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Abstract

World economies, and especially European ones, have become strongly interconnected in the last decades and a joint modelling is required. We propose here the use of Copulas to build flexible multivariate distributions, since they allow for a rich dependence structure and more flexible marginal distributions that better fit the features of empirical data, such as leptokurtosis. We use our approach to forecast industrial production series in the core EMU countries and we provide evidence that the copula-VAR model outperforms or at worst compares similarly to normal VAR models, keeping the same computational tractability of the latter approach.

JEL classification: C13, C32, C51, C53

Keywords: Forecasting, Industrial Production, Copulas, VAR models

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1 Introduction

The increasing globalization of world economies in general, and the introduction of the Euro in particular, have raised the issue of multivariate joint modelling. Given these stylized facts of contemporary economic systems, the assumption of joint normality can be no longer realistic and this leads to the problem of finding a more appropriate specification of multivariate models. Copula theory can be a solution to this problem. In fact, the essential idea of the copula approach is that a joint distribution can be factored into the marginals and a dependence function called copula. The dependence relationship is entirely determined by the copula, while scaling and shape (mean, standard deviation, skewness and kurtosis) are completely determined by the marginals.

Copulae have been successfully used in finance and we refer the interested reader to the book by Cherubini et al. (2004) for a detailed treatment of many financial applications. However, research work dealing with economic issues has been almost inexistent so far, except for a recent work by Granger et al. (2006) who use a copula-VAR-X-ARCH model to test whether a business cycle indicator influences the conditional copula of income and consumption growth.

In this perspective, we propose here a Copula-VAR approach for industrial production modelling and forecasting: we justify this choice because the dynamics of industrial activity is a fundamental indicator of the business cycle and the relevant information is provided without much delay. Besides, since the introduction of the Euro has had important effects mainly on the countries joining the European Monetary Union (EMU), we focus our analysis on EMU data, only.

The first contribution of the paper is a joint empirical analysis, by means of a copula-VAR-X model, of French, German and Italian industrial production, in the period going from the German reunification till the end of 2005, so that the first effects of EMU can be assessed. The second contribution of the paper is a comparison between our copula-VAR-X approach and the usual Normal VAR model. In this perspective, we perform forecasting exercises considering 1-step ahead and 3-steps ahead forecasts, showing that when the marginals are not normally distributed the copula approach yields better forecasts than the Normal VAR modelling, and keeps the same computational tractability of the latter approach. Besides, we remark that while the VAR estimated with OLS is a QML estimator and provides consistent estimates, its behavior in small samples like the one considered in this work can be rather poor. Furthermore, there are cases where QML estimators are biased, too (see Newey and Steigerwald, 1997). Instead, imposing the multivariate t distribution when only the marginals are leptokurtic while the dependence structure is normal, results in biased coefficients and poor forecasting performances.

The rest of the paper is organized as follows. Section 2 presents the copula-VAR model, while an empirical application of the general approach concerning European Industrial

Production data is introduced in Section 3. We perform extensive forecasting exercises in Section 4, comparing the performances of the alternative models chosen. Section 5 briefly concludes.

2 A Copula-VAR model

Consider a general copula-Vector-Auto-Regression model, where the n endogenous variables $x_{i,t}$ are explained by an intercept μ_i , autoregressive terms of order p , and an error term $\sqrt{h_{i,t}}\eta_{i,t}$

$$\begin{aligned} x_{1,t} &= \mu_1 + \sum_{i=1}^n \sum_{l=1}^p \phi_{1,i,l} x_{i,t-l} + \sqrt{h_{1,t}}\eta_{1,t} \\ &\vdots \\ &\vdots \\ x_{n,t} &= \mu_n + \sum_{i=1}^n \sum_{l=1}^p \phi_{n,i,l} x_{i,t-l} + \sqrt{h_{n,t}}\eta_{n,t}. \end{aligned} \tag{2.1}$$

Let the standardized innovations $\eta_{i,t}$ have mean zero and variance one. Furthermore, they have a conditional joint distribution $H_t(\eta_{1,t}, \dots, \eta_{n,t}; \theta)$ with the parameters vector θ , which can be expressed as follows, thanks to the so-called Sklar's theorem (1959):

$$(\eta_{1,t}, \dots, \eta_{n,t}) \sim H_t(\eta_{1,t}, \dots, \eta_{n,t}; \theta) = C_t(F_{1,t}(\eta_{1,t}; \alpha_1), \dots, F_{n,t}(\eta_{n,t}; \alpha_n); \gamma) \tag{2.2}$$

that is the joint distribution H_t of a vector of innovations $\eta_{i,t}$ is the copula $C_t(\cdot; \gamma)$ of the cumulative distribution functions of the innovations marginals $F_{1,t}(\eta_{1,t}; \alpha_1), \dots, F_{n,t}(\eta_{n,t}; \alpha_n)$, where $\gamma, \alpha_1, \dots, \alpha_n$ are the copula and marginals parameters, respectively. A copula is a function that links together two or more marginal distributions to form a multivariate joint distribution: consequently, copulae allow us to model the dependence structure among different variables in a flexible way and, at the same time, to use marginal distributions not necessarily identical. For example, $F_{1,t}(\eta_{1,t}; \nu_1)$ may follow a Student's t distribution with ν_1 degrees of freedom, $F_{2,t}(\eta_{2,t})$ a standard normal distribution, while $F_{3,t}(\eta_{3,t}; \nu_3)$ a Student's t distribution with ν_3 degrees of freedom.

