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On Optimal Monetary Policy in a Liquidity Effect Model

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Abstract

This paper examines the implications of introducing a variable rate of time preference on the role of monetary policy in a dynamic general equilibrium framework explicitly designed to capture *liquidity effects*. Variable time preference is incorporated by allowing the discount factor applied to future utility to be decreasing in contemporaneous utility. The model is a more general one, in the sense that the fixed discount factor economy is nested as a special case. Numerical simulations of the more general model indicate that for a range of parameters optimal monetary policy can be qualitatively different. This is in spite of the fact that there are very small quantitative differences in the magnitude of monetary non-neutralities, such as liquidity effects, in the fixed and flexible discount factor environments. Furthermore, within this range, monetary policy is *less* activist, in the sense that it is *procyclical* to productivity shocks, as opposed to being *countercyclical* as in the fixed time preference model.

1. Introduction

The negative impact of unanticipated monetary injections on nominal interest rates is central to a number of recent papers, including those of Lucas (1990), Fuerst(1992), and Christiano and Eichenbaum(1995). Common to several general equilibrium models that focus on such *liquidity effects* are assumptions about the nature of trading frictions imposed on the economy. The premise of this class of models is that monetary injections are asymmetric, in that they occur through financial intermediaries in the credit market, and that it takes time to move funds from one market to another. In the event of an unanticipated monetary injection, the credit market is temporarily more liquid in comparison to the goods market, and nominal interest rates must fall in order to induce borrowers to absorb the excess supply of cash. Short run asymmetric effects of this type are typically generated by assuming that household savings decisions are made before the monetary injection is realized. Cash-in-advance restrictions on all transactions of borrowers then ensure that the decline in nominal interest rates translates into an increased demand for real goods and services.

The nature of non-neutralities generated by such liquidity models stands in sharp contrast to those obtained in standard cash-in-advance models, which focus primarily on anticipated inflation effects of monetary shocks, and thus generate a negative correlation between monetary injections and real activity. The emphasis on the consequences of *unanticipated* monetary injections essentially arises from the assumptions about the timing of events, and the predictions of the two types of cash-in-advance models are not necessarily in conflict with each other. To be specific, the long run predictions of the two types of models is essentially the same – in the steady state, the correlation between money growth and output is negative. In a liquidity model, the temporary inflexibility in the savings decisions of individuals is the key to the *short run* negative money-output correlation. At the empirical level,

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although evidence on whether liquidity effects are an important "stylized fact" is not sufficiently conclusive, it has been persuasive enough to make liquidity models very popular in recent years.

In view of the differential impact of monetary shocks, another interesting issue that arises in the context of liquidity models relates to the role of monetary policy. While the role of monetary policy has been extensively investigated in standard cash-in-advance models, it is a relatively unexplored issue in the framework of liquidity models. Fuerst's (1994) paper, for example, demonstrates differences in optimal monetary policy arising as a result of trading frictions that are unique to this class of models. In particular, Fuerst demonstrates that, for a range of preference parameters, monetary policy can be strongly activist, in the sense that it is *countercyclical* to productivity shocks. This is an interesting result, since standard cash-in-advance models typically fail to yield an activist role for monetary policy even though significant non-neutralities are present.

This paper examines the implications of introducing an endogenous individual rate of time preference on the role of monetary policy in a liquidity effect framework. The benchmark model studied in this paper is that of Fuerst (1992, 1994), and endogenous time preference is incorporated by replacing the fixed utility discount factor by a discount factor that is a function of contemporaneous utility, and is hence affected by the levels of consumption and leisure. Preferences are then no longer time additive, although they remain recursive, as in fixed discount factor environments. Epstein (1983) postulates conditions under which such preferences are consistent with expected utility, and the dynamic stability of models with such preferences[1]. One of the assumptions involved is that the discount factor is *decreasing* in utility, reflecting the idea that individuals become more "impatient" with increases in current utility benefits.

The motivation for incorporating such preferences is that they are more general, in the sense that the fixed discount factor environment is nested as a special case. It is therefore desirable to check whether conclusions emerging from standard frameworks are retained when the assumption of variable time preference is relaxed. Allowing flexibility in the discount factor in the context of this paper is somewhat similar in spirit to the exercise conducted by Braun (1994), who demonstrates differences in optimal monetary policy arising due to small modifications of preferences in a standard Cooley and Hansen (1991) type cash-in-advance framework. Standard cash-in-advance models typically yield the Friedman rule as the optimal policy: inflation tax distortions are eliminated by deflating at a rate of money growth equal to the rate of time preference. Braun considers a broader class of preferences, which allow for special cases in which a positive inflation tax may be optimal. He also finds the estimates of the preference parameters to be within the range in which such special cases are nested. In liquidity effect models, differences in optimal policy arise due to the presence of an additional distortion – the inability of agents to move funds across sectors immediately after the shock is observed. Fuerst (1994) shows that the presence of this additional distortion can imply an activist countercyclical role for monetary policy, as opposed to the deterministic non-activist role typically implied by models in which the Friedman rule is optimal. An activist contercyclical role for monetary policy is typically difficult to find within cash-in-advance frameworks, even in the case of cash-in-advance models with Keynesian features, such as the endogenous sticky-price model of Ireland (1996)[2]. Modifying preferences by introducing endogenous time preference further changes the role of policy by altering the conditions it needs to satisfy in order to remove *both* inflation tax and liquidity distortions.

In addition to exploring the role of monetary policy, this paper also analyzes the impact of flexibility in the rate of time preference on liquidity effects, and other monetary non-neutralities, such as anticipated inflation effects. It must be emphasized that the analysis of the effects of variable time preference on liquidity effects *per se* is not the focus of this paper. Nevertheless, it is useful and essential in motivating the differences in the role of monetary policy in the fixed and flexible time preference environments. A number of numerical experiments in this paper are therefore devoted to the analysis of liquidity effects and other type of monetary non-neutralities. It turns out that, qualitatively speaking, variable time preference models produce non-neutralities that are similar to fixed time preference models. However, quantitatively, these non-neutralities can be different. For example, liquidity effects of monetary shocks

1 The original formulation is due to Uzawa (1968), which was extended and refined recently in Epstein's work.

2 Here, as in Ireland (1996), were identifying the term "activist" with monetary policy that is countercyclical to productivity shocks. Procyclical and deterministic monetary policy is considered "non-activist".

can be smaller or larger in the presence of variable time preference, depending on the parameters of the model. Also, monetary policy that is procyclical to productivity shocks amplifies fluctuations in output to a greater degree in a variable time preference model, while countercyclical monetary policy dampens fluctuations in output to a greater degree.

The point we want to make here, and this is the main finding of this paper, is that although variable time preference produces qualitatively similar monetary non-neutralities, its implications for the role of monetary policy can be very different. This is confirmed by means of two types of quantitative experiments. In one experiment, we explicitly calculate the optimal monetary policy using a first best approach. For an isoelastic representation of the period utility function, optimal monetary policy is qualitatively different in the variable time preference model for values of the risk aversion parameter s greater than 1 and less than approximately 1.17. In this range, optimal monetary policy is procyclical to productivity shocks for the variable time preference model, but countercyclical to productivity shocks in the fixed time preference model[3]. In another experiment, we compute the welfare costs of different types of monetary policies in which money growth is constrained to be positive. The results of the second experiment also support the conclusions of the first experiment. The analysis of this paper therefore seems to confirm the non-activist nature of monetary policy that is typically found in the literature, as opposed to the activist nature of monetary policy in a fixed time preference liquidity effect framework such as that of Fuerst (1992, 1994). Even in cases where the optimal policy is qualitatively similar, carrying out the optimal policy is a more difficult task if the economy is characterized by variable time preference.

