Interconnection Incentives of a Large Network Facing Multiple Rivals

David A. Malueg

Marius Schwartz

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Georgetown University

Department of Economics

580 Intercultural Center

Washington, DC   20057

Tel: (202) 687–5830

Fax: (202) 687–6102

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Interconnection Incentives of a Large Network Facing Multiple Rivals*

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Abstract

This paper extends Cremer, Rey and Tirole's analysis of whether a firm with the most installed-base customers, in a market exhibiting network externalities, gains by degrading interconnection with rivals that compete with it for new customers. We allow any number of rivals and consider both tipping equilibria and interior equlibria. Degrading interconnection can yield tipping *away* from the largest network even if its installed-base share exceeds one half. For all parameter values (including those that admit interior equilibria), a share above one half is necessary but not sufficient to ensure degradation is profitable. Greater scope for market expansion—a lower marginal cost or smaller installed-base relative to potential additional demand—makes profitable degradation less likely.

JEL: L13, L41, L86, L96

Keywords: Interconnection, Network Externalities, Exclusion

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† Department of Economics and the A.B. Freeman School of Business, Tulane University, New Orleans, LA 70118; email: <david.malueg@tulane.edu>.

‡ Department of Economics, Georgetown University, Washington, DC 20057; email: <schwarm2@georgetown.edu>.

I. Introduction

In telephony, the Internet, and other markets exhibiting positive network effects—the value of the service to any user rises with the total number of compatible users—some cooperation among competing providers is needed to achieve good interconnection so that customers may enjoy fuller network benefits. However, a firm with a sufficiently large share of customers may gain by restricting interconnection with its smaller competitors in order to attain a relative quality advantage over them. This was a central concern, for example, in two prominent mergers involving so-called "Internet Backbone" providers—firms that supply high-capacity links with international reach to smaller Internet service providers and large business customers.[1] European and US competition authorities feared that the merged entity would command such a large share of Internet customers or traffic that it might gain by degrading—or failing to enhance—interconnection with smaller rivals. Concern with interconnection incentives of competing networks is reflected also in the US Telecommunications Act of 1996 that requires all telephone companies to interconnect.

An influential analysis of interconnection incentives is that of Cremer, Rey, and Tirole (2000; hereinafter CRT),[2] extending the framework of Katz and Shapiro (1985). Networks have locked-in installed-base customers and compete for new customers in a Cournot fashion.[3] CRT implicitly assume that networks derive profits only from final customers, not from above-cost sales of access to other networks. This assumption is appropriate, for instance, if regulation holds access prices close to marginal cost (or, given balanced inbound and outbound traffic, allows positive but symmetric margins). Regulators, however, often find it harder to prevent restrictions of rivals' network access through non-price methods—what Beard, Kaserman, and Mayo (2001) term "sabotage"—because of the many ways in which the real cost of access can be raised or its quality lowered. Degrading interconnection then poses the following tradeoff to the largest network: its own "quality" suffers and thus the absolute attractiveness of its service declines, but its attractiveness rises *relative* to the smaller rival, whose quality has suffered even more. When the largest firm faces a single rival (duopoly case), CRT provide a sufficient condition on the size of the installed-base advantage, as a function of the other model parameters, needed to make degradation profitable. Beyond duopoly, CRT consider an example in which the largest firm

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[1] The MCI/WorldCom merger was concluded in 1998 subject to divestiture of MCI's Internet operations, while MCI WorldCom/Sprint was abandoned in 2000 under pressure from the European Commission and U.S. Department of Justice. See European Commission (1998; 2000), US DOJ (2000), and WorldCom and Sprint (2000). For useful background on the Internet, see Cave and Mason (2001).

[2] The Appendix is available in the working paper by the same title issued in May 1999.

[3] Ennis (2002) considers the case where all customers are locked in but derive value from links to other networks, and where a large networks bargains with smaller ones over payment for interconnection. He finds that the direction of payment depends on whether the value of connectivity is linear, convex or concave in the total number of users.

commands half the installed base and faces two symmetric smaller rivals. This fifty-percent share is not enough for the large firm to profitably degrade interconnection with both rivals, but can make it profitable to pursue "targeted degradation" against only one rival.

Focusing on global degradation, whereby the initially largest network chooses the same interconnection policy towards all rivals,[4] we extend CRT's analysis in two directions. First, the largest network, firm 1, may face any number n of smaller rivals, themselves interconnected. Second, we analyze the possibility that degradation could lead to tipping—all new customers gravitating to a single network—a central policy concern in network industries. These extensions are complementary: allowing an arbitrary number of rivals reveals tipping possibilities that do not arise under duopoly. If (and only if) firm 1 faces two or more rivals, then degradation could lead to a unique equilibrium with tipping *away* from firm 1, even if its installed-base share exceeds one half. The parameter region that admits this possibility expands with n, while the region that admits tipping to firm 1 as the unique degradation equilibrium contracts with n. The driving force is that intra-network competition among the interconnected smaller rivals gives their network a strategic advantage in its competition with firm 1's network for new customers, by enabling their network to commit to a greater expansion of output and thus a greater increase in quality. This competition-based advantage can, depending on the model's other parameters such as the scope for market expansion, outweigh firm 1's initial advantage in the installed base.

In parameter regions where degradation would not lead to a unique equilibrium involving tipping (to firm 1 or away from it), it could lead either to a unique interior equilibrium in which all firms attract new customers, or to any one of three equilibria: the interior equilibrium, tipping to 1, or tipping from 1. In the multiple-equilibria case, the outcome depends on consumers' expectations. We characterize the profitability of degradation in each of the cases and summarize the effect of various parameters on the overall likelihood that—looking across all the other parameters—degradation would be profitable. As expected, increasing firm 1's initial market share is conducive to degradation. Increasing the strength of network effects or the number of rivals has an ambiguous effect (the latter, in regions where degradation would lead to the interior equilibrium, rather than tipping as discussed earlier). By contrast, increasing the scope of market expansion relative to the initial installed base works against profitable degradation, suggesting that degradation may be a greater concern in mature than in expanding markets.

The remainder of the paper is organized as follows. Section II presents the model, which modifies CRT's analysis by allowing any number of rivals and, as in Katz and Shapiro (1985), a potential pool of customers large enough that it is not exhausted in any equilibrium. Section III analyzes tipping equilibria. Section IV considers also the interior equilibrium, summarizes the profitability of degradation in various regions, and reports the relevant comparative statics.

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[4] Malueg and Schwartz (2001) analyze the parameter ranges that can support CRT's targeted degradation example.

II. The Model and The Interconnection Benchmark

The model developed in this section is a straightforward extension of CRT's duopoly framework: the largest firm faces *n* smaller rivals (*n* = 1 is CRT's duopoly case). Unless otherwise stated, all other assumptions and parameters mentioned below track CRT's model.

A. The Model

1. Installed Bases and Interconnection Quality

All firms have constant marginal cost *c* of serving additional customers. Firms differ only in the sizes of their installed bases of existing subscribers. These subscribers are locked in: pricing to them is determined by previous contracts, and they will not switch to other networks.[5] The only relevance of installed bases in the model is to potentially affect competition for new customers. The total installed base in the market is equal to β (> 0). Firm 1 has the largest installed base, of size β_1. The remainder, $\beta - \beta_1$, is divided among the rivals, with the size of firm *i*'s installed base denoted by β_i, *i* = 2, 3, . . . , *n* + 1. Throughout this paper, a firm's "market share" refers to its share of the installed base.

Firms compete for new customers in the next period. The number of subscribers added by firm *i* is denoted by q_i. Any customer values equally communication with any other, regardless of their choices of network. Thus, if (and only if) interconnection between two firms is inferior to the quality of connection between subscribers of the same firm, then the firm with the larger installed base will have an advantage in competing for new customers, because its network gives access to a larger number of existing users.

The interconnection quality between two firms is denoted by θ, where $\theta \in \{0, 1\}$. A value of $\theta = 1$ represents perfect interconnection, i.e., the same quality as between subscribers on the same firm's network, while $\theta = 0$ represents perfect degradation.[6] CRT focus mainly on the case in which all interconnection qualities entail the same cost, which is normalized to zero. For this case, they find that firms choose only perfect interconnection or perfect degradation, not intermediate values of θ, and that firms with the same-size installed bases prefer perfect interconnection. For simplicity, we follow this cost assumption and, in light of CRT's findings, we model the interconnection among firms 2, 3, . . . , *n* + 1 as perfect. Furthermore, we consider

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[5] Foros, Kind, and Sand (2002) extend CRT's model by allowing the price paid by a firm's installed-base customers to increase linearly in the number of customers that can be reached through that firm. They find that this modification can decrease or increase the larger firm's incentive to degrade interconnection with its rival.

[6] Values of θ strictly between 0 and 1 would denote imperfect interconnection—subscribers of firm 1 can potentially reach those of the other network, but not as well as they can reach other subscribers on network 1 (e.g., the connection between networks is less reliable or imposes longer delay).

only *global* interconnection policies by firm 1—firm 1 establishes the same interconnection quality with each of the rivals—and ask whether firm 1 will choose $\theta = 0$ or 1.

2. Demand by Potential New Customers

a. Benefits from Subscribing to a Network

If a new subscriber whose type is τ joins network i, that subscriber obtains gross benefit of $\tau + s_i$, where s_i depends on the size of network i (explained shortly) and τ can be viewed as the value of basic access. New subscribers differ only in their values of τ. Following the original approach of Katz and Shapiro (1985), on which CRT's framework builds, we assume that consumers' connection-valuation parameter τ is uniformly distributed over $(-\infty, 1]$, (with density equal to 1 over this interval).[7, 8] Rather than literally envisioning the potential demand as infinite, we interpret this formulation as capturing the realistic feature that demand dispersion is so large that there will always be some unserved consumers. The term s_i is given by

$$s_i = vL_i,$$

where $v > 0$ is a common taste parameter measuring the intensity of preferences for connectivity (hence, the strength of network externalities), and L_i denotes the size of network i, that is, the total number of links offered by network i next period.

The number L_i includes the subscribers (installed-base and new ones) on network i as well as on other networks with which i interconnects. Because we have assumed that all smaller firms will be interconnected, a subscriber of firm i, $i \neq 1$, can reach the subscribers of all the small firms. If the quality of interconnection between firm i and firm 1 is θ, the number of links offered by i is therefore given by

(1) $$L_i = (\beta - \beta_1) + \sum_{j=2}^{n+1} q_j + \theta(\beta_1 + q_1).$$

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[7] Exact agreement with Katz and Shapiro's setting would model parameter τ as uniformly distributed over $(-\infty, A]$, for $A > 0$. Our current results would carry through after slight reinterpretation. For example, τ would be replaced by τ/A and c would be replaced by c/A. It is without loss of generality that we assume $A = 1$.

[8] Katz and Shapiro (1985) specifically introduce this "unbounded below" assumption to eliminate problems with solutions in which all potential consumers are served: "We assume that the support of r has no finite lower limit in order to avoid having to consider corner solutions, where all consumers enter the market" (ft. 2; their r is equivalent to our τ). CRT assume that τ is uniformly distributed over [0, 1]; this finite pool of potential customers leads to the possibility of corner solutions for some combinations of parameters. Corner solutions raise issues of multiple equilibria (Malueg and Schwartz, 2002, fn. 8). Malueg and Schwartz (2002) retain CRT's assumption, but take the perspective that the plausible market outcome is one in which not all potential customers will subscribe; this constraint, that the total number of new subscribers be less than the potential pool, yields a restriction on plausible combinations of the model's parameters.

Thus, if firm i is connected to firm 1 ($\theta = 1$), then a subscriber to i can reach all of the (next-period) subscribers in the market; but if interconnection with 1 is degraded ($\theta = 0$), then only subscribers to firm 1's rivals are reachable from firm i. Similarly, the number of links offered by firm 1 is given by

(2) $$L_1 = (\beta_1 + q_1) + \theta(\beta - \beta_1 + \sum_{j=2}^{n+1} q_j).$$

b. Individual Subscription Decisions and Inverse Demands

The *net* benefit to customer of type τ from subscribing to firm i at price p_i is given by

(3) $$\tau + s_i - p_i.$$

Regardless of τ, all potential subscribers have the same ranking of various networks' desirability. Therefore, firms attracting new customers must offer the same net benefit,

$$\tau + s_i - p_i = \tau + s_j - p_j,$$

which implies that quality-adjusted prices must be equal:

$$p_i - s_i = p_j - s_j,$$

for all networks i, j that acquire new subscribers. The marginal customer, $\bar{\tau}$, just obtains a net surplus of zero from subscription ($\bar{\tau} = p_i - s_i$). All customers with values of τ greater than $\bar{\tau}$ would subscribe to one of the networks, implying a total number of new subscribers equal to $1 - \bar{\tau}$. Therefore, market clearing requires that, for each firm i, we have

(4) $$\sum_{j=1}^{n+1} q_j = 1 - (p_i - s_i),$$

so that, using (2) with $s_1 = \nu L_1$ in (4), the resulting inverse demand facing firm 1 is

(5) $$\begin{aligned} p_1 &= 1 + s_1 - q_1 - \sum_{j=2}^{n+1} q_j \\ &= 1 + \nu(\beta_1 + \theta(\beta - \beta_1)) - (1-\nu)q_1 - (1-\theta\nu)\sum_{j=2}^{n+1} q_j; \end{aligned}$$

similarly, the inverse demand facing any smaller firm i is

(6) $$\begin{aligned} p_i &= 1 + s_i - q_i - \sum_{j \neq i} q_j \\ &= 1 + \nu((\beta - \beta_1) + \theta\beta_1) - (1-\nu)\left(q_i + \sum_{j \neq 1, i} q_j\right) - (1-\theta\nu)q_1, \end{aligned}$$

for $i \geq 2$.

We restrict $v < 1$ to guarantee that demand is downward-sloping (cf., (5) and (6)).[9] Also, we restrict $c \leq 1$ to ensure that some new customers are added under perfect interconnection, no matter how small the values of v or β: among the potential new subscribers, the highest willingness to pay under perfect interconnection is at least $1 + v\beta$, which is sure to exceed marginal cost if and only if $c \leq 1$.

3. Interconnection Choices and Competition for New Customers

Decisions take place in two stages. First, firm 1 chooses whether to interconnect with the rival network of firms 2, . . . , n + 1. Observing this choice, firms then compete in a Cournot fashion for new subscribers: each firm chooses the number of customers it wishes to add, and firms' prices adjust to yield the quality-adjusted price that clears the market, given the expected numbers of new subscribers to each firm. Firm 1 chooses its interconnection quality to maximize its profit in the expected Cournot competition.

The Cournot profit of any firm i (ignoring the installed-base customers) is simply $\pi_i = (p_i - c)q_i$, where the quality choice θ made by firm 1 enters in the inverse demand function p_i discussed earlier. In choosing interconnection quality, firm 1 faces a tradeoff if its installed-base share exceeds one half: decreasing θ makes all networks less attractive, thus drawing fewer new customers in total (market-contraction effect); but it also gives firm 1 an initial quality advantage due to its largest installed base (quality-differentiation effect).

