Marriage as a Rat Race: Noisy Pre-Marital Investments with Assortative Matching*

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Abstract

We study the incentive to invest to improve marriage prospects, in a frictionless marriage market with non-transferable utility. Stochastic returns to investment eliminate the multiplicity of equilibria in models with deterministic returns, and a unique equilibrium exists under reasonable conditions. Equilibrium investment is efficient when the sexes are symmetric. However, when there is any asymmetry, including an unbalanced sex ratio, investments are generically excessive. For example, if there is an excess of boys, then there is parental over-investment in boys and under-investment in girls, and total investment will be excessive.

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1 Introduction

We study the incentives of parents to invest in their children when these investments also improve their marriage prospects. We assume a frictionless marriage market with non-transferable utility. It has usually been thought that ex ante investments suffer from the hold-up problem, since a parent will not internalize the effects of such investments in her own child upon the welfare of the child's future spouse. However, Peters and Siow (2002) argue that in large marriage markets where the quality of one's match depends on the level of investment, a parent has an incentive to invest more in order to improve the match of her offspring. They argue that the resulting outcome will be Pareto efficient. This is a remarkable result, since they assume a marriage market without transferable utility. With transferable utility, Cole et al. (2001) show that in large markets, prices can provide incentives for efficient investment decisions.[1]

In this paper, we argue that the optimism of Peters and Siow (2002) must be somewhat tempered. When the return to investment is deterministic, we show that there is very large set of equilibria. These include efficient outcomes, but also a continuum of inefficient ones. In order to overcome this embarrassment of riches, we propose a model where the returns to investment are stochastic. This is also realistic – talent risk is an important fact of life. Recent studies of the inter-generational transmission of wealth, in the tradition of Becker and Tomes (1979), find an inter-generational wealth correlation of 0.4 in the United States, which is far from 1. Equilibrium in this model is unique and we are therefore able to make determinate predictions. The model also allows us to address several questions of normative importance and social relevance. Are investments efficient, in the absence of prices? What are the implications of biological or social differences between the sexes for investment decision? What are the implications of sex ratio imbalances in countries such as China – Wei and Zhang (2009) argue that marriage market competition for scarce women underlies the high savings rate in China.

Our paper is related to the literature on matching tournaments or contests. This literature typically models a situation where there is a fixed set of prizes, and agents on the one side of the market compete by making investments, with prizes being allocated to agents according the rank order of their investments (see for example, Cole et al., 1992 and Hopkins and Kornienko, 2010). If the "prizes" derive no utility from these investments, e.g. when the prize is social status, then an agent's investment exerts a negative positional externality on the other side of the market, so that there is over-investment. On the other hand, if the "prizes" derive utility from these investments – for example, if men compete for a set of women with fixed qualities, or students compete for university places – then either over-investment or under-investment is possible, depending on how much these investments are valued (Cole et al., 2001; Hopkins, 2010).

In our context, investments are two-sided – the investments of men are valued by

[1] To appreciate the degree of transferability required, note that Mailath, et al. (2010) show that one needs "personalized prices", which depend upon buyer characteristics as well as seller characteristics, in order ensure efficiency of investments.

women and symmetrically, the investments of women are valued by men. Men do not care directly about how their investments are valued by women, they care only about the consequent improvement in match quality that they get. Women are in a similar situation, since they care only about the improvement in the quality of men that they might get. One might expect therefore, that this could give rise to under-investment or over-investment, depending on parameter values.

Surprisingly, our model yields clear conclusions. Under very special circumstances, when the sexes are completely symmetric, with identical distributions of shocks and a balanced sex ratio, investments will be efficient – not merely in the Pareto sense, but also utilitarian efficient. However, if there are any differences between the sexes, whether it be differing returns to investments, different stochastic shocks or an unequal sex ratio, investments are generically excessive, as compared to Pareto-efficient investments. Since the intuition for the overinvestment result is somewhat subtle, and quite distinct from that in one-sided tournaments with positional externalities, we defer explaining this until the model is introduced.

The rest of the paper is set out as follows. Section 2 discusses the issues arising with a model with deterministic returns, as in Peters and Siow (2002), and also discusses other related literature. Section 3 sets out the basic model with noisy investments. We show that a pure strategy equilibrium exists and is unique for general quality functions. We consider, in turn, additive and multiplicative shocks, and our main finding is that investments are generically excessive, relative to Pareto efficiency. Section 4 shows that when the two sexes are identical investments are efficient. Section 5 uses our model to examine the observational implications of gender differences, by considering the case where talent shocks are more dispersed for boys than for girls. It also examines sex ratio imbalances and their effects on investments. The final section concludes.

2 Motivation and Related Literature

The fundamental problem is the following: investment in a child is assumed to benefit the child's spouse if the child marries, but the benefit to the spouse is not considered by the child's parents. There is therefore a gap between the *privately optimal* investment in a child, which we denote $\bar{x}$, and the *socially optimal* level which is naturally greater. In the absence of prices, it is not clear that there are incentives for efficient investment. Peters and Siow (2002) (PS, henceforth) argue that nonetheless equilibrium investments are socially efficient.

Let us consider the PS model, of investment with deterministic returns, but simplify by assuming that families are identical rather than differing in wealth. Assume a unit measure of boys, all of whom are ex ante identical, and an equal measure of girls, who are similarly ex ante identical. Assume that the quality of the child, as assessed by the partner in marriage market, equals the level of parental investment, x. Suppose a boy is matched with a girl. The utility of the boy's parents is increasing in the investment level of the girl x_G, but they have to bear the cost of investment x_B in their son. Thus, if they choose x_B purely to maximize their utility, they would choose only the privately optimal investment $\bar{x}_B$. But the resulting investment levels $(\bar{x}_B, \bar{x}_G)$ are inefficient, since the indifference curves through this point are not mutually tangent (see the left panel, Fig. 1). Both families would be better off at the point E.

Suppose that the family of the boy believes that if they choose investment level x_B, the quality of their partner is given by a smooth, strictly increasing function, $\phi(x_B)$, as sketched in Figure 1. They would choose investments to maximize their overall payoff, given the return function ϕ. Suppose also that the family of a girl believe that the match quality of their girl is also an increasing function of their own investment level x_G. Assume further that this return function equals $\phi^{-1}(x_G)$, the inverse of that for the boys. Consider a profile of investments (x_B^{**}, x_G^{**}) such that x_B^{**} maximize the payoffs of the boys family given returns $\phi(x_B)$ and x_G^{**} maximizes the payoffs of the girl's family given returns $\phi^{-1}(x_G)$, and $x_B^{**} = \phi^{-1}(x_G^{**})$, i.e. these expectations are actually realized. The profile (x_B^{**}, x_G^{**}) must be such that the indifference curves on the two sides of the market are mutually tangent, as in the right panel of Fig. 1.

A problem with this approach is that, while the expectations $\phi(x_B)$ are realized in equilibrium, they cannot be realized if the family of a boy chooses $x_B \neq x_B^{**}$. In particular, if a boy deviates and chooses $x_B < x_B^{**}$, the match $\phi(x_B) < x_G^{**}$ is not *feasible* since every girl in the market has quality x_G^{**}. In other words, while expectations are "rational" at the equilibrium, they are not so for any investment level that is not chosen in equilibrium.

We shall be explicit in this paper about the matching process that follows a profile of investments. Specifically, we will require the matching to be feasible, to be stable (in the sense of Gale and Shapley, 1962) and to be measure preserving. Even given these restrictions, one can support efficient investments. Suppose that in equilibrium all boys invest x_B^{**} and all girls invest x_G^{**}. If a boy deviates and invests a smaller amount, then our restrictions imply that he can be punished by being left unmatched. Since we have continuum populations on both sides of the market, this does not leave any girl to remain single, and the resulting allocation is feasible, stable and measure preserving. In other words, we can construct a matching in the game after investments are realized that is feasible, stable, and measure preserving, such that given this matching rule, a parent has an incentive to invest the efficient amount in a child.

However, despite these restrictions on matching off the equilibrium path, in terms of equilibria there is an embarrassment of riches. Let (x_B, x_G) be a pair of investments that are weakly greater than the individually optimal investments $(\bar{x}_B, \bar{x}_G)$, and where the payoff of gender i from being matched with a partner with investment level x_j is weakly greater than the payoff from choosing the individually optimal investment level $\bar{x}_i$ and being unmatched. Any such pair can be supported as an equilibrium, by specifying that any agent who deviates to a lower investment level will be left unmatched. Furthermore, if the parent of a boy deviates upwards, and chooses a higher level of investment, his son cannot realize a higher match quality, since all the girls are choosing x_G. We therefore have a "folk theorem" – any pair of investments satisfying the above conditions is an equilibrium.

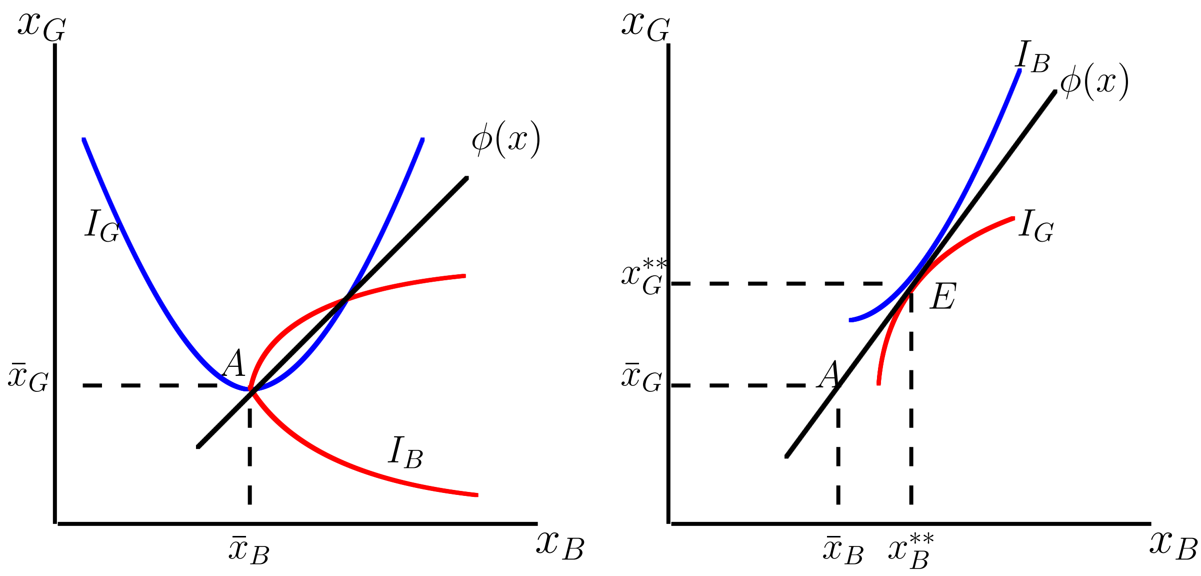

Figure 1: Illustration of basic concepts. I_B, I_G are indifference curves for the boy and the girl respectively. At point A, investments $(\bar{x}_B, \bar{x}_G)$ are privately optimal, but are neither socially efficient nor a rational expectations equilibrium: if parents believe they face a strictly increasing matching function such as $\phi(x)$ they would prefer a higher investment. Equilibrium investments (x_B^{**}, x_G^{**}) at point E are efficient.

Turning now to the original PS environment where families differ in wealth, and thereby in their marginal costs of investment, we still find a continuum of inefficient equilibria.[2] To see this, note that almost all of the continuum of equilibria we have constructed in the homogenous case are strict equilibria – any individual who invests differently does strictly worse. If we perturb wealth levels slightly, and wealth affects payoffs continuously, then these equilibria will continue to be strict. The only thing that is required is that the distribution of wealth is not too dispersed, so that there is a common level of investment that is not so low that it is below the richest family's privately optimal investment and not so high that the poorest family would prefer to deviate downwards and be unmatched. None of these equilibria are efficient. In fact, for all of them, a measure zero of agents make an efficient investment. If $\hat{x}$ is relatively low then all agents underinvest. If $\hat{x}$ is higher, some agents underinvest and some overinvest.