The study of copulae has originated with the seminal papers by Hoeffding (1940) and Sklar (1959) and has seen various applications in the statistics literature. Examples include Clayton (1978), Schweizer and Wolff (1981), Genest and Rivest (1986a,b) and Genest and Rivest (1993). However, only in the last five years or so copulae have been used in economics and finance: see, for instance, the work of Rosenberg (1998, 2003), Bouyé et al. (2001), Embrechts et al. (2003a,b), Patton (2004, 2005b), Fantazzini et al. (2006), Granger et al. (2006). For more details, we refer the interested reader to the recent methodological overviews by Joe (1997) and Nelsen (1999), while Cherubini et al. (2004)

provide a comprehensive and detailed discussion of copula techniques for financial applications.

By applying Sklar's theorem and using the relationship between the distribution and the density function, we can derive the multivariate copula density $c(F_1(x_1), \dots, F_n(x_n))$, associated to a copula function $C(F_1(x_1), \dots, F_n(x_n))$:

$$f(x_1, \dots, x_n) = \frac{\partial^n [C(F_1(x_1), \dots, F_n(x_n))]}{\partial F_1(x_1) \cdots \partial F_n(x_n)} \prod_{i=1}^n f_i(x_i) = c(F_1(x_1), \dots, F_n(x_n)) \prod_{i=1}^n f_i(x_i), \quad (2.3)$$

where

$$c(F_1(x_1), \dots, F_n(x_n)) = \frac{f(x_1, \dots, x_n)}{\prod_{i=1}^n f_i(x_i)}, \quad (2.4)$$

By using this procedure, we can derive the *Normal-copula*, whose probability density function is:

$$\begin{aligned} c^{Normal}(\Phi(x_1), \dots, \Phi(x_n); \Sigma) &= \frac{f^{Normal}(x_1, \dots, x_n)}{\prod_{i=1}^n f_i^{Normal}(x_i)} = \frac{\frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp(-\frac{1}{2} x' \Sigma^{-1} x)}{\prod_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} x_i^2)} = \\ &= \frac{1}{|\Sigma|^{1/2}} \exp\left(-\frac{1}{2} \zeta' (\Sigma^{-1} - I) \zeta\right), \end{aligned} \quad (2.5)$$

where $\zeta = (\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n))'$ is the vector of univariate Gaussian inverse distribution functions, $u_i = \Phi(x_i)$, while Σ is the correlation matrix.

A similar procedure can be followed to derive the *t-copula*, which is the copula of the multivariate Student's t-distribution. Moreover, a model can allow for time-varying dependence structure. That is, we can use a copula function with a dynamic correlation structure (see for example Chen et al. (2004) and Patton (2004)). However, recent literature (again Chen et al. (2005) and Patton (2004)) has shown for financial data that a simple normal copula with no dynamics is sufficient to describe the joint dependence structure in most cases. Only when the number of considered variables is higher than 20, statistically significant differences start to emerge and more complicated copulae than the Normal one may be required. Fantazzini et al. (2006) found similar evidence with monthly operational risk data. Since we work with a small number of endogenous variables and monthly economic data, which are known to have a much simpler dependence structure than daily financial data, we stick to a *constant normal copula* $C_t^{Normal} = C^{Normal}$. This copula belongs to the class of Elliptical copulae.¹ An alternative to Elliptical copulae is given by *Archimedean copulae*: however, they present the serious limitation of modelling only positive dependence (or only partial negative dependence), while their multivariate extensions involve strict restrictions on bivariate dependence parameters. That is why we

¹See Cherubini et al. (2004) for more details.

do not consider them here.

2.1 Copula and Marginals Estimation

Let us suppose to have a set of T empirical data of n economic and financial series, and let $\theta = (\alpha_1, \dots, \alpha_n; \gamma)$ be the parameters vector to estimate, where $\alpha_i, i = 1, \dots, n$ are the parameters of the marginal distribution F_i and γ is the vector of the copula parameters. It follows from (2.3) that the log-likelihood function for the joint conditional distribution $H_t(\cdot; \theta)$ is given by:

$$l(\theta) = \sum_{t=1}^T \log(c(F_1(x_{1,t}; \alpha_1), \dots, F_n(x_{n,t}; \alpha_n); \gamma)) + \sum_{t=1}^T \sum_{i=1}^n \log f_i(x_{i,t}; \alpha_{i,t}). \quad (2.6)$$

Hence, the log likelihood of the joint distribution is just the sum of the log likelihoods of the margins and the log likelihood of the copula. Standard ML estimates may be obtained by maximizing the above expression with respect to the parameters $(\alpha_1, \dots, \alpha_n; \gamma)$. In practice this can involve a large numerical optimization problem with many parameters which can be difficult to solve. However, given the partitioning of the parameters vector into separate parameters for each margin and parameters for the copula, one may use (2.6) to break up the optimization problem into several small optimizations, each with fewer parameters. This multi-step procedure is known as the method of Inference Functions for Margins (*IFM*).

According to the IFM method, the parameters of the marginal distributions are estimated separately from the parameters of the copula. In other words, the estimation process is divided into the following two steps:

1. Estimate the parameters $\alpha_i, i = 1, \dots, n$ of the marginal distributions F_i using the ML method:

$$\hat{\alpha}_i = \arg \max l^i(\alpha_i) = \arg \max \sum_{t=1}^T \log f_i(x_{i,t}; \alpha_i), \quad (2.7)$$

where l^i is the log-likelihood function of the marginal distribution F_i ;

2. Estimate the copula parameters γ , given the estimations performed in step 1):

$$\hat{\gamma} = \arg \max l^c(\gamma) = \arg \max \sum_{t=1}^T \log(c(F_1(x_{1,t}; \hat{\alpha}_1), \dots, F_n(x_{n,t}; \hat{\alpha}_n); \gamma)), \quad (2.8)$$

where l^c is the log-likelihood function of the copula.