B. Equilibrium Under Interconnection

With interconnection by firm 1, all firms offer identical qualities and their services are perfect substitutes. Using inverse demands given in (5) and (6) with $\theta = 1$, firm i's profit can be expressed as

$$\pi_i = (p_i - c)q_i = \left(1 + v\beta - (1 - v)\left(q_i + \sum_{j \neq i} q_j\right) - c\right) q_i, \tag{7}$$

$i = 1, 2, \ldots, n + 1$. At the Cournot equilibrium, each firm i maximizes its profit given in (7), taking as given the outputs of its rivals. Given the assumptions that $v < 1$ and $c \leq 1$, the Cournot equilibrium under interconnection is unique, with each firm adding an identical number of subscribers equal to

[9] The role of $v < 1$ can be understood as follows. Consider perfect interconnection. Suppose at price p the marginal subscriber has personal connection value equal to τ. If $\Delta\tau$ more subscribers are to be added, then the connection value of the new marginal subscriber must be lower by $\Delta\tau$ (since τ is *uniformly* distributed over [0,1]). The quality of the expanded network, however, rises by $v\Delta\tau$, so that the overall value of subscription to the marginal subscriber (which determines the market price) would fall by just $(1 - v)\Delta\tau$. In order for marginal willingness-to-pay actually to fall as the number of subscribers increases, it is necessary that $v < 1$.

(8) $$q^a = \frac{(1-c)+v\beta}{(n+2)(1-v)}.$$

As expected, the equilibrium number of new subscribers increases with the installed base β and valuation parameter v, as both increase network attractiveness, as well as with the number of rivals n, and decreases with marginal cost c. Degradation is profitable if it yields firm 1 greater profit than would this unique interconnection equilibrium.

III. Possible Equilibria Under Degradation

If firm 1 degrades interconnection with its rivals, several types of Cournot equilibria are possible: a) tipping to firm 1—only it obtains new customers; b) tipping away from firm 1—it obtains no new customers; and c) the interior equilibrium, in which all firms acquire new customers. For some parameter values, there exist multiple equilibria—depending on consumers' expectations, the outcome can be any of the above three equilibria. Figure 1(a) illustrates these possibilities in (m_1, v) space, fixing the other parameters β, c, and n. In region A, the unique equilibrium is the interior equilibrium; in B, it is tipping to 1; and in C, it is tipping from 1. In D, all three equilibria exist. As explained below, in Figure 1(a) β is relatively large, while in Figure 1(b) β is relatively small, in which case region B is empty.

This section derives the boundaries, $\underline{M}_1$ and $\overline{M}_1$, between these four regions and shows in each case how the critical value of m_1 varies with the parameters v, β, c, and n. Considering all regions, Section IV examines when degradation is profitable to firm 1.

A. Interior Equilibrium

If firm 1 degrades its interconnection, then its service is no longer a perfect substitute for that offered by the network formed by the n rivals that are themselves interconnected. In this case, firm 1 chooses its number of new customers, q_1, to maximize its profit

(9) $$\pi_1 = (p_1 - c)q_1 = \left(1 + v\beta_1 - (1-v)q_1 - \sum_{j=2}^{n+1} q_j - c\right) q_1.$$

Among the rivals, any firm i chooses its number of new customers, q_i, to maximize

(10) $$\pi_i = (p_i - c)q_i = \left(1 + v(\beta - \beta_1) - (1-v)\left(q_i + \sum_{j\neq 1,i} q_j\right) - q_1 - c\right) q_i.$$

If the Cournot equilibrium has all firms adding new subscribers, then the equilibrium outputs are

(11) $$q_1^d = \frac{(1-c)[1-(1+n)v] + [n+(n+1)(1-v)]v\beta_1 - nv\beta}{2(n+1)(1-v)^2 - n}$$

for firm 1, and

(12) $$q_i^d = \frac{(1-c)(1-2\nu) \; - \; (3-2\nu)\nu\beta_1 \; + 2(1-\nu)\nu\beta}{2(n+1)(1-\nu)^2 - n}$$

for firm i, $i = 2, 3, \ldots, n + 1$. Note that firm 1's rivals all have the same number of new customers, regardless of their individual installed bases, because the rivals are perfectly interconnected and hence offer identical services.

Equations (11) and (12) provide the unique interior equilibrium as long as these expressions are well-defined, $q_1^d > 0$, and $q_i^d > 0$. The denominators of the expressions in (11) and (12) equal zero at

(13) $$\bar{\nu}(n) \equiv 1 - \sqrt{\frac{n}{2(n+1)}},$$

and they are strictly positive for $\nu < \bar{\nu}$ and strictly negative for $\nu > \bar{\nu}$.[10] Therefore, the interior equilibrium exists in two cases—if the numerators of (11) and (12) are strictly positive for $\nu < \bar{\nu}$ or strictly negative for $\nu > \bar{\nu}$:

Case 1: $\nu < \bar{\nu}(n)$

1(i) $$(1-c)[1-(1+n)\nu] \; + \; [n+(n+1)(1-\nu)]\nu\beta_1 \; - \; n\nu\beta > 0$$

and

1(ii) $$(1-c)(1-2\nu) \; - \; (3-2\nu)\nu\beta_1 \; + 2(1-\nu)\nu\beta > 0;$$

or

Case 2: $\nu > \bar{\nu}(n)$

2(i) $$(1-c)[1-(1+n)\nu] \; + \; [n+(n+1)(1-\nu)]\nu\beta_1 \; - \; n\nu\beta < 0$$

and

2(ii) $$(1-c)(1-2\nu) \; - \; (3-2\nu)\nu\beta_1 \; + 2(1-\nu)\nu\beta < 0.$$

The conditions of Cases 1 and 2 for the existence of the interior equilibrium can be converted to conditions on firm 1's market share, m_1, by setting $\beta_1 = m_1\beta$ in the above expressions:

Case 1: $\nu < \bar{\nu}(n)$

(14) 1(i) $$m_1 > \frac{n}{n+(n+1)(1-\nu)} + \frac{\big((n+1)\nu - 1\big)}{\big(n+(n+1)(1-\nu)\big)\nu\beta}(1-c)$$

and

[10] The critical $\bar{\nu}$ depends only on n and decreases in n, with $\bar{\nu}(1) = 1/2$ and with limit $\lim_{n\to\infty} \bar{\nu}(n) = 1 - \sqrt{1/2} \approx 0.293$.

(15) 1(ii) $$m_1 < \frac{2(1-\nu)}{(3-2\nu)} + \frac{(1-2\nu)}{(3-2\nu)\nu\beta}(1-c);$$

or

Case 2: $\nu > \bar{\nu}(n)$

(16) 2(i) $$m_1 < \frac{n}{n+(n+1)(1-\nu)} + \frac{\left((n+1)\nu-1\right)}{\left(n+(n+1)(1-\nu)\right)\nu\beta}(1-c)$$

and

(17) 2(ii) $$m_1 > \frac{2(1-\nu)}{(3-2\nu)} + \frac{(1-2\nu)}{(3-2\nu)\nu\beta}(1-c).$$

In Section III.B we derive conditions for tipping equilibria, and in Section III.C we compare those conditions to the ones for the interior degradation equilibrium discussed above.

B. Tipping Equilibria

1. Tipping To Firm 1

A tipping equilibrium *to* firm 1 if it degrades interconnection with all rivals is described as follows. Suppose each potential new customer expects that any other customer will also choose firm 1 (or no network at all). Given such expectations, firm 1 adds its monopoly number of new subscribers and accepts the corresponding price; taking as given that firm 1 adds this number of new customers, no rival can profitably attract any new customers, despite the fact that firm 1's price is above marginal cost.[11] We now derive the conditions for such a tipping equilibrium.

If $q_2 = \cdots = q_{n+1} = 0$, then firm 1's optimal output (i.e., number of new customers) is

(18) $$q_1^{Tip} = \frac{(1-c)+\beta_1\nu}{2(1-\nu)}.$$

This is simply firm 1's monopoly output—firm 1's Cournot response to its rivals' total new output of zero. Any rival firm *i* will indeed choose zero output if, given q_1^{Tip} and zero output by the other smaller firms, $p_i \le c$ for any $q_i > 0$, where the function p_i is given by (6). At these candidate equilibrium outputs, q_1^{Tip} and $q_2 = \cdots = q_{n+1} = 0$, we indeed have $p_i \le c$ if and only if

[11] Because of the network externalities, entry by a rival on a small scale would deliver a service of low quality, which would have to be compensated by a prohibitively lower price; entry on a large scale would require a large price cut to achieve the requisite market expansion, again driving price below marginal cost.

$$(19) \qquad 1+(\beta-\beta_1)\nu-\left(\frac{1-c+\beta_1\nu}{2(1-\nu)}\right) \le c.$$

Letting $\beta_1 = m_1\beta$, we transform (19) into a condition expressing the *minimum* market share of firm 1, $\underline{M}_1$, for which there exists a tipping equilibrium to firm 1 under degradation:

$$(20) \qquad m_1 \ge \underline{M}_1(c,\nu,\beta) \equiv \frac{2(1-\nu)}{(3-2\nu)} + \frac{(1-2\nu)}{(3-2\nu)\nu\beta}(1-c).$$

Lemma 1: $\partial \underline{M}_1/\partial\nu < 0$ and $\partial \underline{M}_1/\partial n = 0$. Furthermore, $\underline{M}_1\big|_{\nu=1/2} = 1/2$.

The proofs of Lemma 1 and subsequent results are given in Appendix 1. As recorded in the following remark, Lemma 1 establishes the shape of the $\underline{M}_1$ curve shown in Figure 1.

Remark 1: Fix *c*, *β*, and *n*. In (m_1,ν) space, the graph of $\underline{M}_1$ is strictly decreasing in *v* and passes through the point (m_1,ν) = (1/2, 1/2). In addition, this curve is independent of *n*.

The $\underline{M}_1$ curve is decreasing in *v* because tipping to 1 can occur for either of two reasons: network externalities are sufficiently strong or firm 1's installed-base advantage is sufficiently large. Increasing the strength of network externalities thus reduces the installed-base advantage needed for an equilibrium with tipping to 1. The reason $\underline{M}_1$ is independent of *n* is that 1's monopoly output does not vary with *n*—and the condition for tipping to 1 is that, at 1's monopoly output, the rivals' inverse demand be depressed below marginal cost, *c*, which also does not vary with *n*.

2. Tipping From Firm 1

Next consider a tipping equilibrium away *from* firm 1 to the rivals' network. Suppose each potential new customer expects that no new customer will choose firm 1. Given such expectations of $q_1 = 0$, each of the *n* rivals adds the symmetric Cournot-equilibrium number of new customers

$$(21) \qquad q_i^{Tip} = \frac{(1-c)+(\beta-\beta_1)\nu}{(n+1)(1-\nu)},$$

i = 2, . . . , *n* + 1. The outputs $q_1 = 0$ and $q_i = q_i^{Tip}$ indeed form a tipping equilibrium *from* firm 1 if at these outputs $p_1 \le c$, where firm 1's inverse demand function p_1 is given in (5):

(22) $$1+\beta_1 v - \frac{n\left(1-c+(\beta-\beta_1)v\right)}{(n+1)(1-v)} \le c.$$

Therefore, $q_1 = 0$ and $q_i = q_i^{Tip}$ form a tipping equilibrium from firm 1 if condition (22) is satisfied. Letting $\beta_1 = m_1\beta$, we transform (22) into a condition expressing the *maximum* market share of firm 1, $\overline{M}_1$, for which there exists a tipping equilibrium from firm 1, i.e., tipping is to the rivals:

(23) $$m_1 \le \overline{M}_1(c,v,\beta,n) \equiv \frac{n}{n+(n+1)(1-v)} + \frac{\left((n+1)v-1\right)}{\left(n+(n+1)(1-v)\right)v\beta}(1-c).$$

Lemma 2: $\partial\overline{M}_1/\partial v > 0$ and $\partial\overline{M}_1/\partial n > 0$. Furthermore, $\overline{M}_1\big|_{v=1/(n+1)} = 1/2$.

As recorded in the following remark, Lemma 2 establishes the shape of the $\overline{M}_1$ curve shown in Figure 1.

Remark 2: Fix *c*, *β*, and *n*. In (m_1, v) space, the graph of $\overline{M}_1$ is strictly increasing in *v* and passes through the point (m_1, v) = (1/2, 1/(*n* + 1)). In addition, this curve shifts rightward and downward as *n* increases.

The $\overline{M}_1$ curve is increasing in *v* because tipping from 1 can occur if network externalities are sufficiently strong or firm 1's installed-base advantage is not too large. Thus, the greater is m_1, the greater is the *v* needed for tipping from 1. Furthermore, $\overline{M}_1$ increases with *n* because an increase in *n* leads to greater Cournot output of the rivals (given q_1 = 0), thereby further depressing firm 1's inverse demand. Consequently, tipping from 1 could then occur for a somewhat greater installed-base advantage of firm 1 (higher m_1).

Let *c*, *β*, and *n* be given and consider Figure 1. Recall that tipping to 1 can occur in the region above $\underline{M}_1$ and tipping from 1 can occur above $\overline{M}_1$. Because 1/(*n* + 1) ≤ 1/2 for all *n* ≥ 1, Remarks 1 and 2 imply that the $\overline{M}_1$ curve lies below the $\underline{M}_1$ curve for all $m_1 \le 1/2$, and strictly below for $m_1 < 1/2$. This observation yields the following lemma.

Lemma 3: Fix $m_1 \le 1/2$. If (*c*, *v*, *β*, *n*, m_1) supports an equilibrium with tipping to 1 under degradation, then it also supports an equilibrium with tipping away from 1, but not vice versa.

The next result is proved in Appendix 1 (recall that $\overline{v}(n)$ is defined in (13)).

Lemma 4:

(i) Fix $n \geq 1$. If $v = \bar{v}$, then $\underline{M}_1 = \overline{M}_1$; if $v < \bar{v}$, then $\underline{M}_1 > \overline{M}_1$; and if $v > \bar{v}$, then $\underline{M}_1 < \overline{M}_1$.

(ii) Fix $c < 1$ and $n \geq 1$. If β is sufficiently large, then $\underline{M}_1$ and $\overline{M}_1$ intersect at $(m_1, \bar{v})$ for some $m_1 \in [1/2,\ 1)$.

(iii) Fix $c < 1$ and $n \geq 2$. If β is sufficiently small, then $\underline{M}_1$ and $\overline{M}_1$ do not intersect at any $m_1 \in [1/2,\ 1]$.

An implication of Lemma 4 is that if β is sufficiently large, then tipping to firm 1 can be the unique degradation equilibrium—region B in Figure 1 is nonempty, the case shown in Figure 1(a). On the other hand, if $c < 1$ and $n \geq 2$ and if the installed base β is sufficiently small, then the unique degradation equilibrium can be tipping from firm 1, even when 1 has the entire installed base ($m_1 = 1$). This possibility is illustrated in Figure 1(b).

