The miracle as a randomization device☆
A lesson from Richard Wagner's romantic opera *Tannhäuser und der Sängerkrieg auf Wartburg*

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ABSTRACT

In this paper we examine the strategic savviness of the medieval church as portrayed in Richard Wagner's *Tannhäuser*. We show that the church employed an optimal randomization strategy based on arguments of dominance or trembling-hand perfection. Particular attention is paid to the employed randomization device.

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1. Introduction

Crimes are committed because they promise an immediate benefit. The law tries to countervail the criminal's incentives by threatening with deferred punishment and the potential criminal weighs immediate benefits and deferred costs when deciding about whether or not to commit a crime. A crucial role in this decision is assumed by the probability of apprehension. This is different for the sinner.

The (Christian) sinner knows that a punishment is waiting for him *for sure.* He will have to endure eternal sufferings in hell and for normal discount rates this provides strong incentives for good

behavior. However, there is one way out. The sinner can repent and, if met by a forgiving priest, be granted absolution.

Obviously, both enforcement agencies—state and church—want to deter aberrant behavior but the deterrent mechanisms are subtly different for the two. While the state's representatives have to take action to punish, the church's have to take action to forgive. This reflects the difference between an ignorant state (that does not observe the crime and has to exert effort to prosecute) and an all-knowing god (who does not overlook even the slightest misdemeanor). The defaults are exactly opposite. If nothing happens, the criminal goes free, and the sinner is punished. Hence, deterrence is easier to achieve for the church and one wonders why it would establish an institution (absolution) whose purpose it is to weaken default deterrence?

The answer is easily obtained and mirrors arguments against the severest punishment offered by the state, the death penalty (the materialist's equivalent to eternal condemnation). Sinners and criminals alike who know that they will suffer the worst imaginable punishment in any case have no incentive to return to a path of decency. There is nothing more dangerous than a first-time murderer who knows that he will face the gallows if caught. And while this is

particularly bad news for police officers trying to apprehend the criminal, a condemned sinner is obviously bad news for everyone who surrounds him.[1] Hence, both state and church also want to provide incentives to those who have fallen from grace to behave well again. This implies that there are two constraints that define optimal deterrence. Ex ante, an optimal punishment has to be severe enough to deter the crime or sin. Ex post, there must be a "way out" for those who "cooperate with the authorities."[2] In game theoretic terms the second constraint can be derived from dominance arguments or the application of trembling-hand perfection: even with seemingly perfect deterrence, bad things can happen, which requires optimality in *all* information sets.

While the state may have variables to play with to meet both constraints (because both, prosecution and punishment technologies, can be incrementally adjusted), we argue that the church has only one option to balance the two goals: randomization. If absolution is always granted there is no deterrence. And if it is never granted there is no incentive for the sinner to repent. Thus, the choice whether or not to grant absolution must be random.[3]

This point is nicely illustrated in Richard Wagner's *Tannhäuser*.[4] The example is particularly intriguing for the randomization device employed—a miracle.

2. The story and the model

After being disenfranchised from his friends at the Wartburg, Tannhäuser, a medieval Minnesinger, visits the Venusberg, a place governed by the eponymous chieftain of sexual pleasure. Tannhäuser has a jolly good time for a while but then gets bored and leaves. Soon after being back in the green fields he happens to meet his old friends who, not knowing of his sin, invite him back to the Wartburg. Initially hesitating, Tannhäuser decides to join them once reminded of his former love, the immaculate Elisabeth. He is greeted enthusiastically (not least by Elisabeth herself who has missed him badly), and a song contest ensues that is to decide who will get to marry Elisabeth. For Tannhäuser, who is better equipped than all his competitors to win the contest, this is decision time. He can aggravate his sins by winning the contest and marrying Elisabeth without prior absolution, or he can confess.

