

# Informational Integration and FX Trading

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## Abstract

This paper addresses international financial integration in a new way. We focus on informational integration, specifically, the importance of information conveyed by order flow in major currencies for pricing minor currencies. We develop a multi-currency model of portfolio allocation in the presence of dispersed information. We then test the model's implications using four months of concurrent transaction data on nine currencies. The model explains 45 to 78 percent of daily returns in all nine currencies. Moreover, its prediction that order flow in individual markets should be relevant for determining prices in other markets is borne out.

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# Informational Integration and FX Trading

## 1. Introduction

This paper addresses international financial integration in a new way. Existing work on integration can be grouped into two distinct lines, which we term speculative integration and geographic integration. Work on speculative integration is less focused on countries per se, and more focused on whether returns are consistent with speculative efficiency as described by international parity conditions (e.g., uncovered interest parity and real interest parity).<sup>1</sup> By focusing on parity conditions, these studies relate the degree of integration to relative returns across money market instruments. The second line of work on integration, the geographic line, is more focused on national boundaries and more focused on absolute pricing of broad classes of securities.<sup>2</sup> Representative of this geographic line is the recent survey by Karolyi and Stulz (2001), which opens with the following definition: “markets where assets have the same price regardless of where they are traded are said to be integrated, while markets where the price of an asset depends on where it is traded are said to be segmented.” A recent example of analysis in this line is Bekaert and Harvey (1995), who address whether country equity indexes are priced according to covariance with the world market (as an integrated-market CAPM would predict) or according to own variance (as a closed-economy CAPM would predict).

In this paper we focus instead on a dimension of integration that we term informational integration.<sup>3</sup> Specifically, our objective is to determine whether, and to what extent, information revealed via trades in a given currency market is impounded in other currency markets. To the extent it is, we term these currency markets informationally integrated. Important to our working definition is the phrase “revealed in a given currency market,” by which we mean information that is

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<sup>1</sup> See, among others, Frenkel and Levich (1977), Froot and Frankel (1989), Frankel and Okongwa (1996), Marston (1997), and Lothian (2000).

<sup>2</sup> See Solnik (1974), Grauer et al. (1976), Harvey (1991), Chan et al. (1992), and Stulz (1995), among many others)

<sup>3</sup> See also related work by Hasbrouck (1995) and Hasbrouck and Seppi (2001) that addresses the question of where price discovery takes place when related securities are traded on multiple markets.

not otherwise publicly available. We have in mind here dispersed bits of information that get revealed and aggregated in the trading process, for example, information about time-varying risk preferences, hedging demands, or interpretations of macroeconomic announcements. Thus, we depart from the traditional macro approach to exchange rates, in which all private agents share common information, focusing instead on the microeconomics of information aggregation. We develop a model that incorporates micro variables that summarize how, precisely, dispersed information relevant to this market is aggregated. The model identifies interdealer order flow as the medium through which information about asset demands becomes impounded into equilibrium exchange rates.<sup>4</sup> Specifically, our model shows that order flows in a given currency should be relevant not only for pricing that currency, but also for pricing other currencies. This arises due to the presence of dispersed information coupled with a portfolio problem that allocates wealth optimally across all currencies. Alternatively, if public demand for a particular foreign currency depends only on its own rate of return, the exchange rate only responds to changes in order flow in that one currency. In this case, order flows arising from other trades have no impact because they convey no relevant information.

We test the model's implications using four months of concurrent transaction data on nine major currencies that float against the US dollar. Though four months of data is rather little on a macroeconomic time scale, it yields precise estimates in the context of our microeconomic analysis. (We have no reason to believe that these four months—May to August, 1996—are unrepresentative.) In all nine currencies, the model explains 45 to 78 percent of daily returns.<sup>5</sup> Moreover, the model's prediction that order flow information should be relevant for determining prices in multiple markets is borne out. Two major currencies in particular have

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<sup>4</sup> Order flow—a concept from microstructure finance—refers to *signed* volume. Trades can be signed in microstructure models depending on whether the “aggressor” is buying or selling. (The dealer posting the quote is the passive side of the trade.) For example, a sale of 10 units by a trader acting on a dealer's quotes is order flow of  $-10$ . In rational-expectations (RE) models of trading, order flow is undefined because all transactions in that setting are symmetric. One might conclude from RE models that one could never usefully distinguish the “sign” of a trade between two willing counterparties. A large empirical literature in microstructure finance suggests otherwise (Lyons 2001).