C. Uniqueness of Equilibrium Under Degradation

The results of Section III.B collectively show that, for given c, v, β, and n, the possibilities for tipping equilibria fall into four cases:

A: $m_1 < \underline{M}_1$ and $m_1 > \overline{M}_1$ — no tipping equilibrium;

B: $m_1 \geq \underline{M}_1$ and $m_1 > \overline{M}_1$ — tipping to firm 1 but not from firm 1;

C: $m_1 < \underline{M}_1$ and $m_1 \leq \overline{M}_1$ — tipping from firm 1 but not to firm 1;

D: $m_1 \geq \underline{M}_1$ and $m_1 \leq \overline{M}_1$ — tipping to firm 1 and tipping from firm 1.

This taxonomy of possible tipping equilibria can be linked to the possibility of the interior equilibrium discussed earlier. Note that the expressions on the right-hand sides of (14) and (16) are $\overline{M}_1$ and the expressions on the right-hand sides of (15) and (17) are $\underline{M}_1$. Moreover, in light of Lemma 4, case A can arise only if $v < \bar{v}$, and D can arise only if $v \geq \bar{v}$.[12] Consequently, case A admits an interior equilibrium, as does case D if the inequalities in D hold strictly. Therefore, the equilibrium possibilities are as follows, where each of the cases given above corresponds to the so-labeled regions in Figure 1:

region A: $m_1 < \underline{M}_1$ and $m_1 > \overline{M}_1$ — the unique equilibrium is interior;

region B: $m_1 \geq \underline{M}_1$ and $m_1 > \overline{M}_1$ — the unique equilibrium is tipping to firm 1;

region C: $m_1 < \underline{M}_1$ and $m_1 \leq \overline{M}_1$ — the unique equilibrium is tipping from firm 1;

region D: $m_1 \geq \underline{M}_1$ and $m_1 \leq \overline{M}_1$ — both tipping equilibria exist, and if these inequalities are strict, the interior equilibrium also exists.

The foregoing characterization of equilibrium possibilities is central to determining how model parameters affect the prospect for tipping and for profitable degradation.

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[12] For $v \geq \bar{v}$, Lemma 4 shows $\underline{M}_1 \leq \overline{M}_1$, so there do not exist m_1 satisfying $m_1 < \underline{M}_1$ and $m_1 > \overline{M}_1$—that is, case A is impossible. Similarly, for $v < \bar{v}$, case D cannot arise.

D. Tipping Equilibria and the Number of Rivals

As expected, stronger network externalities (higher *v*) increase the likelihood of tipping under degradation—an increase in *v* lowers the minimal market share of firm 1 needed for tipping to 1 ($\underline{M}_1$) and raises the maximal share permitting tipping away from 1 ($\overline{M}_1$).

More interesting is the effect of the number of rivals, *n*, on the possibility that degradation could lead to tipping away from firm 1 even if its initial market share exceeds 1/2. While Lemma 3 focussed on the case of $m_1 \le 1/2$, the following proposition considers the case of $m_1 > 1/2$, i.e. firm 1's installed base is larger than the total base of all its rivals. The following notation is helpful. Define the sets

$$T_1(n) = \left\{(c, v, \beta, m_1) \mid m_1 > 1/2 \text{ and } (c, v, \beta, n, m_1) \text{ supports a tipping equilibrium to } 1\right\}$$

and

$$T_R(n) = \left\{(c, v, \beta, m_1) \mid m_1 > 1/2 \text{ and } (c, v, \beta, n, m_1) \text{ supports a tipping equilibrium from } 1\right\}.$$

Thus, $T_1(n)$ is the set of parameters for which there is tipping to 1, given that 1 faces *n* rivals; similarly, $T_R(n)$ is the set of parameters for which there is tipping from 1. Next define[13]

$$T_1^u(n) \equiv T_1(n) - T_R(n) \quad \text{and} \quad T_R^u(n) \equiv T_R(n) - T_1(n).$$

Thus, $T_1^u(n)$ denotes the set of parameters (together with *n*) for which the unique equilibrium is tipping to 1, and $T_R^u(n)$ denotes the set of parameters for which the unique equilibrium is tipping from 1.

Proposition 1 (Larger Number of Rivals Favors Tipping From 1): Suppose $m_1 > 1/2$.

(i) For any *n*, there exist parameters that admit tipping to 1 as the unique equilibrium, and others for which tipping from 1 is an equilibrium (i.e., the sets $T_1^u(n)$ and $T_R(n)$ are nonempty);

(ii) Tipping from 1 can be the unique equilibrium if and only if firm 1 faces at least two rivals ($T_R^u(n)$ is nonempty iff $n \ge 2$).

(iii) Increasing the number of rivals increases the scope for tipping from 1 and decreases the scope for tipping to 1. That is, if $n'' > n' \ge 1$, then[14]

(a) $T_1(n'') = T_1(n')$ and $T_1^u(n'') \subset T_1^u(n')$;

and

(b) $T_R(n'') \supset T_R(n')$ and $T_R^u(n'') \supset T_R^u(n')$.

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[13] For any two sets *X* and *Y*, we denote by *X* – *Y* the set of elements of *X* that are not elements of *Y*.

[14] For any two sets *X* and *Y*, $X \subseteq Y$ denotes that *X* is a subset of *Y*; $X \subset Y$ denotes that *X* is a strict subset of *Y*. The symbols $\supseteq$ and $\supset$ are interpreted analogously.

The properties described in Proposition 1 are illustrated in Figure 2, which is similar to Figure 1(a) but includes an additional $\overline{M}_1$ contour corresponding to a higher number of rivals. Fix *c*, *β*, and $n' > n \geq 2$. Because $n \geq 2$, the left-side vertical intercept for $\overline{M}_1$ lies strictly below that for $\underline{M}_1$. Figure 2 depicts the case in which *β* is relatively large so that $\underline{M}_1$ and $\overline{M}_1$ intersect at some market share $m_1 < 1$. The region of tipping to 1 lies above the $\underline{M}_1$ curve; the region of tipping from 1 lies below the $\overline{M}_1$ curve. Because $\underline{M}_1$ is independent of *n*, the region of tipping to 1 does not change with *n*. However, the entire $\overline{M}_1$ curve shifts down as the number of rivals increases from *n* to n'. Thus, the region of tipping from 1 strictly expands from $\mathrm{C} \cup \mathrm{D}$ to $\mathrm{C} \cup \mathrm{D} \cup \mathrm{B} \cup \mathrm{C}'$; correspondingly, the region where tipping is uniquely to 1 strictly shrinks from $\mathrm{B} \cup \mathrm{B}'$ to B' and the region where tipping is uniquely from 1 strictly expands to $\mathrm{C} \cup \mathrm{C}'$.

Given the central role of consumers' expectations under network externalities, it is not surprising that there can exist multiple equilibria under degradation (cf. Katz and Shapiro, 1986), including tipping away from firm 1 even if $m_1 > 1/2$. Less obvious is why tipping from 1 can be the unique equilibrium when $m_1 > 1/2$. The answer is that competition among the interconnected rivals confers an advantage to their network, denoted *R*, in the inter-network competition against firm 1. The *n* rivals' intra-network competition can commit their network to a higher expansion of future output, and thus a greater increase of network quality, than can network 1 as a monopolist.[15]

To see the role of competition, recall that a tipping equilibrium to network *j* (*j* = 1 or *R*) exists if and only if the other network *i* cannot profitably attract new customers given *i*'s installed base (which affects *i*'s inverse demand function) and taking as given *j*'s chosen number of new customers. The number of new customers under tipping to firm 1 is 1's monopoly solution conditional on 1's installed base, q_1^m ($\equiv q_1^{Tip}$, defined in (18)); under tipping to the rivals, the number is the *n*-firm Cournot solution conditional on the rivals' total installed base, Q_n^c ($\equiv nq_i^{Tip}$; cf. (21)). For equal installed bases ($m_1 = 1/2$), competition among the *n* rivals implies $Q_n^c > q_1^m$; by continuity, this inequality can also hold for some $m_1 > 1/2$. That is, while firm 1's larger installed base increases its network's attractiveness to new customers and hence its expected output as a monopolist, this advantage can be outweighed by network *R*'s ability to act more aggressively in expanding future output due to competition among its members. The condition $Q_n^c > q_1^m$ in turn makes it possible, for some values of the other parameters *c*, *v*, and *β*, to have a

[15] The strategic benefits of competition as a vehicle of commitment to higher output have been noted in other contexts. For example, Schwartz and Thompson (1986) show that an incumbent may gain from establishing competing divisions for purposes of deterring entry by other firms. Farrell and Galini (1988) show that an innovator may prefer to have more than one licensee despite the profit destruction caused by licensees' competition because, given incomplete contracts, this competition helps assure consumers against future ex post opportunism and thus induces them to undertake specialized investments complementary to the innovator's product.

unique equilibrium with tipping away from firm 1 despite 1's initial demand advantage due to its larger installed base.[16]

Several implications follow. First, when $m_1 > 1/2$, the possibility of a *unique* equilibrium with tipping away from firm 1 requires two or more rivals in order for their network to exhibit competition among its members, as needed to offset 1's installed-base advantage. Thus, region C in Figure 2 can arise only for $n \geq 2$.

Second, rivals' tipping-equilibrium output expands with n. Therefore, the parameter set for $m_1 > 1/2$ consistent with tipping from 1 also expands with n by increasing $\overline{M}_1$, thereby shifting rightward the curve labeled $\overline{M}_1$ in Figure 2. By contrast, firm 1's monopoly output q_1^m is independent of n; hence, the parameter set that admits tipping to firm 1 is independent of n. Thus, an increase in n expands the parameter set for which tipping from 1 is the unique equilibrium and shrinks the set for which tipping to 1 is the unique equilibrium.

IV. Profitability of Degradation to the Largest Firm

It can be shown that in any equilibrium firm 1's profit is $\pi_1^* = (1-v)(q_1^*)^2$, where q_1^* denotes firm 1's corresponding equilibrium output. Therefore, in the first stage firm 1 will choose the interconnection quality ($\theta = 0$ or 1) that gives it the greater equilibrium output.

A. The Interior Equilibrium

First consider parameters values for which the unique equilibrium under degradation is the interior equilibrium, given by (11) and (12). The discussion in Section III.C implies that a necessary (but not sufficient) condition is $v < \bar{v}$. In this case, firm 1 will prefer degradation if and only if $q_1^d > q^a$, where these outputs are given by (8) and (11). Let $\underline{m}_1$ denote the minimum market share for which firm 1 will find degradation (weakly) preferable when it yields the interior equilibrium. This threshold market share, $\underline{m}_1$, is obtained where firm 1's outputs under the two regimes are equal, $q^a = q_1^d$, with the substitution $\beta_1 = \underline{m}_1\beta$. Solving $q^a = q_1^d$ for $\underline{m}_1$ yields

$$(24) \qquad \underline{m}_1(c,v,\beta,n) \equiv \frac{(1-c)n\left[n(1-v)-v\right]+\beta\left[n^2(1-v)+2(1-v)^2+n(3-6v+2v^2)\right]}{\beta(n+2)(1-v)\left[n+(n+1)(1-v)\right]}.$$

[16] Suppose $m_1 = 1/2$ and $n \geq 2$. If $v = 0$, then the absence of network externalities leads to an interior equilibrium in which all firms' outputs are equal. Now increase v slightly. While both network 1 and network R have identical installed bases, the additional competition in network R leads to the interior equilibrium with a greater number of new subscribers to R than to 1. Consequently, network R has higher quality than network 1. Hence, as v is increased further, tipping will first occur in the direction of network R. Because this is true for $m_1 = 1/2$, it is also be true for m_1 slightly larger than 1/2: there is a range of v for which the unique equilibrium is tipping from 1.

Lemma 5: If $v < 1/2$, then $\underline{m}_1 > 1/2$.

The intuition for Lemma 5 has been noted by Cremer, Rey and Tirole (2000, Proposition 5, which discusses their three-firm example). If $m_1 \leq 1/2$, then degradation yields no quality advantage to firm 1—since the interconnected rivals offer access to at least as many customers—but reduces all firms' qualities and thus overall demand by new subscribers. Hence, if degradation can only yield the interior equilibrium, then firm 1 must have an initial market exceeding 1/2 for degradation to be profitable.

Figure 3 is identical to Figure 1(a), except that we confine attention to $m_1 \in [1/2, 1]$ and we have added the contour $\underline{m}_1$, which partitions region A—in which the unique degradation equilibrium is the interior equilibrium—into two sub-regions: A_1, where firm 1's profit is lower than under interconnection, and A_2 where its profit is higher. Furthermore, it can be shown that the $\underline{m}_1$ curve passes through the intersection of the $\underline{M}_1$ and $\overline{M}_1$ curves.[17]

B. Tipping Equilibria

Naturally, firm 1 will forgo degradation if it expects the outcome to be tipping to its rivals. If tipping would be to firm 1, degradation presents a tradeoff: firm 1 obtains all the new subscribers, but the total number of new subscribers falls because degradation reduces product quality (by denying access to the rivals' installed base). Degradation is then profitable if and only if $q_1^{Tip} - q^a > 0$, where

(25) $$q_1^{Tip} - q^a = \frac{(1-c)n + \big((n+2)\beta_1 - 2\beta\big)v}{2(n+2)(1-v)} = \frac{(1-c)n + \big((n+2)m_1 - 2\big)\beta v}{2(n+2)(1-v)}.$$

Because the difference in (25) can be negative, degradation need not be profitable even if it yields tipping to firm 1. However, the following remark provides a sufficient condition for firm 1 to prefer the tipping equilibrium:

Remark 3: The degradation equilibrium with tipping to firm 1 is more profitable to it than the interconnection equilibrium if $m_1 > 2/(n+2)$. Thus, firm 1 prefers the tipping equilibrium if (i) $m_1 > 2/3$ or (ii) $n \geq 2$ and $m_1 > 1/2$.

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[17] To see this, consider v slightly less than $\overline{v}$. Given this value of v, for market shares m_1 satisfying $\overline{M}_1 < m_1 < \underline{M}_1$ the unique degradation equilibrium is the interior equilibrium. But near the endpoints of this interval, firm 1's profit approaches that in the two tipping equilibria. At $(\overline{M}_1, \overline{v})$ the degradation equilibrium is tipping from 1, which is clearly less profitable than interconnection; and at $(\underline{M}_1, \overline{v})$ the degradation equilibrium is tipping to 1, which is usually more profitable than accommodation (see Remark 3 below). By continuity, there is some market share between $\overline{M}_1$ and $\underline{M}_1$ at which firm 1 is indifferent between connection and no connection. As v increases to $\overline{v}$, the interval $(\overline{M}_1, \underline{M}_1)$ shrinks to a single point, the intersection of the $\overline{M}_1$ and $\underline{M}_1$ curves.

C. Unprofitable Degradation

We shall say that degradation is *clearly profitable* at (c, v, β, n, m_1) if degradation yields a unique equilibrium and the corresponding profit to firm 1 strictly exceeds its profit under interconnection. Similarly, degradation is *clearly unprofitable* at (c, v, β, n, m_1) if it yields a unique equilibrium in which firm 1's profit is strictly less than under interconnection. In the other parameter regions the profitability of degradation is ambiguous, depending on which of the multiple equilibria emerges as the outcome under degradation (which, in turn, depends on consumers' expectations).

