quad (15)$$

$$i_t = \rho i_{t-1} + \frac{g_\pi}{4} \sum_{i=0}^3 \pi_{t-i} + g_y y_t + \epsilon_{it}. \quad (16)$$

Equations (13) and (14) denote the Phillips curve and the IS curve, respectively. r_{t-1} represents the real interest rate defined according to equation (15) as a weighted average of an ex-ante one-year real rate and an ex-post

one-year real rate.²³ g_t and z_t are assumed i.i.d. random shocks since in this model inertia is now described by longer leads and lags. Appendix 4 describes how the solution for the system of equations (13)-(16) is derived.

Table 4 shows the benchmark parameter values used for this model. We consider parameter values displayed in Table 1 and in equation (3) of Rudebusch (2002).²⁴ Moreover, we study comovement under the alternative parameter values that characterize the weights on expectational terms in the IS and Phillips curves (that is, μ_r , μ_π and μ_y) displayed in Rudebusch's Table 2. The best fit of the model when trying to replicate the observed comovement is obtained for the following parameter values: $\mu_r = 0.9$, $\mu_\pi = 0.1$ and $\mu_y = 0$. Under this parametrization, Figure 14 shows that for all forecast horizons, except for the 1-quarter ahead forecast error, the correlation coefficients obtained from the model fall within the estimated confidence bands.

Interestingly, the value estimated by Rudebusch (2002) for the smoothing parameter under the Taylor rule considered (Rule 1 in Rudebusch notation), $\rho = 0.73$, is close to the optimal value obtained by him ($\rho = 0.70$) (see Rudebusch's Table 2) when (i) the above parameter values of the weights on expectational terms are considered and (ii) the central bank loss function has the same weight (one) in output and inflation volatility and a weight in interest rate stabilization of 0.5.

Table 4. Rudebusch benchmark parameter values

$\alpha_{\pi 1} = 0.67$	$\alpha_{\pi 2} = -0.14$	$\alpha_{\pi 3} = 0.40$	$\alpha_{\pi 4} = 0.07$
$\alpha_y = 0.13$	$\beta_{y1} = 1.15$	$\beta_{y2} = -0.27$	$\beta_r = 0.09$
$g_\pi = 0.4131$	$g_y = 0.2511$	$\rho = 0.73$	$r^* = 3.01$
$\sigma_g = 0.833$	$\sigma_z = 1.012$	$\sigma_i = 0.36$	$\pi^* = 3.43$

Figure 15 shows the poor performance of the empirical NKM model for the following parameter values: $\mu_r = 0.1$, $\mu_\pi = 0.5$ and $\mu_y = 0.3$ in replicating the observed comovement between output and inflation. In fact, the poor performance is similar to that obtained from the NKM model under the benchmark parametrization in Section 3. These results again support the intuition put forward in Section 3, i.e., the best fit is obtained when the Taylor

²³Equations (13)-(15) correspond to equations (10)-(12) in Rudebusch (2002). Finally, equation (16) is the Taylor rule estimated by Rudebusch, labeled in his paper as equation (3).

²⁴We also take the LS estimated values of r^* and i^* into account in the benchmark parametrization since we consider that the variables in the empirical NKM model are in log deviations from their steady state values.

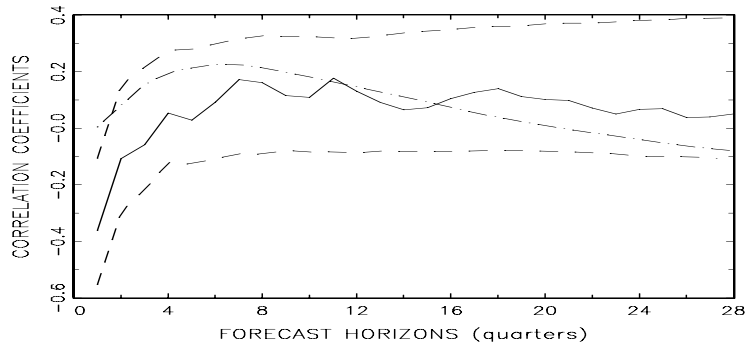


Figure 14: Comovement in the empirical NKM model assuming $\mu_r = 0.9$, $\mu_\pi = 0.1$ and $\mu_y = 0$

rule is consistent with the relative importance of forward- and backward-looking components characterizing IS and New Phillips curves. In the case of the empirical NKM model, the best fit is obtained when a backward-looking Taylor rule such as (13) is combined with IS and Phillips curves dominated by backward-looking forces.²⁵

