Quaderni di Dipartimento

Inflation persistence, Price Indexation and Optimal Simple Interest Rate Rules

Guido Ascari
(Università di Pavia)

Nicola Branzoli
(University of Wisconsin Madison)

129 (11-10)

Dipartimento di economia politica
e metodi quantitativi
Università degli studi di Pavia
Via San Felice, 5
I-27100 Pavia

Novembre 2010

Inflation persistence, Price Indexation and Optimal Simple Interest Rate Rules

Guido Ascari — *University of Pavia*

Nicola Branzoli — *University of Wisconsin Madison*

November 12, 2010

Abstract

We study the properties of the optimal nominal interest rate policy under different levels of price indexation. In our model indexation regulates the sources of inflation persistence. When indexation is zero, the inflation gap is purely forward-looking and inflation persistence depends only on the level of trend inflation, while full indexation makes the inflation gap persistent and it eliminates the effects of trend inflation.

We show that in the former case the optimal policy is inertial and targets inflation stability while in the latter the optimal policy has no inertia and targets the real interest rate. We compare our results with empirical estimates of the FED's policy in the post-WWII era.

JEL classification: E31, E52.

Keywords: Inflation Persistence, Taylor Rule, New Keynesian model, Indexation

1 Introduction

Since the Furher and Moore (1995) seminal contribution, persistence has long been recognized as one of the main properties of the inflation process. Recently a stimulating debate in the literature focused on the sources of inflation persistence and its changes over time.

Inflation persistence was initially assumed to be an "intrinsic" phenomenon of the inflation process. The modelling strategy was therefore to introduce an endogenous lagged term in the New Keynesian Phillips Curve (NKPC henceforth) to match the inflation persistence found in the data (see e.g., Galí and Gertler, 1999, Mankiw, 2001, Rudd and Whelan, 2007). Following this strategy, Christiano et al. (2005) (CEE henceforth) introduced backward-looking price and wage indexation in the Calvo pricing setup. This modelling device is now commonly used in standard workhorse DSGE New Keynesian models of the business cycle (e.g., Schmitt-Grohé and Uribe, 2006, 2007b, Smets and Wouters, 2003, 2007, and Altig et al., 2004).

More recently, however, some papers show that to understand inflation persistence is useful to distinguish between trend inflation, i.e., a low frequency component of the inflation process, and the inflation-gap, i.e., the difference between actual inflation and trend inflation. It is then natural to think of trend inflation as the Federal Reserve's long-run target for inflation. Cogley and Sbordone (2008) show that an NKPC with no intrinsic inflation persistence, but that allows for shifts in trend inflation, successfully describes US inflation dynamics. Hence, if (exogenous) shifts in the level of trend inflation are taken into account, the inflation gap has no persistence and there is no need to assume backward-looking indexation.[1] Cogley et al. (2010) show that the persistence in the inflation-gap increased during the Great Inflation and declined after the Volcker disinflation. The main reason behind this shift in inflation volatility and persistence is the stability of the Fed's long-run inflation target.[2] The findings in Benati (2008) are similar in spirit. Using data from the US, UK, Euro Area, Switzerland, Canada, New Zeland and Japan, Benati (2008) shows that inflation persistence has not been constant across policy regimes, providing empirical evidence about the relationship between infla-

[1] See also Benati (2009) for similar results using international data, and Barnes et al. (2009) for a critique of their results.

[2] Ireland (2007) is the first attempt to endogenize the inflation target of the Central Bank.

tion persistence and trend inflation. When these countries adopted an explicit inflation target, reducing the average level of inflation, the inflation gap showed no persistence. This suggests that inflation persistence is not an "intrinsic" feature of the inflation process, but it depends on the particular monetary policy regime. Thus the empirical evidence suggests that the main driving force behind inflation persistence could be either the inflation-gap or trend inflation, depending on the historical periods and on the monetary policy regimes.[3]

It matters for monetary policy whether the source of inflation persistence is intrinsic or comes from trend inflation. For example, optimal monetary policy differs whether inflation persistence is intrinsic (see Steinsson, 2003) or derives from changes to trend inflation (see Ascari and Ropele, 2007).

Our aim is to investigate how optimal monetary policy, in the form of optimal simple rules, varies with changes in the source of inflation persistence. From a theoretically perspective, there is an easy way to distinguish these two cases in a standard medium-scale New Keynesian DSGE model. In our model there are two sources of inflation persistence: the level of trend inflation and the persistence in the inflation gap. We observe that the degree of backward-looking indexation is the key parameter regulating the strength of these two sources. When indexation to past inflation is full, then inflation persistence is all due to the inflation-gap, and there is no effect of trend inflation on the dynamics of the model (see Ascari, 2004). When past inflation indexation is zero, instead, the inflation-gap is a forward-loooking variable, while trend inflation affects the dynamics of the model inducing inertia in the adjustment (see Amano et al., 2007 and Yun, 2005). Trend inflation increases the dispersion of prices across different sectors, raising the persistence of the inflation process. The intuition starts from the observation that price-resetting firms take into account that many firms are not keeping up with the pace of inflation. The optimal reset price will therefore increase less. Thus the price level takes longer to adjust to its long run value and inflation becomes more persistent. Technically, the inflation process is affected by price dispersion which is a backward-

[3] In the last section of their work, Cogley et al. (2010) provides a very helpful discussion of their results in relation to the previous literature on inflation persistence. So we refer the reader to it for a more comprehensive discussion of the literature. In particular, see Cogley and Sargent (2005), Cogley et al. (2006), Primiceri (2006) and Stock and Watson (2007).

looking variable, and thus induces persistence in inflation.[4] This inertial adjustment mechanism is corrected with full indexation. In this case, the effects of price dispersion on the dynamics of the model is only of second-order (see Schmitt-Grohé and Uribe, 2004a, 2006). Hence, the degree of backward-looking indexation parametrizes how much the persistence of inflation is due to an "intrinsic" inflation-gap component, and how much is induced by trend inflation, through price dispersion. So to ask how the source of inflation persistence affects the optimal policy rule, we will investigate how the degree of indexation affects the optimal policy rule and welfare.

We need to make several modelling choices to answer our research question. First, we use an "operational" medium-scale model, more precisely the model in Schmitt-Grohé and Uribe (2004a) (SGU henceforth) or CEE. This model has been largely used for its empirical success in replicating the behavior of the US and Euro area business cycles. Second, we confine our analysis to optimal, "simple and implementable" monetary policy rules, following closely SGU.

Third, methodologically, we decide for a painstaking grid-search algorithm for the results in the main Section of the paper. As in SGU, we consider nine different cases, combining on the one hand backward-looking, current-looking and forward-looking Taylor rules and, on the other hand, no inertia, inertial and superinertial Taylor rules. This allows us to focus on the implications of the degree of backward-looking indexation for both the shape and the coefficients of the optimal simple rule from the point of view of the stochastic steady state. The grid search method allows us to find the global maximum in our parameter grid, but it is computationally very costly. We therefore switch to a numerical maximization algorithm, as in Schmitt-Grohé and Uribe (2006, 2007a,b), in the robustness section of the paper, i.e., Section 6. Therefore, this section will also indirectly provide a robustness check of our results over the employed methodology.

Fourth, we just focus on the *degree of indexation of prices*, and not of wages. The main reason is that, given our research strategy, the curse of dimensionality is very high, so we could not perform both the analysis.[5] Moreover assuming full wage indexation seems more compelling from both an empirical and theoretical point of view,[6] and also

[4] Section 3 describes the details of this transmission.

[5] For the main Section of the paper, we performed 677448 simulations, each of one take 90 seconds (on a standard Pentium IV (R) 3GHz), reaching a total of 16936.2 computer hours (almost 2 years).

[6] For example, Levin et al. (2006) estimates the average price indexation between 0.11 and 0.19

from anecdotal evidence.

Recall that the empirical literature shows that trend inflation and inflation persistence are related across regimes. This paper highlights the role of price dispersion as the prominent mechanism through which trend inflation affects inflation persistence, and hence the optimal interest rate rule, in a medium-scale DSGE model. Our main results confirm that the shape of the optimal policy is mainly driven by the degree of price dispersion in the economy. For a given level of trend inflation, the lower the degree of indexation, the higher is price dispersion, and the associated costs for the economy. The optimal response of the policy is to stabilize inflation in order to contain price dispersion. Indeed, we show that the variance of price dispersion decreases, while the one of inflation increases with indexation. Moreover, a general prescription for monetary policy is that the lower is the degree of indexation, the larger should be the response of the monetary policy to inflation gap. This confirms the SGU result that inflation volatility under the optimal rule is significant if there is full indexation, while near zero if there is no indexation. Moreover, as in SGU, we find that none of the several optimal policies in the various cases features a substantial reaction to output.[7]

Moreover, we find that the more trend inflation affects inflation persistence, the more inertial is the optimal policy. A robust finding is that the optimal rule is no inertial for high levels of indexation, while it is inertial or superinertial for low levels of indexation. The lower is indexation, therefore, the more important is the ability to exploit the expectational channel of monetary policy in order to stabilize inflation and price dispersion. In general, by changing the source of inflation persistence, the degree of price indexation changes the trade-offs monetary policy is facing in a non-obvious way.

Another important, and not much debated, issue regards the first order effects of changing the degree of indexation.[8] Our results show that the difference in conditional

while the average wage indexation between 0.77 and 0.86 (see also Smets and Wouters, 2003, 2007). Furthermore Ascari and Branzoli (2010) shows that full wage indexation maximizes the steady state welfare for every level of price indexation.

[7] With the exception of the rules maximizing unconditional welfare, see Section 6.

