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Guido Ascari
(Università di Pavia)

Tiziano Ropele
(Banca d'Italia)

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Dipartimento di economia politica
e metodi quantitativi
Università degli studi di Pavia
Via San Felice, 5
I-27100 Pavia

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Trend Inflation, Taylor Principle and Indeterminacy

Guido Ascari[*]

Tiziano Ropele[†]

University of Pavia,
Kiel Institute for the World Economy

Bank of Italy
Kiel Institute for the World Economy

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Abstract

Even low levels of trend inflation substantially affect the dynamics of a basic new Keynesian DSGE model when monetary policy is conducted by a contemporaneous Taylor rule. Positive trend inflation shrinks the determinacy region. Neither the Taylor principle, which requires the inflation coefficient to be greater than one, nor the generalized Taylor principle, which requires that in the long run the nominal interest rate should be raised by more than the increase in inflation, is a sufficient condition for local determinacy of equilibrium. This finding holds for different types of Taylor rules, inertial policy rules and price indexation schemes. Therefore, regardless of the theoretical set up, the monetary literature on Taylor rules cannot disregard average inflation in both theoretical and empirical analysis.

JEL classification: E31, E52.

Keywords: Sticky Prices, Taylor Rules and Trend Inflation

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[*] *Address*: Department of Economics and Quantitative Methods, University of Pavia, Via San Felice 5, 27100 PAVIA, Italy. *Tel: +39 0382 986211; e-mail:* guido.ascari@unipv.it

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1 Introduction[1]

Average inflation in the post-war period in developed countries was moderately different from zero and varied across countries.[2] Nonetheless, much of the extensive literature on monetary policy rules employed models approximated around the zero inflation steady state (see e.g., Clarida et al., 1999; Galí, 2003; Woodford, 2003; or the book edited by Taylor, 1999).

In this article we address this inconsistency by extending the basic small scale new Keynesian DSGE model to allow for positive trend inflation.[3] We add a Taylor rule to describe the monetary authority's behaviour and then examine to what extent the properties of the model economy change as trend inflation varies. We show that *even moderate* levels of trend inflation: (i) modify the conditions under which the rational expectations equilibrium is determinate (or unique); (ii) alter the impulse response functions after a cost-push shock; and (iii) increase the unconditional variances of key variables, such as inflation and output.

Trend inflation has substantial effects on the well-known Taylor principle for the determinacy of the rational expectations equilibrium. This result is driven by the steady state relative prices distortion induced by trend inflation in the Calvo setting. As shown by Ascari (2004) and Yun (2005), the steady state relation between output and inflation is highly nonlinear. The long-run Phillips curve is positively sloped around the zero inflation steady state; however, as soon as trend inflation takes up even moderate positive values, the long-run Phillips curve inverts and becomes negatively sloped reflecting the relative price distortion. In other words, the higher the trend inflation the lower the level of output in steady state. In this article, we demonstrate that this feature has remarkable

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[2] For example, Schmitt-Grohé and Uribe (2004a,b) using US data from 1960 to 1998 calibrate trend inflation at 4.2%. In the same period, Germany, Italy, Spain, and the UK experienced average inflation rates of 3.2%, 8.1%, 7.1% and 9.0% respectively (source: OECD).

[3] Throughout our analysis, we shall use indifferently trend inflation or long-run inflation to denote the inflation rate in the deterministic steady state. As our focus is on the effects of trend inflation, we abstract from other extensions of the model that may modify the key structural equations and thereafter the Taylor principle. For example, Kurozumi (2006) considers a non separable utility function between consumption and real money balances, while Surico (2006) introduces a cost channel.

implications for the celebrated Taylor principle. Therefore, a natural suspicion arises that many of the results in the literature are drawn from a case, namely the zero inflation steady state, which is both empirically unrealistic and theoretically special.

Our key result is generalized and proved to be qualitatively robust to a number of checks: (a) different types of Taylor rules (contemporaneous, backward-looking, forward-looking and hybrid nominal interest rate rules, see e.g., Clarida et al., 2000; Bullard and Mitra, 2002); (b) inertial Taylor rules for all the cases listed in (a); (c) different price indexation schemes (see, e.g., Yun, 1996 and Christiano et al., 2005); and (d) different calibration values.

In a nutshell, research in the field of monetary policy cannot neglect trend inflation, as both the theoretical model and determinacy properties of Taylor rules are sensitive to low and moderate levels of positive trend inflation, as generally observed in western countries.

The seminal analysis in Clarida et al. (2000) can be taken as an example. Clarida et al. (2000) were the first to estimate a Taylor rule on US data. They found the response coefficient of nominal interest rate to inflation was lower than one during the pre-Volcker period, while larger than one afterwards. Strictly speaking, US monetary policy did not satisfy the Taylor principle in the first sub-sample, while it did in the second one. Thus, Clarida et al. (2000) interpreted this evidence as responsible for inflation getting out of control in the Seventies, while getting back on track later. However, the data set used in Clarida et al. (2000) features an average inflation of roughly 4 per cent (see Table II, p. 157). Yet, their analysis is based on a theoretical model that assumes zero trend inflation. When appropriately taken into account, positive trend inflation substantially changes the model's structural equations and the determinacy region, so that one needs to account for trend inflation in order to label the equilibrium determinate or indeterminate. Indeed, using our benchmark parameters calibration in the standard new Keynesian DSGE model, the Clarida et al. (2000) baseline estimates of the Taylor rule coefficients would deliver indeterminacy of the rational expectation equilibrium both in the pre-Volcker and in the Volcker-Greenspan sample period.

Not many articles in the literature investigate the effects of different levels of trend inflation in the standard new Keynesian model.[4] King and Wolman (1996) and As-

[4] A few papers do allow for non-zero steady state inflation in their analysis, but they do not look at what happens when trend inflation changes. Khan et al. (2003) solves the optimal monetary policy problem and then investigates the dynamics of the economy around the given optimal steady state

cari (1998) are early papers that look at the effects of trend inflation on the properties of the steady state of such a model. Following these contributions, Karanassou et al. (2005) studies the long-run relationship between inflation and output in the New Keynesian framework, from both a theoretical and an empirical perspective. Ascari (2004) examines, instead, the effects of trend inflation on the dynamics of the standard new Keynesian model both with Calvo (1983) and Taylor (1979) price setting specification. Ascari (2004), however, assumes an autoregressive process for the money supply and thus the issue of indeterminacy under different policy rules remains unexplored. The analysis in Ascari (2004) is extended by Amano et al. (2007) that studies how the business cycle characteristics of the model (i.e., persistence, correlations, and volatilities) vary with trend inflation. Ascari and Ropele (2007) analyzes how optimal short-run monetary policy changes with trend inflation. Cogley and Sbordone (2005) estimates the New Keynesian Phillips Curve (NKPC, henceforth) allowing for trend inflation. The key finding by Cogley and Sbordone (2008) is that once shifts in trend inflation are properly taken into account, the NKPC is structural. In other words, a Calvo pricing model with constant parameters fits the data very well with no need for indexation or a backward-looking component.

Finally, Kiley (2007) is a very closely related paper. Kiley (2007) investigates how trend inflation influences the determinacy region and the unconditional variance of inflation in a model where prices are staggered a là Taylor (1979) and monetary policy is described by a Taylor rule. Moreover, Hornstein and Wolman (2005) looks at a model similar to Kiley (2007), but allow for firm-specific capital. The results in Kiley (2007) are qualitatively similar to ours, but we extend and complement his analysis in several ways. First, we embed trend inflation in the standard New Keynesian framework (see, e.g., Galí, 2003 or Woodford, 2003) using the more popular Calvo (1983) staggered pricing framework. While the model employed in Kiley (2007) is quite stylized (i.e., two-period Taylor-type staggering), the Calvo pricing scheme allows to vary the average price duration of price contracts. Second, we provide clear analytical results about how trend inflation affects the Taylor principle, while Kiley (2007) presents only numerical results. Third, we generalize the analysis to different kinds of price indexation schemes, different kinds of Taylor rules (contemporaneous, forward and backward looking) and different degrees of inertia in the rules. Indeed, a further contribution of this article is to provide

inflation level. Schmitt-Grohe and Uribe (2004, 2007) simulates the model under different Taylor-type rules calibrating average inflation on US data.

a compact presentation of the basic New Keynesian DSGE model approximated around a general trend inflation level with price indexation. As such, we further generalize the model in Ascari and Ropele (2007) by allowing for different price indexation schemes.

The next section presents the model. Section 3 provides a general formulation of the NKPC allowing for trend inflation and different kinds and degrees of indexation. Section 4 discusses a series of analytical results concerning how trend inflation affects both the determinacy of the rational expectation equilibrium and the dynamic response of the variables to a cost push shock. Section 5 displays numerical results regarding indexation, different kinds of Taylor rules and parameter sensitivity analysis. Section 6 concludes.

2 The Model

In this section we extend the basic new Keynesian DSGE model of Clarida et al. (1999), Galí (2003) and Woodford (2003) to allow for positive trend inflation and price indexation.

Households

Households live forever and their expected lifetime utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma_c} - 1}{1-\sigma_c} + \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1-\sigma_m} - \chi_n \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective rate of time preference and E_0 is the expectation operator conditional on time $t = 0$ information. The instantaneous utility function is increasing in the consumption of a final good (C_t) and real money balances (M_t/P_t) and decreasing in labour (N_t). The parameters σ_c, σ_m and σ_n represent the inverse intertemporal elasticity of substitution in consumption, real money balances and labour respectively; χ_m and χ_n are positive constants. At a given period t, the representative household faces the following nominal flow budget constraint

$$P_t C_t + M_t + B_t \leq P_t w_t N_t + M_{t-1} + (1 + i_{t-1}) B_{t-1} + F_t + T_t \tag{2}$$

where P_t is the price of the final good, B_t represents holding of bonds offering a one-period nominal return i_t, w_t is the real wage, and F_t are firms' profits that are returned to households. In addition, in each period the government makes lump-sum nominal transfers to households of T_t. The representative household's problem is to maximize

(1) subject to the sequence of budget constraints (2), yielding the following first order conditions:

$$\begin{aligned} \textit{labor supply} &: \quad \chi_n N_t^{\sigma_n} C_t^{\sigma_c} = w_t, && (3) \\ \textit{money demand} &: \quad \chi_m \left(M_t/P_t\right)^{-\sigma_m} C_t^{\sigma_c} = i_t/\left(1+i_t\right), && (4) \\ \textit{consumption Euler eq.} &: \quad C_t^{-\sigma_c} = \beta E_t \left[C_{t+1}^{-\sigma_c}\left(1+i_t\right) P_t/P_{t+1}\right]. && (5) \end{aligned}$$

Equations (3), (4) and (5) have the usual economic interpretation.

Final Good Producers

In each period, a final good Y_t is produced by perfectly competitive firms, using a continuum of intermediate inputs $Y_{i,t}$ indexed by $i \in [0,1]$ and a standard CES production function $Y_t = \left[\int_0^1 Y_{i,t}^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}$, with $\theta > 1$. Taking prices as given, the final good producer chooses intermediate good quantities $Y_{i,t}$ to maximize profits, resulting in the usual demand schedule: $Y_{i,t} = \left(P_{i,t}/P_t\right)^{-\theta} Y_t$. The zero profit condition of final good producers leads the aggregate price index $P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{1/(1-\theta)}$.

Intermediate Goods Producers

Intermediate inputs $Y_{i,t}$ are produced by a continuum of firms indexed by $i \in [0,1]$ with technology $Y_{i,t} = N_{i,t}$. Prices are sticky, with intermediate goods producers in monopolistic competition setting prices according to a generalized discrete-time version of the Calvo (1983) mechanism. In each period there is a fixed probability $1-\alpha$ that a firm can re-optimize its nominal price, i.e., $P_{i,t}^*$. With probability α, instead, the firm may either keep its nominal price unchanged or index it. In the latter case the firm may index its nominal price partly to steady state inflation (e.g., Yun, 1996) and/or partly to past inflation rate (e.g., Christiano et al., 2005). In general, the maximization problem of a price-resetting firm can be formulated as

$$\max_{P_{i,t}^*} \quad E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left\{ \frac{P_{i,t}^* \left[\overline{\Pi}^{j(1-\omega)} \Pi_{t,t+j-1}^{\omega}\right]^{\varepsilon}}{P_{t+j}} Y_{i,t+j} - \Gamma_{i,t+j} \right\},$$

$$\text{s.t. } Y_{i,t+j} = \left\{ \frac{P_{i,t}^* \left[\overline{\Pi}^{j(1-\omega)} \Pi_{t,t+j-1}^{\omega}\right]^{\varepsilon}}{P_{t+j}} \right\}^{-\theta} Y_{t+j} \qquad (6)$$

and

$$\Pi_{t,t+j-1} = \begin{cases} 1 & \text{for } j = 0 \\ \left(\frac{P_t}{P_{t-1}}\right)\left(\frac{P_{t+1}}{P_t}\right) \times \cdots \times \left(\frac{P_{t+j-1}}{P_{t+j-2}}\right) & \text{for } j = 1, 2, \cdots. \end{cases}$$

where $\Gamma_{i,t}$ is the real total cost function, $\mathcal{D}_{t,t+j}$ is the stochastic discount factor, $\overline{\Pi}$ is the level of trend inflation (introduced below), and $\Pi_{t,t+j-1}$ represents the *cumulative gross inflation rate* (CGIR, hereafter). The parameter $\varepsilon \in [0,1]$ measures the overall degree of price indexation, while the parameter $\omega \in [0,1]$ allows for any degree of (geometric) combination of indexation to trend or past inflation rate.[5] Moreover, the aggregate price level evolves as

$$P_t = \left[\alpha \left(\overline{\Pi}^{1-\omega}\Pi_{t-1}^{\omega}\right)^{\varepsilon(1-\theta)} P_{t-1}^{1-\theta} + (1-\alpha)\left(P_{i,t}^{*}\right)^{1-\theta}\right]^{1/(1-\theta)}, \tag{7}$$

where $\Pi_t = P_t/P_{t-1}$. The solution to the profit maximization problem (6) returns a formula for the optimal relative price:

$$\frac{P_{i,t}^{*}}{P_t} = \frac{\theta}{\theta-1}\frac{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j}\left\{\Pi_{t+1,t+j}^{\theta} Y_{t+j}\Gamma_{t+j}' \left[\overline{\Pi}^{-\theta j(1-\omega)}\Pi_{t,t+j-1}^{-\theta\omega}\right]^{\varepsilon}\right\}}{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j}\left\{\Pi_{t+1,t+j}^{\theta-1} Y_{t+j} \left[\overline{\Pi}^{(1-\theta)j(1-\omega)}\Pi_{t,t+j-1}^{(1-\theta)\omega}\right]^{\varepsilon}\right\}}, \tag{8}$$

where $\Gamma_t' \equiv \partial\Gamma_t/\partial Y_{i,t}$ denotes the real marginal costs function. Given the linear production technology, it follows that $\Gamma_t' = w_t$. In the deterministic steady state, equation (8) converges to a solution if and only if $\alpha\beta\overline{\Pi}^{\theta(1-\varepsilon)} < 1$. In addition, from equation (7) it must also hold that $\alpha\overline{\Pi}^{(1-\varepsilon)(\theta-1)} < 1$ so that in the steady state the optimal relative price is strictly positive. Given parameter values for α, β, θ and ε, these two inequalities define an upper bound on the level of trend inflation. Throughout our analysis, we work with levels of trend inflation that meet these restrictions.

To fully understand the effects of trend inflation on the optimal reset price, it is insightful to look at the case of no indexation, i.e., $\varepsilon = 0$, for which equation (8) becomes

$$\frac{P_{i,t}^{*}}{P_t} = \frac{\theta}{\theta-1}\frac{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j}\left(\Pi_{t+1,t+j}^{\theta} Y_{t+j}\Gamma_{t+j}'\right)}{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j}\left(\Pi_{t+1,t+j}^{\theta-1} Y_{t+j}\right)}, \tag{9}$$

and then focus on the steady state behaviour of (9). In the standard case of zero trend inflation, $\overline{\Pi} = 1$ and the CGIRs attached to future expected terms are equal to one at all times. Future expected terms are discounted by $\alpha\beta$. With positive trend inflation, $\overline{\Pi} > 1$ and two effects come into play. First, CGIRs at different time horizons shift

[5] For example: the case where $\varepsilon = 1$ and $\omega = 1$ represents full price indexation to the past inflation rate; the combination $\varepsilon = 0.5$ and $\omega = 0.5$ represents the case in which prices are indexed for 25% to trend inflation and for 25% to the past inflation rate; finally, when $\varepsilon = 0$ there is no price indexation (whatever the value of ω). Note that the value of ω does not affect the steady state of the model, as in steady state: $\overline{\Pi} = \pi_{t-1}$, for every t.

upwards, changing the effective discount factors to $\alpha\beta\overline{\Pi}^{\theta}$ and $\alpha\beta\overline{\Pi}^{\theta-1}$ in the numerator and denominator, respectively. Accordingly, when intermediate firms are free to adjust, they will set higher prices to try to offset the erosion of relative prices and profits that trend inflation automatically creates. Second, future terms in (9) are progressively multiplied by larger CGIRs. This means that optimal price-setting under trend inflation reflects future economic conditions more than short-run cyclical variations. Price-setting firms become more "forward-looking". Extending the same reasoning to (8), it is easy to see that indexation mitigates the two effects just described.