Based on the study of the comovement between output and inflation, our analysis of the empirical NKM model suggests that (i) the weights given to the forward-looking components of the IS and Phillips curves (μ_y and μ_π , respectively) must be close to zero, (ii) by contrast, the weight given to the forward-looking component of the real interest rate relevant for output, μ_r , must be close to one, and (iii) as shown by Rudebusch (2002), these parameter values of the weights in expectational terms imply an optimal value for ρ of 0.70 ($\rho < 1$), close to Rudebusch's estimate ($\rho = 0.73$). This result suggests that the monetary policy rule followed by the Fed seems to be near-optimal at least for certain parametrizations of the Fed's loss function.

²⁵Notice that the weight assumed for the only variable dominated by forward-looking behavior in the model, r_{t-1} , is rather small ($\beta_r = 0.09$).

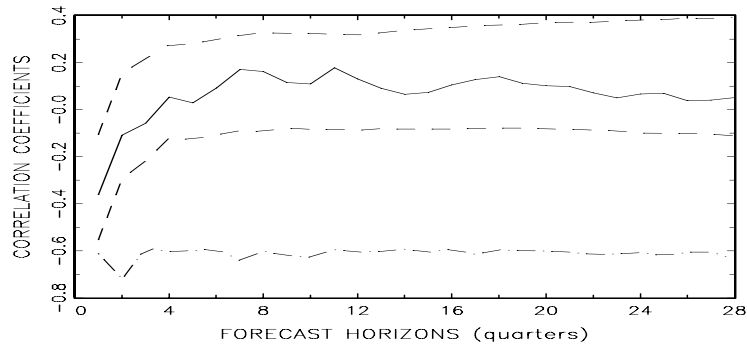


Figure 15: Comovement in the empirical NKM model assuming $\mu_r = 0.1$, $\mu_\pi = 0.5$ and $\mu_y = 0.3$

5 CONCLUSIONS

This paper uses the correlation coefficients of forecast errors at different forecast horizons obtained from estimated *VAR*'s (i) to analyze the comovement between economic activity and inflation in the U.S., and (ii) to evaluate quantitatively the performance of alternative versions of the New Keynesian monetary (NKM) model in replicating the observed comovement between output and inflation.

The empirical results show a rather weak comovement between economic activity and inflation.

In this paper, we study two types of NMN model. On the one hand, we analyze versions derived from first-economic principles where few leads and lags are considered and forward-looking components play a crucial role. On the other hand, we consider an empirical ad-hoc version studied in the literature where many leads and lags are introduced, but backward-looking components dominate. In the two cases a neat result emerges: in order to replicate the observed comovement pattern between output and inflation the type of Taylor rule assumed has to be consistent with the relative importance of forward-looking components characterizing private sector behavior. More precisely, the weak comovement observed in actual data between output and inflation is captured relatively well by a prototype NKM model that combines habit formation à la Furher (this feature introduces extra forward-looking

components in the IS) with a forward-looking Taylor rule. Alternatively, the weak comovement can also be replicated by an empirical version of the NKM model studied by Rudebusch (2002) where IS curve, Phillips curve and Taylor rule are dominated by backward-looking components.

This conclusion provides evidence of a type of ‘internal consistency’ between the behavioral equations characterizing both private and policy actions. It also implies that it is not simple to determine the relative importance of forward-looking behavior characterizing private and policy actions from the analysis of the comovement between output and inflation. So, further research efforts along these lines are warranted.

APPENDIX 1

This appendix briefly describes how to use a *VAR* to study the correlation structure of output and inflation at several forecast horizons.