<sup>5</sup> Our choice of the daily data frequency is motivated by our desire to “integrate out” the transitory price effects from order flow that show up in intraday data. Daily exchange rate changes are well approximated by a martingale, so accounting for daily increments translates into accounting for exchange rate levels over the long run (i.e. accounting for more fundamental factors).

substantial informational relevance in other markets: the German Mark and the Swiss Franc. Because 8 of our 9 currencies are European, we interpret this result as supportive of recent findings from work on information geography, e.g., Hau (2001). Finally, we find that order flow information accounts on average for about 80 percent of the (high) unconditional covariance in currency returns. This suggests that the stylized fact of high covariance in currency returns is not due to the impounding of common public news directly into prices without any role for order flow (which is how the covariance is traditionally modeled). Overall, our findings provide qualitatively new support for the hypothesis that international money markets are indeed integrated.

The remainder of the paper is in four sections. Section 2 presents the specification of our model. Section 3 presents the model's equilibrium and discusses how integration affects exchange rate dynamics. Section 4 describes the data and presents empirical analysis of the link between exchange rates and order flow. Section 5 concludes.

## 2. The Model

The model we use extends the Portfolio Shifts model from Evans and Lyons (2002) to a multi-currency setting. It is designed to show how trading in the foreign exchange market reveals dispersed information that underlies public currency demands. In particular, our aim is to demonstrate that integration implies a link between a given exchange rate and order flows in markets for other currencies.

### 2.1 Environment

Consider an infinitely lived, pure exchange economy with  $K+1$  assets, one of which is riskless and  $K$  having stochastic payoffs, which we take to represent foreign currencies. The return on the riskless asset is normalized to unity and is in zero net supply. The periodic (daily) payoff on foreign exchange, denoted by the  $(K \times 1)$  vector  $R$ , is composed of a series of increments:

$$R_t = \sum_{\tau=1}^t \Delta R_\tau. \quad (1)$$

The vector of increments  $\Delta R_t$  are i.i.d.  $N(0, \Sigma_R)$  over time and are observed before trading in each period. These realized increments represent the flow of publicly available macroeconomic information over time (e.g., changes in interest rates). The matrix  $\Sigma_R$  need not be diagonal (i.e., the contemporaneous increments may be correlated across currencies).

The foreign exchange market has two participant types, customers and dealers. There is a continuum of customers, indexed by  $z \in [0, 1]$  and  $N$  dealers, indexed by  $i$ . Each of these  $N$  dealers makes a market in all  $K$  of the foreign currencies.<sup>6</sup> The mass of customers on  $[0, 1]$  is large (in a convergence sense) relative to the  $N$  dealers. (This will insure that dealers have a comparative disadvantage in holding overnight positions.) Customers and dealers have constant absolute risk aversion (CARA) and maximize utility of the following form:

$$U_t = E_t \left\{ - \sum_{s=0}^{\infty} \delta^s \exp(-\theta c_{t+s}) \right\} \quad (2)$$

where  $E_t$  is the expectations operator conditional on agent's information at time  $t$ , and  $c_{t+s}$  is consumption in period  $t+s$ . We assume that all agents have the same time discount factor  $\delta$ , and risk aversion parameter  $\theta$ . The specifics of the trading environment are described below (with formal setup of the dealer's problem presented in the appendix).

Within each day  $t$ , there are three rounds of trading:

- Round 1:**  $\Delta R_t$  is realized, dealers quote prices and trade with the public.
- Round 2:** Dealers trade among themselves (to share inventory risk)
- Round 3:** Dealers trade again with the public (to share risk more broadly)

The timing of events within each day is shown in Figure 1, which also introduces some notation clarified below.

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<sup>6</sup> Each of the individual dealer problems, then, is perhaps best thought of as representing the problem faced by the whole currency dealing desk at each individual bank.