The IFM estimator verifies the properties of asymptotic normality:

$$\sqrt{T}(\hat{\theta}_{IFM} - \theta_0) \rightarrow N\left(0, \mathbf{H}_0^{-1} \mathbf{B}_0 (\mathbf{H}_0^{-1})'\right) \quad (2.9)$$

where \mathbf{H}_0 and \mathbf{B}_0 are the expected value of the Hessian and of the variance of the Score, respectively (see Joe 1997 and Patton 2005a, for a proof). The matrix $\mathbf{H}_0^{-1}\mathbf{B}_0(\mathbf{H}_0^{-1})'$ is known as the Godambe Information matrix.

Joe (1997) compares the efficiency of the IFM method relative to full maximum likelihood for a number of multivariate models and finds the IFM method to be highly efficient. Therefore, we think it is safe to use the IFM method and benefit from the huge reduction in complexity it implies for the numerical optimization.

3 An Empirical Application: Industrial Production in the main Euro-Zone Economies

The creation of the European Monetary Union (EMU) has increased the importance of business cycle analysis and forecast. Monetary policy is unique for all the 12 countries belonging to EMU and is decided according to the expected future developments of prices and output in the whole Euro area. Indeed, since monetary policy is characterised by operation delays, it must be necessarily forward looking; it follows that a reliable forecast on the pace of economic activity in the subsequent months is of utmost importance in deciding the most appropriate policy stance. As well known, the most commonly used and reliable high-frequency indicator of the business cycle is the industrial production index.² In this section therefore we build a copula-VAR model for describing the dynamics of the monthly seasonally-adjusted industrial production index in the three main countries of EMU, i.e. France, Germany and Italy.³

We follow Stock and Watson's (1998) suggestion to test for unit roots in the variables under scrutiny, as well as to use data-dependent lag-length selection criteria. A careful analysis of the levels and of the first log-differences of the industrial production series, reported in Figure 1 and in Table 1, shows that non-stationarity is the main feature of the variables over the observation period 1990:01 - 2005:12.⁴ We considered various criteria for selecting the most appropriate lag order of our copula-VAR models:⁵ the optimal choice resulted to be a model with three lags. Besides a trace test showed no cointegration at the 5 % confidence level.

We consider Student's t marginals and a normal copula since Jarque-Bera tests on the original Industrial Production series reject normality for all three cases.

With reference to the model structure, recent literature has shown that in explaining past behavior, but especially in making out-of-sample forecasts, models including not

²Industrial production forecasts can be subsequently translated into more meaningful and operationally useful GDP forecasts by using bridge models; see. Baffigi, Golinelli and Parigi (2004).

³As well known, these 3 countries account for about three fourths of the industrial production of the whole Euro area.

⁴We use the Augmented Dickey-Fuller and Dickey-Fuller Test with GLS Detrending (DF-GLS) by Elliott et al. (1996).

⁵See Lütkepohl (1991) for a detailed discussion of the argument.

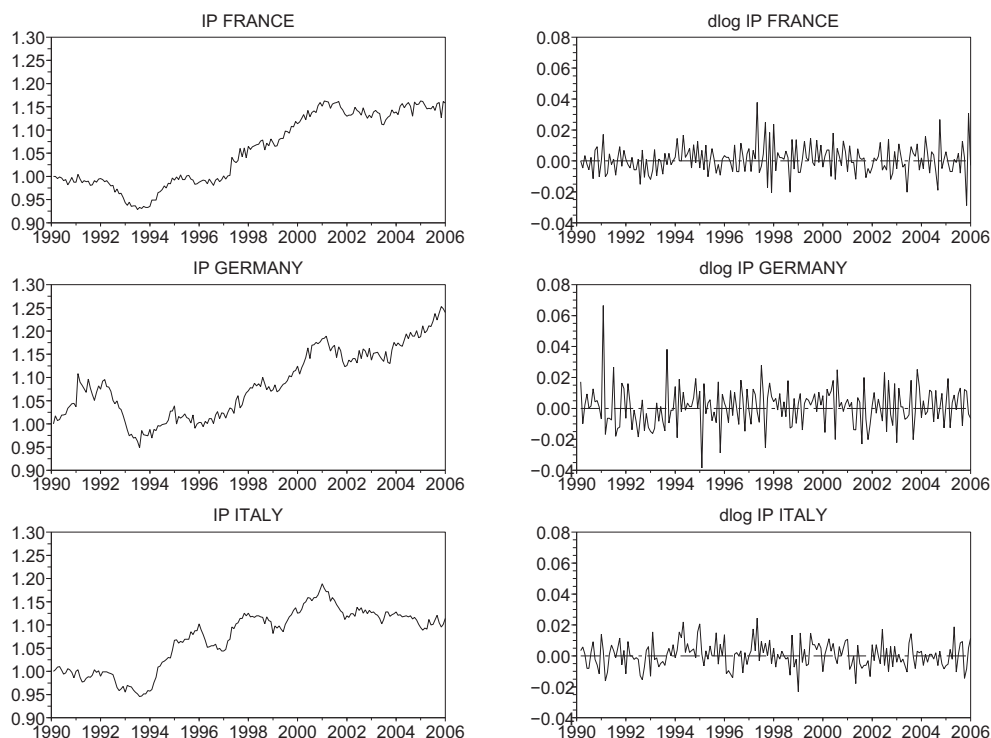


Figure 1: Endogenous variables

		Levels	First differences
France	ADF	-0.08	-20.41**
	DF-GLS	0.71	-20.27**
Germany	ADF	-0.04	-18.72**
	DF-GLS	1.00	-17.82*
Italy	ADF	-1.18	-8.17**
	DF-GLS	-0.28	-7.36**

(*) Significant at the 5% level.