Let us consider an N -vector of random variables X_t . The vector X_t may include any combination of stationary processes and integrated processes of arbitrary order. In order to characterize the comovement of the level of economic activity, Y_t , and inflation, π_t , X_t must contain at least (the log of) Y_t and π_t . Consider the following *VAR*

$$X_t = \alpha + \beta t + \gamma t^2 + \sum_{l=1}^L A_l X_{t-l} + U_t,$$

where α , β , and γ denote fixed N -vectors of constants, A_l represents fixed $N \times N$ coefficient matrices, U_t is an N -dimensional white noise process, that is, $E(U_t) = 0$, $E(U_t U_t') = \Omega_u$ and $E(U_t U_s') = 0$ for $s \neq t$. L is the total number of lags included. The K -period ahead forecast and the K -period ahead forecast error of the random variable Y_t are denoted by $E_t Y_{t+K}$ and $Y_{t+K,t}^{ue}$, respectively. Similarly, we can define $E_t \pi_{t+K}$ and $\pi_{t+K,t}^{ue}$. Let us denote the correlation coefficients between $Y_{t+K,t}^{ue}$ and $\pi_{t+K,t}^{ue}$ by $COR(K)$.

As pointed out by Den Haan (2000), if all time series included in X_t are stationary, then the correlation coefficient of the forecast errors will converge to the unconditional correlation coefficient between Y_t and π_t as K goes to infinity. If X_t includes integrated processes, then correlation coefficients may not converge but they can be estimated consistently for fixed K .

Den Haan (2000) also shows the relationship between correlation coefficients and impulse response functions. Let us denote the covariance between $Y_{t+K,t}^{ue}$ and $\pi_{t+K,t}^{ue}$ by $COV(K)$ and, with no loss of generality, let us assume

that there are M structural shocks driving economic activity and inflation. Den Haan (2000) shows that

$$COV(K) = \sum_{k=1}^K COV^{\Delta}(k)$$

and

$$COV^{\Delta}(k) = \sum_{m=1}^M Y_k^{imp,m} \pi_k^{imp,m},$$

where $z_k^{imp,m}$ is the k -th period impulse response of variable z to a one-standard deviation disturbance of the m -th shock. Therefore, the covariance between economic activity and inflation is simply the sum of the products of economic activity and inflation impulses across the different structural shocks.

By looking at the correlation coefficients of VAR forecast errors at different horizons, the researcher obtains much richer information about system dynamics than by looking only at the unconditional correlation coefficient. As illustrated by Den Haan (2000), considering only one correlation coefficient might be misleading in some cases. Moreover, Den Haan's method avoids the type of *ad-hoc* assumptions necessary to compute impulse response functions. There is a shortcoming, however. This procedure does not identify all the different impulse response functions (that is, it does not identify the response to all the different structural shocks).

APPENDIX 2

This appendix describes the time series considered.

Economic activity indexes:

- CFNAI-MA3: monthly data. Period: 1987:8-2002:4. Chicago Fed website.
- IPI: monthly, seasonally adjusted data. Period: 1987:8-2004:5. Source: Board of Governors of the Federal Reserve System.
- GDP: quarterly, seasonally adjusted data. Period: 1987:3-2002:1. Source: U.S. Department of Commerce, Bureau of Economic Analysis.
- Real potential GDP: quarterly data. Period: 1987:3-2002:1. Source: U.S. Congress, Congressional Budget Office.

Price level indexes:

- Implicit price deflator of GDP: quarterly, seasonally adjusted data. Period: 1987:3-2002:1. Source: U.S. Department of Commerce, Bureau of Economic Analysis.
- CPI for all urban consumers, all items: monthly, seasonally adjusted data. Period: 1987:8-2004:5. Source: U.S. Department of Labor, Bureau of Labor Statistics.
- CPI for all urban consumers, all items: monthly data. Period: 1987:8-2002:4. Source: U.S. Department of Labor, Bureau of Labor Statistics.

Interest rate:

- Federal funds rate: monthly data. Period: 1987:8-2004:5. Source: Board of Governors of the Federal Reserve System.

