

DYNAMIC EFFICIENCY OF ENVIRONMENTAL POLICY: THE CASE OF  
INTERTEMPORAL EMISSIONS TRADING

Talitha Feenstra and Noemi Padrón-Fumero\*

Abstract

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In this paper we analyze the effects of dynamic environmental policies on firms' optimal investment behavior within finite time horizons. We show that when firms are allowed to intertemporally trade their emissions, they invest in abatement in earlier periods, advancing compliance with future environmental standards. Therefore, policies such as emissions banking<sup>1</sup> enhances the dynamic efficiency of the marketable permits and derives substantial cost-savings by itself. We show the dynamics of banking policy and emissions trading when the firm faces a two step emission standard with strict requirements at the end of the program. The firm's optimal trajectory under a pure banking program is compared to command-and-control (CAC), Pigouvian taxes and emissions borrowing, all for a finite time horizon. Banking introduces time flexibility, inducing the firm to over-comply with environmental standards in earlier periods, thus buying a delay in adjustment to future tighter policies. Finally, we analyze the dynamics of a pure emission trading program, where permits are available in a perfect competitive market, but do not last forever. Our results justify the current low trading in the U.S. Acid Rain Program (ARP) and link firm's cost savings to the success of the banking policy.

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Key Words: Dynamic environmental policy, Acid Rain Program, permits, banking.

\* Department of Economics and CentER, Tilburg University, P.O. Box 90153 LE Tilburg, The Netherlands and Departamento de Economía, Universidad Carlos III de Madrid, c/ Madrid 126, 28903 Getafe, Madrid, Spain and Departamento de Economía, Universitat Pompeu Fabra, c/ Ramón Trías Fargas 25-27, 08005 Barcelona, Spain (E-mail: padron@upf.es) respectively. We like to thank Peter Kort, Aart de Zeeuw, Emmanuel Petrakis and Jean-Pierre Benoit for their helpful comments and encouragement. Thanks are also due to Pedro Marin, Alejandro Balbás, Rob Aalbers and José A. García. All responsibilities are of our own.



# 1 Introduction

One often sees that regulators require certain environmental standards to be met at some future point in time, thus creating a period of adjustment for polluters. Global and local pollution targets are generally fixed creating more or less explicitly several phases of emission reduction requirements. The European Union, for example, requires its member countries to reduce sulphur emissions in three phases of five years each. At the end of each phase, all countries should comply with an emission reduction based on their baseline emissions of 1980 (Directive 88/609/CEE) or Second (European) Sulphur Protocol (SSP; Oslo, 1994). More recently, the U.S. Acid Rain Program (Clean Air Act Amendment, 1990) required electric utilities to reduce their rate of emissions for several pollutants according to two reduction phases (1990-95 and 1995-2000) and using a permit market. Other permit programs, such as the Lead in Gasoline Program (1983-86), have generally defined policy dynamics in this terms.<sup>1</sup> This 'step' implementation of abatement targets might be of advantage to the firms for the following reasons: it creates a period of adjustment for regulated firms and induces a planning of abatement actions; it allows countries to implement specific mechanisms to achieve the required standards; it reduces post-deadline uncertainty and, therefore, it drives technical change towards some long run path.

This paper is motivated by two unclear aspects of marketable permit markets related to the US Acid Rain Program (ARP), namely, the step implementation of emission standards and the finite validity of permits. The program creates an emission permits market within the electricity industry that should in principle increase a utility's flexibility in meeting the emission standard in two temporal phases, ending in year 2000. Even though the wide market involved and the serious emissions reductions have created important expectations on the program, a recent evaluation shows negligible trade among firms (Burtraw, 1996) <sup>2</sup>.

On the one hand, we aim to analyze the role of the period of permit's validity and the time horizon of permit's programs to explain current low trade volumes and low prices within the program. On the other hand, we aim to evaluate the benefits derived from the intertemporal distribution of abatement targets. What happens to the overall emission standard and to firms' intertemporal distribution of abatement when intertemporal emission trading is possible? To this purpose, we model environmental policies in finite time horizons and evaluate their impact on a firm's optimal investment behavior.

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<sup>1</sup>This program required refineries to gradually reduce, and finally eliminate, lead content in gasoline production. The program allowed trading reduction quotas among refineries as well as banking – savings of permits during some period that could be used to justify emissions in a later period. It consisted of a stepwise reduction of lead content standards. See EPA (1985).

<sup>2</sup>There is some trade in the market as outsiders are buying permits. However, most firms are reluctant to sell the permits that they don't need yet.

Actual environmental policy designs take into account the specific environmental and market structure. Hence, the implementation of permit programs differs far from theoretical permit systems. The final time horizon of the programs – with limited validity of permits – and the combination of overall and individual emission standards, which characterizes for example the ARP, are two of the aspects that have not been explicitly addressed in the literature. If permits are not valid forever, the regulator implicitly requires firms to actually attain some level of abatement at the end of the program, whatever the net position of the firm in the permit market during the program. Hence at the end of the program, all firms must meet the standard. This raises the question of what are the potential cost savings from a permit market in such a case. It can be argued that, even when allocative efficiency – exploiting abatement cost differences among firms – is only temporal, there are dynamic properties usually overlooked in marketable permits models that determine market performance and give rise to additional cost savings. In this paper we focus on the role of a firm's investment planning under flexible environmental policies.

Environmental policy instruments are often analyzed and compared in three main aspects: static efficiency, institutional feasibility, and the impacts on technological change (Milliman and Prince, 1989). The last aspect is also called dynamic efficiency and, broadly speaking, it concerns the incentives policy gives to polluters to invest in innovation and the direction in which innovative activities are steered. We use dynamic investment in emissions reduction to study the incentives induced by permit trading and banking.

Introducing a system of marketable or transferable pollution permits implies that scarcity on a well defined new good, environmental service, is created (Dales 1968). The regulator establishes an overall standard or it distributes emission permits among firms according to some rule. Then it allows firms to trade their permits or surplus allocations in a well defined market. It has been demonstrated elsewhere that this instrument is a cost-effective strategy for addressing pollution problems within a static once and for all setting (Montgomery, 1972; Tietenberg, 1985; Baumol and Oates, 1988).

The properties of permits are often described using terms such as cost-effectiveness and allocative or market efficiency. *Cost-effectiveness* refers to productive efficiency (Mishan, 1980), in other words, to a firm's ability to minimize costs to achieve an externally determined standard. Cost-effectiveness of marketable permits is proved formally by Montgomery (1972). Permits ensure that any environmental objective is reached at a minimum cost to society and abatement efforts are allocated efficiently over all firms. However nothing ensures that the benefits derived from that standard balance the damage costs at the margin. *Market or allocative efficiency*, on the other hand, makes reference to the internalization of social opportunity cost of the use of the environment as a resource in the price system – social efficiency, – Mishan (1980).

Montgomery's analysis considered a timeless, static framework and concentrated on intrafirm

trading. However, 'time trading' or the distribution of emissions over time is another important aspect to consider when banking permits is allowed, where banking is defined as the option to save surplus allowances for future use or sale. We use the term *dynamic efficiency* to denote an efficient distribution of emissions over time such that (a) an overall standard is averaged during the program and/or (b) firms can distribute their individual emissions over time.

We focus on banking policy and derive the optimal distribution of emissions over time from the point of view of the individual firm – that must decide when to introduce abatement – and from the point of view of the regulator – that creates a market to achieve an efficient intertemporal distribution of an aggregate emission standard<sup>3</sup>.

In the literature, the dynamic properties of permits are, to our knowledge, only analyzed in highly stylized frameworks. The regulator establishes an overall standard for emissions. Firms can trade permits, depending on how their environmental performance relates to the initial distribution of permits. Where a dynamic model of firm's investment behavior is analyzed, the time horizon is taken to be infinite – 'unlimited permit validity'. Kort et al. (1991) analyses subsidies and taxes as alternative instruments to induce pollution abatement to find out the optimal investment path for the firm; Xepapadeas (1992) explores the differences between taxes and CAC policy through firms investment optimal paths; Kort (1996) introduces for the first time permit markets as an option to the firm, comparing permits and taxes within a dynamic framework: under specific investment conditions and infinite time horizons, a uniform tax on emissions is equivalent to a permit system if permit prices are actualized every period according to the interest rate.

Cronshaw and Kruse (1991) analyzes the dynamic properties of permits within a discrete time model and infinite horizon. They show that, under perfect information and unlimited time, an efficient permit price pattern exists for the dynamic market. Firm's abatement is modelled as a recurrent cost, investment in technology is not possible. The introduction of banking does not yield different results in their analysis. Rubin and Kling (1993) builds on that work an intertemporal model of trade and banking to simulate cost savings from the light-duty vehicle manufactures of hydrocarbon emissions.