## 2.2 Trading Round 1

All market participants observe  $\Delta R_t$ , the period's increment to the vector of payoffs  $R$ . On the basis of this increment and other available information, each dealer simultaneously and independently quotes a scalar price for each of the  $K$  assets to his customers at which he agrees to buy and sell any amount.<sup>7</sup> We denote the  $(K \times 1)$  vector of round-one prices of dealer  $i$  on day  $t$  as  $P_{it}$ . (Hereafter we shall suppress the  $t$  subscript for the sake of clarity whenever possible.) Each dealer then receives a net customer-order realization for each of the  $K$  assets, a  $(K \times 1)$  vector  $C_{it}$  that is executed at his quoted price vector,  $P_{it}$ , where negative elements of  $C_{it}$  denote a net customer sale (dealer  $i$  purchase). Each of these  $N$  customer-order realizations of  $C_{it}$  is distributed Normally:

$$C_{it} \sim N(0, \Sigma_C). \quad (3)$$

Customer orders are independent across dealers but not necessarily across currencies (i.e., the off-diagonal elements of  $\Sigma_C$  can be non-zero). Customer orders are also distributed independently of the vector of public-information increments  $\Delta R_t$ . These orders represent liquidity demand shocks from the non-dealer public. Their realizations are not publicly observable.

## 2.3 Trading Round 2

Round 2 is the interdealer trading round. Each dealer simultaneously and independently quotes a scalar price for each of the  $K$  assets to other dealers at which he agrees to buy and sell any amount. These interdealer quotes are observable and available to all dealers in the market. Each dealer then simultaneously and independently trades on other dealers' quotes. Orders for a given asset at a

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<sup>7</sup> While it is true that a bid-ask spread of zero would not induce entry into dealing, introducing a bid-offer spread (or price schedule) in round one to endogenize the number of dealers is a straightforward—but distracting—extension of our model. In equilibrium, expected utility of the commission will just balance the utility cost of marketmaking under risk aversion and asymmetric information. The model's simultaneous-move nature is in the spirit of simultaneous-move games more generally (versus sequential-move games).

given price are split evenly across any dealers quoting that price. Let  $T_{i2}$  denote the  $(K \times 1)$  vector of (net) interdealer trade initiated by dealer  $i$  in round 2. At the close of round 2, all dealers observe the net interdealer order flow, in each of the  $K$  assets from that period:

$$X_t = \sum_{i=1}^N T_{i2}. \quad (4)$$

### 2.4 Trading Round 3

In round 3, dealers share overnight risk with the non-dealer public. Unlike round 1, the public's motive for trading in round 3 is non-stochastic and purely speculative. Initially, each dealer simultaneously and independently quotes a scalar price for each of the  $K$  assets at which he agrees to buy and sell any amount. These quotes are denoted by the  $(K \times 1)$  vector  $P_{3t}$  are observable and available to the public at large.

The mass of customers on the interval  $[0,1]$  is large (in a convergence sense) relative to the  $N$  dealers. This implies that the dealers' capacity for bearing overnight risk is small relative to the public's capacity. Nevertheless, the public is assumed to have finite risk bearing capacity, represented by negative exponential utility in equation (2) with aggregate risk tolerance coefficient  $\theta$ . Dealers therefore set prices so that the public willingly absorbs dealer inventory imbalances, and each dealer ends the day with no net position. These round-3 prices are conditioned on the round-2 interdealer order flow. The interdealer order flow informs dealers of the size of the total inventory that the public needs to absorb to achieve stock equilibrium.

### 2.5 The Public

The public comprises a continuum of non-dealer customers, indexed by  $z \in [0,1]$ . Each member of the public's demand for the risky assets,  $C_{3t}$ , is determined by maximizing expected utility in equation (2) subject to the budget constraint

$$V_{t+1} = C_{3t}' (P_{3t+1} + R_{t+1} - P_{3t}) + V_t - c_t, \quad (5)$$

where  $V_t$  denotes wealth at the start of day  $t$ . The conditioning information available to the public at the beginning of round 3 on day  $t$  is

$$\Phi_t = \left\{ \left\{ \Delta R_\tau \right\}_{\tau=1}^t, \left\{ X_\tau \right\}_{\tau=1}^{t-1}, \left\{ P_{3\tau} \right\}_{\tau=1}^t \right\},$$

where  $P_{3t}$  denotes the common vector of round 3 prices quoted by all dealers (see Proposition 1).