The remaining sections of the paper are organized as follows. Section 2 describes the economic environment, which can be described as an extension of the Fuerst (1992, 1994) models, and briefly discusses the impact of money growth shocks on nominal interest rates. Section 3 analyses the results based on numerical simulations of the model. Section 4 concludes.

2. The Economic Environment

We consider an economy with identical, infinitely lived households, which maximize expected lifetime utility given by

$$E\left\{\sum_{t=0}^{\infty}\left[\prod_{t=0}^{t-1} b\,(u(c_t,1-L_t))\right]u(c_t,1-L_t)\right\} \tag{1}$$

where the endogenous discount factor $b(u)$ must be of the form $e^{-f(u)}$, $f(u)>0$, in order to be consistent with expected utility, as shown in Epstein (1983). Also, $u(c_t,1-L_t)=\log(c_t)-AL_t$, $A>0$, represents the household's time-t momentary utility, defined over consumption c_t, and leisure, $1-L_t$[4]. The function $f(u)=h+tu$, so that an increase in utility causes a decrease in the discount factor - the household becomes more impatient with respect to future utility. The function u must be negative, strictly increasing with $\ln(-u)$ convex in the composite consumption-leisure good. It is also required that f is positive, increasing, strictly concave and that $u'e^{f(u)}$ is nonincreasing[5].

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[3] The estimates of s are usually known to lie between 1 and 2. See, for example, Prescott (1986).

[4] While most of the analysis in this paper assumes logarithmic utility, we also conduct some experiments with the broader class in which $u(c,1-L)=\dfrac{c^{1-s}-1}{1-s}-AL$, where $s=1$ is the log utility case.

[5] These restrictions ensure that a stable steady state distribution for the state variables exists and is unique. Epstein (1983) also shows that, under these conditions, consumption is a normal good in every period , and that deviations from the fixed time preference set up are not too great. Although these conditions are specified for the case in which the utility function has only one argument, *viz.* consumption, results in Epstein (1983) should go through if consumption and leisure are treated as a composite commodity. Restrictions specified in Epstein (1983) should then be satisfied w.r.t. this composite commodity. (See, for example, Gomme and Greenwood (1995) and Mendoza (1991)).

The households in this economy purchase consumption goods from firms, which produce a homogeneous consumption-investment good, using a stochastic production technology. This technology is of the Cobb-Douglas form, given by

$$f(H_t, s_t) = q(s_t) H_t^g, \quad g \in (0,1), \tag{2}$$

where H_t is the labor services firms purchase from households. The vector $s_t \in S$ denotes the state of the economy at time *t*. The positive and continuous variable $q(s_t)$ represents productivity shocks in the goods sector.

To introduce money into the economy, cash in advance constraints are imposed on all purchases carried out by both consumers and firms. The cash in advance constraint on consumers is given by

$$M_t - N_t \geq P_t c_t, \tag{3}$$

where M_t is the amount of money balances the household holds at the beginning of period *t*, and N_t is the portion of these balances that the household deposits with intermediaries in the financial sector.

Firms, on the other hand, use B_t units of cash borrowed from financial intermediaries in the credit market to finance purchases of H_t units of labor services from households. The cash in advance constraint on firms is thus given by

$$B_t \geq W_t H_t. \tag{4}$$

Finally, the financial sector of the economy consists of financial intermediaries who accept cash deposits N_t from households, in addition to receiving the monetary injection X_t from the central bank[6]. The central bank supplies money using the process $M_{t+1}^s = M_t^s + X_t$, where M_t^s is the beginning of period t nominal money supply. The financial intermediaries thus have $N_t + X_t$ units of cash available for loaning out to the firms.

Apart from frictions arising due to the imposition of cash in advance restrictions, an additional friction arises due to the household's inability to alter its savings decision after observing the monetary injection. Since the monetary injection occurs asymmetrically *via* the financial sector, and funds cannot be moved from on location to another, the monetary injection will have distributional effects. This feature of the model is similar to some earlier models of the liquidity effect, studied by Grossman and Weiss (1983), and Rotemberg (1984). In these economies, goods and financial markets are separated, and only half the agents are in the financial market in any given period. The money injection is therefore asymmetric, since only the agents in the financial market receive it. In these economies, however, the distributional effects are allowed to persist forever. Since this makes the analysis of money shocks somewhat intractable, the authors are confined to examining the effects of a one time monetary injection in an otherwise deterministic setting.

To abstract from such wealth effects, we use the representative family assumption of the more recent liquidity models discussed in the previous section. The financial structure of this model is similar to that of Fuerst (1992). It is convenient, at this point, to reiterate Fuerst's (1992) interpretation of the timing of events and the nature of transactions specific to this structure. We assume that the economy is populated with a large number of "families" which consist of members that engage in different trades. A representative family consists of a worker-shopper pair, a firm manager, and a financial intermediary. At the beginning of the period the representative family starts with M_t units of money balances, and chooses to deposit N_t units with the financial intermediary. The family then

[6] Fuerst(1994) assumes that a fixed fraction of the injection goes to the shopper in the goods market. The model here assumes that all of the monetary injection is given only to the financial intermediary. Dropping the assumption that the shopper gets part of the injection is not crucial to the results of the paper, and simplifies the analysis considerably.

separates and each member travels to distinct locations, after which the state of the world - the monetary shock and the technology shock, are revealed. The shopper is in the goods market to purchase goods for consumption, while the worker offers L_t units of labor in the labor market. The firm and financial intermediary are in the credit market. The firm borrows $B_t = N_t + X_t$ dollars from the financial intermediary, to be repaid at the end of the period at a positive nominal interest rate of R_t, and then travels to the labor market. The firm then offers to hire H_t units of labor at the nominal wage rate of W_t. Using the borrowings B_t to finance the wage bill, the firm uses labor services to produce output for sale in the goods market. Liquidity effects arise in this model in the same sense as in Fuerst (1992): A large monetary injection implies that interest rates must fall in order to induce firms to induce firms to absorb the excess cash in the financial market. The excess cash in the hands of the firm also stimulates labor demand, and consequently employment and output.

At the end of the period, the firm repays the loan from the financial intermediary, and all members of the representative family reunite and pool their cash receipts. The family therefore enters period *t+1* with money balances given by

$$M_{t+1} = [M_t + N_t R_t + W_t L_t - P_t c_t] + [X_t(1+R_t)] \quad + [P_t f(H_t, s_t) - W_t H_t - B_t R_t] \tag{5}$$

The representative family's optimization problem thus involves choosing N_t, c_t, L_t, B_t, and H_t to maximize preferences given by (1), subject to constraints (3)-(5).

Since all nominal variables in the economy grow at the same rate as the nominal money supply, we rescale all nominal variables by the beginning of period per capita money stock. Let $m_t = \frac{M_t}{M_t^s}$, $p_t = \frac{P_t}{M_t^s}$, $w_t = \frac{W_t}{M_t^s}$, $b_t = \frac{B_t}{M_t^s}$, and $x_t = \frac{X_t}{M_t^s}$, denote the rescaled nominal variables. We further assume that x_t is i.i.d. The household's dynamic programming problem is then given by

$$V(m_t, s_t) = \max_{n_t \in [0, m_t]} \int \max_{c_t, L_t, b_t, H_t} \{u(c_t, 1-L_t) + b\,(u(c_t, 1-L_t))V(m_{t+1}, s_{t+1})\}\Phi(ds_{t+1}), \tag{6}$$

subject to

$$m_t - n_t \geq p_t c_t \tag{7}$$

$$b_t \geq w_t H_t \tag{8}$$

$$m_{t+1} = \frac{m_t + n_t R_t + w_t L_t - p_t c_t + x_t(1+R_t) + p_t f(H_t, s_{t+1}) - w_t H_t - b_t R_t}{1+x_t}, \tag{9}$$

where $V(m_t, s_t)$ represents the value function corresponding to the family's problem.

