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## ENDOGENOUS FINANCING OF THE UNIVERSAL SERVICE\*

Laura Ilie<sup>1</sup> and Ramiro Losada<sup>2</sup>

### *Abstract*

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We address the question of endogenous financing of the Universal Service Obligations and we compare it with the exogenous financing. A fund is created and fed through a tax firms pay. We show that the way these funds are implemented by the current regulatory regimes at work goes against the social welfare. We propose a new way of implementing the fund that improves welfare. Moreover, we challenge the proportionality payment principle as it leads to lower welfare.

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# Endogenous Financing of the Universal Service\*

Laura Ilie<sup>†</sup>, Ramiro Losada<sup>‡</sup>

April 2004

## Abstract

We address the question of endogenous financing of the Universal Service Obligations and we compare it with the exogenous financing. A fund is created and fed through a tax firms pay. We show that the way these funds are implemented by the current regulatory regimes at work goes against the social welfare. We propose a new way of implementing the fund that improves welfare. Moreover, we challenge the proportionality payment principle as it leads to lower welfare.

**Keywords** : Universal Service Obligations; Social Welfare; Endogenous Financing; Regulation

**JEL Classification** :: D4; L1; L5; L96.

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# 1 Introduction

The last two decades witnessed the large scale deregulation of sectors such as telecommunications, transportation, electricity or gas. Due to technological advances, such network industries have ceased to be natural monopolies and competing operators are starting to enter the markets. This is likely to affect considerably the way the industries operate.

In the markets of public utilities the regulator often values an ‘equal access’ of all consumers to the service at an ‘affordable tariff’.<sup>1</sup> Whereas networks were previously operated by monopolies who were in charge of these universal service obligations, the arrival of new entrants in markets that are now open to competition induces cream skinning on profitable segments of the market and makes previous monopolies unable to finance these obligations through cross-subsidies (see Laffont and Tirole, 2000). The firm is no longer willing to serve high-cost areas at low prices or to subsidize low-income consumers. The industry develops therefore a conflict of interests between operators’ objective of maximizing profits and the objectives related to the public policy. This conflict requires the intervention of a regulator. Moreover, the liberalization movement leads to outcomes that might not be desirable from the social point of view. Without regulatory constraints some users could be excluded from the market or users with different consumption or characteristics would face different tariffs. If the regulator values equality with respect to the access of all user to the market, he must then impose ‘universal service obligations’ (USOs).

In the telecommunications industry universal service obligations, USOs thereafter, have been a central issue in the political debate surrounding regulatory reform in most countries. Any member of the World Trade Organization agreement on basic telecommunications has the right to define the kind of universal service obligations it wishes to maintain.<sup>2</sup> Therefore, the definition of the USOs varies among countries. Nevertheless, different definitions keep common characteristics that allow a general concept of USOs. Ensuring universal service means to provide a defined minimum set of services of

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<sup>1</sup>See, for example, Federal Communications Commission, 1996

<sup>2</sup>For instance, the European Parliament and the Council of the European Union in the Directive 2002/22/EC stipulates the obligation of the member states that the USOs ‘*are made available with the quality specified to all end-users in their territory, irrespective of their geographical location, and, in the light of specific national conditions, at an affordable price*’.

specified quality to all end-users at an affordable price.

The Directive 2002/22/EC of the European Parliament considers the following set of services.

- *Provision of access at a fixed location*, such that any end-user can be connected to the public fixed telephone network. The connection provided shall allow end-users to make and receive local, national and international telephone calls, facsimile communications and data communications, at data rates that are sufficient to permit functional Internet access.
- *Directory enquiry services and directories*. At least one comprehensive directory is available to end-users in a form approved by the relevant authority, whether printed or electronic, or both, and is updated on a regular basis, and at least once a year. At least one comprehensive telephone directory enquiry service is available to all end-users, including users of public pay telephones.
- *Public pay telephones*. National regulatory authorities can impose obligations on undertakings in order to ensure that public pay telephones are provided to meet the reasonable need of end-users in terms of the geographical coverage, the number of telephones, and the quality of services
- *Special measures for disabled users* should be taken in order to ensure access to and affordability of publicly available telephone services, equivalent to that enjoyed by other end-users.

There are many reasons commonly cited for making affordable telephone service available to everyone. A customer who hooks up to a telephone network provide a service not only for his household, but also for everyone else on the telephone network who may ever want to contact him. This **network externality** justifies some public subsidies to promote more extensive use of telephone services. Hooking people to the telephone network may be the cheapest way for the public sector to fulfil obligations such as providing public health, safety and emergency services.

**Redistribution** toward low-income consumers or toward the high cost users is another motivation for universal telephone service. USOs can be

considered a redistributive policy through prices, instead of using mechanisms such as taxation of the income or direct transfers.

**Regional planning** attempts to encourage a more harmonious distribution away from large congested metropolitan areas. This motivation is based on the existence of externalities: noninternalized congestion externalities in the large cities.

There can also be important **political and social reasons** of USOs.<sup>3</sup> A democratic society has an interest in keeping its citizens informed about developments throughout the world.

USOs in a competitive market raise two series of questions. First, who should undertake them (the allocation problem) and second who should pay for USOs (the funding problem). Compared to unconstrained competition between network providers, universal service obligations induce distortions on the competitive entry process and on the equilibrium market structure. These distortions generate both social benefits and social costs. The question of how to share these costs and benefits becomes one of the main concerns for regulators, that try to determine optimal rules for allocating and funding USOs.

On the funding side two methods exist. Most systems share the property that they have to be self funded, that is, mechanisms where the losses are funded through cross-subsidies and through taxes levied on consumers or firms. Taxes may be assessed through different channels: unitary taxes may be levied on consumers or on firms, or taxes may be levied on profits. An alternative method consists in financing USOs through transfers from the government.

The allocation problem refers to the question of which operator will be assigned USOs. In most countries, after opening up the competition, only the incumbent is assigned USOs. Nevertheless, in some cases productive efficiency would require that competitors incur some of USOs. For instance USOs could be auctioned (see Milgrom, 1996).

This paper belongs to a small but growing literature on USOs under competition. Normative foundations to the existence of USOs were investigated in a monopoly context by Riordan (2001) and Laffont and Tirole (2000).

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<sup>3</sup>A case in point was an attempt by the staff of Federal Communications Commission to design policies to help the homeless and the seasonal workers, who certainly count among the least prosperous Americans. Relatively low subsidies could have helped them to have access to voice mail services, thus enabling them to keep contact with others (family, potential employers). Congress never considered this proposal.

Armstrong and Vickers (1993) introduce competition and examine the effects of price discrimination when an incumbent firm faces a price taking entrant in a profitable market while the incumbent also serves customers in a separate market. A ban on price discrimination across the incumbent's markets, which is often part of an universal service requirement, causes the incumbent to be less aggressive in response to entry.

Sorana (2000) analyzes an auction mechanism for the allocation of subsidies for providing the universal service. He highlights both opportunities and dangers that auctions offer to the regulator. He shows that auctions may lead to higher social welfare levels than uniform subsidy schemes, but also shows that auctions designs aimed at stimulating competition may be particularly vulnerable to collusion among bidders.

Chone, Flochel and Perrot (2002) examine, in a market open to competition, various mechanisms for allocating USOs among agents. The obligations they consider are geographic ubiquity and non-discrimination. They analyze from the efficiency and equity points of view, the advantages of a 'restricted-entry' system, where the entrant is not allowed to serve high cost consumers with respect to a 'pay-or-play' system, where the incumbent can choose to serve high cost consumers and not to pay the tax. Under the ubiquity constraint only, 'pay-or-play' regulation dominates restricted entry regulation (leads to higher welfare, is preferred by the consumers and for a given tax level or lump sum transfers, allows the regulator to decentralize the maximization of welfare to the incumbent). Nevertheless, this result no longer holds when the regulator imposes also the non-discrimination constraint.

Valetti, Hoerning and Barros (2002) and Anton, Vander Wide and Vettas (2002) show that under uniform prices there are strategic links between the markets. These former discuss auctions for universal service subsidies, as well as the impact of cross-market price constraints and introduce strategic interaction between competitors. They analyse exogenous financing of the USOs using a lump sum tax, in a multi-market model with an oligopolistic (profitable) urban market and entry auctions for (unprofitable) rural service. Cross-market price restrictions makes the firm operating in both markets to become a 'softer' competitor, thus placing the firm at a strategic disadvantage. Entry incentives must account for this disadvantage and strategic bidding results in an equilibrium subsidy that contains a compensating premium. Consequently, the downstream strategic disadvantage becomes advantageous for insiders, leading to higher prices and profits.

None of these papers consider how the government finances the subsidy.

For instance, Anton, Vander Wide and Vettas use a lump-sum tax determined exogenously, which is just a transfer from the government. This subsidy can be financed by some tax on the firms operating in the industry, essentially creating a cross-subsidy from the profitable to the unprofitable segment of the market. In principle, if welfare maximization is the goal, the government should choose the tax level that minimizes the resulting distortions.

Our paper keeps as a benchmark the model developed by Anton, Vander Wide and Vettas (2002). In particular, we extend their model considering endogenous financing methods. The main contribution of this paper consists in determining the optimal tax. This tax must produce sufficient revenue to cover the subsidy, in other words revenue should be equal to the winning bid in the unprofitable market auction. Taxes may be assessed through different channels: unitary taxes may be levied on consumers or on firms, or taxes may be levied on profits. A further contribution of this paper is to provide a detailed evaluation of different ways to finance the subsidy. We analyse, among the possible endogenous tax rates, which one implies less distortion and which one allows maximization of welfare.

We examine the use of an auction to determine which firm will supply the unprofitable rural market. Oligopoly competition takes place in a profitable urban market and the resulting urban market price determines the ceiling for the rural market price.

The rural market is inherently unprofitable due to large fixed costs. Thus, no firm would independently seek to enter this market. Supply in this market is determined by the outcome of bidding in an upstream procurement auction, in which one firm becomes the single supplier to the rural market. Bids take the form of subsidy requirements and the selected firm is the lowest bidder (smallest subsidy).

The winner of the auction has the obligation to provide service at the same price prevailing in the urban market where, after the auction, the firms will compete *à la Cournot*. This specification captures in a static framework the regulators' dynamic adjustments of the 'affordable price' to the prices charged in low-cost areas and the corresponding reactions of those firms who may be active in both areas.