APPENDIX 3

This appendix describes how to obtain the solution for the NKM model under the optimal monetary plan. The solution is found by solving the following matrix system:

$$\Gamma_0^o \xi_t^o = \Gamma_1^o \xi_{t-1}^o + \Psi^o \epsilon_t^o + \Pi^o \eta_t^o \quad (\text{A.1})$$

where the superscript “o” stands for the NKM model under the *optimal* monetary plan and

$$\xi_t^o = (y_t, \pi_t, i_t, E_t y_{t+2}, E_t y_{t+1}, E_t \pi_{t+1}, E_t \mu_{1t+1}, E_t \mu_{2t+1}, \mu_{1t}, \mu_{1t-1}, \mu_{2t}, g_t, z_t)'$$

$$\epsilon_t^o = (\epsilon_{gt}, \epsilon_{zt})'$$

$$\eta_t^o = (E_t[y_{t+1}] - E_{t-1}[y_{t+1}], y_t - E_{t-1}[y_t], \pi_t - E_{t-1}[\pi_t], \mu_{2t} - E_{t-1}[\mu_{2t}], \mu_{1t} - E_{t-1}[\mu_{1t}])'$$

$$\Gamma_0^\circ = \begin{pmatrix} -a_0 & 0 & -\tau & a_3 & a_2 & \tau & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{\kappa}{1+\beta\omega} & -1 & 0 & 0 & 0 & \frac{\beta}{1+\beta\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & \frac{\omega\beta}{1+\beta\omega} & 0 & \frac{\tau}{\beta} & -1 & 0 & 0 \\ -\lambda_y & 0 & 0 & 0 & 0 & 0 & a_1\beta & 0 & -a_0 & \frac{a_2}{\beta} & \frac{\kappa}{1+\beta\omega} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{\tau}{\lambda_i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma_1^\circ = \begin{pmatrix} -a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\omega}{1+\beta\omega} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{1+\beta\omega} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{a_3}{\beta^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_z & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Pi^o = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Psi^o = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Notice that the system (A.1) is composed by equations (1), (2), (10), (11), (12), (4), (5) and the following appended identities:

$$\begin{aligned} E_t y_{t+1} &= E_{t-1} y_{t+1} + (E_t y_{t+1} - E_{t-1} y_{t+1}), \\ y_t &= E_{t-1} y_t + (y_t - E_{t-1} y_t), \\ \pi_t &= E_{t-1} \pi_t + (\pi_t - E_{t-1} \pi_t), \\ \mu_{2t} &= E_{t-1} \mu_{2t} + (\mu_{2t} - E_{t-1} \mu_{2t}), \\ \mu_{1t} &= E_{t-1} \mu_{1t} + (\mu_{1t} - E_{t-1} \mu_{1t}), \\ \mu_{1t-1} &= \mu_{1t-1}. \end{aligned}$$

These identities show up when implementing the simple rule suggested by Sims (2002): when terms of the form $E_t x_{t+s}$ appear, we simply make a

sequence of those variables and equation creations that involve one period forecast errors.

APPENDIX 4

This appendix describes how to obtain the solution for the empirical NKM model. Equations (13)-(16) can be written in a matrix system as (6) following the simple rule suggested by Sims (2002). This rule amounts to appending the following equations to system (13)-(16):

$$y_t = E_{t-1}y_t + (y_t - E_{t-1}y_t),$$

$$E_{t-1}y_t = E_{t-2}y_t + (E_{t-1}y_t - E_{t-2}y_t),$$

$$E_{t-1}\pi_t = E_{t-2}\pi_t + (E_{t-1}\pi_t - E_{t-2}\pi_t),$$

$$E_{t-1}\pi_{t+1} = E_{t-2}\pi_{t+1} + (E_{t-1}\pi_{t+1} - E_{t-2}\pi_{t+1}),$$

$$E_{t-1}\pi_{t+2} = E_{t-2}\pi_{t+2} + (E_{t-1}\pi_{t+2} - E_{t-2}\pi_{t+2}),$$

$$E_{t-1}\pi_{t+3} = E_{t-2}\pi_{t+3} + (E_{t-1}\pi_{t+3} - E_{t-2}\pi_{t+3}),$$

$$\pi_t = E_{t-1}\pi_t + (\pi_t - E_{t-1}\pi_t),$$

$$E_{t-1}i_t = E_{t-2}i_t + (E_{t-1}i_t - E_{t-2}i_t),$$

$$E_{t-1}i_{t+1} = E_{t-2}i_{t+1} + (E_{t-1}i_{t+1} - E_{t-2}i_{t+1}),$$

$$E_{t-1}i_{t+2} = E_{t-2}i_{t+2} + (E_{t-1}i_{t+2} - E_{t-2}i_{t+2}),$$

$$i_t = E_{t-1}i_t + (i_t - E_{t-1}i_t),$$

$$y_{t-1} = y_{t-1},$$

$$\pi_{t-1} = \pi_{t-1},$$

$$\pi_{t-2} = \pi_{t-2},$$

$$\pi_{t-3} = \pi_{t-3},$$

$$\dot{i}_{t-1} = \dot{i}_{t-1},$$

$$\dot{i}_{t-2} = \dot{i}_{t-2},$$

$$\dot{i}_{t-3} = \dot{i}_{t-3}.$$

Equations (13)-(16) together with these eighteen newly created identities can be written in matrix form as