[8] First order effects derive from a change in the steady state of the model, while second order effects derive from changes that do not influence the steady state (but only the dynamics around the steady state). In this sense, a change of the degree of indexation causes first-order effects because it affects the steady state, while a change in the policy rule parameters cause only second-order effects because it

welfare across the various cases is mainly driven by the first order steady state effects, suggesting that the literature often ends focusing on second-order analysis (as the shape of optimal policy in an approximated model), overlooking important first-order aspects (as the calibration of the degree of indexation) that strongly influence the shape of the optimal policy and have important welfare effects.

Furthermore, one of the main message of this paper is that full indexation is a very special assumption. Full indexation nullifies the effects of trend inflation, and limits to second-order the effects of price dispersion, which is one of the key variables in the Calvo type price staggering models. When prices that are not repotimized are fully anchored to inflation, price dispersion become irrelevant not only from a long-run perspective (as it is obvious), but also in the dynamics of the model. Full indexation is a very special case, almost like a discontinuity point, because it cancels one of the main mechanism of the model.

Finally, we also compare model-predicted optimal policies under different levels of indexation with the empirical estimates of the Taylor rules under different policy regimes. The FED's policy across regimes is very similar to the optimal one implemented in the model where inflation persistence is induced by trend inflation and not by the inflation gap. Therefore, coherently with Cogley et al. (2010), our results also suggest that policy sources of the Great Moderation should be found mainly in movements of other policy instruments, such as the inflation target.

Our analysis is close to Sbordone (2007), who studies optimal policies under different models of inflation persistence and distinguishing two cases. In the first case, inflation persistence is embedded in trend inflation, modeled as a random walk with drift. In the second case persistence is hardwired in the inflation gap. She finds that the optimal policy is sensitive to the model assumed. We extend and complement her findings in two directions. First, we consider simple monetary policy rules rather than the fully optimal ones. Thus our analysis can be interpreted as a discussion on the robustness of her study to the class of policy rules available to the Central Bank. We confirm that the shape of the optimal policy is sensitive to how persistence is induced into the model. This becomes very clear in our analysis, because the degree of indexation is the key parameter that governs the dynamics of the model. Therefore, we share with

does not affect the steady state of the model.

Sbordone (2007) the conclusion that one needs to think deeply about how persistence is hardwired into the model before drawing policy conclusions. Second, in addition to Sbordone (2007), we use a medium-scale model of the business cycle rather than one of only output and inflation, to take the normative analysis to the data.

The rest of the paper is organized as follows. Section 2 briefly sketches the main properties of the model, that is formally described in the Appendix. Section 3 analyzes the role of price indexation in the dynamics of the model, highlighting how it influences the effect of trend inflation and inflation persistence. Section 4 and 5 present the main results of the paper and discusses the similarities between the optimal model-based policy and the empirical estimates from the literature. Section 6 checks the robustness of the main results to different degrees of price stickiness, type of indexation and measure of welfare. The last Section concludes.

2 Model Description

The basic setup is a medium-scale macroeconomic model, obtained by augmenting the standard New Keynesian model with nominal and real frictions. Theses are crucial elements in replicating the dynamics of US business cycle. Since the model is exactly the one described in many papers (e.g., SGU and CEE), we will briefly introduce here the key elements, leaving to the Appendix all the details about the model and calibration.

The real frictions of the model are monopolistic competition, habit persistence in consumption, fixed cost in an otherwise standard Cobb-Douglas production function, variable capacity utilization and adjustment costs in investment. Money is introduced into the model via real balances in the utility function and a cash-in-advance constraint on wage payments of firms. The long-run level of inflation is set equal to the average inflation of the US in the post World-War II period. Wages and prices are sticky à la Calvo-Yun.

Prices that are not re-optimized each period are indexed to past inflation. Woodford (2003) already shows how the degree of backward-looking indexation affects the appropriate microfounded loss function and the dynamics implied by the NKPC in a basic log-linear model. Moreover, in a non-linear model with positive trend inflation, price dispersion is another important channel through which backward-looking indexation affects the dynamics of the model and welfare (see Schmitt-Grohé and Uribe, various papers). Therefore we study a second-order Taylor approximation of the model around the non-stochastic steady state using the method developed in Schmitt-Grohé and Uribe (2004b).

As in SGU, we consider only monetary policy rules that are simple and implementable. Simple because they are a function of a few readily observable macroeconomic variables. Implementable because they must deliver an unique rational expectation equilibrium and induce an equilibrium that satisfy a constraint on the lower bound on the nominal interest rates.[9]

We thus consider monetary policy rules of the following class (where variables expressed in log deviation from steady state's values are denoted with a hat):

$$\hat{R}_t = \alpha_\pi E_t \hat{\pi}_{t-i} + \alpha_y E_t \hat{y}_{t-i} + \alpha_R E_t \hat{R}_{t-i}, \tag{1}$$

where $i \in \{-1, 0, 1\}$. We hence consider nine different cases, combining on the one hand backward-looking, current-looking and forward-looking Taylor rules and, on the other hand, no inertia, inertial and superinertial Taylor rules. This allows us to focus on the implications of the source of inflation persistence for the shape and the coefficients of the optimal simple rule.

3 The Macroeconomic Effects of Price Indexation

In this section we look into the details of the effects of indexation on the dynamics of the model.

Persistence in the inflation gap depends directly upon the degree of indexation. Log-linearizing Eq(32) for the inflation gap (see Appendix) and substituting the steady state values, we can express the inflation gap as a function of its own past values and the expectations component:

$$\hat{\pi}_t = \chi \frac{\bar{\pi}\alpha}{1-\eta} \hat{\pi}_{t-1} + \bar{\pi}^{(\eta-1)} \left[1 - \alpha \bar{\pi}^{(\eta-1)(1-\chi)} \right] \widehat{\tilde{p}}_t \tag{2}$$

where η measures complementarity across consumptions goods, α is the Calvo parameter, χ is the level of price indexation and $\widehat{\tilde{p}}_t$ denotes the price level set by optimizing

[9] Following SGU, we assume commitment to the rules. Results will be generally different under discretion. We leave this interesting question to further research.

firms, which depends solely on expectations about future inflation and output gap. Eq.(2) shows that indexation, by construction, induce inertial adjustment in the inflation level.

By changing the degree of price indexation, we do not only influence the persistence of the inflation gap, but we also regulate the effects of trend inflation on inflation persistence. Consider the case of no indexation. Under the assumption of monopolistic competition, optimizing firms look at the price index, and in particular at the dispersion of prices, to set $\widehat{\tilde{p}}_t$. In particular, since some firms do not move up with the pace of inflation, optimizing firms look also at the price dispersion generated by trend inflation. Many firms have a low relative price. An increase in trend inflation makes this gap bigger, therefore making the optimizing firms adjust slower to a change in current and expected conditions. Full indexation counteracts this effect, because it allows also the non price-resetting firms to adjust with the long-run growth of prices, thus reducing the effect on trend inflation on price dispersion. Partial indexation, instead, increases price dispersion and the effect of thsi mechanism on the persistence of inflation. The higher the degree of indexation, the less trend inflation would impact on the the optimal relative price $\widehat{\tilde{p}}_t$ and on the adjustment of of inflation.[10]

Log-linearizing the Eq.(25) and Eq.(32) in the Appendix, and then substituting the term referring to the newly reset price, it yields the following expression for the log-linearized dynamics of price dispersion, i.e., s :[11]

$$\hat{s}_t = \left[\frac{\eta\alpha\bar{\pi}^{(\eta-1)(1-\chi)} \left(\bar{\pi}^{1-\chi} - 1\right)}{1 - \alpha\bar{\pi}^{(\eta-1)(1-\chi)}} \right] \left(\hat{\pi}_t - \chi\hat{\pi}_{t-1}\right) + \alpha\bar{\pi}^{\eta(1-\chi)}\hat{s}_{t-1}. \tag{3}$$

Note that the higher the level of χ, the lower is persistence in the price dispersion term. If one assumes full indexation, the price dispersion is constant in the log-linearized version of the model ($\chi = 1 \Longrightarrow \hat{s}_t = 0$). On the other hand, the autoregressive term given by $\alpha\bar{\pi}^{\eta(1-\chi)}$ is maximized when $\chi = 0$. Figure 1 shows that full indexation is indeed a very special assumption, because it nullifies the effects of price dispersion, while it matters at first-order with partial indexation.[12] When prices that are not re-optimized

[10] See also Ascari and Ropele (2007) for a thourough discussions of the effects of trend inflation on optimal policy in New Keynesian models.

[11] In (3), $\hat{\pi}_t$ is the log-deviation of inflation, χ is the degree of price indexation, $\bar{\pi}$ is trend inflation, α is the degree of price stickiness and η is the elasticity of substitution across goods.

[12] We shock the model with a 1% increase in aggregate productivity under a standard current looking

are fully anchored to inflation, price dispersion become irrelevant in the dynamics of the model. In Figure 2 we show the impulse response of inflation for different levels of trend inflation, holding indexation constant at 0. The figure shows that the higher is the level of inflation, the slower inflation returns to its long-run value when the economy is hit by an exogenous shock.

To summarize, the calibration of indexation regulates the effects of persistence in the inflation gap and persistence due to trend inflation. When $\chi = 0$, the inflation gap is a purely forward-looking variable and persistence in inflation is induced by the effects of trend inflation, through price dispersion. On the other extreme, full indexation cancels out the effects of positive trend inflation due to the price dispersion term and inflation persistence depends on the backward-looking component of the inflation gap. In what follows we assess the importance of these changes in shaping optimal operational monetary policies.

4 Optimal Interest Rate Policies and the Sources of Inflation Persistence

Our aim is to analyze policy rules à la Taylor, such that these policies can be actually operational and implementable for policy makers. Simple rules, as the ones considered here, are very easy to communicate and to be understood by the public, helping the transparency of central bank behavior.