Relative price dispersion and real marginal costs

At the level of intermediate firms, it holds true that $(P_{i,t}/P_t)^{-\theta}\,Y_t = N_{i,t}$. Integrating this expression over i yields $Y_t s_t = N_t$, where we defined $s_t \equiv \int_0^1 (P_{i,t}/P_t)^{-\theta}\,di$ and $N_t \equiv \int_0^1 N_{i,t}di$. In other words, the variable s_t measures the relative price dispersion across intermediate firms and can be shown to evolve as

$$s_t = (1-\alpha)\left(\frac{P^*_{i,t}}{P_t}\right)^{-\theta} + \alpha\left[\frac{\Pi_t}{\left(\overline{\Pi}^{\omega}\Pi_{t-1}^{1-\omega}\right)^{\varepsilon}}\right]^{\theta} s_{t-1}. \tag{10}$$

?) shows that the variable s_t is bounded below at one. s_t represents the resource costs (or inefficiency loss) due to relative price dispersion under the Calvo mechanism: the higher s_t, the more labour is needed to produce a given level of output. The variable s_t directly affects the real marginal costs via the labour supply equation (3): $\Gamma'_t = w_t = \chi_n Y_t^{\sigma_n} s_t^{\sigma_n} C_t^{\sigma_c}$.

Government

The government injects money into the economy through nominal transfers, so $T_t = M_t^s - M_{t-1}^s$where M^s is the aggregate nominal money supply. Most importantly, we assume that steady state money supply evolves according to the following fixed rule: $M_t^s = \overline{\Pi} M_{t-1}^s$, where $\overline{\Pi}$ is the (gross) steady-state growth rate of the nominal money supply.

Market clearing conditions

The market clearing conditions in the goods, money and labour markets are: $Y_t = C_t$; $Y_{i,t}^s = Y_{i,t}^D = (P_{i,t}/P_t)^{-\theta}\,Y_t$, $\forall i$; $M_t = M_t^s$; and $N_t = \int_0^1 N_{i,t}di$.

3 A generalized New Keynesian Phillips Curve

Log-linearizing (3) and (5), and using the market clearing condition $\widehat{Y}_t = \widehat{C}_t$, yields

$$\sigma_n \widehat{N}_t + \sigma_c \widehat{Y}_t = \widehat{w}_t, \tag{11}$$

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \sigma_c^{-1} \left(\widehat{i}_t - E_t \widehat{\pi}_{t+1} \right), \tag{12}$$

where hatted variables denote percentage deviations from deterministic steady state.

The log-linearization of (7), (8) and (10) is more complicated and leads to the following system of difference equations characterizing the generalized NKPC under trend inflation (and price indexation)

$$\begin{cases} \Delta_t = \beta \overline{\Pi}^{1-\varepsilon} E_t \Delta_{t+1} + \kappa_{(\overline{\pi},\varepsilon)} \widehat{Y}_t + \lambda_{(\overline{\pi},\varepsilon)} \sigma_n \widehat{s}_t + \eta_{(\overline{\pi},\varepsilon)} E_t \left[(\theta - 1) \Delta_{t+1} + \widehat{\phi}_{t+1} \right], \\ \widehat{\phi}_t = (1 - \sigma_c) \left[1 - \alpha \beta \overline{\Pi}^{(\theta-1)(1-\varepsilon)} \right] \widehat{Y}_t + \alpha \beta \overline{\Pi}^{(\theta-1)(1-\varepsilon)} E_t \left[(\theta - 1) \Delta_{t+1} + \widehat{\phi}_{t+1} \right], \\ \widehat{s}_t = \xi_{(\overline{\pi},\varepsilon)} \Delta_t + \alpha \overline{\Pi}^{\theta(1-\varepsilon)} \widehat{s}_{t-1}, \end{cases} \tag{13}$$

where $\Delta_t \equiv \widehat{\pi}_t - \varepsilon (\omega \widehat{\pi}_{t-1})$ and $\widehat{\phi}_t$ is an auxiliary variable. The coefficients $\kappa_{(\overline{\pi},\varepsilon)}$, $\lambda_{(\overline{\pi},\varepsilon)}$, $\eta_{(\overline{\pi},\varepsilon)}$ and $\xi_{(\overline{\pi},\varepsilon)}$ are complicated convolutions of parameters that depend, *inter alia*, on trend inflation and price indexation (for their expressions see Appendix A.1). Of course, our generalization (13) encompasses the standard NKPC: with zero trend inflation (or full price indexation), $\overline{\Pi} = 1$ (or $\varepsilon = 1$) and $\eta_{(\overline{\pi},\varepsilon)} = \xi_{(\overline{\pi},\varepsilon)} = 0$. In this case, both the auxiliary variable and the measure of relative prices dispersion become irrelevant for inflation dynamics (up to the first order). Thus, the system (13) is reduced to the standard specification: $\Delta_t = \beta E_t \Delta_{t+1} + \kappa \widehat{Y}_t$.

Several remarks are noteworthy. As stressed by Ascari and Ropele (2007), trend inflation sensibly alters the inflation dynamics compared to the usual Calvo model with $\overline{\Pi} = 1$ (or $\varepsilon = 1$). Firstly, trend inflation enriches the dynamic structure by adding two new endogenous variables: $\widehat{\phi}_t$, which is a forward-looking variable, and $\widehat{s}_t$, which is a predetermined variable. Secondly, trend inflation directly affects the NKPC coefficients. As price-setting becomes more "forward-looking", higher trend inflation leads to a smaller coefficient on current output and a larger coefficient on future expected inflation. Given the restrictions $\alpha \beta \overline{\Pi}^{\theta(1-\varepsilon)} < 1$ and $\alpha \overline{\Pi}^{(\theta-1)(1-\varepsilon)} < 1$ it holds true that: $\partial \kappa_{(\overline{\pi},\varepsilon)} / \partial \overline{\Pi} < 0$, $\partial \xi_{(\overline{\pi},\varepsilon)} / \partial \overline{\Pi} < 0$ and $\partial \eta_{(\overline{\pi},\varepsilon)} / \partial \overline{\Pi} > 0$ (see Appendix A.1). Consequently, as trend inflation increases, the short-run NKPC flattens when drawn in the plane $(\widehat{Y}_t, \widehat{\pi}_t)$. Hence, the contemporaneous relation between $\widehat{\pi}_t$ and $\widehat{Y}_t$ progressively weakens: the inflation rate becomes less sensitive to variations in current output and more forward looking. Thirdly, trend inflation increases the autoregressive coefficient in the equation of relative prices dispersion. Other things being equal, higher trend inflation yields a more persistent adjustment of the inflation rate. Finally, the effects of trend inflation on the NKPC coefficients are partly counterbalanced by the degree of price indexation. Indeed, one can show: $\partial\kappa_{(\overline{\pi},\varepsilon)}/\partial\varepsilon > 0$, $\partial\xi_{(\overline{\pi},\varepsilon)}/\partial\varepsilon > 0$ and $\partial\eta_{(\overline{\pi},\varepsilon)}/\partial\varepsilon < 0$. In case of full price indexation, the effects of trend inflation are completely neutralized.

To close the model we assume the central bank sets the short run nominal interest rate according to the classic contemporaneous Taylor rule

$$\widehat{i}_t = \phi_\pi \widehat{\pi}_t + \phi_Y \widehat{Y}_t, \tag{14}$$

with $\phi_\pi\,[0,\infty)$, $\phi_Y\,[-1,\infty)$ and at least one different from zero. Note that by letting the policy coefficient ϕ_Y take small negative values too, we also consider pro-cyclical monetary policy rules.

To assess the determinacy of the rational expectations equilibrium (REE henceforth), we first substitute the postulated monetary policy rule (14) into (12) and then write the structural equations in the following matrix format

$$x_t = \mathbf{A}E_t x_{t+1} + \mathbf{B}u_t, \tag{15}$$

where vector x_t includes the endogenous variables of the model while u_t represents a cost-push shock. This stochastic disturbance is simply added to the first equation in (13). Finally, $\mathbf{A}$ and $\mathbf{B}$ are conformable matrices. Determinacy of the REE obtains if the standard Blanchard and Kahn (1980) conditions are satisfied. Next, we analyse how trend inflation affects the determinacy of the REE.

4 Analytical results

This section presents the analytical derivation of our main results. In order to do so, **we assume logarithmic utility in consumption, i.e.,** $\sigma_c \to 1$, indivisible labor (see, Hansen, 1985), i.e., $\sigma_n = 0$, and indexation to trend inflation, i.e., $\omega = 0$. These parameter values greatly simplify the specification of the NKPC given in (13). Firstly, under indexation to trend inflation, $\omega = 0$ and $\Delta_t \equiv \widehat{\pi}_t$; so, the lagged inflation rate does not enter the system (13). Secondly, under indivisible labour, the real marginal costs function is independent from the measure of relative prices dispersion. Then, the variable $\widehat{s}_t$ does not contribute to the joint dynamics of output and inflation, but only determines the path of employment.

Overall, these simplifying assumptions remove the two endogenous predetermined variables present in the model, i.e., $\widehat{\pi}_{t-1}$ and $\widehat{s}_t$, and this allows us to derive several analytical results.

4.1 Determinacy of the REE under trend inflation

Under the above assumptions, vector x_t in the representation (15) includes three non-predetermined variables, i.e., $x_t \equiv \left[\widehat{Y}_t, \widehat{\pi}_t, \widehat{\phi}_t\right]'$. Hence, determinacy of REE obtains if and only if all the eigenvalues of $\mathbf{A}$ lie inside the unit circle.[6] Brooks (2004) demonstrates that necessary and sufficient conditions are

$$|D| < 1, \tag{16}$$

$$|T + D| < M + 1, \tag{17}$$

$$D^2 - TD + M1 < 1, \tag{18}$$

where T, M and D denote the trace, the sum of leading minors of order two and the determinant of matrix $\mathbf{A}$, respectively. Thus, we state the following proposition.

Proposition 1. Necessary and sufficient conditions for determinacy of the REE. *Let $\omega = \sigma_n = 0$, $\sigma_c = 1$, $\varepsilon \in [0,1]$, $\phi_\pi \in [0,\infty)$, $\phi_Y \in [-1,\infty)$ and at least one different from zero. As T, M and D are all positive, determinacy of the REE under positive trend inflation obtains if and only if*

$$\phi_y + \kappa_{(\overline{\pi},\varepsilon)}\phi_\pi > \alpha\beta^2\pi^{\theta(1-\varepsilon)} - 1, \tag{19}$$

$$\phi_\pi + \delta_{(\overline{\pi},\varepsilon)}\phi_y > 1, \tag{20}$$

$$D^2 - TD + M < 1, \tag{21}$$

where $\delta_{(\overline{\pi},\varepsilon)}$ is a complicated convolution of parameters that represents the long-run elasticity of output to inflation (reported in Appendix A.2).

Proof. See Appendix A.3.

[6] See proposition 1 in Blanchard and Kahn (1980). As we work with a linearly approximated model, all our propositions and results relate to local properties of the rational expectations equilibrium.

4.1.1 Determinacy condition under zero inflation steady state

Conditions (19), (20) and (21) generalize the determinacy conditions that obtain in the case zero inflation steady state (or full price indexation to trend inflation). It is useful to briefly recall this case in order to fully understand the effects of trend inflation on determinacy of the REE. Substituting $\overline{\Pi} = 1$ or ($\varepsilon = 1$) into (13), the relevant dynamic system (15) becomes bivariate, i.e., $x_t \equiv \left[\widehat{Y}_t, \widehat{\pi}_t\right]'$. In this case, necessary and sufficient conditions for determinacy of the REE are (see Brooks, 2004),

$$|D| < 1, \tag{22}$$

$$|T| < D + 1. \tag{23}$$

Substituting out for D and T, these two inequalities can be written as

$$\phi_y + \kappa\phi_\pi > \beta - 1, \tag{24}$$

and

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1, \tag{25}$$

which correspond to (19) and (20) for the general case. Figure 1 plots the determinacy region (shaded area) in the plane (ϕ_π, ϕ_y) under zero inflation steady state (or full indexation to trend inflation). This type of picture is very well-known in the literature, although only the positive orthant is usually displayed (e.g., Bullard and Mitra, 2002, Woodford, 2003). Evidently, condition (24) does not bind in the positive orthant while condition (25) does. As stressed by Bullard and Mitra (2002) and Woodford (2001, 2003, see Ch. 4.2.2) among others, condition (25) is a generalization of the standard Taylor principle: to ensure determinacy of the REE, the nominal interest rate should rise by more than the increase of inflation in the long run. Indeed, the coefficient $(1-\beta)/\kappa$ represents the long run multiplier of the inflation rate on output in a standard NKPC log-linearized around the zero-inflation steady state. Hence, the left-hand side of (25) "represents the long-run increase in the nominal interest rate prescribed [...] for each unit of permanent increase in the inflation rate" (Woodford, 2003, p. 254). Therefore, "The Taylor principle continues to be a crucial condition for determinacy, once understood to refer to the *cumulative* response to a *permanent* inflation increase" (Woodford, 2003, p. 256, italics as in the original). In other words, the Taylor principle has to be understood as (where LR stands for long run),

$$\left.\frac{\partial \hat{\imath}}{\partial \hat{\pi}}\right|_{LR} = \phi_\pi + \phi_Y \left.\frac{\partial \hat{Y}}{\partial \hat{\pi}}\right|_{LR} > 1. \tag{26}$$

The intuition is straightforward as provided again by Woodford (2003). Indeed, (12), (14) and the standard NKPC continue to be satisfied if inflation, output and interest rates are increased at all dates by constant factors satisfying (26) with equality. This means that a real eigenvalue of value one corresponds to that equality.

Note that condition (25) has two main implications. Firstly, it implies a sort of trade-off between ϕ_π and ϕ_Y: values of ϕ_π smaller than one may still ensure determinacy provided the central bank responds more aggressively to output. Secondly, in reality this trade-off is very weak: as the subjective discount factor is calibrated very close to one, the coefficient $(1-\beta)/\kappa$ turns out to be roughly zero. Consequently, most researchers, particularly in the empirical monetary policy literature, have concentrated on the value of ϕ_π and on condition $\phi_\pi > 1$, while neglecting the role of ϕ_Y (see e.g., Clarida et al., 2000).

4.1.2 The Taylor principle under trend inflation

Here we extend the discussion in Woodford (2003) to the case of positive trend inflation. Indeed, inequality (20) in Proposition 1 corresponds exactly to (26) in the general case as $\left.\frac{\partial \hat{Y}}{\partial \hat{\pi}}\right|_{LR} = \delta_{(\overline{\pi},\varepsilon)}$. Therefore, even under trend inflation, the Taylor principle, expressed in its generalized form (26), continues to be a crucial condition for determinacy of the REE. What are then the effects of positive trend inflation on the Taylor principle? Clearly, these effects have to come through the coefficient $\delta_{(\overline{\pi},\varepsilon)}$.

Proposition 2. Effect of trend inflation on $\delta_{(\overline{\pi},\varepsilon)}$. *Let $\omega = \sigma_n = 0$, $\sigma_c = 1$, $\varepsilon \in [0,1]$ and $\widehat{i}_t = \phi_\pi \widehat{\pi}_t + \phi_Y \widehat{Y}_t$, with $\phi_\pi \in [0,\infty)$, $\phi_Y \in [-1,\infty)$ and at least one strictly positive. Then, there exists a value of trend inflation $\overline{\Pi}^* \in \left(1, \beta^{\frac{1}{\varepsilon-1}}\right)$ such that*

$$\delta_{(\overline{\pi},\varepsilon)} > 0, \text{ for } \overline{\Pi} \in \left[1, \overline{\Pi}^*\right), \tag{27}$$

$$\delta_{(\overline{\pi},\varepsilon)} \leqslant 0, \text{ for } \overline{\Pi} \in \left[\overline{\Pi}^*, (\alpha\beta)^{\frac{1}{\theta(\varepsilon-1)}}\right). \tag{28}$$

Proof. See Appendix A.4.