In short, we compare a subsidy paid by the firms with a subsidy paid by the government in order to determine, which one is preferred from the regulator's point of view. We describe, among the endogenous financing methods with a subsidy paid by the firms, which one should be chosen by the regulator.



We examine the impact of an endogenous subsidy under cross-market price constraints of the universal service on prices, quantities and profits in each of the rural and urban markets. We employ these market outcomes to analyze welfare and the bidding incentives for the rural market auction. We attempt to answer the question: which regulatory regime should be at work? Which taxable basis produces the minimum distortion? We show that the optimum regulatory regime is one where the intervention of the regulator is stronger than under the current regulatory regime.

We analyze two different regulatory regimes, the regulatory regime at work in Europe and an alternative where the intervention of the regulator is stronger. We show that the regime we propose gives higher social welfare. Given the importance of this new regulatory pattern, in the appendix we provide a generalization in the sense of different taxable basis.

The main policy implications of our paper are that the regulatory regime at work should allow the regulator to figure out the tax. In this context, the proportionality principle is not desirable, as it leads to lower welfare.

The paper is organized as follows. Section 2 lays out the model and section 3 the benchmark. In section 4 we describe the current regulatory frameworks, in section 5, we present two possible choices of the regulation pattern, that are analyzed in detail in sections 6 and 7. Finally we present the conclusions. The appendix contains a generalization of all the taxable basis under the new regulatory regime proposed.

## 2 The Model

Our paper is based on the model developed by Anton, Vander Wide and Vettas (2002). We extend their model considering endogenous financing methods.

Consider two markets, U (urban) and R (rural), linked through the constraint  $p^R \leq p^U$  and two firms, A and B operating in the urban market. Demand functions are the followings:  $D^U(p) = a - p$  in the U market, and  $D^R(p) = b(a - p)$ , where  $p$  is the market price and  $b > 0$  is the size of the unprofitable market.

The marginal cost is  $c \geq 0$  and is the same for both firms and both markets. Moreover,  $c < a$ , so that, ignoring fixed costs, it is always profitable to supply some amount to each market. The fixed costs are  $F^U \geq 0$ , in market U and  $F^R > 0$  in market R. As in Anton et al. we assume  $F^R$  sufficiently

large such that the profits of a monopolist that would operate only in the R market would be negative. This assumption implies the need for subsidies if the government wants consumers in the rural market to be served.

An auction will determine the supplier for the rural market.<sup>4</sup>

### 3 Benchmark

In this paper we define precisely the benchmark situation, where they consider a direct subsidy, which is just a lump-sum transfer from the government to the firm that wins the auction, and then we carefully determine the economic distortions due the tax. We keep as a benchmark the results obtained by Anton, Vander Wide and Vettas (2002).

We consider a complete information game with the following timing.

1. Firms choose their bids. These bids represent lump-sum subsidies that the firm ask from the government in order to serve market R. The lowest bidder (smallest subsidy required) wins, receives a subsidy equal to the winning bid, incurs the fixed costs  $F^R$  and becomes a monopolist in the R market.
2. Firms A and B choose quantities  $q^A$  and  $q^B$  for the U market. Then the price in the U market is determined as  $p^U = a - q^A - q^B$ .
3. The monopolist in the R market can choose a price that can not exceed the price determined in the U market, that is  $p^R \leq p^U$ .
4. Each firm's payoff is the sum of its profits in the two markets, including any subsidies and taxes.

We will call the firm that gains the auction *firm 1* and the one that loses the auction *firm 2* and we will solve the game backwards. Denote by  $q_1$  the quantity supplied in the U market by the firm that operates in *both* markets and by  $q_2$  the quantity supplied in the U market by the firm that only operates in the U market. The maximization problem of firm 1 is

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<sup>4</sup>Our model does not consider intangible benefits resulting from providing universal service. We could add such benefits and make them endogenous by introducing network externalities. Introducing such effect would make the model less general and much more difficult to handle, so we restrict our attention to the regulatory regimes and to the best way of financing USOs .

$$\max_{q_1} \pi_1 - F^U - F^R + S$$

where the operating profit for the firm that operates in both markets, firm 1, is  $\pi_1 = [(a - q_1 - q_2) q_1 - cq_1] + [(a - q_1 - q_2) b (q_1 + q_2) - cb (q_1 + q_2)]$  or, equivalently,  $\pi_1 = (a - c - q_1 - q_2) [q_1 + b (q_1 + q_2)]$ . The maximization problem of firm 2 is

$$\max_{q_2} \pi_2 - F^R$$

and the operating profit of the firm that operates only in the U market, firm 2, is  $\pi_2 = [(a - c - q_1 - q_2) q_2]$ .

Taking into account the price constraint  $p^U = p^R = a - q_1 - q_2$  and maximizing the profits of the two firms, we find the equilibrium quantities and price:

$$q_1 = \frac{a - c}{3 + 2b}$$

$$q_2 = \frac{(a - c)(1 + b)}{3 + 2b}$$

$$p = \frac{(1 + b)(1 - c)}{(3 + 2b)}$$

Therefore, the equilibrium operating profit of the winner firm is:

$$\pi_1 = \frac{(a - c)^2 (1 + b)^3}{(3 + 2b)^2}$$

and for the loser firm is

$$\pi_2 = \frac{(a - c)^2 (1 + b)^2}{(3 + 2b)^2}$$

The equilibrium exogenous subsidy is that one that compensates the winner firm for the disadvantage of winning the auction, therefore it is that one that makes one firm indifferent between winning the auction or not.

$$S_{exo} = \pi_2 - \pi_1 + F^R$$

In the sequel we will refer to this subsidy as the exogenous subsidy, or the direct subsidy, as it is just a transfer from the government to the firm that wins the upstream auction. Notice that we call this subsidy exogenous from the point of view of the method the universal service is financed, because it is paid by the government, not because it is determined exogenously.

## 4 Current regulation

American and European legislation contemplate the possibility to establish a fund to subsidize service to consumers in high cost (rural) areas. Nevertheless, no consensus has yet been reached on the contributions to the fund and different countries follow different rules.<sup>5</sup> In some countries (Germany, Austria) the contributions to the fund are a function of the market share of the operators, in others (France) the contributions are proportional to the operators traffic. As an alternative to the fund, Ireland proposes an increase in the interconnection fee of the operators in order to cover the financing of USOs. In Spain the contributions to the fund depend on the gross revenue of the operators. UK and Portugal did not create any fund but they contemplate this possibility.<sup>6</sup>

The Directive 2002/22/EC of the European Parliament and of the Council stipulates:

*‘Compensating undertakings designed to provide such services (universal service) in such circumstances need not result in any distortion in competition, provided that designed undertakings are compensated for the specific net*

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<sup>5</sup>Telecommunications Act of 1996 and Directive 2002/22/EC of the European Parliament and of the Council

<sup>6</sup>Creating a fund may not always be the best solution, as argued in Gasmi, Laffont and Sharkey (2000): ‘*In some countries, and this is particularly so in less developed countries, the tax system is very inefficient, sometimes even corrupt, to the point where such transfers are socially very costly. This raises the question of what is the best way to introduce competition to limit the deadweight losses due to this transfers. (...) More recently in Argentina the country has been divided into two regions, each one with an urban area and a rural area. Cross-subsidizes are maintained within each region, but some forms of yardstick competition exists between regions.*’

*cost involved and provided that the net cost burden is recovered in a competitively neutral way*'.

The European Parliament defines the minimum distortions as follows:

*'The net cost of the universal service obligations may be shared between all or certain specified classes of undertaking. Member States should ensure that the sharing mechanism respects the principle of transparency, least market distortion, non-discrimination and proportionality. Least market distortions means that contributions should be recovered in a way that, as far as possible minimises the impact of the financial burden falling on end-users, for example by spreading contributions as widely as possible*'.

Another aspect of the current European legislation that we will take into consideration is the *principle of proportionality*.

The Directive 2002/22/EC of the European Parliament and of the Council states:

*'In the case of cost recoveries by means of levies on undertakings, Member States should ensure that the methods of allocation amongst them is based on objective and non-discriminatory criteria and it is in accordance with the principle of proportionality. This principle does not prevent Member States from exempting new entrants which have not yet achieved any significant market presence.'*

## **5 Possible choices of the regulation regime**

We will study endogenous methods of financing the universal service and we will treat the tax rate as endogenous, because the tax must produce sufficient revenue to cover the subsidy, that is revenue should equal the winning bid in the unprofitable market auction. We analyze two possible choices of the regulation pattern.

The first one is the regulation regime at work in most European countries. Under this regime the regulator chooses the taxable basis at the beginning of the game. The firms choose the subsidy in the first stage of the game and the tax the firms should pay is determined endogenously at the end of the game, once the optimum quantities of the firms are settled. The tax rate is adjusted as a consequence of the resolution of the optimization problems of the two firms, so the regulator does not intervene to fix the tax rate. Therefore we will refer to this regime as *'non-intervention regime'*. With this kind of regulation firms have more degrees of freedom when they take

their optimal choices, therefore, the market price is going to be higher than in the case of a direct subsidy paid by the government. This is because firms use the tax as a collusion tool that allows them to increase the final market price. The second regulatory regime is an alternative we propose to the current regulation. Under this regime the government determines the taxable basis at the beginning of the game. The difference with the first regulation pattern is that the intervention of the regulator is stronger, and he acts as a third player of the game. The tax rate that firms should pay is determined by the regulator simultaneously with the equilibrium quantities of the two firms. This regime is more restrictive to the firms, as the tax rate is determined by the regulator at the same time as the firms solve their optimization problems, firms cannot use it anymore as a collusion tool. We will refer to this regime as *'regulated regime'*. The regulator is searching for the highest aggregate social welfare, that is the sum of consumer's net surplus and firms net profits. If welfare maximization is the goal, even if the intervention of the regulator is minimized with the current regulatory regime, we will show that the regulatory regime we propose is preferred because it leads to a higher level of social welfare.

We consider three examples of taxes levied and we analyze all three types of taxes under both regimes. The election of these taxes is justified by real situations and by requirements that have been enacted in a number of countries.

## 6 Non-intervention regime

We use here a method of endogenous financing of the universal service. Therefore tax rate must produce sufficient revenue to cover the subsidy, that is revenue should equal the winning bid in the unprofitable market auction.