Even with a more dispersed distribution of wealth and cost functions, one can construct more complex but still inefficient equilibria. For example, if wealth were more widely distributed then we divide families into two groups, rich and poor, each of which groups has a common level of investment. For example, the matching rule matches those families with sons who choose investment $\hat{x}_L$ to those families with daughters with the same investment, and matches those who choose $\hat{x}_H$ to daughters with the same investment. This suggests that we can construct equilibria even in very dispersed

[2]PS seem to assume that in a rational expectations equilibrium matching must be strictly monotone in investment. The additional equilibria we consider are only weakly increasing. However, the expectations of the agents are met, in equilibrium and off the equilibrium path. In this sense, our equilibria satisfy stronger conditions than rational expectations equilibria.

distributions of wealth provided using multiple levels of investment.[3]

We now show that these problems can be resolved, if we augment the model by adding an idiosyncratic element of match quality. That is, we assume that the quality of an individual on the marriage market is not entirely determined by investments, but also has a stochastic element. This seems realistic, since talent risk is an important fact of life. The simplest case is where the realized value of all investments are subject to a shock that is a random draw from a continuous distribution. Now suppose that the parent of every boy invests the same amount $\hat{x}_B$, while the parent of every girl invests the same amount $\hat{x}_G$. We claim that it is no longer an equilibrium to invest the individually optimal amounts. If the parent of a boy invests $\bar{x}_B$, the marginal private cost of investment must equal the marginal private benefit, i.e. the marginal net cost is zero. However, there is now a benefit to improving match quality, since there is a distribution of qualities on both sides of the market. By investing a little more in a boy, the quality of the boy increases for every realization of the shock, and he ranks higher in the distribution of boy qualities, and will therefore be matched with a better quality girl. Thus, it will be optimal to deviate and increase investment beyond the individually optimal amount. An equilibrium must be characterized by the condition that for a boy, the marginal benefit from investment, in terms of improving match quality, is equal to the marginal net cost. An analogous condition holds for girls as well. In other words, the introduction of a random element of quality eliminates the multitude of equilibria, and ensures that equilibria are unique, under some regularity conditions.

Does this incentive to invest provide for efficient investments? We find that surprisingly if the distribution of shocks is symmetric across genders, the unique equilibrium coincides with the efficient investments. However, if the distribution of shocks is not symmetric, then investments will not in general be efficient. For example, if shocks are more variable for men than for women, then women will overinvest, while men will underinvest, relative to efficiency. Similarly, if the sex ratio is unbalanced so that there are more men than women, there is likely to be overinvestment by men and underinvestment by women.

2.1 Other Literature

Investment in matching models with transferable utility have been studied by Cole et al. (2001) and Felli and Roberts (2000). In large markets, prices ensure that an agent

[3]In contrast, if one assumes that if a boy reduces his investment, he is still matched, but with the lowest type of girl, then all pooling equilibria would seem vulnerable. A deviation by the poorest male family down to the Nash level would not be punished by the poorest female family and so the equilibrium could not be maintained. However, this difficulty applies equally to PS's proposed efficient equilibrium as the efficient level of investment is above the privately optimal level. Thus, under such weak punishment, in the case where all families are ex ante homogenous, the unique equilibrium consists of the individually optimal investments. In the PS setting where families differ in wealth, there seems to be no pure strategy equilibrium. Peters (2007, 2009) acknowledges these difficulties: he finds that equilibria of a non-cooperative game with a finite number of participants do not approach the efficient equilibrium even as the number of participants goes to infinity.

will capture her marginal contribution to social surplus, thereby ensuring the existence of equilibria with efficient investments (inefficient outcomes may also be possible). A recent paper by Mailath, et al. (2010) shows the degree of transferability required – prices must depend on buyer as well as seller characteristics, if efficiency is to be ensured. Iyigun and Walsh (2007) and Chiappori et al. (2009) use a Becker (1973) style transferable utility marriage market to study the distributional consequences of institutional and gender differences for investments.

Matching tournaments with non-transferable utility were first examined by Cole et al. (1992). There is a fixed set of "prizes" on one side of the market, which are allocated to the participants on the other side according to their rank order. Examples include college admissions or competition for status (Hopkins and Kornienko, 2004). The desire to match well provides an incentive to invest and increases equilibrium investments. If the "prizes" derive no utility from these investments, then there is a pure negative positional externality and investments are excessive from a social point of view. In two-sided markets, such as marriage, investments are not inherently wasteful, since the "prizes" also derive utility from investments. Thus, in principle, investments could either be excessive or insufficient.

Hoppe et al. (2009) analyze a signaling model of matching, where an agent cares about his or her match partner's underlying characteristic, that is private information. Since investments are not directly valued, they are inherently wasteful, although they may improve allocative efficiency in the matching process. Hopkins (2010) finds that with one-sided investments, the level of investment can be inefficiently low.

Peters (2007, 2009) investigates two sided investments with finite numbers of participants and finds that the resulting equilibria do not necessarily approach the efficient equilibrium possible with a continuum of agents, even as the number of participants goes to infinity. This contrasts with our results, where the equilibria of the continuum model can be seen as the limit of a finite model (see Appendix B).

Gall et al. (2009) also examine investments, matching and affirmative action in a non-transferable utility setting. Their focus differs from most of the above mentioned literature since they consider a situation where efficiency requires negative assortative matching. However, in the absence of transfers, stable matchings are positively assortative, providing a possible rationale for affirmative action. They allow for investments with stochastic returns and focus on the tradeoff between the positive role of affirmative action on match efficiency versus its possible negative effect upon investment incentives.

The idea of perturbing games as a means to refine the number of equilibria and/or to ensure a pure equilibrium is not new. Lazear and Rosen (1981) is a classic example of a tournament where the return to effort is stochastic. However, in the context of matching tournaments, the approach taken up to now has been to employ incomplete information, see Hoppe et al. (2009), Peters (2009), Hopkins (2010). That is, agents are assumed to differ ex ante in terms of quality or wealth, and equilibrium investment is monotone in one's type. Here, as in Gall et al. (2009), the approach is somewhat simpler. Agents are ex ante identical. Yet still individuals face a smooth optimization problem which has a unique equilibrium.

There is also a literature that considers investment incentives in the presence of search frictions. Acemoglu and Shimer (1999) analyze a model of one-sided investments under transferable utility setting, while Burdett and Coles (2001) analyze a non-transferable utility model with two-sided investments.

3 A Matching Tournament with Noisy Investment

We now set out a model where the return to investments are stochastic. To simplify the analysis, we assume in this section that there is no ex ante heterogeneity – with stochastic returns, there is ex post heterogeneity. Consider a society where all families are ex ante identical, save for the fact that some have boys and others have girls. Assume that a parent i derives a direct private benefit $b_B(x)$ from an investment of x in the quality of a boy, and $b_G(x)$ in the quality of a girl and incurs a cost $\tilde{c}_B(x)$ and $\tilde{c}_G(x)$ respectively. Define the net cost of investment in a boy as $c_B(x) = \tilde{c}_B(x) - b_B(x)$, and assume that this is strictly convex and eventually increasing. Net costs for girls are similarly defined. The quality of a boy, q, depends upon the level of parental investment, x, and the realization of a random shock, ε, that is distributed with a density function $f(\varepsilon)$ and a cumulative distribution function $F(\varepsilon)$ on $[\underline{\varepsilon}, \bar{\varepsilon}]$. Let η denote the realization of the idiosyncratic quality component for a girl; this is distributed with a density function g and cdf G on $[\underline{\eta}, \bar{\eta}]$. We shall assume that the quality of a boy, as assessed by a girl, equals $q(x, \varepsilon)$, where q is continuous and strictly increasing in both arguments. Similarly, a girl's quality on the marriage market is given by $q(x, \eta)$. It is easy to see that our analysis also applies when partners valuation of quality is an increasing concave function of q. Finally, for simplicity, we shall assume that each individual strictly prefers to be matched rather than unmatched, so that the payoff to being unmatched $\bar{u}$ is less than from the lowest possible quality match.

Parents are altruistic and internalize the effects of their decisions on the utility of their own child, but not on the utility of their child's partner. Thus if a girl with parental investment x_G and shock η is matched with a boy whose parent has invested x_B, and who has shock realization ε, her payoff and that of her parents equals

$$U_G(x_G, x_B) = q(x_B, \varepsilon) + b_G(x_G) - \tilde{c}_G(x_G) = q(x_B, \varepsilon) - c_G(x_G). \tag{1}$$

Similarly for a boy of type (x_B, ε) who is matched with a girl of type (x_G, η), his utility would be

$$U_B(x_B, x_G) = q(x_G, \eta) + b_B(x_B) - \tilde{c}_B(x_B) = q(x_G, \eta) - c_B(x_B). \tag{2}$$

Let $\bar{x}_B$ and $\bar{x}_G$ denote the *individually optimal* or Nash investments for boys and girls respectively. That is, for boys it is the investment that minimizes $c_B(x) = \tilde{c}_B(x) - b_B(x)$, or equivalently the investment such that $c'_B(\bar{x}_B) = 0$. The privately optimal investment for girls $\bar{x}_G$ is defined similarly.

Such investments are not Pareto efficient. Consider a social planner who chooses (x_B, x_G) to maximize

$$W(x_B, x_G) = \lambda \left[\int q(x_B, \varepsilon) f(\varepsilon)\, d\varepsilon - c_G(x_G) \right] + (1-\lambda) \left[\int q(x_G, \eta) g(\eta)\, d\eta - c_B(x_B) \right], \tag{3}$$

for some $\lambda \in (0,1)$, where λ is the relative weight placed on the welfare of girls. Differentiating with respect to x_B and x_G, setting to zero and rearranging, we obtain the first order conditions for Pareto efficiency,

$$\frac{c'_B(x_B)}{\int q_x(x_B, \varepsilon) f(\varepsilon)\, d\varepsilon} = \frac{\lambda}{1-\lambda}. \tag{4}$$

$$\frac{c'_G(x_G)}{\int q_x(x_G, \eta) g(\eta)\, d\eta} = \frac{1-\lambda}{\lambda}. \tag{5}$$

This has the interpretation that the investments by boy and girl are such that they must lie on the contract curve in (x_B, x_G) space. That is, the indifference curve of the boy and girl have the same slope equal to $\lambda/(1-\lambda)$. Rearranging the first order condition for welfare maximization, we obtain

$$c'_B(x_B) \times c'_G(x_G) = \int q_x(x_B, \varepsilon) f(\varepsilon)\, d\varepsilon \times \int q_x(x_G, \eta) g(\eta)\, d\eta. \tag{6}$$

In other words, any profile of Pareto-efficient investments satisfies this condition, irrespective of the value of λ.

Suppose that we have a profile of investments (x_B, x_G) such that the product of the marginal costs is smaller than the right hand side of equation (6). Then we have *under-investment* relative to Pareto-efficiency, since it is possible to achieve Pareto efficiency by raising one or both investment levels. Similarly, if the product of marginal costs exceeds the right hand side of the equation, we have overinvestment, since achieving Pareto efficiency requires reducing investments. Note that Pareto efficient investments always exceed the privately optimal level as the under the privately optimal investments, we have $c'_B(\bar{x}_B) = c'_G(\bar{x}_G) = 0$, whereas the right hand side of equation (6) is strictly positive.

Of particular interest is the case where λ, the weight place on girls' welfare, is equal to their proportion in the population, one-half. Let x_B^{**}, x_G^{**} denote the efficient investments in this case. We shall call these the *utilitarian efficient* investments. These are the investments that parents would like the social planner to choose in the "original position", before the gender of their child is realized.

We shall focus upon pure strategy equilibria where the parent of each side of the marriage market choose the same level of investment. Such an equilibrium will be called *quasi symmetric and* consists of a pair (x_B^*, x_G^*). This profile of investments induces distributions of quality on the boys' side, $\tilde{F}(q)$, and on the girl's side, $\tilde{G}(q)$. We require the matching to be stable and measure preserving. Given our specification of preferences, whereby all boys uniformly prefer girls of higher quality, and vice-versa, a stable measure preserving matching is essentially unique, and must be assortative. Since q is strictly increasing in the idiosyncratic shock, and since all agents on the same side of the market choose the same investment level, in equilibrium, there must be matching according to the idiosyncratic shocks alone. For a boy who has shock realization ε, let $\phi(\varepsilon)$ denote the value of η of his match. This satisfies

$$F(\varepsilon) = G(\phi(\varepsilon)), \tag{7}$$

or $\phi(\varepsilon) = G^{-1}(F(\varepsilon))$. That is, if boy is of rank z in the boy's distribution, he is matched with a girl of the same rank z in the girl's distribution. The non-degenerate distribution of qualities on both sides of the marriage market provides incentives of investment above the privately optimal level. If the parent of a boy invests a little more than x_B^*, he increases the boy's rank for any realization of ε. By doing so, he obtains a girl of higher rank. However, he is concerned not with the girl's rank but her quality.

One delicate issue concerns large deviations from the equilibrium, where the quality realization is outside the support of the equilibrium distribution of qualities. For example, if a boy deviates upwards and his quality exceeds $q(x_B^*, \bar{\varepsilon})$, stability implies that he will be matched with the best quality girl, of quality $q(x_G^*, \bar{\eta})$. If he deviates downwards and his quality is below $q(x_B^*, \underline{\varepsilon})$, then stability implies that he could be left unmatched (with payoff $\bar{u}$) or matched with the lowest quality girl. We shall assume that both these outcomes have equal probability. Since we assume that being single has a low payoff, this deters large downward deviations. These assumptions are consistent with the requirement that the matching be stable and measure preserving.