$$\Gamma_0^e \xi_t^e = \Gamma_1^e \xi_{t-1}^e + \Psi^e \epsilon_t^e + \Pi^e \eta_t^e,$$

where the superscript “e” stands for *empirical* NKM model and²⁶

$$\begin{aligned} \xi_t^e &= (y_t, \pi_t, \dot{i}_t, r_t, E_{t-1}y_t, E_{t-1}y_{t+1}, E_{t-1}\pi_{t+4}, E_{t-1}\pi_{t+3}, E_{t-1}\pi_{t+2}, E_{t-1}\pi_{t+1}, E_{t-1}\pi_t, \\ &E_{t-1}\dot{i}_{t+3}, E_{t-1}\dot{i}_{t+2}, E_{t-1}\dot{i}_{t+1}, E_{t-1}\dot{i}_t, y_{t-1}, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \dot{i}_{t-1}, \dot{i}_{t-2}, \dot{i}_{t-3})', \\ \epsilon_t^e &= (z_t, g_t, \epsilon_{it})', \end{aligned}$$

$$\begin{aligned} \eta_t^e &= (y_t - E_{t-1}y_t, E_{t-1}y_t - E_{t-2}y_t, E_{t-1}\pi_t - E_{t-2}\pi_t, E_{t-1}\pi_{t+1} - E_{t-2}\pi_{t+1}, \\ &E_{t-1}\pi_{t+2} - E_{t-2}\pi_{t+2}, E_{t-1}\pi_{t+3} - E_{t-2}\pi_{t+3}, \pi_t - E_{t-1}\pi_t, \\ &E_{t-1}\dot{i}_t - E_{t-2}\dot{i}_t, E_{t-1}\dot{i}_{t+1} - E_{t-2}\dot{i}_{t+1}, E_{t-1}\dot{i}_{t+2} - E_{t-2}\dot{i}_{t+2}, \dot{i}_t - E_{t-1}\dot{i}_t)'. \end{aligned}$$

References

- [1] Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe (2003) “Backward-looking interest rate rules, interest rate smoothing and macroeconomic stability,” National Bureau of Economic Research Working Paper 9558.

²⁶ Γ_0^e and Γ_1^e are 22×22 matrices, so we do not show them to save space. These matrices and the GAUSS code for solving the empirical NKM model are available from the authors upon request.

- [2] Blanchard, Oliver J., and Charles M. Khan (1980) “The solution of linear difference models under rational expectations,” *Econometrica* 48, 1305-1313.
- [3] Cooley, Timothy F., and Lee E. Ohanian (1991) “The cyclical behavior of prices,” *Journal of Monetary Economics* 28, 25-60.
- [4] Den Haan, Wouter J. (2000) “The comovement between output and prices,” *Journal of Monetary Economics* 46, 3-30.
- [5] Den Haan, Wouter J., and Steven Sumner (2004) “The comovement between real activity and prices in the G7,” *European Economic Review* 48, 1333-1347.
- [6] Evans, Charles L., Chin T. Liu, and Genevieve Pham-Kanter (2002) The 2001 Recession and the Chicago Fed National Activity Index: Identifying Business Cycle Turning Points, *Economic Perspectives*, Federal Reserve Bank of Chicago 26, third quarter, 26-43.
- [7] Fuhrer, Jeffrey C. (2000) “Habit formation in consumption and its implications for monetary policy models,” *American Economic Review* 90, 367-390.
- [8] Galí, Jordi (2002) “New perspectives on monetary policy, inflation and the business cycle,” National Bureau of Economic Research Working Paper 8767.
- [9] Galí, Jordi, and Mark Gertler (1999) “Inflation dynamics: a structural econometric analysis,” *Journal of Monetary Economics* 44, 195-222.
- [10] Giannoni, Marc P., and Michael Woodford (2003) “How forward-looking is optimal monetary policy?,” *Journal of Money, Credit, and Banking* 35, 1425-1469.
- [11] Goodfriend, Marvin (1991). “Interest Rates and the Conduct of Monetary Policy”, *Carnegie-Rochester Conference Series on Public Policy* 24, 231-274.
- [12] Goodfriend, Marvin and Robert G. King (1997) “The New Neoclassical synthesis and the role of monetary policy,” *Macroeconomics Annual* 231-283.
- [13] King, Robert G., and Mark Watson (1998) “The solution of singular linear difference systems under rational expectations,” *International Economic Review* 39, 1015-1026.