(**) Significant at the 1% level.

Table 1: Unit root Tests for Industrial Production series.

No. of coint. rel.	Trace stat.	0.05 critical val.	prob.
0	20.66	29.80	0.38
1	6.33	15.49	0.66
2	2.38	3.84	0.12

Table 2: Trace test for cointegration

only autoregressive terms, but also structural explanatory variables and leading indicators are characterized by a definite superior performance.⁶ Hence, we use all these variables together in our empirical model, which is fully specified in eqs. (3.1)-(3.3), and where the variables in upper case letters have the following meaning:⁷

⁶See, as an example of this result, Bodo, Golinelli and Parigi (2000).

⁷The last letter at the end of each variable indicates the country to which we are referring; thus F, G and I stand respectively for France, Germany and Italy. All data used in the empirical application have been retrieved from the Eurostat online database.

- IP = industrial production index;
- ICI = industrial confidence indicator. Business confidence indicators are commonly recognized as significant leading indicators of industrial output. A different leading indicator might have been represented by the index of production expectations months ahead. It can be checked however that these two series are highly correlated and are therefore interchangeable for our purposes. Confidence indicators, moreover, show a slightly better performance in explaining and forecasting industrial output;
- SPREAD = spread between the long term and the short term nominal interest rate. The long term interest rate is the return on 10-years Government bonds; the short term interest rate is the 3-months return on monetary markets;
- XR = excess return on equities, defined as the difference between the percentage change in the share prices index and the short run interest rate;⁸
- LTRIR = long term real interest rate, defined as the difference between the long term rate and the corresponding annual rate of change of producer prices.

The model also includes the following dummy variables:

- DUMGERUN (=1 in 1990 and 1991), to capture the boom in German industrial activity after the process of German unification;
- DUMEMU (=1 after 1999), to capture the effects of the creation of EMU;
- DUMIT96 (=1 in 1996), to capture the effects of countercyclical activity in Italy, due to the effects of the Lira appreciation before its return into the EMS.

$$\begin{aligned}
\Delta \log(IPF)_t &= \mu_F + \sum_{i=1}^3 \phi_{i,F} \Delta \log(IPF)_{t-i} + \sum_{i=1}^3 \phi_{i,F} \Delta \log(IPG)_{t-i} + \sum_{i=1}^3 \phi_{i,F} \Delta \log(IPI)_{t-i} + \\
&+ \alpha_{1,F} \Delta \log(ICIG)_{t-1} + \alpha_{2,F} \Delta \log(ICIG)_{t-2} + \alpha_{3,F} \Delta \log(ICII)_{t-1} + \alpha_{4,F} \Delta \log(ICII)_{t-2} + \\
&+ \alpha_{5,F} XRF_{t-3} + \alpha_{6,F} XRI_{t-2} + \alpha_{7,F} \Delta(LTRIR)_{t-6} + \alpha_{8,F} \Delta SPREADI_{t-3} + \\
&+ \alpha_{9,F} DUMGERUN + \alpha_{10,F} DUMEMU + \alpha_{11,F} DUMIT96 + \sqrt{h_1} \eta_{1,t}, \tag{3.1}
\end{aligned}$$

$$\begin{aligned}
\Delta \log(IPG)_t &= \mu_G + \sum_{i=1}^3 \phi_{i,G} \Delta \log(IPF)_{t-i} + \sum_{i=1}^3 \phi_{i,G} \Delta \log(IPG)_{t-i} + \sum_{i=1}^3 \phi_{i,G} \Delta \log(IPI)_{t-i} + \\
&+ \alpha_{1,G} \Delta \log(ICIG)_{t-1} + \alpha_{2,G} \Delta \log(ICIG)_{t-2} + \alpha_{3,G} \Delta \log(ICII)_{t-1} + \alpha_{4,G} \Delta \log(ICII)_{t-2} + \\
&+ \alpha_{5,G} XRF_{t-3} + \alpha_{6,G} XRI_{t-2} + \alpha_{7,G} \Delta(LTRIR)_{t-6} + \alpha_{8,G} \Delta SPREADI_{t-3} + \\
&+ \alpha_{9,G} DUMGERUN + \alpha_{10,G} DUMEMU + \alpha_{11,G} DUMIT96 + \sqrt{h_1} \eta_{1,t}, \tag{3.2}
\end{aligned}$$

⁸Excess returns are more significant than simple share price indices in the estimates; see also Bradley and Jansen (2004).