Furthermore, as we show in Appendix B, the matching assumption can be justified as the limit of a model with a finite number of agents, as the number of agents goes to infinity. Consider a finite model where the expected numbers of men and women are equal, n. However, the exact numbers of men and women is random, with probability one-half that there are slightly more men than women, and probability one-half that the reverse is the case. Then a boy with the lowest quality is unmatched with probability one-half. As $n \to \infty$, if the probability of being unmatched conditional on choosing equilibrium investments goes to zero, then equilibrium behavior converges to that in the continuum model set out here. Nevertheless, for a deviating boy with a low quality realization, the probability of being unmatched tends to one-half in the limit.[4]

In Appendix A we show that the first order condition for the equilibrium investment in boys can be written as

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} q_\eta(x_G^*, \phi(\varepsilon)) \frac{f(\varepsilon)}{g(\phi(\varepsilon))} \frac{q_x(x_B^*, \varepsilon)}{q_\varepsilon(x_B^*, \varepsilon)} f(\varepsilon)\, d\varepsilon = c_B'(x_B^*). \tag{8}$$

[4]We thank Roger Myerson for suggesting this approach. See also Myerson (1998) for large games with a random set of players.

The intuition for the first order condition is that it balances the marginal cost c' of extra investment on the right hand side with its marginal benefit on the left hand side. The latter principally is determined by the possibility of an improved match from increased investment. Specifically, an increase in ε, a boy's shock, would improve his match, given the matching relation (7) at rate $\phi' = f/g$. Similarly, the first order condition for investment in girls is given by

$$\int_{\underline{\eta}}^{\bar{\eta}} q_\varepsilon(x_B^*, \phi^{-1}(\eta)) \frac{g(\eta)}{f(\phi^{-1}(\eta))} \frac{q_x(x_G^*, \eta)}{q_\eta(x_G^*, \eta)} g(\eta)\, d\eta = c_G'(x_G^*). \tag{9}$$

Existence and uniqueness of equilibrium requires that the conditions A1 are satisfied. These conditions, such as differentiability, convexity of the cost function and concavity of the quality function, are standard and mainly technical, and are set out in detail in Appendix A, as is the proof of the theorem.

Theorem 1 *Assume that conditions A1 are satisfied. There exists investments* (x_B^*, x_G^*) *that are the unique solution of the first order conditions (8) and (9). Suppose either (a) F and G are distributions of the same type (that is,* $G(x) = F(ax + b)$*), or (b)* $f(\varepsilon)$ *and* $g(\eta)$ *are weakly increasing. Then there is a value to not being matched,* $\bar{u}$*, sufficiently low such that* (x_B^*, x_G^*) *is a strict Nash equilibrium. Thus there exists an open set of distributions such that there is a unique quasi-symmetric Nash equilibrium of the matching tournament.*

Are investments Pareto-efficient? What are the factors that lead one sex to invest more than the other? In order to shed more light on these questions, we consider, in turn, different specifications of the quality function $q(\cdot)$.

3.1 Additive Shocks

We first analyse the case where $q(x, \varepsilon) = x + \varepsilon$. One interpretation is that investment or bequest are in the form of financial assets or real estate, while the shocks are to (permanent) labor income of the child. The interpretation is that total household income is like a public good (as in Peters-Siow), which both partners share.

Consider a quasi symmetric equilibrium where all boys invest x_B^* and all girls invest x_G^*. A boy with shock realization ε and of rank z in the distribution $F(.)$ will be matched with a girl of shock $\phi(\varepsilon)$ with same rank z in $G(.)$. Suppose that the parent of a boy invests a little more, $x_B^* + \Delta$, as in Figure 2. If his realized shock is ε, the improvement in the ranking of boys is approximately equal to $f(\varepsilon)\Delta$. The improvement in the quality of the matched girl, $\tilde{\Delta}$ must be such that $g(\eta)\tilde{\Delta} \approx f(\varepsilon)\Delta$, i.e. the improvement in the rank of his match must equal the improvement in his own rank. Thus the marginal return to investment in terms of match quality equals $\frac{f(\varepsilon)}{g(\phi(\varepsilon))}$ at any value of ε.

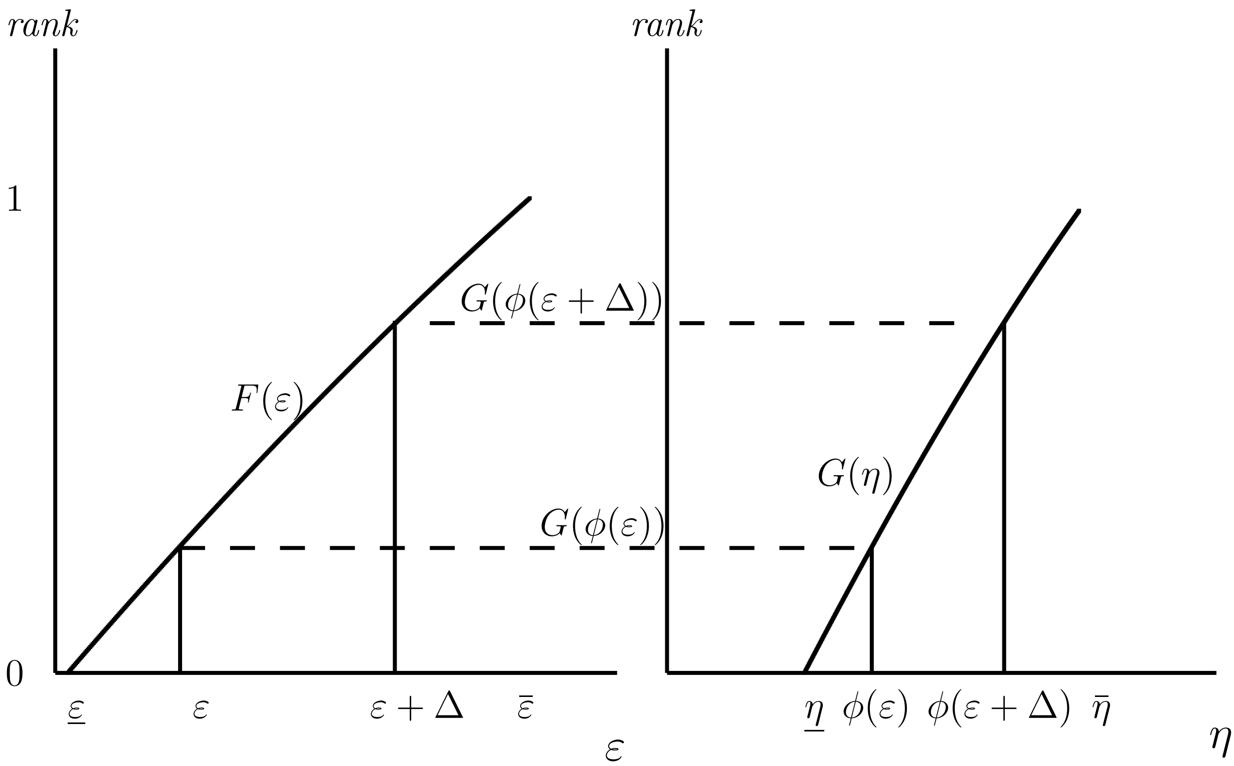

Figure 2: If a boy with shock realisation ε increases investment by an amount Δ, he would overtake other boys with shock realisations between ε and $\varepsilon + \Delta$. The boy's match would improve from the girl with shock value $\phi(\varepsilon)$ to one at $\phi(\varepsilon + \Delta)$.

Integrating over all possible values of ε gives the first order condition for optimal investments in boys,

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\, d\varepsilon = c_B'(x_B^*). \tag{10}$$

Similarly, the first order condition for investment in girls is

$$\int_{\underline{\eta}}^{\bar{\eta}} \frac{g(\eta)}{f(\phi^{-1}(\eta))} g(\eta)\, d\eta = c_G'(x_G^*). \tag{11}$$

The left hand side of the above equations (the marginal benefit) is constant, while the right hand side is strictly increasing in x_i, due to the convexity of the cost function. Thus, there is a unique solution to the first order conditions. Since the optimal investments of the boys do not depend upon the investments of the girls, and vice-verse, there is at most one equilibrium. It remains to show that the first order conditions are sufficient for a maximum. Although the optimization problem faced by agents is not quasi-concave, in Appendix A we provide conditions under which large deviations from x_B^* or x_G^* are unprofitable, thereby ensuring existence of equilibrium.

We now use the first order conditions to examine the efficiency of investments. Under additive shocks, the marginal benefit to a girl from a boy's investment is one, regardless of the realization of the shock. Thus Pareto efficiency requires that $c'_B(x_B) = \frac{\lambda}{1-\lambda}$, and $c'_G(x_G) = \frac{1-\lambda}{\lambda}$. This implies that in any Pareto efficient allocation, $c'_B(x_B) \times c'_G(x_G) = 1$. The condition for utilitarian efficiency (where equal weight is placed on the welfare of boys and girls) is $c'_B(x_B) = c'_G(x_G) = 1$.

Consider the case where $F = G$, i.e. the distribution of shocks is the same. Thus, $\frac{f(\varepsilon)}{g(\phi(\varepsilon))} = 1$ for all values of ε, so that $c'_B(x^*_B) = c'_G(x^*_G) = 1$. Thus investments are utilitarian efficient even if the investment cost functions are different for the two sexes. As we shall see later, this is an example of a more general result – if there are no gender differences whatsoever, this ensures utilitarian efficiency.

In general, if there are any differences between the sexes, $\frac{f(\varepsilon)}{g(\phi(\varepsilon))}$ will differ from 1, and so one cannot expect utilitarian efficiency. The following theorem is the main result of this section.

Theorem 2 *When noise is additive, in a quasi-symmetric equilibrium investments are generically excessive relative to Pareto efficiency.*

Proof. The first order condition for investment for boys and girls are given by (10) and (11). It is useful to make the following change in variables. Since $\eta = \phi(\varepsilon)$,

$$d\eta = \phi'(\varepsilon)\, d\varepsilon = \frac{f(\varepsilon)}{g(\phi(\varepsilon))}\, d\varepsilon. \tag{12}$$

Thus the first order condition for girls is rewritten as

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} g(\phi(\varepsilon))\, d\varepsilon = c'(x^*_G).$$

Consider the product

$$c'(x^*_B) \times c'(x^*_G) \geq \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\, d\varepsilon \right) \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} g(\phi(\varepsilon))\, d\varepsilon \right).$$

By the Cauchy-Schwarz inequality,

$$\begin{aligned} \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\, d\varepsilon \right) \left(\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} g(\phi(\varepsilon))\, d\varepsilon \right) &\geq \left[\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left(\frac{f(\varepsilon)}{(g(\phi(\varepsilon)))^{1/2}} \right) (g(\phi(\varepsilon)))^{1/2}\, d\varepsilon \right]^2 \\ &\geq 1. \end{aligned}$$

Note that Cauchy-Schwarz states that the inequality is strict if the two terms are linearly independent. Thus $c'(x^*_B) \times c'(x^*_G) \geq 1$ with the inequality being strict if $\frac{f(\varepsilon)}{\sqrt{g(\phi(\varepsilon))}}$ and $\sqrt{g(\phi(\varepsilon))}$ are linearly independent functions of ε. Since Pareto efficiency requires $c'(x^*_B) \times c'(x^*_G) = 1$, we have overinvestment generically if the distributions f and g differ. ■

An Example

Let us assume that $F(\varepsilon) = \varepsilon$ on $[0,1]$, i.e. ε is uniformly distributed.[5] Assume that $G(\eta) = \eta^n$ on $[0,1]$. $F(\varepsilon) = G(\phi(\varepsilon))$ implies $\phi(\varepsilon) = \varepsilon^{\frac{1}{n}}, g(\phi(\varepsilon)) = n\varepsilon^{\frac{n-1}{n}}$. The equilibrium conditions are:

$$\begin{aligned} c'(x_B^*) &= \int_0^1 \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\, d\varepsilon = \frac{1}{n}\int_0^1 \varepsilon^{\frac{1-n}{n}}\, d\varepsilon \\ &= 1. \end{aligned} \tag{13}$$

and

$$\begin{aligned} c'(x_G^*) &= \int_0^1 \frac{g(\eta)}{f(\phi^{-1}(\eta))} g(\eta)\, d\eta = n^2 \int_0^1 \eta^{2n-2}\, d\eta \\ &= \frac{n^2}{2n-1}. \end{aligned} \tag{14}$$

The product of the marginal costs equals $\frac{n^2}{2n-1}$ which is positive and greater than one for $n > 0.5$. Efficiency requires that the product equals 1, which it only does for $n = 1$, i.e. when $f = g$.

The example provides additional intuition for the inefficiency result. Let $n = 2$, so that the density function for women, $g(\eta) = 2\eta$ on $[0,1]$. The incentive for investment for a man at any value of ε depends upon the ratio of the densities, $\frac{f(\varepsilon)}{g(\phi(\varepsilon)}$. This ratio exceeds one for low values of ε, but is less than one for high values of ε. Conversely, for women, the incentive to invest depends upon the inverse of this ratio, $\frac{g(\eta)}{f(\phi^{-1}(\eta))}$, which is low at low values of η but high at high values of η. In other words, the ratio of the densities plays opposite roles for the two sexes. However, the weights with which these ratios are aggregated differs between the sexes; high values of η are given relatively large weight in the case of women, since $g(\eta)$ is large in this case, while they are given relatively less weight in the case of men.