Under positive trend inflation the coefficient $\delta_{(\overline{\pi},\varepsilon)}$, which represents the long-run elasticity of output to inflation, switches sign from positive to negative as soon as $\overline{\Pi}$ becomes larger than $\overline{\Pi}^*$. The long-run NKPC is extremely non-linear around $\overline{\Pi} = 1$: it is positively sloped at $\overline{\Pi} = 1$ (because of a discounting effect), but then the slope turns negative, because of the relative prices dispersion effect (see Ascari, 2004, Yun, 2005, Ascari and Ropele, 2007).[7]

Corollary. Effect of trend inflation on condition (20). *In the plane* (ϕ_π, ϕ_y), *as trend inflation increases, the* upper determinacy frontier *defined by* $\phi_y = (1-\phi_\pi)/\delta_{(\overline{\pi},\varepsilon)}$ *progressively turns clockwise tilting around the point* $\phi_\pi = 1$ *and* $\phi_y = 0$. *For* $\overline{\Pi} < \overline{\Pi}^*$ *the* upper determinacy frontier *is negatively sloping, while for* $\overline{\Pi} > \overline{\Pi}^*$ *it is positively sloping.*

Figure 2 visualizes what happens to the Taylor principle (20) as trend inflation increases. The intuition is exactly as described above by Woodford (2003). One has to keep in mind that the Taylor principle relates to the long-run properties of the model, that is, to "*cumulative* responses to a *permanent* inflation increase". The fact that the long-run slope of the NKPC switches sign is evident in (13), where the term $\beta\overline{\Pi}^{1-\varepsilon}$ becomes bigger than one for low levels of trend inflation.[8]

Note that the presence of positive trend inflation overturns the two implications stemming from (25) under zero trend inflation. Firstly, even for low levels of trend inflation, the trade-off between ϕ_π and ϕ_Y disappears as the slope of the upper determinacy frontier switches sign (from negative to positive). Along the upper determinacy frontier, a central bank that wanted to be less strict on inflation (i.e., a lower value of ϕ_π) should be at the same time less aggressive towards output. Similarly, a central bank that wanted to be more aggressive towards output should also be tighter on inflation. Secondly, the higher the level of trend inflation, the larger the absolute value of $\delta_{(\overline{\pi},\varepsilon)}$, hence the flatter the upper determinacy frontier. Now, this implies a crucial role for the policy coefficient ϕ_Y. Indeed, given our postulated Taylor rule, the central bank has to be careful not to over-react to output. Why? Because under positive trend inflation, in the long-run attempts to decrease output via a contractionary monetary policy yields higher inflation.

Therefore, under positive trend inflation the Taylor principle remains valid in its more general formulation, however its implications are radically different. This in turn

[7] Just to give an idea, with the parameter values used in Section 5 and assuming no indexation, i.e., $\varepsilon = 0$, it turns out that $\overline{\Pi}^* = 1.00098$; this corresponds to an annualized value of trend inflation equal to 0.39 per cent.

[8] In this sense, the so-called Taylor principle can be seen as an example of the Samuelson's famous correspondence principle, "whereby the comparative statical behavior of a system is seen to be closely related to its dynamical stability properties" (Samuelson, 1947, p. 351).

casts shadows on the results in most of the literature which are based on a particular case, i.e., $\overline{\Pi} = 1$, which is theoretically special as well as empirically unrealistic.

4.1.3 A second determinacy condition

In general the Taylor principle does not suffice for determinacy of the REE. Also in the standard case of zero inflation steady state, a second condition needs to be fulfilled, namely (24). While this second condition always holds in the positive orthant, generally it may not hold for $\phi_\pi \in [0, \infty)$, $\phi_Y \in [-1, \infty)$, as shown in Figure 1.[9] In particular, for a given $\phi_\pi \in [1, \infty)$ to ensure determinacy of the REE the central bank cannot implement a monetary policy that is excessively pro-cyclical.

Similarly, in the case of positive trend inflation, two more conditions need to be fulfilled. Interestingly, both these conditions can be regarded as generalizations of (22) to the case of trend inflation. Restriction (19) directly corresponds to (22) in the case of the trivariate dynamic system, where $D = \frac{\alpha\beta^2\overline{\Pi}^{\theta(1-\varepsilon)}}{1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}$. Furthermore, condition (21) also collapses to (22) when translated from a trivariate to a bivariate dynamic system.[10] Due to the obscure convolution of parameters in (21), it is not easy to provide a readable expression for it, and hence we resort to the numerical analysis (as discussed in section 5). Notwithstanding, we may provide an intuition of what happens in the simulation. Appendix A.5 shows that, assuming that the coefficient $\eta_{(\overline{\pi},\varepsilon)}$ is small enough (which is quite likely under moderate trend inflation levels), then (21) holds if

$$\phi_y + \kappa_{(\overline{\pi},\varepsilon)}\phi_\pi > \beta\overline{\Pi}^{1-\varepsilon} - 1. \tag{29}$$

Note that condition (29) implies (19), which then becomes redundant. Moreover, it also yields (24) if $\overline{\Pi} = 1$. It is easy to see how trend inflation affects the line described by condition (29) in the plane (ϕ_π, ϕ_y). As visualized in Figure 3, trend inflation reduces $\kappa_{(\overline{\pi},\varepsilon)}$, and thus it flattens the line, and it increases the intercept, which become positive for values of $\overline{\Pi} > \beta^{\frac{1}{1-\varepsilon}}$. As trend inflation increases, therefore, the lower determinacy frontier progressively shifts upwards and eventually crosses the upper determinacy frontier for $\phi_y > 0$. Trend inflation then implies the two determinacy frontiers may cross

[9] More generally, conditions (25) and (24) are not sufficient either for ϕ_y or $\phi_\pi \in (-\infty, +\infty)$, in which case the admissible values of ϕ_y and ϕ_π allow the possibility to $D < 0$.

[10] See Theorem 2 in Brooks (2004). If an eigenvalue is equal zero, the set of inequalities (16)-(18) are the same as the stability ones for a two-dimensional system, where the sum of minors is replaced by the determinant.

in the positive orthant. In other words, while most of the literature discarded condition (22) because it was satisfied for positive values of (ϕ_π, ϕ_y) in the case of zero inflation steady state, this is no longer true under positive trend inflation.

Condition (29) is however only necessary, but not sufficient for (21), and thus to investigate the relevance of this qualitative result we need to resort to numerical simulations. Figure 4 illustrates the numerical determinacy region in the plane (ϕ_π, ϕ_Y) for different levels of annualized trend inflation, i.e., 0, 2, 4, 6 and 8 per cent, showing that the analytical insights of this section holds true.[11]

Result 1. Effect of trend inflation on condition (21). *As trend inflation increases, the* lower determinacy frontier *implicitly defined by* $D^2 - TD + M = 1$ *progressively shifts upwards crossing the* upper determinacy frontier *in the positive orthant of the plane* (ϕ_π, ϕ_Y).

According to our calibration, the intersection in the positive orthant between the upper and lower determinacy frontiers happens for levels of annualized trend inflation greater than 2.42 per cent. For levels of annualized trend inflation greater than this value, not only does the smallest admissible value of ϕ_π positively co-move with $\overline{\Pi}$ (because of the upper shift of the lower frontier) but also the central bank cannot always implement a strict inflation targeting policy. Moreover, Figure 4 visualizes the crucial role that the policy coefficient on output plays with positive trend inflation. As an example, in Figure 4 we highlight with a cross the classical Taylor rule specification, i.e., $\phi_\pi = 1.5$ and $\phi_Y = 0.5$. As (annualized) trend inflation exceeds 2.4 per cent, the classical Taylor rule yields indeterminacy of the REE. Hence, in empirical applications for realistic values of trend inflation the value of ϕ_Y cannot be neglected.

4.1.4 The effects of price indexation

Proposition 3. Effects of price indexation to trend inflation on REE determinacy. *Let* $\omega = \sigma_n = 0$, $\sigma_c = 1$, $\varepsilon \in [0,1]$ *and* $\widehat{i}_t = \phi_\pi \widehat{\pi}_t + \phi_Y \widehat{Y}_t$, *with* $\phi_\pi \in [0, \infty)$, $\phi_Y \in [-1, \infty)$ *and at least one strictly positive. Then, allowing for* partial *price indexation to trend inflation, i.e.,* $\varepsilon \in (0,1)$, *counteracts the effects trend inflation has on REE determinacy properties described above.*

[11] In drawing Figure 4, we set the free parameters as in Section 5: $\alpha = 0.75$, $\beta = 0.99$, $\theta = 11$, $\varepsilon = 0$. Quantitatively, $\eta_{(\overline{\pi},\varepsilon)}$ is indeed very low (see Appendix), so the relevant condition (21) is not very different from (29).

Proof. Notice the indexation parameter only appears in the model coefficients as power to trend inflation, i.e., $\overline{\Pi}^{1-\varepsilon}$. Thus, increasing the value of indexation is equivalent to decrease the level of trend inflation.

So, the whole set of results discussed above carries on, although partial price indexation to trend inflation mitigates the effects of $\overline{\Pi}$ to some extent.

In summary, trend inflation unambiguously affects the determinacy properties of the REE: as $\overline{\Pi}$ increases, the determinacy region shrinks, increasing the possibility of sunspot fluctuations. As trend inflation rises, implementable monetary rules call for increasingly larger and positive coefficients on inflation and smaller coefficients on output. These outcomes are in agreement with the policy prescriptions suggested in Schmitt-Grohé and Uribe (2004, 2007) and in Bullard and Mitra (2002). Although dealing with different issues, these two articles robustly advocate a monetary policy rule characterized by a large response to current inflation and a close to zero coefficient on output. Allowing for positive trend inflation in a basic new Keynesian DSGE model casts some doubts on the *leaning against the wind* prescription in Clarida et al. (1999). As $\overline{\Pi}$ increases, the central bank cannot run the risk of stabilizing the output (in deviation from steady state) but should focus primarily on inflation.

4.2 Closed-form solution under trend inflation

Now we investigate how trend inflation affects the model solution. Without loss of generality, assuming the cost-push shock is purely transitory, i.e., $u_t \sim$ i.i.d $N(0,1)$, allows us to obtain the following closed-form solution[12]

$$\widehat{\pi}_t = \frac{1+\phi_Y}{1+\phi_Y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}u_t, \tag{30}$$

$$\widehat{Y}_t = -\frac{\phi_\pi}{1+\phi_Y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}u_t, \tag{31}$$

$$\widehat{i}_t = \frac{\phi_\pi}{1+\phi_Y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}u_t, \tag{32}$$

$$\widehat{\phi}_t = 0. \tag{33}$$

In the event of a positive cost-push shock that increases inflation, the central bank raises the nominal interest rate. Output falls and inflation returns to steady state. During the

[12] Needless to say, the solution given by (30)-(33) is legitimate if and only if conditions in Proposition 1 are fulfilled.

adjustment path, the variable $\widehat{\phi}_t$ does not move at all from steady state.[13] Equations (30)-(32) exactly parallel the solution that one would obtain in the standard case of zero inflation steady state (see Clarida et al., 1999). However, in our generalized set-up the closed-form coefficients depend, *inter alia*, on trend inflation and the price indexation parameter through the term $\kappa_{(\overline{\pi},\varepsilon)}$. Several results are worth emphasizing.

Proposition 4. Effects of positive trend inflation. *Provided the contemporaneous Taylor rule leads to REE determinacy and* $\varepsilon \in [0,1)$, *higher levels of trend inflation* unambiguously *increase the absolute value of the closed-form coefficients on inflation, output and nominal interest rate.*

Proof It follows immediately from $\partial\kappa_{(\overline{\pi},\varepsilon)}/\partial\overline{\Pi} < 0$.

Corollary. *As trend inflation increases, the impulse response functions of output, inflation and nominal interest rate to a cost-push shock shift outwards.*

As trend inflation increases, the central bank's reaction to a cost-push shock becomes increasingly more aggressive leading to a higher nominal interest rate and a deeper recession; nevertheless, inflation also rises more. Indeed, as already noted above, the degree to which a contraction in output reduces inflation decreases with trend inflation (i.e., $\partial\kappa_{(\overline{\pi},\varepsilon)}/\partial\overline{\Pi} < 0$). So, the contemporaneous output cost for a given reduction in inflation has to increase with $\overline{\Pi}$. In other words, by varying the nominal interest rate, the central bank can engineer a fall in output, which, however, becomes less efficient at stabilizing inflation, as the higher the trend inflation, the flatter the NKPC. In sum, positive trend inflation weakens the interest rate as a policy instrument and worsens the trade-off monetary policy will have to face.

Proposition 5. Effects of price indexation. *For a given level of positive trend inflation, a higher degree of price indexation to trend inflation dampens the absolute value of the closed-form coefficients on inflation, output and nominal interest rate.*

Thus, price indexation to trend inflation counteracts the effects of trend inflation (recall that $\partial\kappa_{(\overline{\pi},\varepsilon)}/\partial\varepsilon > 0$): it slants the short-run generalized NKPC making monetary policy more efficient at stabilizing the economy.

[13] To explain this latter point, note that for $\sigma_c = 1$ the variable $\widehat{\phi}_t$ depends only on future expected variables (see the second equation in (13)).

The effects just described are also reflected in the what is called the efficient policy frontier. The efficient policy frontier links output and inflation variabilities, arguments that typically characterize the central bank's loss function, for different values of ϕ_Y and ϕ_π. In principle, with a Taylor rule such as (14) there should be two distinct efficient frontiers: one arising when varying ϕ_Y and keeping ϕ_π constant; the other one arising when varying ϕ_π and keeping ϕ_Y constant. Under our assumptions in this section, the efficient policy frontier is the same in both cases.

Proposition 6. Efficient policy frontier. *Provided the interest rate rule leads to determinacy of the REE , the efficient policy frontier is given by*

$$\sigma_\pi = 1 - \kappa_{(\overline{\pi},\varepsilon)}\sigma_Y, \tag{34}$$

where σ_Y and σ_π denote the standard deviations of output and inflation respectively.

Proof. From (30) and (31), and since $u_t \sim$ i.i.d $N(0,1)$, it follows

$$\sigma_\pi = \frac{1+\phi_Y}{1+\phi_Y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}\sigma_u = 1 - \frac{\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}{1+\phi_Y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi} = 1 - \kappa_{(\overline{\pi},\varepsilon)}\sigma_Y.$$

Before discussing the effects of trend inflation and price indexation, it is useful to provide the interpretation of equation (34). In the plane (σ_Y, σ_π), equation (34) draws a straight line, which is negatively sloped and with a vertical intercept at 1. Moving along the efficient frontier, say from north-west to south-east, one obtains the effect of increasing the value of ϕ_π (for any ϕ_Y) or equivalently the effect of decreasing the value of ϕ_Y (for any ϕ_π). As the central bank becomes relatively more aggressive on inflation, it delivers more stable inflation and more output variability. Clearly, the length of the efficient policy frontier will differ according to the values of the coefficient that ensure a determinate REE in Figure 3. Trend inflation diminishes the slope of the efficiency frontier, that rotates around the point ($\sigma_Y = 0$, $\sigma_\pi = \sigma_u = 1$).[14] It follows, as shown in Figure 5, that the efficient policy frontier worsens with trend inflation, in the sense that a given output variability can be met only at the cost of a higher inflation variability and vice versa. Points on the zero trend inflation frontier (except for $\sigma_Y = 0$ and $\sigma_\pi = 1$) are no longer attainable as $\overline{\Pi}$ rises, so there must be an increase in σ_Y and/or σ_π as trend inflation increases. As explained above, this is due to the fact that trend inflation worsens the trade-off the monetary authority faces, by changing the slope of the NKPC.

[14] The situtation $\sigma_Y = 0$ and $\sigma_\pi = 1$ obtains in the limit case: $\phi_Y \to \infty$.