Compared to the case of a subsidy paid by the government (a direct subsidy), a subsidy paid by the firms may induce distortions on the strategic decisions of the firms and also on their competitive behavior, being more or less aggressive in their bids for the rural market. These distortions generate both social benefits and social costs. The question of how to share these costs and benefits becomes one of the main questions for regulators and has received various answers in different countries.

Under the *'non-intervention regime'* the tax the firms should pay is determined endogenously at the end of the game, once the firms solved their

maximization problems. This is the regime at work in most European countries.

The game is similar to the one played without taxes. A first and a last step are added. The game starts with a stage in which the regulator chooses the type of endogenous financing, the taxable basis. Then the firm that is going to serve the rural market is determined by an entry auction and the firms compete á la Cournot, exactly like in the game described above. In the last stage of the game, budget balances and the tax rate  $t$  is determined.

We solve for a subgame perfect equilibrium of this game, focusing on pure strategy equilibria in the bidding stage. We solve the game by backward induction.

As the tax rate  $t$  is determined at the end of the game, after the firms solve their maximization problems, they have the freedom to behave strategically using the tax in their favor and rise prices.

## 6.1 Profit tax

To illustrate the dynamic of the endogenous financing, suppose that the regulator introduces a tax rate  $t$  on the profit of the firms. This is an example of a concave taxable basis in the decision variable of each firm, or in other words, is an example of taxable basis that is a function of the price  $p$ , or equivalently the functional form of the taxable basis is of the type  $g(p)h(q_1, q_2)$ , where  $g$  and  $h$  are increasing functions. The current legislation of some countries stipulates this type of tax (In Spain and Italy, for example, the contribution to the fund is a function of the revenue of the operators, which is a concave function on the decision variable of each firm, therefore it belongs to the same class of taxable basis as the profit tax).

When the subsidy is financed through a tax on the firms that are present in the market, a cross-subsidy from the profitable to the unprofitable zone is created. The decision variable of each firm is its quantity, but the results still hold qualitatively if firms' decision variable is the price.

The complete information game is similar to the game played in a framework without taxes, the difference is that now, given  $S$  (the subsidy), in the fourth stage of the game Cournot competition takes place under restrictions. The operators will maximize their respective objective functions subject to the common restriction  $S = t(\pi_1 + \pi_2)$ , as the sum of the taxes levied from the firms present in the market must cover the subsidy required by the winner firm. The maximization problem of firm 1 is:

$$\max_{q_1} \pi_1 - F^U - F^R + S - t\pi_1$$

$$s.t. S = t(\pi_1 + \pi_2)$$

The maximization problem of firm 2 is:<sup>7</sup>

$$\max_{q_2} \pi_2 - F^R - S - t\pi_2$$

$$s.t. S = t(\pi_1 + \pi_2)$$

Substituting the restriction in the objective function, firm 1's maximization problem can be written

$$\max_{q_1} \pi_1 - F^U - F^R + \frac{\pi_2}{\pi_1 + \pi_2} S$$

which is equivalent to

$$\max_{q_1} \pi_1 - F^U - F^R + \frac{q_2}{(1+b)(q_1+q_2)} S$$

Similarly, firm 2's problem is

$$\max_{q_2} \pi_2 - F^U - \frac{\pi_2}{\pi_1 + \pi_2} S$$

which can be written as

$$\max_{q_2} \pi_2 - F^U - \frac{q_2}{(1+b)(q_1+q_2)} S$$

Solving these problems we obtain:

$$q_1 = \frac{(a-c)}{2(3+2b)} + \frac{\sqrt{(1+b)(2+b)^2 \left( (1+b)(2+b)^2 (a-c)^2 - 4(3+2b)S \right)}}{2(1+b)(2+b)^2(3+2b)}$$

$$q_2 = \frac{(1+b)(a-c)}{2(3+2b)} + \frac{\sqrt{(1+b)(2+b)^2 \left( (1+b)(2+b)^2 (a-c)^2 - 4(3+2b)S \right)}}{2(2+b)^2(3+2b)}$$

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<sup>7</sup>Notice that making the loser firm to pay for the subsidy could push the firm out of the market.



**Proposition 1** *Under the profit tax the equilibrium price is higher than in the benchmark case and the level of welfare lower.*

Since U market profit is subject to taxation, for the firm that operates in both markets it becomes more profitable at the margin to decrease supply in the U market. In other words, a profit tax makes more attractive for the firm to sacrifice its U market profit in order to decrease its loss in the R market and thus tends to further increase the price.

As the regulator wishes to get the maximum welfare possible, it is clear that it will not choose to implement an endogenous tax that lowers the welfare. Notice that when the size of the rural market is very big, that is  $b \rightarrow \infty$ , under a profit tax the winner firm is going to ask for a subsidy equal to the direct subsidy.

It is also worth mentioning that if the government imposes a tax just on the urban market profits instead of a tax on the overall profits, this is going to be a particular case of the tax on global profits and it will lead to the same welfare effect as the tax on the overall profits, that means it leads to a decrease in welfare.<sup>8</sup>

We should emphasize here the importance of these conclusions. The increase in price leads to a lower welfare and on the basis of this result we should bring a strong criticism to the Comision del Mercado de Telecomunicaciones (CMT) in Spain. In Spain a fund was defined but it's not operative yet. The contributions to the fund are function of the revenues. The tax on revenue belongs to the same family of concave taxable basis as the tax on profits, as it depends directly on the market price. The CMT settles a tax on the revenue of the operators and this determines a smaller social welfare, so in general this regime is not desirable.

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<sup>8</sup>With a tax on profit on the urban market, the objective functions of the maximization problems become

$$\max_{q_1} \pi_1 - F^U - F^R + \frac{q_2}{q_1 + q_2} S$$

and

$$\max_{q_2} \pi_2 - F^U - \frac{q_2}{q_1 + q_2} S$$

therefore this is just a restriction of the previous case of tax on overall profits (is equivalent with taking  $b = 0$  in the objective functions of the maximization problems).

## 6.2 Quantity tax in the U market

Suppose that the regulator introduces a quantity tax  $t$  in the U market. In some countries (Germany, Austria) the contributions to the fund are functions of the market share of the operators, so it makes sense to consider an example of a quantity tax. This is a way of creating a cross-subsidy from the profitable market to the unprofitable market.

Each firm maximizes its profits subject to the restriction that the subsidy must be equal to the sum of the taxes levied from both operators.

$$S = t(q_1 + q_2)$$

The maximization problems of the two firms are equivalent to

$$\max_{q_1} \pi_1 - F^U - F^R + \frac{q_2}{q_1 + q_2} S$$

and

$$\max_{q_2} \pi_2 - F^U - \frac{q_2}{q_1 + q_2} S$$

By simple comparison with the profit tax objective functions after substituting the constraints it can be seen that this is just a similar case of the situation described there.<sup>9</sup> Under a tax on profit, if  $b = 0$ , the profit tax maximization problems after substituting the constraints become the problems of the two firms under a quantity tax in the U market. Therefore, under a quantity tax in the U market we will obtain the same effects on equilibrium price and social welfare as under the profit tax. The only difference is that these effects are scaled by a factor  $b$ . Now the welfare lowers even more, as under a profit tax the factor  $b$  just mitigates the decrease in welfare. The two cases follow the same dynamic (see the optimization problems).

Notice that the market price increases as a consequence of both operators incentives to lower their market share, to influence their own taxable basis producing less, in order to pay a smaller tax. In terms of welfare, this is the worst endogenous tax of all the taxes described so far.

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<sup>9</sup>We can replicate the results from the profit tax case taxing the quantities that firms sell in the U and R markets.

### 6.3 Tax on the total quantity in the U market

Another type of tax the regulator could impose is a tax on the total quantity in the U market. Each firm maximizes its profits subject to the restriction that the subsidy must be equal to the sum of the taxes levied from both operators.

The maximization problem of the firm 1, the winner of the auction, is:

$$\begin{aligned} \max_{q_1} \quad & \pi_1 - F^U - F^R + S - t(q_1 + q_2) \\ \text{s.t.} \quad & S = 2t(q_1 + q_2) \end{aligned}$$

Under the same restriction firm 2, the loser of the auction solves:

$$\begin{aligned} \max_{q_2} \quad & \pi_2 - F^U - t(q_1 + q_2) \\ \text{s.t.} \quad & S = 2t(q_1 + q_2) \end{aligned}$$

**Proposition 2** *Under a tax on the total quantity in the U market, the equilibrium price, the operating profit of the two firms, the welfare and the subsidy required by the winner firm for provision of USOs are equal to the ones obtained in the benchmark case.*

Notice that with a tax on the total quantity in the U market we obtained the same welfare as in the benchmark case. Notice also that, in the framework of directives of the European Parliament, the tax on the total quantity in the U market presents the advantage of being a competitively neutral tax, which minimizes the distortions. The intuition behind the competitively neutral character of this tax is that, given the subsidy  $S$ , the two firms will try to lower their taxable basis in order to pay less. The subsidy  $S$  is fixed and, by difference with the previous case analyzed, both firms have to share equally its cost, paying the same. Therefore, the firms can not avoid to pay a fixed quantity for the subsidy.

Nevertheless, the regulator pursues the goal of the highest aggregate social welfare, calculated as the sum of consumer's net surplus and firms net profits. When he gives the same weight to the consumers and to the producers, it is not advantageous for him to implement a tax on the total quantity in the U market, as it leads to the same welfare as in the exogenous case. Besides, the loser of the auction could be pushed out of the market due to large tax.

## 6.4 Implications of the current regulatory regime - non-intervention regime

The goal of welfare maximization implies that the government should choose some tax that minimizes the resulting distortions. As we have seen above, all the taxes considered lead to a level of welfare which is at most equal to the welfare under the direct subsidy (the benchmark case). With a tax on the total quantity in the U market we obtain a welfare equal to the welfare of the benchmark case, and with the other two taxes we obtain lower levels of welfare. Therefore, under this regulatory regime, when the tax rate,  $t$ , is determined at the end of the game and the intervention of the regulator is minimized, the objective of the endogenous financing, which is to internalize negative externalities at social level and obtain better welfare is not attained. The best we could get in terms of welfare is at most the same level as under exogenous financing, and besides, in this case there is the possibility that the loser firm will be pushed out of the market, because the subsidy is paid now by the firms.