3.2 Talent Shocks and Complementarities with Investment

Consider next the case where investment is in education and the uncertainty is talent risk. It is plausible that the return to investment depends upon the talent of the child. To model this, we suppose that quality is given by a Cobb-Douglas production function, $q(x,\varepsilon) = x^\alpha \varepsilon$, where $\alpha \leq 1$. All our results apply to a more general Cobb-Douglas form, $q(x,\varepsilon) = x^\alpha \varepsilon^\beta$; we can redefine a new random variable $\tilde{\varepsilon} = \varepsilon^\beta$, and the results that follow will apply. With this production function, $q_x = \alpha x^{\alpha-1}\varepsilon$ and $q_\varepsilon = x^\alpha$, so that the first

[5] The uniform distribution violates our assumption (A1), since $f(.)$ is non-zero at the infimum of its support. This implies that the left hand derivative of a boy's payoffs with respect to investments is strictly greater than the right hand derivative – see the discussion following the proof of Theorem 1, in Appendix A. We focus on the equilibrium where the right hand derivative equals zero, i.e. the one with smallest investments. Any other equilibrium will have strictly larger investments.

order condition for equilibrium reduces to

$$\int (x_G^*)^\alpha \frac{f(\varepsilon)}{g(\phi(\varepsilon))} \frac{\alpha\varepsilon}{x_B^*} f(\varepsilon)\, d\varepsilon = c_B'(x_B^*), \tag{15}$$

and

$$\int (x_B^*)^\alpha \frac{g(\eta)}{f(\phi^{-1}(\eta))} \frac{\alpha\eta}{x_G^*} g(\eta)\, d\eta = c_G'(x_G^*). \tag{16}$$

Consider the case where $f = g$. In this case, the equilibrium investments satisfy the conditions

$$\alpha \frac{(x_G^*)^\alpha}{x_B^*} \mathbf{E}(\varepsilon) = c_B'(x_B^*),$$

$$\alpha \frac{(x_B^*)^\alpha}{x_G^*} \mathbf{E}(\eta) = c_G'(x_G^*).$$

Unlike the additive case, the "reaction function" for the boys is upward sloping in the girl's investments, and vice versa. Intuitively, with if quality is multiplicative, an increase in the girl's investment level increases the dispersion in qualities on the girl's side, thereby increasing investment incentives for boys. However, since $\alpha \leq 1$, the reaction functions have slope less than one, so that there is a unique solution to these equations, and equilibrium is unique. If the cost functions are also identical, then $x_B^* = x_G^* = x^*$. This implies that $c'(x_B^*) = c'(x_G^*) = \alpha\,(x^*)^{\alpha-1}\,\mathbf{E}(\varepsilon)$, which is the same condition as for the first best.

The following theorem shows that there will be generic overinvestment, if the densities are both symmetric, if they are different from each other.

Theorem 3 *Suppose that quality is multiplicative and that the distributions f and g are symmetric. Then in any quasi-symmetric equilibrium investments are generically excessive relative to Pareto efficiency.*

This result is a robust one, in the following sense. Suppose that f and g are symmetric and linearly independent. The Cauchy-Schwarz inequality implies that investment will be strictly too high. Now if we perturb the distributions so the $\tilde{f}$ is close to f and $\tilde{g}$ to g, then $c'(x_B^*) \times c'(x_G^*)$ will still be greater than 1, since the integrals defining this are continuous in the distributions. In other words, we will have excessive investments even with asymmetric distributions as long as the asymmetries are not too large.

Why does the result require that the asymmetry not be too large? To provide some intuition for this, let us return to the example where the distribution of shocks is uniform on $[0, 1]$ for men and where the density function for women, $g(\eta) = 2\eta$ on $[0, 1]$. Here again, the ratio of the densities that is relevant for men, $\frac{f(\varepsilon)}{g(\phi(\varepsilon))}$, is relatively large when ε is low. While these values of ε still have large weight (since $f(\varepsilon)$ is constant in ε), in the multiplicative case, the payoff to investment is low when ε is small. Under symmetry, neither particularly low values nor particularly high values of ε have any special weight and thus the inefficiency result applies.

The Example Re-interpreted

Let us assume that f and g are symmetric around 1 on the interval $[0,2]$. It will suffice therefore to specify the functions on $[0,1]$. Let f be uniform, so that $F(\varepsilon)=\frac{\varepsilon}{2}$ on $[0,1]$, and let $G(\eta)=\frac{\eta^n}{2}$ on $[0,1]$. Let $\alpha=1$, so that quality is multiplicative, $q=x\varepsilon$. It may be useful to note that the shock distributions are "symmetrized extensions" of the shocks in example 3.1. That is, the function g in the current example is constructed by taking the function g in example 3.1, and "reflecting it" in a mirror situated at 1 (the resulting function has to be halved, so as to ensure that it represents a probability distribution).

The equilibrium conditions are now given by

$$\begin{aligned} c'(x_B^*) &= \left(2\int_0^1 \frac{[f(\varepsilon)]^2}{g(\phi(\varepsilon))}\,d\varepsilon\right)\frac{x_G^*}{x_B^*} = \frac{1}{2n}\int_0^1 \varepsilon^{\frac{1}{n}}\,d\varepsilon. \\ &= \frac{1}{n}\frac{x_G^*}{x_B^*}. \end{aligned} \tag{17}$$

Note that in Example 1, where shocks where additive, marginal costs were equal to $1/n$.

$$\begin{aligned} c'(x_G^*) &= \left(2\int_0^1 \frac{g(\eta)}{f(\phi^{-1}(\eta))}g(\eta)\eta\,d\eta\right)\frac{x_B^*}{x_G^*} = n^2\int_0^1 \eta^{2n-1}\,d\eta \\ &= \frac{n^2}{2n-1}\frac{x_B^*}{x_G^*}. \end{aligned} \tag{18}$$

Here again, the expression is identical to the one in Example 1, except for the term in the investment levels. Thus,

$$c'(x_B^*)\times c'(x_G^*) = \frac{n}{2n-1}.$$

For arbitrary Pareto weights λ, the efficiency condition is

$$c'(x_B^{**})\times c'(x_G^{**}) = \left[\frac{\lambda}{1-\lambda}\mathbf{E}(\varepsilon)\right]\left[\frac{1-\lambda}{\lambda}\mathbf{E}(\eta)\right] = 1.$$

We therefore see that the equilibrium investments are always excessive, just as in the additive case. Indeed, the extent of inefficiency is same as in example one, provided that one uses the symmetrized extensions of the original density functions for the multiplicative case.

This example illustrates a more general duality between the additive and multiplicative model. Suppose that we have an additive model with density functions for the noise, f and g, on compact supports. Now consider a multiplicative model with densities $\tilde{f}$ and $\tilde{g}$ that are "symmetrized extensions" of f and g respectively. That is $\tilde{f}=\frac{f}{2}$ on its original support $[a,b]$ and $\tilde{f}$ is symmetric around b. $\tilde{g}$ is defined analogously. Then the overinvestment in the additive model is, up to a constant, the same as in the multiplicative model.

3.3 A Mixed Model

Suppose that shocks are additive for women but multiplicative for women. One example is a traditional society, where women do not work, and so investment in them takes the form of a dowry; while parents invest in their sons' human capital. The marginal benefit to men of an increment in women's investment is unity, while the marginal benefit to women from a man's investment is $\mathbf{E}(\varepsilon)$. Thus Pareto efficient investments satisfy the condition

$$c'(x_B^{**}) \times c'(x_G^{**}) = \mathbf{E}(\varepsilon).$$

Here again, as in the pure multiplicative model, there is generic overinvestment if f and g are symmetric.

Theorem 4 *Suppose that quality is multiplicative for men and additive for women. If the distributions f and g are symmetric, in any quasi-symmetric equilibrium investments are generically excessive relative to Pareto efficiency.*

Here again, we can see an equivalence between symmetric extensions of examples in the additive case and the mixed case.

4 Efficiency in the Absence of Gender Differences

Let us now consider the implications of ex-ante heterogeneity, where individuals differ even before shocks are realized. We begin with an illustrative example. Assume that the sex ratio is balanced. Suppose that we have two classes, H and L, with fractions θ_H and θ_L in the population. Assume that the marginal costs of investment are lower for the upper class, H. Let $c_H(.)$ and $c_L(.)$ be the respective cost functions, where $c'_H(x) < c'_L(x)$ for any x. Let $f_i(.)$ and $g_i(.), i \in \{H, L\}$ denote the density function of shocks for the boys from class i, and the girls from class i respectively. Assume that quality function is additive in the shocks and investment.

Consider a profile of investments $(x_{HB}, x_{LB}, x_{HG}, x_{LG})$ where each individuals who belong to the same class and same gender choose the same investment. This profile induces a distribution of qualities for the boys, $\tilde{F}(q)$, and of girls, $\tilde{G}(p)$. Since any stable measure preserving matching $\tilde{\phi}$ must be assortative, we must have $\tilde{F}(q) = \tilde{G}(\tilde{\phi}(p))$.

Let $x^*_{HB}, x^*_{LB}, x^*_{HG}, x^*_{LG}$ be the equilibrium investment levels. Suppose that the distribution of qualities of in both the sexes has a connected support, without any gaps. In class i, the first order condition for investment in boys is given by

$$\int_{q_{\min}}^{q_{\max}} \frac{\tilde{f}(q)}{\tilde{g}(\phi(q))} f(q - x^*_{iB})\, dq = c'_B(x^*_{iB}). \tag{19}$$

The density function for boy's quality is given by (that for girls is analogous):

$$\tilde{f}(q) = \theta_H f_H(q - x^*_{HB}) + \theta_L f_L(q - x^*_{LB}).$$

Let the class differences be arbitrary, so that f_H can differ from f_L and g_H from g_L. However assume that there are no gender differences, so that $f_H = g_H$ and $f_L = g_L$. Consider a *gender neutral strategy* profile, where investments depend on class but not on gender, so that $x^*_{iB} = x^*_{iG}$ for $i \in \{H, L\}$. Since the shocks do not vary between the sexes, the induced distribution of qualities will be identical in the two sexes. That is, for any value q, $\tilde{F}(q) = \tilde{G}(q)$, implying that $\tilde{\phi}(q) = q$. This in turn implies that $\tilde{f}(q) = \tilde{g}(\phi(q))$. Therefore the left hand side of equation (19) equals one.

Consider a utilitarian social planner who puts equal all types of individual, irrespective of gender or social class. Since the marginal benefit of additional investment in a boy is unity, for any girl who is matched with him, such a planner would set the marginal cost of investment to one. We conclude therefore that investment in boys is utilitarian efficient if there are no gender differences, even if there is large heterogeneity between classes. Similarly, investments in girls is utilitarian efficient.

We show now that this argument is very general – provided that there are no differences between the sexes, equilibrium investments will be utilitarian efficient even if there is wide heterogeneity within each sex. Assume that there is a finite set of types, indexed by $i \in \{1, 2, .., n\}$. Type i has a measure μ^i_B of boys and a measure μ^i_G of girls. We assume that there are equal measures of boys and girls, and normalize each of these to 1, i.e. $\sum_{i=1}^n \mu^i_B = 1$ and $\sum_{i=1}^n \mu^i_G = 1$. A boy of type i has an idiosyncratic component of quality, ε, that is distributed with a density function $f_i(\varepsilon)$ and a cumulative distribution function $F_i(\varepsilon)$. Let η denote the realization of the idiosyncratic quality component for a girl; for type i, this is distributed with a density function g_i and cdf G_i. We shall assume a general quality functions $q(x, \varepsilon)$ and $q(x, \eta)$, where q is continuous and strictly increasing in both arguments. Finally, we shall assume that each individual strictly prefers to be matched rather than unmatched.

A utilitarian efficient profile of investments $(x^{**}_{Bi}, x^{**}_{Gi})^n_{i=1}$ is one that maximizes the sum of payoffs of all individuals, irrespective of type or gender. This satisfies the conditions

$$c'_{Bj}(x_B) = \int q_x(x_B, \varepsilon) f_j(\varepsilon)\, d\varepsilon, \tag{20}$$

$$c'_{Gi}(x_G) = \int q_x(x_G, \eta) g_i(\eta)\, d\eta. \tag{21}$$

We shall focus upon pure strategy equilibria where all agents who are identical choose the same strategy. That is, if two parents belong to the same type i, and have a child of the same gender j, they choose the same level of investment x_{ji}. Thus an equilibrium consists of a profile $((x_{Bi})^n_{i=1}, (x_{Gi})^n_{i=1})$, specifying investment levels for each type of each gender.

