On Skill Heterogeneity, Human Capital, and Inflation

Radhika Lahiri
Queensland University of Technology

and

Elisabetta Magnani
University of New South Wales

December 2005

Abstract
This paper examines the welfare costs of inflation within a monetary dynamic general equilibrium framework with human capital that incorporates endogenous, ex ante skill heterogeneity among workers. Numerical experiments indicate that, overall, welfare costs are more likely to decrease with increases in skill heterogeneity. An implication of this feature is that a greater degree of skill heterogeneity may be associated with a higher tolerance for inflation, consequently implying a positive correlation between agent heterogeneity and inflation. Using a panel of several countries we empirically test this proposition. Our evidence lends some support to this hypothesis.

Introduction

The impact of agents' heterogeneity on the macroeconomic performance of an economy is central to a large and growing body of literature, and remains an open area of research. However, both in the inequality-growth literature and in the inequality-inflation literature, researchers have often focused on one aspects of agents' heterogeneity, namely *income inequality*. For example, studies based on a political economy perspective of the inflation-income heterogeneity link provide a theoretical rationale for a positive as well as a negative correlation between the two aggregates. By developing a theory of the determination of inflation outcomes in democratic societies, Dolmas, Huffman and Wynne (2000), document a positive correlation between income inequality and inflation. Bhattacharya, Bunzel and Haslag (2003), on the other hand, present a theory and empirical evidence that supports a non-monotonic relationship between inequality and inflation. Albanesi (2000) analyzes a model in which the poor are more vulnerable to inflation, and are the weaker party in the political process that determines inflation, leading to a positive inequality-inflation correlation.

In empirical studies on the effect of inequality of aggregate outcomes, the focus on one single dimension of agents' heterogeneity, income inequality, has been motivated by the lack of suitable data. Because human capital inequality has been strongly emphasized as being an important determinant of income inequality, this has led to the common use of income inequality as a good proxy for human capital inequality. See for example, Glomm and Ravikumar (1992), Saint-Paul and Verdier (1993), Galor and Tsiddon (1997), among others. However, recent evidence suggests that income and human capital distribution statistics can show very low correlations (Castello and Domenech, 2002). In addition, Castello and Domenech (2002) also find that human capital inequality measures provide more robust results than income equality measures in the estimation of standard growth and investment equations.

This motivates our interest in an investigation of the relationship between human capital heterogeneity and inflation. This paper considers the effect of the degree of skill-heterogeneity among workers on the welfare costs of inflation. We address this issue in the framework of an equilibrium model with ex-ante heterogeneity of the type studied in Kydland (1984, 1995) and Prasad (1996), with money introduced via a cash-in-advance constraint on the purchases on consumption. However, in our model, skill-heterogeneity is endogenous, and depends on the extent of human capital accumulation undertaken by the representative household. Specifically, the model of this paper suggests that the welfare costs of inflation are likely to *decrease* as skill heterogeneity increases.

The intuition underlying this interesting result can be briefly described as follows. In our model, and important parameter inversely representing heterogeneity is the elasticity of substitution between skilled and unskilled effort. The impact of inflation in cash-in-advance models involves substitution out of activities subject to the inflation tax, such as consumption and work effort, towards leisure. Higher substitutability/low heterogeneity means that substitution out of skilled effort is possible to a greater degree than in the case of low substitutability/high heterogeneity. Combined with the fact that the representative household values the leisure of the skilled worker more than that of the unskilled worker, inflation would entail a greater degree of substitution away from skilled effort. As a result the other variables would also be adversely affected to a greater degree than in the case of high skill heterogeneity. Welfare costs of inflation therefore tend to decrease with increases in heterogeneity.

To empirically test some of the implications of our model, we use a new panel data set of human capital inequality measures, created by Castello and Domenech (2002), to examine the inflation experiences of a large panel of countries between 1960 and 1989. We find that the implications of our model are broadly consistent with the empirical evidence on inflation and human capital heterogeneity.

The remainder of the paper is organized as follows. In Section 2 we describe the economic environment, and in Section 3 we briefly analyze the steady state. The model is parameterized in Section 4, in which we also find welfare costs of inflation associated with different levels of skill heterogeneity. The key implication of the quantitative experiments conducted in Section 4 is that welfare costs of inflation are inversely related to the degree of heterogeneity. In Section 5 we test this implication using a panel of a number of countries. If we focus on the experience of industrialized economies, which, as mentioned above, we believe our model to be more representative of, the data finds supports a positive inflation-heterogeneity correlation. However, this is not true of less developed economies, in which the inflation heterogeneity correlation is found to be negative. Section 6 concludes. A description of the numerical procedure used to solve for the steady state is presented in the Appendix, which also includes the proofs of some of the analytical results of Section 3.

1. The Economic Environment

The economy is populated with a continuum of identical, infinitely lived households that are uniformly distributed along the unit interval [0, 1]. As in Kydland (1984, 1995), we assume that each household consists of two types of workers, skilled (type 1) and unskilled (type 2). However, we make the further assumption that the productivity of the type 1 worker is endogenously determined by the household's skill accumulation, as we will describe later. Household preferences are given by

$$E\sum_{t=0}^{\infty}\beta^t u(c_t, 1-n_{1t}, 1-n_{2t}) \tag{1}$$

where c_t represents household consumption in time *t*, and n_{it} represents labor effort at time *t* of type *i* agent, *i*=1, 2. The functional form used for the momentary utility function is of the "indivisible labor" form as in Hansen (1985) and is given by

$$\log c_t - \psi\, a\, n_{1t} - (1-\psi)\, a\, n_{2t} \tag{2}$$

where ψ and $1-\psi$ are the underlying weights assigned to the leisure of skilled and unskilled workers respectively.[1]

Households enter period *t* with nominal money balances m_{t-1}, carried over from the previous period. The government augments these money balances by a lump-sum transfer equal to the increase in money supply, where the aggregate money supply M_t is determined according the following rule:

$$M_t = g_t M_{t-1}. \tag{3}$$

Thus the total amount of money balances held by a household at the beginning of period *t* is the amount

[1]Specifically skilled and unskilled workers can work some given positive number $h < 1$ or not at all, implying household consumption sets are non convex. However, as in Hansen (1985) and Rogersen (1988), the household consumption set is made convex by allowing agents to trade employment lotteries. As in Prasad (1996), this economy has two independent employment lotteries, one for skilled workers and another for unskilled workers. The expected utility of each household is then defined over total household consumption and the probability of employment of each type of worker.

$$m_{t-1} + (g_t - 1)M_{t-1}. \tag{4}$$

To ensure that money is valued in equilibrium, we assume the presence of cash in advance constraint on the purchase of the non-storable consumption good. Expenditure on the consumption good therefore cannot exceed the total money balances available to the household, i.e.

$$p_t c_t \le m_{t-1} + (g_t - 1)M_{t-1}. \tag{5}$$

The growth rate of money, g_t, evolves according to:

$$\log(g_{t+1}) = \gamma \log(g_t) + \xi_{t+1}. \tag{6}$$

ξ_{t+1} is *i.i.d* normal with mean $(1-\gamma)\log(\overline{g})$ and variance σ_ξ^2 and $\log(\overline{g})$ represents the unconditional mean of $\log(g_t)$.

In every period *t*, household expenditures consist of consumption (c_t), investment in physical capital (x_t), investment human capital (s_t), and the amount of money balances $\left(\frac{m_t}{p_t}\right)$ that are to be carried over to the next period. These expenditures must not exceed total household income, which the sum of income is earned from skilled and unskilled labor, capital, money balances carried over from the previous period, and the lump-sum monetary transfer from the government. Households therefore maximize expected lifetime utility subject to (5) and a sequence of budget constraints of the following form:

$$c_t + x_t + s_t + \frac{m_t}{p_t} \le w_{1t} n_{1t} + w_{2t} n_{2t} + r_t k_t + \frac{m_{t-1} + (g_t - 1)M_{t-1}}{p_t}, \tag{7}$$

where household investment expenditure for physical and human capital in period-*t* is respectively given by

$$x_t = k_{t+1} - (1-\delta_k)k_t; \tag{8}$$

$$s_t = h_{t+1} - (1-\delta_h)h_t. \tag{9}$$

In equation (8) k_t and h_t respectively denote the household's physical and human capital stock in period-*t*; δ_k and δ_h are the corresponding rates of depreciation.

The representative firm in this economy takes the average skill accumulation by the households as given, and hires labor N_t and physical capital K_t to produce

$$Y_t = e^{z_t} K_t^{\theta} N_t^{1-\theta} \tag{10}$$

where z_t is an exogenous productivity shock that follows an AR (1) process of the form:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \qquad 0 < \rho < 1. \tag{11}$$

Here ε_{t+1} is *i.i.d* with zero mean and constant variance σ_z^2.

In (10), the aggregate time-t labor input N_t is a CES function of skilled and unskilled labor, given by

$$N_t = \left\{\omega(h_t) N_{1t}^{1-\nu} + N_{2t}^{1-\nu}\right\}^{\frac{1}{1-\nu}}. \tag{12}$$

The elasticity of substitution between the two types of labor is given by $\frac{1}{\nu}$. The function $\omega(h_t)$ captures the productivity of skilled labor, which is assumed to be increasing and concave, with $\omega(0) = 1$. Note that the degree of heterogeneity in this model is reflected in two parameters: a parameter that impacts directly on the skill differential between the two types of labor effort considered $\omega(h_t)$, and a parameter that describes the elasticity of substitution between them, namely $\frac{1}{\nu}$.

In addition, we make the assumption that the economy wide average stock of human capital equals the stock of human capital accumulated by the household, i.e., $h_t = H_t$.[2] The representative household therefore indirectly influences the relative wage rates of skilled and un-skilled labor and the rental rate of capital through its choice of human capital accumulation.

Taking that choice as given, the firm maximizes profits, which are equal to $Y_t - w_{1t}N_{1t} - w_{2t}N_{2t} - r_t K_t$. The optimality conditions for the firm's problem yield the following functions for the skilled and unskilled wage rates, and the rental rate for capital: [3]

$$w_{1t}(z_t, K_t, N_{1t}, N_{2t}) = 1-\theta)e^{z_t} K_t^{\theta} N_t^{\nu-\theta} \omega(h_t) N_{1t}^{-\nu} \tag{13}$$

[2]With the exception of skill accumulation, capital letters denote aggregate economy wide per capita variables which an individual household regards as being outside its sphere of influence, while lower case letters denote variables specific to the household.

$$w_{2t}(z_t, K_t, N_{1t}, N_{2t}) = (1-\theta)e^{z_t} K_t^{\theta} N_t^{\nu-\theta} N_{2t}^{-\nu} \tag{14}$$

$$r_t(z_t, K_t, N_{1t}, N_{2t}) = \theta e^{z_t}\left(\frac{K_t}{N_t}\right)^{\theta-1}. \tag{15}$$

For a value of g greater than one, both M_t and p_t will grow without bound. In order to make the household's problem stationary, some of the variables need to be transformed. To that end, we define $\hat{m}_t = \frac{m_t}{M_t}$ and $\hat{p}_t = \frac{p_t}{M_t}$. We can then state the household's problem as follows:

$$\max_{c_t, n_{1t}, n_{2t}, \hat{m}_t, k_{t+1}, h_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{\log(c_t) - \psi a\, n_{1t} - (1-\psi) a\, n_{2t}\right\} \tag{16}$$

subject to

$$\frac{\hat{m}_t}{\hat{p}_t} + c_t + k_{t+1} - (1-\delta_k)k_t + h_{t+1} - (1-\delta_h)h_t = w_{1t}n_{1t} + w_{2t}n_{2t} + r_t k_t + \frac{\hat{m}_{t-1} + g_t - 1}{\hat{p}_t g_t}, \tag{17}$$

$$c_t = \frac{\hat{m}_{t-1} + g_t - 1}{\hat{p}_t g_t} \tag{18}$$

the process for technology and monetary shocks, the aggregate capital accumulation rule, given by,

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \tag{19}$$

$$\log(g_{t+1}) = \alpha \log(g_t) + \xi_{t+1}, \tag{20}$$

$$K_{t+1} = (1-\delta)K_t + X_t, \tag{21}$$

as well as the economy-wide aggregate decision rules perceived by the households:

$$N_t = N(z_t, g_t, K_t),$$

$$X_t = X(z_t, g_t, K_t),$$

and,

$$\hat{P}_t = \hat{P}_t(z_t, g_t, K_t). \tag{22}$$

[3]Since the aggregate production technology is of the Cobb-Douglas form, profits will be zero in equilibrium, even though aggregate labor effort is a CES function of skilled and unskilled labor. This is easily verified by substituting the optimal wage and rental rates in the profit function.