The equilibrium conditions in the goods market, the money market, the labor market, the credit market, and the capital market are respectively given by

$$c_t = f(H_t, s_{t+1}), \tag{10}$$

and $m_{t+1} = m_t = 1$, $L_t = H_t$, $b_t = n_t + x_t$. Denote by l_{1t} and l_{2t} the Lagrangian multipliers associated with constraints (7) and (8) respectively. After imposing the equilibrium conditions, the first order conditions for n_t, c_t, L_t, b_t, H_t, k_{t+1}, l_{1t}, and l_{2t} may be expressed as

$$\int \frac{b(u(c_t, 1-L_t))V_m(s_{t+1})R_t(s_{t+1})}{1+x_t(s_{t+1})}\Phi(ds_{t+1}) = \int l_{1t}(s_{t+1})\Phi(ds_{t+1}), \qquad (11)$$

$$u_1(c_t, 1-L_t)\{1+b'(u(c_t, 1-L_t))V(s_{t+1})\}$$

$$-\frac{b(u(c_t, 1-L_t))V_m(s_{t+1})p_t(s_{t+1})}{1+x_t(s_{t+1})} = l_{1t}(k_t, s_{t+1})p_t(s_{t+1}), \qquad (12)$$

$$u_2(c_t, 1-L_t)\{1+b'(u(c_t, 1-L_t))V(s_{t+1})\} = \frac{b(u(c_t, 1-L_t))V_m(s_{t+1})w_t(s_{t+1})}{1+x_t(s_{t+1})}, \qquad (13)$$

$$\frac{b(u(c_t, 1-L_t))V_m(s_{t+1})R_t(s_{t+1})}{1+x_t(s_{t+1})} = l_{2t}(s_{t+1}), \qquad (14)$$

$$\frac{b(u(c_t, 1-L_t))V_m(s_{t+1})\{p_t(s_{t+1})f_L(L_t, s_{t+1}) - w_t(s_{t+1})\}}{1+x_t(s_{t+1})}$$

$$= l_{2t}(s_{t+1})w_t(s_{t+1}), \qquad (15)$$

$$1-n_t(s_{t+1}) \geq p_t(s_{t+1})c_t(s_{t+1}),$$

$$\text{with equality if} \quad l_{1t}(s_{t+1}) > 0, \qquad (16)$$

$$n_t(s_{t+1}) + x_t(s_{t+1}) \geq w_t(s_{t+1})L_t(s_{t+1}),$$

$$\text{with equality if} \quad l_{2t}(s_{t+1}) > 0. \qquad (17)$$

Envelope conditions are given by

$$V_m(s_t) = \int \frac{u_1(c_t, 1-L_t)\{1+b'(u(c_t, 1-L_t))V(s_{t+1})\}}{p_t}\Phi(ds_{t+1}) \qquad (18)$$

The equilibrium conditions above collapse to their fixed time preference versions when we set $b'(u)$ equal to zero, and replace the endogenous discount factor by a fixed discount factor. As shown in Fuerst (1992), we can illustrate the presence of a 'liquidity effect component' in interest rates, in addition to standard Fisherian fundamentals. Using equations (12), (14), and (18), for example, we can derive,

$$1+R_t = \frac{\dfrac{u_1(t)\{1+b'(u(t))V(t+1)\}}{P_t} + \dfrac{l_{2t}-l_{1t}}{M_t^s}}{b(u(c_t, 1-L_t))E\left[\dfrac{u_1(t+1)\{1+b'(u(t+1))V(t+2)\}}{P_{t+1}}\right]}. \qquad (19)$$

In the above expression, and from now on we have surpressed the arguments of various functions for notational convenience, and to economize on space. In the absence of a flexible discount factor, and also of liquidity effects, which are captured by the term $\frac{l_{2t}-l_{1t}}{M_t^s}$, equation (21) implies a Fisherian decomposition of nominal interest rates into the real rate of interest, and an anticipated inflation component. In the fixed time preference economy with liquidity effects, an asymmetric monetary injection through the financial market will change the relative marginal value of cash in the goods market and the financial market, so that $l_2 < l_1$. Holding the anticipated inflation effect fixed, this implies that nominal interest rates will be lower than predicted by Fisherian fundamentals. This is also true of the endogenous time preference economy. However, introducing variability in the discount factor clearly affects the relative roles of the liquidity component and the Fisherian components in determining interest rates: in some sense, the weights assigned to each of these components is now different. The flexible discount factor economy is then potentially consistent with stronger as well as weaker liquidity effects, relative to the fixed discount factor economy. Furthermore, in addition to liquidity effects and Fisherian effects, there is now an additional source of *variability* in nominal interest rates. Since the nature and extent of monetary non-neutralities may be different, it is also natural to expect that the role of monetary policy will not be the same as in the constant time preference framework.

3. Analysis of Quantitative Experiments

We can characterize an equilibrium for this economy in essentially the same fashion as in Fuerst (1992). Consider the case in which the cash-in-advance constraint binds in all states of the world. The state is, as mentioned before, assumed to be i.i.d. Also, for the purpose of the numerical experiments of this section, we let the number of states be four. Two steps are involved in characterizing the economy's equilibrium: First, we obtain expressions for $p_t, w_t, l_{1t}, l_{2t}, R_t$ in terms of n_t, L_t, V_m, and V. These are derived using equations (16), (17), (12), (15), and (14), and are respectively given by

$$p_t = \frac{1-n}{f(t)}, \tag{20}$$

$$w_t = \frac{n+x_t}{L_t} = \frac{A\{1+b'(u(t))V(t+1)\}(1+x_t)}{b(u(t))V_m(t+1)}, \tag{21}$$

$$l_{1t} = \frac{\{1+b'(u(t))V(t+1)\}}{1-n} - \frac{b(u(t))V_m(t+1)}{(1+x_t)}, \tag{22}$$

$$l_{2t} = \frac{(1-n)gb(u(t))V_m(t+1)}{(n+x_t)(1+x_t)} - \frac{b(u(t))V_m(t+1)}{(1+x_t)}. \tag{23}$$

$$R_t = \frac{(1-n_t)g}{n_t + x_t} - 1. \tag{24}$$

Note that, since we have assumed shocks to be i.i.d, it is natural to conjecture that n and V_m are constants. In addition, let us assume $u(c_t, 1-L_t) = \log(c_t) - AL_t$, and $c_t = f(t) = q(s_{t+1})L_t^g$. Then, substituting the expressions for p_t, w_t, l_{1t}, l_{2t}, into equations (11), (13), and (16), we see that the equilibrium is represented by constants $n \in (0,1)$, $V_m > 0$, V, and a function $L_t \in (0,1)$ that satisfy

$$\int \frac{\{1+b'(u(t))V(t+1)\}}{1-n}\Phi(ds_{t+1}) = \int \frac{[b(u(t))V_m(t+1)]^2(1-n)f_L(t)}{f(t)A\{1+b'(u(t))V(t+1)\}(1+x_t)^2}\Phi(ds_{t+1}), \tag{25}$$

$$L_t = \frac{b(u(t))V_m(t+1)(n+x_t)}{A\{1+b'u(t))V(t+1)\}(1+x_t)}, \tag{26}$$

$$V_m(t+1) = \int \frac{\{1+b'(u(t+1))V(t+2)\}}{1-n}\Phi(ds_{t+1}), \tag{27}$$

in addition to the Bellman equation, which is given by,

$$V(t+1) = E\left[u(t+1) + b(u(t+1))V(t+2)\right]. \tag{28}$$

Solving this system involves using an initial guess for $V(t+2)$, c_{t+1}, and L_{t+1} to compute $V(t+1)$ [7]. We can then use (25)-(28) to solve for n, V_m, and the 4×1 vector L and update the guesses for $V(t+2)$, c_{t+1}, and L_{t+1}, for the next iteration. This procedure is repeated until $\left|\frac{V(t+1)-V(t+2)}{V(t+2)}\right| \le 10^{-7}$.