In Figure 3, degradation is clearly profitable in regions A_2 and B (the latter follows from Remark 3 because $n \geq 2$);[18] degradation is clearly unprofitable in regions A_1 and C; and its profitability is ambiguous in region D. Lemmas 3 and 5 show that if $m_1 \leq 1/2$, then under degradation either tipping from 1 is possible (in some cases as the unique equilibrium) or the unique equilibrium is interior and less profitable for firm 1 than is interconnection. This observation yields the next proposition.

Proposition 2 (Unprofitable Degradation): If $m_1 \leq 1/2$, then degradation (a) is clearly unprofitable to firm 1 for some parameter values and (b) is never clearly profitable.

D. Potentially Profitable Degradation

To allow the possibility of clearly profitable degradation, we therefore now focus on $m_1 > 1/2$. In this range, we examine how the parameters v, n, m_1, c, and β affect the likelihood of firm 1's choosing degradation. Specifically, we ask how varying each parameter individually affects the set P of other parameter values for which degradation is clearly profitable and the set U for which it is clearly unprofitable. We say that degradation becomes more likely if P expands and U does not, or if U shrinks and P does not.

1. Network Effects

Increasing the network-effects term v has an ambiguous effect on the likelihood of degradation. This is illustrated in Figure 4, which fixes β and c and plots the contour $\underline{m}_1$ as the number of rivals n takes the values 1, 2, 4, or 6. For a given n, the top of the corresponding contour is shown at $\bar{v}(n)$ $\left(= 1 - \sqrt{n/\left(2(n+1)\right)}\right)$ because higher values of v rule out the interior

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[18] Strictly speaking, degradation is not strictly profitable along the leftmost boundary of region A_2; rather, here firm 1's degradation and accommodation profits are equal ($q_1^d = q^a$). For simplicity, we will gloss over this boundary consideration in the informal discussion but account for it in the formal proofs.

equilibrium being the unique equilibrium under degradation. Figure 4 shows that $\underline{m}_1$ can be decreasing in v (for n = 1 or 2), increasing (n = 6) or backward bending (n = 4). Recall that the unique interior equilibrium under degradation is less profitable than under interconnection to the left of $\underline{m}_1$ and more profitable to the right. Thus, an increase in v can cause a move from unprofitable to profitable degradation—A_1 to A_2 (e.g., for n = 2), the reverse (for n = 6), or two switches (from A_2 to A_1 then back into A_2, for n = 4).

Increasing v has an ambiguous effect on the likelihood of degradation also in the range of v that admits tipping equilibria. Lemmas 1 and 2 showed that increasing v expands the region in which there exists tipping to 1 and that in which there exists tipping away from 1. Thus, as Figure 1 illustrates, an increase in v can cause a move into the region (D) of multiple-equilibria starting from the region (C) of unique tipping away from 1 (thereby increasing the likelihood of degradation) or, for higher values of m_1, starting from the region (B) of unique tipping to 1 (thereby decreasing the likelihood of degradation).

The ambiguous effect of v in our model is partially at odds with CRT's statement that, by focusing on values of v low enough to ensure an interior equilibrium rather than tipping, they have understated the likelihood of degradation.[19] Given that firm 1 has over half the installed base and faces a single rival, it is true that if network effects are strong enough to make tipping possible under degradation, then tipping from 1 cannot be the unique equilibrium. However, if firm 1 faces at least two rivals ($n \geq 2$), then for some values of the other parameters the unique degradation equilibrium can be tipping from 1. Thus, consideration of tipping possibilities do not systematically make degradation more likely.

2. Number of Rivals

Figure 4 also shows that increasing the number of rivals n has an ambiguous effect on the likelihood of degradation in the parameter region where the unique degradation equilibrium would be interior. This follows from the fact that the $\underline{m}_1$ contours corresponding to different values of n intersect. For low v, increasing n expands region A_2 at the expense of A_1, making degradation more likely; the reverse occurs for high v. The ambiguity arises because increasing n serves to reduce firm 1's profit in the unique interior equilibrium both under degradation and under interconnection. Observe that the ambiguous effect of n in the region where degradation leads to the (unique) interior equilibrium is in contrast to the tipping regions, where higher n made degradation less likely (Proposition 1).

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[19] CRT (2000, p. 455) state: " ... Larger network externalities would give rise to 'tipping effects' and make it more likely that the industry would be monopolized."

3. Firm 1's Market Share

An increase in m_1 makes it more likely that $m_1 > \underline{m}_1$ (so the interior degradation equilibrium is more profitable than interconnection), or $m_1 > \underline{M}_1$ (tipping to 1 is possible), or $m_1 > \overline{M}_1$ (tipping from 1 is not possible). Such effects can be seen, for example, in Figure 3, where *n*, *c*, *v*, and *β* are held constant, as a move into $A_2 \cup B$, or out of $A_1 \cup C$. More formally, given market share m_1, we define

$$P\big|_{m_1} \equiv \left\{(c,v,\beta,n) \mid \text{degradation is clearly profitable at } (c,v,\beta,n,m_1)\right\}$$

and

$$U\big|_{m_1} \equiv \left\{(c,v,\beta,n) \mid \text{degradation is clearly unprofitable at } (c,v,\beta,n,m_1)\right\}.$$

An increase in firm 1's market share makes degradation more likely, as reported in the following proposition.

Proposition 3: Suppose $1/2 < m_1' < m_1''$. Then $P\big|_{m_1''} \supset P\big|_{m_1'}$ and $U\big|_{m_1''} \subset U\big|_{m_1'}$.

4. Market Expansion

Increased scope for market expansion *relative* to the installed customer base arises if the base β is lower or if firms' marginal cost c is lower, since the latter allows a greater percentage increase in the equilibrium number of new subscribers. In contrast to the ambiguous effects of *v* and *n*, a reduction in β or c makes degradation less likely in both the tipping regions and interior-equilibrium regions. As we did for market share, we define the following sets:[20]

$$P\big|_{\beta} \equiv \left\{(c,v,n,m_1) \mid m_1 > 1/2,\ n \geq 2, \text{ and degradation is clearly profitable at } (c,v,\beta,n,m_1)\right\};$$

$$U\big|_{\beta} \equiv \left\{(c,v,n,m_1) \mid m_1 > 1/2,\ n \geq 2, \text{ and degradation is clearly unprofitable at } (c,v,\beta,n,m_1)\right\};$$

$$P\big|_{c} \equiv \left\{(v,\beta,n,m_1) \mid m_1 > 1/2,\ n \geq 2, \text{ and degradation is clearly profitable at } (c,v,\beta,n,m_1)\right\};$$

and

$$U\big|_{c} \equiv \left\{(v,\beta,n,m_1) \mid m_1 > 1/2,\ n \geq 2, \text{ and degradation is clearly unprofitable at } (c,v,\beta,n,m_1)\right\}.$$

For $n \geq 2$, degradation is clearly profitable for firm 1 if and only if the unique degradation equilibrium is tipping to 1 or it is interior and $m_1 > \underline{m}_1$; degradation is clearly unprofitable if the unique degradation equilibrium is tipping from 1 or it is interior and $m_1 < \underline{m}_1$. The following lemma shows how the boundaries of these regions vary with the market-expansion parameters.

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[20] In this subsection, the sets of parameters for which degradation is profitable or unprofitable also include the restriction that $n \geq 2$. The reason for this is that when $n = 1$, equilibrium tipping to 1 can be unprofitable if firm 1's market share is less than 2/3. Rather than imposing more complex assumptions in our discussion of the market expansion parameters β and c (see Proposition 4), we restrict attention to the case of $n \geq 2$.

Lemma 6: Fix $c < 1$.

(i) If $v < 1/2$, then $\partial \underline{M}_1 / \partial c < 0$ and $\partial \underline{M}_1 / \partial \beta < 0$.

(ii) If $v > 1/(n+1)$, then, $\partial \overline{M}_1 / \partial c < 0$ and $\partial \overline{M}_1 / \partial \beta < 0$.

(iiii) If $v < 1/2$, then $\partial \underline{m}_1 / \partial c < 0$ and $\partial \underline{m}_1 / \partial \beta < 0$.

Lemma 6 focuses on the case in which firm 1's market share is at least 1/2, a necessary condition for clearly profitable degradation. Therefore, to obtain the comparative statics for $\underline{M}_1$, $\overline{M}_1$, and $\underline{m}_1$, we consider v only over the ranges given. Geometrically, Lemma 6 implies that a reduction in β or c pivots the $\underline{M}_1$ curve counterclockwise through the point $(m_1, v) = (1/2, 1/2)$ and pivots the $\overline{M}_1$ curve clockwise through the point $(m_1, v) = (1/2,\ 1/(n+1))$, with the intersection of these two curves moving to the right. In addition, the $\underline{m}_1$ curve is shifted to the right. These effects are illustrated in Figure 5, which is drawn for the case in which β is sufficiently large that $\underline{M}_1$ and $\overline{M}_1$ intersect at some $m_1 < 1$. The initial scenario is depicted by the curves labeled $\underline{M}_1$, $\overline{M}_1$, and $\underline{m}_1$. In this case, the region of clearly profitable region is to the right of the heavily shaded curve $\alpha\gamma\delta$; after β or c is reduced, the $\underline{M}_1$, $\overline{M}_1$, and $\underline{m}_1$ curves shift in the directions shown, thereby shrinking the region of clearly profitable degradation to that bounded by the heavily shaded curve $\alpha'\gamma'\delta'$. Correspondingly, the region of clearly unprofitable degradation expands from that bounded by $\delta\gamma\varepsilon$ to that bounded by $\delta'\gamma'\varepsilon$. These observations yield the following proposition.

Proposition 4 (Market Expansion and Degradation): Increased scope for market expansion decreases the likelihood of degradation:

(i) If $\beta'' < \beta'$, then $P|_{\beta''} \subseteq P|_{\beta'}$, and the inclusion is strict if $P|_{\beta'}$ is nonempty; also $U|_{\beta''} \supseteq U|_{\beta'}$, and the inclusion is strict if $U|_{\beta''}$ is nonempty.

(ii) If $c'' < c'$, then $P|_{c''} \subseteq P|_{c'}$ and the inclusion is strict if $P|_{c'}$ is nonempty; also $U|_{c''} \supseteq U|_{c'}$, and the inclusion is strict if $U|_{c''}$ is nonempty.

The intuition for the above results is as follows. First, consider tipping. Recall that under tipping to 1 total output equals 1's monopoly output, q_1^m, while under tipping to the rivals, total output is their n-firm Cournot output, Q_n^c. As marginal cost c falls, q_1^m expands by less than does Q_n^c;[22] thus, the competition-based advantage of the rivals' network is stronger the lower is marginal cost. A decrease in c, therefore, can both shrink the range of values v for which there

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[22] This follows because the relevant inverse demands differ only in their intercept terms (see (5) and (6)).

exists a tipping equilibrium to firm 1 and expand the range for which there exists tipping away from 1. Turning to the total installed base, a lower β implies—for given market shares—a smaller advantage to firm 1 in the *absolute* number of installed-base subscribers. A lower β or a lower c thus reduces the likelihood that degradation would be profitable to firm 1, because each effect makes tipping to 1 less likely and tipping away from 1 more likely.

Now consider the parameter region where degradation would lead to the unique interior equilibrium. For this case, and assuming duopoly, CRT decompose the effect of degradation on firm 1 into two terms: a decrease in total industry output, and a shift in output from firm 2 to firm 1. In Appendix 2, we extend this decomposition to the case where firm 1 faces n rivals and the unique interior equilibrium under degradation is stable.[23] We show the following. A fall in β has two opposing effects: it decreases the output-reduction effect of degradation, since new customers lose access to fewer subscribers when the installed base is smaller; but it also decreases the output-shifting effect because, for given market shares, a lower β means a lower absolute advantage to firm 1 in installed-base customers (hence a smaller gain in quality advantage from degradation). This second effect dominates, so that on balance, a lower β makes degradation by firm 1 less likely. Turning to marginal cost, a lower c increases the output-reduction effect of degradation, because a fall in quality is more consequential when firms are otherwise inclined to produce higher outputs (i.e., when marginal costs are low). Less obviously, a lower c also weakly decreases the output-shifting effect (decreasing it strictly with $n > 1$ rivals and leaving it unchanged with $n = 1$, i.e., duopoly). Intuitively, a lower c dampens the installed-base disadvantage of the rivals, because they can more readily compete for new customers. Thus, both effects of a lower c make degradation less attractive to firm 1.

E. Illustrative Calculations: The Role of Potential Market Expansion

The parameters c, v, and β are difficult to interpret empirically—they have no natural unit of measure. For example, c is measured relative to the maximum value for interconnection, which is taken to be $v = 1$. And the installed base β might be understood as measured relative to the number of potential future subscribers. Nevertheless, for services such as Internet connectivity, one might have estimates of expected growth, given the current number of firms and assuming interconnection persists. In this section we show how such estimates can be used to construct safe harbors below which the largest firm in is unlikely to degrade its connection with rivals.

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[23] It can be shown that the interior degradation equilibrium is stable if and only if $v < 1/(n + 1)$. In CRT's duopoly case ($n = 1$), whenever the interior equilibrium is the unique degradation equilibrium ($v < \bar{v}(1) = 1/2$) it is also stable. However, for $n \geq 2$, it is the case that $1/(n+1) < \bar{v}(n)$, so there exists a region in which the unique degradation equilibrium is interior and stable, and another where it is interior and unstable. Appendix 2 focuses on the former case.

Suppose that *under interconnection* the overall network would grow by some factor γ (> 1). Given parameters *c*, *v*, and *n*, this growth rate is achieved if the installed base equals β_γ, which is derived as follows. Given parameters *c*, *v*, *n*, and β, if firm 1 pursues global interconnection, then (8) implies that the number of new subscribers would be

$$\sum q_i = \frac{(n+1)(1-c+\beta v)}{(n+2)(1-v)},$$

so that the growth rate of the overall network would be

$$\text{(26)} \qquad \frac{\beta + \sum q_i}{\beta} = \frac{(n+1)(1-c) + \beta(n+2-v)}{\beta(n+2)(1-v)}.$$

If the network is expected to grow by at least factor γ, then

$$\text{(27)} \qquad \frac{(n+1)(1-c) + \beta(n+2-v)}{\beta(n+2)(1-v)} \geq \gamma,$$

which is equivalent to

$$\text{(28)} \qquad \beta \leq \beta_\gamma \equiv \frac{(n+1)(1-c)}{(n+2)\left[(1-v)\gamma - 1\right] + v}.$$

The function β_γ is decreasing in γ—a higher potential growth rate is possible when the initial installed base is smaller. In this subsection we assume $c < 1$, so that β_γ is well-defined.[24] But even then, β_γ (given in (28)) might not be positive for some values of γ and *v*—this corresponds to low levels of growth that are simply unachievable. Indeed, given $c < 1$, growth by factor $\gamma > 1$ is feasible (by an appropriate choice of $\beta > 0$) if and only if the denominator in (28) is strictly positive, a condition that imposes an upper bound on the feasible *v*:

$$\text{(29)} \qquad v < v_{\max} \equiv \frac{(\gamma - 1)(n+2)}{\gamma(n+2) - 1}.$$

Note that for any $\gamma > 1$ and $n \geq 1$, $v_{\max}$ strictly between 0 and 1.