An operational rule should be implementable in the sense that should both deliver a unique rational expectation equilibrium and satisfy the lower bound on the nominal interest rate.[13] As in SGU, we looked for the optimal monetary policy numerically discretizing the support $[-3, 3]$ in intervals of length 0.0625 for α_π and α_y in the particular class of rules of the form (1). Moreover, in (1): (i) i can take three different values, i.e., $i \in \{-1, 0, 1\}$ corresponding to forward- looking, current-looking and backward-looking policies respectively; (ii) $\alpha_R \in \{0, 1, 2\}$, corresponding to no inertial, inertial and super

Taylor Rule with $\alpha_\pi = 1.5$, $\alpha_y = \alpha_R = 0$.

[13]Following SGU, we require the logarithm of the equilibrium nominal interest rate not to be lower than two times the variance of the nominal interest rate, i.e., $ln(R^*) \geq 2\sigma_{\hat{R}_t}$. If the equilibrium nominal interest rate was normally distributed around its target value, then this constraint would ensure a positive nominal interest rate 98 percent of the time.

inertial rules respectively. On top of that, we study 8 levels of indexation. In addition to the two extreme cases of 0 and 1, we study the welfare-maximizing value of the non-stochastic steady state,[14] 0.8788, and 5 values close to 1, $0.75, 0.80, 0.85, 0.90$ and 0.95. The latters will be useful to analyze even small positive effects of the price dispersion channel on the monetary policy trade-offs.

We hence performed 677448 simulations, solving the model with the perturbation method developed in Schmitt-Grohé and Uribe (2004b). We rank policies using a measure of welfare based on second order approximation of the model around the non-stochastic steady state. We used both an unconditional and a conditional welfare measure, the latter to take into account of transitional dynamics. We use grid-search method for the results in this section.[15] The method allows us to find the global maximum in our parameters' grid.

We organize the presentation of results in this Section as follows. First, we illustrate how indexation changes the optimal rule and the dynamic response of the economy *across all the different types of rules considered*. Second, we analyze in detail how indexation changes the optimal rule and the dynamic response of the economy in the case of *one particular rule*, i.e., forward-looking no inertia. Third, we will illustrate how the optimal rule changes with indexation *within each class of policy rules*.

4.1 The Optimal Simple Rule

In this section we investigate the effect of indexation on the overall optimal simple monetary policy rule. Table 1 shows the type of policy rule, the optimal values of the coefficients and the corresponding welfare levels, for different values of the degree of indexation. Table 2 displays the corresponding unconditional moments for some variables of interest: consumption, output, price dispersion and inflation.

Policy Rule

Table 1 shows that the source of persistence affects the type of optimal operational policy. While the forward looking rule with no inertia is the optimal policy for the highest level of indexation, it turns out that lowering the degree of indexation, thus increasing

[14] See Ascari and Branzoli (2010) for a discussion of this result.

[15] Each simulation took 90 seconds (on a standard Pentium IV (R) 3GHz), so it is about 16936.2 computer hours (almost 2 years). This was made possible by optimizing the functioning of MATLAB symbolic toolbox, and clustering 30 computers.

the role of persistence due to trend inflation via price dispersion, leads the forward-looking rule to be substituted by the current-looking one. The backward-looking policy is never optimal. When there is no indexation then the forward looking inertial policy is optimal.

Looking at the changes in the coefficients of the optimal policy rule, Table 1 shows that when the persistence is entirely due to the inflation gap ($\chi = 1$), the optimal simple rule takes the form of a real interest rate targeting rule, with no degree of inertia. On the contrary, when persistence is entirely due to trend inflation ($\chi = 0$), there is a substantial fall in the reaction to the inflation gap, and policy rule become inertial. On the one hand, the unit root in the policy affects expectations of the long-run interest rate and inflation level. On the other hand, an inertial policy lacks flexibility and therefore would also entails some welfare costs. This may explain why in this case inertial policy is the best choice. Table 3 also provides some further evidence in this direction, showing how the optimal value of α_π in a forward-looking rule changes with the value of α_R and χ : α_π decreases with indexation, unless α_R assumes values close to 1. In other words, an increase in the inertia of the policy keeps inflation under control through the expectation channel.

For intermediate values, the increase in α_π is only modest, due to the fact that a lower indexation makes the inflation gap less persistent and thus, easier to control by a credible forward-looking rule. Moreover, despite the increase in α_π, the ability of the optimal policy to stabilize price dispersion worsen as indexation decreases. It may surprise that this last inertial policy features a very low α_π, but an inertial policy means a permanent change in the nominal interest rate in response to inflation. Indeed, it is interesting to note that, for the inertial forward looking optimal policy when $\chi = 0$, the sum of α_π and α_R is the same as the value of α_π for the forward looking no inertial optimal policy for the highest level of persistence in the inflation gap.

The Importance of Price Dispersion

Table 2 has one clear message: the higher is the persistence due to trend inflation, the lower is the variance of inflation under the optimal policy. First, the column $E(s)$ reports the expected deviation of s from steady state. This is very low, meaning that the mean value of price dispersion is its steady state value. Second, the unconditional variance, σ_s, do not change very much across different degrees of indexation and it is very small. *This means that the main task of the optimal operational rule is to stabilize the degree of price dispersion* around the steady state value. As shown also by SGU, price dispersion is the main inefficiency associated with inflation in New Keynesian models, because it acts like a negative productivity shift in this economy, and thus the optimal policy response calls for its stabilization. A temporary surge in inflation generates an increase in price dispersion, that needs to be stabilized by monetary policy. Moving away from full indexation increases significantly the inertia in price dispersion. Furthermore the lower the degree of indexation, the more current inflation is going to affect current price dispersion. It follows that the lower the degree of indexation, the more important is to stabilize inflation. As a matter of fact under optimal rules the variance of inflation reduces as the degree of indexation decreases.[16]

Table 2 also shows that full indexation is a very special case. Indeed, under full indexation the cost of price dispersion is of second order magnitude.[17] Indeed, despite the rather high volatility in inflation, the volatility of price dispersion is infinitesimal, that is price dispersion is almost always zero. It is interesting to note that, even moving away only slightly from full indexation, i.e. $\chi = 0.95$, considerably worsens the trade-offs monetary policy is facing. Indeed, the volatility of inflation drops by roughly a half, while the one of output increases by one third. Despite the lower volatility of inflation induced by a higher α_π, the volatility of price dispersion is higher by a factor 10^{10}! For the other values of indexation we analyze, instead, the volatility of price dispersion are of similar order of magnitude.[18] This indeed signals that full indexation is a quite special case. Assuming full indexation, however, undoes the role of trend inflation and price dispersion, an important mechanism in New Keynesian models. The full indexation assumption, hence, strongly affects the functioning of the economy, making the task of monetary policy easier.

The case of full indexation, i.e. ignoring the effect of trend inflation on the persistence

[16] See section 4.2.2 for a further discussion of this point.

[17] If $\chi = 1$, there is no first order effect of current inflation on price dispersion, see (3). In this case, the dynamic equation of price dispersion is autonomous from the model and does not influence it.

[18] Note that the volatility of price dispersion in the no indexation case is roughly one hundred times bigger than when $\chi = 0.95$. In this sense also $\chi = 0$ is an extreme case. But while the no indexation case is changing the dynamics of the model quantitatively (i.e., strengthening the effects of price dispersion), the full indexation case is changing the dynamics of the model also qualitatively (i.e., cancelling the price dispersion mechanism).

of inflation, does not imply price stability. The variance of inflation is about 2 per cent per annum, that is, half of its steady state value. However, as said above, the effect of trend inflation on the persistence of the model turns out to be very important in affecting the optimal operational policy. In particular, partial indexation calls for a tighter control of inflation, as a way to stabilize price dispersion and the effects of trend inflation.

Moreover, in the no indexation case, inflation is basically kept fix at the steady state level. Note that this would be the case also if the optimal policy when $\chi = 0$ (i.e., forward looking, $\alpha_\pi = 0.1875, \alpha_y = 0$ and $\alpha_R = 1$) is implemented in the full indexation case. σ_π would then be very small and equal to 0.2416.[19] This is exactly the task accomplished by the inertial policy: stabilize inflation. However, such a policy is not chosen in the full indexation case, because there is no need to stabilize price dispersion: full indexation offsets the the effects of tend inflation and keeps price dispersion constant. In other words, when there are no effects of trend inflation on inflation persistence, indexation take care of the problem of stabilizing price dispersion and stabilizing inflation is no more a fundamental issue for monetary policy.[20]

Welfare

Table 1 shows two different welfare measures: steady state welfare and conditional welfare.

The welfare level of the deterministic steady state does not depend on the persistence in the inflation gap, since the latter is constant by construction. Therefore the different levels of steady state welfare can be thought as a measure of the magnitude of the effects of trend inflation, through price dispersion.

The conditional welfare instead take into account the stochastic steady state of the economy, and therefore the effects of the persistence due to the inflation gap.[21]

[19] In this case the conditional welfare is equal to -156.7254 and $\sigma_s = 4.8039e\text{-}016$.

[20] Schmitt-Grohé and Uribe (2007a) already noted that when price indexation is zero, the variance of inflation is also virtually zero. More generally, without indexation, price dispersion is so costly that a minimum amount of price stickiness suffices to make price stability the central goal of optimal policy. This turns out to be true also in presence of other public finance effects calling for an increase in inflation volatility (see Schmitt-Grohé and Uribe, 2006, 2007b, 2008).