A. Liquidity Effects and Other Monetary Non-neutralities

We now consider the results of numerical simulations of the model, in order to examine the nature of monetary non-neutralities in the fixed and flexible discount factor economies. Tables 1-5 present the simulations for this subsection. The purpose of this subsection is to establish that the nature of monetary non-neutralities can be very similar for both models. The next subsection then demonstrates that optimal monetary policy can be very different in spite of these similarities.

Consider, for example Table 1which examines the liquidity effects of an unanticipated monetary injection. Here, the productivity shock $q \equiv 1$, so the four states of the economy correspond to money growth rates of 3%, 5%, 7%, and 9%, and there are no *real* sources of uncertainty in this economy. In addition, we set $g = .64$, and $A = .75$ [8]. We choose $t = .8$. The parameter h is chosen such that the initial steady state of the discount factor coincides with that of the fixed time preference model. The upper panel in each of these tables presents the flexible discount factor economy, while the lower panel presents the corresponding fixed discount factor version. Comparing the fixed and flexible discount factor economies studied in Table 1, we observe that monetary injections have qualitatively similar effects in both cases. There is a liquidity effect in both economies: interest rates are decreasing in the monetary injection. Lower interest rates imply a lower opportunity cost of using cash to finance purchases of labor, so that equilibrium work effort, and therefore output are increasing in x. To hire more units of labor, the firm has to offer a higher nominal wage, which is thus increasing in x. The current price level, on the other hand, is decreasing in x, since the shopper in the goods market has a fixed supply of cash and the increased supply of output must bring down prices[9]. Another way to illustrate liquidity effects is to compare the marginal value of cash in the goods and financial markets, given by l_{1t} and l_{2t} respectively - in both economies l_{1t} is increasing in the monetary injection while l_{2t} is decreasing in the monetary injection.

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7 We use the steady states of the fixed time preference model as initial guesses. For some parameter values it was not possible to compute an equilibrium.

8 The value of g chosen is the same as that in Fuerst (1992). In Fuerst's (1994) paper g is assumed stochastic, so that there are two types of productivity shocks in the economy. In his analysis of optimal monetary policy Fuerst therefore considers two types of experiments: In one case, g is held fixed and q is allowed to vary, and the opposite in another. The experiments considered in this paper analogous to the former case. The choice of A=.75 was dictated by computational tractability.

9 Allowing variability in capital may cause prices to increase in the money shock - a feature more acceptable in light of the conventional view that money shocks cause the price level to increase. Alternatively if a fraction of the monetary injection is allowed to go into the goods market, as in Fuerst (1994), prices may be increasing in x. In this paper, however, we are focusing only on supply side effects of the money shocks.

However, Table 1 illustrates that the endogenous discount factor economy may be potentially consistent with larger liquidity effects, in the sense that larger than average monetary injections produce a larger than average fall in interest rates compared to the fixed discount factor economy. Compare, for example the rows which present the variables relative to their means. Work effort, and consequently consumption are now a little more responsive to changes in the money growth rate. Monetary injections can therefore have a greater positive impact on real activity. One may wish to assign the following interpretation to this outcome. The discount factor, as shown in Table 1 is decreasing in the monetary injection. This "impatience" with respect to current consumption benefits translates into larger increases in contemporaneous consumption, output and work effort[10]. Liquidity effects in the variable time preference are also amplified in another sense. Note that the monetary injection, $n+x$, is *lower* than in the benchmark model, and yet the model produces a steeper money-interest rate relation. The impatience induced by a positive correlation between monetary injections leads to a *lower* percentage savings of $n_t = \frac{N_t}{M_t}$ compared to the fixed time preference case. As noted by Fuerst (1993), although the monetary injection causes aggregate money supply to increase by $x_t = \frac{X_t}{M_t}\%$, the share of cash in the financial market increases by $\frac{N_t + X_t - N_t / M_t}{N_t / M_t}\% = \frac{x_t}{n_t} > x_t$. Other things being equal, the magnitude of the liquidity effect is strictly decreasing in equilibrium n_t, implying a larger liquidity effect in the variable time preference economy.

It is important at this point to mention that, the opposite case – in which the impact of monetary injections is slightly weaker in the variable discount factor economy – is also possible. Experiments with different combinations of q and A (which are not reported here) showed this to be the case, the results again being qualitatively similar in both economies, with very small quantitative differences. Furthermore, although we observe a positive correlation between monetary injections and real activity there is no exploitable Phillips curve type trade-off in any of the cases presented here. If, for example, we increase the *mean* of the monetary injection, real activity in both economies will be lower in all states of the world. The presence of such *inflation tax* or *anticipated inflation* effects was confirmed using numerical simulations, which are not reported here. Inflation tax effects are also illustrated in Table 2, which presents the deterministic versions of the economies studied in Tables 1. These effects are well known to all cash-in-advance models. In both economies, inflation tax distortions cause substitution out of cash goods (consumption and work effort), towards credit goods (leisure), as money growth increases from 5% to 10%.

The small quantitative differences discussed above may, however, may lead one to expect that the role of monetary policy is not likely to be very different in the presence of endogenous time preference. However, as will be shown in the next subsection, and this is the key point of the paper, for a wide range of parameters a different role for monetary policy is likely to emerge. This difference arises due to the presence of uncertainty in the real sector of the economy. To see this, consider equilibrium work effort in the variable discount factor economy, which is implicitly given by

$$L_t = \frac{b\,(u(t))V_m(t+1)(n+x_t)}{A\{1+b'(u(t))V(t+1)\}(1+x_t)},$$

and compare it with its fixed time preference version

$$L_t = \frac{bV_m(t+1)(n+x_t)}{A(1+x_t)}.$$

[10] Clearly, this is just an interpretation. A different outcome is quite plausible, given that utility depends on both consumption and leisure. The parameters A, q and g are relevant to the labor-leisure decision - lower work effort implies a higher level of leisure and hence utility. On the other hand output is lower, so that utility is lower. The flexible time preference model is thus consistent with higher as well as lower levels of leisure than in the fixed time preference case.

In the case of i.i.d. shocks, $V_m(t+1)$ is a constant, and is independent of the productivity shock q. This implies that work effort in the fixed time preference economy is independent of q, unless the monetary injection varies with q. In the flexible discount factor economy, however, work effort is affected by q via the discount factor. Other things being equal, an increase in utility will cause the numerator to fall and the denominator to increase since $b''(u(t)) > 0$, so that work effort *may* be decreasing in q, since q positively affects utility. In any case, work effort is not independent of q in the variable time preference economy. This suggests that the nature and extent of the impact of monetary injections on the variablity of output will be different in the two economies. Furthermore, the role of monetary policy will not be the same, which motivates some of the analysis in subsection B below.