Observe that in (28) the term $(1 - c)$ is a factor in β_γ. Therefore, upon substitution of β_γ for β and $m_1\beta_\gamma$ for β_1 in firm 1's output formulas at the interior and tipping equilibria ((8), (11), and (18)), *c* enters only through the factor $(1 - c)$. Thus, *c* affects the level of profitability from degradation, but not whether degradation is profitable.[25] Because we are interested only in whether firm 1 finds degradation profitable, the normalization $\beta = \beta_\gamma$ allows us to narrow attention to the parameters *v* and m_1, given a number *n* of rivals facing firm 1 and an expected

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[24] From (26) we see that if $c = 1$, then the growth rate is independent of β.

[25] Alternatively, with the substitution $\beta = \beta_\gamma$, the expression $(1 - c)$ cancels out of the expressions for $\underline{M}_1$, $\overline{M}_1$, and $\underline{m}_1$.

market growth factor γ under interconnection. Thus, for any $c < 1$, Figure 6 displays the equilibrium possibilities under degradation, for the cases in which—under interconnection—the network would grow by 50%, 100%, and 200% when firm 1 faces two or four rivals. As stated in Proposition 4, greater scope for market expansion indeed increases the market share needed for the largest firm to profit by degrading interconnection. This pattern is also reflected in Table 1.

	γ		
	1.5	2.0	3.0
$n = 2$	.529	.595	.731
$n = 4$	.561	.638	.731

Table 1. Safe Harbors: the market share m_1 below which degradation is not clearly profitable for firm 1, given growth factor γ and n rivals to firm 1

If n rivals face firm 1 and the expected growth factor is γ, then degradation is clearly profitable for firm 1 only in regions A_2 and B, where v ranges over the values consistent with n and γ; that is, from 0 to v_{max}. The market shares reported in Table 1 correspond to that at the leftmost point of region $A_2 \cup B$. If the expected growth factor is γ and firm 1 faces n interconnected rivals, then degradation is not clearly profitable for firm 1 if its market share is below the critical value reported. In this sense, the values in Table 1 represent safe harbors. In one respect these safe harbors might be viewed as permissive because firm 1 might be willing to degrade interconnection when multiple equilibria are possible—if firm 1 believes that of the several equilibria possible, the one to be realized will be tipping to 1, then firm 1 may degrade even for market shares below the safe harbors given (see region D in Figure 6). However, in another sense these safe harbors are quite restrictive. For example, Figure 6(d) shows that when $\gamma = 2$ and $n = 4$, even for market shares somewhat above the safe harbor of 0.638, degradation would also be unprofitable for much of the feasible range of v.

V. Conclusion

A central concern in markets with strong network effects is that if one firm attains a high enough share of locked-in customers—for example, through merger—it may restrict interconnection with its smaller competitors because its larger customer base then gives it a quality edge in competing for new customers. A particularly stark risk is that the market would tip to the large firm, resulting in monopoly. Moreover, a common intuition is that when network effects are sufficiently strong and all links have equal value, a fifty-percent share of installed-base customers suffices for tipping to the large firm if it degrades interconnection, because its network then offers access to more links than could the rivals collectively. We showed that this intuition is flawed whenever the large firm faces two or more rivals: it interconnection is degraded, the unique equilibrium can be tipping to the rivals (making degradation clearly unprofitable). Competition among the interconnected rivals serves as a commitment that their network will expand more aggressively than would a single firm (for the same initial base), thereby offering higher quality. This competition-based advantage can compensate for the rivals' smaller installed base. Thus, for a given market share of the large firm, the likelihood of tipping to the rivals rises with their number or, more generally, with the strength of competition among them.

The likelihood that degradation would be profitable—whether the market tips away from rivals or they remain active but contract—decreases as the scope for market expansion relative to the installed base increases, whether due to lower marginal cost or to greater potential for demand growth. With greater scope for market expansion, any share advantage in the installed base becomes less consequential. Thus, for given market shares, the risk that the largest firm could profitably degrade interconnection is higher in relatively mature industries such as traditional telephony than in more rapidly growing industries such as the Internet. In rapidly growing markets, degradation can be unprofitable even if the largest firm controls substantially more than half the installed base. (Malueg and Schwartz (2002) expand on this point for the Internet.)

Several caveats to our analysis should be noted, however. We have been agnostic about what outcome would result when there exist multiple equilibria under degradation, confining our predictions to cases where there is a unique degradation equilibrium. Degradation becomes more likely if consumers' expectations in cases of multiple equilibria would systematically favor the largest firm (a possibility noted by Farrell and Klemperer, 2002). Secondly, we have focused on global degradation; as shown by CRT, targeted degradation may be profitable even when global degradation is not. Finally, and working in the opposite direction, our analysis assumes that the largest network obtains no profit from sales of access to rivals. Relaxing this assumption can make degradation of interconnection less attractive.

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Appendix 1. Proofs

Proof of Lemma 1.

Because n does not enter the expression for $\underline{M}_1$, it follows that $\partial \underline{M}_1 / \partial n = 0$.

Next, we have

(A.1) $$\frac{\partial \underline{M}_1}{\partial \nu} = \frac{G(\nu, c)}{(3 - 2\nu)^2 \beta \nu^2},$$

where $G(\nu, c) \equiv -(1 - c)(3 - 4\nu) - 2(2 - c + \beta)\nu^2$. First observe that $G(\nu, 1) = -2(1 + \beta)\nu^2 < 0$ for all $\nu > 0$. Next consider $c < 1$. The denominator of (A.1) is strictly positive for $\nu \in (0, 1]$; moreover, G is a concave function of ν, achieving its maximum at

$$\nu' \equiv \frac{1 - c}{2(1 - c) + \beta},$$

yielding a maximum of

$$G(\nu', c) = -\frac{(1 - c)(4(1 - c) + 3\beta)}{2(1 - c) + \beta},$$

which is strictly negative for all $0 \le c < 1$. It now follows that $\partial \underline{M}_1 / \partial \nu < 0$ for $0 \le c \le 1$.

Finally, substituting $\nu = 1/2$ into (20) yields $\underline{M}_1\big|_{\nu = 1/2} = 1/2$. ///

Proof of Lemma 2.

First, observe that

$$\frac{\partial \overline{M}_1}{\partial n} = \frac{(1 - \nu)\big(2(1 - c) + \beta\nu\big)}{\beta\nu\big(n + (n + 1)(1 - \nu)\big)^2} > 0$$

for all $0 < \nu < 1$, $\beta > 0$, and $c \le 1$.

Next, we have

(A.2) $$\frac{\partial \overline{M}_1}{\partial \nu} = \frac{G(\nu, c)}{\big(n + (n + 1)(1 - \nu)\big)^2 \beta \nu^2},$$

where

$$G(\nu, c) \equiv (1 - c)(2n + 1) - 2(1 - c)(n + 1)\nu + (n + 1)\big((n + 1)(1 - c) + n\beta\big)\nu^2.$$

First, we see $G(\nu, 1) = n(n + 1)\beta\nu^2 > 0$ for all $\nu > 0$. Next consider $c < 1$. With respect to ν, G assumes its minimum at

$$\nu' \equiv \frac{1 - c}{(n + 1)(1 - c) + n\beta},$$

at which

$$G(v',c) = \frac{(1-c)n\big((2n+1)\beta + 2(1-c)(n+1)\big)}{(n+1)(1-c) + n\beta},$$

which is strictly positive for all $c < 1$. Therefore, we can conclude that $G(v,c) > 0$ for all $v < 1$ and $c \le 1$. Hence, from (A.2), the above cases show that $\partial\overline{M}_1/\partial v > 0$ for all $0 < v < 1$ and $c \le 1$.

Finally, substituting $v = 1/(n + 1)$ into (23) yields $\overline{M}_1\big|_{v=1/(n+1)} = 1/2$. ///

Proof of Lemma 4.

(i) It can be shown that

$$\text{(A.3)} \qquad \underline{M}_1 - \overline{M}_1 = \frac{\big(2(1-c) + \beta v\big)\big(n + 2 - 4(n+1)v + 2(n+1)v^2\big)}{(3-2v)\big(1 + (2-v)n + v\big)\beta v}.$$

Because $\beta > 0$, $c \le 1$, and $0 < v < 1$, it follows that the difference in (A.3) is zero if and only if

$$0 = n + 2 - 4(n+1)v + 2(n+1)v^2.$$

The two roots of this quadratic equation are

$$1 \pm \sqrt{\frac{n}{2(n+1)}}.$$

Because we only consider $v < 1$, we see $\underline{M}_1 = \overline{M}_1$ if and only if

$$v = \bar{v} = 1 - \sqrt{\frac{n}{2(n+1)}}.$$

Because $\underline{M}_1$ is decreasing in v and $\overline{M}_1$ is increasing in v (Lemmas 1 and 2) and $\underline{M}_1 = \overline{M}_1$ at $v = \bar{v}$, it follows that $v > \bar{v}$ implies $\underline{M}_1 < \overline{M}_1$. Similarly, $v < \bar{v}$ implies $\underline{M}_1 > \overline{M}_1$.

(ii) Given c and β, the point of intersection of $\underline{M}_1$ and $\overline{M}_1$ is at $(m_1, v) = (\underline{M}_1\big|_{v=\bar{v}}, \bar{v})$. Moreover,

$$\text{(A.4)} \qquad \underline{M}_1\big|_{v=\bar{v}} = \frac{2\Big[(1-c)\big(\sqrt{2n(n+1)} - n - 1\big) + \big(\sqrt{2n(n+1)} - n\big)\beta\Big]}{\big(2 + \sqrt{2n(n+1)}\big)\beta}.$$

For $n = 1$, it is the case that $\underline{M}_1\big|_{v=\bar{v}} = 1/2$. Moreover, the expression in (A.4) is increasing in n. Therefore, $\underline{M}_1\big|_{v=\bar{v}} \ge 1/2$ for all $n \ge 1$. Taking the limit in (A.4), we have

$$\lim_{\beta\to\infty} \underline{M}_1\big|_{v=\bar{v}} = \frac{2\big(\sqrt{2n(n+1)} - n\big)}{2 + \sqrt{2n(n+1)}},$$

which is strictly positive and less than 1 for all $n \ge 1$.

(iii) Next assume $n \geq 2$ and $c < 1$. In this case, $\sqrt{2n(n+1)} > n+1$ so that, from (A.4) we have

$$\lim_{\beta \to 0} \underline{M}_1\Big|_{v=\bar{v}} = +\infty,$$

which shows that $\underline{M}_1$ and $\overline{M}_1$ do not intersect at any $m_1 < 1$ if β is sufficiently small. ///

Proof of Proposition 1.

(i) Let $n' \geq 1$ be given. Fix $c < 1$ and let $\beta > 0$ be such that $\underline{M}_1(c, \bar{v}\big|_{n'}, \beta) < 1$ (see Lemma 4). Define the market shares

$$m_{1a} \equiv \frac{1}{2}\left(\underline{M}_1(c, \bar{v}\big|_{n'}, \beta) + 1\right)$$

and

$$m_{1b} \equiv \frac{1}{2}\left(\frac{1}{2} + \min\left\{\overline{M}_1(c, 3/4, \beta, n'),\ 1\right\}\right).$$

Observe that $1/2 < m_{1a} < 1$ and $1/2 < m_{1b} < 1$. Furthermore,

$$m_{1a} > \underline{M}_1(c, \bar{v}\big|_{n'}, \beta) = \overline{M}_1(c, \bar{v}\big|_{n'}, \beta, n'),$$

so at $(c, \bar{v}\big|_{n'}, \beta, n', m_{1a})$ the unique degradation equilibrium is tipping to firm 1 (see Section III.C and the conditions defining Region B); thus, $(c, \bar{v}\big|_{n'}, \beta, m_{1a}) \in T_1^u(n')$.

Next, observe that $m_{1b} < \overline{M}_1(c, 3/4, \beta, n')$, so that at $(c, 3/4, \beta, n', m_{1b})$ degradation can yield equilibrium tipping from firm 1. Thus, $(c, 3/4, \beta, m_{1b}) \in T_R(n')$

Because $n' \geq 1$ was arbitrary, the foregoing shows that for any $n \geq 1$, $T_1^u(n)$ and $T_R(n)$ are nonempty.

(ii) If $(c, v, \beta, m_1) \in T_R(1)$, then $v > 1/2$ (see Lemma 2, and recall that here $m_1 > 1/2$). But for $v > 1/2$ it is the case that $\underline{M}_1 < 1/2$, so that $\underline{M}_1 < 1/2 < m_1$, which implies that $(c, v, \beta, 1, m_1)$ supports tipping to 1 as a degradation equilibrium. Thus, if $(c, v, \beta, m_1) \in T_R(1)$, then $(c, v, \beta, m_1) \in T_1(1)$, implying that tipping from firm 1 cannot be the unique degradation equilibrium; that is, $T_R^u(1)$ is empty.

Now suppose $n \geq 2$. Define the market share

$$m_1' \equiv \frac{1}{2}\left(\frac{1}{2} + \min\left\{\underline{M}_1(c, \bar{v}\big|_n, \beta),\ 1\right\}\right).$$

Then $m_1' > 1/2$ and

$$m_1' < \underline{M}_1(c, \bar{v}\big|_n, \beta) = \overline{M}_1(c, \bar{v}\big|_n, \beta, n),$$

so that degradation yields tipping from firm 1 as the unique equilibrium. Thus, $(c, v\big|_n, \beta, m_1') \in T_R^u(n)$.

The two foregoing cases establish that $T_R^u(n)$ is nonempty if and only if $n \geq 2$.

(iii)(a) Fix $n'' > n' \geq 1$. Because $\underline{M}_1$ is independent of *n*, it follows that $T_1(n'') = T_1(n')$. To show that $T_1^u(n'') \subset T_1^u(n')$, we first show that $T_1^u(n'') \subseteq T_1^u(n')$ and then show that $T_1^u(n'') \neq T_1^u(n')$.

Suppose $(c,v,\beta,m_1) \in T_1^u(n'')$. Then at (c,v,β,n'',m_1) the unique degradation equilibrium is tipping to 1, which requires

$$m_1 \geq \underline{M}_1(c,v,\beta)$$

and

$$m_1 > \overline{M}_1(c,v,\beta,n'').$$

From Lemmas 1 and 2 we know that $\overline{M}_1(c,v,\beta,n'') > \overline{M}_1(c,v,\beta,n')$. Consequently,

$$m_1 \geq \underline{M}_1(c,v,\beta)$$

and

$$m_1 > \overline{M}_1(c,v,\beta,n'),$$

which implies that at (c,v,β,n',m_1) the unique degradation equilibrium is tipping to 1; that is, $(c,v,\beta,m_1) \in T_1^u(n')$. Thus, $T_1^u(n'') \subseteq T_1^u(n')$.