## 2.6 Dealers

Each of the  $N$  dealers in the market determines quotes and speculative demand by maximizing the utility function shown in equation (2). Because dealers hold no overnight positions and face a daily stochastic environment that is time invariant, their optimization problem collapses to a series of independent trading problems in equilibrium, one for each period. Within a given period  $t$ , let  $W_{i\tau}$  denote the end-of-round  $\tau$  wealth of dealer  $i$ , where we use the convention that  $W_{i0}$  denotes wealth at the end of period  $t-1$ . (Again, we suppress the period subscript  $t$  when clarity permits.) The dealers' problem can be written as:

$$\begin{aligned} \text{Max} \quad & E[-\exp(-\theta W_{i3} | \Omega_i)] \\ & \{P_{i1}, P_{i2}, P_{i3}, T_{i2}\} \end{aligned} \tag{6}$$

s.t.

$$W_{i3} = W_{i0} + C_{i1}'(P_{i1} - P_{i2}^*) + (D_{i2} + E[T_{i2}^* | \Omega_{i2}])'(P_{i3} - P_{i2}^*) - (T_{i2}^*)'(P_{i3} - P_{i2}^*) .$$

$P_{i\tau}$  is dealer  $i$ 's vector of round- $\tau$  quotes (one for each of the  $K$  risky assets) and a  $*$  denotes a vector of interdealer quotes or trades received by dealer  $i$ . The dealers' problem is defined over four choice variables: the three ( $K \times 1$ ) vectors of quotes  $P_{i1}$ ,  $P_{i2}$ , and  $P_{i3}$ , and the dealer's outgoing interdealer trades in round 2 for each of the  $K$  assets,  $T_{i2}$ . These outgoing interdealer trades in round 2 have three components:

$$T_{i2} = C_{i1} + D_{i2} + E[T_{i2}^* | \Omega_{T_{i2}}], \tag{7}$$



where  $D_{i2}$  is dealer  $i$ 's speculative demand for the  $K$  assets in round 2, and  $E[T_{i2}^*|\Omega_{T_{i2}}]$  is the dealer's attempt to hedge against incoming orders from other dealers. The last three terms in the dealer's budget constraint capture capital gains/losses from round-1 customer orders  $C_{i1}$ , round-2 speculative demand  $D_{i2}$ , and the round-2 position disturbance from incoming interdealer orders  $T_{i2}^*$ . The conditioning information available to dealers,  $\Omega_i$  at each decision node is summarized below:

$$\begin{aligned}
\text{Round 1 quotes:} & \quad \Omega_{P_{i1}} \equiv \{ \{ \Delta R_\tau \}_{\tau=1}^t, \{ X_\tau \}_{\tau=1}^{t-1} \} & (8) \\
\text{Round 2 quotes:} & \quad \Omega_{P_{i2}} \equiv \{ \Omega_{P_{i1}}, C_{i1} \} \\
\text{Round 2 trades:} & \quad \Omega_{T_{i2}} \equiv \{ \Omega_{P_{i2}} \} \\
\text{Round 3 quotes:} & \quad \Omega_{P_{i3}} \equiv \{ \Omega_{P_{i2}}, X_t \}.
\end{aligned}$$

### 3. Exchange Rates, Order Flows and the Structure of Demand

The equilibrium in our model describes the interaction between the  $N$  dealers and the public. Trade between dealers takes the form of a simultaneous-move game: at each decision node, every dealer in the market simultaneously sets quotes or trades. This means that dealers cannot condition on other dealers' quotes or trading decisions when making their own. We identify equilibrium quotes and trades using the concept of a Bayesian-Nash Equilibrium, or BNE. Under BNE, Bayes rule is used to update beliefs and strategies are sequentially rational given those beliefs. Details of the solution are presented in the Appendix. Here we discuss the main features of the equilibrium.

We first consider the properties of the optimal quoting strategies:

**PROPOSITION 1:** A quoting strategy is consistent with symmetric BNE only if the round-one and round-two quotes for each of the  $K$  assets are common across dealers and equal to:

$$P_{1,t} = P_{2,t} = P_{3,t-1} + \Delta R_t, \quad (9)$$

where  $P_{3,t-1}$  is the vector of round-three quote from the previous period, and  $\Delta R_t$  is the vector of public-information payoff increments at the beginning of period  $t$ .

**PROPOSITION 2:** A quoting strategy is consistent with symmetric BNE only if the common round-three quote is:

$$P_{3t} = P_{2t} + \Lambda X_t \quad (10)$$

where  $\Lambda$  is a  $(K \times K)$  matrix of coefficients.

