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Trend Inflation and Firms Price-Setting: Rotemberg vs. Calvo

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Trend Inflation and Firms Price-Setting: Rotemberg vs. Calvo[*]

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Abstract

We compare two widely used pricing assumptions in the New-Keynesian literature: the Calvo and Rotemberg price-setting mechanisms. We show that, once trend inflation is taken into account, the two models are very different. i) The long-run relationship between inflation and output is positive in the Rotemberg model and negative in the Calvo model. ii) The log-linearized NKPCs are very different and the dynamics of the two models differs even to a first order approximation. iii) Positive trend inflation enlarges the determinacy region in the Rotemberg model, while it shrinks the determinacy region in the Calvo model. iv) The responses of output and inflation to a positive technology shock are amplified by trend inflation in Calvo, while they are damped in Rotemberg. v) The two models imply a different non-linear adjustment after a disinflation.

JEL classification: E31, E5.

Keywords: Firms Pricing, Trend Inflation, Determinacy, Disinflation.

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1 Introduction

We consider the two most commonly used approaches to model firms' price-setting behavior within the standard New Keynesian framework of monopolistically competitive firms: the Rotemberg (1982) quadratic cost of price adjustment and the Calvo (1983) random price adjustment signal. The Calvo price-setting mechanism produces relative-price dispersion among firms, while the Rotemberg model is consistent with a symmetric equilibrium. Despite the economic difference between these two pricing specifications, the literature has pointed out that to a first order approximation the implied dynamics are equivalent. As shown by Rotemberg (1987) and Roberts (1995), both approaches imply the same reduced form New Keynesian Phillips Curve (NKPC henceforth). They therefore lead to observationally equivalent dynamics for inflation and output. In particular, both models deliver the well-known result of immediate adjustment of the economy to the new steady state following a disinflation, despite nominal rigidities in price-setting (see, e.g., Ball, 1994 and Mankiw, 2001). Furthermore Nisticò (2007), shows that up to a second order approximation, and provided that the steady state is efficient, both models imply the same welfare costs of inflation. Thus, they imply the same prescriptions for welfare-maximizing Central Banks. Therefore, to the best of our knowledge, but for some exceptions, there is widespread agreement in the literature that the two models are almost equivalent and that up to a first order they imply the same dynamics.[1]

In this work, we show that once trend inflation is taken into account the dynamics of the Calvo and the Rotemberg model differs both quantitatively and qualitatively.[2] Hence, the way in which trend inflation affects the dynamics of a log-linearized New Keynesian model is particularly sensible to the choice of the price-setting mechanisms.

This discrepancy exactly derives from the different kind of nominal rigidities underlying the two models. The Calvo mechanism creates a price dispersion term in the model. The price dispersion term generates a wedge between output and hours and for its backward-looking behavior introduces an inertial mechanism in the model. The Rotemberg model, instead, assumes a quadratic cost of changing prices, that generates a wedge between output and consumption without introducing any inertial mechanism. If trend inflation is zero, these two wedges vanishes and the two models are equivalent up to first-order. Both these wedges,

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[1] The only two exceptions are Kahn (2005) and Lombardo and Vestin (2008). Kahn (2005) shows that even if the reduced form New Keynesian Phillips curve is the same, the impact of competition on the slope of the NKPC differs between the two approaches. Lombardo and Vestin (2008) and Damjanovic and Nolan (2009) show that the two models might imply different welfare costs at a second order of approximation.

[2] We use trend inflation and steady state inflation as synonymous.

however, are quite sensitive to inflation, and then they induce a difference in the two models whenever trend inflation is positive. Trend inflation, hence, brings naturally to light the different implications of the two types of nominal rigidities. Five main results follow.

First, trend inflation has an opposite effect on the long-run relationship between inflation and output in the two models. While the long-run NKPC is negatively sloped in the Calvo model, it is positively sloped in the Rotemberg model.

Second, the log-linear NKPCs implied by the two models are radically different once the model is log-linearized around a generic steady state inflation level. On the one hand, the price dispersion term in the Calvo model generates a backward-looking variable that is absent in the Rotemberg model. On the other hand, the price adjustment term in the Rotemberg model makes inflation to enter the marginal costs.

Third, trend inflation has opposite effects on the determinacy conditions of the two models. Contrary to the Calvo model, where an increase in trend inflation shrinks the determinacy region,[3] positive trend inflation enlarges the determinacy area in the Rotemberg model. This means that when we look for the optimal and implementable rules, e.g., Schmitt-Grohé and Uribe (2007b), the set of the possible rules is going to depend on the pricing assumption. Rules that can be optimal and implementable under Rotemberg pricing, thus, could be not implementable under Calvo.

Fourth, trend inflation has opposite effects on the responses of output and inflation to a positive technology shock. In the Calvo model, the higher is trend inflation the higher are both the decrease in inflation and the increase in output following a positive technology shocks. In the Rotemberg model, the higher is trend inflation the lower are both the decrease in inflation and the increase in output.

Fifth, the two pricing assumptions imply also a different dynamics after a disinflation. As some papers have recently shown (e.g. Ascari 2004, Yun 2005, Ascari and Merkl 2009) non-linear simulations are important because the interplay between long-run effects and short-run dynamics is crucial in the adjustment path after a disinflation. Contrary to the common view, this interaction leads to different results between the implied non-linear dynamics of the Rotemberg and the Calvo model in response to a Central Bank disinflation experiment. Ascari and Merkl (2009) show that in the Calvo model a credible disinflation implies an inertial adjustment (due to the backward-looking price dispersion term) and leads to a permanently higher level of output in the non-linear model. The non-linear dynamics of the Rotemberg

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[3] Ascari and Ropele (2009) show that, under the Calvo model, trend inflation has substantial effects on the well know Taylor principle for determinacy of the rational expectation equilibrium.

model implies that output immediately adjusts to a permanently lower level.

Price indexation may play an important role in the model, above all when trend inflation is high. For this reason, in section five, we consider the effects of price indexation on the long-run properties and on the dynamics of the two pricing models. We show that price indexation dampens the effects of trend inflation in both models and therefore reduces the differences between the two pricing mechanisms. In the particular case of full price indexation the two models are again equivalent as in the case of zero trend inflation. Therefore, the two models imply the same dynamics under two extreme cases: i) the steady state inflation level is zero; ii) prices are fully indexed. Both these two assumptions seems to be against the recent empirical evidence.[4]

The literature on trend inflation, both the theoretical and the empirical one, so far concentrates on NK model with Calvo pricing. Ascari (2004) shows that both the long-run and the short-run properties of DSGE-NK models based on the Calvo staggered price model change dramatically in presence of a trend inflation term. Yun (2005) shows that optimal inflation targets respond to changes in the level of relative price distortion in the presence of initial price dispersion due to trend inflation. Schmitt-Grohé and Uribe (2007a) find a positive relationship between trend inflation and price dispersion. Amano et al. (2007) numerically study the macroeconomic effects of trend inflation and compare three common time-dependent pricing schemes: Calvo, truncated-Calvo, and Taylor. They show that, regardless the price setting mechanism, as trend inflation increases the stochastic means of output, consumption and employment decreases, while the mean of inflation increases. Moreover, they show that the variability of most aggregate variables increases with trend inflation. Damjanovic and Nolan (2010) show that a contractionary monetary shock has a persistent, negative hump-shaped impact on inflation and a positive hump-shaped impact on output. They quantify the utility cost of price dispersion and its impact on optimal monetary policy. Overall, the theoretical literature shows the importance of trend inflation, demonstrating that the results obtained when the model is log-linearized around a zero inflation steady state can be quite misleading.

Moreover, the assumption of nonzero trend inflation is supported by the empirical evidence. First, a low and positive trend inflation seems to be much more realistic, as the post-war economic history of industrialized countries shows. Furthermore, the practice of many central banks suggests that a zero inflation steady state is not an actual target (see for example Sargent et al. 2006 and Primiceri 2006). Cogley and Sbordone (2008) estimates a purely forward-looking Phillips Curve, allowing for shifts in trend inflation. They find that it successfully

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[4] See Sargent et al. (2006), Primiceri (2006), Cogley and Sbordone (2008), Benati (2008) and Benati (2009).

describes US inflation dynamics. Hence, backward-looking price indexation is not necessary to fit the data once trend inflation is taken into account. Benati (2009) finds similar results for other industrialized countries. Benati (2008) finds empirical evidence about the relationship between inflation persistence and trend inflation. Using international data, Benati (2008) shows that persistence has fallen, whenever countries have adopted an explicit inflation target, thereby reducing the average level of inflation. Thus the empirical evidence suggests that the inflation gap is a purely forward looking variable, while the main source of inflation persistence is related to trend inflation.

Therefore, the NK literature cannot disregard the role of trend inflation. The literature on trend inflation, however, so far focuses only on staggered price models (Calvo or Taylor). Despite the Rotemberg (1982) model of price rigidity is widely employed in the NK literature, no work has been done on the effects of trend inflation in such a framework. This is what we do in this paper.

The paper is organized as follows. Section 2 describes the basic New Keynesian model under the two-pricing assumptions. Section 3 presents the log-linear approximation of the models around the very particular case of a zero inflation steady state. Section 4 compare the long-run properties and the dynamics of the two pricing-models under a generic value of trend inflation. Section 5 discusses the role of price indexation. Section 6 discusses an alternative way to introduce the adjustment costs in the Rotemberg model. A final Section concludes.

2 A basic model

In this section we briefly present a very simple and standard cashless New Keynesian model in the two versions of Rotemberg and the Calvo price setting scheme. The model economy is composed of a continuum of infinitely-lived consumers, producers of final and intermediate goods.

2.1 Households and Technology

Consider an economy with a representative household which maximizes the following intertemporal separable utility function

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{C_{t+j}^{1-\sigma}}{1-\sigma} - d_n \frac{N_{t+j}^{1+\phi}}{1+\phi} \right] \tag{1}$$

subject to the period-by-period budget constraint

$$P_tC_t + (1+i_t)^{-1} B_t = W_tN_t - T_t + \Pi_t + B_{t-1}, \tag{2}$$

where C_t is consumption, i_t is the nominal interest rate, B_t are one-period bond holdings, W_t is the nominal wage rate, N_t is the labor input, T_t are lump sum taxes, and Π_t is the profit income. The following first order conditions hold

$$\text{Euler equation :} \quad \frac{1}{C_t^{\sigma}} = \beta E_t\left[\left(\frac{P_t}{P_{t+1}}\right)(1+i_t)\left(\frac{1}{C_{t+1}^{\sigma}}\right)\right], \tag{3}$$

$$\text{Labor supply equation} \quad \frac{W_t}{P_t} = -\frac{U_N}{U_C} = \frac{d_nN_t^{\phi}}{1/C_t^{\sigma}} = d_nN_t^{\phi}C_t^{\sigma}. \tag{4}$$

Final good market is competitive and the production function is given by

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{5}$$

Final good producers demand for intermediate inputs is therefore equal to $Y_{i,t+j} = \left(\frac{P_{i,t}}{P_{t+j}}\right)^{-\varepsilon} Y_{t+j}$.

Intermediate inputs $Y_{i,t}$ are produced by a continuum of firms indexed by $i \in [0,1]$ with the following simple linear technology

$$Y_{i,t} = A_tN_{i,t} \tag{6}$$

where labor is the only input and $\ln A_t = a_t$ is an exogenous productivity shock, which follows an AR(1) process

$$a_t = \rho_a a_{t-1} + v_{a,t} \tag{7}$$

$v_{a,t} \sim WN\left(0, \sigma_v^2\right)$. The labor demand and the real marginal cost of firm i are therefore

$$N_{i,t}^d = \frac{Y_{i,t}}{A_t}, \tag{8}$$

and

$$MC_t^r = \frac{W_t}{P_tA_t}. \tag{9}$$

Given our simple linear production function the marginal cost is the same across firms and simply equal to the productivity-adjusted real wage.

2.2 Price Setting: Rotemberg (1982) and Calvo (1983)

The intermediate good sector is monopolistically competitive and therefore the intermediate-good producer enjoy market power. In what follows we present the Rotemberg (1982) and the Calvo (1983) price-setting mechanisms.

The Rotemberg model

The Rotemberg model assumes that a monopolistic firm faces a quadratic cost of adjusting nominal prices, that can be measured in terms of the final-good and given by

$$\frac{\varphi}{2}\left(\frac{P_{i,t}}{P_{i,t-1}}-1\right)^2 Y_t, \tag{10}$$

where $\varphi > 0$ determines the degree of nominal price rigidity. As stressed in Rotemberg (1982), the adjustment cost accounts for the negative effects of price changes on the customer-firm relationship. These negative effects increase in magnitude with the size of the price change and with the overall scale of economic activity, Y_t. The problem for the firm is then

$$\max_{\{P_{i,t}\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} \mathcal{D}_{t,t+j}\left\{\left(\frac{P_{i,t+j}}{P_{t+j}}-MC^r_{t+j}\right)Y_{i,t+j}-\frac{\varphi}{2}\left(\frac{P_{i,t+j}}{P_{i,t+j-1}}-1\right)^2 Y_{t+j}\right\}, \tag{11}$$

$$\text{s.t. } Y_{i,t+j}=\left[\frac{P_{i,t+j}}{P_{t+j}}\right]^{-\varepsilon} Y_{t+j}. \tag{12}$$

where $\mathcal{D}_{t,t+j} \equiv \beta^j \frac{U_c(t+j)}{U_c(t)}$ is the stochastic discount factor, $MC^r_{t+j} = \frac{W_{t+j}}{P_{t+j}A_{t+j}}$ is the real marginal cost function.