5 Numerical results

In this section, we check the robustness of our analytical results to the simplifying assumptions introduced in Section 4. We remove the assumption of labour indivisibility, which implies that now the dispersion of relative prices enters the real marginal costs, and thus contributes to explain the dynamics of inflation. We also consider both price indexation schemes to trend and the past inflation rate and varying degrees of overall indexation. Furthermore, we investigate the effects of changing the monetary policy rule, by introducing inertial or backward-looking and forward-looking components. For the numerical analysis, we set parameter values as in Galí (2003): $\sigma_n = 1$, $\sigma_c = 1$, $\alpha = 0.75$, $\beta = 0.99$, $\theta = 11$ and $\chi_n = 1$.

5.1 Price indexation

We begin our analysis by comparing the effects of price indexation to trend inflation, i.e., $\omega = 0$, *versus* past inflation, i.e., $\omega = 1$. Note in the latter case the model is further complicated by the presence of another endogenous predetermined variable, namely $\widehat{\pi}_{t-1}$. To analyze the determinacy of the REE we grid-search the region of the plane defined by $\phi_\pi \in [0, 5]$ and $\phi_Y \in [-1, 5]$ and then pick up the pairs (ϕ_π, ϕ_Y) that lead to determinate equilibria. Figure 6 reports the determinacy regions for different levels of trend inflation, i.e. 0, 2, 4, 6 and 8 per cent, in the cases of partial indexation, i.e., $\varepsilon = 0.5$, and full indexation, i.e., $\varepsilon = 1$.

The overall results are in line with the findings presented in previous sections. Firstly, positive trend inflation shrinks the determinacy region. The upper determinacy frontier tilts clockwise, becoming positively sloping even for low levels of trend inflation, while the lower determinacy frontier shifts upwards. However, with respect to Figure 4, partial price indexation visibly counteracts the effects of $\overline{\Pi}$. For example, for $\varepsilon = 0.5$ the basic Taylor specification (marked with a cross in the three panels of Figure 6) ensures determinacy up to levels of trend inflation slightly below 6 per cent. Moreover, the lowest admissible value of ϕ_π becomes relatively less sensitive to trend inflation. Secondly, for a given level of trend inflation, price indexation to past inflation yields a larger number of determinate interest rate rules than under price indexation to trend inflation. While the location of the upper determinacy frontier is similar under both price indexation schemes (see panels A and B in Figure 6), price indexation to past inflation has a different effect on the lower determinacy frontier, which is shifted further downwards. So, the enlargement of the determinacy region moves in favour of more pro-cyclical monetary policy rules, i.e., more negative values of ϕ_Y. Finally, allowing for full price indexation, i.e., $\varepsilon = 1$, which neutralizes any effects of trend inflation, has different implications for the determinacy region. Full price indexation to trend inflation returns the determinacy region that would arise under zero inflation steady state; whereas, full price indexation to past inflation restores the original Taylor (1993) principle, i.e., $\phi_\pi > 1$, making ϕ_Y completely irrelevant for determinacy.[15]

5.2 Dynamic analysis and efficient policy frontier

Next, we study the effects of trend inflation on the model dynamics. We assume the cost-push shock follows an AR(1) process with a 0.8 autoregressive parameter. Figure 8 displays the impulse response functions (IRFs, henceforth) of output, annualized inflation, nominal and real interest rate to a unit cost-push shock both in the case of zero price indexation (the left column) and price indexation to past inflation (the right column).[16] In general, after a shock that boosts inflation the central bank raises the nominal interest rate for several quarters. Such monetary policy increases future expected, and possibly current, short-term (ex-ante) real interest rates making households willing to postpone consumption. Output falls. Then, a long-lasting recession kicks in which decreases the real marginal costs and brings inflation back to steady state. In line with Proposition 4, positive trend inflation shifts outward the IRFs of output, inflation and nominal interest rate, suggesting a deterioration of the short run output/inflation trade-off. Although the central bank implements monetary policies that are progressively more restrictive as trend inflation increases, the flattening of the short-run NKPC makes output have a weaker stabilizing effect on inflation. The right panels of Figure 8 also illustrate the effects of 50 per cent price indexation to past inflation.

Finally, we analyze the effects of trend inflation on the efficient policy frontier. In particular, when we vary ϕ_π in the range $[0, 3]$ we set $\phi_Y = 0.5$, while when varying

[15] Ropele (2007) analytically shows that condition $\phi_\pi > 1$ is indeed the necessary and sufficient condition for the determinacy of REE.

[16] In Figure 8 we use the basic Taylor specification, i.e., $\phi_\pi = 1.5$ and $\phi_Y = 0.5$. In the case of zero price indexation, we can just plot two IRFs for each variable as the REE is not determinate for levels of trend inflation larger than 2 per cent. We do not show IRFs under price indexation to trend inflation, because this indexation rule only yields a rescaling with respect to IRFs with zero indexation. Finally, from a qualitative standpoint, the results do not change if other values of $\phi_\pi$and ϕ_Y are chosen.

ϕ_Y $[0, 3]$ we set $\phi_\pi = 2.5$.[17] In line with Proposition 6, Figure 9 shows that positive levels of trend inflation move the efficient policy frontier north-east, yielding worse outcomes for both inflation and output variability. Moreover, the efficient policy frontier substantially shortens (i.e., it comprises a fewer number of points) as the REE enters the indeterminacy region. Not surprisingly, for a given $\overline{\Pi}$, price indexation to trend inflation shifts the efficient policy frontier south-west, partially offsetting the effects of trend inflation (see panels C and D). Similar results obtain in the case of price indexation to past inflation (see panels E and F).

5.3 Interest rate rules

Inertial interest rate rules

Empirical works on Taylor rules report that central banks tend to adjust the nominal interest rate only gradually (see, e.g., Rudebusch, 1995, Judd and Rudebusch, 1998 or Clarida et al., 2000). Moreover, the recent monetary literature emphasizes the benefit of inertial behavior in the conduct of monetary policy when private agents are forward-looking. So, here we consider specifications of the Taylor rule that allow the nominal interest rate to respond also to its own lagged values, that is $\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t + \phi_i \hat{\imath}_{t-1}$, where the degree of interest rate smoothing is measured by ϕ_i. Generally speaking, cases where $\phi_i \in (0, 1)$ are referred to as partial adjustment; case $\phi_i = 1$ is labelled as a difference rule; cases where $\phi_i \in (1, \infty)$ represent instead superinertial behaviour (Rotemberg and Woodford, 1999, and Woodford, 2003).

Figure 7 illustrates the effects of trend inflation on determinacy when $\phi_\pi \in (0, 5)$, $\phi_Y \in (-1, 5)$ and $\phi_i = 0.5$, 1, 2 and 5. Overall, the figure confirms that interest rate inertia makes indeterminacy less likely, as in the basic New Keynesian model with zero inflation steady state. Moreover, the somewhat counterintuitive feature that explosive rules enlarge the determinacy region survives in the trend inflation. As discussed in Rotemberg and Woodford (1997, see p. 100-101), it is exactly the possibility of the explosiveness of the nominal interest rate that keeps the model on track.[18]

Trend inflation, however, again radically changes the implications for determinacy

[17] In this latter case the value for ϕ_π is different from the one used for the IRFs, only for convenience of presentation. The efficient policy frontiers would otherwise be too short because the REE would quickly become indeterminate as trend inflation increases.

[18] The case of no feedback from inflation and output gap on the nominal interest rate (i.e., $\phi_\pi = \phi_Y = 0$) is of course indeterminate for values of ϕ_i bigger than 1.

regarding the parameters of the monetary policy rule. In a zero trend inflation model, condition (26) becomes $\phi_\pi + \phi_Y (1-\beta)/\kappa > 1 - \phi_i$, such that $\phi_i \geq 1$ is a sufficient condition for a determinate equilibrium in the positive orthant. In other words, a determinate REE necessarily exists for superinertial rules (see Woodford, 2003, p. 256). In the case of positive trend inflation, instead, superinertial rules do not rule out indeterminacy in the positive orthant. Moreover, it is the value of ϕ_Y that actually matters for REE determinacy. Looking at panel B, it is evident that there is no longer a sufficient condition on ϕ_π (provided that is positive) or on ϕ_i. On the contrary, for sufficiently high levels of trend inflation, we can eventually state a sufficient condition on ϕ_Y. As stressed in Section 4.1, this is due to the switch in the sign of $\delta_{(\overline{\pi},\varepsilon)}$. Moreover, $\delta_{(\overline{\pi},\varepsilon)}$ is increasing with trend inflation in absolute value. For values of trend inflation at least as large as 6 per cent, the value of the parameter $\delta_{(\overline{\pi},\varepsilon)}$ becomes so high (in absolute value), that ϕ_Y becomes the crucial monetary policy parameter for condition (26) to be satisfied. To ensure a determinate REE, monetary policy should not respond to the output, when monetary policy is characterized by an inertial (or superinertial) Taylor rule and moderate trend inflation (6 to 8 per cent).

Other interest rate rules

We further explore whether the results of the previous sections are robust to simple variants of the Taylor rule commonly used in the literature (i.e., forward-looking interest rate rule, backward-looking interest rate rule, and various kinds of hybrid interest rate rules) and to changes in the structural parameters of the model. In all these cases, the main result of the paper carries over: moderate levels of trend inflation substantially modify the determinacy region and affect the dynamics of the model economy.

In this section, we just briefly report the results concerning the determinacy conditions in the case of the backward-looking interest rate rule, as for the other policy rules the results are very similar to those presented in previous sections.[19]

When the monetary authority sets the nominal interest rate as a function of lagged values of inflation and output, i.e., $\hat{\imath}_t = \phi_\pi \hat{\pi}_{t-1} + \phi_Y \hat{Y}_{t-1}$, positive levels of trend inflation have some peculiar effects on the determinacy regions. Panel A of Figure 10 illustrates the standard case of zero inflation steady state. Roughly speaking, there are two frontiers that divide the plane into four areas: one frontier is almost horizontal with the ϕ_Y-intercept at two; the other frontier corresponds to the equivalent of condition

[19] The interested reader can download the extended working paper version from the authors' webpage.

(26). Note that above the almost horizontal frontier, the determinacy region now lies *on the left hand side of condition (26) and not on its right*, where the instability region lies. Panels B, C and D of Figure 10 show the effects of positive trend inflation. Once again, the frontier corresponding to (26) again visibly rotates clockwise.[20] However, due to the fact that the determinacy region is partly on the left and partly on the right of this line, the effect of trend inflation is less clear-cut. Roughly speaking, as trend inflation increases: (i) above the almost horizontal frontier, the instability region progressively shrinks and gives way to new determinate equilibria; (ii) below the almost horizontal frontier, the indeterminacy region enlarges. Note that while this latter implication parallels the effect analysed in previous sections, the former effect is specific of the lagged interest rate rule. Moreover, as trend inflation rises a central bank that follows a backward-looking interest rate rule is progressively left with two options to ensure determinacy. It could respond relatively more to inflation and less to output, as in previous sections; or, alternatively, the central bank could just respond with a large coefficient to output, i.e. $\phi_Y > 2$, and discard ϕ_π. Introducing inertial behavior in the backward-looking interest rate rule shifts upward the almost horizontal line in Figure 10. Consequently, the effect described in (i) becomes progressively less important and disappears for superinertial policies.

5.4 Sensitivity Analysis

Finally, we check the robustness of our numerical findings to changes in the structural parametrization. Figure 11 reports the REE determinacy regions, in the case of the contemporaneous interest rate rule and no indexation,[21] when the parameter values of α, θ, σ_n and σ_c are changed in turn.

The Calvo parameter α is a particularly interesting parameter to look at. In a recent paper Cogley and Sbordone (2008) estimates an NKPC similar to (13), allowing for time-varying trend inflation. Their main finding is that once trend inflation is taken into account, the NKPC performs rather well in the data with no need to additional ad hoc persistence terms (such as indexation to past inflation). Moreover, they also

[20] The other almost horizontal line is, in contrast, only slightly sensitive to changes in trend inflation for our calibration values. Finally, note the presence also of the lower frontier that qualitatively moves as in previous cases, shifting upwards with trend inflation.

[21] The qualitative effects of changes in the values of these parameters are in accordance with intuition, and robust across different types of rules, indexation and inertia.

found that the structural parameters of the NKPC are stable, and hence, the Calvo time-dependent pricing model with an exogenous probability of adjustment does seem to fit the data. Cogley and Sbordone's (2008) estimate of α, however, is 0.57, which is lower than the one used in our simulation. Panel A in Figure 11 shows that a lower value of the Calvo parameter mitigates the effects of trend inflation, and thus, in our case it makes the determinacy frontier close less rapidly compared with the baseline case. This leaves room for a relatively larger set of implementable policies for a given trend inflation, but it does not qualitatively change our main results, as evident from the analytical results in Section 4. Lowering the value of the elasticity of substitution across goods, i.e., θ, from 11 to 4 has a similar implication, as shown by Panel B.

In deriving our analytical results in Section 4, for convenience we fix two parameters: $\sigma_n = 0$ and $\sigma_c = 1$. Panel C in Figure 11 shows that considering higher values of the inverse of the intertemporal elasticity of the labour supply ($\sigma_n = 5$, see Pencavel, 1986) has a negligible quantitative effect on the results presented above. Panel D, instead, reveals that setting $\sigma_c = 0.157$, as in Rotemberg and Woodford (1997) and Bullard and Mitra (2002), dramatically strengthens our results from a quantitative point of view. Thus, in choosing a logarithmic utility function in consumption we considered a specification biased against our argument. It is easy to understand why and again it has to do with the slope of the NKPC (i.e., $\kappa_{(\overline{\pi},\varepsilon)} = \lambda_{(\overline{\pi},\varepsilon)} (\sigma_c + \sigma_n) - (1 - \sigma_c)\, \eta_{(\overline{\pi},\varepsilon)}$), which is quite sensitive to σ_c for our benchmark parameters value (i.e., $\sigma_n = 1$).[22]

6 Conclusions

Despite the fact that average inflation in the post-war period in developed countries was moderately different from zero, much of the vast literature on monetary policy rules worked with models approximated around the zero inflation steady state. In this article, we generalize the basic new Keynesian dynamic stochastic general equilibrium model with Calvo staggered prices by taking a log-linear approximation around a general level of trend inflation. Imposing the monetary authority follows a simple contemporaneous Taylor rule, we then look at how the properties of the model economy change as trend

[22] Moreover, the value of σ_c turns out to be quite important for the backward-looking interest rate rule case. As already noted by Bullard and Mitra (2002), the position of the almost horizontal line that characterizes Figure 10 is quite sensitive to σ_c. Indeed, it shifts notably upwards with σ_c and this has strong effects on the size of the determinacy/indeterminacy regions in our parameters' space.

inflation varies.

Trend inflation greatly affects the previous results established in the monetary policy literature. Particularly, moderate levels of trend inflation modify the determinacy region, substantially changing the Taylor principle. Moreover, trend inflation alters the impulse response functions of the model economy after a cost-push shock. In line with Ascari and Ropele (2007), this article shows that the new Keynesian framework is quite sensitive to variations in the trend inflation level, in the sense that higher trend inflation makes monetary policy much less effective in controlling the dynamics of the economy. Our key results are then generalized and proved to be robust to: (a) different kinds of Taylor type rules; (b) inertial Taylor rules for all the cases listed in (a); (c) indexation schemes; (d) different parameter values.

In summary, the literature on monetary policy rules is based on the of the zero inflation steady state, that is both empirically unrealistic and theoretically special. The specification of the theoretical model, and consequently all the results, are quite sensitive to low and moderate levels of trend inflation as empirically observed in western countries. Our analysis therefore shows the literature cannot neglect trend inflation in either empirical or theoretical investigation. As non-superneutrality is a basic feature of the standard model, future work should aim at integrating the long-run properties and the short-run dynamics into a fully non-linear analysis.

In future work, the relationship between price stickiness and trend inflation in this type of analysis should be embedded. In particular, one may argue that α is not a truly structural parameter, and it should decrease with trend inflation. As previously noted, the empirical work of Cogley and Sbordone (2008) justifies the analysis put forward in this work and supports the empirical relevance of the results. From a theoretical perspective, however, a possibility would be to employ the framework in Levin and Yun (2007) that features endogenous contract duration in this analysis. Given the findings in Levin and Yun (2007), our conjecture is that the results for a moderate rate of inflation, as considered in this paper, would not change very much while they would change for high levels of inflation, where the Calvo model is a poor approximation of price setting.

References

Amano, R., S. Ambler, and N. Rebei (2007). The macroeconomic effects of non-zero trend inflation. *Journal of Money, Credit and Banking 39*(7), 1821–1838.