The intuition behind these propositions is straightforward. Within any given round, all dealers quote a common price for each of the  $K$  assets in order to rule out arbitrage opportunities. (Recall that quoted prices are observable by all dealers in round 2, and all members of the public in rounds 1 and 3, and are good for any amount.) This means that the quoted prices can only be conditioned on common information. In rounds one and two, this includes the previous period's round-three price, plus the public-information innovation at the beginning of period  $t$ ,  $\Delta R_t$ . The vector of prices quoted in round 3,  $P_{3t}$ , reflects information in both  $\Delta R_t$  and order flows,  $X_t$  (observed at the end of round 2).

The information compounded in the current payoff increment  $\Delta R_t$  is straightforward: the current increment changes the vector of payoffs  $R$  expected by all dealers and members of the public, so each quoted price changes one-for-one with the payoff increment associated with each risky asset. The information conveyed by the vector of order flows is more complex. Proposition 2 shows that order flows arising from trade involving asset  $k$  (i.e., the  $k$ 'th row of  $X_t$ ) contain information relevant to pricing the  $k$ 'th asset when the  $k$ 'th diagonal element of  $\Lambda$  is non-zero. In addition, order flow relating to the  $k$ 'th asset may also contain information relevant for pricing the  $j$ 'th asset if element  $\{j,k\}$  of  $\Lambda$  is non zero.

To understand the determinants of  $\Lambda$ , we need two further results:

**PROPOSITION 3:** The trading strategy profile:

$$T_{i2} = \alpha C_{i1} \quad (11)$$

$\forall i \in \{1, \dots, N\}$ , where  $\alpha$  is a  $(K \times K)$  matrix, conforms to a BNE.

**PROPOSITION 4:** If dealers quote prices in accordance with Propositions 1 and 2, and members of the public hold rational expectations concerning the distribution of equilibrium round 3 prices, the public's aggregate demand for foreign currency is

$$C_{3t} = \Gamma(E[P_{3t+1} + R_{t+1} | \Phi_t] - P_{3t}), \quad (12)$$

where  $\Gamma = (\theta \Sigma_p)^{-1}$  with  $\Sigma_p$  denoting the covariance of  $P_{3t+1}$  conditioned on information available to the public in round 3 of period t,  $\Phi_t$ .

Proposition 3 states that the optimal trading rule followed by dealers in round 2 for each of the  $K$  risky assets can be expressed as a linear function of the customer orders received in round 1. While trades differ across dealers (indexed by  $i$ ) because they receive different customer orders, the relation between the size of customer order and the dealer's trade is the same across all dealers. In particular, the  $k,j$  element of the matrix  $\alpha$  identified how much any dealer will trade in asset  $k$  conditional on receiving a one unit customer order for asset  $j$ .

Proposition 4 shows the structure of the public's equilibrium demand for currencies in round 3. In equilibrium, the solution to the public's portfolio problem takes the familiar form of equation (12) because the value function depends exponentially on wealth, which, in turn, is conditionally normally distributed. Here the price sensitivity of demand depends on the degree of risk aversion,  $\theta$ , and the conditional covariance of prices  $\Sigma_p$ . When  $\Sigma_p$  is diagonal, the demand for asset  $k$  is insensitive to the expected price change for asset  $j$  ( $\neq k$ ).

With the aid of equations (9) – (12) we can now describe how the link between order flows and prices is related to the structure of the public's demand for currencies.

**PROPOSITION 5:** In a BNE, where members of the public hold rational expectations, the change in the vector of prices quoted in round 3 from day t-1 to day t is:

$$\Delta P_t = \Delta R_t + \Lambda X_t, \quad (13)$$

where the matrix  $\Lambda$  is diagonal if and only if  $\Sigma_C$  and  $\Sigma_R$  are diagonal.

Proposition 5 shows how the link between order flows and price changes relates to the structure of the public's currency demand. In particular, the proposition identifies the conditions under the price change for each foreign currency is affected by own order flows only; namely when  $\Sigma_C$  and  $\Sigma_R$  are both diagonal.<sup>8</sup> When  $\Sigma_C$  or  $\Sigma_R$  is not diagonal, order flows arising from trades between the home and a given foreign currency can convey information that affects the daily price change in each foreign currency. In the case of a non-diagonal  $\Sigma_C$ , the notion of conveying information takes on special significance. Even in common knowledge environments, one expects that supply and demand in one market can affect prices in another. In the case of a non-diagonal  $\Sigma_C$ , however, the demands in other markets are conveying information that is not common knowledge, and this represents a distinct channel through which order flow has price impact.