Firms can change their price in each period, subject to the payment of the adjustment cost. Hence, all the firms face the same problem, and thus will choose the same price, producing the same quantity. In other words: $P_{i,t} = P_t, Y_{i,t} = Y_t$ and $\forall i$. Therefore, from the first order condition, after imposing the symmetric equilibrium, we get

$$1-\varphi\left(\pi_t-1\right)\pi_t+\varphi\beta E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left[\left(\pi_{t+1}-1\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]=\left(1-MC^r_t\right)\varepsilon. \tag{13}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$. Since all the firms will employ the same amount of labor, the aggregate production function is simply given by

$$Y_t = A_t N_t. \tag{14}$$

The aggregate resource constraint should take the adjustment cost into account, that is

$$Y_t = C_t + \frac{\varphi}{2}\left(\pi_t-1\right)^2 Y_t. \tag{15}$$

For what follows, it is important to note that the Rotemberg adjustment cost model creates an inefficiency wedge, Ψ_t, between output and consumption[5]

$$Y_t=\left[\frac{1}{1-\frac{\varphi}{2}\left(\pi_t-1\right)^2}\right]C_t=\Psi_t C_t. \tag{16}$$

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[5] Note that this expression implicitly defines the condition $1 > \frac{\varphi_p}{2}\left(\pi_t-1\right)^2$ for the model to be well-defined, that is: $\pi_t \in \left(1-\sqrt{\frac{2}{\varphi_p}};\ 1+\sqrt{\frac{2}{\varphi_p}}\right)$.

The Calvo model

The Calvo model assumes that each period there is a fixed probability $1-\theta$ that a firm can re-optimize its nominal price, i.e., $P^*_{i,t}$. The price setting problem becomes

$$\max_{\{P_{i,t}\}_{t=0}^{\infty}} \quad E_t \sum_{j=0}^{\infty} \mathcal{D}_{t,t+j} \theta^j \left[\frac{P^*_{i,t}}{P_{t+j}} - MC^r_{t+j} \right] Y_{i,t+j}, \tag{17}$$

$$\text{s.t. } Y_{i,t+j} = \left[\frac{P^*_{i,t}}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j}. \tag{18}$$

The equation for the optimal price is

$$P^*_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \mathcal{D}_{t,t+j} P^{\varepsilon}_{t+j} Y_{t+j} MC^r_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j \mathcal{D}_{t,t+j} P^{\varepsilon-1}_{t+j} Y_{t+j}}, \tag{19}$$

while the aggregate price dynamics is given by

$$P_t = \left[\theta P^{1-\varepsilon}_{t-1} + (1-\theta) \left(P^*_{i,t} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{20}$$

In the Calvo price setting framework, firms charging prices in different periods will generally have different prices. Thus, the model features a distribution of different prices, that is, there will be price dispersion. Price dispersion results in an inefficiency loss in aggregate production. In fact

$$N^d_t = \int_0^1 N^d_{i,t} di = \int_0^1 \frac{Y_{i,t}}{A_t} di = \frac{Y_t}{A_t} \underbrace{\int_0^1 \left[\left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} di \right]}_{s_t} = s_t \frac{Y_t}{A_t}. \tag{21}$$

Schmitt-Grohé and Uribe (2007a) show that s_t is bounded below at one, so that s_t represents the resource costs due to relative price dispersion under the Calvo mechanism. Indeed, the higher s_t, the more labor is needed to produce a given level of output. Moreover, remember that price dispersion is a backward-looking variable, and therefore it introduces an inertial component in the model.

To close the model, the aggregate resource constraint is simply given by

$$Y_t = C_t. \tag{22}$$

In the *Rotemberg* model, the cost of nominal rigidities, i.e., the adjustment cost, creates *a wedge between aggregate consumption and aggregate output*, because part of the output goes in the price adjustment cost. In the *Calvo* model, instead, the cost of nominal rigidities, i.e., price dispersion, creates *a wedge between aggregate hours and aggregate output*, making aggregate production less efficient and introducing an inertial component in the model.

Note that both of these wedges in equations (16) and (21) are *non-linear functions of inflation.* Moreover, they behave very similarly in steady state. Both wedges are minimized at one when steady state inflation equals zero, and both wedges increase as trend inflation moves away from zero.

It is very important to stress that these wedges can vanish in particular cases. In the Rotemberg model, the wedge Ψ_t in (16) equals one when inflation is zero, because firms are not changing their prices and thus there is no adjustment cost to pay. In the Calvo model, the wedge s_t in (21) equals one, when there is no price dispersion, that is, when all the firms have the same price. There is one special case in which both these conditions hold: the zero inflation steady state case.

3 A very particular case: zero steady state inflation

It is well known[6] that the two models deliver equivalent dynamics when log-linearized around a zero inflation steady state. In fact, in this case Calvo-pricing yields the following New Keynesian Phillips curve (NKPC henceforth)

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \widehat{mc}_t, \tag{23}$$

where lower case hatted letters denote log-deviations of the variable with respect to its steady state value. Similarly, under Rotemberg-pricing to a first order approximation the NKPC is

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\varepsilon - 1}{\varphi} \widehat{mc}_t. \tag{24}$$

Therefore, up to a first order approximation the two models are identical, apart the coefficient of the slope of the NKPC. Note that, by imposing

$$\frac{\varepsilon - 1}{\varphi} = \frac{(1-\theta)(1-\beta\theta)}{\theta} \tag{25}$$

and therefore by setting $\varphi = \frac{(\varepsilon-1)\theta}{(1-\theta)(1-\beta\theta)}$, the two models imply the same first-order dynamics.

The zero trend inflation, however, is a very peculiar case. In fact, when the steady state level of inflation is equal to zero, the difference between the two models cancels out. The reason why it happens is that, in this case, the two wedges in equations (16) and (21) disappear, because in steady state $\pi = s = 1$. This is not very surprising, since the zero inflation steady state of both models is equivalent to the steady state of the flexible price version of the model. In all the other, more interesting and realistic, cases the two models entail a different dynamics. The next section will thoroughly investigate these differences.

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[6] See for example Rotemberg (1987), Roberts (1995) and more recently Nisticò (2007) and Lombardo and Vestin (2008).

4 Rotemberg and Calvo under Trend Inflation

In this section, we investigate how the two models differ regarding: (i) the long-run relationship between output and inflation; (ii) the New Keynesian Phillips Curve; (iii) the dynamic response to shocks; (iv) the determinacy properties; (v) the dynamic response to a disinflation. Many results will be analytical, while some will be visualized through numerical simulations.

Calibration In the Figures below in this section, the calibration considers the following rather standard parameters specification: $\sigma = 1$, $\beta = 0.99$, $\varepsilon = 10$, $\phi = 1$, $\theta = 0.75, \varphi = \frac{(\varepsilon-1)\theta}{(1-\theta)(1-\beta\theta)}$, unless explicitly stated. However, none of the Figures qualitatively depends on the parameters values.

4.1 The long-run Phillips Curve

This section investigates the non-linear long-run Phillips curve implied by the two price-setting mechanisms. To understand the differences in the dynamics of the two models, it is necessary to first analyze their steady state properties. In fact, at the root of the different effects of trend inflation on the dynamics of the two price-setting models, lies the fact that trend inflation affects the steady state properties of the two models in two different ways. As we will see, in the Rotemberg model the higher is trend inflation the higher is the steady state level of output, while in the Calvo model the opposite holds.

The Rotemberg model

The Appendix A.2 shows that the long-run Phillips Curve in the Rotemberg model is equal to

$$Y = \left[\frac{\frac{\varepsilon-1}{\varepsilon} + \frac{(1-\beta)}{\varepsilon}\varphi\left(\bar{\pi}^{1-\chi} - 1\right)\bar{\pi}^{1-\chi}}{d_n\left(1 - \frac{\varphi}{2}\left(\bar{\pi}^{1-\chi} - 1\right)^2\right)^{\sigma}} \right]^{\frac{1}{\phi+\sigma}} . \tag{26}$$

Appendix A.2 proves that (if $\beta < 1$)

$$\exists\bar{\pi}^* < 1 \quad s.t. \begin{cases} \bar{\pi} > \bar{\pi}^* \Longrightarrow \frac{dY}{d\bar{\pi}} > 0 \\ \bar{\pi} = \bar{\pi}^* \Longrightarrow \frac{dY}{d\bar{\pi}} = 0 \\ \bar{\pi} < \bar{\pi}^* \Longrightarrow \frac{dY}{d\bar{\pi}} < 0 \end{cases} .$$

Note that this implies that for $\bar{\pi} \geqq 1 \Longrightarrow \frac{dY}{d\bar{\pi}} > 0$, i.e., the higher is trend inflation the more output is produced. The minimum of output occurs at negative rate of steady state inflation, unless $\beta = 1$, in which case $\frac{dY}{d\bar{\pi}} = 0$ for $\bar{\pi} = 1$.

The intuition is straightforward by rewriting the steady state output level as

$$Y = \left(\frac{\Psi^{\sigma}}{d_n \frac{P}{MC}} \right)^{\frac{1}{\phi+\sigma}}, \tag{27}$$

where $\frac{P}{MC}$ is the average markup. Equation (27) shows that there are two effects at works: 1) the "*average markup effect*", due to time discounting and 2) the "*wedge effect*". Both effects go in the same direction of increasing the steady state output. First of all, consider the "*average markup effect*": in changing their price, firms weight today adjustment cost of moving away from yesterday price, relatively more than the tomorrow adjustment cost of fixing a new price away from the today's one, because of discounting. Trend inflation thus reduces the average mark-up. Indeed, the steady state mark-up is given by

$$\frac{P}{MC} = \left[\frac{\varepsilon - 1}{\varepsilon} + \frac{(1-\beta)}{\varepsilon} \varphi \left(\bar{\pi} - 1 \right) \bar{\pi} \right]^{-1} \tag{28}$$

which is monotonically decreasing in $\bar{\pi}$ (for economically relevant values of $\bar{\pi}$). The fact that the mark-up decreases with trend inflation makes output to increase with trend inflation. Secondly, regarding the "*wedge effect*", note that the price adjustment cost increases with trend inflation and so does the wedge Ψ, therefore, given (27), the wedge has a positive effect on output. However, a fraction of output is not consumed, but it is eaten up by the adjustment cost. Given that the wedge between output and consumption, (16), increases with trend inflation, consumption decreases with trend inflation in steady state. Thus, output and hours are increasing with trend inflation, but consumption and welfare are decreasing with trend inflation.

- Figure 1 about here -

The Calvo model

Figure 2 shows the long-run relationship between inflation and output in the standard Calvo model.

- Figure 2 about here -

As well-known (e.g., Ascari 2004, Yun 2005), the long-run Phillips Curve is negatively sloped: positive long-run inflation reduces output, because it increases price dispersion, s. Higher price dispersion acts as a negative productivity shift, because $Y = \frac{AN}{s}$. Thus, the steady state real wage lowers with trend inflation, and so does consumption, while hours increase. As a consequence, steady state welfare decreases.

To grasp the intuition, it is useful to rewrite steady state output level as[7]

$$Y = \left(\frac{1}{d_n \frac{P}{MC} s^{\phi}} \right)^{\frac{1}{\phi+\sigma}} . \tag{29}$$

The symmetry between (29) and (27) in the two models makes the comparison clear. Also in the Calvo model, thus, there are two effects at works: 1) the "*average markup effect*" due to time discounting and 2) the "*wedge effect*". In this case however the two effects go in the opposite direction. The positive slope is due to an "*average markup effect*" similar to the one described above: in setting the new price, firms discount the future, where nominal prices are higher because of trend inflation. Hence, the average mark-up decreases with trend inflation. However, the relationship between steady state output and inflation is non-linear, through the "*wedge effect*" due to price dispersion, s. The effects of non-linearities due to price dispersion are quite powerful and turn up very quickly as trend inflation increases from zero, inverting the relationship from positive to negative.[8] Therefore, while in the Rotemberg model there is no price dispersion that interacts with trend inflation, and both the "*average markup effect*" and the "*wedge effect*" affect the steady state output in the same way, in the Calvo model the price dispersion term inverts very quickly the slope of the long-run Phillips Curve. Thus, for positive trend inflation, this slope is positive in the Rotemberg model, and (mostly) negative in the Calvo model.

The next sections show how the opposite slope of the long-run Phillips Curve between the two models determines their different dynamic properties.

4.2 The generalized NKPC

As we saw above, the relation between trend inflation and the steady state values of the variables is generally non-linear. Therefore, the steady state around which to log-linearize matters for the dynamics of the model. Indeed, we now show that the way in which trend inflation affects the coefficients of the log-linearized equations depends on the specific pricing

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[7] See Ascari and Merkl (2009) for a derivation.

[8] To be more precise, the derivative of the long-run Phillips Curve evaluated at zero inflation, i.e., the tangent at zero inflation of the curve depicted in Figure 1, is positive. Only the "average markup effect" is present in this case. Indeed, this positive slope equals the positive long-run relationship between inflation and output implied by the standard log-linear New Keynesian Phillips Curve (23) popularized by Woodford (2003) among others. See also King and Wolman (1996) and Graham and Snower (2004) .

assumption.