In equilibrium, aggregate per capita quantities turn out to be equal to the choices of the representative household. In particular, it must be the case that $n_t = N_t$, $k_t = K_t$, $x_t = X_t$, and $\hat{m}_{t-1} = \hat{m}_t = 1$. Since the cash in advance constraint is assumed to be binding in equilibrium, we also have $c_t = \frac{1}{\hat{P}_t}$.

3. The Steady State

In this section we show that, since money is introduced in our model via a cash-in-advance constraint, inflation has a negative has impact on the long-run outcomes of several variables, as is typically expected of such models. Furthermore, consistent with some of the theoretical literature on the link between inflation and human capital, we find that inflation has a negative impact on human capital accumulation.[4] The degree of skill heterogeneity in our model has a further impact on the magnitude of distortions associated with inflation, as suggested by some of the analytical results of this section. The subsequent section, based on numerical experiments in fact indicates that it tends to weaken them. Consequently, the welfare costs of inflation tend to decrease with an increase in heterogeneity.

In the non-stochastic steady state, the first order conditions with respect to $c_t, n_{1t}, n_{2t}, \hat{m}_t, k_{t+1}, h_{t+1}$, and equilibrium conditions for this economy imply:

$$\frac{1}{C} = \lambda_1 + \lambda_2 \tag{23}$$

$$\psi a = (1-\theta)\lambda_1 K^{\theta} \left\{ \omega(H) N_1^{1-\nu} + N_2^{1-\nu} \right\}^{\frac{\nu-\theta}{1-\nu}} \omega(H) N_1^{-\nu} \tag{24}$$

$$(1-\psi)\, a = (1-\theta)\lambda_1 K^{\theta} \left\{ \omega(H) N_1^{1-\nu} + N_2^{1-\nu} \right\}^{\frac{\nu-\theta}{1-\nu}} N_2^{-\nu} \tag{25}$$

$$\lambda_1 = \frac{\beta(\lambda_1 + \lambda_2)}{g} \tag{26}$$

$$\theta \left(\frac{K}{N} \right)^{\theta-1} = \frac{1}{\beta} - 1 + \delta_k \tag{27}$$

---

[4]For a survey, see Gillman and Kejak (2005).

$$\frac{1-\theta}{1-\nu} K^{\theta} N^{\nu-\theta} \omega'(H) N_1^{1-\nu} = \frac{1}{\beta} - 1 + \delta_h \tag{28}$$

$$C + \delta_k K + \delta_h H = K^{\theta} N^{1-\theta} \tag{29}$$

Here λ_1 and λ_2 are the Lagrangian multipliers associated with the household budget and cash-in-advance constraints respectively. From equation (27), which is the equilibrium version of the first order condition for capital, it is clear that the capital to "aggregate labor" ratio is independent of inflation, and is given by:

$$\kappa = \frac{K}{N} = \left[\frac{1}{\theta} \left\{ \frac{1}{\beta} - 1 + \delta \right\} \right]^{\frac{1}{\theta - 1}}.$$

Of course, this is not the case for other variables, as a glance at the optimality conditions suggests. Manipulating (23)-(26) and (28) we can express other variables such as consumption and work-effort as functions of human capital:

$$C = \frac{(1-\theta)\beta\kappa^{\theta} (\phi(H))^{\nu} \omega(H)}{\psi\, a\, g}, \tag{30}$$

$$N_1 = \frac{\frac{1}{\beta} - 1 + \delta_h}{\frac{1-\theta}{1-\nu} \kappa^{\theta} (\phi(H))^{\nu} \omega'(H)}, \tag{31}$$

$$N_2 = \left(\frac{\psi}{(1-\psi)\omega(H)} \right)^{\frac{1}{\nu}} \frac{\frac{1}{\beta} - 1 + \delta_h}{\frac{1-\theta}{1-\nu} \kappa^{\theta} (\phi(H))^{\nu} \omega'(H)}, \tag{32}$$

$$N = \frac{\phi(H)\left(\frac{1}{\beta} - 1 + \delta_h \right)}{\frac{1-\theta}{1-\nu} \kappa^{\theta} (\phi(H))^{\nu} \omega'(H)}, \tag{33}$$

$$K = \kappa^{\theta} N = \frac{\phi(H)\left(\frac{1}{\beta} - 1 + \delta_h \right)}{\frac{1-\theta}{1-\nu} (\phi(H))^{\nu} \omega'(H)}. \tag{34}$$

In the above equations $\phi(H)=\left[\omega(H)+\left(\frac{\psi}{(1-\psi)\omega(H)}\right)^{\frac{1-\nu}{\nu}}\right]^{\frac{1}{1-\nu}}$. Also note that for an interior solution to work effort, we need to impose $\nu<1$. Making the necessary substitutions in (29) we can then derive an implicit equation in human capital, given by:

$$\frac{(1-\theta)\beta\kappa^{\theta}(\phi(H))^{\nu}\,\omega(H)}{\psi\,a\,g}+\delta_h H=\frac{(\kappa^{\theta}-\delta_k\kappa)\left(\frac{1}{\beta}-1+\delta_h\right)}{\frac{1-\theta}{1-\nu}\kappa^{\theta}(\phi(H))^{\nu}\,\omega'(H)}. \tag{35}$$

Total differentiation of the above equation with respect to H and g yields a fairly complicated expression for $\frac{dH}{dg}$:

$$\frac{dH}{dg}=\frac{\frac{\Delta_1\left(\phi(H)\right)^{\nu-\theta}}{g^2}}{\frac{\Delta_1}{g}(\nu-\theta)\left(\phi(H)\right)^{\nu-\theta-1}\phi'(H)-\frac{\Delta_2(1-\nu)\left(\phi(H)\right)^{-\nu}}{\omega'(H)}\phi'(H)+\frac{\Delta_2\left(\phi(H)\right)^{1-\nu}\omega''(H)}{(\omega'(H))^2}}. \tag{36}$$

In equation (36) $\Delta_1=\frac{\beta\kappa^{\theta}}{\psi a}$, $\Delta_2=\frac{\kappa^{\theta}-\delta_k\kappa}{\kappa^{\theta}}\left(\frac{1}{\beta}-1+\delta_h\right)\left(\frac{1-\nu}{1-\theta}\right)$, and the expressions $\phi'(H)$ and $\omega'(H)$ are partial derivatives of the respective functions with respect to *H*. How inflation impacts on consumption, work effort, physical and human capital is therefore difficult to discern analytically. Nevertheless, if we impose some restrictions on parameter values and functional forms, it is possible to derive a few weak analytical results. Let $\Omega_{\omega'}=-\frac{\omega''(H)}{\omega'(H)}H$ represent the elasticity of the marginal return to human capital to changes in human capital investment. Also, define $\Omega_{\phi}=\frac{\phi'(H)}{\phi(H)}H$. To interpret the latter expression, note that $\frac{N}{N_1}=\phi(H)$. Then $\phi(H)$ can be regarded as the average contribution of skilled effort to aggregate effort. Consequently, we can interpret

Ω_{ϕ} as the elasticity of this average contribution to changes in human capital investment. We can then summarize our analysis of equations (36) and (31)-(34) in the following propositions, the proof of which is presented in the appendix:

Proposition 1: If $\nu < \theta$ and $\phi'(H) > 0$,

(i) steady state human capital investment is decreasing in the rate of inflation, i.e. $\frac{dH}{dg} < 0$;

(ii) steady state consumption is decreasing in the rate of inflation, i.e., $\frac{dC}{dg} < 0$.

Proposition 2: If the conditions of Proposition 1 hold and $\Omega_{\omega'} > \nu\Omega_{\phi}$,

(i) steady state skilled effort is decreasing in inflation, i.e. $\frac{dN_1}{dg} < 0$;

(ii) the sign of $\frac{dN_2}{dg}$ is ambiguous;

(iii) steady state aggregate effort is decreasing in inflation, i.e. $\frac{dN}{dg} < 0$;

(iv) the steady state capital stock is decreasing in inflation, i.e. $\frac{dK}{dg} < 0$.

First let us consider the interpretation of the conditions $\nu < \theta$ and $\phi'(H) > 0$ in Proposition 1. The first of these places an upper limit on the inverse of the elasticity of substitution between the two types of labor. To interpret the second condition, again recall that $\frac{N}{N_1} = \phi(H)$, i.e. $\phi(H)$ is the average contribution of skilled effort to the "aggregate" work effort in this economy. Then the condition $\phi'(H) > 0$ requires that this average contribution responds positively to changes in human capital accumulation. In Proposition 2 the additional condition requires that the elasticity of the return to human capital investment be greater than the human capital elasticity of the average contribution of skilled effort to aggregate effort, multiplied by the factor ν, which is the inverse of the elasticity of substitution between the two types of labor. One can perhaps interpret the above Propositions as stating conditions under which inflation-tax distortions, as measured by the negative impact of inflation on human capital, skilled effort, and consumption, are important. However, one cannot conclusively say whether the associated welfare costs of inflation will be high or low; that would depend, additionally, on the *magnitude* of the response of variables to increases in the inflation rate. Nevertheless, it should be intuitively clear that experiments varying the heterogeneity parameters will impact on inflation tax distortions, (and consequently the welfare costs of inflation) in interesting ways. In the subsequent section on numerical experiments we will therefore use the conditions of Propositions 1 and 2 to assist in the interpretation of our results.

Note, however, that the conditions in the above proposition are only *sufficient* conditions for the response of human capital, consumption, and other variables to be negative. Numerical simulations conducted and discussed in more detail in the following section indicate that, as is typical in models with a cash-in-advance constraint on consumption purchases, consumption, and work effort, physical and human capital are negatively related to the growth rate of money even when some of the above mentioned conditions do not hold. Interestingly, higher inflation is also shown to be associated with a shift in the percentage composition of the "work force" between skilled and unskilled labor.

The intuition for the negative impact of inflation on economic aggregates is straightforward, and common to several cash-in-advance models in the literature. Inflation acts as a tax on consumption since it requires the use of cash. This leads economic agents to substitute consumption for activities that do not require the use of cash, such as leisure. The decline in work effort causes a decline in output, and consequently consumption, investment and the physical and human capital stock. However, it is also intuitively clear that the *magnitude* of the negative response to inflation in this economy is likely to be affected significantly by the parameters of the functions $\omega(H)$ and $\phi(H)$. Specifically, varying ν, ψ or α, which can be interpreted parameters affecting the extent of ex ante heterogeneity in this economy, has an impact on the conditions of Propositions 1 and 2, and consequently the magnitude of the distortions associated with inflation. It is then natural to expect that welfare costs computations relative to an optimal monetary policy may yield significantly different results as we allow some of these parameters to vary. In the next section, we therefore derive some conclusions regarding welfare costs based on numerical experiments using a plausibly parameterized version of the model.