## 7 Regulated regime

As an alternative to the current regulatory regime we will propose now another one and we will show that the latter one is preferable when the regulator follows the objective of the highest social welfare. We will show, using the same taxes we used above, that under the new regulation pattern it is possible to obtain a higher welfare than with exogenous financing. Under this new regulatory regime, the tax rate is determined simultaneously with the equilibrium quantities by the regulator. The game is similar to the one that we have with the first regulatory pattern, being the way the tax rate,  $t$ , is determined the only difference.

The timing of the model now is the following: first the regulator announces the type of endogenous financing, the taxable basis. Then the firms compete á la Cournot and the tax rate,  $t$ , is determined by the regulator simultaneously with the resolution of the firms' optimization problems, in the same stage of the game. As before, we solve the game by backward induction, focusing on pure strategy equilibria.

With the '*non-intervention regime*' the firms have more freedom, one firm can use its own taxable basis to influence the market price, whereas under

the ‘*regulated regime*’ this can not be done. The only way in which a firm can manipulate the market price in the direction that is convenient to it is through the taxable basis of the rival. In this new regimen we find situations where the consumers are better off, and the improvement in consumers’ surplus overweighs the decrease in the firms’ profits such that the total social welfare is higher.

To exemplify the intuition described above we will consider the three taxes analyzed with the first regulatory regime. Moreover, we will provide with a generalization.

## 7.1 Competitively neutral tax: Quantity tax in the U market

Suppose that the regulator introduces a quantity tax in the U market. The complete information game is similar to the game played in a framework without taxes, the difference is that now, given  $S$ , in the last stage of the game Cournot competition takes place under restrictions. Each firm maximizes its profits subject to the restriction that the subsidy must be equal to the sum of the taxes levied from both operators.

When the subsidy is financed through a tax on the firms that are present in the market, a cross-subsidy from the profitable to the unprofitable zone is created.

Subject to a common restriction, firm 1, the winner, solves the following problem:

$$\max_{q_1} \pi_1 - F^U - F^R + S - tq_1$$

$$s.t. S = t(q_1 + q_2)$$

Similarly, firm 2, the loser of the auction solves:

$$\max_{q_2} \pi_2 - F^U - tq_2$$

$$s.t. S = t(q_1 + q_2)$$

**Proposition 3** *Under a quantity tax in the U market we obtain the same market price and the same level of welfare as under the exogenous subsidy.*

Moreover, in equilibrium the opportunity cost of gaining the auction is zero, so:

$$\pi_1 - F^R + S - tq_1 = \pi_2 - tq_2$$

and we can derive the following result.

**Proposition 4** *Under endogenous financing of the universal service through a quantity tax in the urban market, the operator in charge of providing the USOs receives a subsidy lower than the subsidy he would receive under exogenous financing.*

In the context of the directives of the European Parliament the advantage of a quantity tax versus an exogenous financing is undeniable. Using a quantity tax we obtain the same market price and the same level of welfare as with the exogenous subsidy. The quantity tax is therefore a competitively neutral tax. If the regulator's objective is to obtain the best welfare level, he should choose a tax that minimizes the distortions. Moreover, the subsidy the winner firms gets is lower than under the benchmark case. Besides, if we suppose that in the exogenous case the subsidy is paid by consumers, and that the authorities want to '*minimize the impact of the financial burden falling on end-users*', then the regulator will strictly prefer an endogenous subsidy levied through a quantity tax. Producers' surplus is lower than under the benchmark case, as now firms internalize the cost of the subsidy. Consumers' surplus is higher, as the subsidy is now paid by the firms and not by the consumers.

Nevertheless, there are cases in which very high fixed costs could make endogenous financing of USOs not applicable, as the loser firm would be pushed out of the market. When fixed costs are extremely high, the subsidy is also high and the firm that loses the auction cannot pay its part of tax without getting out of the market.<sup>10</sup> In such a case only exogenous financing makes sense.

## 7.2 Profit tax in the U market

Another type of tax the regulator could impose is a tax on profit of the firms in the urban market. The profit of the winner firm can be written as

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<sup>10</sup>For further details, see proof of proposition 4.

$$\pi_1 = \pi_1^U + \pi_1^R$$

where  $\pi_1^U$  is the profit in the urban area and  $\pi_1^R$  is the profit in the rural area.

Firm 1's maximization problem is

$$\max_{q_1} \pi_1 - F^U - F^R + S - t\pi_1^U$$

$$s.t. S = t(\pi_1^U + \pi_2)$$

Similarly, firm 2, the loser of the auction solves:

$$\max_{q_2} \pi_2 - F^U - t\pi_2$$

$$s.t. S = t(\pi_1^U + \pi_2)$$

**Proposition 5** *Under the profit tax the equilibrium price is higher than in the benchmark case and the level of welfare lower.*

The intuition behind this result is that both firms want to increase their quantities because in this way they pay less in taxes as their competitor's profits are higher. A direct consequence of this result is that the level of welfare is lower than under the benchmark case, and lower than in the previous case of endogenous financing (financing through a quantity tax in U market). Another consequence of a higher price than under the benchmark case is that, although still possible, now it will be more difficult to push out of the market the firm that loses the bid.

The endogenous subsidy is then

$$S_{end} = \frac{\pi_1^U(t) + \pi_2(t)}{2\pi_2(t)} (\pi_2(t) - \pi_1(t) + F^R)$$

and it is smaller than the exogenous subsidy.<sup>11</sup>

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<sup>11</sup>See Gonzalez Gomez P., (2003)

Under a profit tax in the urban market, the rural market becomes more attractive to the operators and they have more incentives to convert into the monopolist in the rural market. Competition becomes therefore more aggressive and the equilibrium subsidy decreases with respect to the exogenous one. Both firms make more operating profits than in the exogenous case, because now they internalize the cost of the subsidy and so they choose to provide a smaller quantity. consumers' surplus lowers more than the increase in the firms' operating profits, hence total welfare decreases.

As the subsidy is an increasing function of  $t$ , as bigger is the subsidy required, as bigger is the tax rate levied by the government. The opportunity cost of gaining the auction is decreasing in  $t$ , this means that as bigger is the tax, as smaller is the opportunity cost of gaining the bid. Both firms have more incentive to be the winner as tax rate,  $t$ , increases, so they compete more aggressively and therefore, the subsidies are lower now than in the benchmark case.

### 7.3 Tax on the total quantity in the U market

Suppose now that the regulator imposes a tax on the total quantity produced in the urban market. The operators will maximize their respective objective functions subject to this common restriction. Firm 1, the winner, solves the maximization problem:

$$\max_{q_1} \pi_1 - F^U - F^R + S - t(q_1 + q_2)$$

$$s.t. S = 2t(q_1 + q_2)$$

Similarly, firm 2, the loser of the bid solves:

$$\max_{q_2} \pi_2 - F^U - t(q_1 + q_2)$$

$$s.t. S = 2t(q_1 + q_2)$$

**Proposition 6** *Under a tax on the total quantity on the U market the equilibrium price is lower than in the benchmark case and the level of welfare higher.*



The intuition behind this result is that the tax on the total quantity on the U market is an equalitarian tax and the operators internalize the negative externality of the competitor. When a firm decides to produce more, the competitor has to pay more in taxes because taxable basis of both firms is the total quantity. Compared with the other cases we get the highest consumer' surplus and the lowest firms' profits.

The endogenous subsidy is in this case

$$S_{end} = \pi_2(t) - \pi_1(t) + F^R$$

**Proposition 7** *The equilibrium subsidy when a tax on the total quantity in the urban market is imposed is higher than the exogenous subsidy.*

Notice that we obtained that under a tax on the total quantity on the U market, an equalitarian tax to cover the subsidy, the level of welfare is higher than under the direct subsidy and higher than under all the endogenous subsidies considered so far. This finding is against the regulatory regime at work that requires the proportionality principle to be satisfied. The Directive 2002/22/EC of the European Parliament and of the Council enacts the *principle of proportionality*.

Due to the lower price than under exogenous subsidy, the firm that loses the bid and operates only in the urban market can be pushed out of the market easier than under the competitively neutral tax.

## 7.4 Welfare analysis

One of the justifications of universal service obligations relies on the fact that it increments social welfare. Given the constraint of the USOs, we are searching for the financing mechanism that guarantees the maximum level of welfare and it is important to analyze the results from this point of view.

For the figures we assume for simplicity the fixed costs of the urban market  $F^U = 0$ , as well as  $c = 0$  and  $a = 100$ .

Recall that we computed the aggregate social welfare as the sum of consumers' net surplus and firms net profit, giving the same weight to the consumers and to the producers. Figure 1 shows the social welfare versus the size of the rural market. Keeping the fixed costs of the rural market,  $F^R$  constant when the size of the rural market  $b$  increases, the welfare under the tax on the total quantity in the urban market is higher than the welfare under

any of the other taxes, even higher than the welfare in the benchmark case. It can be seen that the social welfare is, in any case, an increasing function of the size of the rural market size,  $b$ . Nevertheless, as higher is the social welfare, as higher the market price and therefore, as lower is the probability of pushing out of the market the loser firm.

Figure 2 shows the graph of the social welfare versus the R market fixed costs, holding constant the size of the rural market,  $b$ . Under the competitively neutral tax (the tax on quantity on the urban market) we get exactly the same welfare as with the direct subsidy (the two paths overlap). The only difference is that now producer' surplus is smaller, as the operators pay the subsidy through the tax and consumer's surplus is higher.

Considering the importance of a competitively neutral tax in the context of the European legislation, we take as a second benchmark the case of the competitively neutral tax (the tax on quantity on the urban market). Under the total quantity tax we obtain a higher welfare (see Figures 1 and 2) and under the profit tax a lower one. With respect to the competitively neutral tax, the imposition of a total quantity tax determines a lower producer' surplus, as the producers internalize the cost of the subsidy, and a higher consumers' surplus. The profit tax determines a higher producers' surplus, that overweighs the decrease in the consumers' surplus, such that total welfare decreases.

In the Figures it can be seen that, of all the cases analyzed, the one that leads to the highest social welfare is the case of a total quantity tax in the urban market. This is also the framework for the highest consumer' surplus and the lowest producer' surplus. The second highest welfare is obtained under the competitively neutral tax.