Given that he is a believer, the implications of not confessing are obvious and dismal: the unrepentant sinner will eternally suffer in hell. But what are the consequences of confession? The story has him walking to Rome to confess to the Pope. But before telling the end of the story let us try to model the problem. Employing the most basic structure we can model the game between Tannhäuser and the Pope like this:

Stage 1 Tannhäuser decides about whether to sin or not. Sinning— visiting the Venusberg—promises substantial immediate gratification.[5]
Stage 2 (that is only reached if Tannhäuser has sinned in Stage 1): Tannhäuser has to decide whether to confess or not. Deciding not to confess will bring him on a straight path to hell.

Stage 3 (that is only reached if Tannhäuser has confessed in Stage 2): The church decides about whether to grant absolution or not.

Once Stage 3 is reached the church should always prefer granting absolution to not to—simply because this ensures that the sinner has an incentive not to continue with his aberrant lifestyle. Hence, without commitment there is a unique subgame perfect equilibrium outcome. Anticipating that he will be granted absolution Tannhäuser will decide to go to the Venusberg for a while and then repent (rationally expecting absolution later on). This gives him both, the immediate pleasure and a stab at eternal life in heaven. Consequently, the church has to *commit* itself in order to achieve deterrence and given the church's rigid structures it seems reasonable that it can indeed commit to an absolution strategy for Stage 3. Essentially, this means adding a Stage 0 to the game and dropping Stage 3.

Stage 0 The church commits itself to an absolution strategy, i.e., it chooses a probability $p \in [0,1]$ with which it grants absolution to a sinner who confesses in Stage 2.[6]

What are Tannhäuser's payoffs in this game? There are four possible outcomes:

1. He does not sin, a boring but safe outcome that gives him a payoff that we normalize to zero.
2. He decides to sin, gains the immediate pleasure in the Venusberg, repents and is granted absolution—clearly, the best possible outcome that gives him a strictly positive payoff we denote by b (>0).
3. He gains the immediate pleasure, decides not to repent and suffers in hell; a bad outcome that we normalize to -1.
4. He gains the immediate pleasure, repents, but is not granted absolution; an outcome even worse than the last because he has to bear the costs of atonement without getting any benefit. We denote the payoff in this case by $-1-c$ with $c>0$ denoting the costs of atonement.

Assuming that eternal pains in hell are comparatively large we take it for granted that both b and c are comparatively small. The church can now analyze Tannhäuser's decision problem and then decide on an optimal absolution strategy. From our previous discussion it is clear that, first of all, the church wants to induce incentives for Tannhäuser not to sin. This obviously requires that absolution is not granted too easily and could, in fact, be achieved by never granting absolution. More generally, however, it simply imposes a constraint on the *probability* with which absolution is granted. Denoting this probability by p we can state the first constraint on the church's optimal strategy as

$$pb + (1-p)(-1-c) < 0 \Leftrightarrow p < \frac{1+c}{1+b+c}. \tag{1}$$

As long as this constraint is met, Tannhäuser's expected utility from sinning is strictly negative and he will decide not to go the Venusberg— the first best outcome for the church (whose payoffs we need not model explicitly as only their ordinal structure matters for our main point). Intuitively, the constraint on p gets tougher the more pleasurable the sin (the higher b) and the smaller the costs of atonement (the smaller c). The default, $p=0$, always meets the constraint and would be a good solution if the church could trust on Tannhäuser not making any mistakes. However, with the slightest "trembles", i.e., with the slightest risk that Tannhäuser sins nevertheless, the church wants him to repent. This imposes a second constraint:

$$pb + (1-p)(-1-c) > -1 \Leftrightarrow p > \frac{c}{1+b+c}. \tag{2}$$

In words, the probability of absolution has to be big enough to make confession worthwhile.