4. Inflation and Skill Heterogeneity: Results Based on Quantitative Experiments

In this section, we explore the relationship between inflation and heterogeneity, by examining how long-run aggregate outcomes and welfare costs of inflation change as we vary the levels of the parameters that capture heterogeneity. First we specify $\omega(H) = 1 + H^{\alpha}$; $0 < \alpha < 1$, so that the parameters relevant to the degree of heterogeneity are $\alpha, \psi,$ and ν. The remaining parameters, viz $\beta, \theta, \delta_k, \delta_h, a$ and g are taken directly from relevant papers in the equilibrium business cycle literature, such as Hansen (1985) and Cooley and Hansen (1989), Lahiri (2002), and Canton (2002). The range of values for the parameter ν includes the value 0.4 chosen in the Prasad (1996). The values for α are chosen such that the productivity differential is around "2 or higher" as suggested in Kydland (1995). The parameter ψ, even though it can be interpreted as a parameter representing heterogeneity, is however fixed at 0.59, the value chosen in Prasad (1996). The reason for doing so will be discussed below, with reference to the measure of welfare cost considered in this paper. The other fixed parameters are given by the following: $\beta = .99$; $\theta = .36$; $\delta_k = .025$; $\delta_h = .00375$; $a = 2.86$

The numerical procedure used to calculate the steady state is described in the appendix. To compute welfare costs of inflation, we calculate the increase in consumption that an individual would require to be as well off under the equilibrium allocation associated with the optimal monetary policy. Specifically, we solve for x in the equation $\overline{U} = \log(c^*(1+x)) - \psi n_1^* - (1-\psi)n_2^*$, where c^*, n_1^*, n_2^* are levels of consumption and work effort associated with monetary policy that sets $g > 1$, while $\overline{U}$ is the utility attained under the optimal policy which sets $g = \beta$. We calculate this loss, expressed as a percentage of output and also of consumption, for varying levels of each of the heterogeneity parameters. Note that since ψ is a preference parameter, it obviously affects the *measure* of welfare costs itself. An experiment that considers the effects of varying ψ on welfare costs of inflation is therefore inappropriate.

Table 1 below presents the steady state values of variables and associated welfare costs of inflation as the monetary growth rate increases, with the heterogeneity parameters fixed at $\alpha = 0.1; \nu = 0.4; \psi = 0.59$. Figures 1(a) and 1(b) present the steady state values of variables as ν increases. The 'x' line represents the policy with inflation ($g = 1.15$) and the dotted line represents the optimal policy ($g = \beta$). Figure 1(c) presents welfare costs of inflation as ν increases. Figures 2(a) and 2(b) present the percentage difference in the steady state values of variables in the presence of inflation, relative to their steady state levels when $g = \beta$, for different values of ν. Figure 2(c) presents the elasticity $\Omega_{\omega'}$ and the weighted elasticity $\nu\Omega_{\phi}$ for different values of ν, where the '*' line represents the latter. Figures 3(a)-(c) and Figures 4(a)-(c) present similar experiments with the parameter α.

First, we examine the computations presented in Table 1. As mentioned above, the heterogeneity related parameters in this case are fixed at $\alpha = 0.1; \nu = 0.4; \psi = 0.59$, and the monetary growth rates are set at $g = \beta$, $g = 1.024$; $g = 1.05$; and $g = 1.15$. The usual features of cash-in-advance models are apparent: inflation impacts negatively on consumption, work effort, physical and human capital, and output. [5] The composition of work effort, however, seems to shift slightly in favor of unskilled labor as inflation increases, perhaps due to the relatively higher weight assigned to leisure of the skilled type. In a quantitative sense, the magnitudes of welfare costs are higher than would be observed in a model without endogenous skill heterogeneity or human capital, such as in Cooley and Hansen (1989).

Next we examine the effects of varying the heterogeneity parameters and how this variation impacts on the magnitude of distortions associated with inflation. First consider Figures 1(a) and (b). The response of variables to changes in ν appears similar regardless of the monetary policy in operation, and the magnitude of the negative impact of inflation does not look very striking. The magnitude of the inflation-tax distortions is, however, difficult to discern from these figures and we therefore defer the discussion of

[5] In a qualitative sense, these results hold for other combinations of the heterogeneity parameters as well.

these distortions until the analysis of Figures 1(c) and (d).[6] First, we attempt to gain some intuition for how changes in ν affect the long run values of economic aggregates in general, regardless of what the inflation rate is. Increasing ν, which is the inverse of the elasticity of substitution between different types of labor, amounts to increasing skill heterogeneity in this economy along two dimensions? One dimension is associated with the falling elasticity of substitution – heterogeneity increases in the sense that the two types of labor are substitutable to a lower degree in the production process. Secondly, it is clear from Figure 1(a) that the equilibrium composition of the work force becomes more heterogeneous as ν increases. For high substitutability the work force comprises almost entirely of the skilled type of labor, but as the elasticity of substitution increases, it becomes more heterogeneous. Also, lower substitutability encourages investment in human capital; to the extent that the more expensive type of effort is used, it would be more economical to employ it if its marginal return were higher – and human capital accumulation ensures this. This is also reflected in the increasing skill differential as ν increases. Higher levels of human capital increase the overall productivity of all inputs, and consequently, we see in Figure 1(b) that both the skilled and unskilled wage rates increase. Higher productivity also encourages physical capital accumulation. As a result, output and consumption also increase.

An interesting feature of Figure 1(a) is the inverted U-shaped response of skilled and unskilled work effort. This feature can probably be explained by the same rationale used to interpret backward-bending labor supply curves. Initially, as the wage rate increases, the substitution effect dominates the income effect and work effort increases. After a certain level the income effect takes over and the demand for leisure increases.

So far, we have not considered how inflation-tax distortions are affected as ν increases. From Figures 1(a) and (b) it is easy to discern the negative impact of inflation we discussed earlier. However, we cannot comment on the magnitude of these distortions until we discuss the percentage differences in the levels of variables relative to the case in which the optimal policy prevails. These differences are shown in Figures 2(a) and (b).

[6] The size of the differences is relatively small in comparison to the length of the scale of the vertical axis in all of the graphs. However, as will become clear from the analysis of percentage deviations relative to the optimal policy, presented in Figures 2(a) and (b), these differences can be quite significant.

However, before we discuss Figure 2, we consider the overall measure of inflation tax distortions, as represented by the welfare cost of inflation defined earlier. Figure 1(c) presents such welfare cost computations for different values of ν. It is clear that the welfare cost of inflation, relative to both consumption as well as output decreases as ν increases. In other words, welfare costs of inflation decrease as heterogeneity increases.

In order to interpret this result we turn to Figures 2(a), (b), and (c). Recall that $\nu < \theta$, $\phi'(H) > 0$, and $\Omega_{\omega'} > \nu\Omega_{\phi}$ were some of the conditions stated in Propositions 1 and 2. For the functional form of the productivity function considered in this paper it is easy to check that $\Omega_{\omega'} = -(\alpha - 1)$. Since we do not vary α in this experiment, the horizontal line in Figure 2(c) at 0.9 represents this elasticity. The upward sloping ' *' line represents $\nu\Omega_{\phi}$. Furthermore, it is easy to read the sign of $\phi'(H)$ from this figure as $\nu\Omega_{\phi} > 0 \;\; iff \;\; \phi'(H) > 0$. Obviously, the conditions $\nu < \theta$ and $\phi'(H) > 0$ do not appear to hold simultaneously for this experiment, while the condition $\Omega_{\omega'} > \nu\Omega_{\phi}$ does not hold for values of ν greater than 0.4. As mentioned above these were only sufficient conditions for the impact of inflation to be negative and numerical experiments confirmed that the impact of inflation on all of the variables in this model is negative, even when the said conditions do not hold? However, some of these conditions particularly $\nu < \theta$ appear to be of some importance in determining the magnitude of inflation tax distortions in this economy. For example, note that in Figure 2(a) and (b) the largest distortions, measured in terms of the percentage deviation from the optimal level of the variables in question, appear to be for values of ν much smaller than θ. As ν increases past this range, the decline in welfare costs is more gradual. The other conditions do not seem to be playing a significant role in the sense that they do not correlate strongly with the magnitude of distortions shown in the figures.

Why are inflation tax distortions high for smaller values of ν? The answer to this question may be related to the fact that the impact of inflation in cash-in-advance models involves substitution out of activities subject to the inflation tax, such as consumption and work effort, towards leisure. Higher substitutability in the lower range of values for ν means that substitution out of skilled effort is possible to a greater degree in this case. Combined with the fact that the representative household values the leisure of the skilled worker more than that of the unskilled worker (since $\psi = 0.59$), inflation would entail a greater degree of substitution out of skilled effort. As a result the other variables would also be adversely affected to a greater degree than in the case of low substitutability.

The other condition that may be of some relevance is $\Omega_{\omega'} > \nu \Omega_{\phi}$; one can perhaps assert that the distortions to the left of $\nu = 0.4$,where this condition holds, are larger than the distortions beyond this value. The left hand side of this inequality can be interpreted as a factor of importance in of the "supply side" of the skill accumulation decision; one would expect a larger *leftward* shift in supply of human capital in response to the inflation tax if the elasticity of its marginal return was large. [7] The "demand side" response, on the other hand, depends on the human capital elasticity of the average contribution of skilled effort to aggregate effort, appropriately weighted by the inverse of the elasticity of substitution. If Ω_{ϕ} is negative and large, the average contribution of skilled effort *increases* when human capital accumulation falls in response to the inflation tax. The demand for skilled effort may increase due to this increase in average productivity or alternatively, fall given that a smaller amount of skilled effort may be needed in the production process. When there is high substitutability, the extent of shifts in demand may not be too large and the corresponding weight assigned to this elasticity is low. In any case, our numerical experiments suggest that the supply side response of work effort dominates and the overall impact is negative. For larger values of ν, Ω_{ϕ} increases and becomes positive. This means that the average contribution of skilled effort decreases when human capital accumulation falls. Again, demand may fall or rise depending on the extent to which skilled effort is needed relative to the decline in its average productivity. The scenario in which inflation tax distortions weaken is when the demand shifts positively to counter the leftward shift in supply so that equilibrium skilled effort does not fall too much. Re-examining the percentage differences in skilled effort and unskilled effort relative to the optimal case in Figure 2(a), there is evidence to suggest that this interpretation may be plausible.

[7] Recall that the marginal return is assumed to be decreasing in human capital. As the inflation tax encourages a *reduction* in human capital accumulation the marginal return to it would increase as a result. If the elasticity of this marginal return is high, a greater extent of substitution out of skilled leisure is possible, thus enhancing the negative response of skilled effort to increases in inflation.

We now turn to the discussion of experiments that vary α. First we interpret the changes in the steady state variables in Figures 3(a) and (b) as α is increased from 0.05 to 0.35. Higher α represents a higher marginal return to human capital, and therefore a greater steady state level of human capital. As a result, the skill differential increases, as does the productivity of skilled and unskilled effort leading to increases in wage rates. Physical capital also increases as it is more productive when higher levels of human capital are used in the production process. The increase in output and consumption is another obvious implication of the increase in α. The income effect of wage increases appears to dominate the substitution effect in this case; both skilled and unskilled effort decline as α increases.

Looking at the welfare cost estimates in Figure 3(c), we find that welfare costs of inflation relative to consumption increase very gradually, while welfare costs relative to output increase and then decrease. Note that the magnitude of changes in this case is very small. To interpret these changes, we examine Figures 4(a) and 4(b). In this case ν is fixed at its calibrated value of 0.4, so the condition $\nu < \theta$ is not satisfied for this experiment. The size of the difference relative to the optimal policy, shown in Figures 4(a) and (b), seems to increase monotonically for most variables, except for work effort and human capital accumulation. From Figure 4(c), we see that $\Omega_{\omega'} > \nu \Omega_{\phi}$ holds for a small range of relatively low values of α, while $\phi'(H) > 0$ holds for values of α greater than or equal to 0.1. The work effort response appears consistent with the interpretation given earlier with reference to the experiment with variations in ν.

Now the welfare cost measure, which is by definition the consumption compensation you have to give the representative household to make it as well off in terms of utility as in the case when the optimal policy is in place, is directly affected by the variables that enter the utility function – consumption and leisure. In absolute levels, this compensation obviously has to increase as α increases. This is because, as is clear from Figures 4(a) and (b), the percentage difference in consumption levels relative to the optimal policy increases with α, while the percentage difference in leisure decreases with α. Expressed relative to consumption or output, however, welfare costs may increase or decrease, depending on how fast output or consumption increase as α increases. This is what seems to be happening in Figure 3(c); welfare costs relative to consumption increase very gradually, while welfare costs relative to output increase and then decrease. In fact, in simulations for higher values of α, not reported here, the welfare costs of inflation relative to consumption also start to decrease as α increases.