Finite time horizons, besides coming closer to current environmental programs, introduce interesting conditions in the specification of dynamic models for permits and, in general, for environmental policy. To evaluate performance and effectiveness of intertemporal trading, we focus on a permit policy designed to achieve an emissions objective by all firms (say a total emission limit or rate standard) at some point in time. Whatever is the position of the firm

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<sup>3</sup>If the distribution of emissions over time does not matter in terms of environmental impact, the overall standard defined by the regulator can be averaged by firms for the duration of the program. Then, the sum of all emissions during the whole period  $\int_0^T S(t)dt$  can be the overall objective of the regulator, instead of each  $S(t)$ . This is usually the case for stock pollutants such as greenhouse gases and chlorofluorocarbons.

during each period, it must comply in the last year (when permits are no valid anymore) with that objective. This definition implies: first, the combination of two instruments, permits and individual standard; and second, the introduction of a terminal condition into the optimal control problem (fixed time with fixed end point problem). Given limited time horizons and positive discount rates, banking induces firms to start building their abatement capital stock in earlier periods and result in significant cost savings as compared to a command-and-control policy. A pigouvian tax is equivalent to a banking policy if the tax rate can be adjusted over time. Emissions borrowing, on the other hand, delays abatement investment of the firm. Finally, the optimal investment trajectory of firms participating in a pure permit market depends on the path of permit's price.

The paper is organized as follows. Section 2 formulates the environmental policy scenario and derives the conditions under which the individual firm finds banking advantageous. We compute cost savings for the firm from individual intertemporal emission trading – when it is free to choose the time-distribution of its emissions during the program and, therefore, its investment in emissions abatement. Optimal trajectories are compared to CAC and tax policies. The possibility of a borrowing policy – defined as an earlier use of future endowments of permits – is explored in section 3. Section 4 derives optimal investment paths under a similar limited time horizon program, when the firm may only trade permits in a perfectly competitive permit market within a finite period. Banking or borrowing of emissions is not allowed. We characterize permit price paths under alternative institutional arrangements. Section 5 concludes.

## 2 Banking as an intertemporal transfer of emissions

Consider a firm that is faced with a system of tradeable permits during the period  $[0, T]$ . The firm is assumed to have minimization of abatement costs as an objective<sup>4</sup>. In order to focus on time flexibility, it is assumed that the firm is allowed exclusively to bank the permits: it can save for its own later use those permits that have not been used, but it cannot trade its emission surplus with other polluters. This deliberately limits the use of permits to capture only the effects of time flexibility. In the next section we extend the model and allow trade in permits.

We assume that the permit program has the following characteristics: the regulator introduces a permit system at time 0. A permit is defined as a license to emit some amount of pollution at any time during the program, say  $x$  tons of  $SO_2$ . The regulator announces to the firm that in the final period  $T$  it has to comply with a specific (stricter) emission standard  $\ell(T)$ . Hence, at some point in time each firm must technically adapt (or reduce output) to meet environmental requirements. Permit banking allows the firm to adjust at its own speed.

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<sup>4</sup>Since often the regulated firm is a public utility it is not always appropriate to describe it as a profit maximizer.

The firm's production process causes emissions,  $E(t)$ . Emissions can be decreased by investment in abatement capital. Let emissions be denoted by  $E$  and abatement capital by  $K$ , then emissions at time  $t$  are given by  $E(K(t))$ . It is assumed that a higher abatement capital stock results in lower emissions,  $E'(\cdot) < 0$ , and that it is more costly in terms of capital to decrease emissions when these are already low,  $E''(\cdot) \geq 0$ . Given the regulation, the firm will try to minimize the amount of money it has to spend on abatement. For a certain standard  $\ell$ , we can invert  $E(\cdot)$  to find  $K = E^{-1}(\ell)$ , the amount of capital that is required to satisfy this standard. This is denoted by  $k(\ell)$ .

The regulator announces at time 0 what standard  $\ell(t)$  will hold during the period  $[0, T]$ . The standard becomes stricter over time, therefore  $\ell(t)$  is a decreasing function of time. Specifically the following scheme is assumed:

$$\ell(t) = \ell_1 \quad t \in [0, s] \tag{2.1}$$

$$\ell(t) = \ell_2 \quad t \in (s, T] \tag{2.2}$$

with  $\ell_1 > \ell_2$

The firm is assumed to have enough abatement capital at time 0 to satisfy the standard  $\ell(0)$ :

$$K(0) = k(\ell(0)). \tag{2.3}$$

After this period the emission standard is tightened to the final level. This way of standard setting is almost equivalent to the ARP<sup>5</sup>. The firm is allowed to create permits and save these in a bank. The firm creates permits if it emits less than the standard  $\ell(t)$ . It can bank these and use them later. The firm is allowed to emit more than the standard if it has previously banked permits to cover the additional emissions. Banking permits increases the stock of permits at the bank,  $A(t)$ . This stock neither depreciates nor earns interest<sup>6</sup>.

Most regulators are vague about the continuation of the program after time  $T$ . For example, the ARP does not specify what will happen with existing permits or the 2005 standard after the year 2005. The Second (European) Sulphur Protocol (SSP: Oslo, 1994) specifies new international targets for  $\text{SO}_2$ , but leave the implementation of future stringent standards open. A reasonable assumption is that the standard  $\ell(T)$  will remain valid. Another possibility is that stricter standards are implemented. A linear scrap value function  $S(K, A) = v_a A + v_k K$  is

<sup>5</sup>See for a description of this program for example N. Kete (1992). The announcement of the ARP induced firms to anticipate the first period standard. When the program started in 1995 most of the firms already complied with the first period limit.

<sup>6</sup>In the US Emissions Trading Program (1981) banked permits could depreciate according to required environmental quality improvements in the areas implemented (Padrón, 1991).

introduced to reflect the expectations of the firm on future regulations. When the firm expects stricter future regulation, it attaches a positive scrap value to both  $K$  and  $A$ . On the other hand, when the firm expects that a new cheaper technology will be developed it attaches more value to  $A$ , rather than to physical capital,  $K$ .

The amount of permits banked (or withdrawn from the bank) at a certain time – the rate of permit banking – is denoted by  $a(t)$ . The increase in the stock of banked permits is given by:

$$\dot{A}(t) = a(t) \quad (2.4)$$

The bank requires firms to hold a non-negative stock of permits, loans are not provided.

$$A(t) \geq 0 \quad (2.5)$$

Furthermore, the firm is assumed to start with no banked permits at time 0:

$$A(0) = 0 \quad (2.6)$$

Abatement capital can be accumulated according to a standard capital accumulation function:

$$\dot{K} = I(t) - \delta K(t) \quad (2.7)$$

where  $\delta$  is the rate of depreciation and  $I(t)$  is gross investment in abatement capital. If the firm invests it is subject to adjustment costs. Together with the pure costs of investment these are included in the investment cost function  $C(I)$ . It is assumed that  $C(0) = 0$ ,  $C'(\cdot) \geq 0$  and  $C''(\cdot) > 0$ . The firm determines its investment in abatement and permits banked at each time  $t$  to minimize its discounted stream of investment costs minus scrap values at the final time  $T$ .

Given that the firm has to satisfy the standard at any period, the rate at which it can bank permits at time  $t$  is given by:

$$a(t) = \ell(t) - E(K(t)), \quad (2.8)$$

which are the units it emits below the standard. At time  $T$ , when the program ends, the firm is required to satisfy the standard without using permits:  $E(K(T)) \leq \ell(T)$ . The permit system is introduced to allow the firm to choose its own time path for accumulation of abatement capital and obedience of a stricter standard  $\ell_2$ . It is intermediate between a system with complete time flexibility, where the firm is only required to satisfy  $\ell_2$  at the final time  $T$  and a system with no time flexibility, where the firm must satisfy the standard  $\ell(t)$  at any time. Summarizing, the firm has to solve the following constrained dynamic optimization problem: to minimize the

present value of investment costs during the whole program minus the scrap value of abatement capital and permits in the last period,

$$\min_{a,I} \left\{ \int_0^T e^{-rt} [C(I)] dt - [v_k K(T) + v_a A(T)] e^{-rT} \right\} \quad (2.9)$$

subject to the following constraints,

$$\dot{A}(t) = a(t) \quad (2.10)$$

$$\dot{K}(t) = I(t) - \delta K(t) \quad (2.11)$$

$$a(t) = \ell(t) - E(K(t)) \quad (2.12)$$

$$A(t) \geq 0 \quad (2.13)$$

$$I(t) \geq 0 \quad (2.14)$$

$$A(0) = 0; \quad K(0) = k(\ell_1) \quad (2.15)$$

$$E(K(T)) \leq \ell(T) \quad (2.16)$$

When environmental policy is given by the scheme of standards (2.1 ) to (2.2), the standard  $\ell(t)$  is discontinuous at  $s$ . This implies that one must split the optimization problem into two parts in order to apply the maximum principle. However, first order optimality conditions in both periods are similar. They can be rewritten in the following form (see Appendix A):

$$\lambda_1 \leq C'(I) \quad I \geq 0 \quad I[C'(I) - \lambda_1] = 0 \quad (2.17)$$

$$\lambda_2 = \frac{\dot{\lambda}_1 - (r + \delta)\lambda_1}{E'(K)} \quad (2.18)$$

$$\mu = r\lambda_2 - \dot{\lambda}_2 \quad (2.19)$$

$$\mu \geq 0 \quad A \geq 0 \quad \mu A = 0 \quad (2.20)$$

and the terminal conditions,

$$\lambda_1(T) \geq v_K; \quad K(T) \geq k(\ell_2); \quad (K(T) - k(\ell_2))(\lambda_1 - v_K) = 0 \quad (2.21)$$

$$\lambda_2(T) \geq v_a ; A(T) \geq 0 ; A(T)(\lambda_2(T) - v_a) = 0 \quad (2.22)$$