$$\begin{aligned}
\Delta \log(IPI)_t = & \mu_I + \sum_{i=1}^3 \phi_{i,I} \Delta \log(IPF)_{t-i} + \sum_{i=1}^3 \phi_{i,I} \Delta \log(IPG)_{t-i} + \sum_{i=1}^3 \phi_{i,I} \Delta \log(IPI)_{t-i} + \\
& + \alpha_{1,I} \Delta \log(ICIG_{t-1}) + \alpha_{2,I} \Delta \log(ICIG_{t-2}) + \alpha_{3,I} \Delta \log(ICII_{t-1}) + \alpha_{4,I} \Delta \log(ICII_{t-2}) + \\
& + \alpha_{5,I} XRF_{t-3} + \alpha_{6,I} XRI_{t-2} + \alpha_{7,I} \Delta(LTRIRI_{t-6}) + \alpha_{8,I} \Delta SPREADI_{t-3} + \\
& + \alpha_{9,I} DUMGERUN + \alpha_{10,I} DUMEMU + \alpha_{11,I} DUMIT96 + \sqrt{h_1} \eta_{1,t}, \tag{3.3}
\end{aligned}$$

$$(\eta_{1,t}, \dots, \eta_{n,t}) \sim C^{Normal}(t_{1,t}(\eta_{1,t}; \nu_1), t_{2,t}(\eta_{2,t}; \nu_2), t_{3,t}(\eta_{3,t}; \nu_3); \Sigma),$$

As Bodo, Golinelli and Parigi (2000) show, a useful explanatory variable for output behaviour in the Euro area might be represented by the US industrial production index. Actually, in our estimates, this variable turns out to be significant only for Italy and moreover both marginally and with no lag. Therefore, despite the fact that the US index is usually published one month ahead of European data, its use would reduce the forecasting horizon of the model by one month, with no substantial improvement in the results. Furthermore, Marchetti and Parigi (2004) show that, in the case of Italy, a useful indicator of industrial production might be represented by electricity consumption. Despite that indication, this variable has however the same disadvantage of being a coincident indicator, thus reducing the forecasting horizon of the estimates; moreover electricity consumption data of the same kind are not available for France and Germany. Finally, it might be noticed that despite its economic relevance, the price of oil does not result significant in influencing the dynamics of industrial activity in the period under consideration.

We report in Table 3 the estimation output for a copula-VAR model with Student's t marginals and lagged exogenous variables.

In the equation for *France* the only specifically significant exogenous variable is constituted by the excess return on equities, with a lead of 3 months on the industrial production index. The sign is positive as expected, since an increasing excess return should provide an indication of an upsurge of economic activity in the immediate near future. No index of industrial confidence, instead, appears significant in predicting the dynamics of French industrial production; this effect is then somehow caught and replaced by the interaction with other countries' variables, particularly the Italian and German confidence indicators. As to *Germany*, the dynamics of the industrial production index depends positively, as predicted, upon firms' confidence, with a lag of 1 and 2 months. Besides, two dummies appear significant: a first one, meant to capture the boom in economic activity following the process of reunification (boom which appears countercyclical with respect to other countries experience), and a second one, that records the positive effects on German exports, and thus on industrial activity, of the creation of the European Monetary Union.

The dynamics of the *Italian* industrial production index depends positively, as predicted, upon firms' confidence, with a lag of 1 and 3 months. Also the excess return on Italian equities, with a lead of 2 months, appears to provide some information about the dynamics

France		Germany		Italy	
Constant	0.0015	Constant	-0.0019	Constant	0.0044*
$\Delta \log(\text{Ipi}(-1))$	0.0369	$\Delta \log(\text{Ipi}(-1))$	0.1643	$\Delta \log(\text{Ipi}(-1))$	-0.0790
$\Delta \log(\text{Ipi}(-2))$	0.2081**	$\Delta \log(\text{Ipi}(-2))$	-0.0039	$\Delta \log(\text{Ipi}(-2))$	0.0143
$\Delta \log(\text{Ipi}(-3))$	0.1672*	$\Delta \log(\text{Ipi}(-3))$	0.0426	$\Delta \log(\text{Ipi}(-3))$	-0.2680**
$\Delta \log(\text{Ipf}(-1))$	-0.6075**	$\Delta \log(\text{Ipf}(-1))$	0.1734	$\Delta \log(\text{Ipf}(-1))$	-0.1524*
$\Delta \log(\text{Ipf}(-2))$	-0.3037**	$\Delta \log(\text{Ipf}(-2))$	0.3513**	$\Delta \log(\text{Ipf}(-2))$	-0.0533
$\Delta \log(\text{Ipf}(-3))$	-0.0953	$\Delta \log(\text{Ipf}(-3))$	0.3321**	$\Delta \log(\text{Ipf}(-3))$	-0.0033
$\Delta \log(\text{Ipg}(-1))$	0.0132	$\Delta \log(\text{Ipg}(-1))$	-0.5141**	$\Delta \log(\text{Ipg}(-1))$	0.0236
$\Delta \log(\text{Ipg}(-2))$	-0.0171	$\Delta \log(\text{Ipg}(-2))$	-0.3375**	$\Delta \log(\text{Ipg}(-2))$	-0.0176
$\Delta \log(\text{Ipg}(-3))$	0.0215	$\Delta \log(\text{Ipg}(-3))$	-0.0999	$\Delta \log(\text{Ipg}(-3))$	0.1204*
$\Delta \log(\text{ICIG}(-1))$	0.0653*	$\Delta \log(\text{ICIG}(-1))$	0.1365**	$\Delta \log(\text{ICIG}(-1))$	0.0322
$\Delta \log(\text{ICIG}(-2))$	0.0921**	$\Delta \log(\text{ICIG}(-2))$	0.01144*	$\Delta \log(\text{ICIG}(-2))$	0.0339
$\Delta \log(\text{ICII}(-1))$	-0.0019	$\Delta \log(\text{ICII}(-1))$	-0.0511	$\Delta \log(\text{ICII}(-1))$	0.0497*
$\Delta \log(\text{ICII}(-3))$	-0.0065	$\Delta \log(\text{ICII}(-3))$	-0.0556	$\Delta \log(\text{ICII}(-3))$	0.0517*
XRF(-3)	0.0003*	XRF(-3)	0.0002	XRF(-3)	0.0001
XRI(-2)	0.0000	XRI(-2)	-0.0001	XRI(-2)	0.0002*
$\Delta(\text{LTRIRI}(-6))$	-0.0014	$\Delta(\text{LTRIRI}(-6))$	-0.0021	$\Delta(\text{LTRIRI}(-6))$	-0.0034**
$\Delta(\text{SpreadI}(-3))$	-0.0005	$\Delta(\text{SpreadI}(-3))$	0.0005	$\Delta(\text{SpreadI}(-3))$	0.0033*
DUMGERUN	0.0030	DUMGERUN	0.0092**	DUMGERUN	0.0004
DUMEMU	0.0015	DUMEMU	0.0049**	DUMEMU	-0.0039**
DUMIT96	0.0037	DUMIT96	0.0045	DUMIT96	-0.0052*
Degrees Of Freedom	4.79**	Degrees Of Freedom	5.42*	Degrees Of Freedom	9.16
LB(12) Std. Res.	0.29	LB(12) Std. Res.	0.57	LB(12) Std. Res.	0.71
LB(12) Std.Res. Sq.	0.99	LB(12) Std. Res.Sq.	0.97	LB(12) Std. Res.Sq.	0.19