The intuition underlying these results is as follows. Both types of work effort are decreasing in inflation, but an increase in the productivity differential offsets the output loss associated with a given rate of inflation. In a relative sense, therefore, the welfare costs of inflation are nor likely to be high.

Overall, we may conclude that high levels of heterogeneity are likely to be associated with lower welfare costs of inflation. An implication of this result is that economies in which agents are characterized by a higher degree of heterogeneity experience lower costs of inflation, and as such are likely to experience higher inflation rates. In other words, skill heterogeneity could contribute toward explaining variations in the inflation experiences of different countries at any given point in time. The scope of the next section is to empirically estimate the correlation between inflation and skill heterogeneity.

5. Inflation and Skill Heterogeneity: An Empirical Analysis

In order to test whether agents' heterogeneity indeed affects the policy maker's decision over the optimal inflation level we compare the experiences of a number of countries over a period of time starting in 1960 and ending, in our most comprehensive case, in 2000. Our empirical strategy is to control for differences in institutional arrangements across countries so as to shed light on the correlation between human capital inequality and inflation. The data on inflation are drawn from The International Financial Statistics published by the International Monetary Fund. The sample comprises 108 countries, of which 33 are defined as developed economies (LDC = 0) and 44 are defined by Cukierman and Webb (1995) as democracies (dummy for authoritarian regime=0). However, the number of countries actually used in the estimation procedure is much smaller due to data availability constraints.

5.1 The Explanatory Variables.

The type of heterogeneity at work in the theoretical model is correlated with agents' productivity, and affects the agents' substitutability in the production process. A natural candidate for a measure of such heterogeneity is human capital inequality. Differences in human capital attainment indeed produce heterogeneity that affects productivity, and the substitutability between agents in addition to the value assigned to non-working activities. Data for agents' heterogeneity are provided by Castello and Domenech (2002) and refer to human capital inequality. Using the recent information contained in Barro and Lee's (2001) data set about educational attainments, Castello and Domenech calculate a human capital Gini coefficient $G = 1 - 2\int_0^1 A(y)dy$ where $A(y)$ is the Lorenz curve of the educational attainment distribution. The Lorenz curve plots the cumulative percentage of educational attainment (human capital) reached by the bottom y-percent of the population. The Gini coefficient is a measure of human capital inequality that ranges from zero to one: in the case of perfectly equal distribution the Lorenz curve would coincide with the 45-degree lines and the Gini coefficient would be zero. Castello and Domenech (2002) propose two Gini coefficients, namely G25, the Gini coefficient computed using the population aged 25 and plus, and G15, the Gini coefficient computed using the population aged 15 and plus. While for the most part we will use the former, we will use the latter to check the robustness of our results. Both measures of human capital inequality are available for all 108 countries in the data set at times of 5-year interval starting from 1960.

It is now well established that the conduct of monetary policy and specifically the rate of growth of the money stock is the primary factor determining a country's inflation rate. The actual policy pursued by the monetary authority depends on a number of factors some of which have an exquisitely political flavor. For instance there is now a large body of literature that relates central banks' decisions to their independence from, or vulnerability to, political pressure, which may work to deviate the central bank's attention from the pursuit of a price stability goal (e.g., Cukierman, Webb and Neyapti, 1992; Cukierman and Webb, 1995). The other variables we include in our data set reflect this type of argument. The measures of Central Bank independence (CBI), central bank vulnerability (vulnerability) and political instability (political change) come from the Cukierman and Webb (1995) data set. The CBI variable measures legal independence of central bank from political power. Cukierman et al., (1992) code central bank independence following two main principles. First of all, they code only a few narrow but relatively precise legal characteristics. Secondly, they only use the written information from central banks' charters. The legal characteristics as described in the charters define a few important issues, namely:

(i) the appointment, dismissal and term of office of the central bank's chief officer;

(ii) the policy formulation cluster and the resolution of possible conflicts over monetary policy between monetary and fiscal authorities;

(iii) the objectives of the central bank;

(iv) limitations on the ability of the central bank to lend to the public sector and regulation of the modalities with which such lending can take place.

The way the single components of central bank's legal independence are aggregated is fully described in Cukierman et al., (1992).

The Cukierman-Webb (1995) vulnerability variable takes its origin from raw data on the actual dates of changes of the governors of the central banks. To measure central bank vulnerability to political instability, Cukierman and Webb estimate the probability per month of a change in central bank governor conditional on being a time interval that follows a political transition. They show that although this probability decreases monotonically with the number of months that have elapsed since the last political transition, the estimated probability of a change in governor at the central bank is more than two times larger in periods within six months after a political transition than in periods that are more removed from political change. They then compute an index of the political vulnerability of the central bank (vulnerability), defined for each country in the Cukierman-Webb (1995) sample as the fraction of political transitions that are followed with a lag of 0 to 6 months by a replacement of the central bank governor. Cukierman and Webb (1995) illustrate that the highest level of central bank vulnerability occurs in the face of high level political transitions, which is then included among the explanatory variables.

The last variable we include is the degree of openness (openness) of an economy to the rest of the world. We measure this as the ratio of the sum of imports and exports over a country's GDP. The argument is that the degree of exposure to international trade may increase the ability of a central bank to pre-commit to a given (low) inflation target.

The Cukierman-Webb variables described above are available for 67 countries from 1950 to 1989 although data for economies that achieved political independence or established a central bank after the 1950 start later. The Cukierman-Webb data set includes all the major industrial and developing economies, but excludes most Easter European countries. Table 2 reports the summary statistics for the main variables.

5.2 The Empirical Specification.

We estimate a model of the form

$$\pi_{it} = \alpha + x_{it}\beta + \eta_i + \varepsilon_{it} \tag{37}$$

where π_{it} is the inflation measure in country i in time t, x_{it} is a set of explanatory variables specific to country i in time t and $\eta_i + \varepsilon_{it}$ is the residual. We are interested in estimating the βs. While the error component ε_{it} has the usual properties (mean zero, uncorrelated with itself, uncorrelated with the vector x), the characteristics of the error component η_i define the estimation strategy we will adopt. In particular, given the extreme heterogeneity of inflation experiences we observe in our sample, and the extreme differences of the institutional features of the countries considered, we opt for treating the country specific error component, η_i, as a fixed effect rather than a random variable. This amounts to estimating the following equation,

$$\pi_{it} - \overline{\pi}_i = \alpha + (x_{it} - \overline{x}_i)\beta + \varepsilon_{it} - \overline{\varepsilon}_i \tag{38}$$

where $\overline{\pi}_i = \sum_t \pi_{it} / T$, $\overline{x}_i = \sum_t x_{it} / T$, $\overline{\varepsilon}_i = \sum_t \varepsilon_{it} / T$. In the actual estimation the dependent variable has been transformed to reduce heteroskedasticity of the error and thus improve the efficiency of the estimate. Also, since a few countries had three-digit inflation rates in some years, using the untransformed inflation rate as a dependent variable would give undue weight to these outlying observations. Instead, we use

$D = \frac{\pi}{1+\pi}$ as the dependent variable, as in Cukierman et al., (1992, 1995). The variable D takes a value from zero to one.

5.3 The empirical results.

We begin by reproducing some of the results from the previous literature using our data set. In this way the actual impact of human capital inequality on inflation will be better evaluated. When the dependent variable D is regressed on openness only, using a FE estimator or simply OLS on the pooled cross-section observations, the openness coefficient is negative and highly statistically significant, a results often highlighted in the empirical literature (Romer, 1993; Lane, 1995). The FE coefficient and standard error of openness is reported below

$$D = -0.0007(openness) + 0.15$$
$$(0.0001) \qquad (39)$$

The negative correlation between openness and inflation is robust to the inclusion of CBI among the explanatory variables, although it becomes statistically non-significant when variables representing the vulnerability of the central bank and high-level political change are included among the regressors.

The degree of independence of the central bank from political pressure CBI has often been found to have positive although a hardly statistically significant effect on inflation. Using our full sample we find a positive and statistically significant coefficient in the OLS and FE regressions. The CBI coefficients turn statistically non-significant and negative in the case of developed democracies for which the FE regression results are as follows

$$D = -0.07(CBI) + 0.09$$
$$(0.06) \qquad (40)$$

For this very restricted group of countries the OLS estimate of CBI is negative and highly statistically significant, a result that reproduces the one found by Cukierman et al., (1992).

Our new empirical results are illustrated in tables 3-5. Table 3 reports the Fixed Effect estimation results obtained by using the full sample to estimate equation (38). The left hand side panel illustrates results where the dependent variable is D, while in the right hand side panel the dependent variable is the logarithmic transformation of the inflation rate π. The Gini coefficient computed using the population of those aged 25 and over is consistently negative and statistically significant. The sign of these estimates suggests that countries where agents are differently endowed with human capital tend to have better inflation records, once we keep constant those institutional factors that may impact upon the commitment to price stability. However, this suggested link between human capital inequality and inflation does not consistently apply to all countries. For instance, Table 4 illustrates the Fixed Effect estimation results obtained by splitting the sample in Non-Authoritarian and Authoritarian regimes, left hand side panel and right hand side panel, respectively, of Table 4. When such a distinction is made we notice that the negative correlation between G25 and inflation does not hold for Authoritarian regimes where a mildly statistically significant positive correlation between these two variables emerges.

The results illustrated in Table 5 further illustrates that the relationship between human capital inequality and inflation may depend on other features of the economy that are broadly captured by the dummy variable for the state of development. For Less Developed Countries (LDC=1) we again find the negative correlation between the Gini coefficient of the human capital distribution and inflation we initially found in the full sample estimates. However, in a sample of more developed countries (LDC=0) we find that human capital inequality increases inflation, a result that can be explained by our model where human capital inequality decreases the welfare costs of inflation thus opening a space where the commitment to price stability may be relaxed.

Note that these results are robust to (i) changes in the dependent variable (as illustrated in table 3 above), (ii) changes in the population used to compute the human capital inequality measure (G15 rather than G25). Also in most cases the use of OLS as an estimation technique does not radically change the results. For instance our OLS estimates for Central Bank vulnerability (vulnerability), which reproduce the results by Cukierman and Webb (1995), are not dramatically altered when the Gini measure of human capital inequality is included among the regressions. In such a regression the G25 variable has a negative, mildly statistically significant coefficient, which appears to be consistent with our FE estimates reproduced in table 3. (These results are available upon request).

6. Concluding Remarks

The objective of this paper was to examine the link between skill heterogeneity and the costs of inflation. This issue was addressed within a dynamic general equilibrium framework that incorporated ex-ante, endogenous skill heterogeneity among workers. Numerical experiments based on a plausible parameterization of this model indicate that welfare costs of inflation relative to an optimal monetary policy are likely to decrease as skill heterogeneity increases. An implication of this feature is that a greater degree of skill heterogeneity would be associated with a greater tolerance for inflation, consequently implying a positive correlation between agent heterogeneity and inflation. An empirical study based on a panel of several countries lends some support to this hypothesis. If we focus on the experience of industrialized economies, the data supports a positive inflation-heterogeneity correlation. On the other hand, this is not true of less wealthy economies, in which the inflation-heterogeneity correlation if found to be negative. However, the model economy we study in this paper, and its parameterization, is representative of developed economies, and to that end is only capable of explaining the long run or cyclical features of such economies. We may therefore interpret the results as supportive of the theoretical implications of our model.

Appendix.

A. Proof of Proposition 1.

It is easy to check that the numerator of the expression for $\frac{dH}{dg}$ is positive. The sign of $\frac{dH}{dg}$ therefore depends on the sign of the denominator. Note that the parameters Δ_1 and Δ_2 are positive. Given that we have assumed $\omega''(H) < 0$, it is then clear that the sign of the denominator will be negative if $\nu < \theta$ and $\phi'(H) > 0$. To see that $\frac{dC}{dg} < 0$, we take the total derivative of equation (30) and obtain

$$\frac{dC}{dg} = -\frac{(1-\theta)\beta\kappa^{\theta}\left(\phi(H)\right)^{\nu}\omega(H)}{\psi a g^{2}} + \left[\frac{(1-\theta)\beta\kappa^{\theta}\{\nu\left(\phi(H)\right)^{\nu-1}\phi'(H)\omega(H) + \left(\phi(H)\right)^{\nu}\omega'(H)}{\psi a g}\right]\frac{dH}{dg}.$$

The first term is obviously negative. Since $\frac{dH}{dg}$ is negative, and the term inside the brackets is positive under our assumptions, part (ii) of the proposition follows.