[21] The conditional measure of welfare assumes a initial state of the economy and takes into consideration the transitional dynamics from that initial condition to the stochastic steady state implied by the policy rule. We will assume that the initial condition is always the deterministic steady state (recall

Table 1 shows that the conditional welfare is always lower than the correspondent steady state welfare since the transitional dynamics (from the deterministic to the stochastic steady state) are taken into account. The difference between the two, however, is tiny and basically invariant across indexation levels. The losses across optimal policies are therefore determined by the steady state one, that is by the effects of trend inflation. The optimal simple rules maintain the conditional welfare very close to the steady state one. For example, given our calibration, the best indexation degree is 0.8788. If instead, the economy features full indexation the steady state welfare loss amounts to 0.0023%, while the loss in terms of conditional welfare amounts to 0.0020%. If instead, the economy features no indexation the steady state welfare loss amounts to 0.14%, and the loss measured in terms of conditional welfare is basically the same. Hence, the effects due to trend inflation are much more important than the those induced by the inflation gap. This suggests that an optimal simple monetary policy does a good job in stabilizing the cycle around the deterministic steady state, but cannot do much in compensating the first order effects deriving from trend inflation. Figure 3 displays the percentage welfare gain of the different indexation levels with respect to zero indexation. Each bar displays the steady state welfare gain and the overall conditional welfare gain net of the former.[22] The graph shows that an increase in the level of indexation reduces both the steady state losses and the losses associated with the stochastic steady state under the optimal rule, since indexation acts as a partial correcting mechanism for those firms that can not optimize their price. However, the effects of persistence in the inflation gap on losses due to movements in the exogenous variables are very small. This result holds also for any given level of χ considered.[23] Therefore, *conditional on choosing the optimal policy, persistence in trend inflation matters much more that persistence in the inflation gap.*

that the deterministic steady state varies with the degree of indexation).

[22]That is, define ssw_χ and cw_χ the steady state welfare and the conditional welfare, respectively, associated with a given value of χ. Then, for all levels of χ analyzed, the percentage conditional welfare gain is defined as: $\frac{cw_\chi - cw_0}{cw_0}$, and the percentage steady state gain (normalized over the conditional one) as: $\frac{ssw_\chi - ssw_0}{cw_0}$. Then the black area in the graph is $\frac{cw_\chi - cw_0}{cw_0} - \frac{ssw_\chi - ssw_0}{cw_0}$.

[23]For a given level of χ, conditional on choosing the optimal rule for each one of the different class of policies, the differences in conditional welfare levels are low. Results are available upon requests.

4.2 The Forward-looking Rule

In this section we concentrate on a particular rule: the forward-looking rule with no inertia (FLNI), i.e., $i = -1$ and $\alpha_R = 0$. We look at this particular rule because it turns out to be representative of all the other cases analyzed. This way we can focus on the effects of different sources of persistence within a single policy rule, leaving the comparison across rules to the next sections. These results let use describe more in details the mechanisms affecting monetary policy.

4.2.1 Implementability

Figure 4 shows how changes in the sources of inflation persistence can affect the determinacy and implementability regions. The graphs visibly display an increase in both the determinacy and implementability areas with a reduction in the effect of persistence due to trend inflation. Indeed low levels of indexation tend to reduce the parameter space available for policy options. In particular, if trend inflation affects the persistence of the model, i.e. indexation is not full, the Taylor principle ($\alpha_\pi > 1$) does not define a condition for determinacy. Indeed the effect of varying the degree of indexation on the implementability region are qualitatively similar to the effect of changing the trend inflation level, as in Ascari and Ropele (2007). Persistence in trend inflation, thus, increases the likely of sunspots fluctuations. *Ceteris paribus*, in fact, an increase in inflation leads to an increase in price dispersion, which in turn rises the marginal costs, and hence inflation. This mechanism gets stronger the lower is the indexation, and therefore the policy response needs to be tougher to induce determinacy of the rational expectation equilibrium.

4.2.2 Indexation, Optimal Policy and Unconditional Moments

Table 4 and 5 are equivalent to Table 1 and 2 for the FLNI policy rule. They show the optimal values of the coefficients of the FLNI policy, the corresponding welfare levels and unconditional moments for some variables of interest.

An increase in the effects of persistence due to trend inflation calls for a policy that further reduce the variance of inflation. The optimal policy does it in a straightforward way: by increasing the response to inflation, i.e., α_π, from 1.125 to 2.6875. If the policy $\alpha_\pi = 2.6875$ and $\alpha_y = 0.1875$ is implemented in the full indexation case, then $\sigma_\pi = 1.03$.

Again in the full indexation case monetary policy could stabilizes inflation through a higher α_π, but it chooses not to do so, because price dispersion is zero. Note, however, that, when $\chi = 0$, the variance of inflation is even higher than the one in $\chi = 0.85$,despite the value of α_π that is twofold. This signals that price dispersion inertia induced by trend inflation makes inflation more difficult to control. As said above, this may explain why for sufficiently low levels of indexation, the inertial policy rule may be preferred. Under full indexation, the optimal policy rule resembles a real interest rate targeting rule, while, as indexation decreases, the optimal policy rule shift to a pure inflation targeting rule with a stronger reaction to inflation deviation from target.

Finally, two results already noted above are still present. Firts, optimal policies are not responding to the output gap. Second, as Table 2, Table 5 again shows that full indexation is a rather special case. While the partial indexation cases are all similar in terms of order of magnitude of the second moments of the variables, the full indexation tends to cancel the effects of price dispersion, as evident from the variances of s, π and y.

4.3 Optimal rule across classes of policy rules

In this section we present how indexation affects the optimal operational rule also for each of the other policy class: current looking, backward looking and inertial policies.

Tables 6 to 8 display the results for the optimal operational simple policy rules within each different class of policies. Given the large number of policies we analyzed, Tables 6 to 8 show the optimal policy rules for just 3 levels of indexation: full indexation (i.e., $\chi = 1$), no indexation (i.e., $\chi = 0$) and the optimal steady state indexation level (i.e., $\chi = 0.8788$)[24].

The no inertial policy rules exhibit the same features explained above. The main message is that the degree of indexation modifies the trade-off monetary policy is facing, due to the interaction between trend inflation and inflation persistence. The higher is the effect of trend inflation: (i) the higher is price dispersion and its costs; (ii) the higher price dispersion inertia and its variance; (ii) the less persistent is inflation. Therefore Table 6 confirm the following facts (i) the variance of price dispersion decreases with indexation, while the one of inflation increases; (ii) the difference in conditional welfare

[24] For the other level of indexation analyzed results are available upon requests.

across the various cases is mainly driven by the first order steady state effects; (iii) the case $\chi = 1$ eradicate the effects of price dispersion from the model (iv) the optimal rule is not responding to output; (v) the lower the degree of indexation, the larger α_π.

Points (i)-(iv) hold true also for the inertial and super inertial policies.[25] The inertial and super inertial policies, instead, exhibit quite a different pattern regarding the parameter α_π. In particular, α_π is surprisingly decreasing with the degree of indexation. In the case of super inertial policy rules and no indexation it even becomes substantially negative. Since the value of α_R is different for inertial and super inertial policy rules, it may not surprise to find different values for α_π and α_y, but we do not have an intuition of the effects of indexation on α_π in these cases.[26] The no inertial policy rules always perform the best when there are small effects of trend inflation on the persistence of the model, while the inertial ones generally perform the best when indexation is zero. This confirms our arguments presented in the previous section.

5 Interpreting US monetary policy

In this Section we compare the above results with the US monetary policy in the postwar era. Our task is to determine whether one of the models better describes the FED's behavior over the last fifty years. To do so, our optimal policies are compared with empirical estimates of the same monetary rule. We use our results for the two benchmark cases of $\chi = 0$ and $\chi = 1$ to focus on sources of inflation persistence. Recall that when $\chi = 0$, the Central Bank has optimizes the movement in the nominal interest rate under the model in which inflation persistence is completely determined by the effects of trend inflation. When $\chi = 1$, the Central Bank sets the policy using the model in which inflation is persistent because the inertia in the inflation gap (see Section 3).

Table 9 shows our optimal Taylor rules in the two reference models with the estimates in Benati (2008), Smets and Wouters (2007) and Boivin and Giannoni (2006).[27] All these

[25] There are two exceptions among the superinertial policy rules with full indexation: the current looking and forward looking policy rules, where α_y is equal to 0.3125 and 0.625, respectively.

[26] Moreover, while the policy rules in the Tables satisfy the requirements for an operational policy, they are quite close to the boundaries of the determinacy frontiers. In particular, it is clear that a combinations of value for $\{\alpha_\pi, \alpha_y, \alpha_R\}$ as {0,0,1} or {0,0,2} would immediately lead to an explosive path for the nominal interest rate.

[27] The values for Boivin and Giannoni are taken from Table 2. The values for Smets and Wouters and

papers estimate a current-looking version of the policy rule, thus we report the models' best policies for current-looking rules with the globally optima.

Benati reports empirical estimates for the structural parameter for the whole sample and for the post-Volker period. Smets and Wouters (2005) and Boivin and Giannoni (2006) report the estimates for the pre- and post-Volker period. There is a remarkable similarity with our optimal policy under no persistence in the inflation gap. In particular, a level of inertia in the interest rate close to unity and a moderate response to inflation makes the model under $\chi = 0$ the likely environment that influenced the interest rate policy. The level of inertia in the policy rule estimated by Benati and Smets and Wouters is influenced by the AR(1) structure of the monetary policy shock assumed in their empirical models. In general, the empirical version of Eq.(1) contains an additional shock ε_t to better fit the data. Benati and Smets and Wouters assume an AR(1) process while Boivin and Giannoni assume i.i.d shocks. As a result, Benati and Smets and Wouters estimate α_R close to 0.8 and the autoregressive coefficient of the shock around 0.2, Boiving and Giannoni find instead an estimate of α_R equal to 1. Hence, independently of the particular empirical specification of the policy, the persistence in the Taylor rule is reasonably close to unity. The lack of inertia and the response to the inflation gap greater than one of the optimal policy in the model with no effect of trend inflation makes this model an unlikely candidate to explain the FED behavior. Note that we are considering the two benchmark models, while the true policy environment is probably somewhere in the middle. For example, Smets and Wouters (2005) estimate a level of indexation in prices between 0.21 and 0.45 depending on the sample period.