To see that $T_1^u(n'') \neq T_1^u(n')$, fix $c < 1$ and let $\beta > 0$ be such that $\overline{M}_1(c,\bar{v}|_{n'},\beta,n') < 1$ (see Lemma 4). Define the market share

$$m_1' \equiv \frac{1}{2}\left(\overline{M}_1(c,\bar{v}|_{n'},\beta,n') + \min\left\{\overline{M}_1(c,\bar{v}|_{n'},\beta,n''),1\right\}\right).$$

Then

$$m_1' > \overline{M}_1(c,\bar{v}|_{n'},\beta,n') = \underline{M}_1(c,\bar{v}|_{n'},\beta)$$

so that $(c,\bar{v}|_{n'},\beta,m_1') \in T_1^u(n')$. However, because $\overline{M}_1(c,v,\beta,n'') > \overline{M}_1(c,v,\beta,n')$, we also have

$$m_1' > \underline{M}_1(c,\bar{v}|_{n'},\beta)$$

and

$$m_1' < \overline{M}_1(c,\bar{v}|_{n'},\beta,n''),$$

so that at $(c,\bar{v}|_{n'},\beta,n'',m_1')$ both tipping equilibria are possible under degradation. Hence, $(c,\bar{v}|_{n'},\beta,m_1') \notin T_1^u(n'')$. Thus, $T_1^u(n'') \neq T_1^u(n')$.

(iii)(b) The proof that $T_R(n'') \supset T_R(n')$ and $T_R^u(n'') \supset T_R^u(n')$ follows the same lines given for (iii)(a) and so is omitted. ///

Proof of Lemma 5.

It can be verified that

$$\text{(A.5)} \qquad \underline{m}_1 - \frac{1}{2} = \frac{2(1-c)\big(n(1-v)-v\big)n + \beta\left[2(1-v)^2 + (1-4v+v^2)n + (1-v)vn^2\right]}{2\beta(2+n)(1-v)\left[n+(n+1)(1-v)\right]}.$$

Because $c \leq 1$ and $v < 1/2$ (recall that $v < \bar{v}(n) \leq 1/2$ is necessary for the degradation equilibrium to be interior and unique), it is clear that the expression in (A.5) is strictly positive if the coefficient of β in the numerator is strictly positive over the relevant range. Let the function f denote the coefficient of β; that is,

$$f(v,n) \equiv 2(1-v)^2 + (1-4v+v^2)n + (1-v)vn^2.$$

Observe that

$$\text{(A.6)} \qquad f(v,1) = 3 - 7v + 2v^2 = (1-2v)(3-v) > 0$$

for $v < 1/2$. Also,

$$\text{(A.7)} \qquad f(v,2) = 4 - 8v > 0$$

for $v < 1/2$. Next observe that

$$\text{(A.8)} \qquad \left.\frac{\partial f}{\partial n}\right|_{n=2} = 1 - 3v^2 > 0$$

for all $v < 1/2$. Because f is convex in n for $0 < v < 1$ and $f(v,2) > 0$ for all $v < 1/2$, it now follows from (A.8) that

$$\text{(A.9)} \qquad f(v,n) \geq f(v,2) > 0$$

for all $v < 1/2$ and all $n \geq 3$. Inequalities (A.6), (A.7), and (A.9) now establish that the coefficient of β in (A.5) is strictly positive for all $c \leq 1$ and $v < 1/2$. ///

Proof of Proposition 3.

Fix $1/2 < m_1' < m_1''$. We will prove only $P|_{m_1''} \supset P|_{m_1'}$. The proof that $U|_{m_1''} \subset U|_{m_1'}$ is analogous. We first show that $P|_{m_1''} \supseteq P|_{m_1'}$. Suppose $(c, v, \beta, n) \in P|_{m_1'}$. Then the degradation equilibrium is unique and strictly more profitable than the accommodation equilibrium (so firm 1's degradation output strictly exceeds q^a). There are two cases to consider: either the degradation equilibrium is tipping to 1 or it is the interior equilibrium. First, suppose that at (c, v, β, n, m') the unique degradation equilibrium is tipping to 1; then

$$m_1' \geq \underline{M}_1 \text{ and } m_1' > \overline{M}_1 \qquad \text{(unique degradation equilibrium is tipping to 1)}$$

and

$$\left. q_1^{Tip}\right|_{m_1'} > q^a \qquad \text{(degradation is strictly profitable).}$$

Then for $m_1'' > m_1'$ it is the case that $m_1'' > m_1' \geq \underline{M}_1$ and $m_1'' > m_1' > \overline{M}_1$, so the unique degradation equilibrium at share m_1'' continues to be tipping to 1. Moreover, $q_1^{Tip}\big|_{m_1''} > q_1^{Tip}\big|_{m_1'} > q^a$, where the first inequality follows from (18), so the degradation equilibrium at share m_1'' is strictly more profitable than the accommodation equilibrium. Therefore, $(c, v, \beta, n) \in P\big|_{m_1''}$.

The second case to consider is that at (c, v, β, n, m') the unique degradation equilibrium is interior; then

$$m_1' < \underline{M}_1 \text{ and } m_1' > \overline{M}_1 \qquad \text{(unique degradation equilibrium is interior)}$$

and

$$q_1^d\big|_{m_1'} > q^a \qquad \text{(degradation is strictly profitable).}$$

Now for $m_1'' > m_1'$ it is the case that $m_1'' > m_1' > \overline{M}_1$, so there are two subcases to consider: either $m_1'' < \underline{M}_1$, in which case the unique degradation equilibrium is interior; or $m_1'' \geq \underline{M}_1$, in which case the unique degradation equilibrium is tipping to 1. In the first case, $m_1'' < \underline{M}_1$, firm 1's degradation output is $q_1^d\big|_{m_1''}$, with

$$q_1^d\big|_{m_1''} > q_1^d\big|_{m_1'} > q^a,$$

(where the first inequality follows from (11)) implying degradation is strictly profitable. In the second case, $m_1'' \geq \underline{M}_1$, firm 1's degradation output is $q_1^{Tip}\big|_{m_1''}$, with

$$q_1^{Tip}\big|_{m_1''} \geq q_1^{Tip}\big|_{m_1 = \underline{M}_1} = q_1^d\big|_{m_1 = \underline{M}_1} > q_1^d\big|_{m_1'} > q^a,$$

where the first inequality follows from (18), the equality follows from substitution of $m_1 = \underline{M}_1$ into (18) and (11), and the second inequality follows from (18), implying degradation is strictly profitable. Thus, if at (c, v, β, n, m') the unique degradation equilibrium is interior and strictly profitable, then at (c, v, β, n, m'') the degradation equilibrium is unique and strictly more profitable than accommodation. Hence, $(c, v, \beta, n) \in P\big|_{m_1''}$.

The above analysis shows $P\big|_{m_1''} \supseteq P\big|_{m_1'}$. To complete the proof, it remains only to show that $P\big|_{m_1''} \neq P\big|_{m_1'}$. Continuing with $1/2 < m_1' < m_1''$, fix $c = 0$, $n = 1$, and $\beta > 0$ such that $1 + (3m_1'' - 2)\beta > 0$ (this last condition guarantees that tipping to 1 is strictly more profitable than accommodation when firm 1's share is m_1''—see (25)). Next define v' and v'' as the unique solutions in v to

$$m_1' = \overline{M}_1(0, v, \beta, 1) \qquad \text{and} \qquad m_1'' = \overline{M}_1(0, v, \beta, 1).$$

Then it follows from Lemma 2 that $1/2 < \nu' < \nu'' < 1$. Now define $\nu^* \equiv (\nu' + \nu'')/2$. Then at $(0, \nu^*, \beta, 1, m'')$ the unique degradation equilibrium is tipping to 1 and it is strictly more profitable than accommodation, so $(0, \nu^*, \beta, 1) \in P|_{m_1''}$. However, at $(0, \nu^*, \beta, 1, m')$, tipping *from* 1 is possible, so degradation is not clearly profitable; that is, $(0, \nu^*, \beta, 1) \notin P|_{m_1'}$. Thus, $P|_{m_1''} \neq P|_{m_1'}$. ///

Proof of Lemma 6.

(i) It is easily calculated that

$$\frac{\partial \underline{M}_1}{\partial c} = -\left(\frac{1-2\nu}{(3-2\nu)\beta\nu}\right) \qquad \text{and} \qquad \frac{\partial \underline{M}_1}{\partial \beta} = -\left(\frac{(1-2\nu)(1-c)}{(3-2\nu)\beta^2\nu}\right),$$

both of which are strictly negative for $0 \le \nu < 1/2$ and $0 \le c < 1$.

(ii) Also,

$$\frac{\partial \overline{M}_1}{\partial c} = \frac{\left(1-(n+1)\nu\right)}{\left(n+(n+1)(1-\nu)\right)\beta\nu} < 0,$$

where the inequality follows because the denominator of the fraction is strictly positive for all $\nu \in (0,1)$ and the numerator is strictly negative for all $\nu > 1/(n+1)$. Similarly,

$$\frac{\partial \overline{M}_1}{\partial \beta} = \frac{\left(1-(n+1)\nu\right)}{\left(n+(n+1)(1-\nu)\right)\beta^2\nu} < 0$$

for all $\nu \in (1/(n+1), 1)$.

(iii) Finally, we determine the effects of c and β on $\underline{m}_1$:

$$\frac{\partial \underline{m}_1}{\partial c} = -\frac{n\left[n(1-\nu)-\nu\right]}{\beta(n+2)(1-\nu)\left(n+(n+1)(1-\nu)\right)} < 0$$

and

$$\frac{\partial \underline{m}_1}{\partial \beta} = -\frac{(1-c)n\left[n(1-\nu)-\nu\right]}{\beta^2(n+2)(1-\nu)\left(n+(n+1)(1-\nu)\right)} < 0,$$

where the inequalities follow because $n \ge 1$ and we are restricting attention to $\nu < 1/2$. ///

Proof of Proposition 4.

We will prove that if $\beta'' < \beta'$, then $P|_{\beta''} \subseteq P|_{\beta'}$, and the inclusion is strict if $P|_{\beta'}$ is nonempty. The other conclusions of the proposition are proved analogously.

Let $\beta'' < \beta'$ be given and suppose $(c, \nu, n, m_1) \in P|_{\beta''}$. Then the degradation equilibrium at $(c, \nu, \beta'', n, m_1)$ is unique and either tipping to 1 or interior. Suppose it is tipping to 1. Then

$$m_1 \ge \underline{M}_1(c, \nu, \beta'') \text{ and } m_1 > \overline{M}_1(c, \nu, \beta'', n).$$

By Lemma 6 we have $\underline{M}_1(c,v,\beta'') > \underline{M}_1(c,v,\beta')$ and $\overline{M}_1(c,v,\beta'',n) > \overline{M}_1(c,v,\beta',n)$, so that

$$m_1 > \underline{M}_1(c,v,\beta') \text{ and } m_1 > \overline{M}_1(c,v,\beta',n),$$

implying that at (c,v,β',n,m_1) the degradation equilibrium is unique and interior. Because $n \geq 2$, it follows that at (c,v,β',n,m_1) this tipping to 1 is strictly more profitable to firm 1 than accommodation. Hence, $(c,v,n,m_1) \in P\big|_{\beta'}$.

Suppose instead that at (c,v,β'',n,m_1) the degradation equilibrium is unique and interior. Then

$$m_1 < \underline{M}_1(c,v,\beta'') \text{ and } m_1 > \overline{M}_1(c,v,\beta'',n).$$

Again by Lemma 6 we have $\underline{M}_1(c,v,\beta'') > \underline{M}_1(c,v,\beta')$ and $\overline{M}_1(c,v,\beta'',n) > \overline{M}_1(c,v,\beta',n)$. Consequently, $m_1 > \overline{M}_1(c,v,\beta',n)$ and either (a) $m_1 \geq \underline{M}_1(c,v,\beta')$ or (b) $m_1 < \underline{M}_1(c,v,\beta')$. In case (a), the degradation equilibrium is unique and it is tipping to 1, which is strictly more profitable than accommodation because $n \geq 2$. In case (b), the degradation equilibrium is unique and interior, with

$$m_1 > \underline{m}_1(c,v,\beta'',n) > \underline{m}_1(c,v,\beta',n),$$

where the first inequality follows because the interior degradation equilibrium is strictly more profitable to firm 1 than accommodation at (c,v,β'',n,m_1), and the second inequality follows from Lemma 6. Hence, in case (b) firm 1 also finds degradation strictly more profitable than accommodation. Therefore, $(c,v,n,m_1) \in P\big|_{\beta'}$.

The foregoing establishes that $P\big|_{\beta''} \subseteq P\big|_{\beta'}$. Now suppose $P\big|_{\beta'}$ is nonempty. We will show this inclusion is strict. Suppose that $(c,v,n,m_1) \in P\big|_{\beta'}$. Then under degradation at (c,v,β',n,m_1), the unique equilibrium is either tipping to 1 or the interior equilibrium. First suppose it is the tipping equilibrium to 1. Then it is also the case that $(c,\overline{v},n,m_1) \in P\big|_{\beta'}$ and at $(c,\overline{v},\beta',n,m_1)$ the unique equilibrium is tipping to 1, where $\overline{v} = \overline{v}(n)$ for the particular value of n under consideration. Define the market share

$$m_1^* \equiv \frac{1}{2}\Big(\underline{M}_1(c,\overline{v},\beta',n) + \min\Big\{\underline{M}_1(c,\overline{v},\beta'',n), 1\Big\}\Big).$$

Then at $(c,\overline{v},\beta',n,m_1^*)$ the unique degradation equilibrium is tipping to 1, so $(c,\overline{v},n,m_1^*) \in P\big|_{\beta'}$. In contrast, at $(c,\overline{v},\beta'',n,m_1^*)$ tipping from 1 is possible, so degradation is not clearly profitable, so $(c,\overline{v},n,m_1^*) \notin P\big|_{\beta''}$.

Next suppose that under degradation at (c,v,β',n,m_1), the unique equilibrium is the interior equilibrium. Because degradation is strictly profitable, it must be that $m_1 > \underline{m}_1(c,v,\beta',n)$. Now define the market share

$$m_1^* \equiv \frac{1}{2}\Big(\underline{m}_1(c,v,\beta',n) + \min\Big\{\underline{m}_1(c,v,\beta'',n), 1\Big\}\Big).$$

Then $m_1^* > \underline{m}_1(c, v, \beta', n)$ and $m_1^* < \underline{m}_1(c, v, \beta'', n)$, so $(c, v, n, m_1^*) \in P\big|_{\beta'}$ but $(c, v, n, m_1^*) \notin P\big|_{\beta''}$.