Figures 3 and 4 show the subsidy as function of the fixed cost and of the size of the rural market,  $b$ , respectively. From these graphs we can observe the following regularities. When we hold constant the fixed costs in the rural market, under all taxes analyzed the subsidy required by the winner firm is a decreasing function of the rural market size,  $b$ . Notice also that when we hold constant the rural market size  $b$ , the subsidy increases with the fixed costs of the rural market. The lowest subsidy is attained with a profit tax and the highest one with the tax on the total quantity in the rural area. This latter one is also the only subsidy that exceeds the direct subsidy.

In figure 5 it can be seen that when the rural market size increases and the fixed costs are hold constant, the profit of the loser firm increases also. Notice that as much as the regulator wants to benefit the consumers, as

sooner it will determine the firm that loses the bid to get out of the market. Under the tax on total quantity in the U market, the loser firm makes the lowest profit, so in this case it will be the easiest to push it out of the market.

In Figure 6 we can observe that, when the rural market size is constant, the profit of the loser firm under a direct subsidy is flat, as the profit does not depend on the rural market fixed costs. In all the endogenous cases the profit of the loser firm decreases with the rural market fixed costs, and is smaller than with the direct subsidy.

Recall that the European legislation stipulates that the proportionality principle should be fulfilled. Following this principle, in France for example, the contributions to the fund are proportional to operators' traffic, and in Spain they are proportional to the revenue. Nevertheless, among the three cases that we analyzed here, the highest welfare is given by a tax that does not respect the proportionality principle, the tax on total quantity in the U market.

If the regulator gives equal weight to the consumers and to the firms, no matter what the geographical location of the consumer is, then the regulator should choose the total tax on quantity on the urban market, as it leads to the biggest social welfare. Nevertheless, with this tax is more likely that the loser firm gets out of the market and the subsidy the winner firm will require for provision of USOs is the highest.

Recall the importance of a competitively neutral tax in the context of the directives of the European Parliament. The European legislation stipulates that the market distortions should be minimum, as well as the *financial burden falling on end-users*. With a competitively neutral tax, a tax on quantity in the U market, these objectives are fulfilled. Moreover, as it can be seen in the Figures, the subsidy the winner firms receives is lower than the direct subsidy or than the subsidy with a tax on total quantity. The competitively neutral tax does not maximize the welfare (like the tax on the total quantity does), but it is less likely that the loser firm is going to be pushed out of the market.

## 8 Conclusions

This paper contributes to the debate on the financing of the universal service in the context of deregulation of telecommunications and price restriction across markets. The subsidy required by the firm in charge of provision of

the universal service is determined endogenously through a tax to be paid by the operators. The tax must produce sufficient revenue to cover the subsidy, that is revenue should equal the winning bid in the unprofitable market auction.

We analyze two possible choices of the regulation pattern. Each possibility induces a particular game between the operators. These two scenarios differ in the moment of the election of the tax rate. In one case, this happens at the end of the game while in the other case the tax rate is determined simultaneously with the equilibrium quantities. Under both regimes the tax rate,  $t$ , is set such that to cover the cost of USOs, the subsidy  $S$ . In the ‘*non-intervention regime*’ the government only chooses the taxable basis while under the ‘*regulated regime*’ it chooses taxable basis as well as the tax rate. In first regulatory regime the tax the firms should pay is determined endogenously at the end of the game, once the optimum quantities of the firms are settled. In this case, the firms can use it in their favor as a tool for increasing their profit to consumers’ detriment. The decrease in the consumers’ surplus outweighs the increase in the producers’ surplus and the total welfare is lower than with a direct subsidy. The second regulatory regime, when the tax rate is determined at the same time as the equilibrium quantities of the two firms, is more restrictive to the firms. In this situation, for certain types of taxes, the consumers are better off and the social welfare is higher than in the benchmark case. The regulator maximizes aggregate social welfare, that is the sum of consumer’s net surplus and firms net profits. Even if the intervention of the regulator is minimized with the first regulatory regime, the second regulatory regime is preferred because for certain taxes leads to a higher level of social welfare.

Given the importance of a competitively neutral tax in the framework of the directives of the European Parliament, we show that under the first regulatory regime a competitively neutral tax (the tax on total quantity in the urban market) is the best from the point of view of the social welfare. For the other two types of taxes analyzed lower levels of social welfare are obtained. The second regulatory regime is superior to the first one in terms of welfare, as among the same taxes as before determined a lower social welfare, now we can find taxes that keep the same welfare (the competitively neutral tax) or increase it (the tax on total quantity). Therefore the best regulatory regime is the one that determines the tax simultaneously with the equilibrium quantities, and under this regime the best tax is the tax on the total quantity. We also give a generalization of this regime and we

determine under what conditions the welfare can be improved with respect to the benchmark case.

Notice that the tax on the total quantity is an equalitarian tax over the subsidy, therefore it does not fulfil the proportionality principle that the directives of the European Parliament recommend. Nevertheless, when the consumers and the firms have equal weight in the social welfare, the tax on the total quantity leads to the highest welfare. The analysis presented here suggests that, when the consumers and the firms are equal weighted in the social welfare function the proportionality principle is not desirable.

It is also worth mentioning that, according to the regulatory regime at work, the identity of the competitively neutral tax changes. Under the first regime the competitively neutral tax is the tax on the total quantity and under the second one the competitively neutral tax is the tax on the individual quantity. Both type of taxes lead to the same equilibrium price, and hence, to the same welfare as with the direct subsidy. We also give a generalization of the second regime and we determine under what conditions the welfare can be improved with respect to the benchmark case.

The main policy implications of our paper are that the regulation regime at work should be the one where the tax rate is chosen by the regulator and that the proportionality principle is not desirable, as it leads to lower welfare.

Of course, imposing USOs also generates welfare benefits, the representation of which deserves further analysis. Endogenous financing of the universal service when an entrant firm is allowed to bid for the rural market is also left for further research. The presence of outside firms in the rural market auction will affect bidding and the resulting market structure.

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# A Appendix

Given the importance of the regulatory regime we propose as an increasing welfare regime and the importance of a competitively neutral tax in the context of the directives of the European Parliament, we will provide with a generalization of the new regulatory pattern proposed, the ‘*regulated regime*’. The generalization we provide below allows us to see the behavior of the economic agents under different types of taxable basis (it comprises the case of tax on quantity, tax on profits, tax on revenue or any other type of tax).

## A.1 Generic tax in the U market

We have seen that taxes may be assessed through different channels and we considered for our analysis three types of taxes in the U market: a unitary tax on the quantity produced ( $\frac{\partial p}{\partial t} = 0$ ), a taxes levied on profits ( $\frac{\partial p}{\partial t} > 0$ ) and an equalitarian tax, a tax on the total quantity ( $\frac{\partial p}{\partial t} < 0$ ). We could generalize the previous analysis of the different types of endogenous subsidies.

Let  $t$  be the tax rate the government imposes and let  $B_i$  be the taxable basis of firm  $i$  (where  $i = 1, 2$ ), the variable that we levy the tax on (it could be quantity, price, profit, revenue or any other variable).<sup>12</sup>

Firm 1, the winner solves the following problem:

$$\max_{q_1} \pi_1 - F^U - F^R + S - tB_1$$

$$s.t. S = t(B_1 + B_2)$$

Firm 2, the loser of the auction, solves:

$$\max_{q_2} \pi_2 - F^U - tB_2$$

$$s.t. S = t(B_1 + B_2)$$

The first order conditions for these two problems are:

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<sup>12</sup>Recall that the taxable basis  $B_i$  is imposed by the regulator at the beginning of the game.



$$\frac{\partial \pi_1}{\partial q_1} + t \frac{\partial B_2}{\partial q_1} = 0$$

$$\frac{\partial \pi_2}{\partial q_2} + t \frac{\partial B_1}{\partial q_2} = 0$$

On the other hand, in equilibrium the opportunity cost of gaining the auction should be zero for any firm. So

$$\pi_1(t) - F^U - F^R + S - tB_1(t) = \pi_2(t) - F^U - tB_2(t)$$

We also should take into account that the contributions of the operators should sum up the total value of the subsidy, that is:

$$S = t(B_1 + B_2)$$

Substituting the latter equation in the opportunity cost equation, we get

$$t = \frac{F^R + [\pi_2(t) - \pi_1(t)]}{2B_2(t)}$$

which can be rewritten as

$$t = \frac{\pi_2(t) - [\pi_1(t) - F^R]}{B_T(t) + [B_2(t) - B_1(t)]}$$

where  $B_T$  is the total taxable base and  $\pi_2^*(t)$ ,  $\pi_1^*(t)$  are the equilibrium profits. The numerator is the opportunity cost without taxes. The term  $B_2(t) - B_1(t)$  indicates that the firms internalize the fact that now the firms pay themselves the provision of the universal service in the rural market.

Substituting the tax obtained above in the opportunity cost equation, we get the endogenous subsidy

$$S_{end} = \frac{B_1(t) + B_2(t)}{2B_2(t)} [F^R + (\pi_2(t) - \pi_1(t))]$$

Recall that  $B_1(t)$  and  $B_2(t)$  have the same functional form, as they represent the taxable basis for each of the two firms.

Notice the fundamental difference between the two regulatory regimes analyzed. With the ‘*non-intervention regime*’ the firms have more freedom, one firm can use its own taxable basis to influence the market price, whereas under the ‘*regulated regime*’ this can not be done. The only way in which a firm can manipulate the market price in the direction that is convenient to it is through the taxable basis of the rival. The derivative  $\frac{\partial B^i}{\partial q_j}$  is therefore essential as it shows if one firm can or cannot influence the market price using the taxable basis of the rival.

The three cases studied above can be summarized as follows.

1.  $\frac{\partial B^i}{\partial q_j} = 0$ ,  $i = 1, 2$ ,  $i \neq j$ . This is the case of a competitively neutral tax, as it could be a tax on quantity.
2.  $\frac{\partial B^i}{\partial q_j} < 0$ ,  $i = 1, 2$ ,  $i \neq j$ . This condition is satisfied by a tax on profit of the operators. As the objective of the regulator is to maximize social welfare, we will not consider this case for generalization as under this condition the social welfare decreases with respect to the exogenous subsidy.
3.  $\frac{\partial B^i}{\partial q_j} > 0$ ,  $i = 1, 2$ ,  $i \neq j$ . This is the case of a tax on the total quantity.