Boivin and Giannoni (2006) report a slight increase in the policy response to inflation between the two sub-samples.[28] This result is their key evidence to argue that monetary policy has become more effective in the in the post-1980 period. Although we agree with their general message, their result should be interpreted cautiously for two main reasons. First, their model is very stilized lacking the typical frictions used in medium-scale models to empirically fit the persistence in the macrodata. Indeed, Boivin and Giannoni

Benati are taken from Table 5 and 12 respectively. Results are reported considering that our parameters α_π and α_y are given by $(1-\rho)\,\phi_\pi$ and $(1-\rho)\,\phi_y$. The difference is in how our version of the Taylor rule is written.

[28] A similar increase in the policy parameters, from 0.266 to 0.452, can be found also in Clarida et al. (2000).

find a value of indexation equal to 1 in both samples, a result that is in sharp contrast with all the other papers discussed in this section. Second, as stressed by Mavroeidis (2004), the identification of forward-looking Taylor rules depends on the stability of inflation forecasts. The more inflation forecasts converge to the actual inflation target, as in the post-1980 period, the less the policy parameters are identified.[29]

6 Robustness

We here check the robustness of our results along four main dimensions: (i) the interrelation between the Calvo parameter and the degree of indexation; (ii) the level of trend inflation; (iii) the type of indexation; (iv) the welfare measure.

In order to do this exercise, we need to perform another large number of new simulations. We therefore employ an optimization algorithm, as in Schmitt-Grohé and Uribe (2006, 2007a,b). Both methods have advantages and disadvantages. The grid-search will always find the global maximum, but it discretizes the parameter space. The optimization algorithm, instead, will always find a local maximum, but it does not guarantees global convergence. Therefore, this section will provide an analysis of the different performance of the two algorithms and hence an indirect check of the robustness of our results over the employed methodology.

The Calvo parameter: Table 10a,b shows the optimal forward looking no inertia policies for different values of the degree of both price stickiness and indexation. We take six values of α between 0.55 and 0.8,[30] and two values of $\chi = 0, 1$. The results confirm that the shape of the parameters in the optimal policy are mainly driven by the persistence in trend inflation. Looking at Table 11a, when there is no persistence in the inflation gap, it is evident that the higher is α, that is the higher is the effect of trend inflation on the forward-looking decisions of price-resetting firms, the more the optimal

[29] See also Benati (2008) for a discussion of the identification of the parameters of the model.

[30] This interval is suggestd by Schmitt-Grohè and Uribe (2008). The most recent evidence on the micro data suggests that prices change on average approximately between 7 (see Klenow and Kryvstov, 2008) and 8 to 11 months (see Nakamura and Steinsson, 2008), implying a value of α around 0.5. The estimates of macroeconomic models, instead, are usually higher: CEE estimates α to be 0.6, Altig et al. (2005) to be 0.8. The 90-percent posterior probability interval for α estimated in Del Negro et al. (2005) is (0.51, 0.83).

policy need to be aggressive on inflation.

On the contrary, Table 10b shows that when the effects of trend inflation are offset by indexation and only the inflation gap induces persistence in inflation, the effects of varying α roughly disappear. Indeed, under full indexation, varying the value of α has only marginal effects on the optimal policy parameters and on conditional welfare. As argued above, full indexation is a very special case: it eradicates the effects of the persistence due to trend inflation, and hence it makes the value of price stickiness basically unimportant for optimal policy and welfare.

Trend inflation: Table 13 shows the optimal policies across the 9 types of policies considered here, when steady state inflation is reduced to 2%, instead of 4.2% as in the main Section of the paper. Results are qualitatively very similar (also in terms of moments, not shown) to the benchmark case. Clearly, the welfare costs associated to trend inflation are smaller, because smaller are the effects of trend inflation on the persistence in the inflation process.

Type of Indexation: It is also often assumed in the literature an hybrid indexation scheme, where fixed prices are indexed both to past inflation and to trend inflation. Such assumption implies full indexation in the long-run, and, hence, no effect of trend inflation on the persistence of inflation. For us, this case corresponds to the analysis of different degree of persistence in the inflation gap under no effect of trend inflation. Table 12c shows the optimal policies in this case. Two main points are worth stressing. First, the welfare effects are obviously very small, since there are no long-run effects irrespective of the value of indexation. Second, Table 12c shows once again how assuming persistence only in the inflation gap is a very special case: inflation volatility is roughly 6 times higher with respect to all the other cases considered . The intuition is clear: while full indexation to trend inflation cancels the effects of trend inflation on the persistence of the model, lack of persistence in the inflation gap induces the optimal policy to be inertial and not respond to the inflation gap.

Another common assumption is to assume that the prices that can not be changed are automatically indexed to trend inflation. Table 12a and b show the optimal policies, across all type of policies considered, for five different levels of the degree of indexation from 0 to 1, under the two different indexation schemes. When there is full indexation to trend inflation the dynamics of the model is similar to the one of a model approximated around a zero inflation steady state, and the result of the optimality of price stability is restored, as already stressed by SGU. Thus in this case, even if there are no long-run effects, the optimal inflation volatility is very low. The latter then decreases even further when the steady state level of price dispersion increases because of partial indexation. Contrary to the case of backward-looking indexation, the optimal policy changes very little with the degree of indexation. The forward-looking superinertial policy is always optimal, simply because inertial policies are the most effective in stabilizing inflation.

Unconditional Welfare: Table 6 to 8 also display the unconditional welfare levels implied by the policies. The unconditional welfare measure is the most commonly employed in the literature and is the expected value of welfare given the unconditional distribution of the variables, i.e. it is independent of the initial conditions of the state vector. Therefore one can see it as the weighted average of the conditional welfare levels associated with all possible values of the initial state vector, with weights given by their unconditional probabilities. Hence unconditional welfare may imply different optimal policies from the ones obtained using conditional welfare as the ranking measure. As stressed in SGU, the different ranking implied by the two measures demonstrates the importance of considering the transitional dynamics and the initial condition and it indicates the fact that the optimal operational rule lacks time consistency.

Therefore, Table 14 presents the optimal policy for each level of indexation using unconditional welfare.[31] The optimal policies are different from the ones presented in Table 1, not only quantitatively, but also qualitatively. The current looking no inertial policy is optimal for high level of indexation, while the backward looking no inertia is optimal for low ones. Recall that the backward looking policy was never optimal according to conditional welfare. Besides, all the optimal policies are very close to the upper bound for α_π in our set of values for the grid search (i.e., $\alpha_\pi \in [-3, 3]$). It is very likely that the optimal policy would have implied an higher level of α_π. Furthermore, the value of α_y is actually sizably different zero. As a result, the volatility of inflation implied by these optimal policies is higher than the one implied by the optimal policies under conditional welfare (the same is true for the other variables, not shown).

Finally, since the optimal policies are close to the upper bound of our grid-search

[31] These results are based on the grid-search method used in the main Section of the paper.

interval, we calculate the optimal unconstrained parameter values for α_π and α_y using the same optimization algorithm of the previous subsections and for the usual five values of χ between 0 and 1. Table 15 shows that the results are indeed quite different with respect to the previous Table: (i) the values of α_π and α_y are extremely high; (ii) the unconditional welfare is the lowest welfare measure across the Tables of the paper; (iii) the forward-looking super inertia policy is always optimal. The implied optimal inflation volatility is, however, very similar across methods.

Figure 5 replicates Figure 1 for the unconditional welfare ranking. It shows that the unconditional welfare gains, net of the steady state ones, are quite sizeable although still lower than their long-run counterpart.

The results in this subsection show, once again, that the type of optimal policies depends very much on the sources of inflation persistence in the model. However, maximizing conditional rather than unconditional welfare deliver very different insights, in terms both of optimal simple rules and of implied volatilities of the variables.

7 Conclusions

In this paper we linked the level of indexation to the sources of inflation persistence and we have analyzed how the optimal interest rate rule changes with it.

We used a standard medium-scale New Keynesian model to show that trend inflation and the inflation gap are the main sources of inflation persistence and that their effects are regulated by the level of price indexation. Similarly to Sbordone (2007), we distinguished among two polar cases. In the first scenario, inflation persistence is induced by the effects of trend inflation, while the inflation gap is purely forward-looking. In the second scenario, persistence is entirely due to the effects of the inflation gap. Using numerical simulations, we characterized the effects on optimal monetary policy rules. An increase in the persistence due to trend inflation worsen the inflation-output trade-off and makes the task of stabilizing inflation harder. Moreover, inflation stabilization is not the main objective of monetary policy when the only source of persistence in the inflation gap.

We chose a model that replicates a variety of evidences about business cycles to compare our results with empirical estimates of US monetary policy. The optimal policy from the model in which the inflation persistence is due to trend inflation is remarkably similar to the FED's Taylor rules estimated by different papers. Given the empirical evidence of the relationship between trend inflation and inflation persistence, our results suggest that models based that take into account trend inflation are more likely to capture the main trade-offs used by the FED's to set the nominal interest rate in the past 50 years.

References

Altig, D., L. J. Christiano, M. Eichenbaum, and J. Linde (2004). Firm-specific capital, nominal rigidities, and the business cycle. Technical Report wp 04-16, Federal Reserve Bank of Clevelend.