Any strategy profile induces a distribution of qualities of boys and girls in the population. In particular, $\mathbf{x}_B \equiv (x_{Bi})_{i=1}^n$, in conjunction with the realization of idiosyncratic shocks, induces a cumulative distribution function of qualities, $\tilde{F}$, in the population of boys. Since ε is assumed to be atomless, and q is continuous, $\tilde{F}$ admits a density function $\tilde{f}$, although its support may not be connected if the investment levels of distinct types are sufficiently far apart (i.e. there may be gaps in the distribution of qualities). Similarly, let $\tilde{G}$ denote the cumulative distribution function of girl qualities. Since stable measure matchings will be assortative, i.e. if a boy of type q is matched to a girl of type $\phi(q)$ if and only if

$$\tilde{F}(q) = \tilde{G}(\phi(q)) \tag{22}$$

Thus the distributions $\tilde{F}$ and $\tilde{G}$ define the match payoffs associated with equilibrium investments for each type of boy and each type of girl.

The following assumption will play an important role in our results:

Symmetry Assumption: Men and women are symmetric with regards to costs of investment and the idiosyncratic component of quality. Specifically, for any type i: i) $\mu_B^i = \mu_G^i$, ii) the investment cost functions and quality functions do not differ across the sexes, and iii) $f_i = g_i$, the idiosyncratic component of quality has the same distribution.

The symmetry assumption is strong, but there are reasonable conditions under which it is satisfied. Suppose that investment costs or the idiosyncratic component depend upon the "type" of the parent (e.g. parental wealth, human capital or social status), but not directly upon gender. If the gender of the child is randomly assigned, with boys and girls having equal probability, then the symmetry assumption will be satisfied.

We shall call a strategy profile *gender neutral* if $x_{Bi} = x_{Gi} \forall i$, so each type of parent invests the same amount regardless of the gender of their child.

A profile of investments, $((x_{Bi})_{i=1}^n, (x_{Gi})_{i=1}^n)$, has *no quality gaps* if the induced distributions of qualities, $\tilde{F}(q)$ and $\tilde{G}(p)$, have connected supports.

Theorem 5 *If $((x_{Bi}^*)_{i=1}^n, (x_{Gi}^*)_{i=1}^n)$ is an equilibrium where type i of gender j is matched with positive probability, then $x_{ji}^* > \bar{x}_{Bi}$, the privately optimal investment level. Suppose that the symmetry assumption is satisfied. The utilitarian efficient profile of investments is a gender neutral equilibrium if it has no quality gaps. A gender neutral strategy profile that has no quality gaps is an equilibrium if and only if it locally maximizes utilitarian payoffs.*

The efficiency result applies plausibly to a non-marriage context. Consider a single-population matching model, where quality is a one-dimensional scalar variable. An example is partnership formation, e.g. firms consisting of groups of lawyers. Theorem 5 implies that one has efficient ex ante investments, even absent transferable utility. While the formal proof restricts attention to pair-wise matching, the extension to matches consisting of more than two partners is immediate.

The intuition for the efficiency result is as follows. Consider a gender neutral profile of investments, where there are no quality gaps. Then a boy of quality q will be matched with a girl of the same quality, i.e. $\tilde{\phi}(q) = q$. Thus the marginal return to investment equals the increment to his own quality, and thus private incentives and utilitarian welfare are perfectly aligned.

This result does require the no quality gap assumption, which will be satisfied if the support of the shocks is large enough, or if sufficient similarly between adjacent types so that their equilibrium investments are not too far apart. Interestingly, if there are quality gaps, then there is a tendency to overinvestment, rather than under investment. Let us return to the two-class illustrative example at the beginning of this section, and suppose that the differences in utilitarian investments between the rich and the poor are so large that there is a quality gap. Suppose that an individual boy deviates from this profile and has a quality realization that is greater than the best poor boy, and smaller than the worst rich boy. Assume that the deviator is assigned either the match of the former or that of the latter, each with probability one-half.[6] Under such a matching rule, the poor boys would have an incentive to deviate upwards – the rich boys have no incentive to deviate downwards.[7]

5 Applications: Gender Differences and Sex Ratio Imbalances

We have established that investments will be utilitarian efficient if there are no gender differences and if the sex ratio is balanced. On the other hand, if there are any gender differences, investments will be excessive. We now turn to the observational implications of gender differences and of sex ratio imbalances.

5.1 Gender Differences

We use our model to examine a contentious issue – what are the implications of gender differences. Let us assume that the shocks to quality constitute talent shocks. One issue, that excites great controversy, is whether the distributions differ for men and women. For example, Baron-Cohen (2004) and Pinker (2003) argue that there are intrinsic gender differences that are rooted in biology, while Fine (2010) has attacked this view. In any case, in a study based on test scores of 15-year-olds from 41 OECD countries, Machin and Pekkarinen (2008) find that boys show greater variance than

[6] Such a rule could be justified as the limit of a model with a large but finite number of agents of each type, along the lines of argument in Appendix B.

[7] The efficiency result can be obtained without the no quality gap assumption if we modify the matching rule so that a deviator is always assigned the match of the next worst individual. While such a matching rule is formally correct in continuum model, it does not seem to correspond to the limit of a reasonable finite model, such as the one set out in Appendix B.

girls in both reading and mathematics test scores in most countries. We now explore the implications of differences in variability between the sexes.

Suppose that the shocks are more variable for men then for women. One way to formalize the idea of a distribution being more variable than another is the dispersive order. A distribution F is larger in the dispersive order than a distribution F, or $F \geq_d G$ if

$$g(G^{-1}(z)) \geq f(F^{-1}(z)) \text{ for all } z \in (0,1), \tag{23}$$

with the inequality being strict on a set of z values with positive measure (see Shaked and Shanthikumar (2007, pp148-9)). For example, if one has two uniform distributions where the support of one is a strictly longer interval than the other, then the first is larger than the second in the dispersive order. A second example is of two normal distributions with different variances. These measures of dispersive order do not rely upon an equality of means (see Hopkins and Kornienko (2010) for further examples and discussion).

Proposition 1 *Assume shocks are additive or multiplicative. If the distribution of shocks for boys is more dispersed than that for girls, that is $F \geq_d G$, then there is under-investment in boys and there is over-investment in girls relative to the utilitarian efficient level.*

Some intuition for this result is provided in Figure 2. There, as assumed in the above proposition, the distribution of shocks for boys is more dispersed than it is for girls (the range $[\underline{\varepsilon}, \bar{\varepsilon}]$ is clearly larger than $[\underline{\eta}, \bar{\eta}]$). Notice that in the diagram quite a large deviation Δ by the boy, given the relatively dispersed distribution of boys, leads to a relatively small increase in the match quality of the girl obtained. The point is that the dispersion of boys means that it is relatively difficult for a boy to overtake his competitors and therefore the return to increased investment is relatively low. In contrast, a girl would have a relatively high return to greater investment as the high density of her competitors around her would make them relatively easy to overtake. Therefore in equilibrium investment in girls is greater than in boys.

It has been observed that frequently the average performance of girls in school is better than that of boys. Our model provides a possible partial explanation for this – the incentives to invest for girls are greater, from marriage market matching considerations.

It is interesting that while differences in the dispersion of shocks have strong implications, differences in the mean play a lesser role. Suppose that f is a translation of g, i.e. $f(\varepsilon) = g(\varepsilon + k)$ for some k, but cost functions are the same for the two sexes. If $k > 0$, this corresponds to the case where women are uniformly better than men. If quality is additive, investments will still be utilitarian efficient, and this difference has no implications for investment incentives. If quality is multiplicative, the first order conditions imply that there must be underinvestment by women relative to the utilitarian efficient amount, and overinvestment by men, since $c'(x_G^*) < \alpha\,(x_G^*)^{\alpha-1}\,\mathbf{E}(\eta)$ while $c'(x_B^*) > \alpha\,(x_B^*)^{\alpha-1}\,\mathbf{E}(\varepsilon)$. Thus, in the multiplicative case, investment behavior partially offsets differences in mean quality.

5.2 Sex Ratio Imbalances

Sex ratio imbalances are an important phenomenon in countries such as China and parts of India. These imbalances are extremely large in China, where it is estimated that one in five boys born in the 2000 census will be unable to find a marriage partner (see Bhaskar, 2011). Wei and Zhang (2009) argue that the high savings rate in China is partly attributable to the sex ratio imbalance. They argue that parents of boys feel compelled to invest more, in order to improve their chances of finding a partner, thus raising the overall savings rate. However, one might conjecture that this might be counter-balanced by the reduced pressure felt by the parents of girls. We therefore turn to our model to provide an answer to this question.

Assume that each sex is ex ante identical, and let the relative measure of girls equal $r < 1$. For simplicity and brevity, we specialize to the model with additive shocks. At the matching stage, since $r \leq 1$, all girls should be matched, and the highest quality boys should be matched. Since every girl is matched, the investment in her generates benefits for herself and for her partner (for sure). Thus the first best investment level in a girl, x_G^{**}, satisfies $c'(x_G^{**}) = 1$. Now consider investment in a boy. If we assume that the idiosyncratic component of match values is sufficiently small, then welfare optimality requires that only a fraction r of boys invest, and that their investments also satisfy $c'(\cdot) = 1$. However, if we restrict attention to symmetric investment strategies, then investment will take place in all boys, and since investment occurs before ε is realized, each boy has a probability r of being matched, and thus the first best efficient level of investment in a boy, x_B^{**}, must satisfy $c'(x_B^{**}) = r$, i.e. the marginal cost must equal the expected marginal benefit. Similarly, one can show that the condition for Pareto efficiency, with arbitrary weights on the welfare of boys and girls is

$$c_B'(x_B^{**}) \times c_G'(x_G^{**}) = r. \tag{24}$$

We now turn to equilibrium. Suppose that all boys invest the same amount x_B^* and all girls invest the same amount x_G^*. Since the top r fraction of boys will only be matched, this corresponds to those having a realization of $\varepsilon \geq \hat{\varepsilon}$ where $F(\hat{\varepsilon}) = 1 - r$. In this case, a boy of type $\varepsilon \geq \hat{\varepsilon}$ will be matched with a girl of type $\phi(\varepsilon)$, where

$$1 - F(\varepsilon) = r[1 - G(\phi(\varepsilon))]. \tag{25}$$

The derivative of the matching function is given by

$$\phi'(\varepsilon) = \frac{f(\varepsilon)}{rg(\varepsilon)}. \tag{26}$$

That is, an increase in ε increases a boy's match quality relatively more quickly, since the distribution of girls is relatively thinner, since $r < 1$.

Those boys with realisations below $\hat{\varepsilon}$ will not be matched and receive a payoff $\bar{u}$. Let us call this the "misery effect" – it reflects the disutility (misery) from remaining single. As Hajnal (1982) has noted, in Asian societies such as China and India, marriage rates have historically been extremely high (over 99%, as compared to the traditional "European marriage pattern" with marriage rates around 90%). Thus the misery effect is likely to be large in Asian societies.

The first order condition for boys in an equilibrium where all boys invest the same amount x_B^*, while all girls invest the same amount x_G^* is given by

$$\frac{1}{r}\int_{\hat{\varepsilon}}^{\bar{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\, d\varepsilon + f(\hat{\varepsilon})(\underline{\eta} + x_G^* - \bar{u}) = c_B'(x_B^*). \tag{27}$$

As compared to our previous analysis, we notice two differences. The first term is the improvement in match quality, and the sparseness of girls increases the investment incentives, due to the term in $1/r$. Additionally, an increment in investment raises the probability of one's son getting matched, at a rate $f(\hat{\varepsilon})$, and the marginal payoff equals the difference between matching with worst quality girl and receiving $\underline{\eta} + x_G^*$, and not being matched and receiving $\bar{u}$. An unbalanced sex ratio tends to amplify investments in boys, for two reasons. First, a given increment in investment pushes boys more quickly up the distribution of girls, and second, there is an incentive to invest in order to increase the probability of match taking place at all, since there is discontinuous payoff loss from not being matched at $\hat{\varepsilon}$, due to the misery effect.

Similarly, the first order condition for investment in girls is given by

$$r\int_{\underline{\eta}}^{\bar{\eta}} \frac{g(\eta)}{f(\phi^{-1}(\eta))} g(\eta)\, d\eta = c'(x_G^*). \tag{28}$$

Notice here that the role of $r < 1$ is to reduce investment incentives, since an increment in investment pushes a girl more slowly up the distribution of boy qualities. Furthermore, there is no counterpart to the misery effect for the scarcer sex, and the only reason to invest arises from the consequent improvement in match quality.

Since girls are in excess supply, a girl whose parents invest and whose quality realization is discretely lower than every other girl will still be able to find a partner. Such a girl will get a match payoff of $x_B^* + \hat{\varepsilon}$, no matter how low her own quality. Thus, the conditions for the existence of a quasi-symmetric equilibrium are more stringent than in the balanced sex ratio case. Large downward deviations in investment will not be profitable provided that the dispersion in the qualities of boys is sufficiently large, and the cost function for girls is sufficiently convex.