Ascari, G. (1998). On superneutrality of money in staggered wage setting models. *Macroeconomics Dynamics 2*, 383–400.

Ascari, G. (2004). Staggered prices and trend inflation: Some nuisances. *Review of Economic Dynamics 7*, 642–667.

Ascari, G. and T. Ropele (2007). Optimal monetary policy under low trend inflation. *The Journal of Monetary Economics 54*, 2568–2583.

Blanchard, O. J. and C. M. Kahn (1980). The solution of linear difference models under rational expectations. *Econometrica 48*, 1305–1311.

Brooks, B. P. (2004). Linear stability conditions for a first-order three-dimensional discrete dynamic. *Applied Mathematics Letters 17*, 463–466.

Bullard, J. and K. Mitra (2002). Learning about monetary policy rules. *Journal of Monetary Economics 49*, 1105–1129.

Calvo, G. A. (1983). Staggered prices in a utility-maximising framework. *Journal of Monetary Economics 12*, 383–398.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Clarida, R., J. Galí, and M. Gertler (1999). The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature 37*, 1661–1707.

Clarida, R., J. Galí, and M. Gertler (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics 115*, 147–180.

Cogley, T. and A. Sbordone (2008). Trend inflation, indexation and inflation persistence in the New Keynesian Phillips Curve. *American Economic Review 98*(5), 2101–2126.

Galí, J. (2003). New perspectives on monetary policy, inflation, and the business cycle. In M. Dewatripont, L. Hansen, and S. Turnovsky (Eds.), *Advances in Economic Theory*, pp. 151–197. Cambridge University Press.

Hansen, G. D. (1985). Indivisible labor and the business cycle. *Journal of Monetary Economics 16*, 309–327.

Hornstein, A. and A. L. Wolman (2005). Trend inflation, firm-specific capital and sticky prices. *Federal Reserve Bank of Richmond Economic Quarterly Review 91*, 57–83.

Judd, J. P. and G. D. Rudebusch (1998). Taylor rule and the Fed: 1970-1997. *Federal Reserve Bank of San Francisco Economic Review 3-16*, 3.

Karanassou, M., H. Sala, and D. Snower (2005). A reappraisal of the inflation-unemployment tradeoff. *European Journal of Political Economy 21*, 1–32.

Khan, A., R. King, and A. L. Wolman (2003). Optimal monetary policy. *Review of Economic Studies 70*, 825–860.

Kiley, M. T. (2007). Is moderate-to-high inflation inherently unstable? *International Journal of Central Banking 3*(2), 173–201.

King, R. G. and A. L. Wolman (1996). Inflation targeting in a st. louis model of the 21st century. *Federal Reserve Bank of St. Louis Quarterly Review*, 83–107.

Kurozumi, T. (2006). Determinacy and expectational stability of equilibrium in a monetary sticky-price model with Taylor rule. *Journal of Monetary Economics 53*, 827–846.

Levin, A. and T. Yun (2007). Reconsidering the natural rate hypothesis in a New Keynesian framework. *Journal of Monetary Economics 54*(5), 1344–1365.

Pencavel, J. (1986). Labor supply of men: A survey. In O. C. Ashenfelter and R. Layard (Eds.), *Handbook of Labor Economics*, pp. 3–102. Amsterdam, North-Holland.

Ropele, T. (2007). Hybrid new keynesian phillips curve and equilibrium determinacy conditions. Mimeo, University of Pavia.

Rotemberg, J. and M. Woodford (1999). Interest rate rules in an estimated sticky price model. In J. B. Taylor (Ed.), *Monetary Policy Rules*, pp. 57–119. University of Chicago Press.

Rotemberg, J. J. and M. Woodford (1997). An optimization-based econometric framework for the evaluation of monetary policy. In J. J. Rotemberg and B. S. Bernanke

(Eds.), *NBER Macroeconomics Annual 1997*, pp. 297–346. Cambridge, MA, The MIT Press.

Rudebusch, G. D. (1995). Federal reserve interest rate targeting, rational expectations and the term structure. *Journal of Monetary Economics 35*, 245–274.

Samuelson, P. A. (1947). *Foundations of Economic Analysis*. Cambridge, MA: Harvard University Press.

Schmitt-Grohé, S. and M. Uribe (2004). Optimal operational monetary policy in the Christiano-Eichenbaum-Evans model of the U.S. business cycle. NBER wp No. 10724.

Schmitt-Grohé, S. and M. Uribe (2007). Optimal simple and implementable monetary and fiscal rules. *Journal of Monetary Economics 54*(6), 1702–1725.

Surico, P. (2008). The cost channel of monetary policy and indeterminacy. *Macroeconomic Dynamics 12*, 724–735.

Taylor, J. B. (1979). Staggered wage setting in a macro model. *American Economic Review 69*, 108–113.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

Taylor, J. B. (1999). A historical analysis of monetary policy rules. In J. B. Taylor (Ed.), *Monetary Policy Rules*, pp. 319–341. Chicago: University of Chicago Press.

Woodford, M. (2001). The Taylor rule and optimal monetary policy. *American Economic Review Papers and Proceedings 91*, 232–237.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

Yun, T. (1996). Nominal price rigidity, money supply endogeneity and business cycle. *Journal of Monetary Economics 37*, 345–370.

Yun, T. (2005). Optimal monetary policy with relative price distortions. *American Economic Review 95*, 89–108.

A Appendix

A.1 The coefficients of the generalized NKPC

We report below the coefficients of the generalized NKPC (see the system (13) in the text):

$$\lambda_{(\overline{\pi},\varepsilon)} = \frac{\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]\left[1-\alpha\beta\overline{\Pi}^{\theta(1-\varepsilon)}\right]}{\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}, \tag{35}$$

$$\eta_{(\overline{\pi},\varepsilon)} = \beta\left(\overline{\Pi}^{1-\varepsilon}-1\right)\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right], \tag{36}$$

$$\kappa_{(\overline{\pi},\varepsilon)} = \lambda_{(\overline{\pi},\varepsilon)}\left(\sigma_c+\sigma_n\right)-\left(1-\sigma_c\right)\eta_{(\overline{\pi},\varepsilon)}, \tag{37}$$

$$\xi_{(\overline{\pi},\varepsilon)} = \frac{\theta\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\left(\overline{\Pi}^{1-\varepsilon}-1\right)}{1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}. \tag{38}$$

Note that, given our restrictions in the main text, $\lambda_{(\overline{\pi},\varepsilon)}$, $\eta_{(\overline{\pi},\varepsilon)}$ and $\xi_{(\overline{\pi},\varepsilon)}$ are positive for positive trend inflation, i.e., $\overline{\Pi}>1$, while the sign of $\kappa_{(\overline{\pi},\varepsilon)}$ is surely positive only for $\sigma_c \geq 1$ and ambiguous otherwise.

A.1.1 The coefficient $\lambda_{(\overline{\pi},\varepsilon)}$ is decreasing in $\overline{\Pi}$, i.e., $\frac{\partial\lambda_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}}<0$

From equation (35) compute the partial derivative with respect to $\overline{\Pi}$,

$$\frac{\partial\lambda_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} = -\frac{(1-\varepsilon)\left[\theta\left(\overline{\Pi}^{2\theta\varepsilon+1}-\alpha^2\beta\overline{\Pi}^{2\theta+\varepsilon}\right)-\overline{\Pi}^{2\theta\varepsilon+1}+\alpha\beta\overline{\Pi}^{\theta+\theta\varepsilon+1}\right]}{\alpha\overline{\Pi}^{(\theta+1)(\varepsilon+1)}}.$$

Notice the expression in square brackets can be factorized as follows

$$\begin{aligned}
&\theta\left(\overline{\Pi}^{2\theta\varepsilon+1}-\alpha^2\beta\overline{\Pi}^{2\theta+\varepsilon}\right)-\overline{\Pi}^{2\theta\varepsilon+1}+\alpha\beta\overline{\Pi}^{\theta+\theta\varepsilon+1}\\
=\;&\theta\overline{\Pi}^{2\theta\varepsilon+1}\left[1-\alpha^2\beta\overline{\Pi}^{\theta+\theta+\varepsilon-(2\theta\varepsilon+1)}\right]-\overline{\Pi}^{2\theta\varepsilon+1}\left[1-\alpha\beta\overline{\Pi}^{\theta+\theta\varepsilon+1-(2\theta\varepsilon+1)}\right]\\
=\;&\overline{\Pi}^{2\theta\varepsilon+1}\left\{\theta\left[1-\alpha^2\beta\overline{\Pi}^{\theta+\theta+\varepsilon-(2\theta\varepsilon+1)}\right]-\left[1-\alpha\beta\overline{\Pi}^{\theta+\theta\varepsilon+1-(2\theta\varepsilon+1)}\right]\right\}
\end{aligned}$$

And moreover, the expression in curly brackets can be written as

$$\overline{\Pi}^{2\theta\varepsilon+1}\left\{\theta\left[1-\alpha^2\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\overline{\Pi}^{\theta(1-\varepsilon)}\right]-\left[1-\alpha\beta\overline{\Pi}^{\theta(1-\varepsilon)}\right]\right\}.$$

Given restrictions $\alpha\beta\overline{\Pi}^{\theta(1-\varepsilon)} < 1$ and $\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)} < 1$, the last expression is positive. Hence, $\frac{\partial\lambda_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} < 0$.

A.1.2 The coefficient $\eta_{(\overline{\pi},\varepsilon)}$ is increasing in $\overline{\Pi}$, i.e., $\frac{\partial\eta_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} > 0$

From equation (36) compute the partial derivative with respect to $\overline{\Pi}$,

$$\frac{\partial\eta_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} = (1-\varepsilon)\frac{\overline{\Pi}^{-\varepsilon}\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]+\alpha(\theta-1)\overline{\Pi}^{(\theta-1)(1-\varepsilon)-1}\left(\overline{\Pi}^{1-\varepsilon}-1\right)}{\beta\left\{\left(\overline{\Pi}^{1-\varepsilon}-1\right)\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]\right\}^2} > 0,$$

which is positive given positive trend inflation (i.e., $\overline{\Pi} > 1$) and the restriction $\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)} < 1$.

A.1.3 The coefficient $\kappa_{(\overline{\pi},\varepsilon)}$ is decreasing in $\overline{\Pi}$, i.e., $\frac{\partial\kappa_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} < 0$

This result immediately follows from the fact that $\frac{\partial\lambda_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} < 0$ and $\frac{\partial\eta_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} > 0$.

A.1.4 The coefficient $\xi_{(\overline{\pi},\varepsilon)}$ is increasing in $\overline{\Pi}$, i.e., $\frac{\partial\xi_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} > 0$

From (38) compute the partial derivative with respect to $\overline{\Pi}$,

$$\begin{aligned}\frac{\partial\xi_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} &= \frac{(1-\varepsilon)\theta\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\left[(\theta-1)\overline{\Pi}^{-1}\left(\overline{\Pi}^{1-\varepsilon}-1\right)+\overline{\Pi}^{-\varepsilon}\right]}{1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}} \\ &\quad +\frac{\alpha^2\theta(\theta-1)(1-\varepsilon)\overline{\Pi}^{2(\theta-1)(1-\varepsilon)-1}\left(\overline{\Pi}^{1-\varepsilon}-1\right)}{\left[1-\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]^2}.\end{aligned}$$

Again, assuming positive trend inflation (i.e., $\overline{\Pi} > 1$) and the restriction $\alpha\overline{\Pi}^{(\theta-1)(1-\varepsilon)} < 1$, it follows that $\frac{\partial\xi_{(\overline{\pi},\varepsilon)}}{\partial\overline{\Pi}} > 0$.

A.2 The long-run multiplier of trend inflation on output

We derive the long-run multiplier of trend inflation on output, i.e., the partial derivative $\partial\widehat{Y}/\partial\Delta$, where $\Delta \equiv (1-\varepsilon\omega)\widehat{\pi}$. To begin with, we eliminate from (13) all time subscripts and expectation operators, and then collect terms,

$$
\begin{aligned}
&\left[1-\beta\overline{\Pi}^{1-\varepsilon}-\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\right]\Delta\equiv\kappa_{(\overline{\pi},\varepsilon)}\widehat{Y}+\lambda_{(\overline{\pi},\varepsilon)}\sigma_n\widehat{s}+\eta_{(\overline{\pi},\varepsilon)}\widehat{\phi},\\
&\left(1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right)\widehat{\phi}=\left(1-\sigma_c\right)\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]\widehat{Y}+\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\left(\theta-1\right)\Delta,\\
&\left(1-\alpha\overline{\Pi}^{\theta(1-\varepsilon)}\right)\widehat{s}=\xi_{(\overline{\pi},\varepsilon)}\Delta.
\end{aligned} \tag{39}
$$

Then, we compute the derivatives:

$$
\frac{\partial\widehat{Y}}{\partial\Delta}=\frac{1}{\kappa_{(\overline{\pi},\varepsilon)}}\left\{1-\beta\overline{\Pi}^{1-\varepsilon}-\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)-\lambda_{(\overline{\pi},\varepsilon)}\sigma_n\frac{\partial\widehat{s}}{\partial\Delta}-\eta_{(\overline{\pi},\varepsilon)}\frac{\partial\widehat{\phi}}{\partial\Delta}\right\}, \tag{40}
$$

$$
\frac{\partial\widehat{\phi}}{\partial\Delta}=\frac{\left(1-\sigma_c\right)\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]}{1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}\frac{\partial\widehat{Y}}{\partial\Delta}+\frac{\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\left(\theta-1\right)}{1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}, \tag{41}
$$

$$
\frac{\partial\widehat{s}}{\partial\Delta}=\frac{\xi_{(\overline{\pi},\varepsilon)}}{1-\alpha\overline{\Pi}^{\theta(1-\varepsilon)}}. \tag{42}
$$

Therefore, substituting (41) and (42) into (40) yields

$$
\frac{\partial\widehat{Y}}{\partial\Delta}=\frac{\left\{\frac{1-\beta\overline{\Pi}^{1-\varepsilon}-\eta_{(\overline{\pi},\varepsilon)}(\theta-1)}{\kappa_{(\overline{\pi},\varepsilon)}}-\frac{\lambda_{(\overline{\pi},\varepsilon)}\sigma_n}{\kappa_{(\overline{\pi},\varepsilon)}}\frac{\xi_{(\overline{\pi},\varepsilon)}}{1-\alpha\overline{\Pi}^{\theta(1-\varepsilon)}}-\frac{\eta_{(\overline{\pi},\varepsilon)}}{\kappa_{(\overline{\pi},\varepsilon)}}\frac{\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}(\theta-1)}{1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}\right\}}{1+\frac{\eta_{(\overline{\pi},\varepsilon)}}{\kappa_{(\overline{\pi},\varepsilon)}}\frac{(1-\sigma_c)\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]}{1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}}}, \tag{43}
$$

Finally, setting $\sigma_c=1$ and $\sigma_n=0$ yields,

$$
\frac{\partial\widehat{Y}}{\partial\Delta}=\underbrace{\frac{\left(1-\beta\overline{\Pi}^{1-\varepsilon}\right)\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]-\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)}{\kappa_{(\overline{\pi},\varepsilon)}\left[1-\alpha\beta\overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right]}}_{\delta_{(\overline{\pi},\varepsilon)}}, \tag{44}
$$

A.3 REE Determinacy Conditions

With price indexation to trend inflation, i.e., $\omega=0$, and infinite labour supply elasticity, i.e., $\sigma_n=0$, the vector x_t includes only non-predetermined variables, namelyY_t, π_t and ϕ_t. To ensure determinacy of REE all eigenvalues of matrix $\mathbf{A}$ must lie inside the unit circle.