To understand the intuition behind this result, consider the case where  $\Sigma_C$  and  $\Sigma_R$  are diagonal. Here the customer orders received by each dealer in round 1 are uncorrelated across currencies and the public's demand for currency  $k$  in round 3 depends only on the expected price change for currency  $k$  (see equation 12). In equilibrium, the vector of outgoing interdealer trades in round 2,  $T_{i2}$ , comprise customer orders from round 1,  $c_{i1}$ , and the dealers speculative demand,  $D_{i2}$  (because  $E[T_{i2}^* | \Omega_{T_{i2}}] = 0$ , see equation 7). When  $\Sigma_C$  and  $\Sigma_R$  are diagonal, there is no loss of expected utility from a dealer choosing his speculative demand for each currency separately. Consequently, as in Evans and Lyons (2002), the dealers trade during round 2 is proportional to the customer order he received in round 1 on a currency by currency basis (i.e.,  $T_{i2} = \alpha C_{i1}$  where  $\alpha$  is diagonal). This implies that dealers can infer the size of the aggregate portfolio shift for currency  $k$  on the part of the public in round 1 from the order flow from trades between the home and  $k$ 'th currency alone (i.e., the  $k$ 'th. element of  $X$ ). Dealers also know that the public needs to be induced to re-absorb this portfolio shift in round 3 and that the public's demand for

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<sup>8</sup> There has to be a strong a priori presumption that  $\Sigma_R$  is not diagonal: for example, when information is relevant to the U.S. then one would expect it to affect the payoffs on all bilateral exchange rates that involve the dollar. Our specification of  $K$  risky assets, each with its own random payoff, does not embed this possibility of nation-specific information explicitly (though a non-diagonal  $\Sigma_R$  can accommodate it). Note too that triangular arbitrage will also introduce non-diagonal structure to  $\Sigma_R$ .

currency  $k$  depends only on the expected price change for currency  $k$  (see equation 12). This inducement requires an adjustment in the price of the  $k$  currency alone. Hence, only order flows associated with trade between the home and  $k$ 'th. foreign currency affect the price of the  $k$ 'th. currency.

When both  $\Sigma_c$  and  $\Sigma_R$  are not diagonal, the relation between prices and order flows change for two reasons. First, when  $\Sigma_R$  is not diagonal, the price of each currency needs to be adjusted in response to *every* element of the aggregate round-1 customer order. This is necessary to induce the public to absorb the realized portfolio shift, which can be seen from inverting the public's round 3-demand curve:

$$P_{3t} = E[P_{3t+1} + R_{t+1} | \Phi_t] - \Theta \Sigma_p C_{3t}.$$

In equilibrium,  $E[P_{3t+1} | \Phi_t] = P_{3t-1} + \Delta R_t$ , so the aggregate portfolio shift,  $C_{3t}$ , must be completely accommodated by changing the vector of round 3 prices,  $P_{3t}$ . To clear the market, the price of the  $k$ 'th. currency must therefore be set with regard to the portfolio shifts in every currency according to the elements in the  $k$ 'th row of  $\Sigma_p$ .

The second difference arises because it is no longer optimal for dealer speculative demands in round 2 to be proportional to round 1-customer orders on a currency-by-currency basis. This means that a dealer's trade in currency  $k$  is no longer proportional to his customer order for currency  $k$ . As a result, it is not possible to infer the size of the aggregate portfolio shift for currency  $k$  on the part of the public in round 1 from the order flow from trades between the home and  $k$ 'th currency alone. In general, all the order flows are needed to make inferences about the size of the aggregate portfolio shift in each of the  $k$  currencies. Under these circumstances, all order flows will affect each price even if  $\Sigma_R$  is diagonal because they contain information about the size of the portfolio shift that the public must absorb, which affects the setting of each round 3 price.

To summarize, our model shows how interdealer order flows act as medium through which information about the public's asset demands becomes impounded into equilibrium exchange rates. In particular, the model shows that the information contained in individual order flows varies according to structure of the public's

portfolio demand and dealers' speculative demands. In special cases, order flows arising from trades between the home and a particular foreign currency act as a sufficient statistic for the aggregate portfolio shift that must be absorbed by the public, and this shift can be induced by the change in just one exchange rate. In the more general case, many order flows contain information relevant for determining the aggregate portfolio shifts in any one currency, and many prices must be adjusted to induce the public to absorb the shift. In these circumstances, multiple order flows appear as proximate determinants of daily exchange rate changes.