Equation 2.17 refers to optimal investment in abatement capital. Except for corner solutions, the shadow value of an additional unit of abatement capital ( $\lambda_1$ ) must equal marginal investment costs. The second equation, (2.18) links the shadow value of a permit at the bank ( $\lambda_2$ ) to the firm's abatement. The more the firm has invested in abatement capital, the more costly it is in terms of capital to reduce emissions even further (since  $E'(K) \leq 0$ ) and, hence, the higher the value of a permit at the bank. The shadow value of a permit at the bank ( $\lambda_2$ ) is positive (i.e. it is valuable to have permits at the bank) when the shadow value of abatement capital does not grow, or grows at rate lower than the depreciation corrected interest rate ( $(r + \delta)\lambda_1$ ). Equations 2.19 and 2.20 link the stock of permits at the bank to its shadow value. If this shadow value is decreasing in real terms (i.e.  $r\lambda_2 - \dot{\lambda}_2 > 0$ ), then from (2.20) it follows that  $A = 0$  – it is not optimal to hold a positive stock of permits. On the other hand, if an optimal solution is characterized by positive amounts of permits at the bank, the permits' shadow value must grow at exactly the interest rate. Along an optimal path the shadow value of a permit in the bank will never grow at rate higher than the interest rate. This follows from (2.20). If it would grow faster, the firm would want to bank an infinite number of permits, hence no optimal solution exists.

**Lemma 1** *An additional permit at the bank is never valued negatively by the firm:*

$$\lambda_2 \geq 0 \quad \forall t \in [0, T] \quad (2.23)$$

This is reasonable since the firm can choose to use the permit at any time and there are no costs incurred by keeping the permit at the bank.

*Proof:* Assume that this would not hold for all  $t$ , so that  $\lambda_2(t_1) < 0$  for some  $t_1 \in [0, T]$ . From equation (2.19) and (2.20) it follows that  $\dot{\lambda}_2 < 0$  for all  $t \geq t_1$ . This contradicts the terminal condition  $\lambda_2(T) \geq v_A \geq 0$ .

Equation (2.18) then implies that along an optimal path, the shadow value of abatement capital grows at most at the rate  $(r + \delta)$ . Furthermore if  $\lambda_2 = 0$  for some  $t_1$ , then  $\lambda_2 = 0$  for all  $t \geq t_1$ . From (A.18)  $\dot{\lambda}_2(t_1) = -\mu_1 \leq 0$  and it was just derived that  $\lambda_2 \geq 0$ . Hence,  $\dot{\lambda}_2(t_1) = 0$ . The same reasoning applies for all  $t \geq t_1$  and results in  $\lambda_2 = 0$  for all  $t \geq t_1$ .

We now show that this system of banking is always advantageous to the firm compared to with a command-and-control policy. Consider a firm that is not allowed to bank permits, but is subject to a strict standard policy: it has to satisfy the standard  $\ell(t)$  at any moment. In terms of the permit system, the firm is not allowed to use any permits. This is equivalent to adding the constraint

$$a(t) \geq 0 \quad (2.24)$$

to the cost minimization problem above. First order conditions for the CAC problem and its formal specification are given in the Appendix. It is immediately clear that allowing a firm to bank permits for later use decreases overall abatement expenditures to the firm. Allowing for the use of permits implies that the constraint (2.24) is removed from the firm's decision problem. The optimal solution with permits will be at least as good as the optimal solution under command-and-control policy. It is very likely to be better, causing cost savings, since the firm can smooth its investment.

Let us assume now that  $S(K(T), A(T)) = 0$  (or  $v_k = 0$  and  $v_a = 0$ ), that is, there is no value of abatement capital and permits after  $T$ . The firm starts to invest at a rate higher than  $I = \delta k(\ell_1)$  (or  $\dot{K} = 0$ ) at time  $s_1$ , with  $0 \leq s_1 < s$ . Before that time it was investing at rate  $\delta k(\ell_1)$  to make up for depreciation. From  $s_1$  onwards the capital stock increases. The firm emits less than the standard and accumulates permits. At time  $s$ , the stricter standard  $\ell_2$  is imposed, while the firm has not accumulated enough abatement capital yet to satisfy it. The firm uses its banked permits to cover too high emissions, while it continues to invest at a high rate and to build abatement capital. At time  $s_2$ , with  $s < s_2 \leq T$ , the firm has accumulated enough capital and its emissions satisfy the strict standard. It decreases its investments to invest only for depreciation and stops using permits. At time  $T$ , the program stops and the firm receives the scrap value of abatement capital and permits, if any. This is an optimal path for low enough scrap values.

If either  $v_k$  or  $v_a$  is high, then the firm will continue to invest in abatement capital even if it satisfies the new stricter standard. Therefore, the definition of the optimal path will depend on the definition of the policy target or post-program policy.

We can compare the optimal path under the permit policy described above (see *figure 6*) to the investment strategy when the firm faces a binding standard CAC (see *figure 7*). The optimal path under a command and control policy is given by:

- (i)  $I(t) = \delta k(\ell_1)$  for  $t < s_3$  for all  $s_3 \in [0, s)$ ,
- (ii)  $I(t) > \delta k(\ell_1)$  for  $s_3 \leq t < s$  such that at  $s$ ,  $K(s) = k(\ell_2)$ ,
- (iii)  $I(t) = \delta k(\ell_2)$  for  $s < t \leq T$ .

This path is a feasible solution to the optimization problem (2.9), but it does not satisfy the first order conditions. This is not surprising; allowing the firm to bank permits improves its flexibility in planning its investment expenditures. Even in the highly simplified setting we use, banking permits results in cost savings. In reality, due to variances in adjustment costs over time, even higher cost savings can be expected from the banking policy.

## 2.1 Emissions tax for limited time

It is in principle possible for a regulator with perfect information on firms abatement costs to set up a system of emission taxes that leads the firm to the same time path of investment as it would choose under the permit program described above. The regulator should compute the optimal investment path and set a tax on emissions equal to the shadow value of banked permits,  $\lambda_2$ , at any time. In the case of a system of emission taxes, the firm has to pay a tax,  $\tau$ , for every unit of emissions. Thus, it also pays explicitly for the emissions that are below the standard. This is in contrast with the system of permits just described, where the firm only pays an implicit opportunity cost. The optimization problem that the firm has to solve under taxes is to minimize the discounted stream of abatement costs plus the emissions tax minus the scrap value of abatement capital and excess of abatement (which again represent firm's expectations on policy developments after T).

$$\min_I \left\{ \int_0^T e^{-rt} [\tau E(K) + C(I)] dt - [v_k K(T)] e^{-rT} \right\} \quad (2.25)$$

subject to constraints 2.10 to 2.16. Optimality conditions for the firm are:

$$C'(I) \leq \tilde{\lambda}_1 \quad I \geq 0 \quad I[\tilde{\lambda}_1 - C'(I)] = 0 \quad (2.26)$$

$$\tau = \frac{\tilde{\lambda}_1 - (r + \delta)\tilde{\lambda}_1}{E'(K)} \quad (2.27)$$

and terminal conditions

$$\tilde{\lambda}_1(T) \geq v_k; \quad E(K(T)) \leq \ell(T); \quad [\tilde{\lambda}_1(T) - v_k][E(K(T)) - \ell(T)] = 0 \quad (2.28)$$

Assuming that an interior solution exists, these conditions are equivalent to those for an optimum under the permits system, (2.17) to (2.22), when the tax is set such that

$$\tau(t) = \lambda_2(t), \quad (2.29)$$

where  $\lambda_2$  is the shadow value of permits from section 2. If the regulator sets taxes equal to this rate, the investment path chosen by an optimizing firm subject to these taxes equals that chosen by a firm subject to the banking permit described in section 2.

One can conclude that it is possible to arrive at the same results with a tax as with a permit system. But note that this requires the regulator to have perfect knowledge of the cost functions of the firm. Whenever this is not the case it is impossible for the regulator to calculate the correct level of taxes. Moreover, the tax must be adjusted every period to follow the optimal path. On the contrary, for the permit system to reach the optimal path of investment it is only

necessary that the regulator sets the level of standards,  $\ell_1$  and  $\ell_2$ . This difference in information requirements is a well known difference between taxes and permits, see for example Baumol and Oates (1988).

### 3 Borrowing emissions

One could go a step further than the system described thusfar and allow the firm more flexibility. Consider a firm that must comply with the environmental policy defined by scheme (2.1) to (2.2), as described above, and assume now that the regulator endows the firm at time 0 with enough permits to comply with this emissions scheme. This leaves the firm completely free to allocate its emissions over time, or in formal terms, removes the  $A \geq 0$  constraint (however, the requirement that  $A(T) \geq 0$  remains). Borrowing is defined as allowing the firm to emit more than the standard at some time  $t$  even if it has no stock of saved permits, provided that it balances this with emission reductions below the standard later on. The structure described above and formalized below allows for such borrowing.