(*) Significant at the 5% level; (**) Significant at the 1% level.

Table 3: Copula-VAR model with Student's t marginals and lagged exogenous variables.

1	0.1484**	0.2859**
0.1484**	1	0.1420**
0.2859**	0.1420**	1

(**) Significant at the 1% level.

Table 4: Normal copula - Correlation matrix.

of industrial activity. The spread between the long and the short term interest rate also appears to be a good leading indicator of the pace of economic activity in the case of the Italian economy: the sign is positive, as expected, indicating that an opening spread supplies information about a likely upsurge in business cycle trends. The long term real interest rate, with a lag of six months, appears to have a significant negative effect on industrial output, as expected: in the case of Italy, this effect may be a consequence of the high level of firms' external indebtedness, mainly used to finance their investment plans. Finally, in the Italian case, two dummies appear significant in explaining the dynamics of industrial production: a first one, in an opposite way with respect to Germany, indicates the negative effects of the creation of EMU on exports and production (mainly through the loss of competitiveness); a second one, instead, is meant to capture the countercyclical negative effects on industrial activity of the appreciation of the Lira in 1996, subsequent to the strong depreciation of 1995 and targeted to the goal of re-entering the EMS in November 1996 and joining EMU two years later.

Besides, we note that for all three countries the current growth rate of industrial activity

depends negatively upon its own lagged rates and positively upon foreign lagged IP growth rates: such evidence is probably attributable to the beneficial effects of the deepening process of European economic integration and interdependence that has taken place after 1990.

3.1 Specification Tests of Marginals and Copula Models

The Ljung-Box tests on the standardized residuals in levels and squares reported in Table 3 highlight no misspecification in the conditional mean and variance.

We tested the goodness-of-fit of the models employed for the conditional marginal distributions by using the specification tests discussed in Granger et al. (2006): we used the Kolmogorov-Smirnov test for density specification, together with the “Hit” test in order to test jointly for the adequacy of the dynamics and the density specifications in the marginal distribution models. The latter test divides the support of the density into regions, R_i , and then applies interval forecast evaluation techniques to each region separately, and then to all regions jointly. Following Granger et al. (2006), we break the support of the three probability integral transforms $t_{1,t}(\eta_{1,t}; \nu_1)$, $t_{2,t}(\eta_{2,t}; \nu_2)$, and $t_{3,t}(\eta_{3,t}; \nu_3)$ into 5 regions: $[0,0.1]$, $(0.1,0.25]$, $(0.25,0.75]$, $(0.75,0.9]$ and $(0.9,1]$. We then construct “hit” variables for each region and marginal series, as $Hit_{i,t}^{t_1} = \mathbf{1}\{t_{1,t}(\eta_{1,t}) \in R_i\}$, $Hit_{i,t}^{t_2} = \mathbf{1}\{t_{2,t}(\eta_{2,t}) \in R_i\}$ and $Hit_{i,t}^{t_3} = \mathbf{1}\{t_{3,t}(\eta_{3,t}) \in R_i\}$, which take the value 1 if the realized value is in the region, and 0 otherwise. Under the null of correct specification, each of these Hit variables should be iid Bernoulli($U - L$), where L and U are the lower and upper boundaries of the region. To test individual regions we estimate a logistic regression of the hit variables on a constant and variables that should, if the model is well specified, have no influence on the hit variable. We used the first three lags of the three hit variables for the same region (i.e., $Hit_{i,t-i}^{t_1}$, $Hit_{i,t-i}^{t_2}$, and $Hit_{i,t-i}^{t_3}$, with $i = 1, 2, 3$) to capture serial correlation⁹. Under the null hypothesis that the density models are well specified the test statistic is a χ_{10}^2 random variable. To test all regions jointly we estimated a multinomial logit model, with the same specifications for each region as for the individual tests. The test statistic for the joint test is a χ_{40}^2 under the null hypothesis. The p -values for each test statistic are presented in Table 5.

All the three marginals passed the joint tests and all individual region tests at the 0.05 level, thus highlighting that the three marginals are correctly specified. Testing for marginal distribution model misspecification is a critical step in constructing multivariate distribution models using copulas, since if we use a misspecified model for the marginal distributions, then the probability integral transforms will not be Uniform(0, 1), and so any copula model will automatically be misspecified.