Proposition 2 *If $r < 1$ and the noise is additive, there exists a unique quasi-symmetric equilibrium, provided that $f(.)$ is sufficiently dispersed and $c_B(.)$ is sufficiently convex. Investments are excessive relative to Pareto efficiency, for any distributions of noise.*

It is worth pointing out that even absent the misery effect, there will be strictly excessive investments, even if the noise distributions are identical, unless they happen to be uniform. When $r < 1$, $g(\phi(\varepsilon))$ is not a linear transformation of $f(\varepsilon)$ unless f

and g are uniform. Thus the inequality in (50) will be strict, and excessive investment follows.

As we have already noted, if F and G have the same distributions, and $r = 1$, investments are utilitarian efficient. Thus a balanced sex ratio is sufficient to ensure efficiency of investments in this case. This provides an additional argument for the optimality of a balanced sex ratio, over and above the congestion externality identified in Bhaskar (2011).

We may use our model to evaluate the theoretical basis of the empirical work by Wei and Zhang (2009), attributing the high savings rate in China to the sex ratio imbalance. As an example, consider the case where the noise distributions are both uniform. Let k be the ratio of the densities, i.e. $k = \frac{f(\varepsilon)}{g(\eta)}$. The equilibrium investment levels are defined by

$$c'(x_B^*) = k + f(\hat{\varepsilon})(\underline{\eta} + x_G^* - \bar{u}).$$

and

$$c'(x_G^*) = \frac{r}{k}.$$

If $k = 1$ so that the two distributions have the same dispersions, we see that there are excessive investment in boys relative to utilitarian levels – the marginal cost of investment exceeds one, while efficiency at the investment stage requires $c'(x_B^*) = r < 1$. There are insufficient investments in girls, since $c'(x_G^*) = r$ rather than the utilitarian level, one. If we assume that the costs of investment are quadratic (so that marginal costs are linear), aggregate investment in the economy, $X^*(r)$, will be proportional to the weighted sum of the right hand side of the optimality conditions, i.e.

$$X^*(r) = \frac{r}{1+r}x_G^* + \frac{1}{1+r}x_B^* \propto \frac{1+r^2}{1+r} + \frac{x_G^*}{\bar{\varepsilon}(1+r)}.$$

Note that the first term, on the right hand side, $\frac{1+r^2}{1+r}$, is increasing in r, while the derivative of sign of the second term is ambiguous, since both the numerator and denominator are increasing in r. However, if the incentive for boys to ensure a match is large enough, total investment will increase with sex ratio imbalances.

While the uniform example is useful for getting an explicit solution for total investments, it is slightly misleading – $f(\hat{\varepsilon})$, the value of the density function for the marginal type of boy, does not vary with r. If the density function is unimodal, the $f(\hat{\varepsilon})$ would increase as r declines in the empirically relevant range, so that the misery effect would be stronger, thereby increasing investments.

We can also use this model to examine the implications of sex ratio imbalances for investments in the presence of wealth inequalities. For reasons of space, we restrict ourselves to a verbal discussion – details of the analysis are available from the authors. Consider a society with two social classes, the rich and the poor. Since rich parents have higher wealth, the marginal costs of investment, in terms of the utility of foregone consumption, will be lower for them than the poor. Thus, in equilibrium, the rich will invest more than the poor. Now suppose that r, the ratio of boys to girls, is less than one in both classes. Thus rich boys who have a low realization of the idiosyncratic shock will marry poor girls who have a high realization. This aggravates the marriage market imbalance facing poor boys. Investment incentives for rich and poor boys are greater due to the sex ratio imbalance. Interestingly, poor girls now have an additional incentive to invest since this improves their chances of being matched with rich boys. Thus, wealth inequalities may interact with gender imbalances, to further raise investments.

6 Conclusions

In this paper, we examined a matching tournament model of marriage with competing investments that have stochastic returns. This approach has the advantage that it assures the existence of a unique pure equilibrium, in an area where many models have multiple equilibria or equilibrium only exists in mixed strategies. We find that investment in this equilibrium is generically inefficiently high, with efficiency only occurring for sure when the two sexes are absolutely identical.

Of course, claims of differences between the sexes are controversial. However, there is no need to assume such differences are intrinsic rather than arising from particular forms of social or market organization. Further, there is one difference that we show implies excessive investment which is entirely objectively measurable: an unequal sex ratio. The fact that the sex ratio is now far from one in both the world's two most populous nations has attracted increased attention from economists. For example, Wei and Zhang (2009) present evidence that the uneven sex ratio drives the high saving rate in China. Our current results suggest that, if indeed the observed increased saving results from an unbalanced sex ratio, saving may be inefficiently high. We hope that the current approach may provide a relatively tractable model that will help in further investigation of this and related issues.

Appendix A: Proofs

Before proving Theorem 1, we state our assumptions.

Assumption A1:

1. Assumptions on shocks: let $f(\varepsilon)$ and $g(\eta)$ be differentiable on their bounded supports $[\underline{\varepsilon}, \bar{\varepsilon}]$ and $[\underline{\eta}, \bar{\eta}]$ respectively, with $f(\underline{\varepsilon}) = 0$ and $g(\underline{\eta}) = 0$.
2. Assumptions on costs: let $c_B(x_B)$ and $c_G(x_G)$ be twice differentiable and strictly increasing. Further assume
   (a) Convexity: $c''_B(x_B) > 0$ and $c''_G(x_G) > 0$.

(b) There exist privately optimal levels of investment $\bar{x}_B > 0$, $\bar{x}_G > 0$ such that $c'_B(\bar{x}_B) = 0$ and $c'_G(\bar{x}_G) = 0$.

(c) Bounded maximum rational investment: there are investment levels $\tilde{x}_B$, $\tilde{x}_G$ such that $\lim_{x \to \tilde{x}_B} c'_B(x) = \infty$ and $\lim_{x \to \tilde{x}_G} c'_G(x) = \infty$.

3. Assumptions on quality: let $q(x_B, \varepsilon)$ and $q(x_G, \eta)$ be strictly increasing and three times differentiable, with $q_{xx}(x_B, \varepsilon) \leq 0$, $q_{\varepsilon\varepsilon}(x_B, \varepsilon) \leq 0$, $q_{xx}(x_G, \eta) \leq 0$, $q_{\eta\eta}(x_G, \eta) \leq 0$. Let either

   (a) $q(x_B, \varepsilon)$ and $q(x_G, \eta)$ be additive so that q_ε and q_η are constants.

   (b) If $q(x_B, \varepsilon)$ and $q(x_G, \eta)$ are not additive, then let $q_\varepsilon(0, \varepsilon) = 0$ and $q_\eta(0, \eta) = 0$ and $q_{x\varepsilon}(x_B, \varepsilon) > 0$, $q_{xx\varepsilon}(x_B, \varepsilon) \leq 0$ and $q_{x\eta}(x_G, \eta) > 0$, $q_{xx\eta}(x_G, \eta) \leq 0$.

   (c) The value of not being matched is fixed at $\bar{u}$ which satisfies $\bar{u} < q(0, \underline{\varepsilon})$ and $\bar{u} < q(0, \bar{\eta})$.

Proof of Theorem 1: We first derive the first order conditions as given in (8) and (9). We then show that these first order conditions are sufficient, in the sense that no individual can benefit from large deviations. Next, we show that the first order conditions define a unique symmetric best response for the boys as a function of the investment level of the girls, and vice versa, i.e. we have well-behaved reaction functions. Finally, we show that these reaction functions cross exactly once, so that equilibrium exists and is unique.

Consider a quasi-symmetric equilibrium where all boys invest x_B^* and all girls invest x_G^*. Suppose that a parent of a boy deviates from this equilibrium and invests $x_B^* + \Delta$ in his son. If the realization of the shock for his son is ε, combined with the higher level of investment, the son will hold the same rank in the population of boys as a boy with a shock level ξ, where $\xi(\varepsilon, \Delta)$ is defined by the equation

$$q(x_B^* + \Delta, \varepsilon) = q(x_B^*, \xi(x_B^* + \Delta, \varepsilon)). \tag{29}$$

For example, in the additive case $\xi(x_B^* + \Delta, \varepsilon) = \varepsilon + \Delta$. Given this deviation, the boy now holds rank $F(\xi)$ in the population of boys and can expect a match with a girl holding rank $G(\phi(\xi))$ in the population of girls. She would be of quality $q(x_G^*, \phi(\xi))$.

For $\Delta > 0$, define $\hat{\varepsilon}$ by $\xi(x_B^* + \Delta, \hat{\varepsilon}) = \bar{\varepsilon}$. That is, if the boy who deviates upwards has a high shock realization, specifically on the interval $[\hat{\varepsilon}, \bar{\varepsilon}]$, he will match with the highest ranking girl who has quality $q(x_G^*, \phi(\bar{\varepsilon}))$, for sure. Similarly for downward deviations, $\Delta < 0$, define $\tilde{\varepsilon}$ by the relation $q(x_B^*, \tilde{\varepsilon}) = q(x_B^* + \Delta, \underline{\varepsilon})$.

If all boys invest an amount x_B^* and girls x_G^*, then the expected match quality or benefit $B(\Delta)$ of a boy investing $x_B^* + \Delta$, where $\Delta > 0$, is given by

$$B(\Delta) = \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} q(x_G^*, \phi(\xi(x_B^* + \Delta, \varepsilon))) f(\varepsilon)\, d\varepsilon + (1 - F(\hat{\varepsilon})) q(x_G^*, \bar{\eta})). \tag{30}$$

The derivative of the expected match or benefit with respect to $\Delta > 0$ is then given by

$$B'(\Delta) = \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} q_\eta(x_G^*, \phi(\xi(x_B^* + \Delta, \varepsilon)))\phi'(\xi(x_B^* + \Delta, \varepsilon))\xi_x(x_B^* + \Delta, \varepsilon) f(\varepsilon)\, d\varepsilon. \tag{31}$$

To evaluate this at $\Delta = 0$, note that ϕ' and ξ_x, evaluated at $\Delta = 0$, are given by

$$\phi'(\varepsilon) = \frac{f(\varepsilon)}{g(\phi(\varepsilon))},$$

$$\xi_x(x_B^*, \varepsilon) = \frac{q_x(x_B^*, \varepsilon)}{q_\varepsilon(x_B^*, \varepsilon)}.$$

$\xi(x_B^* + \Delta, \varepsilon)$ evaluated at $\Delta = 0$ is simply ε. Since $\xi = \varepsilon$ when $\Delta = 0$, $\hat{\varepsilon} = \bar{\varepsilon}$ when $\Delta = 0$. Thus, evaluating (31) at $\Delta = 0$ gives us

$$B'(\Delta)\,|_{\Delta=0} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} q_\eta(x_G^*, \phi(\varepsilon)) \frac{f(\varepsilon)}{g(\phi(\varepsilon))} \frac{q_x(x_B^*, \varepsilon)}{q_\varepsilon(x_B^*, \varepsilon)} f(\varepsilon)\, d\varepsilon. \tag{32}$$

We have therefore shown that the right hand derivative, evaluated at $\Delta = 0$, equals the left hand side of the first order condition (8).

Turning to downward deviations, $\Delta < 0$, $B(\Delta)$ is given by

$$B(\Delta) = \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} q(x_G^*, \phi(\xi(x_B^* + \Delta, \varepsilon))) f(\varepsilon)\, d\varepsilon + F(\tilde{\varepsilon}) \frac{[\bar{u} + q(x_G^*, \underline{\eta})]}{2}. \tag{33}$$

That is, if the deviating boy has a low shock realization, specifically on the interval $[\underline{\varepsilon}, \tilde{\varepsilon}]$, his overall quality will be below the support of the equilibrium distribution of qualities. With equal probability, he will be left unmatched and have utility $\bar{u}$, or match with the lowest quality girl. We can therefore calculate the derivative with respect to a downward deviation $\Delta < 0$ as

$$B'(\Delta) = \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} q_\eta(x_G^*, \phi(\xi(\cdot)))\phi'(\xi(\cdot))\xi_x(x_B^* + \Delta, \varepsilon) f(\varepsilon)\, d\varepsilon + f(\tilde{\varepsilon}) \frac{(q(x_G^*, , \underline{\eta}) - \bar{u})}{2} \tag{34}$$

Now, because $\tilde{\varepsilon}$ is defined by the relation $q(x_B^*, \tilde{\varepsilon}) = q(x_B^* + \Delta, \underline{\varepsilon})$, then $\tilde{\varepsilon}$ goes to $\underline{\varepsilon}$ as Δ approaches zero, and because $f(\underline{\varepsilon})$ is zero by assumption, the above derivative approaches (32) as Δ goes to zero. Thus, the left and right derivatives exist and are equal at $\Delta = 0$ and give the first order condition (8). The first order condition for girls (9) can similarly be derived.