## 4. Empirical Analysis

### 4.1 Data

The dataset contains daily measures of actual transactions for nine spot markets over a four-month period, May 1 to August 31, 1996: Mark, Yen, Pound, Belgium Franc, French Franc, Swiss Franc, Krona, Lira and Guilder; all verses the Dollar. As noted in the introduction, we chose the daily frequency in order to “integrate out” the transitory price effects from order flow that show up in intraday data. (Daily exchange rate changes are well approximated by a martingale.) The data were collected from the Reuters Dealing 2000-1 system via an electronic feed customized for the purpose. Dealing 2000-1 is the most widely used electronic dealing system. According to Reuters, over 90 percent of the world's direct inter-dealer transactions take place through the system.<sup>9</sup> All trades on this system take the form of bilateral electronic conversations. The conversation is initiated when a dealer uses the system to call another dealer to request a quote. Users are expected to provide a fast two-way quote with a tight spread, which is in turn dealt or declined quickly (i.e., within seconds). To settle disputes, Reuters keeps a temporary record of all bilateral conversations. This record is the source of our data. (Reuters was unable to provide the identity of the trading partners for confidentiality reasons.)

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<sup>9</sup> In 1996, interdealer transactions accounted for about two-thirds of total trading in major spot markets. This two-thirds from interdealer trading breaks into two transaction types—direct and brokered. Direct trading accounted for about half of interdealer trade and brokered trading accounted for the other half. For more detail on the Reuters Dealing 2000-1 System, see Lyons (1995) and Evans (2001).

For these trades executed on D2000-1 our data set includes a bought/sold indicator that allows us to measure signed volume, i.e., order flow, directly. This is a major advantage: we do not have to use the noisy algorithms used elsewhere in the literature for signing trades. One drawback is that it is not possible to identify the size of individual transactions. For model estimation, order flow is therefore measured as the difference between the number of buyer-initiated and seller-initiated trades.<sup>10</sup>

The variables in our empirical model are measured daily. We take the spot rate, as the last purchase-transaction price (foreign currency/\$) before 4:00 pm London time.<sup>11</sup> To facilitate comparisons of our results across currencies, we focus on the daily change in the log transaction price rather than the change in the actual price. Order flow for currency  $k$ , is the difference between the number of buyer- and seller-initiated trades between currency  $k$  and the dollar (in thousands, negative sign denotes net dollar sales) during the 24 hours before 4:00 pm. Market activity after 4 PM on Friday is aggregated until 4 PM on Monday.

#### *4.2 Characteristics*

Sample statistics for daily exchange rate changes and order flows are reported in Table 1. As expected, the mean change in (log) prices is very close to zero and appears serially uncorrelated for all currencies. Thus, there is no evidence that daily exchange rate changes can be predicted from the history of past changes. The mean of daily order flow is also close to zero and serially uncorrelated for 7 of the 9 currencies. The exceptions are the order flows associated with the Pound and French Franc where the order flows display significant positive autocorrelations at lags 1 and 2.

Table 2 reports statistics on the contemporaneous correlations in daily exchange rate changes (i.e., daily changes in log prices) and order flows. The upper panel of the table shows the sample correlation between each of the exchange rates listed in the rows and columns. In all cases, the correlations are positive. The table

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<sup>10</sup> This is common in the literature; see, for example, Hasbrouck (1991). See also Jones et al. (1994) for analysis suggesting that trade size conveys no additional information (beyond that conveyed by the number of buys minus sells).

<sup>11</sup> The last purchase transaction is generally within a few seconds of the end of the hour. Using purchase transactions eliminates bid-ask bounce.

also reports the asymptotic p-value for the null hypothesis of a zero correlation in parenthesis below each sample correlation. With the exception of the pound-yen correlation, all the p-values are extremely small indicating that the positive sample correlations are indeed highly statistically significant. The lower panel of Table 2 reports analogous statistics for order flows. Overall, order flows are less strongly correlated across currency pairs than exchange rates. In some cases, the correlations are negative, but not statistically significantly. Correlations between the order flows associated with European currencies are generally positive, and statistically significant.