The Rotemberg model

The log-linearization of equation (13) yields the following generalized NKPC under Rotemberg pricing

$$\hat{\pi}_t = \gamma_f \beta \hat{\pi}_{t+1} + \gamma_{dy} \beta \left(1-\sigma\right) \Delta \hat{y}_{t+1} + \gamma_{mc} \widehat{mc}_t \tag{30}$$

and

$$\widehat{mc}_t = \left(\sigma + \phi\right) \hat{y}_t - \varsigma_c \sigma \hat{\pi}_t - \left(1+\phi\right) a_t. \tag{31}$$

are the log-linearized real marginal costs. Moreover, log-linearizing equations (3), (4), (14), (16) and combining them together delivers the following log-linearized IS curve,

$$\hat{y}_t = E_t \hat{y}_{t+1} - \varsigma_c \Delta E_t \hat{\pi}_{t+1} - \frac{1}{\sigma} E_t \left(\hat{\imath}_t - \hat{\pi}_{t+1}\right) \tag{32}$$

γ_f, γ_{dy}, γ_{mc} and ς_c are complicated convolution parameters that depend on trend inflation,

$$\begin{aligned}
\varsigma_c &= \frac{\varphi\left(\bar{\pi}-1\right)\bar{\pi}}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}-1\right)^2\right]}, \\
\frac{C}{Y} &= \left(1-\frac{\varphi}{2}\left(\bar{\pi}-1\right)^2\right) \\
\varrho &\equiv \left(2\bar{\pi}^2-\bar{\pi}\right)\frac{C}{Y} + \beta\left[\left(\bar{\pi}-1\right)\bar{\pi}\right]^2 \sigma\varphi, \\
\gamma_p &= \frac{\left(2\bar{\pi}^2-\bar{\pi}\right)\frac{C}{Y} + \beta\left[\left(\bar{\pi}-1\right)\bar{\pi}\right]^2 \sigma\varphi}{\varrho}, \\
\gamma_f &= \frac{\left(2\bar{\pi}^2-\bar{\pi}\right)\frac{C}{Y} + \left[\left(\bar{\pi}-1\right)\bar{\pi}\right]^2 \sigma\varphi}{\varrho} \\
\gamma_{dy} &= \frac{\left(\bar{\pi}^2-\bar{\pi}\right)\frac{C}{Y}}{\varrho}, \\
\gamma_{mc} &= \frac{\left(\varepsilon-1+\varphi\left(\bar{\pi}^2-\bar{\pi}\right)\left(1-\beta\right)\right)\frac{C}{Y}}{\varphi\varrho}.
\end{aligned}$$

Equation (30) encompasses the standard NKPC, because, under a zero steady state inflation (i.e., $\bar{\pi}=1$), $\varsigma_c = \gamma_{dy} = 0$, $\gamma_f = 1$, and $\gamma_{mc} = \frac{\varepsilon-1}{\varphi}$, so that equation (30) boils down to (24).

The Calvo model

As shown by Ascari and Ropele (2009) the log-linearization of the Calvo model is described by the following first-order difference equations: [9]

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[9] For a detailed derivation and description of the reduced form solution of the Calvo model under trend inflation see Ascari and Ropele (2009). See also Cogley and Sbordone (2008).

$$\begin{aligned}
\Delta_t &= \left[\beta\bar{\pi}^{1-\chi} + \eta\left(\theta - 1\right)\right] E_t\Delta_{t+1} + \kappa\hat{y}_t - \lambda\phi a_t + \lambda\phi\hat{s}_t + \eta E_t\hat{\psi}_{t+1}, && (33)\\
\hat{\psi}_t &= \left(1-\sigma\right)\left(1-\theta\beta\bar{\pi}^{(\varepsilon-1)(1-\chi)}\right)\hat{y}_t + \theta\beta\bar{\pi}^{(\varepsilon-1)(1-\chi)}\left[\left(\varepsilon-1\right)E_t\Delta_{t+1} + E_t\hat{\psi}_{t+1}\right], && (34)\\
\hat{s}_t &= \xi\Delta_t + \theta\bar{\pi}^{\varepsilon(1-\chi)}\hat{s}_{t-1}, && (35)\\
\hat{y}_t &= E_t\hat{y}_{t+1} + \hat{y}_{t-1} - \sigma^{-1}\left(\hat{\imath}_t - E_t\hat{\pi}_{t+1}\right), && (36)
\end{aligned}$$

where $\Delta_t \equiv \hat{\pi}_t - \chi\mu\hat{\pi}_{t-1}$, and $\hat{\psi}_t$ is an auxiliary forward-looking variable, λ, η, κ, and ξ are complicated convolution parameters that depend on trend inflation,

$$\begin{aligned}
\lambda &\equiv \frac{\left(1-\theta\bar{\pi}^{(\varepsilon-1)(1-\chi)}\right)\left(1-\theta\beta\bar{\pi}^{\varepsilon(1-\chi)}\right)}{\theta\bar{\pi}^{(\varepsilon-1)(1-\chi)}},\\
\eta &\equiv \beta\left(\bar{\pi}^{1-\chi}-1\right)\left[1-\theta\bar{\pi}^{(\varepsilon-1)(1-\chi)}\right],\\
\kappa &\equiv \lambda_{(\bar{\pi},\varepsilon)}\left(\sigma+\varphi\right) + \eta_{(\bar{\pi},\varepsilon)}\left(1-\sigma\right),\\
\xi &\equiv \frac{\varepsilon\theta\bar{\pi}^{(\varepsilon-1)(1-\chi)}\left(\bar{\pi}^{1-\chi}-1\right)}{1-\theta\bar{\pi}^{(\varepsilon-1)(1-\chi)}}.
\end{aligned}$$

Notice that, trend inflation alters the inflation dynamics compared to the usual Calvo model in three ways. Firstly, trend inflation enriches the dynamic structure by adding two new endogenous variables: a forward looking auxiliary variable, i.e., $\hat{\psi}_t$, and a predetermined variable, i.e., $\hat{s}_t$, which represents price dispersion. Secondly, trend inflation directly affects the NKPC coefficients. Higher trend inflation makes the NKPC more "forward-looking", leading to a smaller coefficient on current output and a larger coefficient on future expected inflation. The short-run NKPC, hence, flattens when drawn in the plane $(\hat{y}_t, \hat{\pi}_t)$. Thirdly, trend inflation increases the inertia of the equation of the relative price dispersion $\hat{s}_t$. This means that, *ceteris paribus,* higher trend inflation yields a more persistent adjustment of inflation rate.

The two log-linearized systems present three main differences. First of all, in the Calvo model the presence of a price dispersion wedge creates an endogenous predetermined variable in the NKPC, which is absent in the Rotemberg model. Secondly, in the Rotemberg model, the presence of price adjustment costs causes the real marginal cost to depend also on actual inflation (see the additional term $\varsigma_c\sigma\hat{\pi}_t$ in (31)). Finally, the price adjustment cost generates a wedge between output and consumption in the resource constraint, that appears in the IS curve as the additional term $\varsigma_c\Delta E_t\hat{\pi}_{t+1}$ (see (32)).

Not surprisingly these differences in the log-linear model will deliver different dynamic responses and determinacy properties.

4.3 The Dynamics

In this section we compare the dynamics of the two price setting models. We assume that the central bank follows a Taylor-type feedback rule and we study the responses of output and inflation to a technology shock. It is well-known that the dynamics of the two model will be equivalent under zero trend inflation. We investigate to what extent the dynamics will, instead, differ between the two models as trend inflation varies.

We simply assume that the central bank sets the short run nominal interest rate according to the following standard Taylor-type rule

$$\hat{\imath}_t = \alpha_\pi \hat{\pi}_t + \alpha_y \hat{y}_t. \tag{37}$$

and we set $\alpha_\pi = 1.5$ and $\alpha_y = 0.5/4$, in the simulation.

The Rotemberg model

Figure 3 shows the impulse response functions (IRFs henceforth) of output and inflation to a positive technology shock, for different values of trend inflation, when prices are set à la Rotemberg.

- Figures 3 about here -

As expected, in response to a positive technology shock output increases on impact while inflation decreases. Then, after some periods they return to their initial level. Note that, the higher trend inflation, the lower are both the decrease in inflation and the increase in output. The effects of varying trend inflation, however, are quantitatively minor. Moreover, also the persistence of output and inflation is substantially unaffected by the level of trend inflation.

The Calvo model

Figures 4 shows the IRF of output and inflation to a positive technology shock for different levels of trend inflation, in the Calvo model.

- Figures 4 about here -

As in the Rotemberg model, in response to a positive productivity shock output increases and inflation decreases. Then, after some periods they return to their initial level. Actually, the IRF coincides when the model is log-linearized around zero inflation. Unlike the Rotemberg model, however, the IRF are very sensitive to varying trend inflation in the Calvo model.

As trend inflation increases, the responses of output and inflation amplify and become more persistent. As shown in Ascari (2004), this happens because of the strong effects that trend inflation has on the coefficient of the NKPC in the Calvo model. Moreover, trend inflation increases the inertia in the dynamic equation of the relative price dispersion $\hat{s}_t$, which is a predetermined variable.[10]This means that, *ceteris paribus,* higher trend inflation yields a more persistent adjustment of the inflation rate. As a consequence also the response of output becomes more persistent. In the Rotemberg model, instead, there is no price dispersion and the model is completely forward looking.

Overall these results show that, if moderate levels of trend inflation are considered, the two models exhibit different dynamics in response to a productivity shock, even to a first order approximation. Trend inflation has opposite effects on the adjustment dynamics of output and inflation in the two models.

4.4 Determinacy and the Taylor Principle

To assess the determinacy of the rational expectations equilibrium (REE henceforth), we first substitute the Taylor rule (37) into the IS curve and then we write the structural equations in the following matrix format

$$x_t = \mathbf{A} E_t x_{t+1} + \mathbf{B} a_t, \tag{38}$$

where vector x_t includes the endogenous variables of the model while a_t is the technology shock. Determinacy of REE obtains if the standard Blanchard and Kahn (1980) conditions are satisfied. Next, we analyze how trend inflation affects the determinacy of REE.

The Rotemberg model

We first present the analytical derivation of our main results under Rotemberg pricing. Then, we compare our results with those obtained by Ascari and Ropele (2009) for the Calvo model. In order to derive simple analytical results, in this section we will assume that: $\phi = 0$, $\sigma = 1$, $\alpha_\pi \in [0, \infty)$, $\alpha_Y \in [0, \infty)$. In particular, we are able to state the following proposition[11]

Proposition 1. Necessary and sufficient conditions for determinacy of REE. *Let* $\phi = 0$, $\sigma = 1$, $\alpha_\pi \in [0, \infty)$, $\alpha_y \in [0, \infty)$ *and at least one different from zero. Determinacy*

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[10] In a recent paper, Damjanovic and Nolan (2010) show that, in a model with low trend inflation, a negative monetary shock can have a persistent and hump-shaped impact on output and a positive impact on inflation.

[11] In the Rotemberg model vector x_t in the representation (38) includes two non-predetermined variables, i.e., $x_t \equiv [\hat{y}_t, \hat{\pi}_t,]'$. Hence, determinacy of REE obtains if and only if all eigenvalues of $\mathbf{A}$ lie inside the unit circle.

of REE under positive trend inflation obtains if and only if

$$\alpha_\pi + \frac{\left(1 + \varsigma_c \gamma_{mc} - \beta \gamma_f\right)}{\gamma_{mc}} \alpha_y > 1, \tag{39}$$

where $\frac{\left(1+\varsigma_c \gamma_{cm} - \beta \gamma_f\right)}{\gamma_{cm}}$ *is the long-run elasticity of output to inflation* (see Appendix A.4).

With zero steady state inflation, i.e. with $\bar{\pi} = 1$, condition (39), becomes:

$$\alpha_\pi + \frac{1 - \beta}{\kappa} \alpha_y > 1, \tag{40}$$

where $\kappa = \frac{\varepsilon - 1}{\varphi}$ is the slope of the NKPC. We also know that in this particular case, by imposing condition (25), i.e., by imposing that the Rotemberg and the Calvo model coincides up to first order, then the conditions to ensure determinacy of REE are identical under the two pricing models. As stressed by Woodford (2001, 2003, see chp. 4.2.2) among others, condition (40) is a generalization of the standard Taylor principle: to ensure determinacy of REE the nominal interest rate should rise by more than the increase of inflation in the long run. Indeed, the coefficient $(1 - \beta)/\kappa$ represents the long run multiplier of the inflation rate on output in a standard NKPC log-linearized around the zero-inflation steady state (see (24)). In other words, the Taylor principle has to be intended as,

$$\left.\frac{\partial \hat{\imath}}{\partial \hat{\pi}}\right|_{LR} = \alpha_\pi + \left.\frac{\partial \hat{y}}{\partial \hat{\pi}}\right|_{LR} \alpha_y > 1. \tag{41}$$

The generalized Taylor principle in its formulation (41) is still a crucial condition for determinacy of REE in the Rotemberg model with trend inflation. Indeed, the coefficient $\frac{\left(1+\varsigma_c \gamma_{cm} - \beta \gamma_{for}\right)}{\gamma_{cm}}$ in (39) represents the long-run elasticity of output to inflation of the generalized model with trend inflation (see Appendix A.4). Hence Proposition 1 corresponds exactly to (41) in the general case of trend inflation.

What are then the effects of trend inflation on the determinacy region in the Rotemberg model?

Proposition 2. The effects of trend inflation on the determinacy region. *Let* $\phi = 0$, $\sigma = 1$, $\alpha_\pi \in [0, \infty)$, $\alpha_y \in [0, \infty)$ *and at least one different from zero. Then*

$$\left.\frac{d\left[\frac{\left(1+\varsigma_c \gamma_{cm} - \beta \gamma_f\right)}{\gamma_{cm}}\right]}{d\bar{\pi}}\right|_{\bar{\pi}=1} = \varphi + \frac{\varphi(1-\beta)}{\varepsilon - 1}\left[3 - \frac{\varphi\,(1-\beta)}{\varepsilon - 1}\right] \tag{42}$$

which is positive for β *sufficiently close to 1. (see Appendix A.4.3)*

Corollary. *Let* $\phi = 0$, $\sigma = 1$, $\alpha_\pi \in [0, \infty)$, $\alpha_y \in [0, \infty)$ *with at least one different from zero, and* β *sufficiently close to 1. Then, the determinacy region widens in the parameter space* (α_π, α_y).