The next three simulations examined are presented in Tables 3-5, all of which allow q to vary across states . Furthermore, the average monetary injection is 5% in all of the cases considered. In Table 3 we illustrate a case in which monetary policy is deterministic, so that $Cov(x,q) = 0$. Tables 4 and 5 represent countercyclical ($Cov(x,q) = -.0033$) and procyclical monetary policy ($Cov(x,q) = .0033$) respectively. Again, in both economies procyclical policy has the effect of amplifying fluctuations in output (consumption), while countercyclical policy dampens them. Flexibility in the rate of time preference, however, affects the extent to which these fluctuations are amplified or dampened. Moving from deterministic to procyclical monetary policy amplifies fluctuations by 27.6% in the variable discount factor economy, and by 25.1% in the fixed time preference case[11]. Moving from deterministic to countercyclical policy decreases fluctuations in the variable discount factor economy by 25.4%, and in the fixed discount factor economy by 22.9%. In some sense, there is an additional channel through which monetary policy affects fluctuations in the variable time preference economy - household decisions are now indirectly affected by changes in their utility discount factor.

Again, note that simulations 3-5 confirm work effort in the flexible time preference economy to be decreasing in q, while it is independent of q in the fixed time preference economy. The intuition for this behavior is fairly straightforward. In both economies, the representative family, in effect, allocates cash for employment purposes before q is observed. The firm thus has a supply of cash ($n+x$) for hiring workers that is independent of q. The worker in the fixed time preference economy bases his current work effort decision on the nominal wage rate, since the state of the economy is i.i.d. The nominal wage is in turn determined by the fixed supply of cash. The worker and the firm in the endogenous time preference economy, however, behave differently, since they are members of a family that becomes more impatient for current utility as q increases. For a fixed level of q, increasing leisure has a direct and positive effect on utility, and also an indirect negative effect via the fall in output. If q, on the other hand is increasing across states, an increase in leisure implies less of a sacrifice in terms of the decrease in current utility. Work effort in the variable discount factor framework is thus decreasing in q. This feature of the model has interesting implications for optimal monetary policy, as will become clear from the discussion below.

B. Welfare Costs and Optimal Monetary Policy

In this subsection we discuss monetary policies designed to make competitive equilibrium Pareto optimal. We also compute welfare costs associated with policies that deviate from the optimum. We emphasize that the purpose of computing welfare costs is not intended for comparison of the *levels* of these costs across the fixed and variable discount factor economies. Such a comparison would be inappropriate since the preferences in the two economies are different. The reason for performing a second best exercise of this type is essentialy the same as that in Fuerst (1994): the optimal policies, as we will see below, need not be unique, but may nevertheless be taken seriously, since they are qualitatively similar to solutions of the second best problem.

In order to compute welfare costs, we first need to discuss what conditions must be imposed on the economy for monetary policy to be optimal. The analysis here is similar to that of Fuerst (1994). As in the fixed time preference economy, welfare costs arise due to two types of frictions. Firstly, there are cash-in-advance constraints on various transactions, so that there are inflation tax distortions that can only be eliminated by deflation. Secondly, there is an additional friction imposed due to the inability of the representative family to transfer funds between goods and

[11] We can get larger differences for some parameter values.

financial markets, which has the effect of making the financial market relatively more liquid. Intuitively, then, one would expect optimal monetary policy to be one that varies with productivity shocks, since monetary shocks must be such that the marginal value of cash in the goods and financial markets is equated.

To eliminate inflation tax effects, we must allow x to take negative values, i.e., impose, following Fuerst (1994), $x > -1$. In addition, optimal work effort, by definition is given by L_t that solves

$$\frac{u_2(t)}{u_1(t)} = f_L(t) \tag{29}$$

From the first order conditions for this economy, the competitive equilibrium is Pareto optimal (or work effort is equal to the above expression), *iff* $l_{1t} = l_{2t} = 0, \forall t$. It can be easily verified that $l_{1t} = l_{2t} = 0$ *iff*[12]

$$\frac{b(u(t))V_m(t+1)}{(1+x_t)} \geq \frac{u_1(t)\{1+b'(u(t))V(t+1)\}f(t)}{1-n}, \tag{30}$$

$$\frac{b(u(t))V_m(t+1)}{(1+x_t)} \geq \frac{gu_1(t)\{1+b'(u(t))V(t+1)\}f(t)(1+x_t)}{(n+x_t)b(u(t))V_m(t+1)}. \tag{31}$$

An additional condition is needed to ensure stationary equilibrium - cash in advance constraints must bind in at least one state so that the above expressions hold with equality for at least one state of the world. If the above two conditions are satisfied then the equilibrium condition above (27) collapses to

$$\int \frac{b(u(t))}{(1+x_t)}\Phi(ds_{t+1}) = 1, \tag{32}$$

and L_t is implicitly given by

$$L_t = \frac{gf(t)u_1(t)}{u_2(t)}. \tag{33}$$

Optimal monetary equilibrium in the endogenous time preference economy is thus given by a function $x\colon S \to R$, and constants $n \in (0,1)$, $V_m > 0$, such that

$$\frac{b(u(t))V_m(t+1)}{(1+x_t)} \geq \frac{u_1(t)\{1+b'(u(t))V(t+1)\}f(t)}{1-n}, \tag{34}$$

with equality for some $s_{t+1} \in S$,

$$\frac{b(u(t))V_m(t+1)}{(1+x_t)} \geq \frac{gu_1(t)\{1+b'(u(t))V(t+1)\}f(t)(1+x_t)}{(n+x_t)}, \tag{35}$$

with equality for some $s_{t+1} \in S$,

$$\int \frac{b(u(t))}{(1+x_t)}\Phi(ds_{t+1}) = 1, \tag{36}$$

where the arguments in the utility function are given by $c(s_{t+1}) = q(s_{t+1})L(s_{t+1})^g$, and $L: S \to (0,1)$ is given by (37). These conditions collapse to the definition of optimal monetary policy in the fixed time preference version if we set

[12] We use equations (12), (15), and the non-binding constraints.

$b'(u(t)) = 0$, and $b(u(t)) \equiv b$. Equation (36), of course, represents the endogenous time preference equivalent of the stochastic version of the *Friedman rule*, which arises in several cash-in-advance models. Equations (34) and (35), on the other hand, are of the type that have been shown by Fuerst (1994) to arise in the context of liquidity effect models. As in the case of fixed time preference models, the conditions above allow a lot of flexibility, so that optimal monetary policy need not be unique.

To illustrate the differences in optimal monetary policy in the fixed and flexible time preference models, let us first consider the deterministic version of the model discussed above. Since there is no uncertainty, we have $l_1 = l_2$[13], so that there are no distortions arising because of frictions associated with liquidity effects. Optimal monetary policy is therefore required only to take care of inflation tax distortions. Equation (360) implies that optimal monetary policy is given by $x = b(u) - 1$. The fixed time preference version is simply the Friedman rule $x = b - 1$, and it is quite clear that optimal monetary policy can, at least in a quantitative sense, be different in the presence of endogenous time preference. Equilibrium n and V_m are given by deterministic versions of (34) and (35), in which these equations must hold with equality in all states.