We now show that large deviations are not profitable. We first consider upward deviations. Given the strict convexity of costs, it suffices to show that $B'(\Delta) \leq B'(0)$ for any $\Delta > 0$. A sufficient condition is that $B(\Delta)$ as given in (30) is concave in Δ. This will hold if all the three functions $q(\cdot), \phi(\cdot)$ and $\xi(\cdot)$ are concave. We have seen earlier in this proof that ξ is concave, and q is concave by assumption. This leaves $\phi(\cdot)$. If a) F and G are identical or if they are of the same type (that is, $G(x) = F(ax + b)$), then $\phi(\varepsilon)$ is linear and therefore weakly concave. Thus, $\phi(\varepsilon)$ and hence $B(\Delta)$ is concave for an open set of distributions. We now consider b), $f'(\varepsilon) \geq 0$ and $g'(\eta) \geq 0$. While this does not imply that ϕ is concave, note that (31) can be written as

$$B'(\Delta) = \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} q_\eta(x_G^*, \phi(\varepsilon))\phi'(\varepsilon)\xi_x(x_B^* + \Delta, \gamma(x_B^* + \Delta, \varepsilon))f(\gamma(x_B^* + \Delta, \varepsilon))\, d\varepsilon. \qquad (35)$$

where $\gamma(x_B^* + \Delta, \varepsilon)$ is defined by the relation $q(x_B^*, \varepsilon) = q(x_B^* + \Delta, \gamma)$, which therefore implies that $\gamma_x < 0$. We then have

$$\begin{aligned} B''(\Delta) &= -\frac{\partial \tilde{\varepsilon}}{\partial \Delta} q_\eta(x_G^*, \phi(\varepsilon))\phi'(\varepsilon)\xi_x(x_B^* + \Delta, \gamma(x_B^* + \Delta, \varepsilon))f(\gamma(x_B^* + \Delta, \varepsilon)) \qquad (36)\\ &+ \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} [q_\eta(x_G^*, \phi(\varepsilon))\phi'(\varepsilon)\xi_x(x_B^* + \Delta, \gamma(x_B^* + \Delta, \varepsilon))f'(\gamma(x_B^* + \Delta, \varepsilon))\gamma_x(x_B^* + \Delta, \varepsilon) \\ &+ q_\eta(x_G^*, \phi(\varepsilon))\phi'(\varepsilon)\xi_{xx}(x_B^* + \Delta, \gamma(x_B^* + \Delta, \varepsilon))f(\gamma(x_B^* + \Delta, \varepsilon)) \\ &+ q_\eta(x_G^*, \phi(\varepsilon))\phi'(\varepsilon)\xi_{x\varepsilon}(x_B^* + \Delta, \gamma(x_B^* + \Delta, \varepsilon))f(\gamma(x_B^* + \Delta, \varepsilon))\gamma_x(x_B^* + \Delta, \varepsilon)]\, d\varepsilon \end{aligned}$$

which is negative as $f' \geq 0$, $\xi_{xx} \leq 0$, $\xi_{x\varepsilon} \geq 0$, $\gamma_x < 0$ and $\partial\tilde{\varepsilon}/\partial\Delta > 0$.

Turning finally to downward deviations, comparing (34) with $B'(0)$ as given in (32), we can see that we can find $\bar{u}$ sufficiently low such that $B'(\Delta) > B'(0)$ for $\Delta < 0$. Thus, given the convexity of costs, large downward deviations are not profitable.

We now look at the reaction functions implied by the first order conditions. We have

$$\xi_{xx} = \frac{d}{dx_B}\frac{q_x(x_B, \varepsilon)}{q_\varepsilon(x_B, \varepsilon)} = \frac{q_{xx}q_\varepsilon - q_{\varepsilon x}q_x}{q_\varepsilon^2} \leq 0,$$

given our assumptions that $q_{xx} \leq 0$, $q_{\varepsilon\varepsilon} \leq 0$ and $q_{\varepsilon x} \geq 0$. So, the left hand side of the first order condition (8) is concave in x_B, with the right hand side being convex by assumption. Clearly therefore, for each value of x_G there is a unique value of x_B that solves (8), or in other words the first order conditions (8) and (9) implicitly define reaction functions with $R_B(x_G)$ and $R_G(x_B)$ being the reaction functions implied by (8) and (9) respectively. Crossing of these functions will represent equilibrium points. We now prove that, under the above assumptions that we have made, the crossing must exist and be unique and thus there is a unique equilibrium. We first show that the reaction functions are both increasing. Differentiating (8) we have

$$\left.\frac{dx_B}{dx_G}\right|_{R_B} = \frac{\int q_{x\eta}(x_G, \phi(\varepsilon))\xi_x(x_B, \varepsilon)\theta_B(\varepsilon)\, d\varepsilon}{-\int q_\eta(x_G, \phi(\varepsilon))\xi_{xx}(x_B, \varepsilon)\theta_B(\varepsilon)\, d\varepsilon + c_B''(x_B)} \geq 0 \qquad (37)$$

as $\xi_{xx} \leq 0$ as established above and using $\theta_B(\varepsilon) = f^2(\varepsilon)/g(\phi(\varepsilon))$. Similarly, from (9), we have

$$\left.\frac{dx_G}{dx_B}\right|_{R_G} = \frac{\int q_{x\varepsilon}(x_B, \phi^{-1}(\eta))\xi_x(x_G, \eta)\theta_G(\eta)\, d\eta}{-\int q_\varepsilon(x_B, \phi^{-1}(\eta))\xi_{xx}(x_G, \eta)\theta_G(\eta)\, d\varepsilon + c_G''(x_G)} \geq 0, \qquad (38)$$

where $\theta_G(\eta) = g^2(\eta)/f(\phi^{-1}(\eta))$. Now, we show that $R_B(x_G)$ and $R_G(x_B)$ are both concave. We differentiate (38) to obtain, suppressing arguments,

$$\left.\frac{d^2 x_G}{dx_B^2}\right|_{R_G} = \frac{(\int q_{xx\varepsilon}\xi_x\theta_G\,d\eta)(-\int q_\varepsilon\xi_{xx}\theta_G\,d\varepsilon + c_G'') - (-\int q_{x\varepsilon}\xi_{xx}\theta_G\,d\varepsilon)(\int q_{x\varepsilon}\xi_x\theta_G\,d\eta)}{(-\int q_\varepsilon\xi_{xx}\theta_G\,d\varepsilon + c_G'')^2} \leq 0. \tag{39}$$

We can make a similar calculation for R_B.

We now examine where these reaction functions might cross. First, we address the case where quality is additive in the shock so that $q_{x\varepsilon}(x_B, \varepsilon) = 0$ and $q_{x\eta}(x_G, \eta) = 0$. Then, we can see from inspection of (37) that $R_B(x_G) = x_B^*$, a constant. Further, it is clear that $x_B^* > \bar{x}_B$. Similarly, $R_G(x_B) = x_G^*$. Thus, the equilibrium exists and is unique.

Second, if $q_{x\varepsilon}(x_B, \varepsilon) > 0$ and $q_{x\eta}(x_G, \eta) > 0$ note that now $R_B(x_G)$ and $R_G(x_B)$ are both strictly increasing and therefore invertible, and also both are strictly concave. Let $Q_B(x_B) = R_B^{-1}(x_B)$ be the inverse reaction function for boys. Further, given $q_\eta(0, \eta) = 0$ by assumption 2, then when $x_G = 0$ the first order condition (8) reduces to $c'(x_B) = 0$. That is, we have $R_B(0) = \bar{x}_B > 0$, and $Q_B(\bar{x}_B) = 0$. Similarly, $R_G(0) = \bar{x}_G > 0$. Thus, if we consider $Q_B(x_B)$ and $R_G(x_B)$ on the (x_B, x_G) plane, the first crossing point if any (and an equilibrium if it occurs) will be Q_B crossing R_G from below. Now, as $R_B(x_G)$ is strictly increasing and strictly concave, it follows that $Q_B(x_B)$ is strictly increasing and strictly convex. Clearly, as at such a first point of crossing we have $Q_B' \geq R_G'$ and as Q_B is convex and R_G is concave, to the right of the first crossing, it must hold that $Q_B' > R_G'$. Thus, there can be no further crossing as at a further crossing, it would have to hold that $Q_B' \leq R_G'$. So, any crossing is unique. Lastly, there must be at least one crossing, as by assumption 2c on costs and examination of the first order conditions we have $\lim_{x \to \tilde{x}_B} Q_G(x) = \infty$ and $\lim_{x \to \infty} R_G(x) = \tilde{x}_G$. Thus, there exists a unique pair (x_B^*, x_G^*) that solve (8) and (9). We have therefore proved the existence and uniqueness of quasi-symmetric equilibrium. ∎

In some of our examples, e.g. those with a uniform distribution, the densities f or g are not continuous at the lower bound of their support. In this case, the left hand derivative of the benefit function, (32) is strictly greater than the right hand derivative, (34). Since optimality requires that the left hand derivative of benefits is greater than or equal to marginal costs, and that the right hand derivative is less than or equal, there is a continuum of equilibria in this case. We focus in these examples upon the equilibria with the smallest investments, i.e. where the right hand derivative equals marginal costs. Given that our results demonstrate overinvestment, investments will only be greater in any other equilibrium.

Proof of Theorem 3: Let f and g be symmetric functions around their means, $\tilde{\varepsilon}$ and $\tilde{\eta}$. Symmetry implies that $f(\tilde{\varepsilon} - \Delta) = f(\tilde{\varepsilon} + \Delta)$ for any Δ. If f and g are both symmetric, then $g(\phi(\varepsilon))$ is also symmetric around $\tilde{\varepsilon}$. Further, $\frac{f(\varepsilon)}{g(\phi(\varepsilon))}$ is also symmetric around $\frac{\tilde{\varepsilon}}{\phi(\tilde{\varepsilon})}$. Finally, symmetry implies that $\phi(\tilde{\varepsilon} + \Delta) + \phi(\tilde{\varepsilon} - \Delta) = 2\phi(\tilde{\varepsilon})$. Using these facts,

$$\begin{aligned}
\int \phi(\varepsilon)g(\phi(\varepsilon))\,d\varepsilon &= \int^{\tilde{\varepsilon}} \phi(\varepsilon)g(\phi(\varepsilon))\,d\varepsilon + \int_{\tilde{\varepsilon}} \phi(\varepsilon)g(\phi(\varepsilon))\,d\varepsilon \\
&= \int^{0} \phi(\tilde{\varepsilon}-\Delta)g(\phi(\tilde{\varepsilon}-\Delta))\,d\Delta + \int_{0} \phi(\tilde{\varepsilon}+\Delta)g(\phi(\tilde{\varepsilon}+\Delta))\,d\Delta \\
&= \left(\int^{0} \phi(\tilde{\varepsilon}-\Delta)+\phi(\tilde{\varepsilon}+\Delta)\right) g(\phi(\tilde{\varepsilon}-\Delta))\,d\Delta \\
&= 2\phi(\tilde{\varepsilon})\int^{0} g(\phi(\tilde{\varepsilon}-\Delta))\,d\Delta \\
&= 2\phi(\tilde{\varepsilon})\int^{\tilde{\varepsilon}} g(\phi(\varepsilon))\,d\varepsilon.
\end{aligned}$$

Similarly,

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\varepsilon\,d\varepsilon = 2\tilde{\varepsilon}\int^{\tilde{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\,d\varepsilon.$$

Thus we may write the product of the marginal costs as

$$c'(x_B^*)\times c'(x_G^*) = \left(2\tilde{\varepsilon}\int^{\tilde{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\varepsilon d\varepsilon.\right)\left(2\tilde{\eta}\int^{\tilde{\varepsilon}} g(\phi(\varepsilon))\,d\varepsilon.\right)\left(\alpha^2\left(x_B^* x_G^*\right)^{\alpha-1}\right). \quad (40)$$

By the Cauchy-Scwharz inequality,

$$\begin{aligned}
c'(x_B^*)\times c'(x_G^*) &\geq 4\tilde{\varepsilon}\tilde{\eta}\left(\int^{\tilde{\varepsilon}}\left(\frac{f(\varepsilon)}{(g(\phi(\varepsilon)))^{1/2}}\right)(g(\phi(\varepsilon)))^{1/2}\,d\varepsilon.\right)^2\left(\alpha^2\left(x_B^* x_G^*\right)^{\alpha-1}\right) \quad (41) \\
&\geq 4\tilde{\varepsilon}\tilde{\eta}\left(\frac{1}{2}\right)^2\left(\alpha^2\left(x_B^* x_G^*\right)^{\alpha-1}\right). \quad (42)
\end{aligned}$$

Thus the product of marginal costs exceeds $\tilde{\varepsilon}\tilde{\eta}\left(\alpha^2\left(x_B^* x_G^*\right)^{\alpha-1}\right)$, and investments are excessive relative to Pareto efficiency. ∎

Proof of Theorem 4: Using the same notation as for the multiplicative model, we may write the product of the marginal costs as

$$\begin{aligned}
c'(x_B^*)\times c'(x_G^*) &= \left(\int \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\varepsilon\,d\varepsilon\right)\left(\int g(\phi(\varepsilon))\,d\varepsilon\right), \\
&\quad \left(2\tilde{\varepsilon}\int^{\tilde{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon) d\varepsilon.\right)\left(2\int^{\tilde{\varepsilon}} g(\phi(\varepsilon))\,d\varepsilon.\right) \\
&\geq \tilde{\varepsilon}, \quad (43)
\end{aligned}$$

which shows our result. ∎

Proof of Theorem 5: Since all types have positive measure, and since the shocks are atomless, each type must be matched with probability one in a quasi symmetric equilibrium. If type i of a boy invests x_{Bi}^* and has a shock realization ε, his quality is equal to $q(x^*_{Bi}, \varepsilon)$.Denote his match quality type by $\tilde{\phi}(q(x^*_{Bi}, \varepsilon))$. $\tilde{\phi}$ is defined by the equation $\tilde{F}(q(x^*_{Bi}, \varepsilon)) = \tilde{G}(\tilde{\phi}(q(x^*_{Bi}, \varepsilon))$, and is strictly increasing in ε. Since q is strictly increasing in both arguments, this implies that $\tilde{\phi}(q(x^*_{Bi}, \varepsilon))$ is strictly increasing in x^*_{Bi} for ε in the interior of this interval. Thus the derivative of $c_{Bi}(.)$ must be strictly negative at x^*_{Bi}.