The derivative in (42) reveals the effects of trend inflation on the condition (39). Recall that (39) is equivalent to (41) in the case of the Rotemberg model. Hence (42) demonstrates that $\left.\frac{\partial \hat{y}}{\partial \hat{\pi}}\right|_{LR}$ increases with trend inflation around the point $\bar{\pi} = 1$.[12] As from the corollary, if $\left.\frac{\partial \hat{y}}{\partial \hat{\pi}}\right|_{LR}$ increases, then the region in the parameter space (α_π, α_y) that guarantees determinacy of the REE enlarges. In fact, for a given α_y the condition (39) is satisfied for lower values of α_π.

Figures 5a and 5b visualizes the content of Proposition 2. Figure 5a shows the usual graph of the Taylor principle in the space (α_π, α_y) in the case $\bar{\pi} = 1$ which is identical to the one we get under Calvo pricing with zero trend inflation. In the case $\bar{\pi} = 1$ in fact, condition (41) implies $\alpha_y > (1 - \alpha_\pi) / \left.\frac{\partial \hat{y}}{\partial \hat{\pi}}\right|_{LR}$, where $\left.\frac{\partial \hat{y}}{\partial \hat{\pi}}\right|_{LR, \bar{\pi}=1} = \frac{1-\beta}{\kappa} = \varphi \frac{1-\beta}{\varepsilon - 1}$. As trend inflation increases, Proposition 2 shows that $\left.\frac{\partial \hat{y}}{\partial \hat{\pi}}\right|_{LR}$ increases, and the line rotates anti-clockwise (see figure 5b).

- Figures 5 about here -

Moreover, from a quantitative perspective, Figures 6 depicts the determinacy regions for different levels of trend inflation, i.e. from 0 to 4%, resulting from simulating the model for the values $\alpha_\pi \in [0, 5]$ and $\alpha_Y \in [-1, 5]$ (for the calibration see the beginning of Section 4). The determinacy frontier rotates anti-clockwise enlarging the determinacy region and remaining negatively sloped, as suggested by Proposition 2.

- Figure 6 about here -

The Calvo model

In a recent paper Ascari and Ropele (2009) show that trend inflation shrinks the determinacy region in the Calvo model. This means that in the Calvo model trend inflation affects

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[12] In general, the derivative in (42) yields a very cumbersome expression that would not allow to derive any analitycal insights. We were able, however, to derive the condition in (42) evaluating the derivative at $\bar{\pi} = 1$, to understand how trend inflation affects the Taylor principle when $\bar{\pi}$ slightly moves from one. By continuity argument, one may argue that the result holds for the values of $\bar{\pi}$ very close to one, such as the ones we consider (recall that $\bar{\pi}$ is the gross quarterly inflation rate). The simulations below, indeed, confirm such conjecture.

the determinacy region in the opposite way with respect to the one described above for the Rotemberg model. In particular, Ascari and Ropele (2009) show that the generalized Taylor principle, (41), is still a necessary condition in the Calvo model. However, as trend inflation increases, $\left.\frac{\partial \hat{y}}{\partial \hat{\pi}}\right|_{LR}$ decreases, and then very rapidly switches sign from positive to negative, such that the determinacy frontier rotates clockwise (figure 5c shows the equivalent of preposition 2 in the Calvo model). So trend inflation strongly shrinks the determinacy region in the space (α_π, α_y) in the Calvo model, while it does the opposite in the Rotemberg model.

Moreover, the two authors, show that the generalized Taylor principle is a necessary, but not sufficient condition for local determinacy of the REE in the positive orthant of the parameter space (α_π, α_y). This is because, generally, there is a second determinacy frontier that needs to be satisfied. This frontier lies entirely below the positive orthant when $\bar{\pi} = 1$, such that it is usually disregarded in the literature (see Figure 5a). Trend inflation, however, moves this second determinacy frontier upwards, making it crossing the positive orthant for moderate rate of trend inflation. Hence, this condition becomes necessary, even if it looks only at positive values of α_π and α_y. Figure 6 above shows that, also in the Rotemberg model, this second determinacy frontier is relevant and it lies entirely below the positive orthant when $\bar{\pi} = 1$ (being the Rotemberg model equivalent to the Calvo model in this case). However, the simulation shows that trend inflation shifts this frontier upwards as in the Calvo model, but the effects are very minor and the frontier never crosses the positive orthant, given our calibration. Therefore, contrary to the Calvo model, in the Rotemberg model the generalized Taylor principle remains not only a necessary, but also a sufficient condition for the determinacy of the REE in the positive orthant of the space (α_π, α_y).

To sum up, the determinacy conditions in the two models are equivalent when the model is log-linearized around zero trend inflation, i.e., $\bar{\pi} = 1$, but they are different in presence of moderate level of inflation. In particular, trend inflation has opposite effects on the condition defining the generalized Taylor principle in the two models. Moderate inflation enlarges the determinacy region in the Rotemberg model, while it shrinks it in the Calvo model. Moreover, form a quantitative perspective, these effects are small in the former case, and large in the latter.

4.5 Disinflation Dynamics

In this section we look at an unanticipated and *permanent* reduction in the inflation target of the Central Bank. The Central Bank follows the standard Taylor rule (37). In particular, we employ a non-linear simulation method by using the package DYNARE.[13] We plot the path for output, inflation, nominal interest rate, real wages, consumption and hours in response to such a change in the Central Bank policy regime. We consider three cases: a disinflation from 4%, 6% and 8% trend inflation to zero.

The Rotemberg model

When prices are set à la Rotemberg, the economy would immediately adjust to the new steady state without any transitional dynamics (see Figure 7). Thus, the non-linear version of the simple New Keynesian model above with Rotemberg pricing is completely forward-looking. Note that this is the same results that would be obtained in the log-linear model.

Taking into account the role of trend inflation, however, reveals the long-run effects of such a policy. A disinflation policy permanently decreases output and hours (together with the real wage), but it increases consumption. As explained in Section 4.1, a disinflation causes an increase in firms' markup, and a fall in output (and hence in hours). Moreover, a disinflation reduces the size of the adjustment costs, so it reduces the wedge between output and consumption, as shown by (16). Consumption increases because the decrease in the fraction of output wasted for adjusting prices more than compensates the decrease in output. Thus, a disinflation would cause output and consumption to move in opposite directions.

So two main results stem from this analysis of the effects of a disinflation policy in the Rotemberg model. First, there is no transitional dynamics and the economy immediately adjusts to the new steady state level, because the non-linear model is completely forward-looking. Second, there are, however, long-run effects of such a policy: output and hours decrease, while consumption increases.

- Figure 7 about here -

The Calvo model

As Figure 7 above, Figure 8 plots the responses of the main economic variables to disinflation policies from 4%, 6% and 8% trend inflation to zero in the case of the Calvo model.

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[13] Figures 7 and 8 are obtained using the software DYNARE developed by Michel Juillard and others at CEPREMAP, see http://www.cepremap.cnrs.fr/dynare/. The paths in the Figures display the movement from a deterministic steady state to another one. DYNARE solves for these paths by stacking up all the equations of the model for all the periods in the simulation (which we set equal to 100). Then the resulting system is solved en bloc by using the Newton-Raphson algorithm, by exploiting the special sparse structure of the Jacobian blocks. The non-linear model thus is solved in its full-linear form, without any approximation.

As shown by Ascari and Merkl (2009), when nonlinear simulations are employed, the adjustment path of the Calvo model is completely different from the one described above for the Rotemberg model. Indeed, the two main results above are turned around.

First, the dynamic adjustment of the non-linear Calvo model after a disinflation is inertial. The Calvo model implies price dispersion, i.e., s_t, that is a backward-looking variable that adjusts sluggishly after a disinflation. Thus, the non-linear solution of the model features a new endogenous state variable, and the model dynamics is inertial. The Rotemberg model, instead, does not feature any price dispersion.

Second, output and consumption increase, while hours decreases. Output increases sluggishly to the new higher steady state level (see Section 4.1). Since output is entirely consumed, consumption and output show the same adjustment path. The adjustment dynamics in hours worked is, instead, different. Hours jump up on impact, because output increases, but then they decrease. As explained in Section 2.2, inflation in the Calvo model creates a wedge between aggregate hours and aggregate output, through price dispersion in (21). The lower price dispersion, the less the hours that are needed for a given output. For all the cases considered, price dispersion decreases monotonically to the new lower steady state level. This is why hours thus peak on impact, and then start decreasing. Indeed, along the adjustment, output is increasing, while price dispersion is decreasing. From period 2 onwards, the latter effect then dominates, making aggregate production more efficient and thus saving hours worked, despite the rise in output.

- Figure 8 about here -

We therefore show that, when the economy is hit by a permanent and unanticipated inflation target shock, the two nonlinear models, based on the two different price setting mechanisms, show very different and opposite dynamics. The Calvo model implies that output and consumption closely move together, while output and hours move in opposite directions during the adjustment, after the impact period. The opposite is true for the Rotemberg model. Moreover, while in the non-linear Calvo model the adjustment is inertial, in the non-linear Rotemberg model the adjustment is immediate. The intuition for these differences is straightforward, and lies in the two different wedges that nominal rigidities create in the two models. Both wedges decrease after a disinflation. In the Rotemberg model, however, a disinflation reduces the wedge between output and consumption, so that they move in opposite directions, while in the Calvo model a disinflation reduce the wedge between output and hours, so that they move in opposite directions.

Finally, the results in the Rotemberg model are qualitatively similar to the ones of the standard linear model. The version of the New Keynesian model (e.g., Woodford, 2003) log-linearized around zero steady state inflation would imply an immediate adjustment after a disinflation. Indeed, if log-linearized around a zero inflation steady state, then price dispersion would not matter for the model dynamics up to first-order. So nothing prevents the model to jump to the new steady state.[14] In other words, the results in the Rotemberg model are qualitatively robust to trend inflation and non-linear analysis, why this is not the case for the Calvo model.

5 Indexation

Our results show that, with non zero trend inflation, even to a first order approximation the two models are quite different models. In fact, they imply the same dynamics only under a very particular assumptions: a zero steady state inflation. For all the other cases, the long-run properties and the implied dynamics of the two models are very different.

The next section investigates what is the effect of indexation on the difference between the two models. Not surprisingly, it shows that partial indexation tends to mitigate this difference, that however, qualitatively is very robust, because it vanishes only in the case of full indexation. We now assume that firms have the possibility to index their price. We look at two types of indexation: to long-run inflation $\bar{\pi}$ and to past inflation π_{t-1}. In particular, we consider the effects of price indexation on the long-run properties and on the dynamics of the two pricing models. We show that in both models, price indexation is able to dampen the effects of trend inflation and therefore to reduce the differences between the two pricing mechanism. In the very particular case of full price indexation the two models are again equivalent as in the case of zero trend inflation.

Under the Rotemberg model, the equivalent of indexation would be a cost adjustment rule that decreases the cost of automatically adjusting prices either to trend and/or to past inflation. The cost of adjusting prices can be rewritten in the more general specification

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[14] Moreover, output would decrease, as implied by the non-linear Rotemberg model. The NKPC, in fact, is positively sloped when the Calvo or Rotemberg model are log-linearized around zero inflation steady state.

considered by Ireland (2007) among others, i.e.,

$$\frac{\varphi}{2}\left(\frac{P_{i,t}}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}P_{i,t-1}}-1\right)^{2}Y_{t}, \tag{43}$$

where, as before, $\varphi > 0$ determines the degree of nominal price rigidity. This definition is the correspondent of the general specification of the Calvo price setting scheme (adopted by Smets and Wouters, 2003 among others), within the Rotemberg one. Notice that: (i) $\chi \in [0,1]$ allows for any degree of price indexation; (ii) $\mu \in [0,1]$ allows for any degree of (geometric) combination of the two types of indexation usually employed in the Calvo pricing literature, i.e., to steady state inflation (e.g., Yun, 1996) and to past inflation rates (e.g., Christiano et al., 2005). In particular, when $\mu = 0$ ($\mu = 1$) firms find it costless to adjust their prices in line with the central bank inflation target (the previous period's inflation rate).

For a given price inflation, the adjustment cost (43) decreases with price indexation to a degree given by χ. In fact, the higher χ, the lower, *ceteris paribus,* is $\frac{P_{i,t}}{\left(\pi_{t-1}^{\chi}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}P_{i,t-1}}$ and the lower is the cost of adjusting prices. Since trend inflation increases the cost of adjusting prices (thus increasing the wedge between consumption and output), by allowing for price indexation the effects of trend inflation would be damped. Thus, price indexation offsets the effects of trend inflation both in the long-run and in the short-run.

The long-run

The Appendix A.2 shows that, assuming the adjustment cost (43), the long-run Phillips curve in the Rotemberg model is equal to

$$Y=\left[\frac{\frac{\varepsilon-1}{\varepsilon}+\frac{(1-\beta)}{\varepsilon}\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}}{d_{n}\left(1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^{2}\right)^{\sigma}}\right]^{\frac{1}{\phi+\sigma}}. \tag{44}$$

The long run Phillips curve is still positive sloped, i.e., the higher is trend inflation $\bar{\pi}$ the higher is the amount of output produced in the long run. However, the higher is the parameter χ the lower is the increase in output following an increase in $\bar{\pi}$. The firms steady state cost of adjusting prices is equal to $\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^{2}Y$, where it is evident that indexation counteracts the effect of trend inflation. At the limit, when price are fully indexed, i.e. $\chi = 1$, the economy steady state is the same as in flexible price economy. Indeed, in the case of full indexation, the steady state wedge $\Psi = \frac{1}{1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^{2}}$ is no more a function of trend inflation and it is minimized at one. In this case the equality between consumption and output is restored, i.e., $C = Y$.