- [14] Klein, Paul (2000) "Using the generalized Schur form to solve a multivariate linear rational expectations model," *Journal of Economic Dynamics and Control* 24, 1405-1423.
- [15] Kydland, Finn E., and Edward. C. Prescott (1977) "Rules rather than discretion: the inconsistency of optimal plans," *Journal of Political Economy* 85, 473-491.
- [16] Kydland, Finn E., and Edward. C. Prescott (1990) "Business cycles: real facts and a monetary myth," Federal Reserve Bank of Minneapolis *Quarterly Review* 14, 3-18.
- [17] Lowe, P. and Ellis, L. (1997) "The smoothing of official interest rates" in Lowe, P.(ed.) *Monetary Policy and Inflation Targeting Proceedings of a Conference*, Reserve Bank of Australia.
- [18] Lubik, Thomas A., and Frank Schorfheide (2003) "Computing sunspot equilibria in linear rational expectations models," *Journal of Economic Dynamics and Control* 28, 273-285.
- [19] Lubik, Thomas A., and Frank Schorfheide (2004) "Testing for indeterminacy: an application to U.S. monetary policy," *American Economic Review* 94, 190-217.
- [20] Mankiw, N. Gregory (2001) "The inexorable and mysterious trade-off between inflation and unemployment," *Economic Journal* 111, C45-61.
- [21] McCallum, Bennett T., and Edward Nelson (1999) "An optimizing IS-LM specification for monetary policy and business cycle analysis," *Journal of Money, Credit, and Banking* 31, 296-316.
- [22] Rotemberg, Julio J., and Michael Woodford (1995) "Dynamic general equilibrium models with imperfectly competitive product markets," in Timothy F. Cooley (ed.) *Frontiers of Business Cycle Research*, Princeton: Princeton University, pp. 243-293.
- [23] Rotemberg, Julio J., and Michael Woodford (1997) "An optimizing-based econometric model for the evaluation of monetary policy," *NBER Macroeconomics Annual*, Cambridge, MA: MIT Press, pp. 297-346.
- [24] Rudebusch, Glenn D. (2002) "Term structure evidence on interest rate smoothing and monetary policy inertia," *Journal of Monetary Economics* 49, 1161-1187.

- [25] Sack, Brian (1997) “Uncertainty and gradual monetary policy,” *Federal Reserve Board, mimeo*.
- [26] Sims, Christopher A. (2001) “Solving linear rational expectations models,” *Computational Economics* 20, 1-20.
- [27] Stock, James, and Mark Watson (1999) “Forecasting inflation,” *Journal of Monetary Economics* 44, 293-335.
- [28] Svensson, L.(1997) “Inflation forecast targeting: implementing and monitoring inflation targets”, *European Economic Review* 41, 1111-1146.
- [29] Vázquez, Jesús (2002a) “The comovement between output and prices in the EU15 countries: An empirical investigation,” *Applied Economics Letters* 9, 957-966.
- [30] Vázquez, Jesús (2002b) “The comovement between output and inflation in the EU15 countries and the U.S.,” (in Spanish) in Joaquín Auriolés y Elena Manzanera (eds.) *Cuestiones Clave de la Economía Española*, Centra, Sevilla: Ediciones Pirámide, pp. 145-161.
- [31] Woodford, Michael (2003) “Optimal interest-rate smoothing,” *Review of Economic Studies* 70, 861-886.
- [32] Yun, Tack (1996) “Nominal price rigidity, money supply endogeneity, and business cycles,” *Journal of Monetary Economics* 37, 345-370.