Amano, R., S. Ambler, and N. Rebei (2007). The macroeconomic effects of non-zero trend inflation. *Journal of Money, Credit and Banking 39*(7), 1821–1838.

Ascari, G. (2004). Staggered prices and trend inflation: Some nuisances. *Review of Economic Dynamics 7*, 642–667.

Ascari, G. and N. Branzoli (2010). The long-run optimal degree of indexation in the New Keynesian Model models with price staggering à la Calvo. *Economics Bullettin 30*(1), 482–493.

Ascari, G. and T. Ropele (2007). Optimal monetary policy under low trend inflation. *The Journal of Monetary Economics 54*, 2568–2583.

Benati, L. (2008). Investigating inflation persistence across monetary regimes. *The Quarterly Journal of Economics 123(3)*, 1005–1060.

Benati, L. (2009). Are "intrinsic inflation persistence" models structural in the sense of Lucas (1976)? ECB working paper No. 1038.

Boivin, J. and M. Giannoni (2006). Has monetary policy become more effective? *Review of Economics and Statistics 88(3)*, 445–462.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Cogley, T. and T. J. Sargent (2005). Drifts and volatilities: Monetary policies and outcomes in the post WWII U.S. *Review of Economic Dynamics 8*(2), 262–302.

Cogley, T. and A. Sbordone (2008). Trend inflation, indexation and inflation persistence in the new keynesian phillips curve. *American Economic Review 98(5)*, 2101–2126.

Cogley, T. W., G. Primiceri, and T. J. Sargent (2010). Inflation-gap persistence in the U.S. *American Economic Journal: Macroeconomics 2*(1), 43–69.

Del Negro, M., F. Schorfheide, F. Smets, and R. Wouters (2005). On the fit and forecasting performance of New-Keynesian models. ECB working paper No. 491.

Fuhrer, J. and G. Moore (1995). Inflation persistence. *Quarterly Journal of Economics 110*, 127–159.

Galí, J. and M. Gertler (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics 44*, 195–222.

Ireland, P. (2007). Changes in Federal Reserve's inflation target: Causes and consequences. *Journal of Money, Credit and Banking 39(8)*, 1851–1882.

Klenow, P. J. and O. Kryvtsov (2008). State-dependent or time-dependent pricing: Does it matter for recent U.S. inflation? *The Quarterly Journal of Economics 123*(3), 863–904.

Levin, A. T., A. Onatski, J. Williams, and N. Williams (2006). Monetary policy uncertainty in microfounded macroeconometric models. In M. Gertler and K. Rogoff (Eds.), *NBER Macroeconomic Annual*, pp. 229–287. Cambridge, MA: MIT Press.

Mankiw, G. N. (2001). The inexorable and mysterious tradeoff between inflation and unemployment. *The Economic Journal*, C45–C61.

Mavroeidis, S. (2004). Weak identification of forward-looking models in monetary economics. *Oxford Bulletin of Economics Statistics 66*(S1), 609–635.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *Quarterly Journal of Economics 123*(4), 1415–1464.

Primiceri, G. E. (2006). Why inflation rose and fell: PolicymakersŠ beliefs and U.S. postwar stabilization policy. *Quarterly Journal of Economics 121*, 867–901.

Rudd, J. and K. Whelan (2007). Modeling inflation dynamics: A critical review of recent research. *Journal of Money, Credit and Banking 39(1)*, 155–170.

Sargent, T., N. Williams, and T. Zha (2006). Shocks and government beliefs: The rise and fall of american inflation. *American Economic Review 96(4)*, 1193–1224.

Sbordone, A. (2007). Inflation persistence: Alternative interpretations and policy implications. *Journal of Monetary Economics 54*, 1311–1339.

Schmitt-Grohé, S. and M. Uribe (2004a). Optimal operational monetary policy in the

Christiano-Eichenbaum-Evans model of the U.S. business cycle. NBER wp No. 10724.

Schmitt-Grohé, S. and M. Uribe (2004b). Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control 28*, 755–775.

Schmitt-Grohé, S. and M. Uribe (2006). Optimal fiscal and monetary policy in a medium-scale macroeconomic model. In M. Gertler and K. Rogoff (Eds.), *NBER Macroeconomics Annual*, pp. 383–425. Cambridge, MA: MIT Press.

Schmitt-Grohé, S. and M. Uribe (2007a). Optimal inflation stabilization in a medium-scale macroeconomic model. In K. Schmidt-Hebbel and R. Mishkin (Eds.), *Monetary Policy Under Inflation Targeting*, pp. 125–186. Santiago, Chile: Central Bank of Chile.

Schmitt-Grohé, S. and M. Uribe (2007b). Optimal simple and implementable monetary and fiscal rules. *Journal of Monetary Economics 54*, 1702–1725.

Schmitt-Grohé, S. and M. Uribe (2008). Policy implications of the New Keynesian Phillips Curve. *Federal Reserve Bank of Richmond Economic Quarterly 94*(4), 435–465.

Smets, F. and R. Wouters (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association 1*, 1123–1175.

Smets, F. and R. Wouters (2007). Shocks and frictions in US business cycles: A bayesian DSGE approach. *American Economic Review 97*(3), 586–606.

Steinsson, J. (2003). Optimal monetary policy in an economy with inflation presistence. *Journal of Monetary Economics 50*(7), 1425–1456.

STOCK, J. H. and M. W. WATSON (2007). Why has U.S. inflation become harder to forecast? *Journal of Money, Credit and Banking 39*, 3–33.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.

Yun, T. (2005). Optimal monetary policy with relative price distortions. *American Economic Review 95*, 89–108.

8 Tables

Table 1. Optimal Operational Monetary Policies Rules						
χ	Policy Class	α_R	α_π	α_y	SS Welf.	Conditional Welf.
1	Forward looking	0	1.1250	-0.0625	-156.7143	-156.7220
.95	Forward looking	0	1.1875	0	-156.7119	-156.7199
.90	Forward looking	0	1.1875	0	-156.7107	-156.7189
.8788	Forward looking	0	1.1875	0	-156.7106	-156.7188
.85	Current looking	0	1.0625	0	-156.7108	-156.7191
.80	Current looking	0	1.0625	0	-156.7122	-156.7205
.75	Current looking	0	1.0625	0	-156.7149	-156.7232
0	Forward looking	1	0.1875	0	-156.9351	-156.9428

Table 2. Unconditional Moments under Optimal Operational Rules (x10^{-2})								
χ	$E(c)$	$E(y)$	$E(s)$	$E(\pi)$	σ_c	σ_y	σ_s	σ_π
1	0.13	-0.19	3.9926(x10^{-6})	4.06	1.6722	3.7602	0	2.0483
.95	0.17	-0.38	5.9022(x10^{-6})	4.20	1.9147	5.1102	6.9(x10^{-4})	1.2012
.90	0.17	-0.38	1.2596(x10^{-5})	4.19	1.9127	5.0978	1.7(x10^{-3})	1.2808
.8788	0.17	-0.38	7.9551(x10^{-6})	4.20	1.9080	5.0661	2.0(x10^{-3})	1.2635
.85	0.16	-0.36	3.5347(x10^{-6})	4.19	1.9016	4.9235	1.6(x10^{-3})	0.7360
.80	0.16	-0.36	4.1604(x10^{-6})	4.19	1.8989	4.9050	2.4(x10^{-3})	0.7371
.75	0.16	-0.37	4.9830(x10^{-6})	4.19	1.8964	4.8863	3.3(x10^{-3})	0.7371
0	0.15	-0.34	2.9616(x10^{-6})	4.20	1.8065	4.7709	9.6(x10^{-3})	0.1716

Note: variables are expressed in deviation from steady state, except π that is in levels and annualized.

Table 3. Optimal Monetary Policies for different values of α_R Forward Looking No Inertia						
	α_R= 0.1		α_R= 0.5		α_R= 0.9	
χ	α_π	α_y	α_π	α_y	α_π	α_y
1	1	0.0625	0.5	0.0625	0.6875	0.1875
.8788	1.0625	0.0625	0.5625	0.0625	0.5625	0.125
0	2.1875	0.1875	1	0.0625	0.3125	0.0625

Table 4. Optimal Monetary Policies - Forward Looking No Inertia				
χ	α_π	α_y	SS Welf.	Conditional Welf.
1	1.1250	-0.0625	-156.7143	-156.7220
.95	1.1875	0	-156.7119	-156.7199
.90	1.1875	0	-156.7107	-156.7189
.8788	1.1875	0	-156.7106	-156.7188
.85	1.2500	0	-156.7108	-156.7193
.80	1.4375	0	-156.7122	-156.7216
.75	1.6250	0	-156.7149	-156.7267
0	2.6875	0.1875	-156.9351	-156.9767

Note: variables are expressed in deviation from steady state, except π that is in levels and annualized.

Table 5. Unconditional Means under Optimal Forward Looking No Inertia Rule (x10^{-2})								
χ	$E(c)$	$E(y)$	$E(s)$	$E(\pi)$	σ_c	σ_y	σ_s	σ_π
1	0.13	-0.19	3.99(x10^{-6})	4.06	1.67	3.76	0	2.05
.95	0.17	-0.38	5.90(x10^{-6})	4.20	1.91	5.11	6.8987(x10^{-4})	1.20
.90	0.17	-0.38	1.26(x10^{-6})	4.19	1.91	5.10	1.7(x10^{-3})	1.28
.8788	0.17	-0.38	7.96(x10^{-6})	4.20	1.91	5.07	2.0(x10^{-3})	1.26
.85	0.16	-0.37	6.40(x10^{-6})	4.20	1.89	5.01	2.2(x10^{-3})	1.04
.80	0.16	-0.37	4.29(x10^{-6})	4.20	1.87	4.96	2.4(x10^{-3})	0.75
.75	0.17	-0.45	1.18(x10^{-5})	4.26	1.93	5.4	4.8(x10^{-3})	1.16
0	0.13	-0.56	2.05(x10^{-4})	4.28	1.95	5.73	7.78(x10^{-2})	1.16

Note: variables are expressed in deviation from steady state, except π that is in levels and annualized.