The above analysis shows that if $P\big|_{\beta'}$ is nonempty, then $P\big|_{\beta''} \subset P\big|_{\beta'}$. ///

Appendix 2. Decomposing the Effects of β and c on the Largest Network's Incentives for Global Degradation

In this Appendix we provide a decomposition of how c and β affect firm 1's incentives for degradation. To understand the effects of the interconnection-quality choice on firm 1's equilibrium output, CRT (2000, p. 451) decompose the effect into a "demand expansion effect," $q_1^* + q_2^*$, that captures the effect of quality on total output; and a "quality differentiation effect," $q_1^* - q_2^*$, that depicts the effect of interconnection quality on firm 1's advantage in acquiring new customers. In this Appendix we introduce a similar decomposition that allows us to analyze, for $n \geq 1$, how firm 1's incentives for global degradation depend on β and c.

Firm 1's (interior) equilibrium output, q_1^*, can be represented as follows:

$$q_1^* = \frac{1}{2}\left\{\left(q_1^* + \sum_{i=2}^{n+1} q_i^*\right) + \left(q_1^* - \sum_{i=2}^{n+1} q_i^*\right)\right\}.$$

Here $q_1^* + \sum_{i=2}^{n+1} q_i^*$ is simply total equilibrium output. Also, $q_1^* - \sum_{i=2}^{n+1} q_i^*$ is firm 1's advantage in acquiring new customers, which translates into the increase in its advantage in overall network quality after new subscribers are added (for $n = 1$, this effect is positive; for $n \geq 2$, it may be negative).

The role of c. Let $TotQ(\theta, c)$ denote the equilibrium total output, $q_1^* + \sum_{i=2}^{n+1} q_i^*$, given interconnection quality θ (either 0 or 1) and installed base β (all other variables, such as market share and marginal cost, are held constant). Similarly, let $QDE(\theta, c)$ denote the quality differentiation effect, $q_1^* - \sum_{i=2}^{n+1} q_i^*$. Letting $q_1^*(\theta, c)$ denote firm 1's equilibrium output when it chooses quality θ (either 0 or 1) and the common marginal cost is c, we can express firm 1's output as a sum of the two terms described above: $2q_1^* = TotQ + QDE$. If the difference $q_1^*(0, c) - q_1^*(1, c)$ is positive, then for cost parameter c, firm 1 would prefer global degradation over accommodation. Next we determine how an increase in c affects this difference, holding the other parameters constant. The terms *TotQ* and *QDE* come into play in expressing the effect of c on these incentives for degradation as follows, with $c_h > c_l$:

$$2\left\{\left(q_1^*(0,c_h)-q_1^*(1,c_h)\right)-\left(q_1^*(0,c_l)-q_1^*(1,c_l)\right)\right\}$$

$$=\left[\left(TotQ(0,c_h)+QDE(0,c_h)\right)-\left(TotQ(1,c_h)+QDE(1,c_h)\right)\right]$$

$$-\left[\left(TotQ(0,c_l)+QDE(0,c_l)\right)-\left(TotQ(1,c_l)+QDE(1,c_l)\right)\right]$$

$$=\left\{\left(TotQ(0,c_h)-TotQ(1,c_h)\right)-\left(TotQ(0,c_l)-TotQ(1,c_l)\right)\right\}$$

$$+\left\{\left(QDE(0,c_h)-QDE(1,c_h)\right)-\left(QDE(0,c_l)-QDE(1,c_l)\right)\right\}.$$

It can be shown that

$$\frac{\partial\left[TotQ(0,c)-TotQ(1,c)\right]}{\partial c}>0,$$

for any parameters for which the equilibrium under degradation is interior and stable (recall that stability requires $\nu<1/(n+1)$); that is, degradation reduces total output by less when cost is high than when it is low. Thus, if parameters yield a stable interior equilibrium under degradation, then for $c_h>c_l$, it follows that

$$\left(TotQ(0,c_h)-TotQ(1,c_h)\right)-\left(TotQ(0,c_l)-TotQ(1,c_l)\right)>0.$$

Similarly, it can be shown that

$$\frac{\partial\left[QDE(0,c)-QDE(1,c)\right]}{\partial c}\geq 0$$

for any parameters such that the degradation equilibrium is an interior equilibrium (the inequality is strict if $n>1$); that is, degradation increases firm 1's network quality advantage more when cost is high than when it is low. Consequently, for $c_h>c_l$ we have

$$\left(QDE(0,c_h)-QDE(1,c_h)\right)-\left(QDE(0,c_l)-QDE(1,c_l)\right)\geq 0,$$

where the inequality is strict for $n>1$. Thus, increases in c produce reinforcing effects that favor degradation: first, there is a smaller loss in total output from degradation at the higher cost level; second, firm 1's increase in its absolute network quality advantage is greater at the higher cost level. Hence, the above decomposition yields

$$2\left\{\left(q_1^*(0,c_h)-q_1^*(1,c_h)\right)-\left(q_1^*(0,c_l)-q_1^*(1,c_l)\right)\right\}$$

$$=\left\{\left(TotQ(0,c_h)-TotQ(1,c_h)\right)-\left(TotQ(0,c_l)-TotQ(1,c_l)\right)\right\}$$

$$+\left\{\left(QDE(0,c_h)-QDE(1,c_h)\right)-\left(QDE(0,c_l)-QDE(1,c_l)\right)\right\}$$

$$>0.$$

Thus, if at marginal cost c_l firm 1 is just indifferent between degradation and accommodation, then at a slightly higher cost level it will strictly prefer degradation.

The role of β. As with the analysis of marginal cost, we let $TotQ(\theta,\beta)$ denote the equilibrium total output, $q_1^* + \sum_{i=2}^{n+1} q_i^*$, given interconnection quality θ and installed base β, and let $QDE(\theta,\beta)$ denote the quality differentiation effect, $q_1^* - \sum_{i=2}^{n+1} q_i^*$. The effect of β can be decomposed analogously to that above, here with $\beta_h > \beta_l$:

$$2\left\{\left(q_1^*(0,\beta_h) - q_1^*(1,\beta_h)\right) - \left(q_1^*(0,\beta_l) - q_1^*(1,\beta_l)\right)\right\}$$
$$= \left\{\left(TotQ(0,\beta_h) - TotQ(1,\beta_h)\right) - \left(TotQ(0,\beta_l) - TotQ(1,\beta_l)\right)\right\}$$
$$+ \left\{\left(QDE(0,\beta_h) - QDE(1,\beta_h)\right) - \left(QDE(0,\beta_l) - QDE(1,\beta_l)\right)\right\}.$$

It can be shown that

$$\frac{\partial\left[TotQ(0,\beta) - TotQ(1,\beta)\right]}{\partial\beta} < 0,$$

for any parameters for which the equilibrium under degradation is interior and stable; that is, degradation reduces total output by more when the installed base is large than when it is small. Thus, if parameters yield a stable interior equilibrium under degradation, then for $\beta_h > \beta_l$, it follows that

$$\left(TotQ(0,\beta_h) - TotQ(1,\beta_h)\right) - \left(TotQ(0,\beta_l) - TotQ(1,\beta_l)\right) < 0.$$

Also, it can be shown that for all relevant cases[26],

$$\frac{\partial\left[QDE(0,\beta) - QDE(1,\beta)\right]}{\partial\beta} > 0,$$

which says that the increase in firm 1's network quality advantage from degradation is greater when the installed base is large than when it is small. Thus, increases in β have opposing effects on firm 1's incentives for degradation. However, it can be shown that the second effect dominates the first, so that if at β_l firm 1 is indifferent between accommodation and degradation, then, holding constant firm 1's share of the installed base, it will strictly prefer degradation for $\beta_h > \beta_l$.

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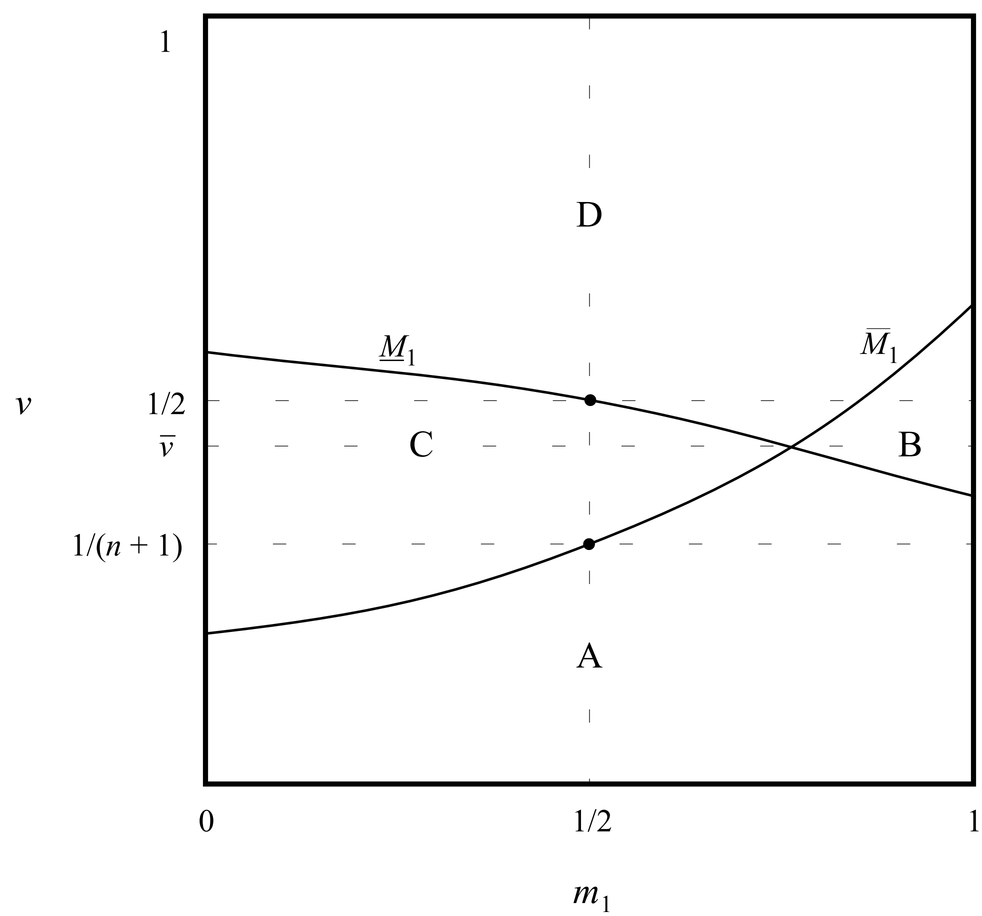
[26] The only cases in which, for stable equilibria, this derivative is negative arise when $n = 2$ and firm 1's market share is less than 53/104. But in these situations, when comparing the stable interior equilibrium under degradation with that under accommodation, firm 1 always prefers accommodation, for all $\beta > 0$. For such cases, changes in β do not affect firm 1's choice of interconnection quality (perfect interconnection), which is our primary concern.



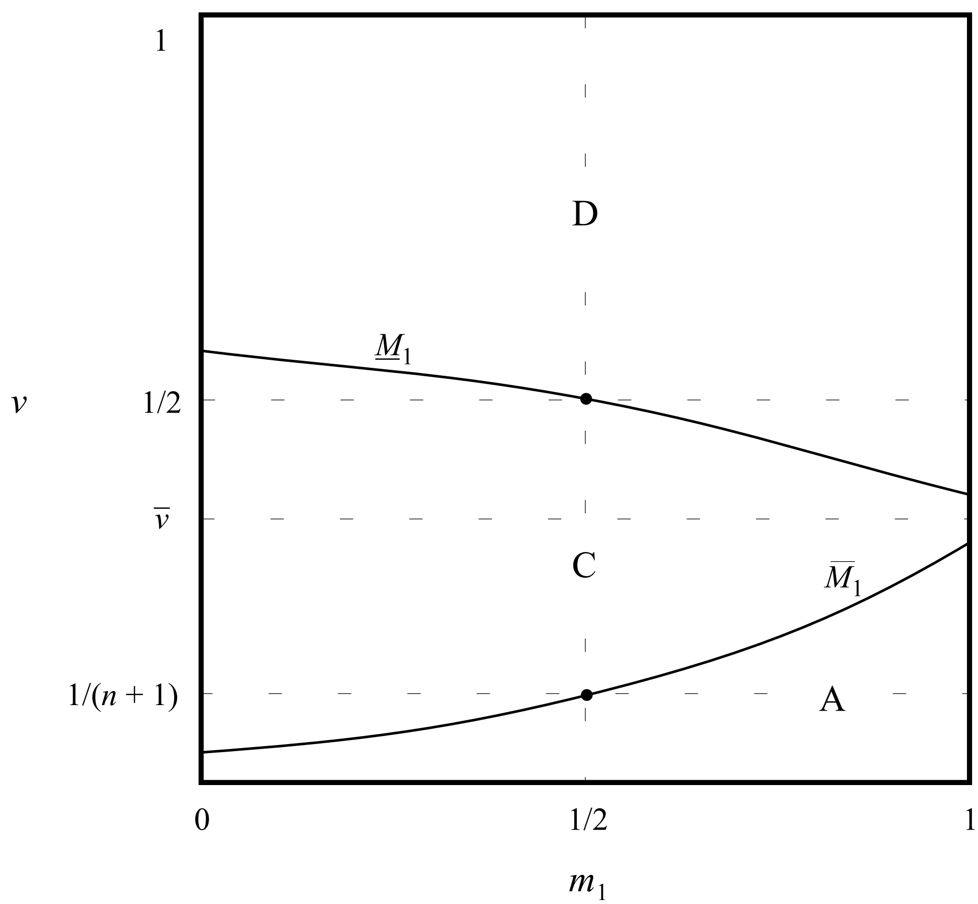
Region A: The unique equilibrium is interior

Region B: The unique equilibrium is tipping to 1

Region C: The unique equilibrium is tipping from 1

Region D: Three equilibria exist: tipping to 1, tipping from 1, and the interior equilibrium

Figure 1(a). Equilibria Under Degradation (large β)



Region A: The unique equilibrium is interior

Region C: The unique equilibrium is tipping from 1

Region D: Three equilibria exist: tipping to 1, tipping from 1, and the interior equilibrium

Figure 1(b). Equilibria Under Degradation (small β)