The characteristic polynomial associated with a cubic matrix reads as

$$
p\left(\lambda\right)=-\lambda^3+T\lambda^2-M\lambda+D, \tag{45}
$$

where T, M and D denote the trace, the sum of leading minors of order two and the determinant of matrix $\mathbf{A}$, respectively. Setting $\sigma_c = 1$, the dynamic systems reads

$$\begin{cases} \pi_t \equiv \beta \overline{\Pi}^{1-\varepsilon} E_t \pi_{t+1} + \kappa_{(\overline{\pi},\varepsilon)} \widehat{Y}_t + \eta_{(\overline{\pi},\varepsilon)} E_t \left[(\theta - 1) \pi_{t+1} + \widehat{\phi}_{t+1} \right], \\ \widehat{\phi}_t = \alpha \beta \overline{\Pi}^{(\theta-1)(1-\varepsilon)} E_t \left[(\theta - 1) \pi_{t+1} + \widehat{\phi}_{t+1} \right], \\ \widehat{Y}_t = E_t \widehat{Y}_{t+1} - \left(\phi_y \widehat{Y}_t + \phi_\pi \widehat{\pi}_t - E_t \widehat{\pi}_{t+1} \right) \end{cases} \tag{46}$$

where $\kappa_{(\overline{\pi},\varepsilon)} = \lambda_{(\overline{\pi},\varepsilon)}$. In matrix form,

$$\begin{bmatrix} 1 & 0 & -\kappa \\ 0 & 1 & 0 \\ \phi_\pi & 0 & 1 + \phi_y \end{bmatrix} \begin{bmatrix} \pi_t \\ \widehat{\phi}_t \\ \widehat{Y}_t \end{bmatrix} = \begin{bmatrix} \beta \overline{\Pi}^{1-\varepsilon} + \eta (\theta - 1) & \eta & 0 \\ (\theta - 1) \alpha \beta \overline{\Pi}^{(\theta-1)(1-\varepsilon)} & \alpha \beta \overline{\Pi}^{(\theta-1)(1-\varepsilon)} & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t+1} \\ \widehat{\phi}_{t+1} \\ \widehat{Y}_{t+1} \end{bmatrix} \tag{47}$$

and thus

$$A = \frac{1}{1 + \phi_y + \kappa \phi_\pi} \begin{bmatrix} \kappa + \left(\theta \eta + \overline{\Pi}^{1-\varepsilon} \beta - \eta \right) (1 + \phi_y) & \eta (1 + \phi_y) & \kappa \\ \frac{1 + \phi_y + \kappa \phi_\pi}{\overline{\Pi}^{(\theta-1)(\varepsilon-1)}} \alpha \beta (\theta - 1) & \frac{1 + \phi_y + \kappa \phi_\pi}{\overline{\Pi}^{(\theta-1)(\varepsilon-1)}} \alpha \beta & 0 \\ \eta \phi_\pi (1 - \theta) - \overline{\Pi}^{1-\varepsilon} \beta \phi_\pi + 1 & -\eta \phi_\pi & 1 \end{bmatrix} \tag{48}$$

It follows that

$$M = \frac{\beta \overline{\Pi}^{1-\varepsilon} + \mu_{(\overline{\pi},\varepsilon)} \left[1 + \kappa_{(\overline{\pi},\varepsilon)} + q_{(\overline{\pi},\varepsilon)} (\theta - 1) + \beta \overline{\Pi}^{1-\varepsilon} (1 + \phi_y) \right]}{1 + \phi_y + \kappa_{(\overline{\pi},\varepsilon)} \phi_\pi}, \tag{49}$$

$$T = \mu_{(\overline{\pi},\varepsilon)} + \frac{1 + \kappa_{(\overline{\pi},\varepsilon)} + \beta \overline{\Pi}^{1-\varepsilon} (1 + \phi_y) + \mu_{(\overline{\pi},\varepsilon)} q_{(\overline{\pi},\varepsilon)} (\theta - 1) (1 + \phi_y)}{1 + \phi_y + \kappa_{(\overline{\pi},\varepsilon)} \phi_\pi}, \tag{50}$$

$$D = \frac{\beta \overline{\Pi}^{1-\varepsilon} \mu_{(\overline{\pi},\varepsilon)}}{1 + \phi_y + \kappa_{(\overline{\pi},\varepsilon)} \phi_\pi}, \tag{51}$$

where $\mu_{(\overline{\pi},\varepsilon)} \equiv \alpha \beta \overline{\Pi}^{(\theta-1)(1-\varepsilon)} < 1$, and $q_{(\overline{\pi},\varepsilon)} \equiv \frac{\left[1 - \alpha \overline{\Pi}^{(\theta-1)(1-\varepsilon)}\right] \left(\overline{\Pi}^{1-\varepsilon} - 1 \right)}{\alpha \overline{\Pi}^{(\theta-1)(1-\varepsilon)}}$.

For standard calibration values and $\phi_\pi \in [0, \infty)$ and $\phi_Y \in [-1, \infty)$, one can show that $T \in (0, \infty)$, $M \in (0, \infty)$ and $D \in (0, \infty)$.

Theorem 1 in Brooks (2004) demonstrates that necessary and sufficient conditions for 3X3 matrix as A to have all the eigenvalue within the unit circle are

$$|D| < 1, \tag{52}$$

$$|T + D| < M + 1, \tag{53}$$

$$D^2 - TD + M < 1. \tag{54}$$

Substituting the expressions for T, M and D in gives (19), (20) and (21) in Proposition 1 in the main text.

A.4 Proof of Proposition 2: Effects of trend inflation on $\delta_{(\overline{\pi},\varepsilon)}$

First, we prove there exists a value of trend inflation, denoted by $\overline{\Pi}^{*}$, such that $\delta_{(\overline{\pi}^{*},\varepsilon)} = 0$. Notice that $\delta_{(1,\varepsilon)} = (1-\beta)/\kappa_{(1,\varepsilon)} > 0$ and $\delta_{\left(\beta^{1/(\varepsilon-1)},\varepsilon\right)} < 0$. Therefore, as $\delta_{(\overline{\pi},\varepsilon)}$ is a continuous function and $\overline{\Pi} \in \left[1, (\alpha\beta)^{1/[\theta(\varepsilon-1)]}\right)$, there exists a value of trend inflation $\overline{\Pi}^{*} \in \left(1, \beta^{1/(\varepsilon-1)}\right)$ such that $\delta_{(\overline{\pi}^{*},\varepsilon)} = 0$. Second, notice the sign of $\delta_{(\overline{\pi},\varepsilon)}$ depends only on the sign of its numerator, as its denominator is always positive. Given that the numerator of $\delta_{(\overline{\pi},\varepsilon)}$ monotonically decreases with $\overline{\Pi}$, for $\overline{\Pi} \in \left[1, (\alpha\beta)^{1/[\theta(\varepsilon-1)]}\right)$, it follows that $\overline{\Pi}^{*}$ is unique and therefore the proposition follows.

A.5 Factorization of (21)

Substituting into condition (21)

$$1 - D^2 + TD - M > 0$$

the relevant terms, it yields

$$\begin{aligned}
0 \quad < \quad & 1 - \left(\frac{\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}}{1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}\right)^2 \\
& \frac{\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}}{1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}\left(\mu_{(\overline{\pi},\varepsilon)} + \frac{1+\kappa_{(\overline{\pi},\varepsilon)}+\beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)+\mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left(1+\phi_y\right)}{1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}\right) + \\
& -\frac{\beta\overline{\pi}^{1-\varepsilon}+\mu_{(\overline{\pi},\varepsilon)}\left[1+\kappa_{(\overline{\pi},\varepsilon)}+q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)+\beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)\right]}{1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi}
\end{aligned} \tag{55}$$

Using simple algebra

$$\begin{aligned}
0 \quad < \quad & \left(1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right)^2 - \left(\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}\right)^2 \\
& \left(1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right)\beta\overline{\pi}^{1-\varepsilon}\mu^2_{(\overline{\pi},\varepsilon)} + \\
& +\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}\left[1+\kappa_{(\overline{\pi},\varepsilon)}+\beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)+\mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left(1+\phi_y\right)\right] + \\
& -\left(\beta\overline{\pi}^{1-\varepsilon}+\mu_{(\overline{\pi},\varepsilon)}\left[1+\kappa_{(\overline{\pi},\varepsilon)}+q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)+\beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)\right]\right)\left(1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right),
\end{aligned}$$

multiplying and, then, factorizing $\left(1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right)$ and $\left(\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}\right)$, it delivers

$$
\begin{aligned}
0 \;<\; & \left(1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right)^2 + \\
& \left(1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right)\left[\beta\overline{\pi}^{1-\varepsilon}\mu^2_{(\overline{\pi},\varepsilon)} - \left(\beta\overline{\pi}^{1-\varepsilon} + \mu_{(\overline{\pi},\varepsilon)}\left[1+\kappa_{(\overline{\pi},\varepsilon)} + q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right) + \beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)\right]\right)\right] \\
& + \left(\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}\right)\left[-\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)} + 1 + \kappa_{(\overline{\pi},\varepsilon)} + \beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right) + \mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left(1+\phi_y\right)\right].
\end{aligned}
$$

The same expression can also be written as

$$
\begin{aligned}
0 \;<\; & \left[1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi - \beta\overline{\pi}^{1-\varepsilon}\right] \\
& \left\{\mu_{(\overline{\pi},\varepsilon)}\left[\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)} - 1 - \kappa_{(\overline{\pi},\varepsilon)} - \beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)\right] + 1 + \phi_y + \kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right\} \\
& + \mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left[\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right) - 1 - \phi_y - \kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right]
\end{aligned}
$$

Now add and subtract $\pm\beta\overline{\pi}^{1-\varepsilon}$ in the last square bracket to write

$$
\begin{aligned}
0 \;<\; & \left[1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi - \beta\overline{\pi}^{1-\varepsilon}\right] \\
& \left\{\mu_{(\overline{\pi},\varepsilon)}\left[\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)} - 1 - \kappa_{(\overline{\pi},\varepsilon)} - \beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)\right] + 1 + \phi_y + \kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right\} \\
& + \mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left[\beta\overline{\pi}^{1-\varepsilon} - 1 - \phi_y - \kappa_{(\overline{\pi},\varepsilon)}\phi_\pi\right] \\
& + \mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left[\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right) - \beta\overline{\pi}^{1-\varepsilon}\right].
\end{aligned}
$$

Then group $\left[1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi - \beta\overline{\pi}^{1-\varepsilon}\right]$ to get

$$
\begin{aligned}
0 \;<\; & \left[1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi - \beta\overline{\pi}^{1-\varepsilon}\right] \\
& \left\{\mu_{(\overline{\pi},\varepsilon)}\left[\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)} - 1 - \kappa_{(\overline{\pi},\varepsilon)} - \beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)\right] + 1 + \phi_y + \kappa_{(\overline{\pi},\varepsilon)}\phi_\pi - \mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\right\} \\
& + \mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left[\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right) - \beta\overline{\pi}^{1-\varepsilon}\right].
\end{aligned}
$$

Moreover, note that $\mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)} = \eta_{(\overline{\pi},\varepsilon)}$, since $\mu_{(\overline{\pi},\varepsilon)} \equiv \alpha\beta\overline{\pi}^{(\theta-1)(1-\varepsilon)}$ and $q_{(\overline{\pi},\varepsilon)} \equiv \frac{1-\alpha\overline{\pi}^{(\theta-1)(1-\varepsilon)}}{\alpha\overline{\pi}^{(\theta-1)(1-\varepsilon)}}\left(\overline{\pi}^{1-\varepsilon}-1\right)$. Thus, the condition (21) can be expressed as

$$
\begin{aligned}
0 \;<\; & \left[1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi - \beta\overline{\pi}^{1-\varepsilon}\right] \\
& \left\{\mu_{(\overline{\pi},\varepsilon)}\left[\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)} - 1 - \kappa_{(\overline{\pi},\varepsilon)} - \beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)\right] + 1 + \phi_y + \kappa_{(\overline{\pi},\varepsilon)}\phi_\pi - \eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\right\} \\
& + \mu_{(\overline{\pi},\varepsilon)}q_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\beta\overline{\pi}^{1-\varepsilon}\left[\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right) - 1\right] \qquad (56)
\end{aligned}
$$

By sum and subtract $\pm\left[\kappa_{(\overline{\pi},\varepsilon)}\left(\phi_\pi-1\right)+\phi_y\left[\left(1-\beta\overline{\pi}^{1-\varepsilon}\right)-\frac{\eta_{(\overline{\pi},\varepsilon)}(\theta-1)}{1-\mu_{(\overline{\pi},\varepsilon)}}\right]\right]$ to the curly bracket in (56), and rearranging the terms there, one can write it as

$$\begin{aligned}
&\left\{\mu_{(\overline{\pi},\varepsilon)}\left[\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}-1-\kappa_{(\overline{\pi},\varepsilon)}-\beta\overline{\pi}^{1-\varepsilon}\left(1+\phi_y\right)\right]+1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi-\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\right\}\\
= &\ \kappa_{(\overline{\pi},\varepsilon)}\left(\phi_\pi-1\right)+\phi_y\left[\left(1-\beta\overline{\pi}^{1-\varepsilon}\right)-\frac{\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)}{1-\mu_{(\overline{\pi},\varepsilon)}}\right]+\frac{\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)}{1-\mu_{(\overline{\pi},\varepsilon)}}\left[\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right)-1\right]+\\
&+\left(1-\mu_{(\overline{\pi},\varepsilon)}\right)\left[1-\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}+\kappa_{(\overline{\pi},\varepsilon)}+\phi_y\beta\overline{\pi}^{1-\varepsilon}\right]+\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\phi_y.
\end{aligned}$$

Substituting the above expression in (56), it yields

$$\begin{aligned}
0 < &\left[1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi-\beta\overline{\pi}^{1-\varepsilon}\right]\\
&\left\{\kappa_{(\overline{\pi},\varepsilon)}\left(\phi_\pi-1\right)+\phi_y\left[\left(1-\beta\overline{\pi}^{1-\varepsilon}\right)-\frac{\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)}{1-\mu_{(\overline{\pi},\varepsilon)}}\right]+\frac{\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)}{1-\mu_{(\overline{\pi},\varepsilon)}}\left[\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right)-1\right]\right.\\
&\left.+\left(1-\mu_{(\overline{\pi},\varepsilon)}\right)\left[1-\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}+\kappa_{(\overline{\pi},\varepsilon)}+\phi_y\beta\overline{\pi}^{1-\varepsilon}\right]+\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\phi_y\right\}\\
&+\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\beta\overline{\pi}^{1-\varepsilon}\left[\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right)-1\right]. \qquad (57)
\end{aligned}$$

Recall the Taylor principle (20)

$$\phi_\pi+\phi_y\frac{\left(1-\alpha\beta\overline{\pi}^{(\theta-1)(1-\varepsilon)}\right)\left(1-\beta\overline{\pi}^{1-\varepsilon}\right)-\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)}{\kappa_{(\overline{\pi},\varepsilon)}\left[1-\alpha\beta\overline{\pi}^{(\theta-1)(1-\varepsilon)}\right]}>1 \qquad (58)$$

which can also be written as

$$\left[\kappa_{(\overline{\pi},\varepsilon)}\left(\phi_\pi-1\right)+\phi_y\left[\left(1-\beta\overline{\pi}^{1-\varepsilon}\right)-\frac{\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)}{1-\mu_{(\overline{\pi},\varepsilon)}}\right]\right]>0$$

and so (56) becomes

$$\begin{aligned}
0 < &\left[1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi-\beta\overline{\pi}^{1-\varepsilon}\right]\\
&\left\{\kappa_{(\overline{\pi},\varepsilon)}\left(\phi_\pi+\delta_{(\overline{\pi},\varepsilon)}\phi_y-1\right)+\frac{\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)}{1-\mu_{(\overline{\pi},\varepsilon)}}\left[\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right)-1\right]\right.\\
&\left.+\left(1-\mu_{(\overline{\pi},\varepsilon)}\right)\left[1-\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}+\kappa_{(\overline{\pi},\varepsilon)}+\phi_y\beta\overline{\pi}^{1-\varepsilon}\right]+\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\phi_y\right\}\\
&+\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\beta\overline{\pi}^{1-\varepsilon}\left[\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right)-1\right] \qquad (59)
\end{aligned}$$

or,

$$\begin{aligned}
0 &< \left[1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi-\beta\overline{\pi}^{1-\varepsilon}\right] \\
&\left\{\kappa_{(\overline{\pi},\varepsilon)}\left(\phi_\pi+\delta_{(\overline{\pi},\varepsilon)}\phi_y-1\right)+\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left[\frac{\phi_y}{1-\mu_{(\overline{\pi},\varepsilon)}}-1\right]\right. \\
&\left.+\left(1-\mu_{(\overline{\pi},\varepsilon)}\right)\left[1-\beta\overline{\pi}^{1-\varepsilon}\mu_{(\overline{\pi},\varepsilon)}+\kappa_{(\overline{\pi},\varepsilon)}+\phi_y\beta\overline{\pi}^{1-\varepsilon}\right]\right\} \\
&+\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\beta\overline{\pi}^{1-\varepsilon}\left[\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right)-1\right]. \qquad (60)
\end{aligned}$$

A necessary, but not sufficient condition for this last expression to hold is

$$0<\left[1+\phi_y+\kappa_{(\overline{\pi},\varepsilon)}\phi_\pi-\beta\overline{\pi}^{1-\varepsilon}\right]$$

which is exactly condition (29) in the main text.