The dynamics

Considering the adjustment cost in (43), then the log-linear approximations of the generalized NKPC (30), the real marginal costs (31) and the log-linearized IS curve (32) become

$$\hat{\pi}_t = \gamma_p \hat{\pi}_{t-1} + \gamma_f \beta \hat{\pi}_{t+1} + \gamma_{dy} \beta \left(1 - \sigma\right) \Delta \hat{y}_{t+1} + \gamma_{mc} \widehat{mc}_t \tag{45}$$

$$\widehat{mc}_t = \left(\sigma + \phi\right) \hat{y}_t - \varsigma_c \sigma \hat{\pi}_t + \varsigma_c \sigma \mu \chi \hat{\pi}_{t-1} - \left(1 + \phi\right) a_t \tag{46}$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \varsigma_c \Delta \hat{\pi}_{t+1} + \varsigma_c \mu \chi \Delta \hat{\pi}_t - \frac{1}{\sigma} E_t \left(\hat{\imath}_t - \hat{\pi}_{t+1}\right) \tag{47}$$

where, γ_p,γ_f, γ_{dy}, γ_{mc} and ς_c are complicated convolution parameters that depend both on trend inflation and indexation (see Appendix A.3). Again the degree of indexation counteracts the effects of trend inflation on the log-linearized coefficients of the model equations. For a given value of trend inflation, as the degree of price indexation increases the dynamics of the Rotemberg model converges to the dynamics of a standard New Keynesian model.[15]

As shown in Ascari and Ropele (2009), also in the Calvo model, price indexation counteracts the effect of trend inflation, because it reduces price dispersion by allowing also the non price-resetting firms to keep up with the pace of inflation. By dampening the effects of trend inflation, indexation diminishes the difference between the two pricing models.

Regarding the determinacy of the model, Figure 9 and 10 compares the effects of price indexation to trend inflation, i.e., $\mu = 0$, *versus* past inflation, i.e., $\mu = 1$ in the Rotemberg model. In both cases we assume three different values for χ: $\chi = 0.5$ (partial indexation), $\chi = 1$ (full indexation) and $\chi = 0$ (no indexation). Notice in the case of indexation to past inflation (i.e. with $\mu = 1$) the model is further complicated by the presence of an endogenous predetermined variable, namely $\hat{\pi}_{t-1}$. As before, we numerically analyze the determinacy of REE in the region of the plane defined by $\alpha_\pi \in [0, 5]$ and $\alpha_Y \in [-1, 5]$. We consider a constant value of annual trend inflation equal to 4%.

- Figure 9 and 10 about here -

Figure 9 and 10 show that partial indexation shrinks the determinacy region. *In the Rotemberg model, hence, the higher is the degree of price indexation (both to trend and to past inflation), the smaller is the determinacy region.* This result stands in sharp contrast to what happen in the Calvo model, where price indexation enlarges the determinacy region. While

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[15] In particular, as shown in the Appendix A.3.3, under full indexation to trend inflation, i.e., $\chi = 1$ and $\mu = 0$, the dynamic system collapses to the standard New Keynesian model log-linearized around a zero inflation steady state. Under full indexation to past inflation, i.e., $\chi = 1$ and $\mu = 1$, the dynamic system is equivalent to the hybrid Phillips curve in Christiano et al (2005).

under Rotemberg price indexation shrinks the determinacy region, thus, under Calvo pricing the opposite holds. This is just the mirror image of the fact that the effect of trend inflation on the determinacy region is the opposite in the two pricing models. While under Rotemberg trend inflation enlarges the determinacy region, under Calvo pricing trend inflation shrinks the determinacy region. Since indexation counteracts the effects of trend inflation in both models, then, indexation will have opposite effects in the two models.

Moreover, with full indexation (both to trend and to past inflation) the two models converge to the same area of determinacy. In fact, with full indexation, the two models are again equivalent as in the case of no trend inflation (zero steady state inflation), because the two wedges in equations (16) and (21) disappear. This is not very surprising, since with full indexation the steady state of both models is equivalent to the steady state of the flexible price version of the model. This is exactly the reason why the dynamics of Rotemberg and Calvo models are identical under full indexation.

Finally, comparing the two types of indexation in the Rotemberg model, it turns out that, for any given level of trend inflation, price indexation to past inflation yields a smaller number of determinate interest rate rules than under price indexation to trend inflation.

6 Conclusion

This paper analyzes the dynamics of a New Keynesian model with two firms' price-setting mechanisms: the Rotemberg (1982) quadratic cost of price adjustment and the staggered price setting introduced by Calvo (1983). Despite assuming two quite different forms of nominal rigidities, the conventional wisdoms is to consider these two models as observationally equivalent, because they deliver the same log-linear NKPC.

Contrary to the conventional wisdom, we show that the two models are quite different models, once trend inflation is considered. Indeed, the two different nominal rigidities assumptions generates two different wedges in the two models. Price dispersion in the Calvo model generates a wedge between output and hours and introduces an inertial component in the model, while the adjustment cost in the Rotemberg model generates a wedge between output and consumption and the model remains a pure forward looking model. These two different wedges makes the Calvo and Rotemberg models different. However, these two wedges vanish under the particular case of zero steady state inflation, simply because there is no cost of price rigidities in steady state in this peculiar case. On the contrary, trend inflation alters the cost of the nominal rigidities in the two models. It thus affects the magnitude of these wedges, revealing the difference between the two pricing rigidities assumptions.

In particular: (i) the long-run NKPC is negatively sloped in the Calvo model and positively sloped in the Rotemberg model; (ii) the log-linear NKPC in the two models is qualitatively very different, implying two different dynamic systems; (iii) positive trend inflation shrinks the determinacy region in the Calvo model, while it enlarges the determinacy region in the Rotemberg model; (iv) positive trend inflation amplifies the impulse response functions to a technology shock in the Calvo model, while it dampens them in the Rotemberg model; (v) a permanent and credible disinflation implies inertial adjustment and output gains in the Calvo model, while it implies immediate adjustment and output losses in the Rotemberg model.

Throughout the paper we assume that the Rotemberg adjustment cost represents a pure waste for the economy and therefore it goes in the resource constraint. This is the most common assumption in the NK literature. Nevertheless, to understand the robustness of our results on the difference between the Rotemberg and the Calvo model we also considered the case in which the adjustment costs are rebated to consumers.[16] In this case, what we have defined as the "wedge effect" is shut down and the aggregate resource constraint becomes $C_t = Y_t$. The results, however, remain qualitatively unaffected, even if quantitatively mitigated.[17] Moreover, consumption and output move in the same direction after a disinflation in this case.

Summing up, as a general point, this paper stresses the importance of the interplay between long-run effects and short-run dynamics. The two models are non-linear in trend inflation. Therefore, the two price-setting mechanisms imply a very different dynamics even to a first order approximation, once the non-linearities due to trend inflation are considered. Log-linearizing the model around a zero inflation steady state, instead, removes these interesting and intrinsic differences between the two models.

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[16] This means that, as in the standard model, $\frac{\varphi_p}{2}\left(\frac{P_t}{\left(\pi_{t-1}^{\chi}\right)^{\gamma}(\bar{\pi}^{\chi})^{1-\gamma}P_{t-1}}-1\right)^2 Y_t$ is a cost for the intermediate good producing firm and therefore it lowers firms profits Π_t. However, we now assume that the cost of adjusting prices is paid to the representative consumer. Then, $\frac{\varphi_p}{2}\left(\frac{P_t}{\left(\pi_{t-1}^{\chi}\right)^{\gamma}(\bar{\pi}^{\chi})^{1-\gamma}P_{t-1}}-1\right)^2 Y_t$ enters the household budget constraint increasing his revenues. When markets clear the household budget constraint becomes: $C_t = \frac{W_t N_{,t}}{P_t} + \frac{\varphi_p}{2}\left(\frac{P_t}{\left(\pi_{t-1}^{\chi}\right)^{\gamma}(\bar{\pi}^{\chi})^{1-\gamma}P_{t-1}}-1\right)^2 Y_t + \Pi_t$. Therefore, substituting for the representative firms profits, Π_t, it straightforward to find that market clearing conditions imply that the entire output is consumed.

[17] For a detailed description of these results we refer to a previous version of the paper (see Ascari and Rossi 2008).

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A Technical Appendix - The Rotemberg model with general indexation

A.1 Firms Price-Setting Problem

The Rotemberg model assumes that a monopolistic firm faces a quadratic cost of adjusting nominal prices, that can be measured in terms of the final-good and given by

$$\frac{\varphi}{2}\left(\frac{P_{i,t}}{\left(\pi^{\chi}_{t-1}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}P_{i,t-1}}-1\right)^{2}Y_{t}, \tag{48}$$

where $\varphi > 0$ determines the degree of nominal price rigidity. Also (48) is a general specification for the adjustment cost used by, e.g., Ireland (2007), among others. The problem for the firm is then

$$\max_{\{P_{i,t}\}_{t=0}^{\infty}} E_{t}\sum_{j=0}^{\infty}\mathcal{D}_{t,t+j}\left\{\begin{array}{c}\frac{P_{i,t+j}}{P_{t+j}}Y_{i,t+j}-MC^{r}_{i,t}Y_{i,t+j}+\\ -\frac{\varphi}{2}\left(\frac{P_{i,t+j}}{\left(\pi^{\chi}_{t+j-1}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}P_{i,t+j-1}}-1\right)^{2}Y_{t+j}\end{array}\right\}, \tag{49}$$

$$\text{s.t. } Y_{i,t+j}=\left[\frac{P_{i,t+j}}{P_{t+j}}\right]^{-\varepsilon}Y_{t+j}. \tag{50}$$

where $\mathcal{D}_{t,t+j}\equiv\beta^{j}\frac{U_{c}(t+j)}{U_{c}(t)}$ is the stochastic discount factor, $MC^{r}_{t+j}=\frac{W_{t+j}}{P_{t+j}A_{t+j}}$ is the real marginal cost function. Firms can change their price in each period, subject to the payment of the adjustment cost. Therefore, all the firms face the same problem, and thus will choose the same price, producing the same quantity. In other words: $P_{i,t}=P_{t}, Y_{i,t}=Y_{t}$, and $MC^{r}_{i,t}=MC^{r}_{t}\ \forall i$. Therefore, from the first order condition, after imposing the symmetric equilibrium, we get:

$$\begin{aligned}
&1-\varphi\left(\frac{\pi_{t}}{\left(\pi^{\chi}_{t-1}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}}-1\right)\frac{\pi_{t}}{\left(\pi^{\chi}_{t-1}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}}+\\
&+\varphi\beta E_{t}\left(\frac{C_{t+1}}{C_{t}}\right)^{-\sigma}\left[\left(\frac{\pi_{t+1}}{\left(\pi^{\chi}_{t-1}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}}-1\right)\frac{\pi_{t+1}}{\left(\pi^{\chi}_{t-1}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}}\frac{Y_{t+1}}{Y_{t}}\right]\\
=\ &\left(1-MC^{r}_{t}\right)\varepsilon.
\end{aligned} \tag{51}$$

In the Rotemberg model, the adjustment cost enters the aggregate resource constraint that is given by

$$Y_{t}=C_{t}+\frac{\varphi}{2}\left(\frac{P_{t}}{\left(\pi^{\chi}_{t-1}\right)^{\mu}\left(\bar{\pi}^{\chi}\right)^{1-\mu}P_{t-1}}-1\right)^{2}Y_{t}, \tag{52}$$

A.2 The Steady State

The deterministic steady state is obtained by dropping the time indices. The steady state inflation is equal to the Central Bank inflation target: $\pi = \bar{\pi}$.

The aggregate resource constraint implies

$$C = \left(1 - \frac{\varphi}{2}\left(\bar{\pi}^{1-\chi} - 1\right)^2\right) Y, \tag{53}$$

from the aggregate production function

$$Y = N, \tag{54}$$

where we put $A = 1$ in steady state without loss of generality. The real marginal costs are

$$MC^r = \frac{W}{P}. \tag{55}$$

Equation (51) becomes

$$\left(1 - \varphi\left(\bar{\pi}^{1-\chi} - 1\right)\bar{\pi}^{1-\chi}\right) + \varphi\beta\left[\left(\bar{\pi}^{1-\chi} - 1\right)\bar{\pi}^{1-\chi}\right] = \left(1 - MC_t^r\right)\varepsilon, \tag{56}$$

then solving for the steady state value of aggregate real marginal costs yields

$$MC^r = \frac{\varepsilon - 1}{\varepsilon} + \frac{(1-\beta)}{\varepsilon}\varphi\left(\bar{\pi}^{1-\chi} - 1\right)\bar{\pi}^{1-\chi}. \tag{57}$$

The markup, defined as $\frac{1}{MC^r}$, is therefore

$$markup = \left[\frac{\varepsilon - 1}{\varepsilon} + \frac{(1-\beta)}{\varepsilon}\varphi\left(\bar{\pi}^{1-\chi} - 1\right)\pi^{1-\chi}\right]^{-1}, \tag{58}$$

and the labor supply equation is

$$\frac{W}{P} = d_n N^{\phi} C^{\sigma}. \tag{59}$$

Euler Equation gives

$$1 + \bar{\imath} = \frac{1}{\beta}. \tag{60}$$

(54), (55) and (59) imply

$$MC^r = d_n Y^{\phi} Y^{\sigma}, \tag{61}$$

then, substituting the aggregate resource constraint, (53) and , and combining it with real marginal costs in (57) yields the steady state level of output

$$Y = \left[\frac{\frac{\varepsilon-1}{\varepsilon} + \frac{(1-\beta)}{\varepsilon}\varphi\left(\bar{\pi}^{1-\chi} - 1\right)\bar{\pi}^{1-\chi}}{d_n\left(1 - \frac{\varphi}{2}\left(\bar{\pi}^{1-\chi} - 1\right)^2\right)^{\sigma}}\right]^{\frac{1}{\phi+\sigma}}. \tag{62}$$

Define: $a \equiv \frac{\varepsilon-1}{\varepsilon}$, $b \equiv \frac{(1-\beta)}{\varepsilon}\varphi$, $c \equiv d_n, d \equiv \frac{1}{\phi+\sigma}$, which are constants independent of the steady state inflation rate $\bar{\pi}$. Then

$$\frac{d}{d\bar{\pi}}\left(\left[\frac{a+b\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}}{c\left(1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right)^{\sigma}}\right]^{d}\right) =$$

$$= d\left[Y\left(\bar{\pi}\right)\right]^{d-1}\frac{b(1-\chi)\bar{\pi}^{-\chi}\left(2\bar{\pi}^{1-\chi}-1\right)+\sigma\left(1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right)^{-1}\left(\varphi\left(\bar{\pi}^{1-\chi}-1\right)(1-\chi)\bar{\pi}^{-\chi}\right)\left[a+b\left(\pi^{1-\chi}-1\right)\pi^{1-\chi}\right]}{c\left(1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right)^{\sigma}}$$

This expression implies:

- $\chi = 1 \Longrightarrow \frac{dY}{d\bar{\pi}} = 0$

- $\bar{\pi} \geqq 1 \Longrightarrow \frac{dY}{d\bar{\pi}} > 0$, so that the minimum of output occurs at negative rate of steady state inflation, unless $\beta = 1$, that implies $b = 0$.