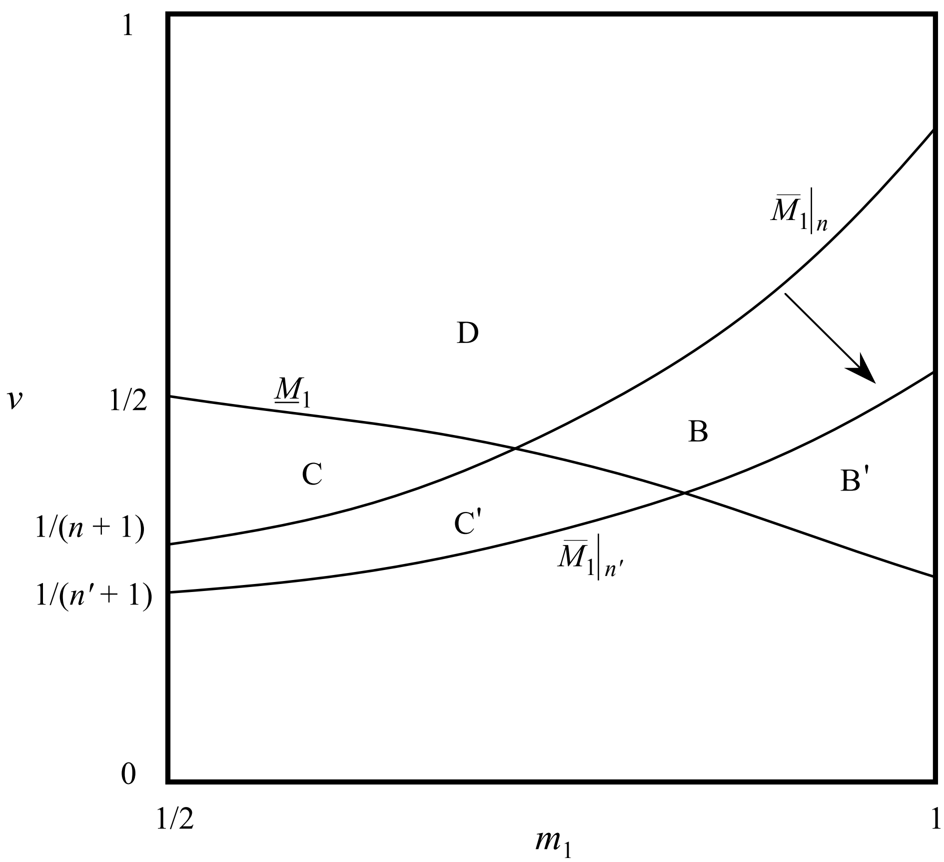
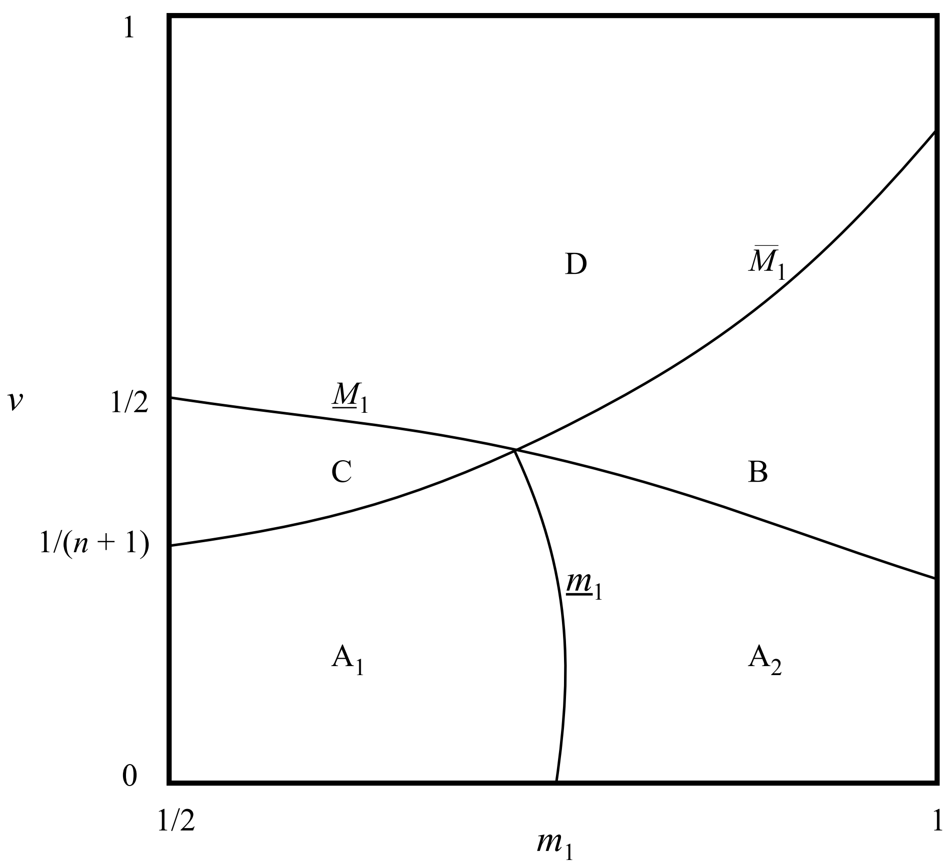


Figure 2. The Effect of n on Possibilities for Tipping Equilibria (large β)



No Degradation: region A_1 — degradation leads to **worse interior equilibrium for firm 1**

region C — degradation leads to **tipping away from firm 1**

Degradation: region A_2 — degradation leads to **better interior equilibrium for firm 1**

region B — degradation leads to **tipping to firm 1**

Ambiguous: region D — degradation can lead to **either tipping equilibrium**

Figure 3. Equilibrium Possibilities and Interconnection Choice

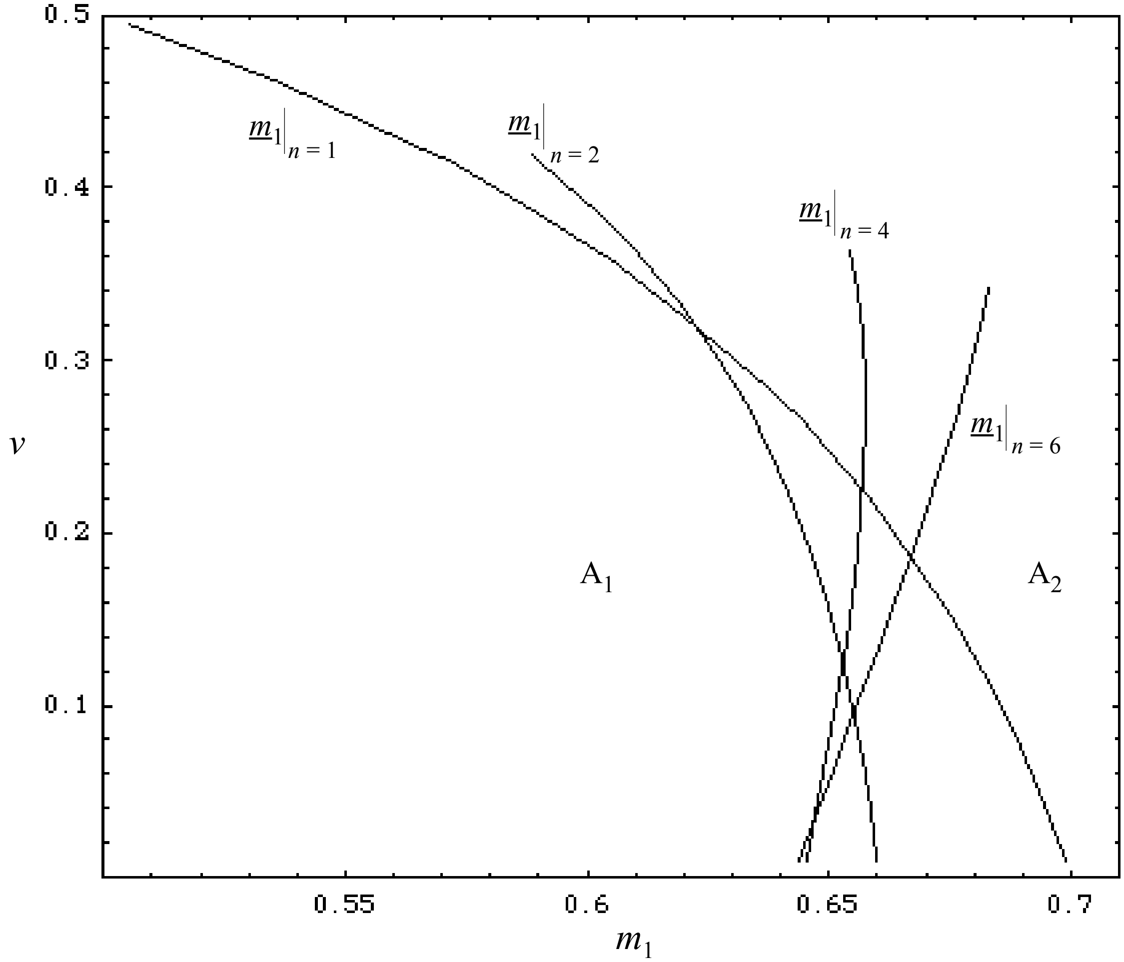


Figure 4. Effects of n on Profitability of Degradation When the Unique Equilibrium Is Interior ($\beta = 1$, $c = 0.7$)

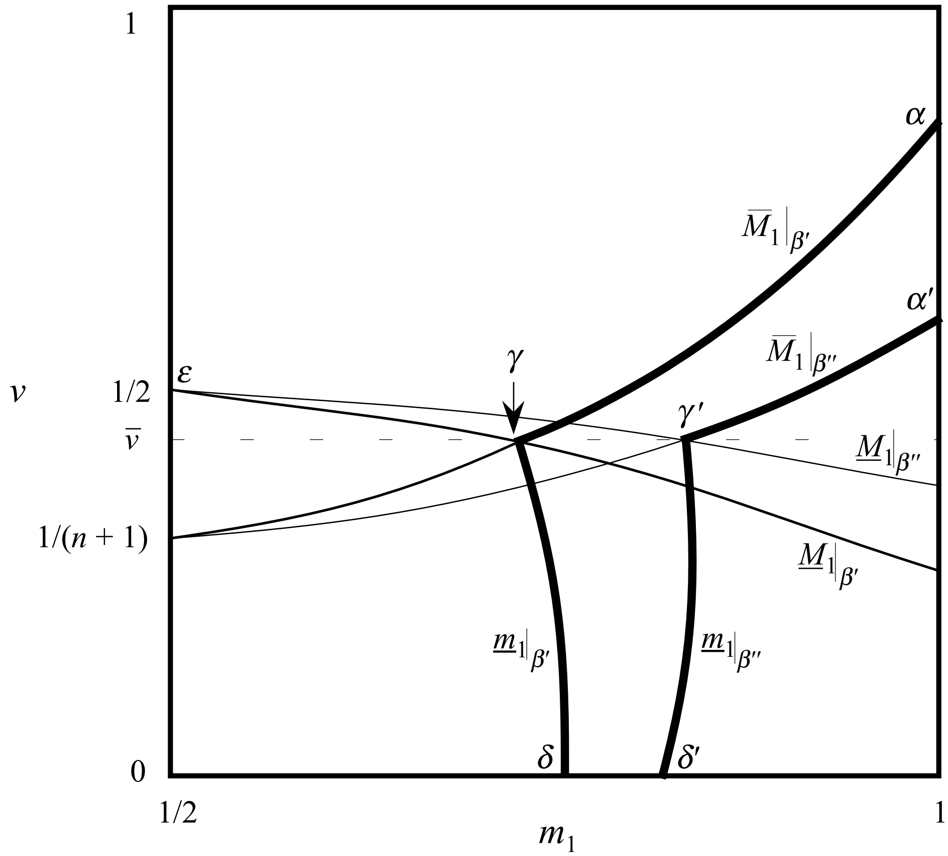
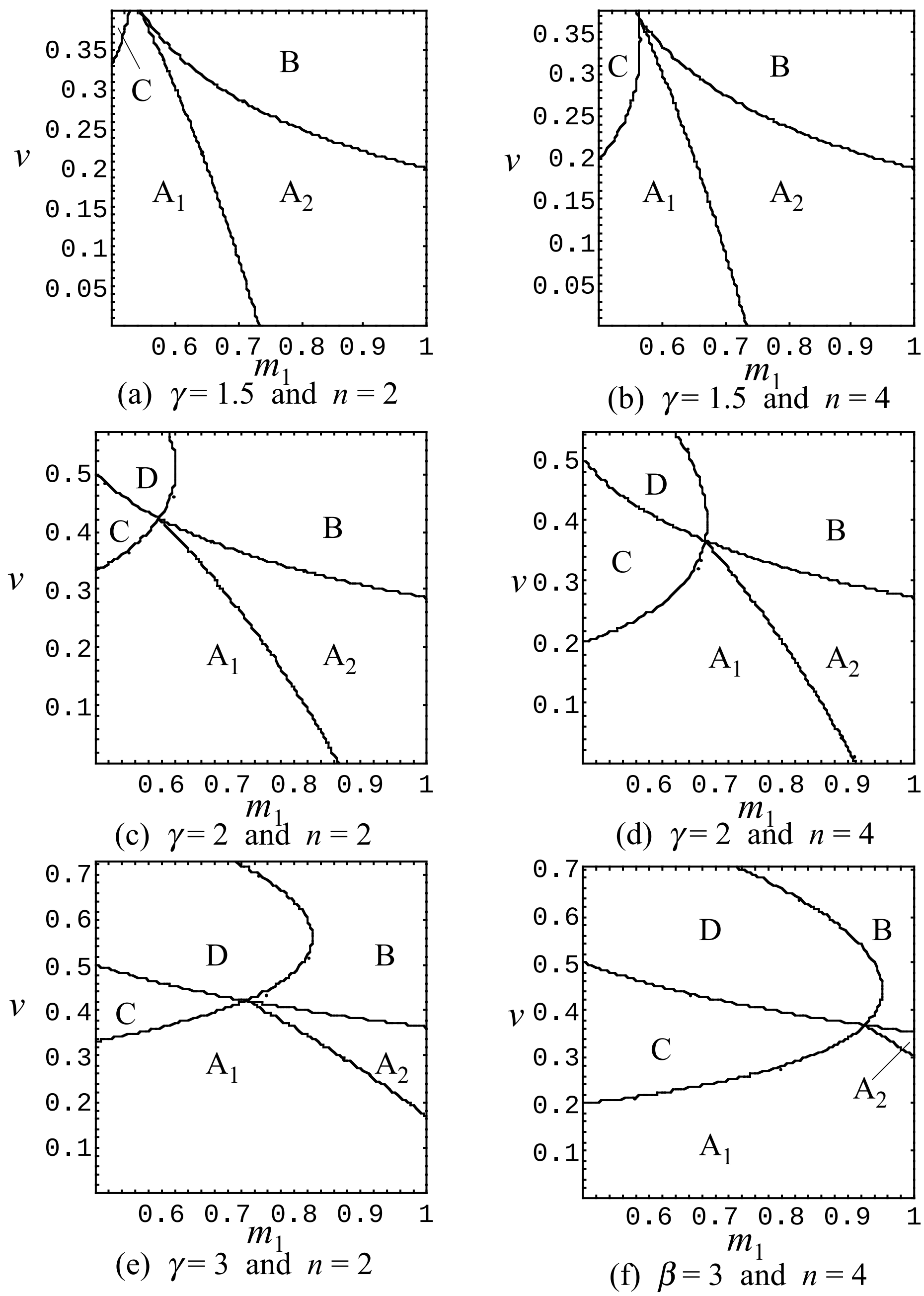


Figure 5. Effects of Increased Scope for Market Expansion: $\beta'' < \beta'$



No Degradation: region A_1 — degradation leads to **worse interior equilibrium for firm 1**

region C — degradation leads to **tipping from firm 1**

Degradation: region A_2 — degradation leads to **better interior equilibrium for firm 1**

region B — degradation leads to **tipping to firm 1**

Ambiguous: region D — degradation can lead to **either tipping equilibrium**

Figure 6. Equilibrium Possibilities and Interconnection Choice: Market Expansion Scenarios