Let  $B(t)$  denote total permit holdings of the firm, such that:

$$B(0) = \int_0^T \ell(t) dt$$

Emissions reduce this stock:

$$\dot{B}(t) = -E[K(t)] \tag{3.1}$$

so that:

$$B(t) = L - \int_0^t E[K(s)] ds$$

gives the stock of permits owned by the firm at time  $t$ .

At the end of the program the firm must comply with the stricter standard,  $\ell(T) = \ell_2$  and therefore  $K_T \geq k(\ell_2)$ , must hold. Also the firm must have paid off its "loans":  $B(T) \geq 0$  is required. Like in section 2, the firm starts with abatement capital  $K(0) = k(\ell_1)$  at  $t = 0$ . In summary, the firm has to solve the following constrained dynamic optimization problem:

$$\min_I \left\{ \int_0^T e^{-rt} [C(I)] dt - [v_k K(T) + v_b B(T)] e^{-rT} \right\} \tag{3.2}$$

subject to the following constraints,

$$\dot{B}(t) = -E(K(t)) \tag{3.3}$$

$$\dot{K}(t) = I(t) - \delta K(t) \quad (3.4)$$

$$B(T) \geq 0 \quad (3.5)$$

$$I(t) \geq 0 \quad (3.6)$$

$$B(0) = L; \quad K(0) = k(\ell_1) \quad (3.7)$$

$$E(K(T)) \leq \ell(T) \quad (3.8)$$

First order conditions are:

$$C'(I) \leq \lambda_1 \quad I \geq 0 \quad I[\lambda_1 - C'(I)] = 0 \quad (3.9)$$

$$\lambda_3 = \frac{\dot{\lambda}_1 - (r + \delta)\lambda_1}{E'(K)} \quad (3.10)$$

$$\dot{\lambda}_3 = r\lambda_3 \quad (3.11)$$

together with terminal conditions:

$$\lambda_1(T) \geq v_K, \quad E(K_T) \leq l_T, \quad [\lambda_1(T) - v_K][l_T - E(K(T))] = 0 \quad (3.12)$$

$$\lambda_3(T) \geq v_b, \quad B(T) \geq 0, \quad B(T)[\lambda_3(T) - v_b] = 0 \quad (3.13)$$

Since the firm always needs to keep some permits in order to justify its final emissions, it follows that  $B(t)$  will always be positive.

**Lemma 2** *An additional permit at the bank is never valued negatively by the firm:*

$$\lambda_3 \geq 0 \quad \forall t \in [0, T] \quad (3.14)$$

*Proof:*

We can solve equation (3.11) for  $\lambda_3$ :

$$\lambda_3(t) = e^{r(t-T)}\lambda_3(T)$$

$\lambda_3$  is the shadow value of one unit added to  $B$ , the stock of permits available to the firm, which cannot be negative if at  $T$  emission permits have some nonnegative scrap value. The firm can

always keep them and receive this scrap value. Or otherwise it can use them and save some investment costs. If  $v_B > 0$  then  $\lambda_3(T) > 0$  follows from the terminal condition (3.13). Since  $\lambda_3(t) = e^{-rT} e^{rt} \lambda_3(T)$ , it follows that  $\lambda_3(t) > 0$  for all  $t$ .

From equation (3.10) it then follows that along an optimal path,  $\dot{\lambda}_1 < (r + \delta)\lambda_1$  must hold. Which means that  $\lambda_1$ , the shadow value of additional abatement capital, should not grow too fast. The optimal final path depends, again, on the scrap value function. If  $v_K$  is high enough, the firm will invest higher quantities than the standard would require. If  $v_K$  is low enough, say  $v_K = 0$ , then  $K(T) = k[\ell(T)]$  and the firm accumulates just enough capital to satisfy the requirements at  $T$ . The optimal path will depend on the discount rate and the shape of  $E(K)$ .

Finally, as long as  $\ell_1 < \ell_2$  and, as we have assumed, the firm has convex emission and adjustment costs  $E''(K) > 0$  and  $C''(I) > 0$ , it is relatively cheap to invest at the lower standard (lax standards) and more expensive to do so at stricter ones. Therefore, it is not optimal for the firm to borrow in the first phase of the program, for that would imply that the firm lets its capital depreciate. Then, along  $\ell_1$ , the optimal path is determined by investment to maintain the capital stock.

During the second period, where  $\ell_2$  is binding, the firm will have to invest more. This saves some costs, as delaying investment is always cheaper, but it also implies higher adjustment costs. Smooth investment is cheaper if the firm faces a convex investment cost function. Therefore, along  $t \in [s, T]$ , the firm will distribute investment to achieve  $\ell_2$ : borrowing emissions during  $[s_1, s_2)$  as it will be over the standard for that period, and compensating for the overuse of permits during  $[s_2, T]$  by building a capital stock greater than required by the standard.

Summarizing, borrowing tends to delay investment in emissions reduction. Only when the standard becomes stricter, the firm starts building or increasing its abatement capital stock. During this time, the firm is borrowing emissions against future reductions above the final standard. The rate of investment in the second period depends on the length of the period, adjustment costs and the rate of discount. Indeed, the higher the discount rate, the lower adjustment costs or the longer this first period, the later and slower the firm builds its capital stock. Finally, the scrap value function, together with the borrowing rate in the earlier period, will determine the final position of the firm with respect to the standard.

Comparing banking and borrowing policies dynamics we can conclude the firm might incur in the same net costs of abatement. However, banking is a preferred policy both in terms of emissions distribution over time, as pollution is reduced earlier, and in terms of irreversibilities, as borrowing emissions against the future can yield "pollute-and-close" behavior of firms.

## 4 Competitive emission trading program with terminal time

In this section only permit trade is considered. Again consider a firm that minimizes the discounted stream of abatement costs over the period  $[0, T]$ . It is subject to some emission standard  $\ell(t)$  during the entire period but it may exceed this standard at some  $t$  if it buys sufficient permits from other firms. If the firm emits less than the standard it saves some permits that can be sold in the same period to other firms in a competitive permit market. The market price of a permit is given by  $p(t)$ . Let  $y(t)$  be the amount of permits the firm sells or buys in the market at time  $t$ . Then,  $y(t)$  is given by:

$$y(t) = \ell(t) - E(K(t)), \quad (4.1)$$

the difference between the standard and the firm's emission rate. If  $y(t) > 0$ , the firm is a net seller of permits and if  $y(t) < 0$ , the firm is a net buyer of permits. The firm invests in abatement capital in order to decrease its emission rate, increasing revenues of permit sales, if the firm is net seller, or decreasing costs of permit purchases, if the firm is a net buyer. At the end of the period it is required that

$$E(K(T)) \leq \ell(T),$$

hence  $y(T) \geq 0$ . The possibility of trade in permits may also allow firms some flexibility in the timing of investments. This flexibility is large when firms in the permit market differ in initial capital stock, adjustment costs or depreciation rate. For example, a firm facing adjustment costs that are close to linear has incentives to delay its investments until the last period. Until it has enough capacity to satisfy the standard, the firm buys permits from firms with more convex adjustment cost functions.

Cost savings from this policy are derived from temporary allocative efficiency, since all firms must comply at the end of the program with the standard. Complete allocative efficiency would be reached if firms could maintain their net position in the permit market beyond  $T$ .

Firm's must pay their permit purchases if they are net buyers and receive some benefits if they are net sellers. We consider the same standard scheme specified in the above section. The control problem for the firm then becomes:

$$\min_I \left\{ \int_0^T e^{-rt} [C(I) - p(t)y(t)] - [v_k K(T)] e^{-rT} \right\} \quad (4.2)$$

subject to the following constrains:

$$\dot{K} = I(t) - \delta K(t) \quad (4.3)$$

$$y(t) = l(t) - E(K(t)) \quad (4.4)$$

$$I \geq 0 \quad (4.5)$$

$$K(0) = k(\ell_1) \text{ and } K(T) \geq k(\ell_2) \quad (4.6)$$

The main difference with the banking model is that firms must sell excess abatement at the prevailing price. First order conditions are derived in the Appendix and can be summarized as follows:

$$C'(I) \leq \lambda_1 \quad I \geq 0 \quad I[\lambda_1 - C'(I)] = 0 \quad (4.7)$$

$$\dot{\lambda}_1 = \lambda_1(t)(r + \delta) + E'(K)p(t) \quad (4.8)$$

with the transversality condition:

$$\lambda_1(T) \geq v_k; \quad E(K(T)) \leq \ell(T); \quad [\lambda_1(T) - v_k][E(K(T)) - \ell(T)] = 0 \quad (4.9)$$

From equation (4.7), it follows that the shadow price of abatement capital must at least equal the marginal cost of investment along an optimal path. If an interior solution is assumed, then  $\lambda_1 = C'(I)$ . Differentiating this with respect to time and rewriting (4.8) gives:

$$\dot{I} = \frac{C'(I)(r + \delta) + p(t)E'(K)}{C''(I)} \quad (4.10)$$

It holds that  $C'(I) > 0$ ,  $C''(I) > 0$ ,  $p(t) \geq 0$  and  $E'(K) \leq 0$ . Therefore,  $C'(I)(r + \delta) > -p(t)E'(K)$  implies that investment increases over time. Since  $E''(K) > 0$ , it follows that when the permit price,  $p$ , is constant and not too large so that  $C'(I)(r + \delta) > -p(t)E'(K)$  holds for some  $K(t^*)$ , one can conclude that investment is monotonically increasing for all  $t \geq t^*$ . Additionally, a constant investment function implies that:

$$C'(I) = \frac{p(t)E'(K)}{(r + \delta)} \quad (4.11)$$

or in words, the marginal cost of investment equals the discounted marginal value of additional abatement capital. The firm increases investment if marginal investment costs are lower than the discounted marginal cost of the last unit of emissions abated. In that case, the firm reduces emissions and sells the permits it does not need in the market. The firm pays an opportunity cost for every unit it emits. It always pays to reduce emissions if marginal costs are lower than marginal benefits; it always pays to buy them if costs are high and the firm has to comply with the standard.