If $\beta < 1$, then

- $\exists \pi^* < 1 s.t. \begin{cases} \bar{\pi} > \bar{\pi}^* \Longrightarrow \frac{dY}{d\bar{\pi}} > 0 \\ \bar{\pi} = \bar{\pi}^* \Longrightarrow \frac{dY}{d\bar{\pi}} = 0 \\ \bar{\pi} < \bar{\pi}^* \Longrightarrow \frac{dY}{d\bar{\pi}} < 0 \end{cases}$.

Finally, given (58) and the definition of Ψ in the main text in (16), (62) can be written as

$$Y = \left(\frac{\Psi^{\sigma}}{d_n \frac{P}{MC}}\right)^{\frac{1}{\phi+\sigma}} \tag{63}$$

which is (27) in the main text.

A.3 Derivation of the Log-Linear Model

A.3.1 The IS Curve and the Real Marginal Costs

By log-linearizing the household Euler equation (3) and the household labor supply (4) we get

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}E_t\left(\hat{\imath}_t - \hat{\pi}_{t+1}\right) \tag{64}$$

$$\hat{w}_t = \phi\hat{n}_t + \sigma\hat{c}_t \tag{65}$$

where lower case hatted letters denote log-deviations of the variable with respect to its steady state value.

Considering now the log-linearization of the economy resource constraint (16) and simplifying we get:

$$\hat{c}_t = \hat{y}_t - \varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\frac{Y}{C}\hat{\pi}_t + \varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\mu\chi\frac{Y}{C}\hat{\pi}_{t-1} \tag{66}$$

note that: $\frac{Y}{C}=\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]^{-1}$, then

$$\hat{c}_t=\hat{y}_t-\frac{\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]}\hat{\pi}_t+\frac{\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\mu\chi}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]}\hat{\pi}_{t-1} \tag{67}$$

Given (67) and considering that the log-linearized production function implies that $\hat{y}_t=a_t+\hat{n}_t$, we can now rewrite equation (64) and (65) as:

$$\begin{aligned}\hat{y}_t &= E_t\hat{y}_{t+1}-\frac{\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]}\Delta\hat{\pi}_{t+1}+\\ &\quad+\frac{\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\mu\chi}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]}\Delta\hat{\pi}_t-\frac{1}{\sigma}E_t\left(\hat{\imath}_t-\hat{\pi}_{t+1}\right) \qquad (68)\\ \hat{w}_t &= \left(\phi+\sigma\right)\hat{y}_t-\phi a_t-\frac{\sigma\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]}\hat{\pi}_t+\\ &\quad+\frac{\sigma\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\mu\chi}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]}\hat{\pi}_{t-1} \qquad (69)\end{aligned}$$

Note that, equation (68) is the generalized IS curve of the Rotemberg model with trend inflation and price indexation

Labor demand implies $w_t=MC_t^r A_t$, can be rewritten in log-linear terms as follows:

$$\hat{w}_t=\widehat{mc}_t+a_t \tag{70}$$

Imposing the labor market equilibrium, i.e. (69) = (70) we get the log-linear real marginal costs:

$$\widehat{mc}_t=\left(\sigma+\phi\right)\hat{y}_t-\left(1+\phi\right)a_t-\frac{\sigma\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]}\hat{\pi}_t+\frac{\sigma\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\mu\chi}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]}\hat{\pi}_{t-1} \tag{71}$$

A.3.2 The Log-linearized NKPC

From firms' optimal price setting problem we get equation (13). Log-linearizing equation (13) and considering the log-linearization of the economy resource constraint, we get the following NKPC:

$$\hat{\pi}_t=\gamma_p\hat{\pi}_{t-1}+\gamma_f\beta\hat{\pi}_{t+1}+\gamma_{dy}\beta\left(1-\sigma\right)\Delta\hat{y}_{t+1}+\gamma_{mc}\widehat{mc}_t \tag{72}$$

where

$$
\begin{aligned}
\gamma_p &= \frac{\left(2\bar{\pi}^{2(1-\chi)} - \bar{\pi}^{(1-\chi)}\right)\chi\mu\frac{C}{Y} + \beta\left[\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\right]^2\sigma\varphi\mu\chi}{\varrho}, \\
\gamma_f &= \frac{\left(2\bar{\pi}^{2(1-\chi)} - \bar{\pi}^{(1-\chi)}\right)\frac{C}{Y} + \left[\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\right]^2\sigma\varphi}{\varrho} \\
\gamma_{dy} &= \frac{\left(\bar{\pi}^{2(1-\chi)} - \bar{\pi}^{1-\chi}\right)\frac{C}{Y}}{\varrho}, \\
\gamma_{mc} &= \frac{\left(\varepsilon - 1 + \varphi\left(\bar{\pi}^{2(1-\chi)} - \bar{\pi}^{1-\chi}\right)(1-\beta)\right)\frac{C}{Y}}{\varphi\varrho}. \\
\frac{C}{Y} &= \left(1 - \frac{\varphi}{2}(\bar{\pi}-1)^2\right) \\
\varrho &\equiv \left(2\bar{\pi}^{2(1-\chi)} - \bar{\pi}^{(1-\chi)}\right)(1+\beta\chi\mu)\frac{C}{Y} + \beta\left[\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\right]^2\sigma\varphi(1+\mu\chi),
\end{aligned}
$$

Under full indexation to trend inflation, i.e., $\chi = 1$ and $\mu = 0$,

$$
\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{\varepsilon - 1}{\varphi}\widehat{mc}_t \tag{73}
$$

the NKPC collapses to the standard NKPC log-linearized around a zero inflation steady state.

Under full indexation to past inflation, i.e., $\chi = 1$ and $\mu = 1$,

$$
\hat{\pi}_t = \frac{1}{(1+\beta)}\hat{\pi}_{t-1} + \frac{\beta}{(1+\beta)}\hat{\pi}_{t+1} + \frac{\varepsilon - 1}{\varphi(1+\beta)}\widehat{mc}_t \tag{74}
$$

the dynamic system is equivalent to the hybrid Phillips curve in Christiano et al (2005).

A.3.3 The Dynamic System

The reduced dynamic system of the model is given by five equations: 1) the IS curve; 2) The NKPC; 3) the equation of the real marginal cost; 4) the Taylor rule adopted by the monetary authority; 5) the AR (1) process of the technology shock.

$$
\begin{aligned}
\hat{y}_t &= E_t\hat{y}_{t+1} - \varsigma_c\Delta\hat{\pi}_{t+1} + \varsigma_c\mu\chi\Delta\hat{\pi}_t - \frac{1}{\sigma}E_t\left(\hat{\imath}_t - \hat{\pi}_{t+1}\right) && (75) \\
\hat{\pi}_t &= \gamma_p\hat{\pi}_{t-1} + \gamma_f\beta\hat{\pi}_{t+1} + \gamma_{dy}\beta(1-\sigma)\Delta\hat{y}_{t+1} + \gamma_{mc}\widehat{mc}_t && (76) \\
\widehat{mc}_t &= (\sigma+\phi)\hat{y}_t - \varsigma_c\sigma\hat{\pi}_t + \varsigma_c\sigma\mu\chi\hat{\pi}_{t-1} - (1+\phi)a_t && (77) \\
\hat{\imath}_t &= \alpha_i\hat{\imath}_t + (1-\alpha_i)\left[\alpha_\pi\hat{\pi}_t + \alpha_y\hat{y}_t\right] && (78) \\
a_t &= \rho_a a_{t-1} + \upsilon_{a,t} && (79)
\end{aligned}
$$

where

$$
\begin{aligned}
\varsigma_c &= \frac{\varphi\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}}{\left[1-\frac{\varphi}{2}\left(\bar{\pi}^{1-\chi}-1\right)^2\right]}, \\
\frac{C}{Y} &= \left(1-\frac{\varphi}{2}\left(\bar{\pi}-1\right)^2\right) \\
\varrho &\equiv \left(2\bar{\pi}^{2(1-\chi)}-\bar{\pi}^{(1-\chi)}\right)\left(1+\beta\chi\mu\right)\frac{C}{Y}+\beta\left[\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\right]^2\sigma\varphi\left(1+\mu\chi\right), \\
\gamma_p &= \frac{\left(2\bar{\pi}^{2(1-\chi)}-\bar{\pi}^{(1-\chi)}\right)\chi\mu\frac{C}{Y}+\beta\left[\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\right]^2\sigma\varphi\mu\chi}{\varrho}, \\
\gamma_f &= \frac{\left(2\bar{\pi}^{2(1-\chi)}-\bar{\pi}^{(1-\chi)}\right)\frac{C}{Y}+\left[\left(\bar{\pi}^{1-\chi}-1\right)\bar{\pi}^{1-\chi}\right]^2\sigma\varphi}{\varrho} \\
\gamma_{dy} &= \frac{\left(\bar{\pi}^{2(1-\chi)}-\bar{\pi}^{1-\chi}\right)\frac{C}{Y}}{\varrho}, \\
\gamma_{mc} &= \frac{\left(\varepsilon-1+\varphi\left(\bar{\pi}^{2(1-\chi)}-\bar{\pi}^{1-\chi}\right)\left(1-\beta\right)\right)\frac{C}{Y}}{\varphi\varrho}.
\end{aligned}
$$

Note that for $\chi = 0$ and $\mu = 0$ the parameters become:

$$
\begin{aligned}
&\varsigma_c = \frac{\varphi(\bar{\pi}-1)\bar{\pi}}{\left[1-\frac{\varphi}{2}(\bar{\pi}-1)^2\right]}, \\
&\gamma_p = 0, \\
&\gamma_f = \left[\frac{\left(2\bar{\pi}^2-\bar{\pi}\right)\frac{C}{Y}+[(\bar{\pi}-1)\bar{\pi}]^2\sigma\varphi}{(2\bar{\pi}^2-\bar{\pi})\frac{C}{Y}+\beta[(\bar{\pi}-1)\bar{\pi}]^2\sigma\varphi}\right], \\
&\gamma_{dy} = \frac{\left(\bar{\pi}^2-\bar{\pi}\right)\frac{C}{Y}}{(2\bar{\pi}^2-\bar{\pi})\frac{C}{Y}+\beta[(\bar{\pi}-1)\bar{\pi}]^2\sigma\varphi}, \\
&\gamma_{mc} = \frac{\left(\varepsilon-1+\varphi\left(\bar{\pi}^2-\bar{\pi}\right)(1-\beta)\right)\frac{C}{Y}}{\varphi\left[(2\bar{\pi}-\bar{\pi})\frac{C}{Y}+\beta[(\bar{\pi}-1)\bar{\pi}]^2\sigma\varphi\right]}.
\end{aligned}
$$

and the system of equation coincides with the one considered in the main text.

Note that for $\chi = 1$ and $\mu = 0$, with full indexation to trend inflation, the parameters become:

$$
\begin{aligned}
&\varsigma_c = 0, \\
&\gamma_{past} = 1, \\
&\gamma_{for} = 1, \\
&\gamma_{dy} = 0, \\
&\gamma_{mc} = \tfrac{\varepsilon-1}{\varphi}.
\end{aligned}
$$

it is easy to see that the dynamic system collapses to the one obtained by log-linearizing around a steady state inflation equal to zero.

Finally note that, with full indexation to past inflation, i.e. with $\chi = 1$ and $\mu = 1$ the parameters becomes:

$$\begin{aligned}
&\varsigma_c = 0,\\
&\gamma_p = \tfrac{1}{1+\beta},\\
&\gamma_f = \tfrac{1}{1+\beta},\\
&\gamma_{dy} = 0,\\
&\gamma_{mc} = \tfrac{\varepsilon-1}{\varphi(1+\beta)}.
\end{aligned}$$

so that the dynamic system becomes:

$$\begin{aligned}
\hat{y}_t &= E_t\hat{y}_{t+1} - \frac{1}{\sigma}E_t\left(\hat{\imath}_t - \hat{\pi}_{t+1}\right) && (80)\\
\hat{\pi}_t &= \frac{1}{1+\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta}\hat{\pi}_{t+1} + \frac{\varepsilon-1}{\varphi\left(1+\beta\right)}\widehat{mc}_t && (81)\\
\widehat{mc}_t &= \left(\sigma+\phi\right)\hat{y}_t - \left(1+\phi\right)a_t && (82)\\
\hat{\imath}_t &= \alpha_i\hat{\imath}_t + \left(1-\alpha_i\right)\left[\alpha_\pi\hat{\pi}_t + \alpha_y\hat{y}_t\right] && (83)\\
a_t &= \rho_a a_{t-1} + \upsilon_{a,t} && (84)
\end{aligned}$$

which coincides with the standard New Keynesian model where the Phillips curve is the hybrid one well described in Christiano et al (2005).