## A.2 Competitively neutral tax: $\frac{\partial B^i}{\partial q_j} = 0$ , $i = 1, 2$ , $i \neq j$

The idea of a competitively neutral endogenous tax is that the presence of USOs is designed not to affect competition and the profit of the firms with respect to the benchmark case. We have already emphasized the importance of a competitively neutral tax in the context of the Directives of the European Parliament and of the Council. As it can be seen, regulatory authorities follow the objective of a ‘*minimum distortion to the market and to undertakings*’. Taking into account this objective we derive the next results.

**Proposition 8** *The tax the operators pay is a competitively neutral tax for the market if and only if  $\frac{\partial B^i}{\partial q_j} = 0$ ,  $i = 1, 2$ ,  $i \neq j$ .*

**Corollary 9** *The tax the operators pay is a competitively neutral tax as long as the taxable basis does not depend on the price, either on any function of the price. In general the taxable basis should not depend on any of the decision variables of the other firm.*

**Proposition 10** *Under endogenous financing of the universal service through a competitively neutral tax, if the taxable basis has an increasing functional form, the operator in charge of providing the USOs receives a subsidy lower than or equal to the subsidy he would receive under exogenous financing.*

A direct consequence of latter result we get the following propositions.<sup>13</sup>

**Proposition 11** *When  $\frac{\partial B_i}{\partial q_j} = 0$ ,  $i = 1, 2$ ,  $i \neq j$ , the subsidy required by the winner firm is minimized when the winner does not contribute with anything to the fund.*

**Proposition 12** *When  $\frac{\partial B_i}{\partial q_j} = 0$ ,  $i = 1, 2$ ,  $i \neq j$ , and the winner firm does not contribute with anything to the fund, the endogenous subsidy reaches its minimum which is half of the direct subsidy.*

Notice also that the endogenous financing we considered leads to the same level of welfare as the direct subsidy and leaves unchanged the decision variables of the two firms and also the market price. The advantage of the endogenous financing is that the subsidy required for providing USOs is smaller than in the exogenous case. Moreover, if we suppose that in the direct case the subsidy is paid by consumers, and that the authorities want to ‘*minimises the impact of the financial burden falling on end-users*’, then the regulator will strictly prefer an endogenous indirect subsidy.

Nevertheless, there are cases in which very high fixed costs could make endogenous financing of USOs not applicable, as the loser firm would be pushed out of the market. When fixed costs are extremely high only exogenous financing (direct subsidy) makes sense.

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<sup>13</sup> *When the firm operates in both markets, the incentive to relax the cross-market restriction makes the firm a ‘softer’ competitor and places the firm at a strategic disadvantage relative to the urban market competitors.*

*A firm that supplies both markets would like to set a rural price in excess of the oligopolistically determined urban price and, as a result, the reaction function of this firm shifts in the direction that makes this firm a ‘softer’ competitor. The shift (...) is downwards under quantity setting.’- Anton, Vander Wide and Vettas (2002)*

### A.3 Increasing welfare tax: $\frac{\partial B_i}{\partial q_j} > 0, i = 1, 2, i \neq j$

We will analyze now the case when  $\frac{\partial B_i}{\partial q_j} > 0, i = 1, 2, i \neq j$ , which is the third case of the above generalization. Such a condition is satisfied for example by the tax on the total quantity in the U market. We have shown that under a tax on total quantity we obtain the highest welfare among all the cases studied, as the market price decreases with respect to the other tax regimes. We will show now that this result can be generalized for any type of tax basis that satisfies the conditions mentioned below.

**Proposition 13** *When  $\frac{\partial B_i}{\partial q_j} > 0, i = 1, 2, i \neq j$ , and the taxable basis is a concave function with positive cross second derivatives, than the social welfare is higher than under the competitively neutral tax ( $\frac{\partial B_i}{\partial q_j} = 0, i = 1, 2, i \neq j$ ), and therefore higher than when  $\frac{\partial B_i}{\partial q_j} < 0, i = 1, 2, i \neq j$ .*

Hence, among all endogenous tax regimes this is the one that maximizes the social welfare.

For a convex functional form of the taxable basis, we will consider two functional forms and we will show that for a large class of functions, we obtain, as in the above case, an increase in welfare with respect to the benchmark cases.

1. Suppose now that the functional form of the taxable basis is  $B(q_1, q_2) = (q_1 + q_2)^\alpha$ , where  $\alpha \geq 2$  is any integer number. Applying the same procedure as in the proof of proposition 13, we can show that

$$\frac{dq_1 + dq_2}{dt} = \frac{2\alpha (q_1 + q_2)^{\alpha-1}}{3 + 2b - 2t\alpha(\alpha - 1)(q_1 + q_2)^{\alpha-2}}$$

As  $3 + 2b > 2t\alpha(\alpha - 1)(q_1 + q_2)^{\alpha-2}$  is a necessary condition for the firms maximization problems to be well defined, we obtain that also in this case inequality  $\frac{dq_1 + dq_2}{dt} > 0$  is satisfied, therefore the social welfare is superior to the welfare in the benchmark cases.

2. Let the functional form of the taxable basis be  $B(q_1, q_2) = q_1^\alpha + q_2^\alpha$ . Solving the firms maximization problems, we obtain

$$q_1 + q_2 = \frac{(2 + b)(a - c) + \alpha t (q_1^{\alpha-1} + q_2^{\alpha-1})}{3 + 2b}$$

Differentiating with respect to  $t$  we get

$$\frac{dq_1 + dq_2}{dt} > \frac{\alpha (q_1^{\alpha-1} + q_2^{\alpha-1})}{3 + 2b - \alpha t(\alpha - 1)q_1^{\alpha-2}} > 0$$

The last inequality is a condition for the firms maximization problems to be well defined. Hence we obtain an increase in welfare.

We showed that under certain conditions, when the taxable basis satisfies  $\frac{\partial B_i}{\partial q_j} > 0$ ,  $i = 1, 2$ ,  $i \neq j$ , we obtain always a bigger welfare than under competitively neutral tax ( $\frac{\partial B_i}{\partial q_j} = 0$ ,  $i = 1, 2$ ,  $i \neq j$ ). Nevertheless, among all the taxes that fulfil this condition ( $\frac{\partial B_i}{\partial q_j} > 0$ ,  $i = 1, 2$ ,  $i \neq j$ ) there is no taxable basis strictly better than others in terms of welfare, for any value of the rural market size,  $b$  and for any value of the rural market fixed cost  $F^R$ .

To exemplify this, in Figure 7 we will compare a quadratic taxable basis (QT),  $B_i = (q_1 + q_2)^2$  with a linear tax (LT),  $B_i = q_1 + q_2$ . Figure 7 shows that, for very big rural market size and small fixed costs, the universal service could be provided by a monopolist that would operate only in the R market, so in region E of the graph USOs would not be needed. In region A of the graph both type of taxes lead to negative profit of the loser firm, so we will not consider this area. In region B the quadratic tax lead also to negative profit for the loser firm, so here only a linear tax could be operative.

Hence the regions where both types of taxes are operative are C and D. Notice that the quadratic taxable basis determines higher welfare than the linear one in region C of the graph, therefore, the quadratic taxable basis is preferable in a framework where the fixed costs in the rural market are high and the size of the rural market is low. The linear taxable basis is preferable in situations where the rural market fixed costs  $F^R$  are low and the size of the unprofitable market,  $b$  is very big (region D in the graph).

The aggregate quantity under the linear tax is

$$q_1 + q_2 = \frac{(2 + b)(a - c) + 2t}{3 + 2b}$$

and under the quadratic tax is

$$q_1 + q_2 = \frac{(2 + b)(a - c)}{3 + 2b - 2t}$$

Both aggregate quantities are increasing in the tax rate,  $t$ . As bigger the tax rate  $t$  is, as bigger the aggregate quantity and so as smaller the equilibrium price. Since to high fixed costs  $F^R$  correspond high tax rates  $t$  ( $S_{end} = \frac{B_1(t)+B_2(t)}{2B_2(t)}[F^R + (\pi_2(t) - \pi_1(t))]$ ), under a linear tax, when  $b$  is small the aggregate quantity is also small, so the price is high and in this case the quadratic tax is preferable, as it leads to higher welfare. When  $F^R$  is small the tax rate  $t$  is low, and with  $b$  high, the aggregate quantity with a linear tax is big, so this taxation regime is better as it leads to higher welfare.

As it can be seen in Figure 7, there is no taxation regime strictly better than the other one for any value of  $b$  and  $F^R$ . For certain values of these parameters, we can determine whether the quadratic tax is better or the linear one, but in general this is difficult to be said.

## A.4 Proof Proposition 1

As  $S \leq \frac{(1+b)(2+b)^2(a-c)^2}{4(3+2b)}$  is a necessary condition for the loser firm not to be out of the market, we obtain positive equilibrium quantities. It can be easily verified that  $\frac{\partial q_i}{\partial S} < 0$ ,  $i = 1, 2$ , so  $q_1$  and  $q_2$  are decreasing functions of the subsidy  $S$ . When  $S = 0$  we are back in the benchmark case. Hence, for any subsidy  $S > 0$  we will obtain bigger quantities  $q_1$  and  $q_2$ . The equilibrium price is

$$p = a - q_1 - q_2$$

so the price is higher than with the exogenous subsidy and then the social welfare decreases with respect to the benchmark case.

## A.5 Proof Proposition 2

The maximization problems of the two operators lead to

$$\max_{q_1} \pi_1 - F^U - F^R + \frac{S}{2}$$

and

$$\max_{q_2} \pi_2 - F^U - \frac{S}{2}$$

As  $F^U$ ,  $F^R$  and  $\frac{S}{2}$  are constants, it is straightforward that the solution of these maximization problems is the same as in the benchmark case. Therefore we obtain the same equilibrium price and the same level of welfare. Moreover, in equilibrium the opportunity cost of gaining the auction is zero, so:

$$\pi_1 - F^R + S - t(q_1 + q_2) = \pi_2 - t(q_1 + q_2)$$

From the above equation and the restriction of the maximization problem, it can be seen that the subsidy obtained is equal to the direct subsidy.