B. Proof of Proposition 2.

From condition (31) we can derive:

$$\frac{dN_1}{dg} = -\frac{\frac{1}{\beta} - 1 + \delta_h}{\frac{1-\theta}{1-\nu}\kappa^{\theta}}\left[\frac{\nu(\phi(H))^{\nu-1}\phi'(H)\omega'(H) + \omega''(H)(\phi(H))^{\nu}}{(\omega'(H)(\phi(H))^{\nu})^{2}}\right]\frac{dH}{dg}.$$

Again, since $\frac{dH}{dg}$ is negative given the conditions of Proposition 1 hold, the sign depending on the term in brackets. We can then check that $\frac{dN_1}{dg} < 0$ *iff* $-\frac{\omega''(H)}{\omega'(H)} > \nu\frac{\phi'(H)}{\phi(H)}$. Multiplying both sides by *H*, this amounts to $\Omega_{\omega'} > \nu\Omega_{\phi}$.

Also, $\frac{dN_2}{dg} = -\frac{1}{\nu}\left(\frac{\psi}{(1-\psi)\omega(H)}\right)^{\frac{1}{\nu}-1}\left(\frac{\psi\omega'(H)}{(1-\psi)(\omega(H))^{2}}\right)N_1\frac{dH}{dg} + \left(\frac{\psi}{(1-\psi)\omega(H)}\right)^{\frac{1}{\nu}}\frac{dN_1}{dg}.$

The first term on the right hand side is positive since $\frac{dH}{dg}$ is negative. The second term is negative if $\frac{dN_1}{dg} < 0$, i.e. if $\Omega_{\omega'} > \nu\Omega_{\phi}$. Overall the sign of $\frac{dN_2}{dg}$ is ambiguous. Also note that

$$\frac{dN}{dg} = \phi'(H)N_1\frac{dH}{dg} + \phi(H)\frac{dN_1}{dg},$$

and,

$$\frac{dK}{dg} = \kappa^{\theta}\frac{dN}{dg}.$$

It is easy to check that parts (iii) and (iv) of the propositions follow from the given assumptions.

C. Numerical Procedure

The numerical procedure used to solve for the steady state of the model involves the construction of a "grid" of values for human capital, and searching this grid for a value that satisfies equation (35). Once this is found, equations (30)-(34) can be used to find the steady state values of other variables. Results are accurate up to three decimal places.

References

Albanesi, S., (2000), "Inflation and Inequality," Universita Bocconi, IGIER Working Paper # 199.

Barro, R.J. and J. Lee, (2001), "International Data on Educational Attainment: Updates and Implications," Oxford Economic Papers, July 2001, 53(3), 541-63.

Bell, L.A., and R.B. Freeman, (2001), "The Incentive for Working Hard: Explaining Hours Worked Differences in the U.S. and Germany," Labour Economics, 8(2), 181-202.

Bhattacharya, J., Bunzel, H., and J. Haslag, (2003), "Inflationary Finance in a Simple Voting Model," Iowa State University, Working Paper # 03012.

Campillo M., and Miron J.A., (1996), "Why Does Inflation Differ Across Countries?" NBER Working Paper #5540.

Canton, E., (2002), "Business Cycles in a Two-Sector Model of Endogenous Growth," Economic Theory, 19, 477-492.

Castello, A., and R. Domenech, (2002), "Human Capital Inequality and Economic Growth: Some New Evidence," Economic Journal, 112(478), 187-200.

Chakraborty, S. & Mausumi Das, 2003. "Mortality, Human Capital and Persistent Inequality", University of Oregon Economics Department Working Papers 2003-11.

Cooley, T.F. and G.D. Hansen, (1989), "The Inflation Tax and the Business Cycle," American Economic Review, 79, 733-748.

Cukierman, A., and S. Webb, (1995), "Political Influence on the Central Bank: International Evidence," The World Bank Economic Review, 9(3), 397-423.

Cukierman, A., Webb, S., and B. Neyapti, (1992), "Measuring the Independence of Central Banks and its Effects on Policy Outcomes," The World Bank Economic Review, 6, 353-98.

Dolmas, J., Huffman, G.W., and M.A. Wynne, (2000), "Inequality, Inflation, and Central Bank Independence," Canadian Journal of Economics, 33(1), 271-87.

Easterly, W., and S. Fischer, (2001), "Inflation and the Poor," Journal of Money, Credit, and Banking, 33(2), Part I, 160-78.

Fischer, S., (1981), "Towards an Understanding of the Costs of Inflation," Carnegie-Rochester Conference Series on Public Policy, 15, 5-42.

Galor, O. and D. Tsiddon, (1997), "The Distribution of Human Capital and Economic Growth," Journal of Economic Growth, 2(1), 93-124.

Glomm, G. and B. Ravikumar, (1992), "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," Journal of Political Economy, 100(4), 1127-70.

Hansen, G.D., (1985), "Indivisible Labor and the Business Cycle," Journal of Monetary Economics, 16, 309-28.

Kydland, F.E., (1984), "Labor-Force Heterogeneity and the Business Cycle," Carnegie-Rochester Conference Series on Public Policy, 21(0), 173-208.

Lane, P., (1995), "Inflation in Open Economies," manuscript, Harvard University.

Lahiri, R., (2002), "The Inflation Tax, Variable Time Preference and the Business Cycle," Macroeconomic Dynamics, 6 (4), 496-522.

Lucas, R.E., Jr., (1981), "Discussion of Towards an Understanding of the Costs of Inflation: II," Carnegie-Rochester Conference Series on Public Policy, 15, 43-52.

Mulligan, C.B., and X. Sala-i-Martin, (1996), "Adoption of Financial Technologies: Implications for Money Demand and Monetary Policy," Working Paper # 5504, Cambridge, Mass.: NBER.

Prasad, E., (1996), "Skill Heterogeneity and the Business Cycle," Canadian Journal of Economics, 29(4), 910-29.

Romer, D., (1993), "Openness and Inflation: Theory and Evidence," Quarterly Journal of Economics, CVII (4), 869-904.

Saint-Paul, G. and T. Verdier, (1993), "Education, Democracy and Growth," Journal of Development Economics, 42(2), 399-407.

Table 1: Steady state values as g increases.

	$g = \beta$ (Optimal Policy)	$g = 1.024$	$g = 1.05$	$g = 1.15$
Human capital	10.7655	10.3780	10.0995	9.1495
Consumption	2.3580	2.2796	2.2230	2.0294
Skilled effort N_1	.1455	.1408	.1375	.1260
Unskilled effort N_2	.0466	.0454	.0445	.0413
$\frac{N_1}{N_1 + N_2} \times 100$	75.72	75.63	75.56	75.31
$\frac{N_2}{N_1 + N_2} \times 100$	24.98	24.37	24.44	24.69
Aggregate effort(N)	.8708	.8417	.8208	.7492
Capital stock	33.0794	31.9772	31.1833	28.4630
Output	3.2253	3.1179	3.0405	2.7752
Skill differential	2.2682	2.2636	2.2602	2.2478
Skilled wages	11.0004	10.9714	10.9499	10.8727
Unskilled wages	7.6444	7.6242	7.6093	7.5556
Utility	.7529	.7223	.6995	.6165
Welfare cost	0	.0136	.0548	.1461
Welfare cost as % of consumption	0	1.36	2.46	7.20
Welfare cost as % of output	0	1.00	1.80	5.27

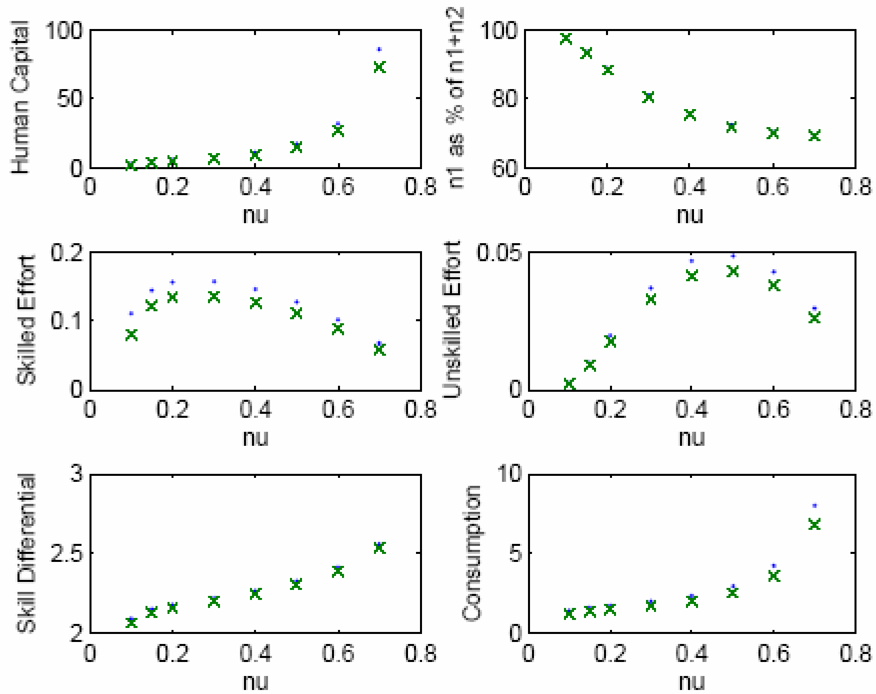
Human Capital
n1 as % of n1+n2
Skilled Effort
Unskilled Effort
Skill Differential
Consumption
nu
100
50
0
0.2
0.4
0.6
0.8
80
60
0.1
0.05
3
2.5
2
10
5