A.4 Determinacy

In order to derive simple analytical results, in this section we set $\chi = 0$, $\sigma = 1, \phi = 0$, and $\alpha_\pi \in [0,\infty)$, $\alpha_Y \in [0,\infty)$.

Then, system of equation is:

$$\begin{cases}
\hat{y}_t = E_t\hat{y}_{t+1} - \varsigma_c\hat{\pi}_{t+1} + \varsigma_c\hat{\pi}_t - \hat{\imath}_t + \hat{\pi}_{t+1}\\
\hat{\pi}_t = \gamma_f\beta\hat{\pi}_{t+1} + \gamma_{mc}\widehat{mc}_t\\
\widehat{mc}_t = \hat{y}_t - \varsigma_c\hat{\pi}_t\\
\hat{\imath}_t = \alpha_\pi\hat{\pi}_t + \alpha_y\hat{y}_t
\end{cases} \qquad (85)$$

where:

$\varsigma_c = \frac{Y}{C}\varphi\left(\bar{\pi}-1\right)\bar{\pi} = \frac{\varphi(\bar{\pi}-1)\bar{\pi}}{\left[1-\frac{\varphi}{2}(\bar{\pi}-1)^2\right]}$;

$\gamma_p = 0$,

$\gamma_f = \frac{\left(2\bar{\pi}^2-\bar{\pi}\right)+[(\bar{\pi}-1)\bar{\pi}]^2\varphi}{(2\bar{\pi}^2-\bar{\pi})+\beta[(\bar{\pi}-1)\bar{\pi}]^2\varphi}$;

and

$\gamma_{mc} = \frac{\varepsilon-1+\varphi\left(\bar{\pi}^2-\bar{\pi}\right)(1-\beta)}{\varphi\left[(2\bar{\pi}^2-\bar{\pi})+\beta[(\bar{\pi}-1)\bar{\pi}]^2\varphi\right]}$.

Substituting $\widehat{mc}_t$ in the NKPC and $\hat{\imath}_t$ in the IS curve we get:

$$\begin{cases}
\hat{y}_t = E_t\hat{y}_{t+1} + \left(1-\varsigma_c\right)\hat{\pi}_{t+1} + \left(\varsigma_c-\alpha_\pi\right)\hat{\pi}_t - \alpha_y\hat{y}_t\\
\hat{\pi}_t = \gamma_f\beta\hat{\pi}_{t+1} + \gamma_{mc}\hat{y}_t - \gamma_{mc}\varsigma_c\hat{\pi}_t
\end{cases}. \qquad (86)$$

We consider two cases: 1) determinacy with zero inflation steady state; 2) determinacy with trend inflation; 3) the effects of trend inflation on the determinacy region.

A.4.1 Determinacy with zero inflation steady state

Note that in the case $\bar{\pi} = 1$, the system becomes:

$$\begin{cases} \hat{y}_t = E_t \hat{y}_{t+1} - \alpha_\pi \hat{\pi}_t - \alpha_y \hat{y}_t + E_t \hat{\pi}_{t+1} \\ \hat{\pi}_t = \beta \hat{\pi}_{t+1} + \kappa \hat{y}_t \end{cases} \tag{87}$$

we call $\kappa = \frac{\varepsilon - 1}{\varphi}$.

The system can be rewritten in matrix form as follows

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{y}_t \end{bmatrix} = \begin{bmatrix} \frac{\kappa}{\alpha_y + \kappa \alpha_\pi + 1} + \beta \frac{\alpha_y + 1}{\alpha_y + \kappa \alpha_\pi + 1} & \frac{\kappa}{\alpha_y + \kappa \alpha_\pi + 1} \\ \frac{1}{\alpha_y + \kappa \alpha_\pi + 1} - \beta \frac{\alpha_\pi}{\alpha_y + \kappa \alpha_\pi + 1} & \frac{1}{\alpha_y + \kappa \alpha_\pi + 1} \end{bmatrix} \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{y}_{t+1} \end{bmatrix}. \tag{88}$$

Conditions for having two positive roots within the unit circle are

1) $\det B < 1$

2) $trB - \det B < 1$

3) $trB + \det B > -1$

Condition (1) implies that

$$\alpha_y > \beta - 1 - \alpha_\pi \frac{\varepsilon - 1}{\varphi}. \tag{89}$$

Condition (2) implies that

$$\alpha_\pi + \alpha_y \frac{(1 - \beta)\varphi}{(\varepsilon - 1)} > 1. \tag{90}$$

Notice that, given the long-run log-linearized Phillips curve, $\hat{y} = \frac{(1-\beta)\varphi}{\varepsilon - 1} \hat{\pi}$, condition (90) has the following interpretation

$$\alpha_\pi + \alpha_y \left. \frac{\partial \hat{y}}{\partial \hat{\pi}} \right|_{LR} = \left. \frac{\partial \hat{\imath}}{\partial \hat{\pi}} \right|_{LR}$$

where $\left. \frac{\partial \hat{y}}{\partial \hat{\pi}} \right|_{LR}$ is the multiplier of inflation in the long-run log-linear NKPC.

Condition (3) requires

$$\frac{(\kappa + 2\beta + \beta \alpha_y + 1)}{\alpha_y + \kappa \alpha_\pi + 1} > -1 \tag{91}$$

that is always satisfied if $\frac{\kappa + \beta \alpha_y + 1}{\alpha_y + \kappa \alpha_\pi + 1}$ and $\frac{2\beta}{\alpha_y + \kappa \alpha_\pi + 1}$ are both positive and if condition 2 is verified.

A.4.2 Determinacy with trend inflation and no indexation

As before, we rewrite the system of equations in the matrix form

$$\begin{bmatrix} \hat{\pi}_t \\ \hat{y}_t \end{bmatrix} = B' \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{y}_{t+1} \end{bmatrix} \tag{92}$$

where

$$B' = \begin{bmatrix} \beta\left(\alpha_y+1\right)\frac{\gamma_f}{\alpha_y+\alpha_\pi\gamma_{cm}+\varsigma_c\alpha_y\gamma_{cm}+1} - \left(\varsigma_c-1\right)\frac{\gamma_{cm}}{\alpha_y+\alpha_\pi\gamma_{cm}+\varsigma_c\alpha_y\gamma_{cm}+1} & \frac{\gamma_{cm}}{\alpha_y+\alpha_\pi\gamma_{cm}+\varsigma_c\alpha_y\gamma_{cm}+1} \\ -\left(\varsigma_c-1\right)\frac{\varsigma_c\gamma_{cm}+1}{\alpha_y+\alpha_\pi\gamma_{cm}+\varsigma_c\alpha_y\gamma_{cm}+1} - \beta\gamma_{for}\frac{\alpha_\pi-\varsigma_c}{\alpha_y+\alpha_\pi\gamma_{cm}+\varsigma_c\alpha_y\gamma_{cm}+1} & \frac{\varsigma_c\gamma_{cm}+1}{\alpha_y+\alpha_\pi\gamma_{cm}+\varsigma_c\alpha_y\gamma_{cm}+1} \end{bmatrix}$$

Again, conditions for having two positive roots within the unit circle are

1) $\det B' < 1$

2) $trB' - \det B' < 1$

3) $trB' + \det B' > -1$

Since

$$\det B' = \frac{\beta\gamma_f}{\alpha_y + \alpha_\pi\gamma_{cm} + \varsigma_c\alpha_y\gamma_{cm} + 1} \tag{93}$$

and

$$trB' = \frac{\beta\gamma_f + \gamma_{cm} + \beta\alpha_y\gamma_f + 1}{\alpha_y + \alpha_\pi\gamma_{cm} + \varsigma_c\alpha_y\gamma_{cm} + 1} \tag{94}$$

then,

1) $\det B' < 1$ requires

$$\alpha_y > \beta\gamma_f - 1 - \alpha_\pi\gamma_{cm}. \tag{95}$$

2) $trB - \det B < 1$ requires

$$\alpha_\pi + \frac{\left(1 + \varsigma_c\gamma_{cm} - \beta\gamma_f\right)}{\gamma_{cm}}\alpha_y > 1 \tag{96}$$

that can also be rewritten as:

$$\alpha_\pi + \frac{\left(1 + \varsigma_c\gamma_{cm} - \beta\gamma_f\right)}{\gamma_{cm}}\alpha_y = \alpha_\pi + \alpha_y \left.\frac{\partial\hat{y}}{\partial\hat{\pi}}\right|_{LR} = \left.\frac{\partial\hat{\imath}}{\partial\hat{\pi}}\right|_{LR} \tag{97}$$

because the long-run log-linearized Phillips curve is

$$\hat{\pi} = \gamma_f\beta\hat{\pi} + \gamma_{mc}\hat{y} - \gamma_{mc}\varsigma_c\hat{\pi}. \tag{98}$$

3) $trB + \det B > -1$ requires

$$\frac{2\beta\gamma_f + \gamma_{cm} + \beta\alpha_y\gamma_f + 1}{\alpha_y + \alpha_\pi\gamma_{cm} + \varsigma_c\alpha_y\gamma_{cm} + 1} > -1, \tag{99}$$

that is redundant provided that $\gamma_f, \gamma_{cm}, \varsigma_c > 0$, $\bar{\pi} \geq 1$ and condition (96) satisfied.

A.4.3 Trend inflation and Determinacy

We now study the effect of trend inflation on the determinacy region. In order to obtain analytical results we compute the value of the derivative of the coefficient $\frac{\left(1+\varsigma_c\gamma_{cm}-\beta\gamma_f\right)}{\gamma_{cm}}$ for $\bar{\pi}=1$. We study the derivative in three steps by splitting our coefficient in three parts, i.e.,

$$\frac{\left(1+\varsigma_c\gamma_{cm}-\beta\gamma_f\right)}{\gamma_{cm}}=\frac{1}{\gamma_{cm}}+\frac{\varsigma_c\gamma_{cm}}{\gamma_{cm}}-\frac{\beta\gamma_f}{\gamma_{cm}}=\frac{1}{\gamma_{cm}}+\varsigma_c-\frac{\beta\gamma_f}{\gamma_{cm}} \tag{100}$$

1) $\frac{d\varsigma_c}{d\bar{\pi}}=\frac{\varphi(2\bar{\pi}-1)+\frac{\varphi^2}{2}(\bar{\pi}-1)^2}{\left[1-\frac{\varphi}{2}(\bar{\pi}-1)^2\right]^2}$.Evaluated at $\bar{\pi}=1:$

$$\frac{d\varsigma_c}{d\bar{\pi}}_{\bar{\pi}=1}=\varphi>0. \tag{101}$$

2) $\frac{d(1/\gamma_{cm})}{d\bar{\pi}}=\frac{\varphi\left(2\bar{\pi}^2-\bar{\pi}\right)}{(\varepsilon-1+\varphi(\bar{\pi}^2-\bar{\pi})(1-\beta))}+\frac{\varphi^2\beta[(\bar{\pi}-1)\bar{\pi}]^2}{(\varepsilon-1+\varphi(\bar{\pi}^2-\bar{\pi})(1-\beta))\left(1-\frac{\varphi}{2}(\bar{\pi}-1)^2\right)}$. Evaluated at $\bar{\pi}=1$

$$\frac{d\left(1/\gamma_{cm}\right)}{d\pi}_{\bar{\pi}=1}=\frac{\varphi 3\left(\varepsilon-1\right)-\varphi^2\left(1-\beta\right)}{\left(\varepsilon-1\right)^2}=\frac{\varphi}{\left(\varepsilon-1\right)}\left[3-\frac{\varphi\left(1-\beta\right)}{\left(\varepsilon-1\right)}\right] \tag{102}$$

3)

$$\frac{d\left(\frac{\beta\gamma_f}{\gamma_{cm}}\right)}{d\pi}_{\bar{\pi}=1}=\frac{\varphi\beta}{\varepsilon-1}\left[3-\frac{\varphi\left(1-\beta\right)}{\varepsilon-1}\right] \tag{103}$$

Therefore, the total derivative of $\frac{\left(1+\varsigma_c\gamma_{cm}-\beta\gamma_f\right)}{\gamma_{cm}}$ with respect to $\bar{\pi}$ evaluated at $\bar{\pi}=1$ is

$$\frac{d\left[\frac{\left(1+\varsigma_c\gamma_{cm}-\beta\gamma_f\right)}{\gamma_{cm}}\right]}{d\bar{\pi}}_{\bar{\pi}=1}=\varphi+\frac{\varphi\left(1-\beta\right)}{\varepsilon-1}\left[3-\frac{\varphi\left(1-\beta\right)}{\varepsilon-1}\right] \tag{104}$$

as in Proposition 2.