## A.6 Proof Proposition 3

The solution of the maximization problems is:

$$q_1 = \frac{a - c}{3 + 2b}$$

$$q_2 = \frac{(a - c)(1 + b)}{3 + 2b}$$

The equilibrium profit of the winner firm is:

$$\pi_1 = \frac{(a - c)^2 (1 + b)^3}{(3 + 2b)^2}$$

and for the loser firm is

$$\pi_2 = \frac{(a - c)^2 (1 + b)^2}{(3 + 2b)^2}$$

Market price is

$$p = \frac{(1 + b)(a + c) + c}{3 + 2b}$$

Notice that  $\frac{\partial p}{\partial t} = 0$ , as the price does not depend on the tax  $t$ . Therefore in equilibrium we obtain the same quantities, profits, welfare and price as in the benchmark case.

## A.7 Proof Proposition 4

Substituting the constraint into the objective function we can solve it for  $t$  and get the equilibrium subsidy:

$$S_{end} = \frac{q_1 + q_2}{2q_2} (\pi_2 - \pi_1 + F^R)$$

The term in the bracket is the subsidy in the exogenous case. As  $\frac{q_1 + q_2}{2q_2} = \frac{2+b}{2(1+b)} < 1$ , we obtain  $S_{end} < S_{exo}$ .

## A.8 Proof Proposition 5

Solving the optimization problems we get the equilibrium price

$$p = \frac{a(1 + b) + (1 - t)c(2 + b) - at(b + t)}{3 + 2b - 2t - 2bt - t^2}$$



which is an increasing function of  $t$ , as  $\frac{\partial p}{\partial t} > 0$ .<sup>14</sup> When  $t = 0$ , we get the market price under the benchmark case, so the equilibrium price is going to be higher than with exogenous subsidy. To a higher price it corresponds a lower social welfare.<sup>15</sup>

## A.9 Proof Proposition 6

Solving these problems we derive the equilibrium price

$$p = \frac{a(1+b) + c(2+b) - 2t}{3+2b}$$

which is a decreasing function of  $t$ , as  $\frac{\partial p}{\partial t} < 0$ . As  $t = 0$  in the above equation leads to the price under the benchmark case, for any positive  $t$  the market price is now lower than in the benchmark case. Hence, the social welfare is higher than with exogenous subsidy, and higher than in any of the endogenous cases previously analyzed.

## A.10 Proof Proposition 7

Solving the maximization problems we get the profits of the two firms in the equilibrium:

$$\pi_1 = \frac{(1+b)((a-c)(1+b) - bt)^2}{(3+2b)^2}$$

$$\pi_2 = \frac{(a-c+2tb)(1+b)[(a-c)(1+b) - bt]}{(3+2b)^2}$$

We denote by  $\pi_2^{end}(t)$ ,  $\pi_1^{end}(t)$  the profit of the firms in the endogenous case of a tax on total quantity on the U market and by  $\pi_2^{exo}$ ,  $\pi_1^{exo}$  the profit of the firms under exogenous subsidy. Then it can be checked that the inequality

$$S_{end} = \pi_2^{end}(t) - \pi_1^{end}(t) + F^R < S_{exo} = \pi_2^{exo} - \pi_1^{exo} + F^R$$

is satisfied for any  $t < a - c + \frac{a-c}{2b}$ , hence for any  $t < a - c$ , which a necessary condition for the firms not to incur in losses.

<sup>14</sup>See Gonzalez Gomez P., (2003)

<sup>15</sup>The same conclusions apply to other taxes than the tax on profit, as the tax on total revenue or the tax on price. This is because these taxes belong to the same family of taxable basis that involves the market price.

## A.11 Proof Proposition 8

Plugging the conditions in the first order conditions of both firms maximization problems we get

$$\frac{\partial \pi_i}{\partial q_i} = 0, i = 1, 2$$

## A.12 Proof Proposition 10

We have already found the subsidy under endogenous financing is

$$S_{end}^{neu} = \frac{B_1(t) + B_2(t)}{2B_2(t)} [F^R + (\pi_2^*(t) - \pi_1^*(t))]$$

Using Proposition 1 we get that equilibrium profits do not depend on the tax  $t$ , and as the term in the bracket is just the exogenous subsidy, comparing the two subsidies we obtain that  $S_{end}^{neu} \leq S_{exo}$  if and only if

$$\frac{B_1(t) + B_2(t)}{2B_2(t)} \leq 1.$$

Consider  $f()$  to be the functional form of the taxable basis, an increasing function. Then the previous relation can be written

$$\frac{f(q_1) + f(q_2)}{2f(q_2)} \leq 1$$

As  $q_2 > q_1$ , then the above inequality is clearly fulfilled by any increasing function  $f()$ . Therefore,  $S_{end}^{neu} \leq S_{exo}$

## A.13 Proof Proposition 11

The lowest endogenous subsidy would be attained when the ratio  $\frac{f(q_1) + f(q_2)}{2f(q_2)}$  is minimized, so we have to solve the problem

$$\min_{f(q_1), f(q_2)} \frac{f(q_1) + f(q_2)}{2f(q_2)}$$

$$s.t. q_1 < q_2$$

$$f(q_1) \geq 0$$

$$f(q_2) \geq 0$$

This is equivalent to

$$\min_{f(q_1), f(q_2)} \frac{f(q_1)}{f(q_2)}$$

$$s.t. \ q_1 < q_2, f(q_1) \geq 0, f(q_2) \geq 0$$

As we supposed the function  $f$  is increasing, we get that the objective function in the minimization problem belongs to the interval  $[0,1]$ . Therefore the solution to this problem is any increasing function that makes  $f(q_1) = 0$ ,  $f(x) > 0$  for any  $x \neq q_1$ . This means that the subsidy required by the firm that wins the auction is minimized when the firm does not contribute with anything to the fund.<sup>16</sup>

## A.14 Proof Proposition 12

The previous proposition shows that when the subsidy is paid entirely by the loser operator, the endogenous subsidy is minimized. The opportunity cost equation becomes

$$\pi_1(t) - F^R + S = \pi_2(t) - tB_2(t)$$

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<sup>16</sup>Notice that this case, when the winner of the bid does not pay anything to the subsidy, is similar to one analyzed by P.Chone, L. Flochel and A. Perrot (2002). Although they use another framework, they consider a ‘*taxation regime*’ under which the incumbent which is the winner of the bid, serves high cost consumers, and the entrant serves the low cost consumers and pays a tax  $t$  for the subsidy. As in our model, depending on the tax level  $t$ , the firm that loses the auction could be pushed out of the market. Nevertheless, our model differs from the one studied by P.Chone, L. Flochel and A. Perrot (2002) as they suppose the regulator have perfect information. In general, in the market for a network product there is asymmetric information between the regulator and the operators. The advantage of our model is that it uses auctions to allocate the USOs and determine which firm will serve the market, reducing in this way the informational rents, distortions created by asymmetric information between the regulator and the operators.

and

$$S = tB_2(t)$$

It can be easily checked that  $S_{end}^{neu} = \frac{S_{exo}}{2}$ .

### A.15 Proof Proposition 13

Notice that if the functional form of the taxable basis is of the type  $g(p)h(q_1, q_2)$ , where  $g$  and  $h$  are increasing functions, then the taxable basis does not fulfil the condition  $\frac{\partial B_i}{\partial q_j} > 0$ ,  $i = 1, 2$ ,  $i \neq j$ . An example of such a tax it could be a tax on the profit of the operators or a tax on revenue.

Swartz's theorem (or Young's theorem) states that if at least one of the two partial derivatives of a function  $f(q_2, q_1)$  is continuous, then  $\frac{\partial^2 f}{\partial q_2 \partial q_1} = \frac{\partial^2 f}{\partial q_1 \partial q_2}$ .

Totally differentiating the first order conditions of both firms maximization problems we get the following system of equations

$$\left( \frac{\partial^2 \pi_1}{\partial q_1^2} + t \frac{\partial^2 B_2}{\partial q_1^2} \right) dq_1 + \left( \frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} + t \frac{\partial^2 B_2}{\partial q_1 \partial q_2} \right) dq_2 + \frac{\partial B_2}{\partial q_1} dt = 0$$

$$\left( \frac{\partial^2 \pi_2}{\partial q_2^2} + t \frac{\partial^2 B_1}{\partial q_2^2} \right) dq_2 + \left( \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} + t \frac{\partial^2 B_1}{\partial q_2 \partial q_1} \right) dq_1 + \frac{\partial B_1}{\partial q_2} dt = 0$$

Notice that

$$\pi_1 = (a - q_1 - q_2)(q_1 + b(q_1 + q_2))$$

$$\pi_2 = (a - q_1 - q_2)q_2$$

In the sequel we use the following notations:

$$\frac{\partial B_2}{\partial q_1} = B'_1, \quad \frac{\partial B_1}{\partial q_2} = B'_2, \quad \frac{\partial^2 B_1}{\partial q_2^2} = B_{22}$$

$$\frac{\partial^2 B_2}{\partial q_1 \partial q_2} = B_{12}, \quad \frac{\partial^2 B_1}{\partial q_2 \partial q_1} = B_{21}, \quad \frac{\partial^2 B_2}{\partial q_1^2} = B_{11}$$

Solving the above system of equations, we obtain  $\frac{dq_1}{dt}$  and  $\frac{dq_2}{dt}$  and summing them up we get

$$\frac{dq_1 + dq_2}{dt} = \frac{B'_2(1 + tB_{12} - tB_{11}) + B'_1(1 + tB_{21} - tB_{22})}{A}$$

where

$$A = 3 + 2b + tB_{21}(1 + 2b) + tB_{12} + t^2(B_{11}B_{22} - B_{12}B_{21}) - 2tB_{11} - 2t(1 + b)B_{22}$$

As the taxable basis is a concave function with positive cross second derivatives, that is  $B_{12}$  and  $B_{21}$  are both positive, then

$$\frac{dq_1 + dq_2}{dt} > 0$$

As the price in the U market is determined as  $p = a - q_1 - q_2$ , the latter inequality implies that

$$\frac{dp}{dt} < 0$$

This shows that the price decreases with  $t$ , therefore the social welfare is higher now than in the benchmark cases. Under the competitively neutral tax, as well as under the exogenous subsidy the price is flat with respect to  $t$ .