Figure 1(a): Steady State Values of Variables as ν increases

Optimal Policy ($g = \beta$)	.
Policy with $g = 1.15$	x

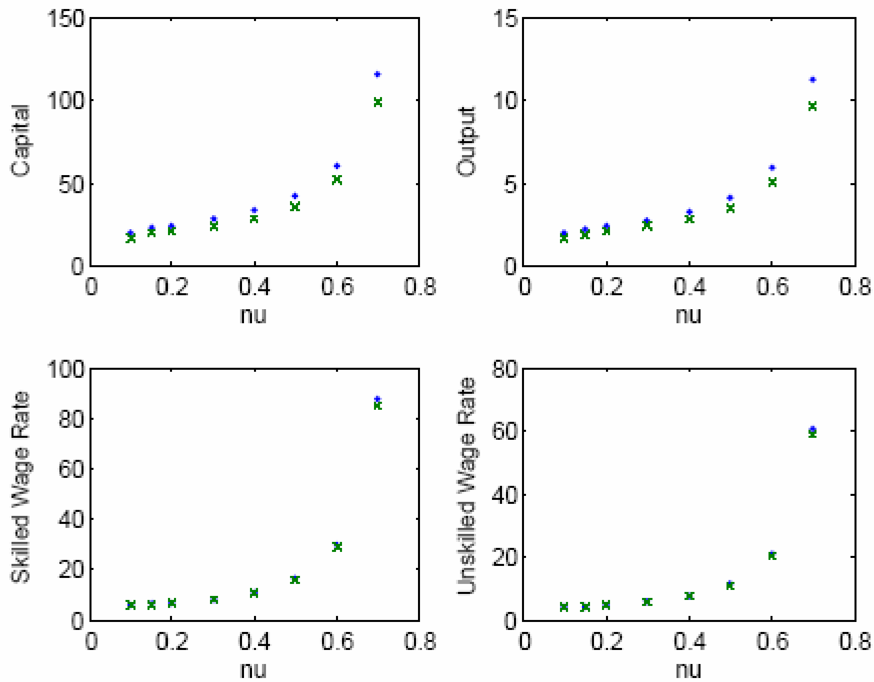

Figure 1(b): Steady state values of variables as ν increases

Optimal Policy ($g = \beta$)	.
Policy with $g = 1.15$	x

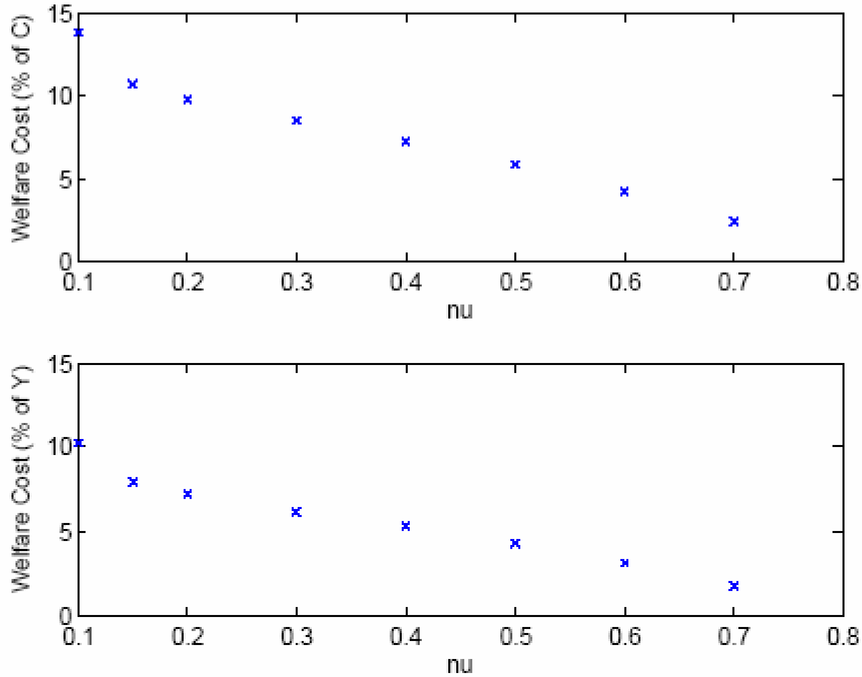

Figure 1(c): Welfare costs of inflation as ν increases

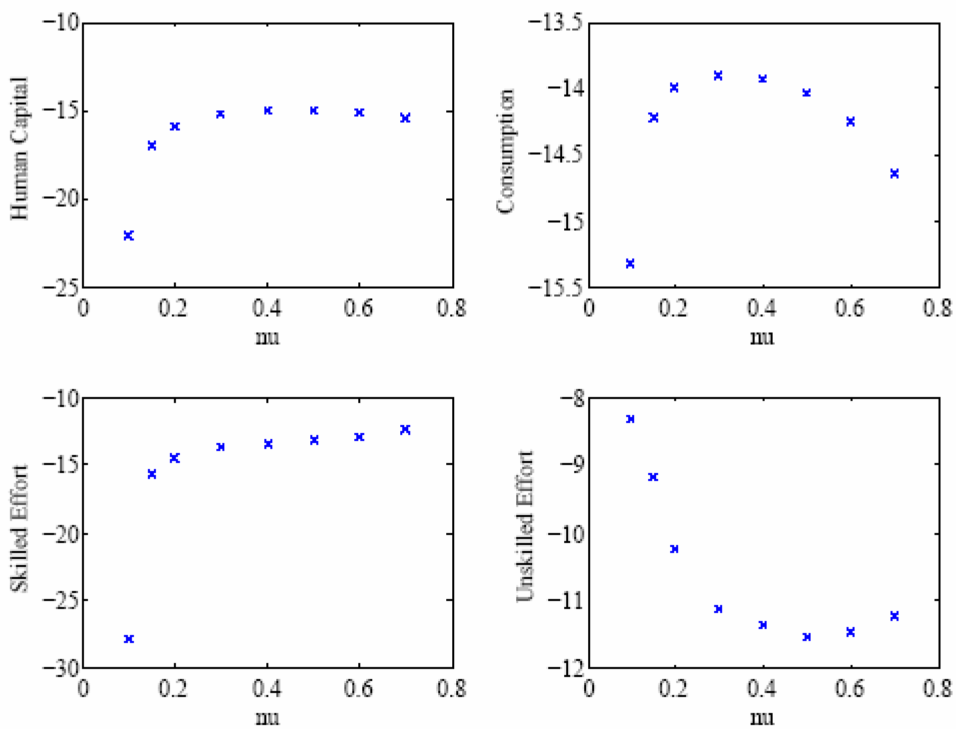

Figure 2(a): Percentage difference relative to the optimal policy as ν varies.

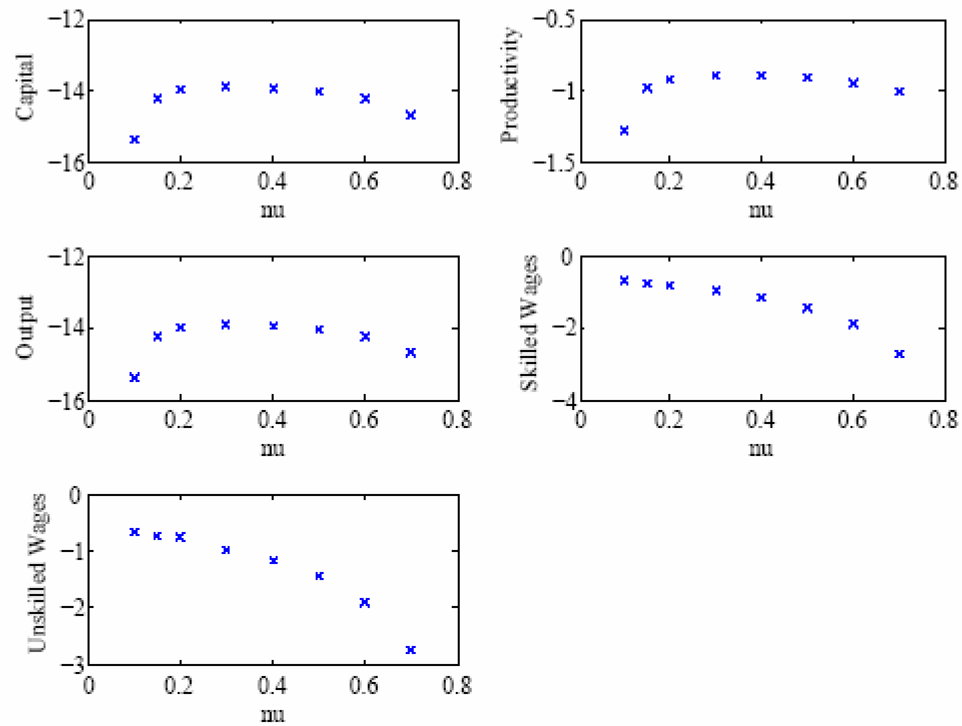

Figure 2(b): Percentage difference relative to the optimal policy as ν varies.

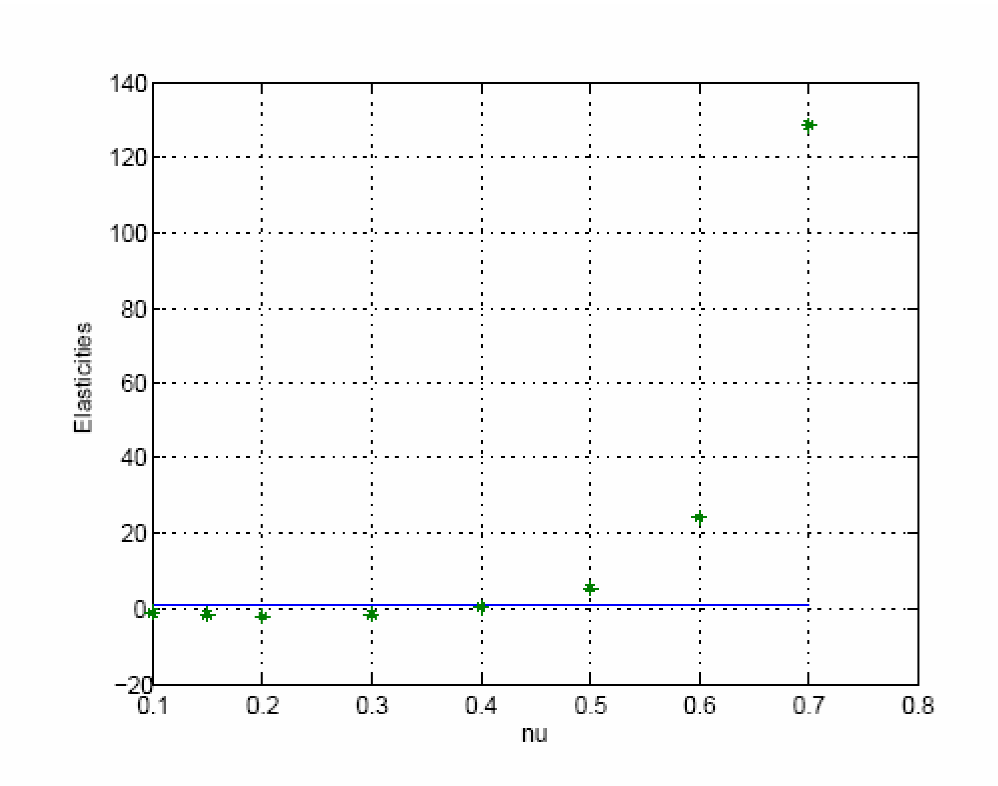

Figure 2(c): Elasticities $\Omega_{\omega'}$ and $\nu\Omega_{\phi}$ as ν varies.