The dynamics of investment capital are determined by the permit price. First, if the permit price increases at the rate of interest  $p(t) = e^{rt}p(0)$ , then  $\dot{\lambda}_1 = (r + \delta)\lambda_1 + e^{rt}p(0)E'(K(t))$  determines the rate of investment of the firm in abatement capital. Second, if the permit price is constant (it decreases in real terms)  $p(t) = p(0)$ , then  $\dot{\lambda}_1 = (r + \delta)\lambda_1 + p(0)E'(K(t))$  determines investment in abatement. Finally, if the permit price increases in real terms  $p(t) = e^{(r+\alpha)t}p(0)$ , where  $\alpha$  is a positive constant, given that the firm can not save permits for later use or sale, the shadow price of abatement will be increasing and the firm will increase the rate of investment: whether it does so to buy less permits or to sell more permits, depends on the net position of the firm in the market.

A limited time horizon in permit markets suggests that, given that permits are not valid after  $T$ , the price of the permit may decrease over time and fall to zero when program reaches the terminal period. Note that the price of the permit acts as the scrap value of a permit in the final period. When permits are not expected to be valid after  $T$ ,  $p(T) = 0$ , that is, the shadow value of investment above the level required by the final standard will be zero. Therefore, the final position on investment in abatement capital of the firm will depend exclusively on its expectations of future tighter emission limits. Indeed, the definition of permit programs with finite time horizons, where permits explicitly expire in the last period or the weak definition of the program induces firms to believe so, will vanish any incentive to invest further than required by the standard.

## 5 Conclusions

We used a dynamic optimal control model with finite time horizons to characterize firms' emissions-reduction-investment when environmental policy targets are defined by the government with adjustment periods. Several rules of intertemporal emissions trading have been combined to compare firms' abatement investment paths in a dynamic setting with finite time horizons. We show that intertemporal distribution of emissions, even through the simple banking policy, results in cost savings for the firm.

We obtained optimal trajectories of firms' investment when the banking policy allows the firm to decide its own intertemporal emissions distribution. Compared to a standard command-and-control setting, banking cost savings' justify firm's earlier investment. This result holds whenever adjustment costs increase with investment levels. A Pigouvian tax could determine the same optimal investment paths of banking if the tax is adjusted periodically. However, it requires perfect information on firms' abatement costs and adjustments of the tax rate over time. Finally, investment paths in a pure permit market, where any excess or deficit of permits must be instantly cleared, depends on the path of price of a permit.

Some comments on the implications of these results related to the failure of the Acid Rain

Program for US electricity industry are needed. The Program establishes an emissions permit market to achieve an overall emissions standard within two temporal phases. Even though low trade among participating firms has occurred during the first phase, the program has induced firms's abatement and compliance with future emission standards. The low trade and low permit prices might be compensated by the success of the banking policy. If the firms had already advanced the first emission standard of the program established for the first period, banking now permits to delay investment in the second period is not only the optimal behavior of the firm but it also results in cost savings, whatever the development of the permit price. Moreover, firms' expectations on future technology changes, specially in an industry characterized with high investment and irreversible costs, may also additionally supports this delay in investment. In this sense, the banking policy enhances the dynamic efficiency of marketable permits, defined here as the incentives permits gives to polluters to invest in innovation and the direction in which innovative activities are steered.

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## A Optimal trajectory for banking

Consider the optimization problem as given by equations 2.9 to 2.16. This is an optimal control problem with two state variables,  $K$  and  $A$ , two control variables,  $a$  and  $I$  and a pure state constraint, equation 2.13. To obtain the optimality conditions for the optimal control problem we apply Pontryagin's maximum principle for the two intervals  $[0, s]$  and  $[s, T]$  (See Feichtinger and Hartl, 1986). We find for each time interval that the Lagrangian is given by:

$$L = -\lambda_0 C(I) + \lambda_1(t)(I - \delta K) + \lambda_2 a(t) + \mu_1 A(t) + \mu_2(t)[E(t) - \ell(t) + a(t)] + \mu_3(t)[-E(t) + \ell(t) - a(t)] \quad (\text{A.1})$$

Here  $\lambda_0 \in R$ ,  $\lambda_0 \geq 0$ ,  $\lambda_i(t)$  are two co-state variables and  $\mu_j(t)$  are three dynamic Lagrange multipliers. Necessary conditions for an optimal solution are:

$$-\lambda_0 C'(I) + \lambda_1(t) \leq 0; I \geq 0; I(\lambda_1 - C'(I)) = 0 \quad (\text{A.2})$$

$$\lambda_2(t) + \mu_4 - \mu_5 = 0 \quad (\text{A.3})$$

$$\dot{\lambda}_1 = \lambda_1(t)(r + \delta) - (\mu_4 - \mu_5)E'(K) \quad (\text{A.4})$$

$$\dot{\lambda}_2 = \lambda_2(t)r - \mu_1 \quad (\text{A.5})$$

$$\mu_1 \geq 0; \mu_1 A(t) = 0 \quad (\text{A.6})$$

$$\mu_4 \geq 0; \quad \mu_4(E(K) - \ell + a) = 0 \quad (\text{A.7})$$

$$\mu_5 \geq 0; \quad \mu_5(-E(K) + \ell - a) = 0 \quad (\text{A.8})$$

$$E(K) - \ell + a = 0 \quad (\text{A.9})$$

at points  $\tau$  of discontinuity, there may be a jump  $\eta$  in  $\lambda_2$  and it must hold:

$$\eta(\tau) \geq 0 \quad A(\tau) \geq 0 \quad \eta(\tau)A(\tau) = 0 \quad \lambda_2(\tau^-) = \lambda_2(\tau^+) + \eta(\tau) \quad (\text{A.10})$$

For the period  $[0, s]$  these conditions must hold and additionally

$$K(0) = k(\ell_1); \quad A(0) = 0 \quad (\text{A.11})$$

$$K(s) = K_s; \quad A(s) = A_s \quad (\text{A.12})$$

For the period  $[s, T]$  the conditions 2.17 to A.8 must hold and additionally:

$$K(s) = K_s; \quad A(s) = A_s \quad (\text{A.13})$$

$$\lambda_1(T) \geq S_K; \quad E(K(T)) \leq \ell(T); \quad [\lambda_1(T) - S_K][E(K(T)) - \ell(T)] = 0 \quad (\text{A.14})$$

$$\lambda_2(T) \geq S_A; \quad A(T) \geq 0; \quad [\lambda_2(T) - S_A]A(T) = 0 \quad (\text{A.15})$$

The first order conditions can be rewritten, using (A.3) to substitute for  $(\mu_4 - \mu_5)$  and assuming that  $\lambda_0 \neq 0$ , as:

$$\lambda_1 \leq C'(I); \quad I \geq 0; \quad I(C'(I) - \lambda_1) = 0 \quad (\text{A.16})$$

$$\dot{\lambda}_1 = (r + \delta)\lambda_1 + \lambda_2 E'(K) \quad (\text{A.17})$$

$$\dot{\lambda}_2 = r\lambda_2 - \mu_1 \quad (\text{A.18})$$

$$\mu_1 \geq 0 \quad A \geq 0 \quad \mu_1 A = 0 \quad (\text{A.19})$$

together with the discontinuity condition (A.10) and the conditions (A.11) to (A.15).