We finally performed a goodness-of-fit test of our parametric Normal copula model by

⁹We also tried adding other lagged variables as well as exogenous variables. None of these changes affected the final conclusion.

Region	$t_{1,t}(\eta_{1,t}; \nu_1)$	$t_{2,t}(\eta_{2,t}; \nu_2)$	$t_{3,t}(\eta_{3,t}; \nu_3)$
[0,0.1]	0.32	0.10	0.25
(0.1,0.25]	0.93	0.98	0.85
(0.25,0.75]	0.84	0.13	0.89
(0.75,0.9]	0.08	0.45	0.89
(0.9,1]	0.16	0.89	0.05
Joint Test	0.37	0.35	0.93
KS Test	0.99	0.72	0.84

Table 5: Tests of the marginal distribution models (p -values)

using the second test proposed in Chen et al. (2004)¹⁰, which is based on the multivariate probability integral transform and kernel density estimation of univariate density functions. The normal copula is not rejected with a p -value of 0.66. This result confirmed an (unreported) estimation of the t -copula whose degrees of freedom were equal to 32, which is no more distinguishable from a normal copula.

4 Forecasting Exercises

In this section we perform some forecasting exercises about the monthly growth rates of IP series in order to assess the goodness of our approach. The competing models are the following ones:

1. A copula-VAR model with constant normal copula and Student's t marginals;
2. A standard VAR model estimated with OLS;
3. A VAR model with a multivariate Student's t distribution;
4. A simple ARMA(1,1) model.

We remark that the multivariate Student's t distribution is a particular restriction on equation (2.2) that imposes a t -copula and Student's t marginals with the same degrees of freedom: imposing such a restriction when it is not the case results in biased coefficients. We use the observations ranging between 1990:01 and 2002:12 as the first initialization sample, while those from 2003:01 till 2005:12 are used to perform 1-step ahead and 3-steps ahead forecasts. A summary of the forecasting performances is reported in Table 6 -7.

The two tables show that the copula-VAR model yields better forecasting statistics than the competing models for 1-step ahead and 3-step ahead forecasts with regards to France and Germany, while this is not the case for Italy: this result can be explained if we consider that the former two countries present the most leptokurtic distributions, as shown in table 3. Besides, while the OLS estimator is a QML estimator and provides consistent estimates,

¹⁰The first test can be used only with bivariate copulas.

<i>Model</i>	RMSE			MAE		
	France	Germany	Italy	France	Germany	Italy
COPULA-VAR	0.936	0.992	0.732	0.837	0.928	0.759
NORMAL-VAR	1.010	1.027	0.733	0.862	0.938	0.753
STUDENT'S T VAR	1.046	1.013	0.789	0.881	0.905	0.802
ARMA	1.056	1.009	0.719	0.908	0.913	0.755

Table 6: One month ahead forecast

<i>Model</i>	RMSE			MAE		
	France	Germany	Italy	France	Germany	Italy
COPULA-VAR	1.047	1.121	1.348	0.862	0.935	1.099
NORMAL-VAR	1.132	1.179	1.366	0.824	0.945	1.145
STUDENT'S T VAR	1.161	1.246	1.380	0.840	1.000	1.143
ARMA	1.208	1.246	1.151	0.950	0.995	0.936

Table 7: Three months ahead forecast

its behavior in small samples like ours can be rather poor. Furthermore, there are cases where QML estimators are biased, too (see Newey and Steigerwald, 1997). Instead, it is interesting to notice the poor performance of the VAR model estimated with a multivariate t distribution, thus highlighting that such a restriction is not adequate.

In order to compare the predictive accuracy of our models, we perform the Hansen and Lunde's (2005) and Hansen's (2005) Superior Predictive Ability (SPA) test, which compares the performances of two or more forecasting models. The forecasts are evaluated using a loss function like the MAE and the RMSE. The best forecasting model is the model that produces the smallest expected loss. The SPA test compares for the best standardized forecasting performance relative to a benchmark model, and the null hypothesis is that none of the competing models is better than the benchmark.

Let $L(Y_t; \hat{Y}_t)$ denote the loss if one had made the prediction, \hat{Y}_t , when the realized value turned out to be Y_t . The performance of model k relative to the benchmark model (at time t), can be defined as:

$$X_k(t) = L(Y_t, \hat{Y}_{0t}) - L(Y_t, \hat{Y}_{kt}), \quad k = 1, \dots, l; \quad t = 1, \dots, n.$$

The question of interest is whether any of the models $k = 1, \dots, l$ is better than the benchmark model. To analyze this question Hansen (2005) formulates the testable hypothesis that the benchmark model is the best forecasting model. This hypothesis can be expressed parametrically as

$$\mu_k = E[X_k(t)] \leq 0, \quad k = 1, \dots, l.$$

For notational convenience, Hansen (2005) defines an l -dimensional vector μ by

$$\mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_l \end{pmatrix} = E \begin{pmatrix} X_1(t) \\ \vdots \\ X_l(t) \end{pmatrix}$$

Since a positive value of μ_k corresponds to model k being better than the benchmark, Hansen (2005) wants to test the hypothesis $H_0 : \mu_k \leq 0$ for $k = 1, \dots, l$. Therefore, the equivalent vector formulation is

$$H_0 : \mu \leq \mathbf{0}$$

One way to test this hypothesis is to consider the test statistic

$$T_n^{sm} = \max_k \frac{n^{1/2} \bar{X}_k}{\hat{\sigma}_k}$$

where

$$\bar{X}_k = \frac{1}{n} \sum_{t=1}^n X_k(t), \quad \hat{\sigma}_k^2 = \widehat{\text{var}}(n^{1/2} \bar{X}_k).$$