We now turn to the case in which monetary injections are i.i.d.. For the class of utility functions given by $\frac{c^{1-s} - 1}{1-s} - AL$, equation (33) implies that optimal labor is given by

$$L(s_{t+1}) = \left[\frac{gq(s_{t+1})^{1-s}}{A}\right]^{1/(1-g(1-s))} \tag{37}$$

and this is also true of the fixed time preference economy[14]. Optimal monetary policy in both economies then varies with the degree of risk aversion s. In the log utility case, $s = 1$, and income and substitution effects exactly offset each other, so that optimal work effort is independent of productivity shocks. For $s < 1$, the substitution effect is dominant implying that optimal work effort is increasing in productivity shocks, while for $s > 1$, income effects dominate, so that optimal work effort is decreasing in productivity shocks.

Now consider competitive equilibrium work effort in the two economies. As discussed before, competitive equilibrium work effort in the fixed time preference economy is independent of q, while in the endogenous time preference economy it is likely to be decreasing in q - this is also confirmed by the numerical simulations studied above. Optimal work effort for both economies, on the other hand is given by equation (37). In the fixed time preference economy it is clear that for $s < 1$, optimal monetary policy is likely to be procyclical, as we want work effort to be increasing in q. Furthermore, for $s > 1$, countercyclical policy is required, while for $s = 1$, optimal monetary policy is likely to be deterministic (- i.e. fixed independently of q). However, given that competitive equilibrium work effort is *decreasing* in q in the endogenous discount factor economy, optimal monetary policy will evidently not be deterministic, as in the fixed time preference model. Consider for example the log utility case presented in Tables 3-5. Since optimal labor is independent of q, optimal monetary policy would entail counteraction of the effect of productivity shocks, which cause work effort to fall in the good states. Since competitive equilibrium work effort is *increasing* in the money injection, this would be achieved by procyclical monetary policy. This intuition is confirmed when we look at Table 6, in which welfare costs expressed relative to mean consumption are computed for the alternative monetary policies presented in Tables 3-5[15]. While the reduction

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13 To see this, note that equations (11) and (14) imply $E\,l_{1t} = E\,l_{2t}$.

14 In the log utility case studied in this paper, optimal L is simply given by $\frac{g}{A}$.

15 The welfare cost under any monetary policy is implicitly defined by Δc that solves:

$$E[u(c^* + \Delta c, 1 - L^*)] = E[u(\hat{c}, 1 - \hat{L})],$$

in welfare losses in moving from deterministic to activist monetary policy are extremely marginal in the fixed time preference case, procyclical policy is clearly the best alternative in the endogenous time preference framework[16].

To explicitly calculate an optimal monetary policy that achieves a level of labor given by (33), and has the characteristics described above, we follow the strategy pursued in Fuerst (1994). Since optimal monetary policy is obviously not unique, we search for optimal monetary equilibrium within the class in which the cash-in-advance constraint in the financial market is "just binding", so that $l_1 = l_2 = 0$, and $n + x = wL$. In terms of the definition of optimal policy above this means that (31) holds with for all $s_{t+1} \in S$. Then (30) and (31) imply

$$\frac{g(1-n)}{n+x_t} \geq 1 \qquad \forall s_{t+1} \in S$$

and with equality for some $s_{t+1} \in S$. Equilibrium n in the flexible (as well as fixed) discount factor economy is then given by

$$n \leq \frac{g - x_t}{1+g},$$

with equality for some $s_{t+1} \in S$. Since the R.H.S. is decreasing in x,

$$n = \frac{g - \tilde{x}}{1+g}, \tag{38}$$

where $\tilde{x} = \max\left(x_t(s_{t+1})\right)$. Equations (35) and (36) imply

$$V_m(t) = \int \frac{b(u(t))V_m(t+1)}{(1+x_t)} \Phi(ds_{t+1}),$$

$$= \int \frac{gu_1(t)\{1+b'(u(t))V(t+1)\}f(t)}{(n+x_t)} \Phi(ds_{t+1}) \tag{39}$$

$$\frac{b(u(t))V_m(t+1)(n+x_t)}{(1+x_t)} \geq gu_1(t)\{1+b'(u(t))V(t+1)\}f(t), \tag{40}$$

Optimal monetary policy can then be numerically computed by iterating on the Bellman equation (28) and using (38)-(40) to compute n, V_m, and x at each step of the iteration, until convergence is achieved. Table 7 presents optimal monetary policy for the log utility case, which, as expected, is procyclical for the endogenous time preference model, (given by $x = [-.1060, -.0989, -.0919, -.0848]$), and deterministic for the fixed time preference model, (given by $x = [-.05, -.05, -.05, -.05]$). Simulations for the more general case of utility functions $\frac{c^{1-s} - 1}{1-s} - AL$ are summarized in Figure 1. Since an endogenous discount factor appears to have the same effect in the log utility case as would be the impact of decreasing the level of risk aversion, we would expect that for the $s < 1$ case, optimal monetary policy will be even more procyclical. The case $s > 1$, on the other hand, may reduce

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where c^* and L^* denote competitive equilibrium consumption and labor, and $\hat{c}$ and $\hat{L}$ denote optimal consumption and labor.

[16] An interesting extension would be to study how welfare costs would be affected by the interaction of monetary and fiscal policies. In cash-in-advance models that focus on inflation tax effects, the presence of other distortionary taxes has been shown to double welfare costs of inflationary policies. See for example Cooley and Hansen (1991).

the role for procyclical policy, and for relatively large s we would expect optimal monetary policy to be countercyclical, but to a lesser degree than in the fixed time preference case. We can confirm this intuition numerically as shown in Figure 1. Also note that the interval in which monetary policy is qualitatively different corresponds roughly to $s \in (1, 1.17)$. According to Prescott (1986) the range in which estimates of this parameter lie corresponds to [1, 2], with more plausible values being "close to 1". An activist countercyclical role for monetary policy is typically difficult to find within cash-in-advance frameworks, even in the case of cash-in-advance models with Keynesian features such as the endogenous sticky-price model of Ireland (1996). The analysis here, in some sense, confirms earlier conclusions about non-activist monetary policy. There are, however, plausible ranges for s for which optimal policy is qualitatively similar in both economies. For $s > 1.17$, for example, activism on part of the central bank ha a welfare improving role in both economies. However, as we have seen above, the conditions required for optimality are far more complicated in the variable time preference economy. That is, it may not be possible to implement the optimal policy, since more information is required relative to the fixed time preference case.

4. Conclusions

This paper can be described as an extension of Fuerst's (1994) work, which characterizes the nature of optimal monetary policy in a general equilibrium model with two types of distortions. One of these frictions is due to the presence of cash-in-advance restrictions. The second friction arises due to the assumption that goods and financial markets are segmented, so that expansionary monetary policy shocks can temporarily lower the equilibrium interest rate, and thus lead to a liquidity effect. Fuerst shows that these features can imply, (for a range of parameters), that the appropriate monetary policy is one which responds in an activist countercyclical manner to real shocks that hit the economy. This paper extends Fuerst's by allowing the representative agent's discount factor to be endogenous, in a manner suggested by Epstein (1983). The paper demonstrates that while conclusions regarding the response of the economy to monetary shocks are not at risk from this generalization, the nature of optimal monetary policy can change in some cases. Furthermore, even in cases where the nature of optimal policy does not change, carrying out the appropriate policy is a more complicated exercise. The central bank now requires information on the preference parameters relating to subjective discount rates, which are not directly observable, in addition to other fundamentals such as the nature of productivity shocks.

The conclusions here are, of course, subject to similar caveats as the Fuerst's (1994). As in that paper, we assume that shocks to the economy are i.i.d, and that there is no capital accumulation. These assumptions, however allow the analysis to be more tractable, in addition to maintaining comparability with Fuerst's analysis.