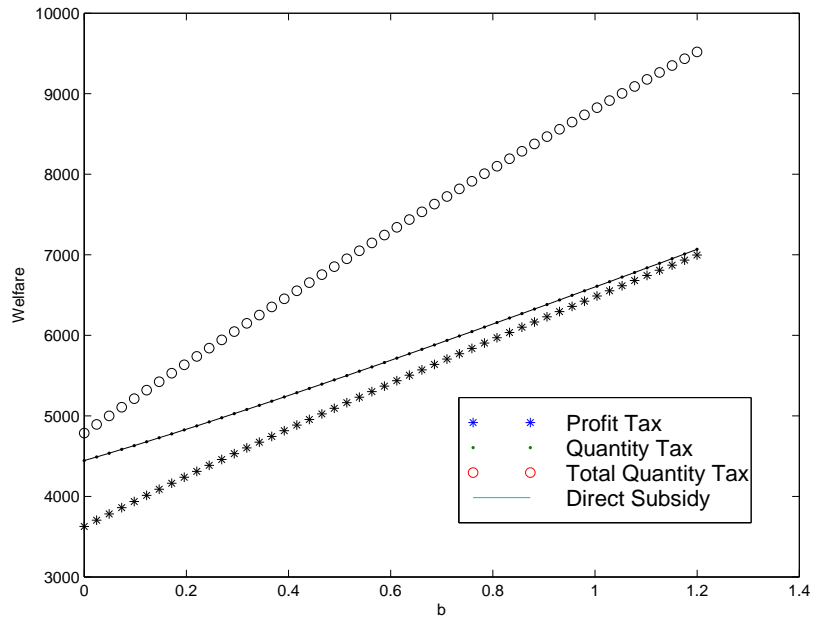


Figure 1: Social welfare versus rural market size (with  $F^R = 3000$ ).

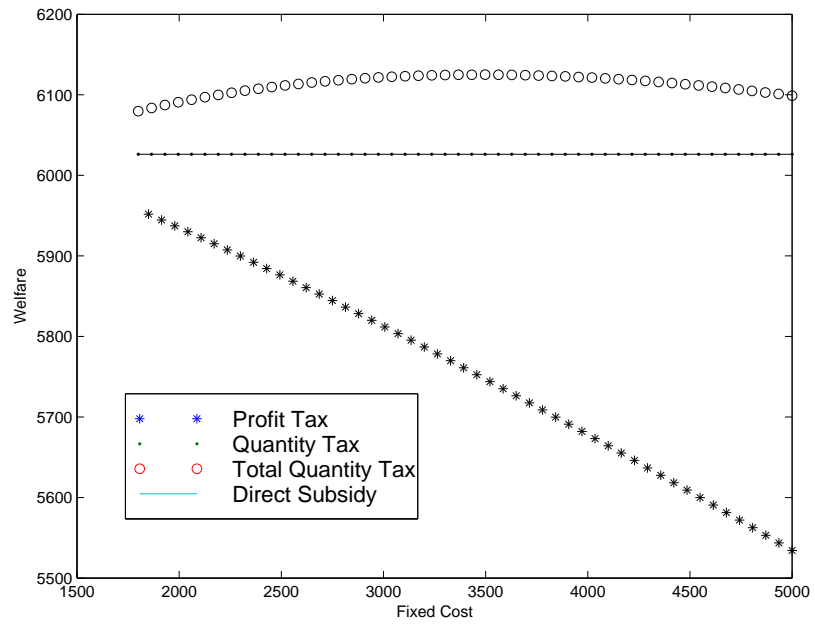


Figure 2: Social welfare versus rural market fixed costs (with  $b = 0.75$ ).

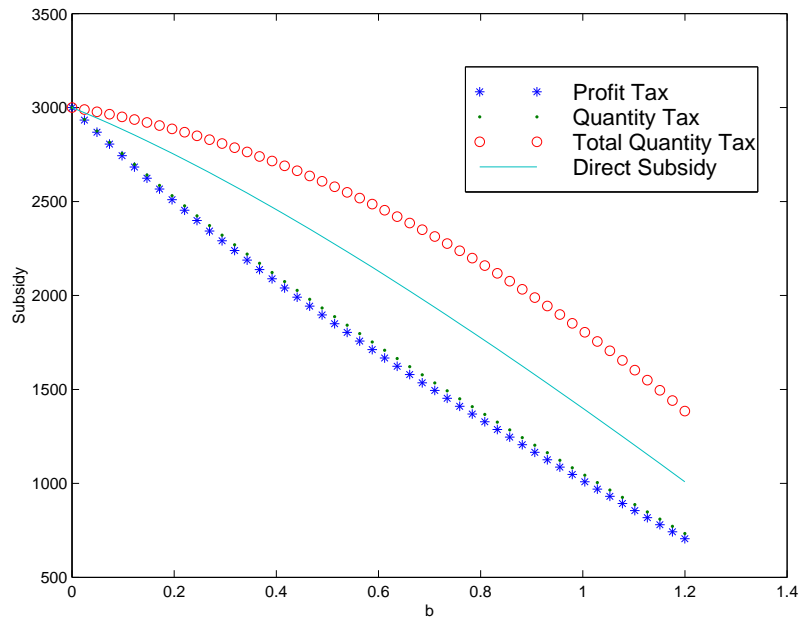


Figure 3: Subsidy versus rural market size (with  $F^R = 3000$ ).

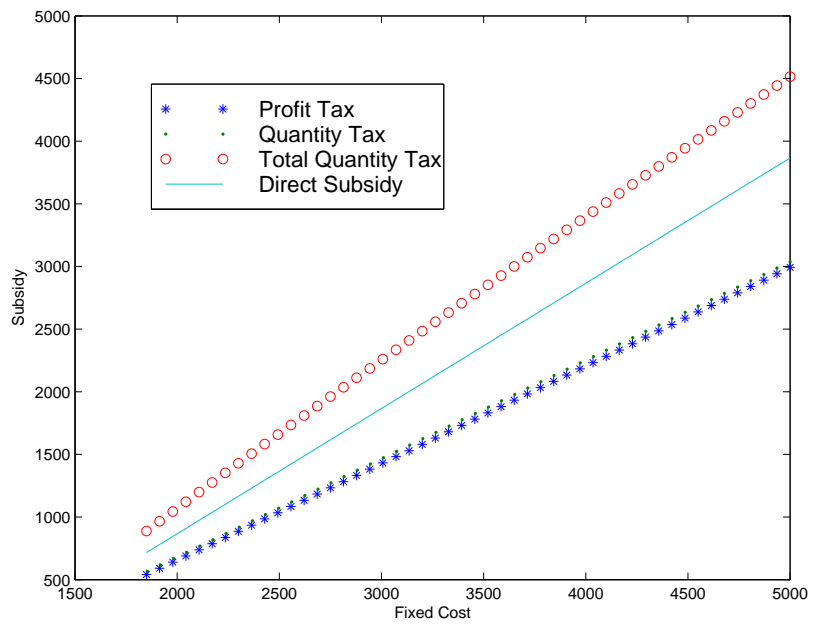


Figure 4: Subsidy versus rural market fixed costs (with  $b = 0.75$ ).

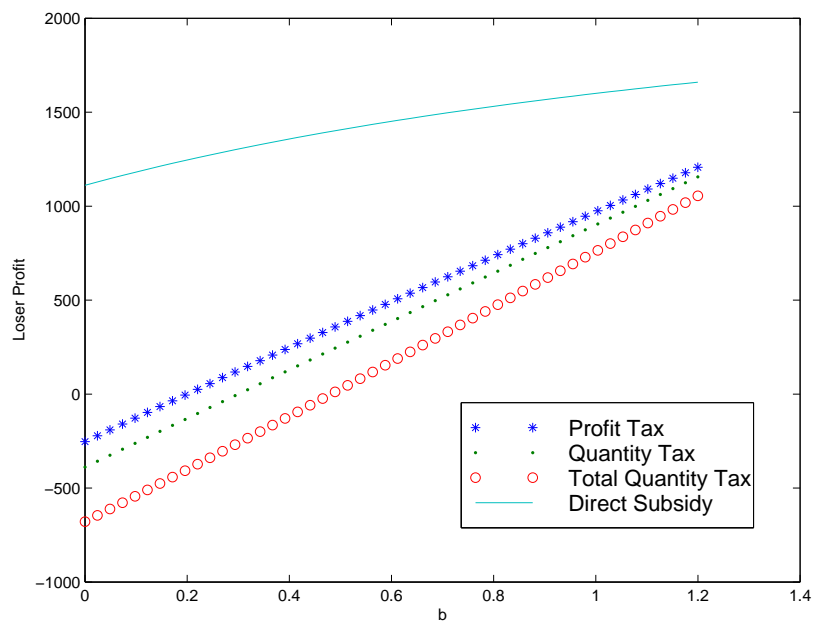


Figure 5: Profit of the loser firm versus rural market size (with  $F^R = 3000$ ).



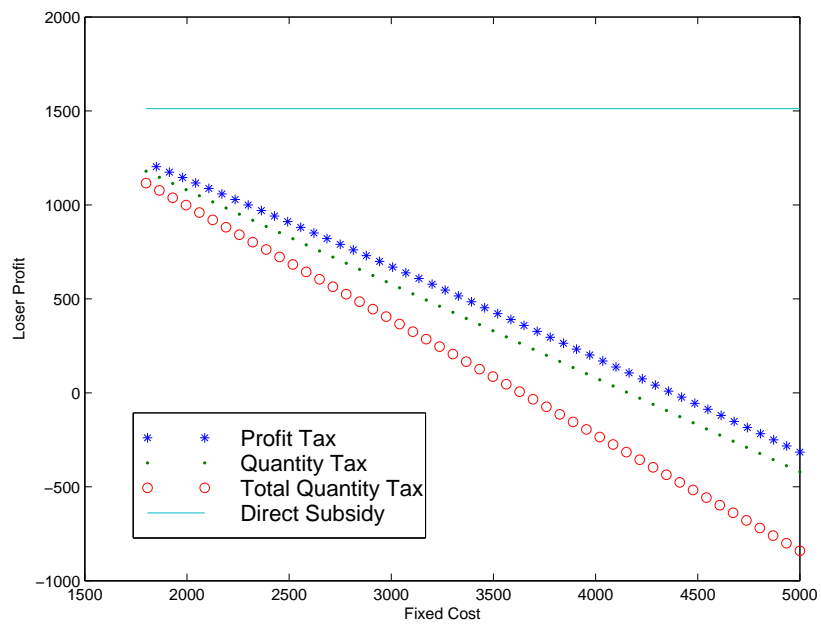


Figure 6: Profit of the loser firm versus rural market fixed costs (with  $b = 0.75$ ).

Figure 7: Comparison of a linear tax with a quadratic tax