Table 6. Optimal Operational Monetary Policies - No Inertia						
χ	α_π	α_y	σ_s	σ_π	Conditional Welf.	Unconditional Welf.
Forward Looking						
1	1.1250	-0.0625	0	2.0483	-156.7220	-156.5252
0.8788	1.1875	0	0.0020	1.2635	-156.7188	-156.4169
0	2.6875	0.1875	0.0778	1.1574	-156.9767	-156.6381
Current Looking						
1	1.0625	0	0	0.7278	-156.7227	-156.4289
0.8788	1.0625	0	0.0014	0.7645	-156.7189	-156.4299
0	1.625	0	0.0212	0.3353	-156.9456	-156.6679
Backward Looking						
1	1.3125	0.0625	0	1.5648	-156.7233	-156.3729
0.8788	1.375	0.0625	0.0015	1.3164	-156.7203	-156.3810
0	1.3125	0	0.0180	0.2919	-156.9451	-156.4442

Table 7. Optimal Operational Monetary Policies - Inertia, $\alpha_R = 1$						
χ	α_π	α_y	σ_s	σ_π	Conditional Welf	Unconditional Welf.
Forward Looking						
1	0.8125	0.1875	0	1.9911	-156.7232	-156.3456
0.8788	0.5	0.0625	0.0012	1.0058	-156.7199	-156.3975
0	0.1875	0	0.0096	0.1716	-156.9428	-156.6751
Current Looking						
1	0.4375	0.0625	0	1.6442	-156.7237	-156.3639
0.8788	0.4375	0.0625	0.0013	1.1334	-156.7203	-156.3911
0	0.0625	0	0.0076	0.1390	-156.9431	-156.6733
Backward Looking						
1	0.75	0.1250	0	1.3195	-156.7243	-156.3790
0.8788	0.5	0.0625	0.0011	0.9584	-156.7209	-156.3975
0	0.0625	0	0.0074	0.1355	-156.9432	-156.6730

Note: variables are expressed in deviation from steady state, except π that is in levels and annualized.

Table 8. Optimal Operational Monetary Policies - Super Inertia, $\alpha_R = 2$						
χ	α_π	α_y	σ_s	σ_π	Conditional Welf	Unconditional Welf.
Forward Looking						
1	1.6875	0.625	0	1.9939	-156.7237	-156.3435
0.8788	0.5625	0.125	8.40(x10^{-4})	0.6401	-156.7203	-156.4155
0	-0.9375	0	0.0207	0.3095	-156.9426	-156.6931
Current Looking						
1	0.8125	0.3125	0	1.3833	-156.7248	-156.3753
0.8788	0.25	0.0625	6.48(x10^{-4})	0.4206	-156.7210	-156.4254
0	-0.8750	0	0.0214	0.3206	-156.9450	-156.6871
Backward Looking						
1	-0.75	-0.0625	0	1.5262	-156.7250	-156.5191
0.8788	-0.75	-0.0625	0.0018	1.5077	-156.7203	-156.5230
0	-0.6875	0	0.0182	0.2738	-156.9453	-156.6809

Note: variables are expressed in deviation from steady state, except π that is in levels and annualized.

Table 9. Empirical Estimates over Different Policy Regimes and Optimal Model-based Policies			
	α_R	α_π	α_y
Benati	Sample Period 1951-2005		
	0.827 [0.774 ; 0.857]	0.208 [0.187 ; 0.377]	0.122 [0.076 ; 0.253]
	Sample Period 1951-2005		
	0.808 [0.760 ; 0.861]	0.348 [0.2 ; 0.566]	0.132 [0.061 ; 1.127]
Smets-Wouters	Sample Period 1966-1979		
	0.81 [0.78 ; 0.84]	0.31 [0.23 ; 0.40]	0.03 [0.02 ; 0.04]
	Sample Period 1984-2004		
	0.84 [0.82 ; 0.86]	0.28 [0.21 ; 0.37]	0.01 [0 ; 0.02]
Boivin-Giannoni	Sample Period 1959-1979		
	1.011 [0.994 ; 1.028]	0.276 [0.269 ; 0.283]	0 [-0.004 ; 0.004]
	1979-2002		
	0.602 [0.587 ; 0.617]	0.508 [0.458 ; 0.558]	0 [-0.038 ; 0.038]
Model	$\chi = 0$		
Current-looking rules	1	0.062	0
Globally Optimal: fw-looking	1	0.188	0
	$\chi = 1$		
Current-looking rules	0	1.062	0
Globally Optimal: fw-looking	0	1.125	-0.0625

Table 11a. Optimal Forward Looking Policies No Inertia - $\chi = 0$			
Calvo Parameter (α)	α_π	α_y	Conditional Welfare
0.55	2.3015	0.2094	-156.9321
0.60	2.5157	0.1629	-156.9749
0.65	2.7819	0.1211	-157.0614
0.70	3.1083	0.0845	-157.2192
0.75	3.5048	0.0538	-157.5175
0.80	3.9931	0.0302	-158.1520

Table 11b. Optimal Forward Looking Policies No Inertia - $\chi = 1$			
Calvo Parameter (α)	α_π	α_y	Conditional Welfare
0.55	1.1442	-0.0239	-156.7227
0.60	1.0888	-0.0336	-156.7213
0.65	1.0542	-0.0329	-156.7206
0.70	1.0538	-0.0225	-156.7202
0.75	1.0628	-0.0132	-156.7197
0.80	1.0920	-0.0034	-156.7192

Table 12a. Optimal Monetary Policies, Backward-looking indexation						
χ	Policy Class	α_R	α_π	α_y	Conditional Welf.	σ_π
1	Forward Looking	0	1.0888	-0.0336	-156.7214	1.83
.75	Forward Looking	2	0.1286	0.0199	-156.7240	0.32
.50	Forward Looking	2	0.0478	0.0063	-156.7576	0.28
.25	Forward Looking	2	0.4965	0.0054	-156.8289	0.25
0	Forward Looking	2	-0.0712	-0.0049	-156.9425	0.23

Table 12b. Optimal Monetary Policies, Trend inflation indexation						
χ	Policy Class	α_R	α_π	α_y	Conditional Welf.	σ_π
1	Forward Looking	2	0.0522	0.0051	-156.7223	0.27
.75	Forward Looking	2	0.0531	0.0052	-156.7227	0.26
.50	Forward Looking	2	0.0546	0.0049	-156.7569	0.24
.25	Forward Looking	2	0.0547	0.0049	-156.8287	0.24
0	Forward Looking	2	0.0712	0.0049	-156.9425	0.23

Table 12c. Optimal Monetary Policies, Hybrid indexation $\chi =$ degree of backward-looking indexation						
χ	Policy Class	α_R	α_π	α_y	Conditional Welf.	σ_π
1	Forward Looking	0	1.0888	-0.0336	-156.7214	1.83
.75	Forward Looking	1	0.2450	0.0109	-156.7235	0.46
.50	Forward Looking	1	0.1394	0.0021	-156.7230	0.33
.25	Forward Looking	2	0.0478	0.0060	-156.7226	0.30
0	Forward Looking	2	0.0521	0.0051	-156.7224	0.27

Table 13. Optimal Monetary Policies, Trend Inflation = 2%					
χ	Policy Class	α_R	α_π	α_y	Conditional Welfare
1	Forward Looking	0	1.0927	-0.0340	-156.5008878
.75	Forward Looking	0	1.4070	0.0241	-156.4974982
.50	Forward Looking	0	1.4921	0.1136	-156.5049626
.25	Forward Looking	2	0.0473	-0.0074	-156.5196983
0	Forward Looking	2	0.0437	0.0047	-156.5390798

Table 14. Optimal Monetary Policies ranked by unconditional welfare							
χ	Policy Class	α_R	α_π	α_y	Unconditional Welf.	Conditional Welf.	σ_π
1	Current Looking	0	2.8750	0.6250	-156.2653	-156.7267	3.26
.95	Current Looking	0	2.8750	0.6250	-156.2654	-156.7256	3.27
.90	Current Looking	0	2.8750	0.6250	-156.2669	-156.7276	3.27
.8788	Current Looking	0	2.8750	0.6250	-156.2681	-156.7277	3.28
.85	Current Looking	0	2.8750	0.6250	-156.2703	-156.7298	3.28
.80	Backward Looking	0	3	0.6875	-156.2778	-156.7319	3.35
.75	Backward Looking	0	3	0.6875	-156.2857	-156.7398	3.36
0	Backward Looking	0	3	0.2500	-156.6265	-156.9858	1.13

Table 15. Optimal Monetary Policies ranked by unconditional welfare							
χ	Policy Class	α_R	α_π	α_y	Unconditional Welf.	Conditional Welf.	σ_π
1	Forward Looking	2	8.0957	4.1546	-155.9998	-156.8276	3.33
.75	Forward Looking	2	8.7518	3.8128	-156.0332	-156.7935	3.30
.50	Forward Looking	2	10.8519	3.7786	-156.1456	-156.8252	3.16
.25	Forward Looking	2	11.9724	2.5497	-156.2991	-156.8945	1.76
0	Forward Looking	2	13.0999	1.7394	-156.4398	-156.9948	1.06