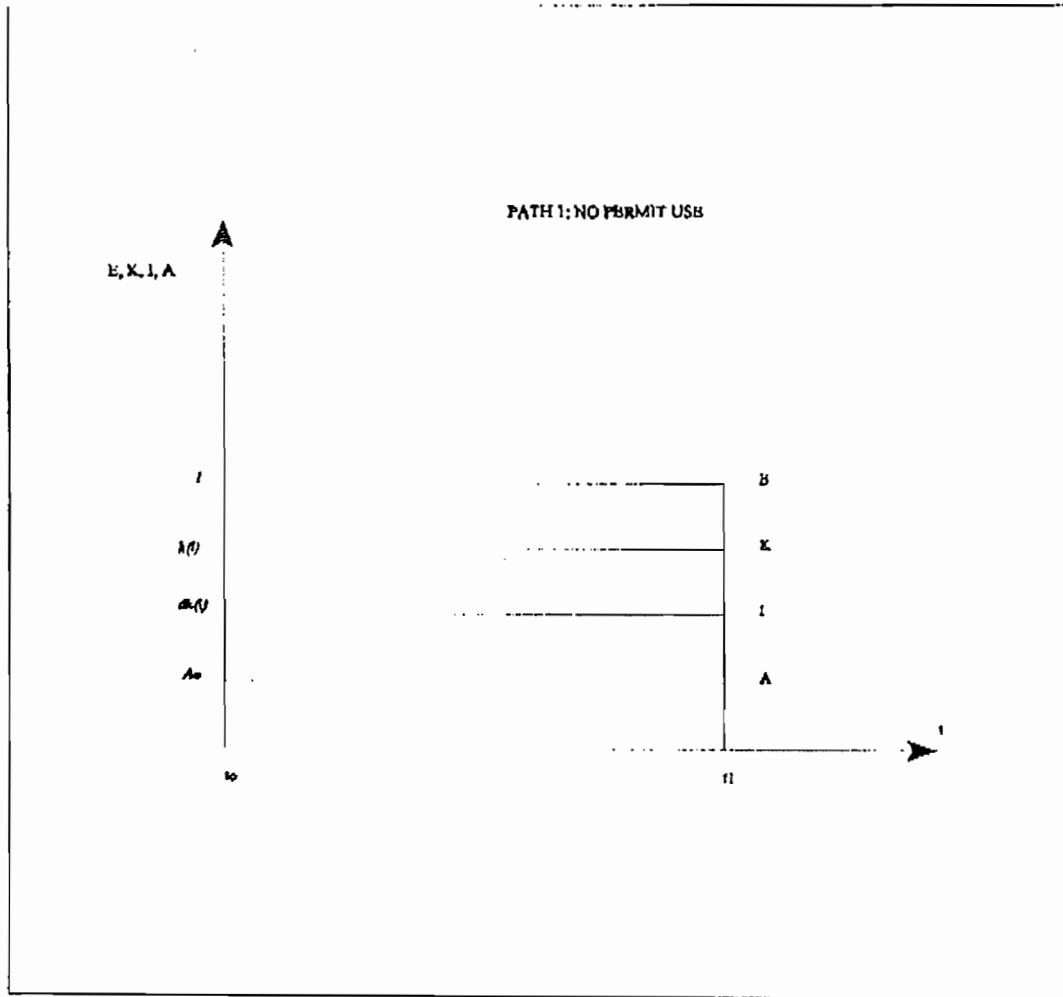


Figure 1. No permit use.

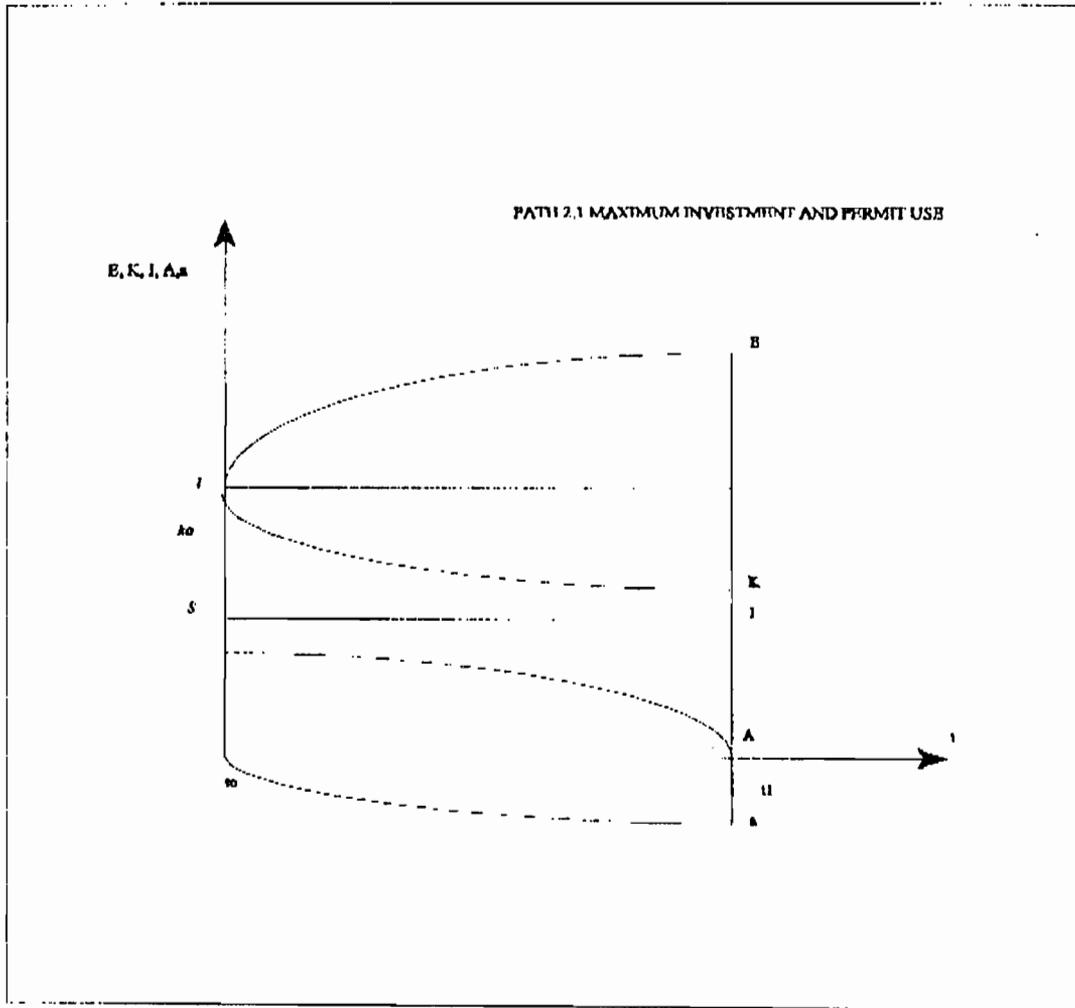


Figure 2. Maximum investment with permit use.

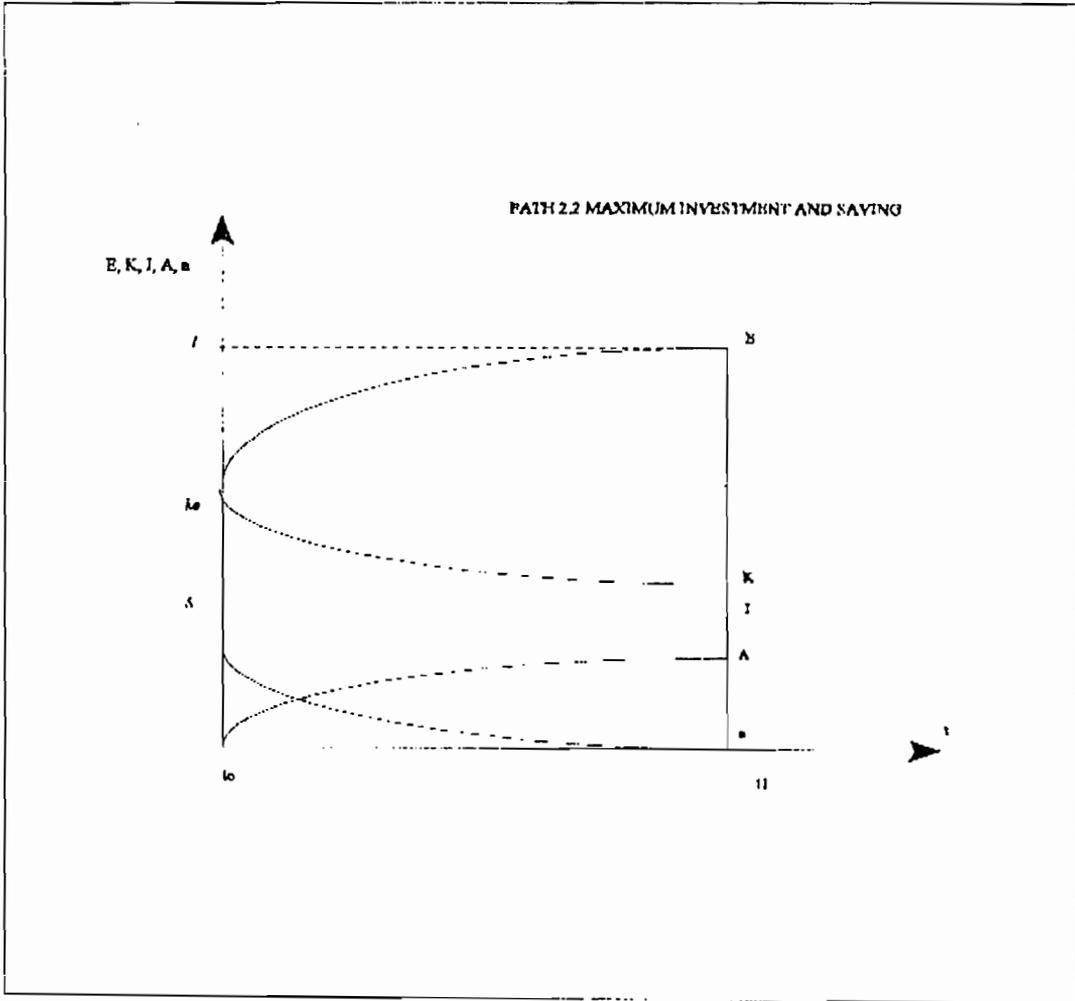


Figure 3. Maximum investment with permit use.