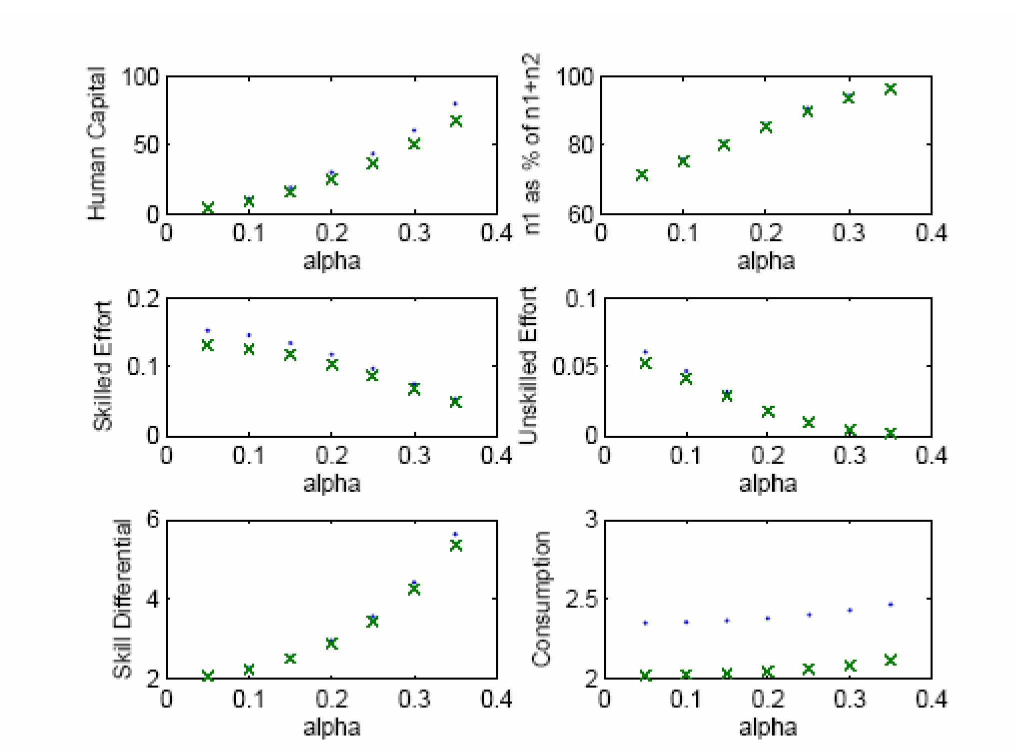

Figure 3(a): Steady state values of variables as α increases

Optimal Policy ($g=\beta$)	.
Policy with $g=1.15$	x

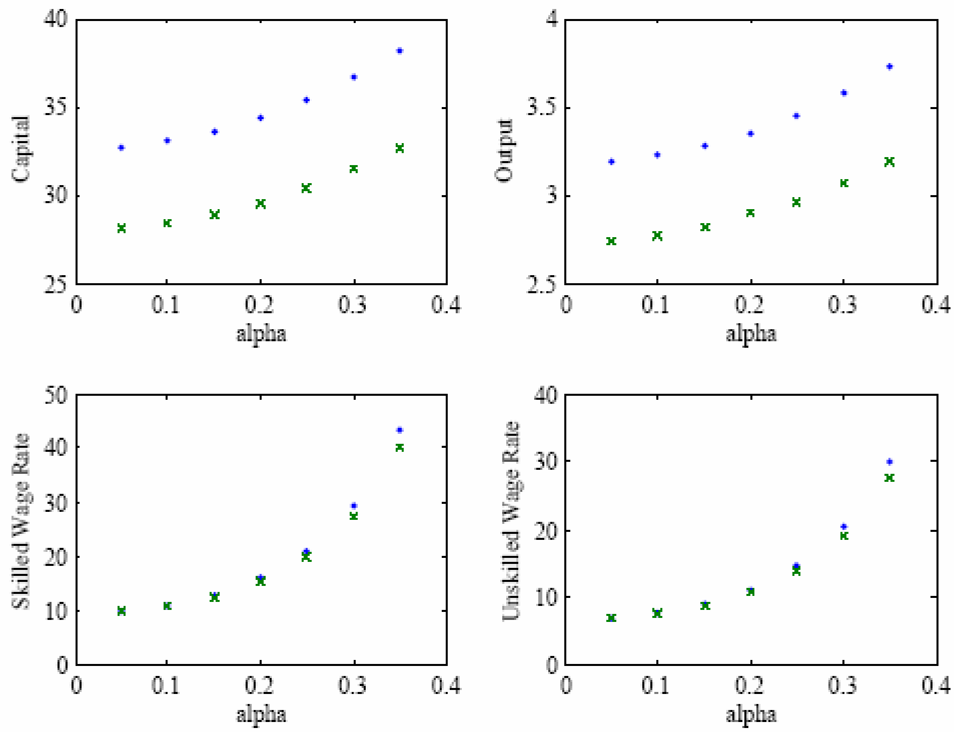

Figure 3(b): Steady state values of variables as α increases

Optimal Policy ($g = \beta$)	.
Policy with $g = 1.15$	x

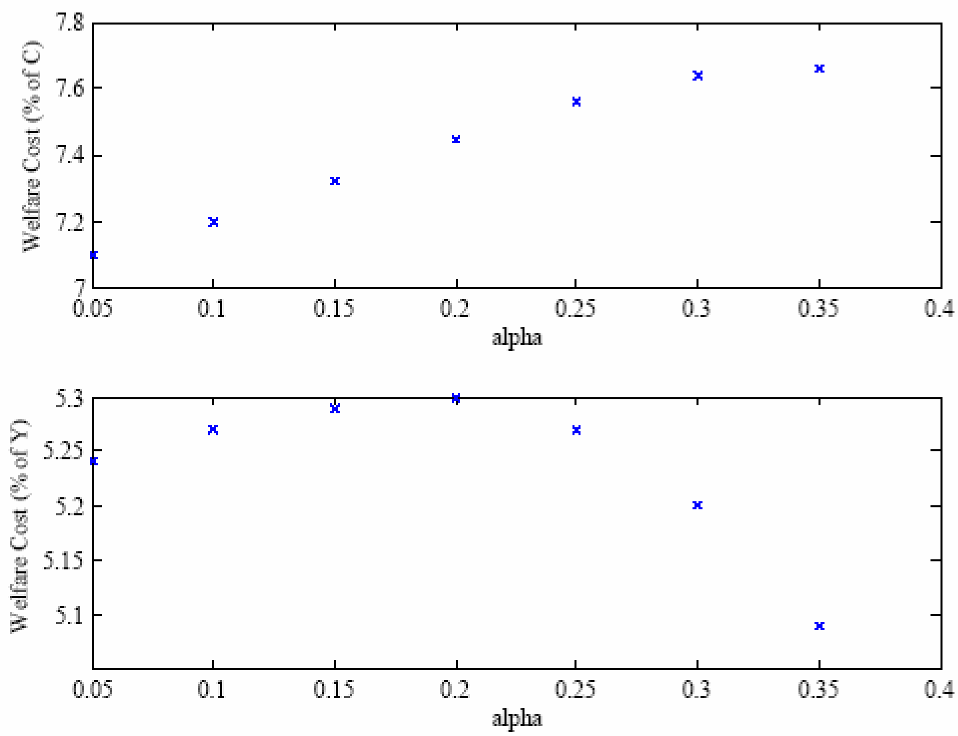

Figure 3(c): Welfare costs of inflation as α increases.

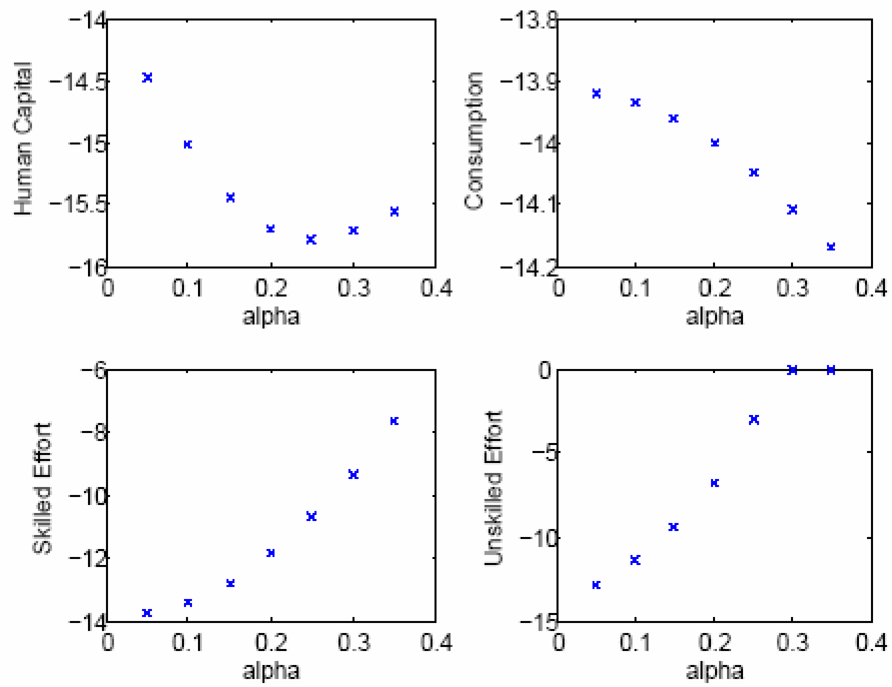

Figure 4(a): Percentage change relative to optimal policy as α varies.

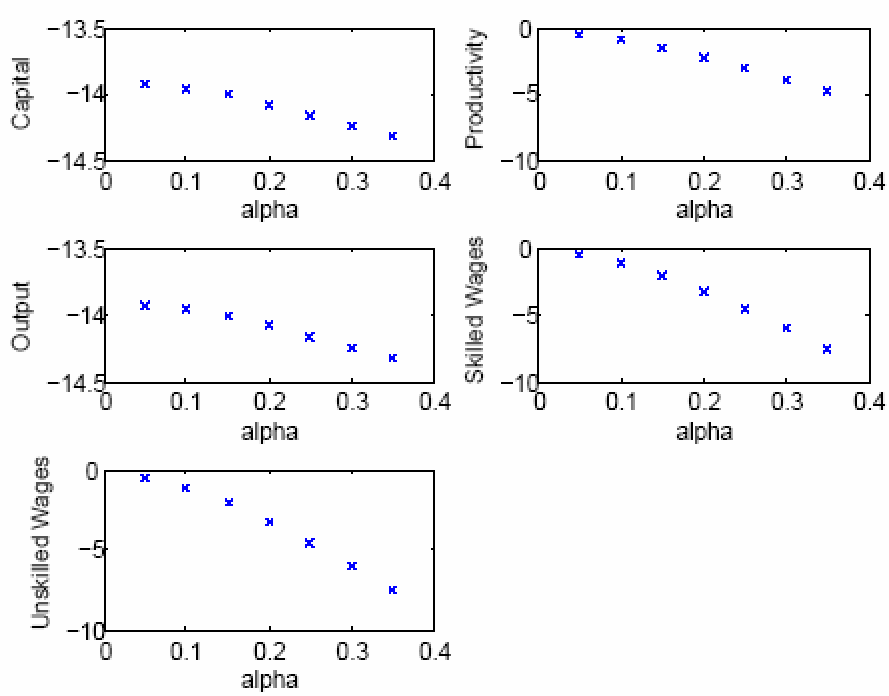

Figure 4(b): Percentage change relative to optimal policy as α varies.

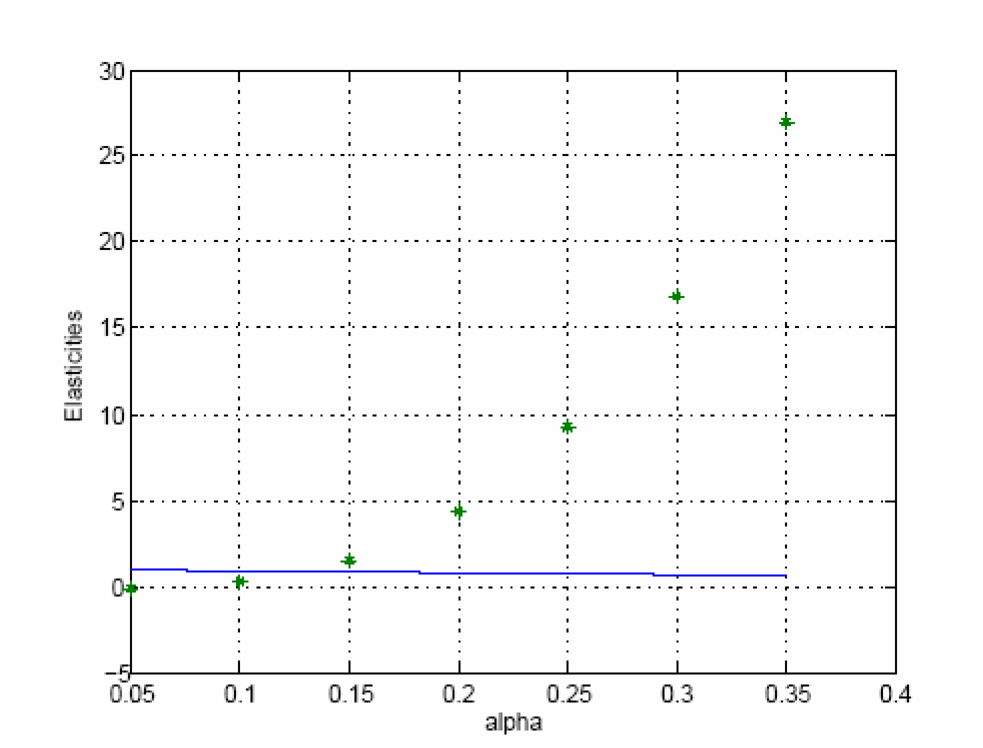

Figure 4(c): Elasticities $\Omega_{\omega'}$ and $\nu\Omega_{\phi}$ as α varies.

Table 2. Summary statistics for the main variables					
Variable name	Obs.	Mean	Stand. Dev.	Min.	Max.
Inflation π	100	36.3	114.0	2.68	920.5
$D = \frac{\pi}{1+\pi}$	100	0.11	0.09	0.03	0.52
G25	105	0.48	0.24	0.13	0.94
G15	108	0.45	0.23	0.11	0.91
Openness	98	66.2	42.9	10.9	243.0
CBI	57	0.34	0.12	0.12	0.69
Vulnerability	56	0.29	0.31	0	1.28
High-level pol. change	56	0.03	0.05	0	0.18

Table 3. Inflation and human capital inequality, 1960-2000. The dependent variable is $D = \frac{\pi}{1+\pi}$ in the left hand panel and $\log \pi$ in the right hand panel, where π is the inflation rate. Fixed Effect Estimation results.