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Table 1. Liquidity Effects

Endogenous Time Preference $n = .3187$; $V_m = 11.24$	$b(u) = e^{-(h+tu)}$, $t = 0.8$; $u(c_t, 1-L_t) = \log(c_t) - AL_t$, $A = 0.75$; $c_t = q(s_{t+1})L_t^g$, $g = 0.64$				Mean	Standard Deviation
x	.03	.05	.07	.09	.06	.0258
q	1.0	1.0	1.0	1.0	1.0	0
Consumption	.7268	.7436	.7597	.7750	.7513	.0208
Labor	.6073	.6295	.6509	.6715	.6398	.0276
Interest Rates	1.2502	1.1824	1.1216	1.0667	1.1552	.0790
Price Level	.9374	.9161	.8967	.8790	.9073	.0251
Wage Rate	.5742	.5858	.5972	.6087	.5915	.0418
l_1	1.2458	1.4375	1.6203	1.7952	1.5247	.2364
l_2	2.5165	1.7905	1.1660	.6257	1.5247	.8147
$b(u)$	.9219	.9173	.9134	.9101	.9157	.0051
Cons./Mean Cons.	.9674	.9898	1.0112	1.0316		
Labor/Mean Labor	.9492	.9839	1.0173	1.0495		
Int. Rate/Mean Int. Rate	1.0822	1.0235	.9709	.9234		
Prices/Mean Prices	1.0331	1.0097	.9883	.9688		
Wages/Mean Wages	.9708	.9903	1.0097	1.0291		
l_1/Mean l_1	.8171	.9428	1.0627	1.1774		
l_2/Mean l_2	1.6505	1.1743	.7648	.4103		
Fixed Time Preference $n = .3276$; $V_m = 1.4873$	$A = .75$; $g = 0.64$; $t = 0$.				Mean	Standard Deviation
x	.03	.05	.07	.09	.06	.0258
q	1.0	1.0	1.0	1.0	1.0	0
Consumption	.7621	.7795	.7960	.8197	.7873	.0213
Labor	.6541	.6775	.7001	.7218	.6884	.0291
Interest Rates	1.2033	1.1396	1.0823	1.0305	1.1139	.0744
Price Level	.8823	.8626	.8448	.8284	.8545	.0232
Wage Rate	.5467	.5573	.5680	.5786	.5627	.0137
l_1	.1155	.1416	.1667	.1910	.1537	.0325
l_2	.2789	.1879	.1087	.0395	.1537	.1032
b	.95	.95	.95	.95	.95	0
Cons./Mean Cons.	.9680	.9900	1.0110	1.0310		
Labor/Mean Labor	.9502	.9842	1.0170	1.0486		
Int. Rate/Mean Int. Rate	1.0803	1.0231	.9716	.9251		
Prices/Mean Prices	1.0325	1.0095	.9886	.9694		
Wages/Mean Wages	.9717	.9906	1.0094	1.0283		
l_1/Mean l_1	.7513	.9213	1.0849	1.2426		
l_2/Mean l_2	1.8144	1.2220	.7068	.2560		

Table 2. Inflation Tax Effects

Endogenous Time Preference	$b(u)=e^{-(h+tu)}$, $t=0.8$; $u(c_t,1-L_t)=\log(c_t)-AL_t$, $A=0.75$; $c_t=q(s_{t+1})L_t^g$, $g=0.64$, $q\equiv 1$	
	$x\equiv 0.05$	$x\equiv 0.10$
Consumption	.7600	.7113
Labor	.6513	.5873
Interest Rates	1.1417	1.2054
Price Level	.8862	1.0101
Wage Rate	.5782	.6496
l_1	1.4580	1.7839
l_2	1.4580	1.7839
$b(u)$	.9173	.9126
n	.3265	.2815
V_m	11.5354	10.4693
Fixed Time Preference	$A=.75$; $g=0.64$; $t=0$, $q\equiv 1$	
	$x\equiv 0.05$	$x\equiv 0.10$
Consumption	.7948	.7489
Labor	.6985	.6364
Interest Rates	1.1053	1.1579
Price Level	.8366	.9460
Wage Rate	.5512	.6152
l_1	.1432	.1925
l_2	.1432	.1925
b	.95	.95
n	.3350	.2916
V_m	1.5039	1.4116

Table 3. Deterministic Monetary Policy with Real Uncertainty. ($Cov(x,q)=0$).

Endogenous Time Preference $n=.3271$; $V_m=10.02$	$b(u)=e^{-(h+tu)}$, $t=0.8$; $u(c_t,1-L_t)=\log(c_t)-AL_t$, $A=0.75$; $c_t=q(s_{t+1})L_t^g$, $g=0.64$				Mean	Standard Deviation
x	.05	.05	.05	.05	.05	0
q	1.0	1.1	1.2	1.3	1.15	.1291
$b(u)$	1.0196	.9459	.8834	.8296	.9196	.0819
Cons./Mean Cons.	.8788	.9602	1.0407	1.1203	.8757	.0910
Labor/Mean Labor	1.0155	1.0049	.9947	.9849	.6541	.0086
Int. Rate/Mean Int. Rate	1.0	1.0	1.0	1.0	1.1418	0
Prices/Mean Prices	1.1286	1.0330	.9531	.8854	.7746	.0812
Wages/Mean Wages	.9846	.9950	1.0052	1.0152	.5767	.0026
l_1/Mean l_1	.9788	.9944	1.0077	1.0191	1.2440	.0216
l_2/Mean l_2	1.1087	1.0286	.9606	.9021	1.2440	.1107
Fixed Time Preference $n=.3350$; $V_m=1.5039$	$A=.75$; $g=0.64$; $t=0$.				Mean	Standard Deviation
x	.05	.05	.05	.05	.05	0
q	1.0	1.1	1.2	1.3	1.15	.1291
b	.95	.95	.95	.95	.95	0
Cons./Mean Cons.	.8696	.9565	1.0435	1.1304	.9141	.1026
Labor/Mean Labor	1.0	1.0	1.0	1.0	.6985	0
Int. Rate/Mean Int. Rate	1.0	1.0	1.0	1.0	1.1053	0
Prices/Mean Prices	1.1391	1.0355	.9492	.8762	.7345	.0832
Wages/Mean Wages	1.0	1.0	1.0	1.0	.5512	0
l_1/Mean l_1	1.0	1.0	1.0	1.0	.1432	0
l_2/Mean l_2	1.0	1.0	1.0	1.0	.1432	0

Table 4. Countercyclical Monetary Policy ($Cov(x,q) = -.0033$).