Suppose that the profile is symmetric so that for each type i, $x^*_{Bi} = x^*_{Gi}$. This implies that $\tilde{F} = \tilde{G}$, and so $\phi(q(x^*_{Bi}, \varepsilon)) = q(x^*_{Bi}, \varepsilon)$. Consider now the benefit for a boy of type i from investing $x^*_{Bi} + \Delta$. For any realization of the shock ε, his quality equals $q(x^*_{Bi} + \Delta, \varepsilon)$. If $q(x^*_{Bi} + \Delta, \varepsilon)$ belongs to the support of $\tilde{F}$, he will be matched with a type $\phi(q(x^*_{Bi} + \Delta, \varepsilon)) = q(x^*_{Bi} + \Delta, \varepsilon)$. Thus, if $q(x^*_{Bi} + \Delta, \varepsilon)$ belongs to the support of $\tilde{F}$ for almost all realizations of ε, given the distribution F_i, the payoff gain from investing $x^*_{Bi} + \Delta$ equals:

$$\int q(x^*_{Bi} + \Delta, \varepsilon) f_i(\varepsilon)\, d\varepsilon - c_{Bi}(x^*_{Bi} + \Delta). \tag{44}$$

If $q(x^*_{Bi} + \Delta, \varepsilon)$ is greater than the highest quality in the support of $\tilde{F}$, then the deviating individual is matched with the partner of the highest quality girl. If $q(x^*_{Bi} + \Delta, \varepsilon)$ is less than the lowest type in the support of $\tilde{F}$, the deviator is left unmatched with probability one-half. Thus for any Δ, the payoff from choosing $x^*_{Bi} + \Delta$ is less than or equal to the expression in (44). The payoff from the equilibrium investments is given by

$$\int q(x^*_{Bi}, \varepsilon) f_i(\varepsilon)\, d\varepsilon - c_{Bi}(x^*_{Bi}). \tag{45}$$

If the symmetric profile $(x^*_{Bi})^n_{i=1}$ is efficient, then the expression in (44) is less than that in (45) for all Δ, and so is also an equilibrium. This establishes "only if". To show "if", note that for $(x_{Bi})^n_{i=1}$ to be an equilibrium, (44) must be less than (45) for all Δ such that $q(x^*_{Bi} + \Delta, \varepsilon)$ belongs to the support of $\tilde{F}$ for almost all realizations of ε. Taking the limit as $\Delta \to 0$, and noting that in this case the support condition satisfied, a necessary condition for equilibrium is that

$$\int q_x(x^*_{Bi}, \varepsilon) f_i(\varepsilon)\, d\varepsilon - c'_{Bi}(x^*_{Bi}) = 0. \tag{46}$$

This is the first order condition for efficient investments. Similarly, the second order condition must also be satisfied. In other words, if we have a symmetric equilibrium, then the investments for each type of boy must locally maximize the difference between his benefit in terms of expected quality and investment costs. ∎

Proof of Proposition 1: With equal measures of men and women, if $\eta = \phi(\varepsilon)$, then ε and η have the same rank z in the two distributions. So, if $F \geq_d G$, then, by the definition (23), we have $\frac{f(\varepsilon)}{g(\phi(\varepsilon))} \leq 1$ for all values of ε, and is strictly less on a set values of ε of positive measure. Thus, in the additive case, the integral on the left-hand side of equation (10) is strictly less than one, and the integral on the left-hand side of equation (11) is strictly greater than one. If the investment cost functions are the same for the sexes, then girls invest more than boys. But in any case as utilitarian efficiency requires

$c'_B(x_B) = 1 = c'_G(x_G)$, boys' investment is lower and girls' investment is higher than the utilitarian levels.

Now suppose that quality is multiplicative for both sexes. Assume that the shocks have the same mean, so that cost functions are the same. If $F \geq_d G$, then

$$c'_B(x_B^*) = \int (x_G^*)^\alpha \frac{f(\varepsilon)}{g(\phi(\varepsilon))} \frac{\alpha\varepsilon}{x_B^*} f(\varepsilon)\, d\varepsilon < \frac{\alpha\,(x_G^*)^\alpha}{x_B^*} \mathbf{E}(\varepsilon), \tag{47}$$

and

$$c'_G(x_G^*) = \int (x_B^*)^\alpha \frac{g(\eta)}{f(\phi^{-1}(\eta))} \frac{\alpha\eta}{x_G^*} g(\eta)\, d\eta > \frac{\alpha\,(x_B^*)^\alpha}{x_G^*} \mathbf{E}(\eta). \tag{48}$$

Thus, investment in boys is less than the utilitarian level, while that in girls exceeds it. ∎

Proof of Proposition 2: This follows the proof of Theorem 1 except in the case of downward deviations by girls. A girl choosing $x_g^* + \Delta$ for some $\Delta < 0$ faces a marginal benefit of

$$B_{\underline{\eta}}^{\prime \bar{\eta}+\Delta} \frac{g(\eta)}{f(\phi^{-1}(\eta))} g(\eta - \Delta)\, d\eta \tag{49}$$

We can find a distribution $F(\varepsilon)$ sufficiently dispersed in the sense of the dispersion order (see (23) for a definition) so that $f(\cdot)$ is small enough to ensure that $B'(\Delta) > c_G'^*(x_G^*+\Delta)$ for $\Delta < 0$.

We have $f(\hat{\varepsilon})(\underline{\eta} + x_G^* - \bar{u}) > 0$, reflecting our assumption that the misery effect is strictly positive. Thus, combining the first order conditions (27) and (28),

$$c'(x_B^*) \times c'(x_G^*) > \left(\frac{1}{r} \int_{\hat{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\, d\varepsilon\right) \left(r \int \frac{g(\eta)}{f(\phi^{-1}(\eta))} g(\eta)\, d\eta\right).$$

Making a change of variables, from η to ε,

$$c'(x_B^*) \times c'(x_G^*) > \frac{1}{r} \left(\int_{\hat{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\, d\varepsilon\right) \left(\int_{\hat{\varepsilon}} g(\phi(\varepsilon))\, d\varepsilon\right).$$

By the Cauchy-Schwarz inequality,

$$\frac{1}{r} \left(\int_{\hat{\varepsilon}} \frac{f(\varepsilon)}{g(\phi(\varepsilon))} f(\varepsilon)\, d\varepsilon\right) \left(\int_{\hat{\varepsilon}} g(\phi(\varepsilon))\, d\varepsilon\right) \geq \frac{1}{r} \left[\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left(\frac{f(\varepsilon)}{(g(\phi(\varepsilon)))^{1/2}}\right) (g(\phi(\varepsilon)))^{1/2}\, d\varepsilon\right]^2 = \frac{1}{r}\left(r^2\right). \tag{50}$$

Thus $c'(x_B^*) \times c'(x_G^*) > r$ while efficiency requires equality. ∎

Appendix B: Finite Numbers

In this appendix, we show some results with a finite number of boys and girls analogous to those obtained in the main part of the paper with a continuum. We look at the limit as the number of boys and girls goes to infinity. We show that if there is uncertainty about the relative number of boys and girls, then the finite model approaches the continuum model analysed in the main body of the paper. In particular, this gives a foundation for the assumption we make in the continuum that downward deviations can result in the deviator being unmatched.

We assume here that quality is additive in investment and in the idiosyncratic shock. We further simplify considerably by assuming that shocks for girls are uniform on [0,1], so that $G(\eta) = \eta$. As before, matching will be assortative in quality. Assume there are n boys and m girls. Again we consider a quasi-symmetric equilibrium where all girls have the same level of investment x_G^* and all boys invest x_B^*. When $m \geq n$ so that every boy will be matched, the expected equilibrium match of a boy with shock ε can be calculated as therefore

$$x_G^* + \Pr[1]E[1] + \Pr[2]E[2] + ... + \Pr[n]E[n] = x_G^* + \phi_m(\varepsilon) \tag{51}$$

where $\Pr[j]$ is the probability of a boy's shock ε being j -th highest and $E[i]$ is the expected value of the $i-$th highest shock amongst girls. In equilibrium, the probability that a boy is higher ranked than another with another randomly chosen candidate is $\Pr[\varepsilon_i > \varepsilon_j] = F(\varepsilon)$. Note also that the expected value of the top prize, that is, $E[1]$, will be the expected value of the highest order statistic for the distribution $G(\eta)$ given m draws. Thus, $\phi_m(\varepsilon)$ is defined as the expected match value of a boy with a shock realisation ε when faced with a population of m girls.

Now, let us assume that $m = n - 1$, there is one fewer girl than there are boys, so that in equilibrium the worst boy will remain unmatched and get payoff v. Given the simplifying assumption that the girls' shocks are uniform, the expected values of their shocks will be $(n-1, n-2, ..., 1)/n$. Thus, the expected match of a boy choosing the equilibrium investment x_B^* will be

$$\begin{aligned} \phi_{n-1}(\varepsilon) = \ & (F(\varepsilon)^{n-1}\tfrac{n-1}{n} + (n-1)F(\varepsilon)^{n-2}(1-F(\varepsilon))\tfrac{n-2}{n} + \dots \\ & \dots + (n-1)F(\varepsilon)(1-F(\varepsilon))^{n-2}\tfrac{1}{n} + (1-F(\varepsilon))^{n-1}v) \end{aligned} \tag{52}$$

which simplifies to

$$F\frac{n-1}{n}\left(F^{n-2} + (n-2)F^{n-3}(1-F) + ...\right) + (1-F)^{n-1}v = F(\varepsilon)\frac{n-1}{n} + (1-F(\varepsilon))^{n-1}v. \tag{53}$$

This gives us, for a downward deviation $\Delta < 0$, an expected match value of (see (33) for the equivalent in a continuum)

$$B_{n-1}(\Delta) = \int_{\underline{\varepsilon}-\Delta}^{\bar{\varepsilon}} \left[\frac{(n-1)}{n} F(\varepsilon+\Delta) + (1-F(\varepsilon+\Delta))^{n-1} v + x_G^* \right] f(\varepsilon)\, d\varepsilon + F(\underline{\varepsilon}-\Delta) v. \tag{54}$$

We can also repeat this for $m = n+1$, so that all boys will be matched. One can calculate that $\phi_{n+1}(\varepsilon) = (2 + (n-1)F(\varepsilon))/(n+2)$. Next, suppose that there is uncertainty about the exact numbers of boys and girls. Specifically, there is an equal chance of the number of girls being $n+1$ and of being $n-1$. Then, a boy choosing $x_B^* + \Delta$ has expected benefit

$$B_{n-1,n+1}(\Delta) = \frac{1}{2} \int_{\underline{\varepsilon}-\Delta}^{\bar{\varepsilon}} (\phi_{n-1}(\varepsilon+\Delta) + \phi_{n+1}(\varepsilon+\Delta) + 2x_G^*) f(\varepsilon)\, d\varepsilon + F(\underline{\varepsilon}-\Delta) \frac{2/(n+2)+v}{2}. \tag{55}$$

However, what we are really interested in is the limit, taking $n \to \infty$. By inspection it is clear that $\lim_{n\to\infty} \phi_{n-1}(\varepsilon) = \lim_{n\to\infty} \phi_{n+1}(\varepsilon) = F(\varepsilon)$. Thus,

$$\lim_{n\to\infty} B_{n-1,n+1}(\Delta) = \int_{\underline{\varepsilon}-\Delta}^{\bar{\varepsilon}} [F(\varepsilon+\Delta) + x_G^*]\, f(\varepsilon)\, d\varepsilon + F(\underline{\varepsilon}-\Delta) v/2. \tag{56}$$

If we choose $v = 2\bar{u}$, then the above matches with the continuum case, as given in (33), but here specialized to additive shocks.

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