The latter is estimated by using the bootstrap method. The superscript “ sm ” refers to standardized maximum. Under the regularity condition, Hansen (2005) shows that

$$T_n^{sm} = \max_k \frac{\bar{X}_k}{\hat{\sigma}_k} \xrightarrow{p} \max_k \frac{\mu_k}{\sigma_k}$$

which is greater than zero if and only if $\mu_k > 0$ for some k . So one can test H_0 using the test statistic T_n^{sm} . The only remaining problem is to derive the distribution of T_n^{sm} under the assumption of a true null hypothesis. Testing multiple inequalities is more complicated than testing equalities (or a single inequality) because the distribution is not unique under the null hypothesis. Nevertheless, a consistent estimate of the p-value can be obtained by using a bootstrap procedure, as well as an upper and a lower bound¹¹.

The results produced by Hansen’s SPA test with the RMSE as the loss function are presented in Tables 8-9.

<i>Benchmark</i>	FRANCE			GERMANY			ITALY		
	<i>Lower</i>	<i>Cons.</i>	<i>Upper</i>	<i>Lower</i>	<i>Cons.</i>	<i>Upper</i>	<i>Lower</i>	<i>Cons.</i>	<i>Upper</i>
COPULA-VAR	0.5822	0.5822	0.9883	0.8159	0.9201	0.9201	0.5587	0.5815	0.6804
NORMAL-VAR	0.0811	0.0811	0.1625	0.5449	0.5449	0.5449	0.7108	0.7108	0.8023
STUDENT’S T VAR	0.0719	0.083	0.083	0.5909	0.6361	0.6361	0.0629	0.0629	0.0629
ARMA	0.0283	0.0283	0.0283	0.4905	0.5026	0.5026	0.5312	0.6079	0.6782

Table 8: Hansen Tests between all models for $\Delta \log(IP)$ 1-step ahead forecasts.

<i>Benchmark</i>	FRANCE			GERMANY			ITALY		
	<i>Lower</i>	<i>Cons.</i>	<i>Upper</i>	<i>Lower</i>	<i>Cons.</i>	<i>Upper</i>	<i>Lower</i>	<i>Cons.</i>	<i>Upper</i>
COPULA-VAR	0.5328	0.5328	0.9061	0.8401	0.8401	0.9916	0.1282	0.1282	0.2135
NORMAL-VAR	0.2429	0.2429	0.3968	0.2394	0.2842	0.3757	0.1434	0.1672	0.1672
STUDENT’S T VAR	0.2713	0.3164	0.3164	0.1289	0.1294	0.1294	0.0839	0.0839	0.0839
ARMA	0.0173	0.0173	0.0173	0.2816	0.2816	0.2816	0.5328	0.5328	0.9159

Table 9: Hansen Tests between all models for $\Delta \log(IP)$ 3-steps ahead forecasts.

¹¹The authors would like to thank Peter Hansen for supplying the Ox code that calculates the SPA test statistics and associated p-values.

The reported Hansen's SPA-consistent p-values show whether there is evidence against the hypothesis that the benchmark model is the best forecasting one. A low p-value (less than 0.10) means that the benchmark model is inferior to one or more of the competing models: the empirical results seem to highlight that the sample being analyzed does not yield evidence that any of the benchmarks can be outperformed, except for the case of France with 1-step ahead forecasts where only the copula model cannot be outperformed, and the multivariate t distribution with France and Italy. This confirms our previous discussion with forecasting performances reported in tables 6 -7.

Therefore, we can conclude that, overall, our forecasting exercises highlight the fact that a copula-VAR approach is a valuable tool when we deal with small samples and some (if not all) of the considered variables are strongly leptokurtic. Besides, differently from the multivariate t distribution, we remark that the copula approach keeps the computational tractability of a standard normal-VAR estimated with OLS, thanks to the 2-steps procedure described in section 2.1 which allows for the separate estimation of the parameters of the marginal distributions from the parameters of the copula.

5 Conclusions

This paper proposed a copula-VAR approach for the joint modelling of industrial production in the main EMU countries with a multivariate distribution different from the normal one. The empirical analysis examined French, German and Italian Industrial Production data. We found strong evidence that confidence variables are important leading indicators of industrial output for all three economies, while the long term real interest rate and the spread between long and short term interest rates were found to be marginally significant for Italy, only. Besides, the analysis highlighted the negative effects of the creation of EMU on Italian exports and production, and the opposite positive effects on German industrial activity, instead.

The forecasting exercises showed that a copula-VAR approach yields better forecasts when the considered marginals are strongly leptokurtic and we deal with small samples. Besides, such an approach allows for computational tractability thanks to 2-steps estimation procedures. While the VAR model estimated with OLS is a QML estimator and provides consistent estimates, its behavior in small samples like the one considered in this work can be rather poor. Furthermore, there are cases where QML estimators are biased, too (see Newey and Steigerwald, 1997). Instead, imposing the multivariate t distribution when only the marginals are leptokurtic while the dependence structure is normal, results in biased coefficients and poor forecasting performances.

Our approach may be applied to a wide variety of situations that go beyond the specific empirical framework examined in this work. An avenue of future research would be to consider much longer datasets, starting from the end of World War II or even earlier. The

use of dynamic copulae, not considered here, could be of help in explaining the time-varying interdependencies between European and world economies, in general. A second avenue of research could be an analysis conducted at the micro level in order to better understand the opposite impact of the introduction of EMU on Italian and German industrial activity.

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