Endogenous Time Preference $n = .3273; V_m = 9.998$	$b(u) = e^{-(h+tu)}$, $t = 0.8$; $u(c_t, 1-L_t) = \log(c_t) - AL_t$, $A = 0.75$; $c_t = q(s_{t+1})L_t^g$, $g = 0.64$				Mean	Standard Deviation
x	.075	.075	.025	.025	.05	
q	1.0	1.1	1.2	1.3	1.15	.1291
$b(u)$	1.0146	.9411	.8878	.8338	.9193	.0771
Cons./Mean Cons.	.9043	.9880	1.0250	1.0927	.8733	.0679
Labor/Mean Labor	1.0576	1.0464	.9527	.9434	.6583	.0394
Int. Rate/Mean Int. Rate	.9337	.9337	1.0663	1.0663	1.1463	.0877
Prices/Mean Prices	1.1008	1.0025	.9807	.9110	.7739	.0608
Wages/Mean Wages					.5766	.0101
l_1/Mean l_1	1.1527	1.1552	.8354	.8567	1.2527	.2229
l_2/Mean l_2	.5296	.4913	1.5362	1.4429	1.2527	.7100
Fixed Time Preference $n = .3366$; $V_m = 1.5073$	$A = .75$; $g = 0.64$; $t = 0$.				Mean	Standard Deviation
x	.075	.075	.025	.025	.05	
q	1.0	1.1	1.2	1.3	1.15	.1291
b	.95	.95	.95	.95	.95	0
Cons./Mean Cons.	.8944	.9838	1.0185	1.0133	.9149	.0791
Labor/Mean Labor	1.0409	1.0409	.9591	.9591	.7022	.0332
Int. Rate/Mean Int. Rate	.9353	.9353	1.0647	1.0647	1.1030	.0824
Prices/Mean Prices	1.1118	1.0107	.9763	.9012	.7293	.0637
Wages/Mean Wages	1.0238	1.0238	.9762	.9762	.5500	.0151
l_1/Mean l_1	1.2275	1.2275	.7725	.7725	.1428	.0375
l_2/Mean l_2	.2951	.2951	1.7049	1.7049	.1428	.1163

Table5. Procyclical Monetary Policy ($Cov(x,q) = .0033$).

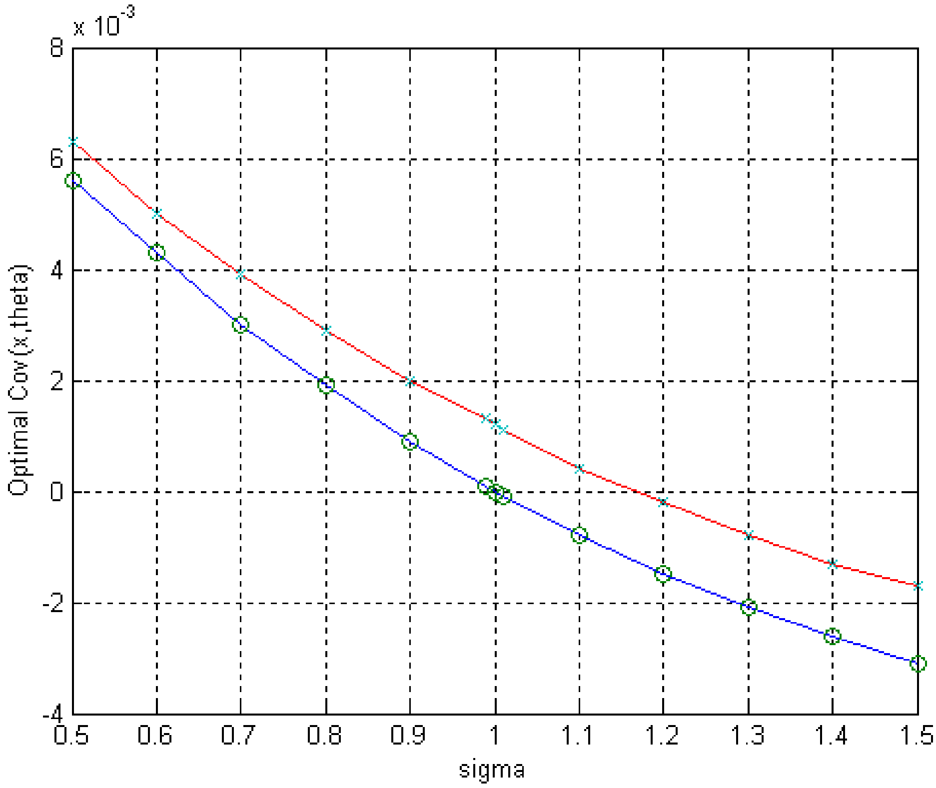
Endogenous Time Preference $n = .33$; $V_m = 9.9206$	$b(u) = e^{-(h+tu)}$, $t = 0.8$; $u(c_t, 1-L_t) = \log(c_t) - AL_t$, $A = 0.75$; $c_t = q(s_{t+1})L_t^g$, $g = 0.64$				Mean	Standard Deviation
x	.025	.025	.075	.075	.05	
q	1.0	1.1	1.2	1.3	1.15	.1291
$b(u)$	1.0229	.9491	.8778	.8243	.9185	.0863
Cons./Mean Cons.	.8539	.9329	1.0659	1.1473	.8828	.1161
Labor/Mean Labor	.9742	.9639	1.0362	1.0258	.6600	.0239
Int. Rate/Mean Int. Rate	1.0658	1.0658	.9342	.9342	1.1333	.0861
Prices/Mean Prices	1.1559	1.0580	.9259	.8602	.7690	.1019
Wages/Mean Wages	.9600	.9702	1.0296	1.0401	.5752	.0234
l_1/Mean l_1	.8003	.8307	1.1833	1.1856	1.22223	.2608
l_2/Mean l_2	1.6834	1.5619	.3892	.3655	1.22223	.8810
Fixed Time Preference $n = .3366$; $V_m = 1.5073$	$A = .75$; $g = 0.64$; $t = 0$.				Mean	Standard Deviation
x	.025	.025	.075	.075	.05	
q	1.0	1.1	1.2	1.3	1.15	.1291
b	.95	.95	.95	.95	.95	0
Cons./Mean Cons.	.8449	.9293	1.0684	1.1574	.9191	.1284
Labor/Mean Labor	.9591	.9591	1.0409	1.0409	.7022	.0332
Int. Rate/Mean Int. Rate	1.0647	1.0647	.9353	.9353	1.1030	.0824
Prices/Mean Prices	1.1662	1.0602	.9222	.8513	.7326	.1031
Wages/Mean Wages	.9762	.9762	1.0238	1.0238	.5500	.0151
l_1/Mean l_1	.7725	.7725	1.2275	1.2275	.1428	.0375
l_2/Mean l_2	1.7049	1.7049	.2951	.2951	.1428	.1163

Table 6. Welfare Cost as % of Mean Consumption.

	Endogenous Time Preference	Fixed Time Preference
Procyclical Monetary Policy	1.9673	1.1833
Deterministic Monetary Policy	2.0845	1.1966
Countercyclical Monetary Policy	2.1817	1.1941

Table 7. Optimal Monetary Policy.

Endogenous Time Preference	$b(u) = e^{-(h+tu)}$, $t = 0.8$; $u(c_t, 1-L_t) = \log(c_t) - AL_t$, $A = 0.75$; $c_t = q(s_{t+1})L_t^g$, $g = 0.64$			
q	1.0	1.1	1.2	1.3
x	-.1060	-.0989	-.0919	-.0848
Consumption	.9035	.9938	1.0842	1.1745
Labor	.8533	.8533	.8533	.8533
Interest Rates	0	0	0	0
Price Level	.5810	.5393	.5045	.4751
Wage Rate	.3937	.4020	.4103	.4185
l_1	0	0	0	0
l_2	0	0	0	0
$b(u)$	1.0039	.9302	.8677	.8183
Fixed Time Preference	$A = .75$; $g = 0.64$; $t = 0$.			
q	1.0	1.1	1.2	1.3
x	-.05	-.05	-.05	-.05
Consumption	.9035	.9938	1.0842	1.1745
Labor	.8533	.8533	.8533	.8533
Interest Rates	0	0	0	0
Price Level	.6411	.5829	.5343	.4932
Wage Rate	.4344	.4344	.4344	.4344
l_1	0	0	0	0
l_2	0	0	0	0
b	.95	.95	.95	.95



×	Endogenous Time Preference
o	Fixed Time Preference

Figure 1. Optimal Monetary Policy.