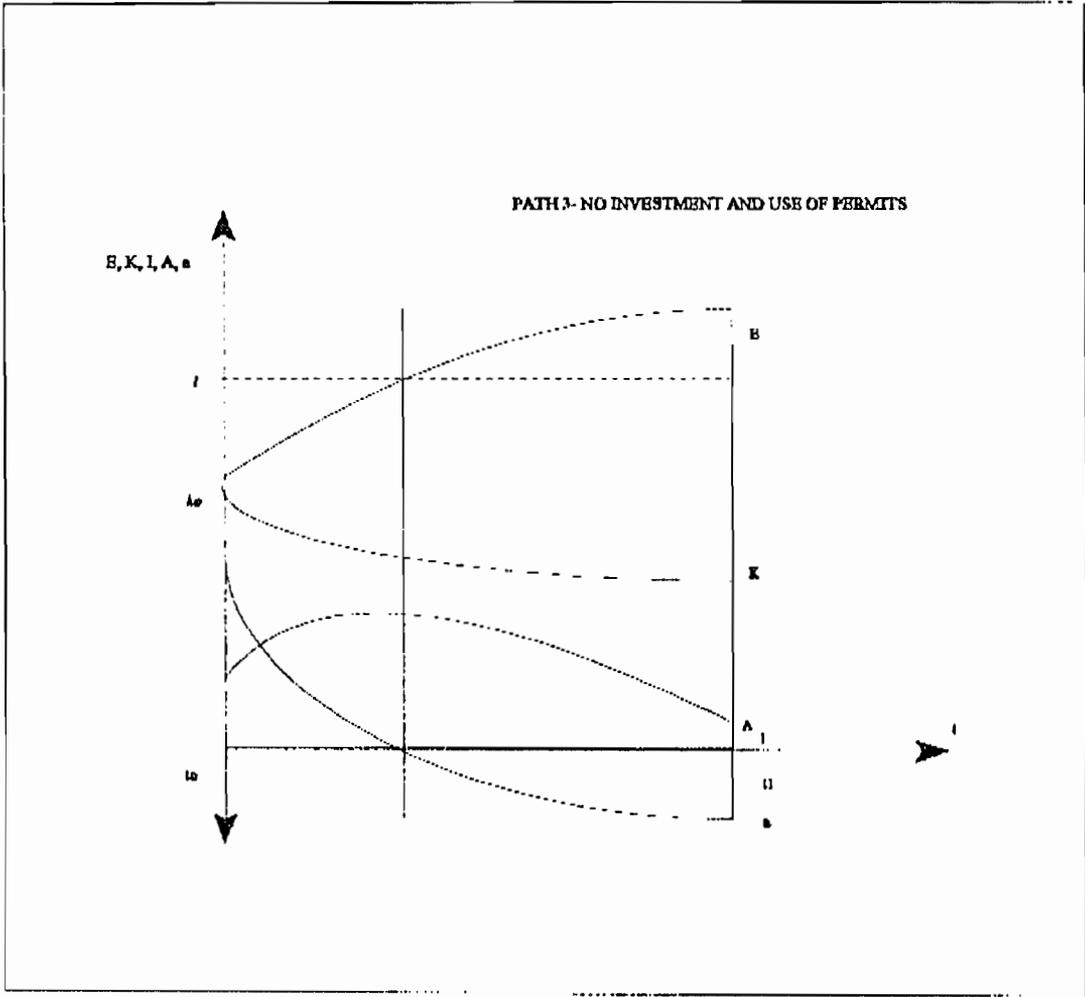


Figure 4. No investment with permit use.

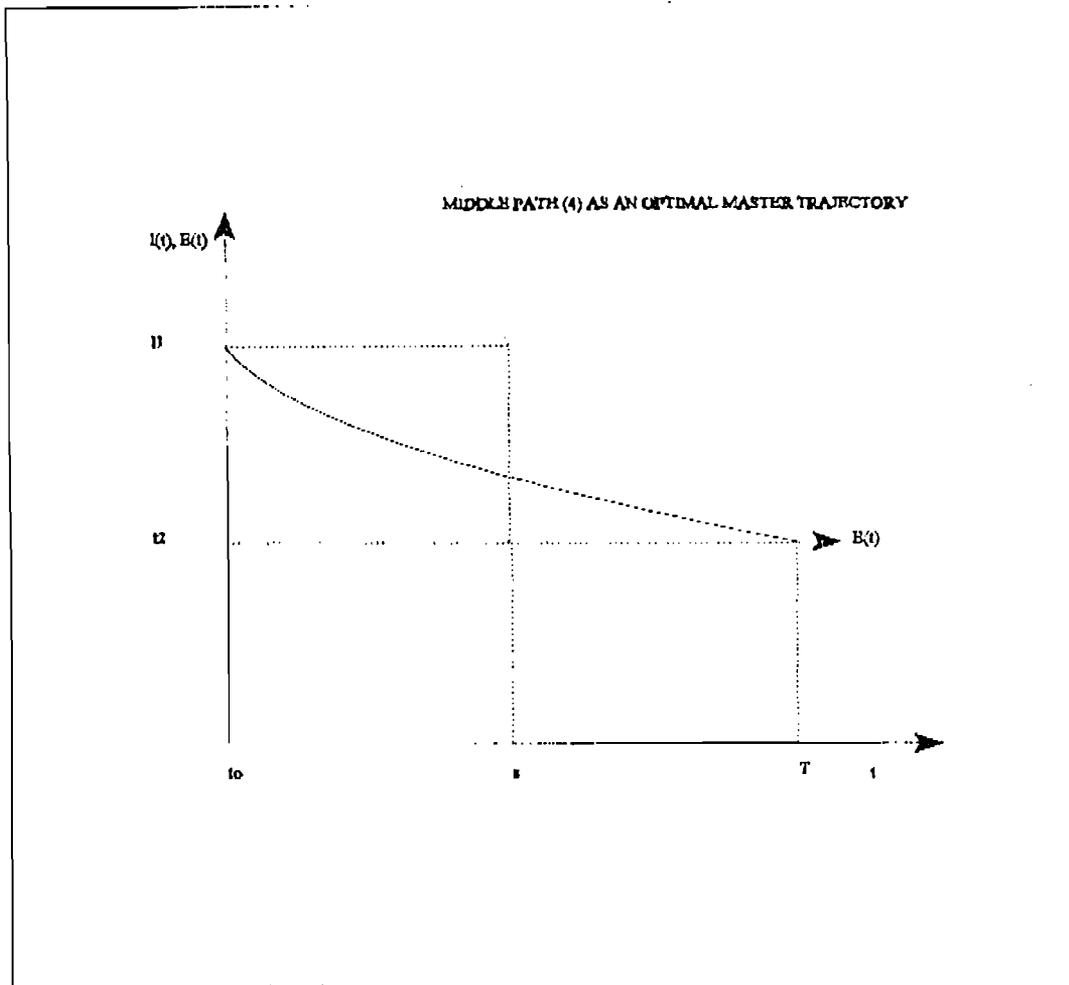


Figure 5. Middle path as an optimal trajectory.