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[1] Similar perverse incentives are created by debt overhang, see, for example, Krugman (1989).
[2] A worldly institution that offers rebates to "repentent" criminals is that of plea bargaining (see, for example, Reinganum, 1988).
[3] Notice that the church cannot offer a "reduced" punishment for repentant sinners. It has only one "big" punishment at its disposal—hell—and this necessitates randomization. Dante's *Divina Commedia*, of course, knows different circles of hell, but for each sin, there is exactly one pre-specified circle and again there are no rebates. A couple of hundred years after Tannhäuser's death the Catholic church found, of course, another way to fine tune its deterrence mechanisms—By introducing purgatory (first defined in the Council of Trent, 1545–1563).
[4] For other papers that study literary sources with game theoretic tools, see Brams (1994) and the literature cited therein.
[5] The music as well as Wagner's stage directions leaves very little doubt about how pleasurable a stay in the Venusberg is.

[6] Notice that any such p is a *pure* strategy, i.e., the choice of a particular p means that the church will randomize with that probability *for sure*.

Taking the two constraints together, we get

$$\frac{c}{1+b+c} < p < \frac{1+c}{1+b+c}. \tag{3}$$

Thus, there is an entire range of strategies fulfilling the two constraints, all of which involve *randomization*. Assuming that *b* and *c* are fairly small, the range is rather large. For a derivation of the *optimal p* we would need further assumptions. Heuristically, we might guess that the church might feel more comfortable to make sure that its first-order target (to deter Tannhäuser from sinning) is achieved. Hence, it might wish to choose a rather small, albeit positive *p*.

In Wagner's opera we observe the use of such a small *p*. To see this, let us now tell the rest of Tannhäuser's story.

He walks to Rome, always seeking out the most stony paths and avoiding the shelter of the shadows, as he wants to make sure that the Pope takes his atonement seriously. Alas, it is to no avail. When the Pope hears that Tannhäuser has been to the Venusberg he shows his most unforgiving side. As Tannhäuser reports:

"And he whom I so begged began: − / 'If you have enjoyed such sinful delights / and enflamed your passions at the fires of hell, / if you have sojourned in the Venusberg, / then, now from henceforth, you are eternally damned!' /"

This sounds like *p=0* but the Pope continues.

"As this staff in my hand / no longer bedecks itself in fresh green, / so from the burning brands of hell / deliverance can never blossom for you!"

There is still hope for Tannhäuser. The Pope's staff—a piece of dead wood—has to blossom again. If it does, so the implicit ruling, Tannhäuser will be pardoned after all.

Of course, the chances of this happening are slim. But, as believers know, miracles can and do happen, so there is no doubt that *p*>0. And, of course, we might already anticipate how the story ends—after all, what would be the point of elaborating on this sophisticated scheme if it doesn't come to effect? And so it does. With a little help of Elisabeth (who dies in grief for Tannhäuser) the miracle happens: The Pope's staff blooms again.

"The salvation of grace is the penitent's reward, / now he attains the peace of the blessed!"

Tannhäuser dies and goes straight to heaven—a payoff of +*b* after all.

3. Conclusion

In his famous book, Schelling (1960) discussed various reasons for why agents might want to employ randomization when it comes to threats and promises, in particular, reducing expected costs of threats.[7] The reason we discuss here, to mitigate the effects of deterrence and offer the one who has fallen a chance to return on a path of doing good, is not among them. In fact, we were not able to find any formalized argument of the type exemplified here even though it is derived easily. It simply follows from invoking trembling-hand perfection (or requiring dominance) and the observation that randomization convexifies the church's set of possible punishments. However, the implementation of the optimal deterrence strategy requires a credible randomization device. In Wagner's opera we see a very effective one—the miracle.[8]

References

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[7] Cost reduction was probably also the logic behind the idea of decimation. a punishment strategy used in the Roman army that involved randomization. Those selected for punishment were divided into groups of ten; each group cast lots, and the soldier on whom the lot fell was executed by his nine comrades.

[8] Notice that "staff miracles" were more frequently referred to in the medieval literature. In fact, there are several other examples where the church explicitly demanded a miracle before granting salvation. Moser (1977) tells two intriguing stories in one of which the required miracle involves a black sheep that has to become white for salvation to be granted. In a second story a new-born child has to demand to be baptized by the sinner.