Table 16. Calibration		
β	$1.03^{-0.25}$	Time discount rate
θ	0.36	Share of capital
ψ	0.5827	Fixed cost (guarantee zero profits in steady state)
δ	0.025	Depreciation of capital
υ	1	Fraction of wage bill subject to CIA constraint
η	6	Elasticity of substitution of different varieties of goods
$\tilde{\eta}$	21	Elasticity of substitution of labour services
α	0.6	Probability of not setting a new price each period
$\tilde{\alpha}$	0.64	Probability of not setting a new wage each period
b	0.65	Degree of habit persistence
ϕ_0	1.1196	Preference parameter
ϕ_1	0.5393	Preference parameter
σ_m	10.62	Intertemporal elasticity of money
κ	2.48	Investment adjustment cost parameter
$\tilde{\chi}$	1	Wage indexation
γ_1	0.0324	Capital utilization cost function parameter
γ_2	0.000324	Capital utilization cost function parameter
z	1	Steady state value of technology shock
λ_z	0.979	Serial correlation of technology shock (in log-levels)
η_z	0.0072	Standard deviation of technology shock
λ_g	0.96	Serial correlation of demand shock (in log-levels)
η_g	0.02	Standard deviation of demand shock
σ	0.18	Parameter scaling all exogenous shocks

9 Figures

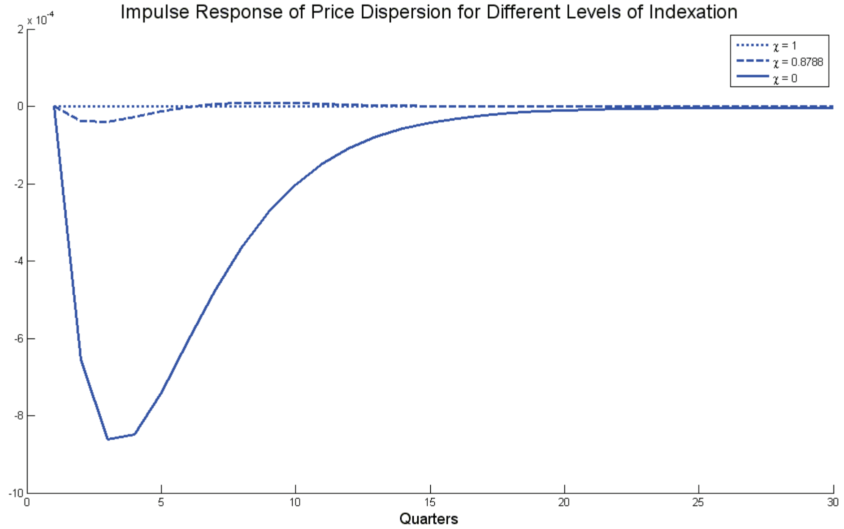

Figure 1. Impulse Response Functions of Price Dispersion after a 1% increase in the TFP for different levels of Indexation; $\alpha_\pi = 1.5$, $\alpha_y = 0$ and $\alpha_R = 0$ in the Taylor Rule (1)

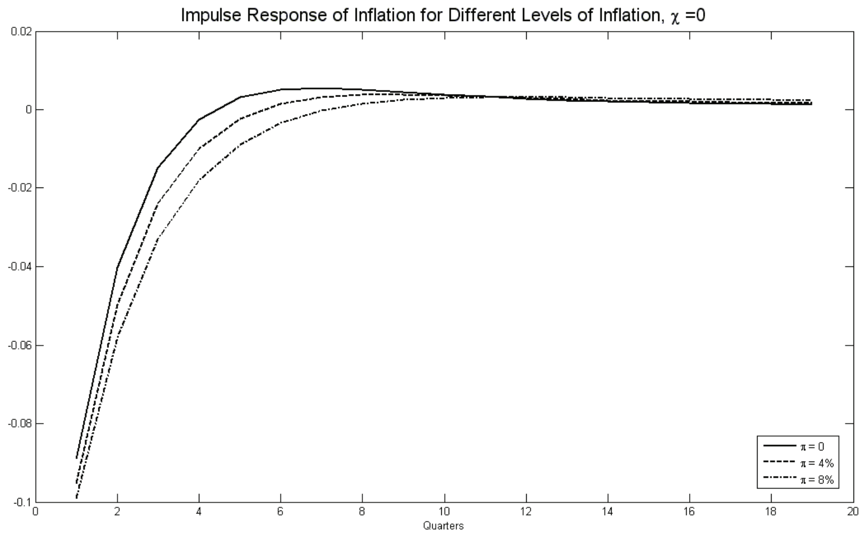

Figure 2. Impulse Response Functions of Inflation after a 1% increase in the TFP for different levels of Inflation; $\alpha_\pi = 1.5$, $\alpha_y = 0$ and $\alpha_R = 0$ in the Taylor Rule (1)

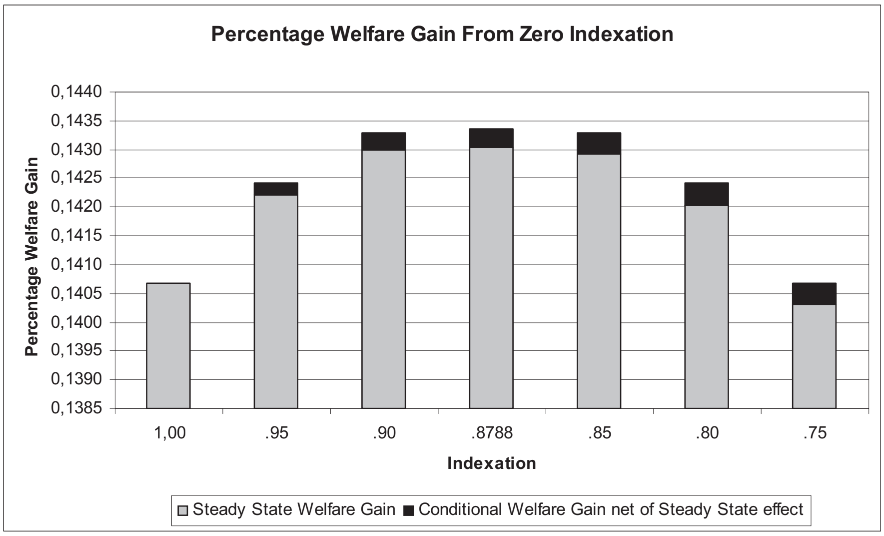

Figure 3. steady state and conditional percentage gain with respect to the 0 indexation case for the best policies ranked according to conditional welfare.

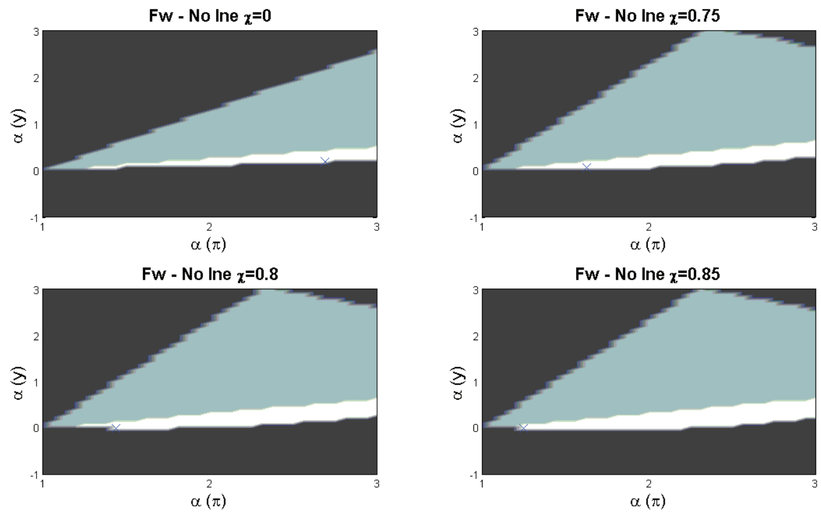

Figure 4. Indeterminacy regions

Note: Each panel shows three regions: the white one displays the values of α_y and α_π that deliver determinate rational expectation equilibria, the grey one signals that the equilibrium is not implementable in the sense described in footnote 13, and the black region represents indeterminate rational expectation equilibria. All the values of both $\alpha_y < -1$ and $\alpha_\pi < 1$ yield indeterminacy and are not shown in the figure.

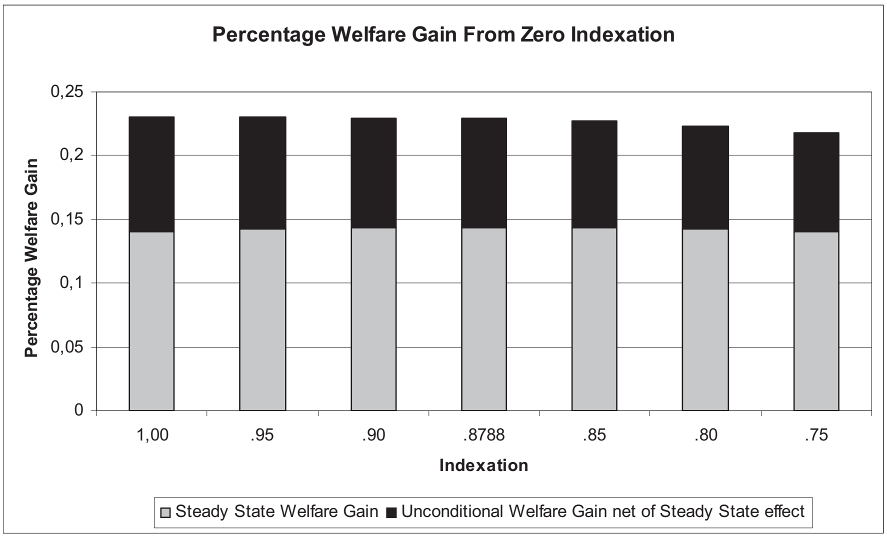

Figure 5. steady state and unconditional percentage gain with respect to the 0 indexation case for the best policies ranked according to unconditional welfare.