B Figures

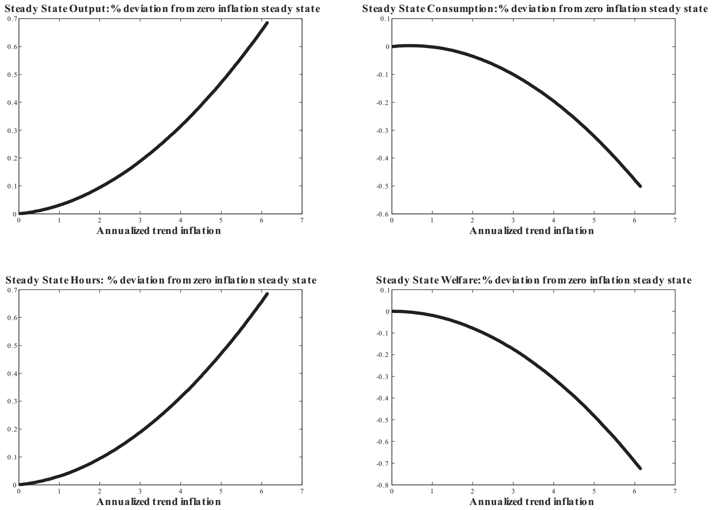

Fig. 1 Steady state and the long-run Phillips curve in the Rotemberg model

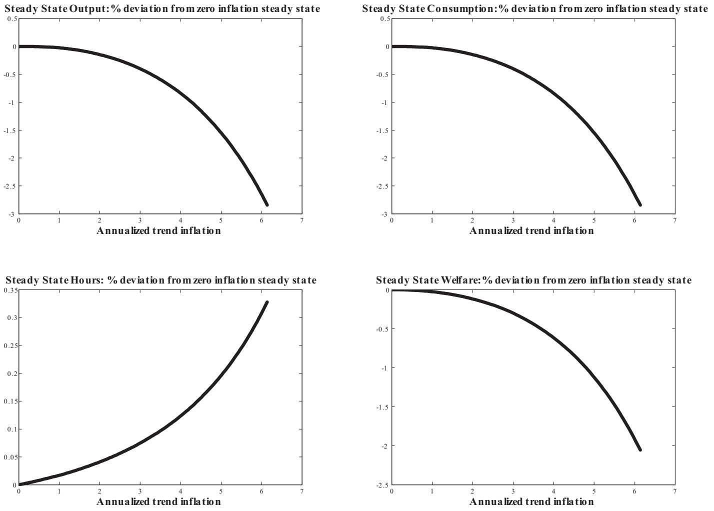

Fig. 2 Steady state and the long-run Phillips curve in the Calvo model

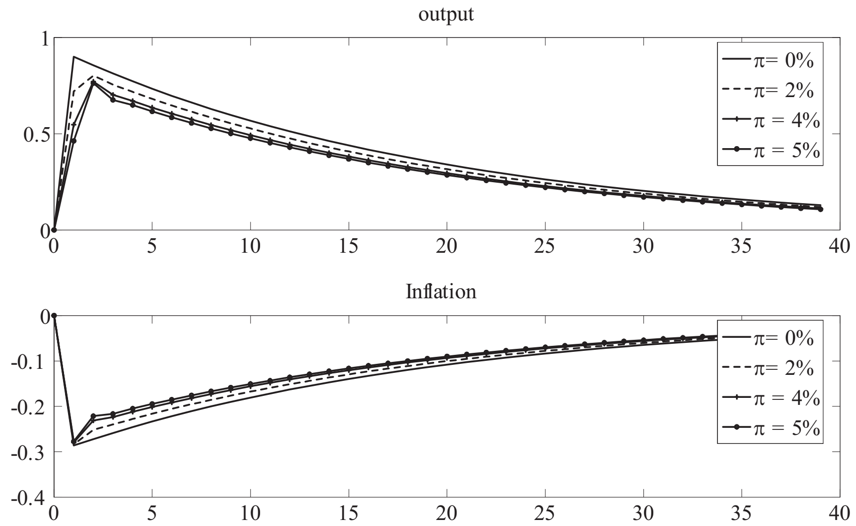

Fig. 3 IRFs to a positive technology shock under Rotemberg model.

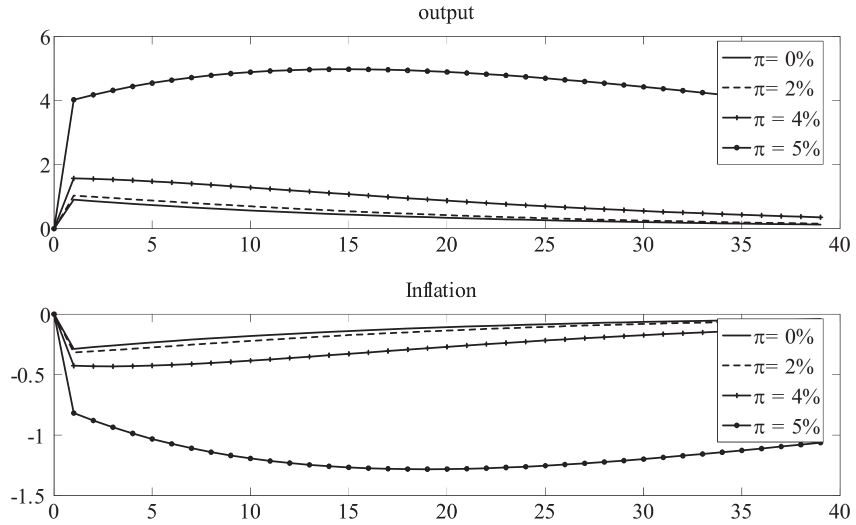

Fig. 4 IRFs to a positive technology shock under Calvo model.

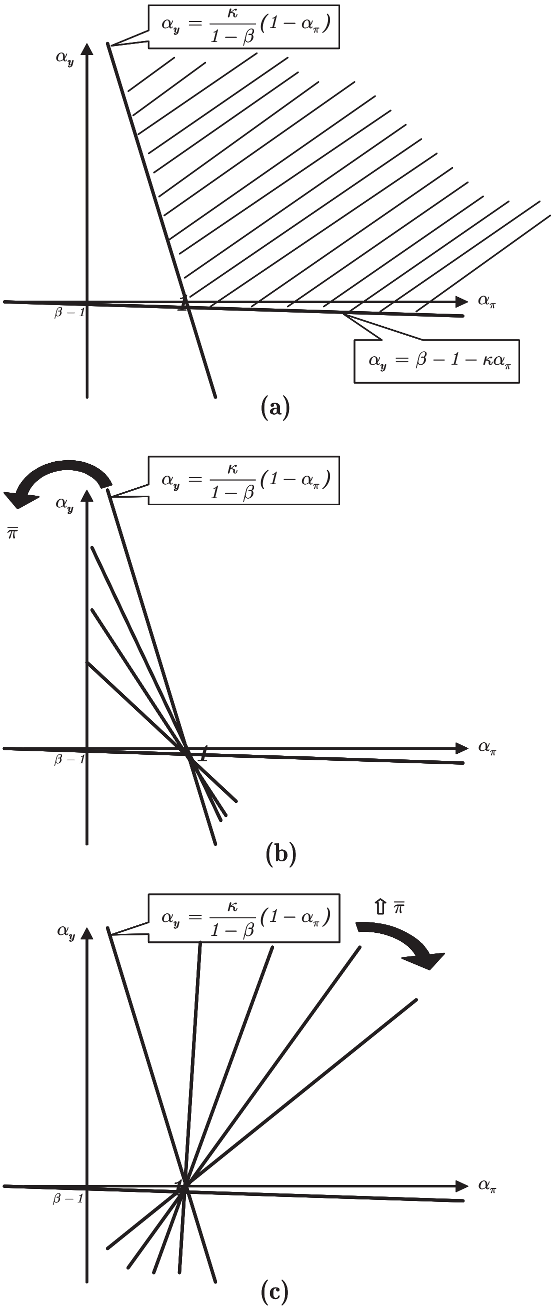

Fig. 5 The effect of trend inflation on the Taylor Principle. (b) Rotemberg vs. (c) Calvo pricing.

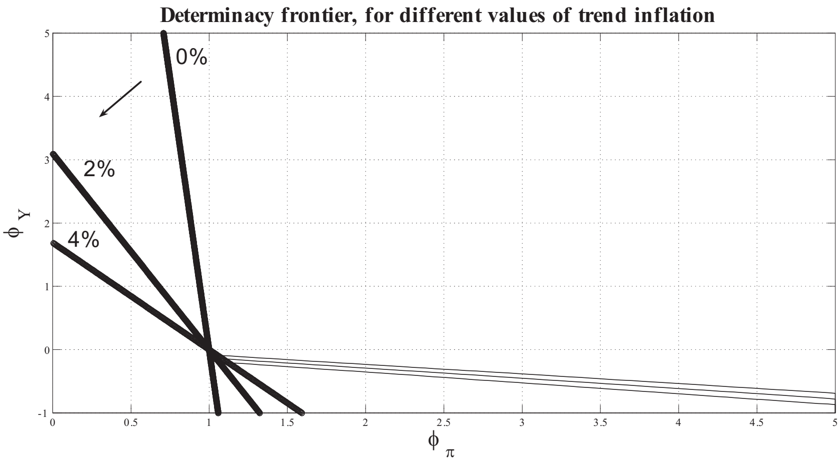

Fig. 6 The effect of trend inflation on the determinacy region in the Rotemberg model.

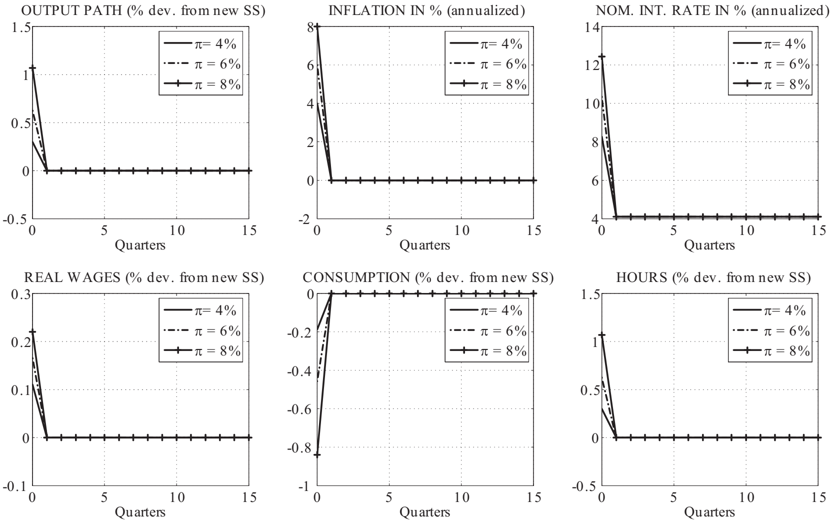

Fig. 7 Disinflation in the Rotemberg model.

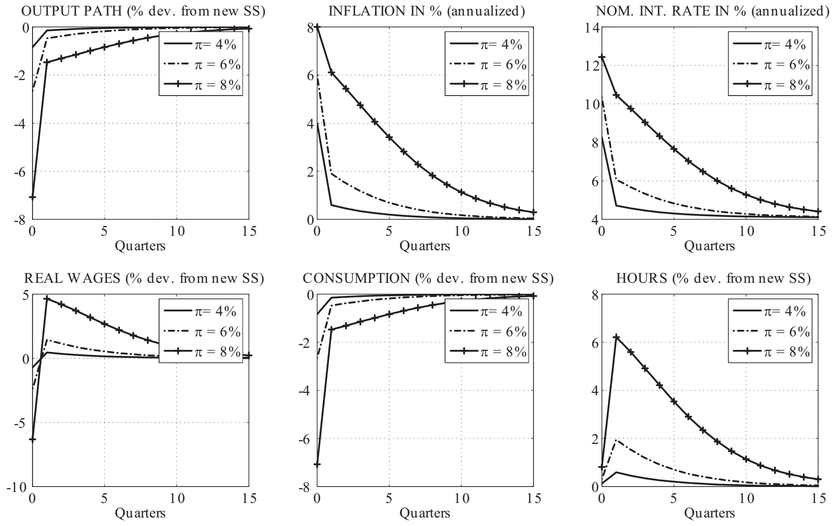

Fig. 8 Disinflation in the Calvo model.

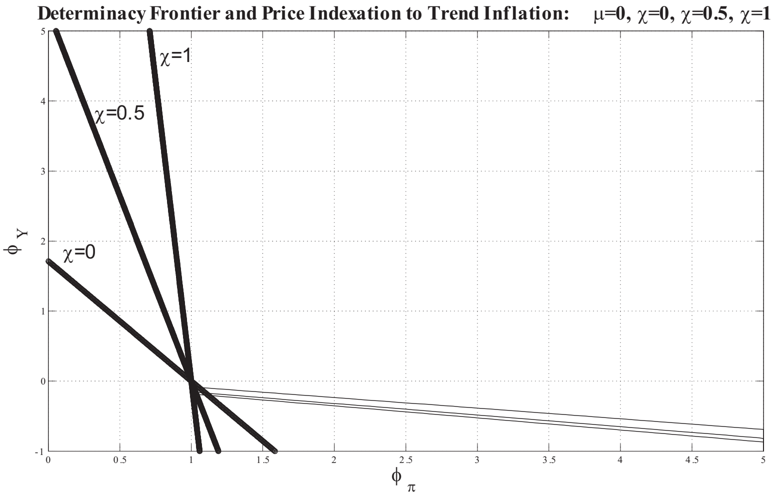

Fig. 9 Price indexation to trend inflation and determinacy in the Rotemberg model.

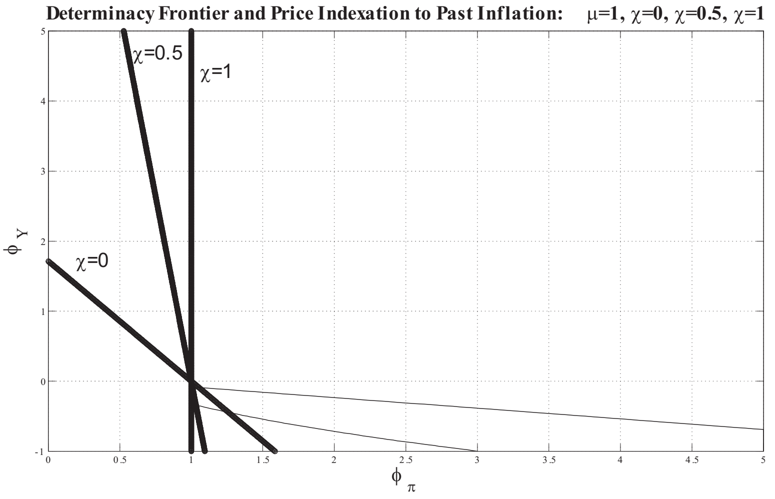

Fig. 10 Price indexation to past inflation and determinacy in the Rotemberg model.