Moreover note all the terms and parentheses in (60) are positive, apart two ambiguous terms: (i) $\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\left[\frac{\phi_y}{1-\mu_{(\overline{\pi},\varepsilon)}}-1\right]$ in the curly bracket; (ii) the last term $\eta_{(\overline{\pi},\varepsilon)}\left(\theta-1\right)\beta\overline{\pi}^{1-\varepsilon}\left[\mu_{(\overline{\pi},\varepsilon)}\left(1+\phi_y\right)-1\right]$. Both of them are multiplied by $\eta_{(\overline{\pi},\varepsilon)}$. So assuming $\eta_{(\overline{\pi},\varepsilon)}$ is small enough, then (29) is the relevant condition. In our numerical exercises, $\eta_{(\overline{\pi},\varepsilon)}$ is indeed very small (2.0811×10^{-3} is the highest value for the 8% annual inflation), so that the necessary condition (29) approximates quite well the necessary and sufficient condition.

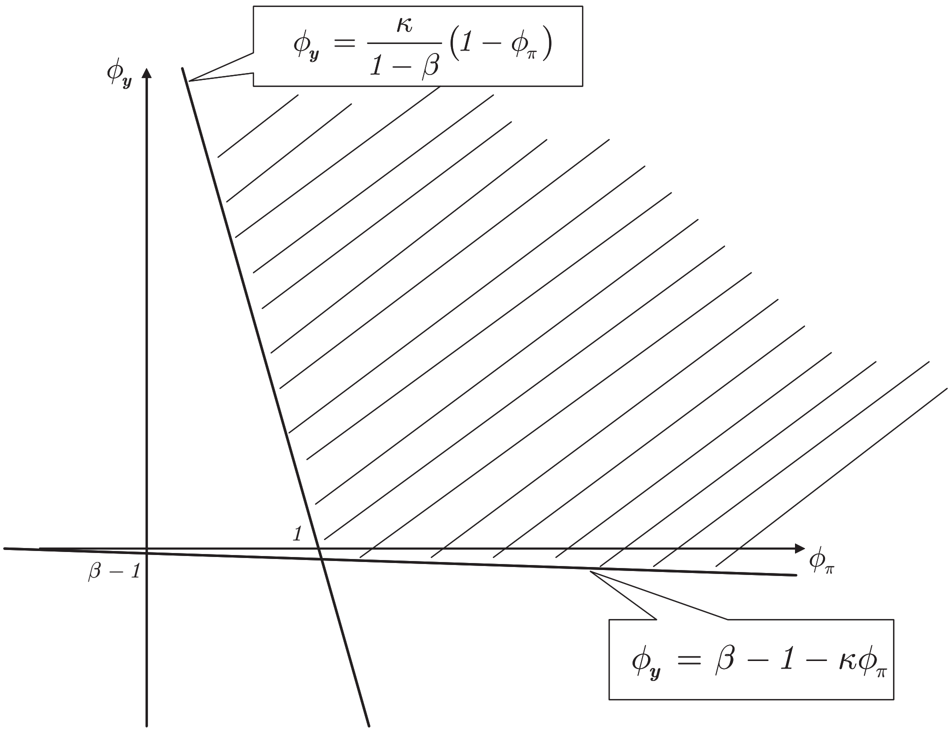

Figure 1: The determinacy region in the zero inflation steady state case.

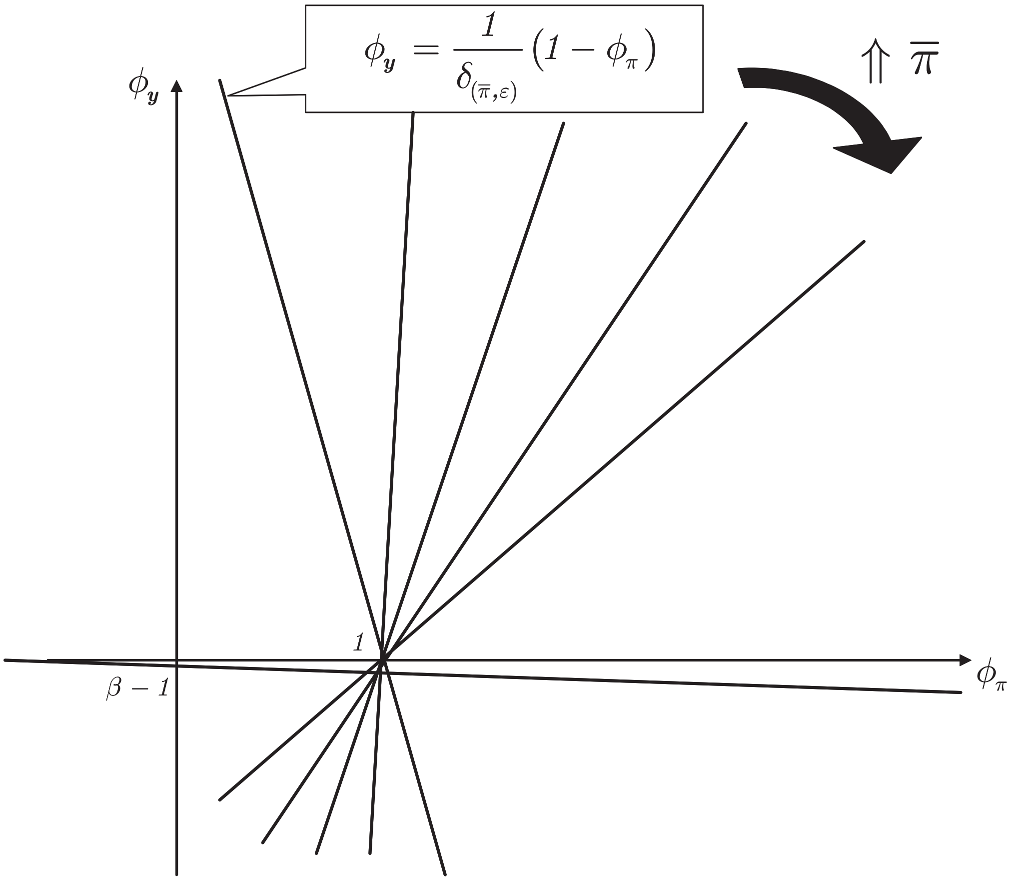

Figure 2: The effect of trend inflation on the Taylor principle.

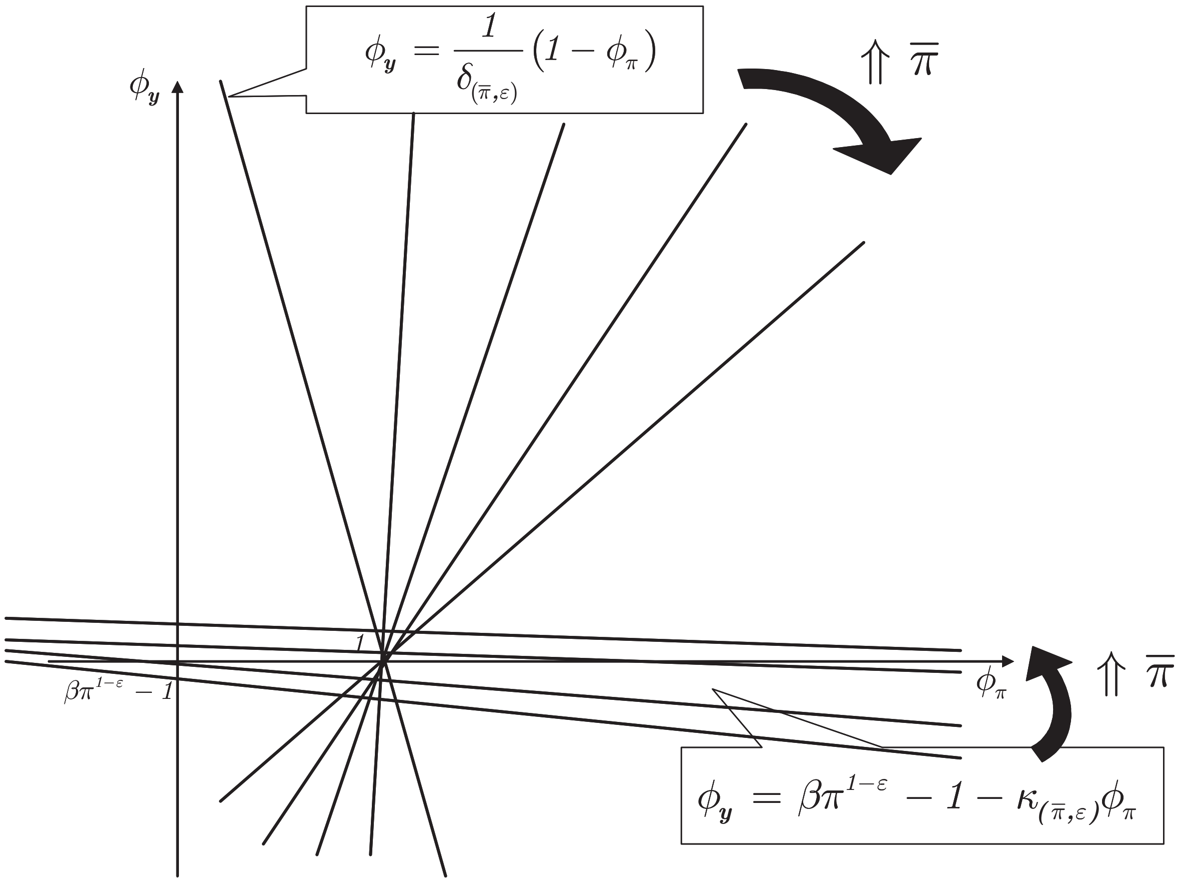

Figure 3: The effects of trend inflation on the determinacy conditions.

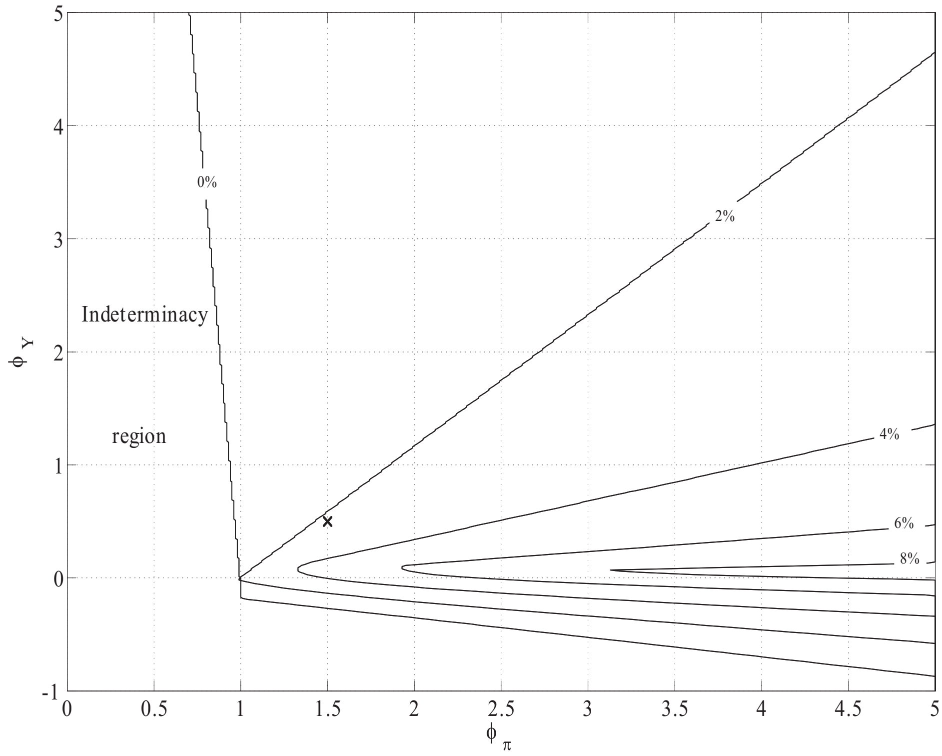

Figure 4: Contemporaneous nominal interest rate rule and the effects of trend inflation on REE determinacy. The cross marker identifies the classic Taylor rule specification, i.e. $\phi_\pi = 1.5$ and $\phi_Y = 0.5$.

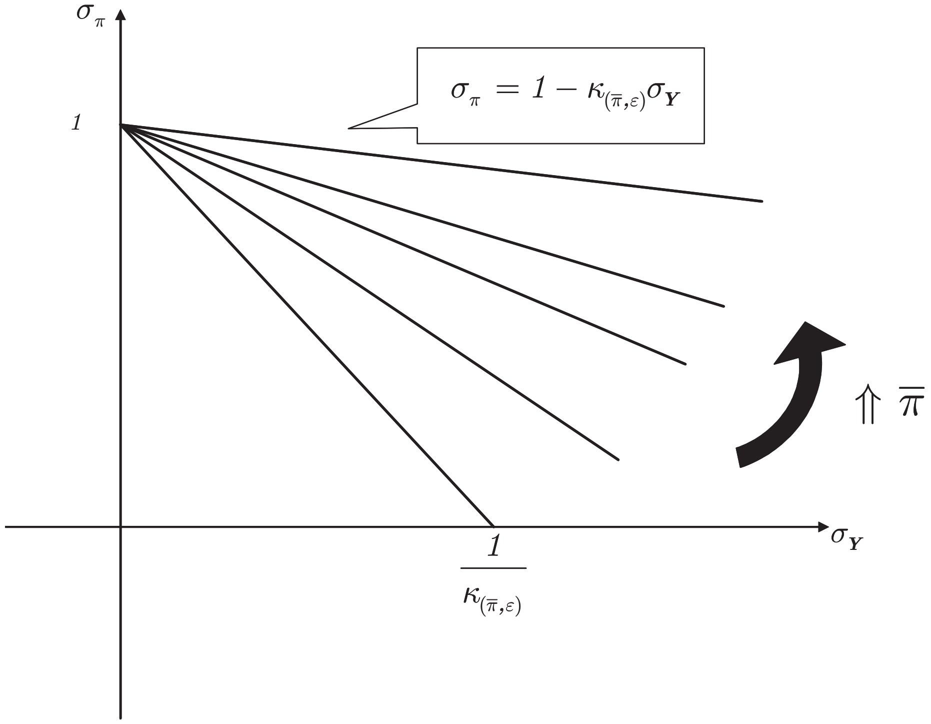

Figure 5: The effects of trend inflation on the efficient policy frontier.

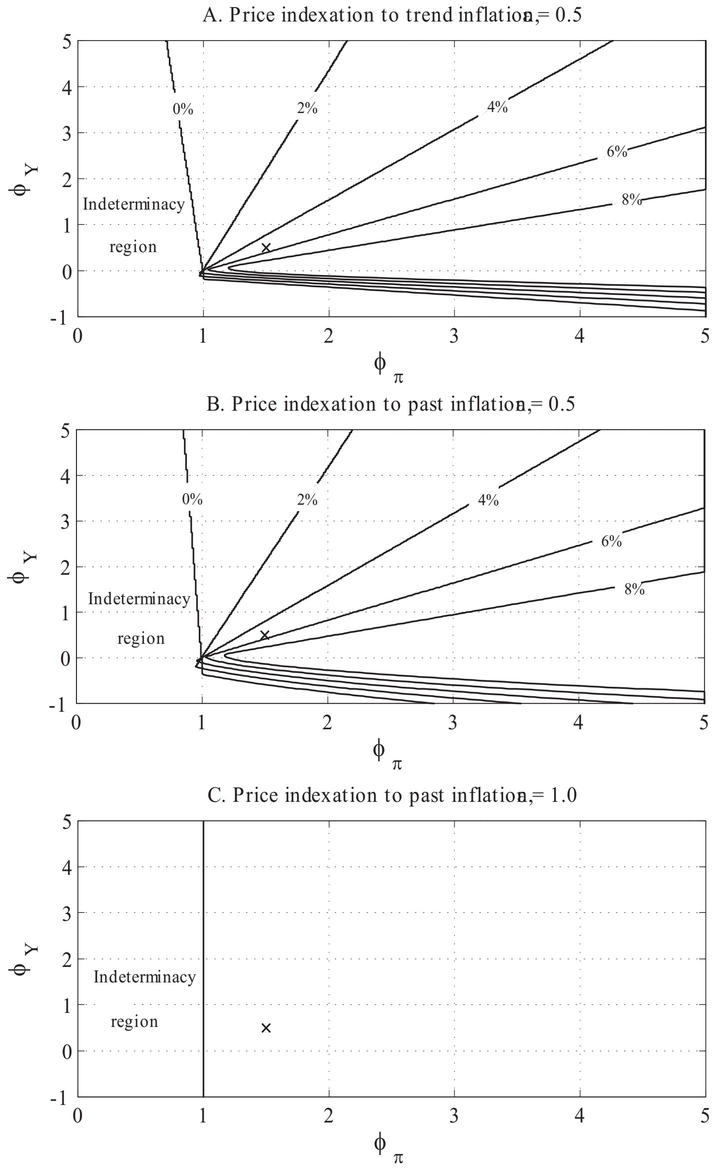

Figure 6: Contemporaneous interest rate rule, price indexation and the effects of trend inflation. The cross marker identifies the canonical Taylor rule, i.e. $\phi_\pi = 1.5$ and $\phi_Y = 0.5$.