Explain. variables	Inflation is D, FE 1960-2000			Inflation is $\log \pi$, FE 1960-2000		
Gini (pop. 25+)	-0.14**	-0.36**	-0.44**	-1.94**	-4.71**	-6.27***
	(0.06)	(0.15)	(0.16)	(0.66)	(1.61)	(1.64)
Openness	----	----	0.001**	----	----	0.014**
	----	----	(0.0004)	----	----	(0.004)
Central Bank Ind.	----	0.31*	0.27	----	2.76	1.98
	----	(0.17)	(0.17)	----	(1.72)	(1.70)
Vulnerability (lag 0-6 months)	----	0.04	0.04	----	0.007	0.07
	----	(0.04)	(0.04)	----	(0.40)	(0.40)
High-level political change	----	-0.06	-0.04	----	-2.75	-2.33
	----	(0.19)	(0.19)	----	(2.18)	(2.14)
Constant	0.17***	0.13	0.14	2.87***	2.89**	3.06***
	(0.03)	(0.09)	(0.09)	(0.29)	(0.93)	(0.91)
No. of observations	745	264	262	711	260	258
No. of groups	97	50	50	97	50	50
F test	5.10**	3.45**	3.62**	8.58**	3.91**	5.47**

Exp. Var.	FE Non-Authoritarian			FE Authoritarian (*a*)		
Gini (pop. 25+)	-0.27**	-0.48**	-0.57**	0.22	0.41	0.75*
	(0.12)	(0.16)	(0.16)	(0.22)	(0.39)	(0.45)
Openness	----	----	0.001**	----	----	-0.005
	----	----	(0.0004)	----	----	(0.004)
Central Bank Ind.	----	0.29*	0.24	----	-0.78	-0.61
	----	(0.16)	(0.16)	----	(1.43)	(1.44)
Vulnerability (0-6 months)	----	0.01	0.02	----	0.20	0.28
	----	(0.04)	(0.04)	----	(0.21)	(0.21)
High-level political change	----	-0.04	-0.02	----	-0.55(*b*)	0.35(*b*)
	----	(0.19)	(0.19)	----	(1.72)	(1.80)
Constant	0.20***	0.18**	0.18**	0.04	0.22	0.11
	(0.04)	(0.09)	(0.09)	(0.10)	(0.58)	(0.59)
No. of observations	386	239	237	46	25	25
No. of groups	44	43	43	8	7	7
F test	5.33**	4.15**	4.07**	1.02	X(4)=3.7	X(5)=6.2

Table 4. Inflation and human capital inequality. Democratic and non-democratic regimes, 1960-2000. The dependent variable is $D = \frac{\pi}{1+\pi}$ where π the inflation rate is. Fixed Effects Estimation results.

Notes:

(a) Because of the lack of variability of some variables within the sample of non-democratic countries, the last two columns illustrate Random Effect estimation results.

(b) The “High-level political change” variable has been replaced with “Low-level political change”.

Table 5. Inflation and human capital inequality. Developed and less developed countries, LDC = 0 and LDC = 1, respectively, 1960-2000. The dependent variable is $D = \frac{\pi}{1+\pi}$ where π the inflation rate is. Fixed Effects (FE) Estimation results.

Exp. Var.	FE, LDC = 1			FE, LDC = 0		
Gini (pop. 25+)	-0.17**	-0.53**	-0.60**	0.31**	0.53**	0.38**
	(0.08)	(0.21)	(0.21)	(0.12)	(0.19)	(0.19)
Openness	----	----	0.0008	----	----	0.002**
	----	----	(0.0006)	----	----	(0.0006)
Central Bank Ind.	----	1.04**	1.02**	----	-0.14	-0.20
	----	(0.39)	(0.39)	----	(0.20)	(0.19)
Vulnerability (0-6 months)	----	0.004	-0.010	----	0.07	0.05
	----	(0.05)	(0.06)	----	(0.05)	(0.05)
High-level political change	----	-0.04	-0.03	----	0.89*	0.88*
	----	(0.25)	(0.25)	----	(0.50)	(0.48)
Constant	0.21***	0.06	0.06	-0.006	0.0002	-0.02
	(0.04)	(0.18)	(0.18)	(0.02)	(0.09)	(0.09)
No. of observations	534	144	144	182	120	118
No. of groups	73	31	31	21	20	20
F test	4.62**	4.54**	4.02**	6.80**	3.08**	4.65***

LISTING OF DISCUSSION PAPERS - 2003

Worthington A, & Valadkhani A, Measuring the Impact of Natural Disasters on Capital Markets: An Empirical Application Using Intervention Analysis, No 154, September 2003

Robinson M, The Output Concept and Public Sector Services, No 155, September 2003

Worthington A, Brown K, Crawford M, & Pickernell D, Socio-Economic and Demographic Determinants of Household Gambling in Australia, No 156, September 2003

Worthington A, & Higgs H, Tests of Random Walks and Market Efficiency in Latin American Stock Markets: An Empirical Note, No 157, September 2003

(Replacing Previous No 158) Worthington A, & Higgs H, Systematic Features of High-Frequency Volatility in Australian Electricity Markets: Intraday Patterns, Information Arrival and Calendar Effects, No 158, November 2003

Worthington A, & Higgs H, Weak-form Market Efficiency in European Emerging and Developed Stock Markets, No 159, September 2003

West T, & Worthington A, Macroeconomic Risk Factors in Australian Commercial Real Estate, Listed Property Trust and Property Sector Stock Returns: A Comparative Analysis using GARCH-M, No 160, October 2003

Lee B, Interstate Comparison of Output and Productivity in the Australian Agricultural Sector – 1991 – 1999, No 161, October 2003

McCarthy S, Hedging Versus not Hedging: Strategies for Managing Foreign Exchange Transaction Exposure, No 162, November 2003

Worthington A, Emergency Finance in Australian Households:
An Empirical Analysis of Capacity and Sources, No 163, November 2003

Worthington C, Debt as a Source of Financial Stress in Australian Households, No 164, November 2003

Robinson M, The Australian Budgeting System: On the Cusp of Change, No 165, November 2003

Lahiri R, Cooperation v/s Non-cooperation in R&D Competition with Spillovers, No 166, December 2003

Wolff R, Yao Q, & Tong H, Statistical Tests for Lyapunov Exponents of Deterministic Systems, No 167, December 2003

Wolff R, Barnett A, A Time Domain Test for Some Types of Non-Linearity, No 168 December 2003

LISTING OF DISCUSSION PAPERS - 2004

Drew M, Veeraraghavan M, Ye M, Do Momentum Strategies Work? - Australian Evidence, No 169, January 2004

Drew M, Mallin M, Naughton T, Veeraraghavan M, Equity Premium: - Does it Exist? – Evidence from Germany and United Kingdom, No 170, January 2004

Layton A, Valadkhani A, Measures of National Export Price Volatility Based on the Capital Asset Pricing Model, No 171, January 2004

Drew M, Marsden A, Veeraraghavan M, Small Firm Effect, Liquidity and Security Returns: Australian Evidence, No 172, February 2004

Drew M, Stanford J, Portability of Superannuation Balances, No 173, March 2004

Drew M, Naughton T, Veeraraghavan M Pricing of Equities in China: Evidence from the Shanghai Stock Exchange No 174, May 2004

Valadkhani A, Worthington A, Layton A, An Analysis of the Rising Cost of Education in Australia, No 175, April 2004

Li S, Worthington A, The Relationship Between The Adoption of Internet Banking and Electronic Connectivity: - An International Comparison, No 176, May 2004

Drew M, Marsden A, Veeraraghavan M, Does Idiosyncratic Volatility Matter? – New Zealand Evidence, No 177, May 2004

Guégan D, How Can We Dane the Concept of Long Memory? - An Econometric Survey, No 178, April 2004

Clements A, Hurn S, White S, Discretised Non-Linear Filtering for Dynamic Latent Variable Models: with Application to Stochastic Volatility - No 179, May 2004

McCarthy S, Ispriani A, An Operating/Economic Exposure Australian Case Study: Foster's Group Limited, No 180, May 2004

Lahiri R, On Skill Heterogeneity and Inflation, No 181, June 2004

Bianchi R, Drew M, Polichronis J, A Test of Momentum Trading Strategies in Foreign Exchange Markets: Evidence from the G7, No 182, June 2004

Jeisman S, Exchange Market Pressure in Australia, No 183, June 2004

Higgs H, Worthington A, Financial Returns and Price Determinants in the Australian Art Market - 1973-2003, No 184, July 2004

Worthington, A, The distribution of Financial Literacy in Australia, No 185, November 2004

Higgs H, Worthington A, Systematic Features of High-Frequency Volatility in Australian Electricity Markets: Intraday Patterns, Information Arrival and Calendar Effects, No 186, December 2004

LISTING OF DISCUSSION PAPERS - 2005

Behrens P. Economic Development Strategies: Examples from Europe and Australia, No 187, January 2005

Wilson C, Tidsell C. What Role Does Knowledge of Wildlife Play in Providing Support for Species' Conservation? No 188, February 2005

Zhang S. Consumption Behaviour Under Institutional Transitions in China, No 189, March 2005

Collet J & Fadili J Simulation of Gegenbauer Processes Using Wavelet Packets - No 190, March 2005

Clements A & White S. Non-Linear Filtering With State Dependant Transition Probabilities: A Threshold (Size Effect) SV Model. No 191, April 2005

Clements A & White S. Non-Linear Filtering For Stochastic Volatility Models with Heavy Tails and Leverage - No 192, April 2005

Lee B. Significance of Employing a Multilateral Index Formula for Interstate Comparisons: A Case Study of the Australian Farm Sector - No 193, June 2005

Li S, Alfay E. Evidence on the Arbitrage Efficiency of SPI Index Futures and Options Markets – No 194, June 2005

Worthington A, Lee B. Efficiency, Technology and Productivity Change in Australian Universities: 1998-2003 – No 195, June 2005

Hoffman M, Worthington A, & Higgs H. Modelling Residential Water Demand with Fixed Volumetric Charging in a Large Urban Municipality: The Case of Brisbane, Australia – No 196, June 2005

Wilson, C. Exposure to Pesticides, Ill-health and Averting Behaviour: Costs and Determining the Relationships – No 197, July 2005

Wilson C & Tidsell C. Globalisation, Concentration of Genetic Material and Their Implications for Sustainable Development – No 198, July 2005

Pollard G. The Cost Decision: A New Discount Approach for Net Cost Projects – No 199, July 2005

Layton A & Smith D. Testing of the Power of Leading Indicators into Predicting Business Cycle Phase Changes? – No 200, August 2005

Worthington A C & Higgs H. Market Risk in Demutualised Self-Listed Stock Exchanges: An International Analysis of Selected Time-Varying Betas – No 201, September 2005

Handley-Schachler M & Li S. International Effects of the Andersen Accounting and Auditing Scandals: Some Evidence from the US, UK and Australian Stock Markets – No 202, September 2005

Lazarov Z. Assessing the Economic Significance of the Intra-Daily Volatility Seasonalities – No 203 – October 2005

Chen L, Li S & Chen Y. Corporate Governance and Corporate Performance: Some Evidence from Newly Listed Firms on Chinese Stock Markets – No 204, November 2005

ISSN 1324-5910

All correspondence to:

Dr Steven Li
Editor, *Discussion Papers in Economic, Finance and International Competitiveness*
School of Economics and Finance
Queensland University of Technology
GPO Box 2434, BRISBANE QLD 4001, Australia

Telephone: 61 7 3864 2521
Facsimile: 61 7 3864 1500
Email: s.li@qut.edu.au

Queensland University of Technology

DISCUSSION PAPERS IN ECONOMICS, FINANCE AND INTERNATIONAL COMPETITIVENESS

On Skill Heterogeneity, Human Capital, and Inflation

Radhika Lahiri
&
Elisabetta Magnani

Discussion Paper No 205, December 2005

Series edited by
Dr Steven Li

School of Economics and Finance