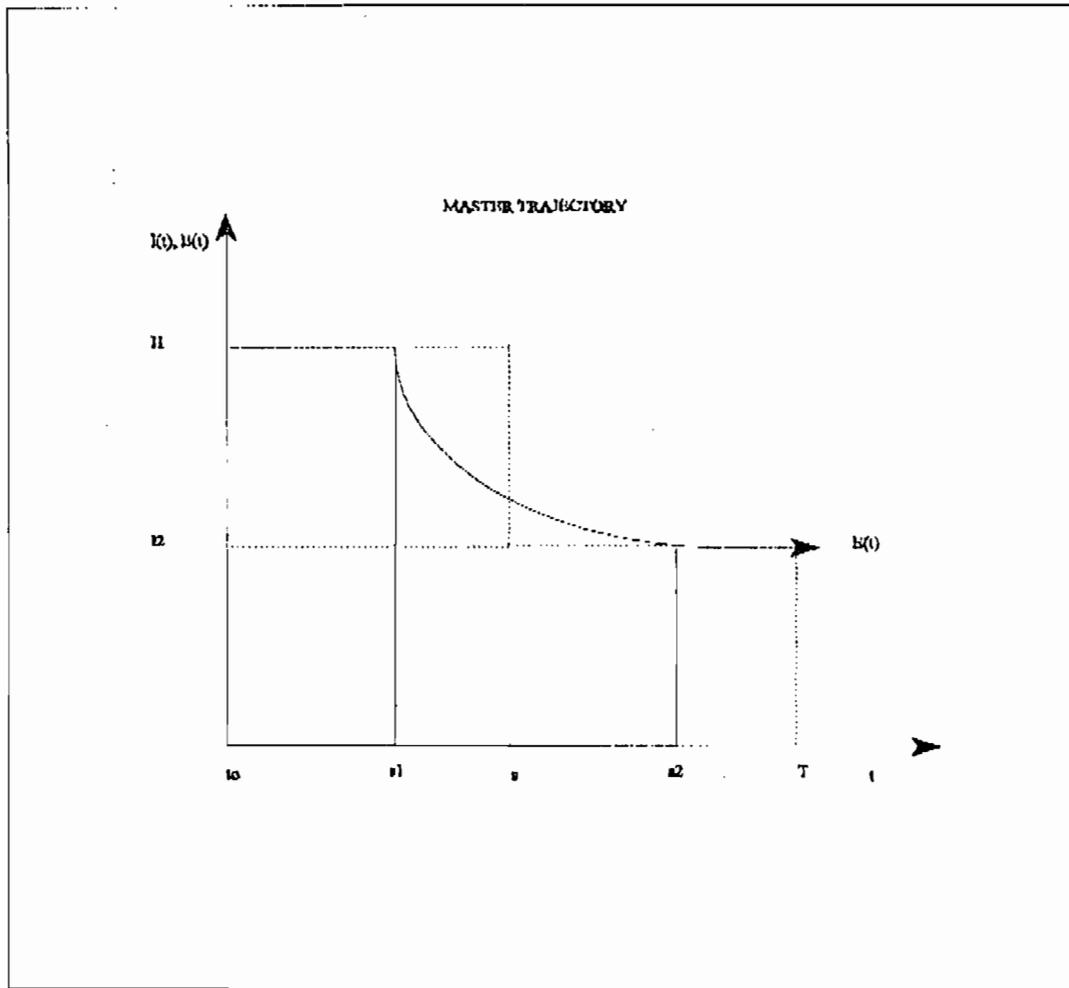


Figure 6. Optimal trajectory: path 1, path 4 and path 1.

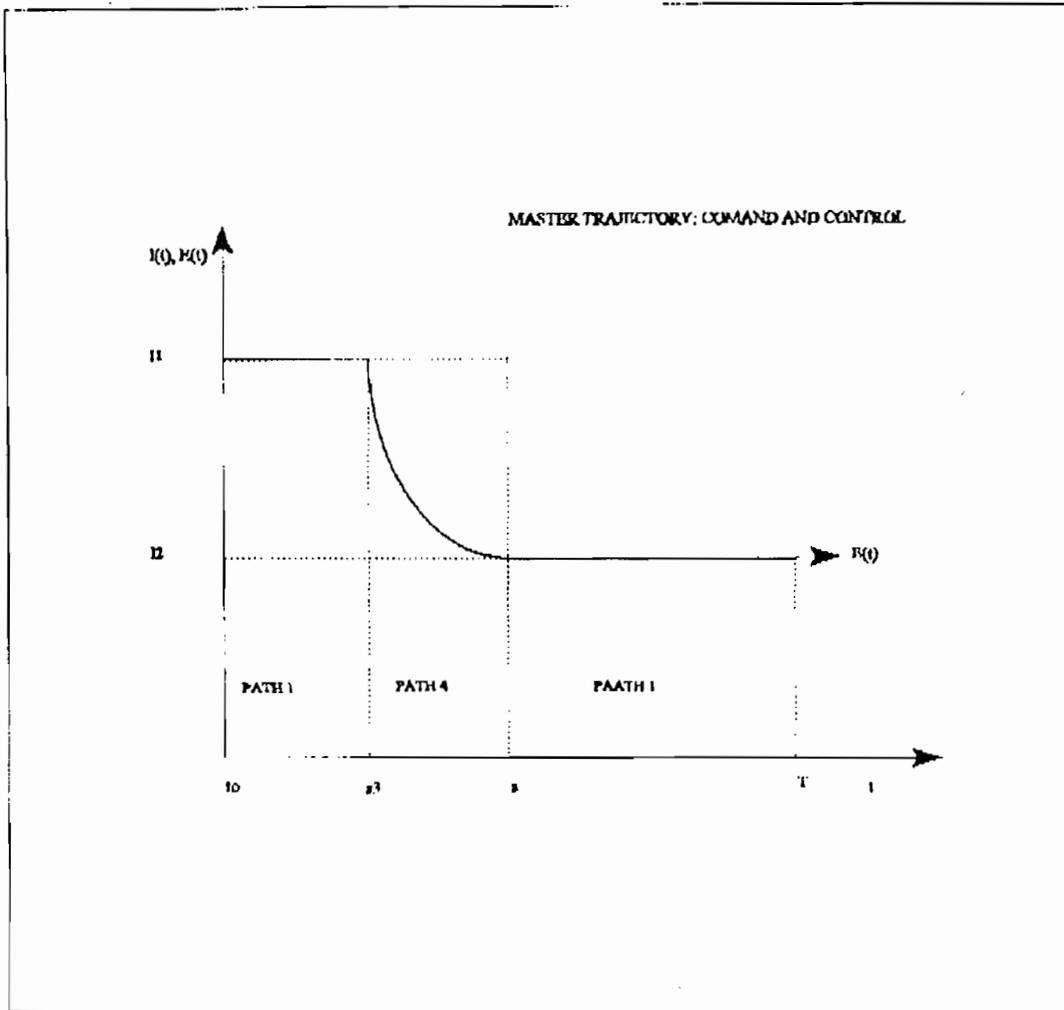


Figure 7. Master trajectory under command-and-control.

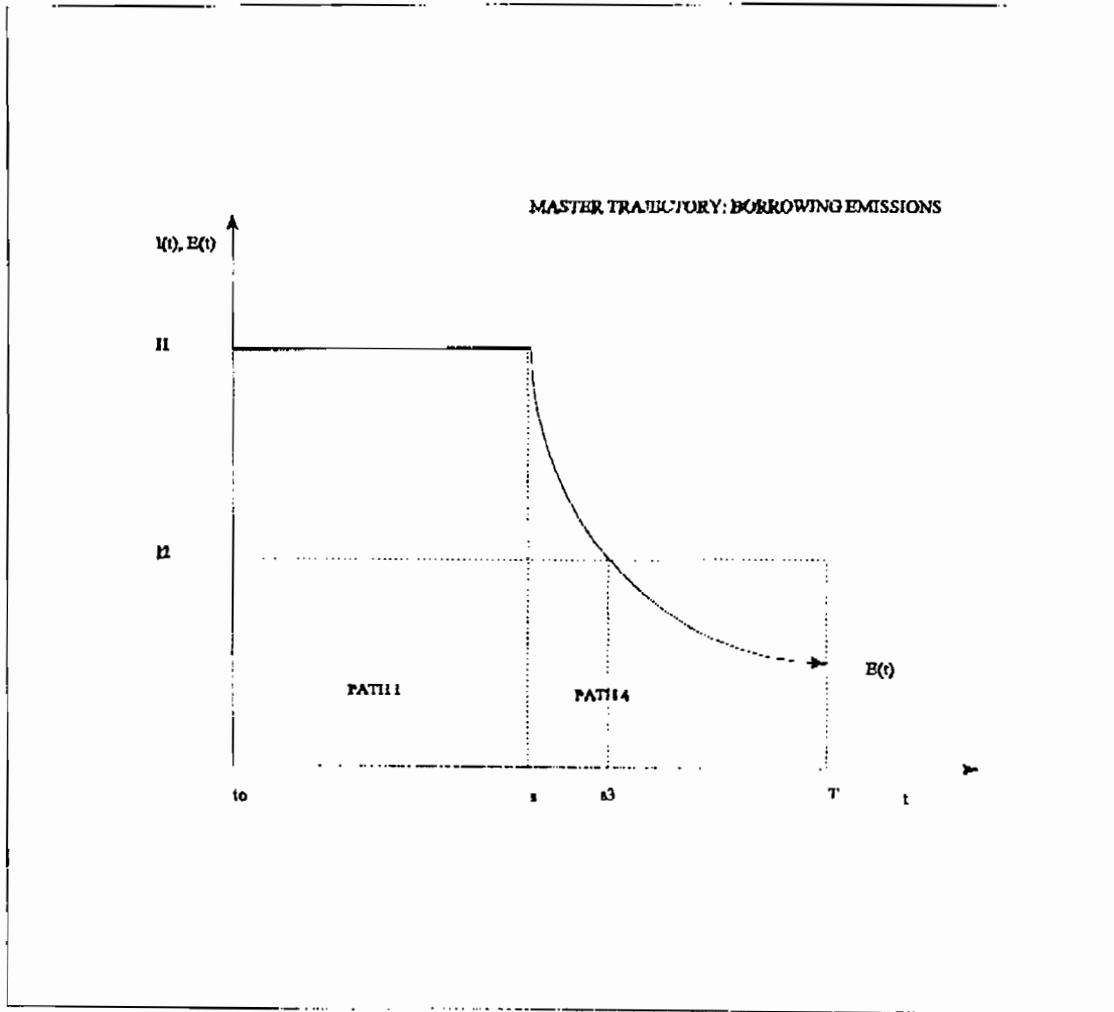


Figure 8. Master trajectory under borrowing policy.