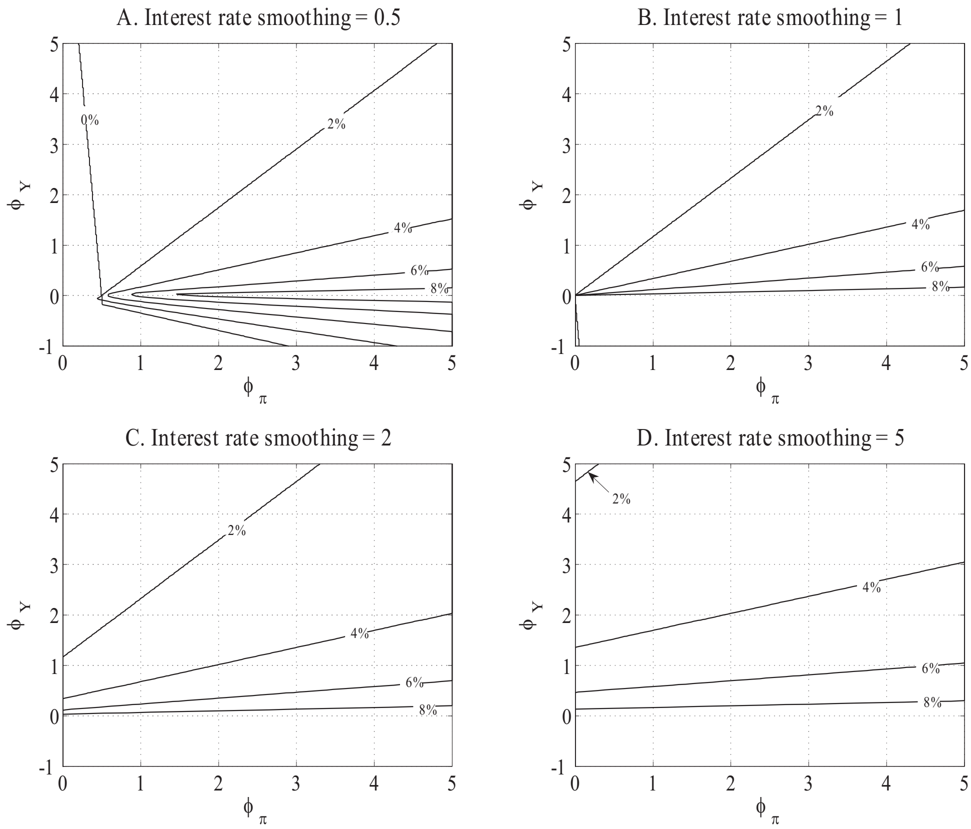

Figure 7: Inertial contemporaneous interest rate rule and the effects of trend inflation.

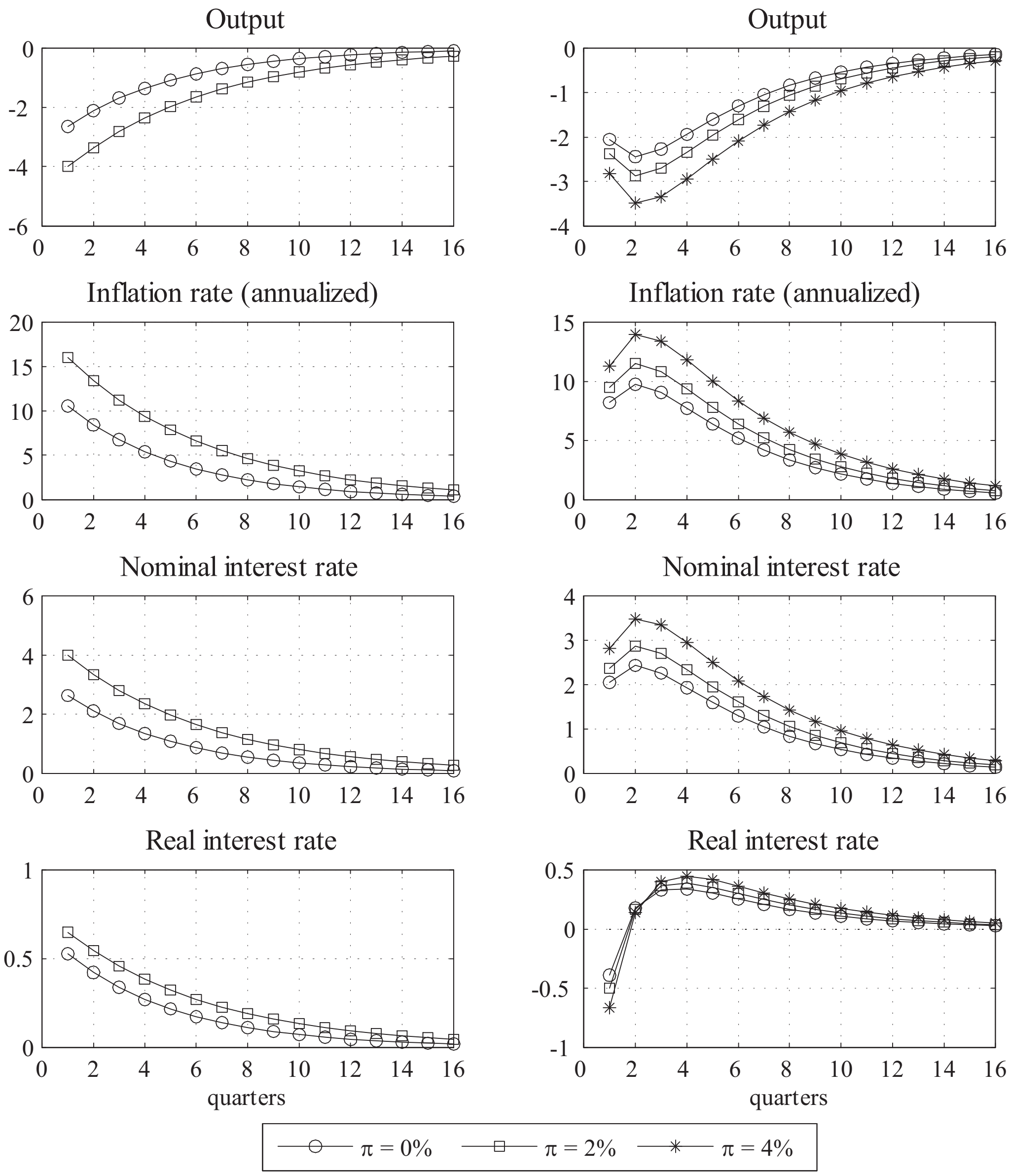

Figure 8: Impulse response functions to a unit cost push shock ($\phi_\pi = 1.5$ and $\phi_Y = 0.5$). Left column: zero price indexation. Right column: 50% price indexation to past inflation.

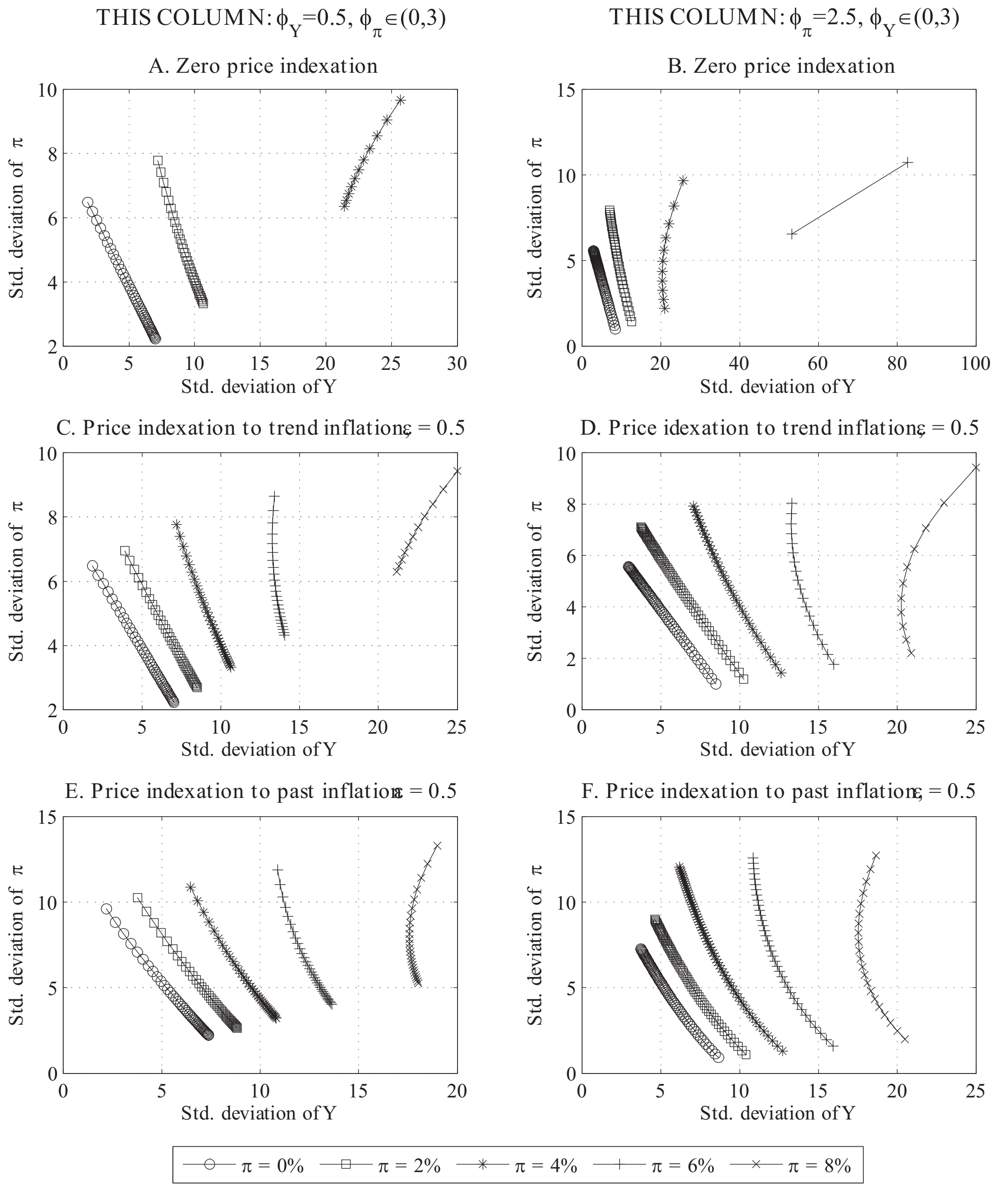

Figure 9: Efficient policy frontiers with contemporaneous interest rate rule and different rates of trend inflation.

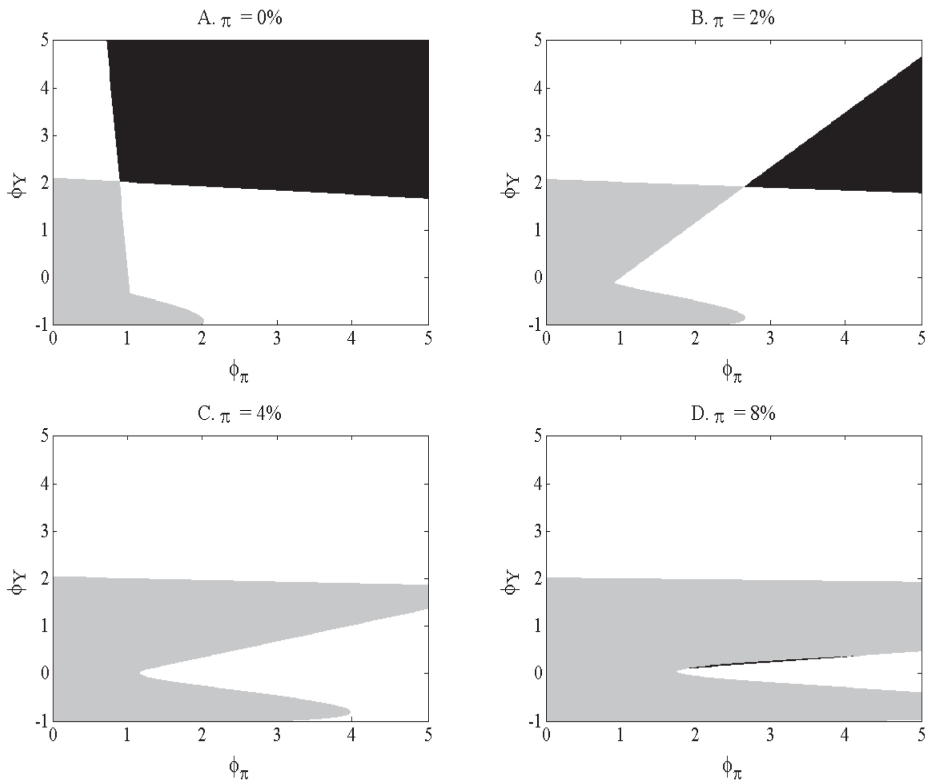

Figure 10: Backward looking interest rate rule and the effects of trend inflation (Black area = REE instability; Grey = REE indeterminacy; White = REE determinacy).

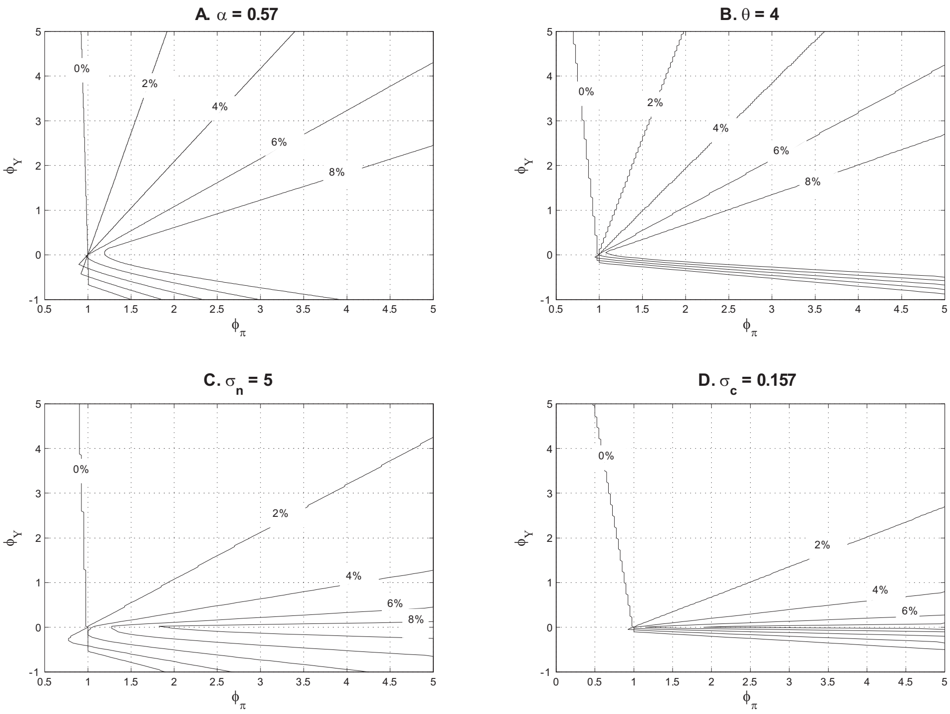

Figure 11: Sensitivity analysis. Contemporaneous interest rate rule and no indexation