### 4.3 Model Estimates

The focus of our empirical analysis is equation (13) from Proposition 5, which links exchange rates to order flows and public information increments,  $\Delta R_t$ . For our empirical model, we assume the latter take the form of serially uncorrelated shocks that affect the exchange rate directly (i.e., these shocks have no effect on order flow, as is commonly assumed; see, e.g., Hasbrouck 1991). Our empirical version of (13) for exchange rate  $k$  is

$$\Delta p_t^k = \lambda_{kk} X_t^k + \sum_{j \neq k} \lambda_{kj} X_t^j + v_t^k, \quad (14)$$

where  $\Delta p_t^k$  is the daily change in the log  $k$  currency price of dollars, and  $X_t^j$  is the daily order flow associated with currency  $j$ . The coefficients,  $\lambda_{kj}$ , correspond to the elements from row  $k$  of the matrix  $\Lambda$  (from equation 13) identifying the price-impact of order flow  $j$  on exchange rate  $k$ . The effects of the public information increment are captured by the residual  $v_t^k$ . Theoretically speaking, these increments must be serially uncorrelated, and uncorrelated with all the order flows. They may, however, be correlated across exchange rates. For example, public news directly impacting the international value of the Dollar, will affect all the exchange rates in our sample. To account for this possibility, we estimate (14) as part of a GLS system.

Table 3 reports estimates for two versions of (14): one including the own order flow only,  $X_t^k$ ; the other including all the sample order flows. In six of the nine



exchange rates, the coefficient on own order flow,  $\lambda_{kk}$ , is positive and highly statistically significant when  $X_t^k$  is the only order flow included in the regression. Interestingly, the estimates of  $\lambda_{kk}$  are much larger in the cases of the less heavily traded currencies, such as the Lira, French Franc, Swiss Franc and Pound, than for the Mark and the Yen. (Recall that the dependent variable for each equation is the difference in the log foreign currency price of Dollars.) This observation is consistent with idea that the price impact of order flow should be large in less liquid markets. (see, for example, Kyle 1985). It is also worth noting that in the cases where the price impact of own order flow is statistically significant, order flow accounts for a substantial fraction of the variance in daily exchange rates. The  $R^2$  statistics range from 0.33 (in the case of the Pound) to 0.68 (in the case of Mark).

Estimates of (14) including all the order flows are reported in the lower portion of each cell in Table 3. Two striking results emerge from these estimates. First, for every exchange rate the array of order flows account for a substantial fraction of returns: the majority of the  $R^2$  statistics are over 65 percent, ranging between 45 and 78 percent. When judged in light of past empirical disappointments (Meese and Rogoff 1983, Frankel and Rose 1995), this is a remarkable finding. It provides strong confirmation that order flows are an important (proximate) determinant of daily exchange rate changes. Second, at least two order flows have a statistically significant impact on the exchange rate in every case. For every currency save the Yen, order flows associated with the Mark have a positive and statistically significant price impact even after accounting for the price impact of own order flow. For some exchange rates, these cross-currency effects are economically quite significant. For example, in the case of the Krona, the price impact of Mark and S Franc are highly significant and the  $R^2$  rises from less than 1 percent to 68 percent when all order flows are introduced. Note too that for all three of the currencies that show a weak own-flow effect (the B Franc, Krona, and Guilder), all three are strongly affected by flows in the two dominant regional currencies, the Mark and S Franc. This is consistent with recent findings of “information geographies” in financial markets (see, e.g., Hau 2001).

#### 4.4 Covariance Decompositions

What fraction of the high contemporaneous correlation in currency returns (upper panel of Table 1) can be accounted for by order-flow-induced price movements?<sup>12</sup> Table 4 presents an answer to this question. The upper entry in each cell reports the fraction of the covariance between  $\Delta p_t^j$  and  $\Delta p_t^i$  (for column currency  $j$  and row currency  $i$ ) that is attributable to order flows. This is calculated as the statistic  $1 - \left( \text{Cov}(\hat{v}_t^j, \hat{v}_t^i) / \text{Cov}(\Delta p_t^j, \Delta p_t^i) \right)$ , where  $\hat{v}_t^k$  for  $k = i, j$ , is the estimated residual from regression equation (14), i.e., the portion of price variation unaccounted for by order flow. The lower entry reports in parenthesis the asymptotic p-value for the null hypothesis that the fraction equals unity (i.e.,  $\text{Cov}(\hat{v}_t^j, \hat{v}_t^i) = 0$ ). Note that order flow information accounts on average for about 80 percent of the (high) unconditional covariance in currency returns. This suggests that the stylized fact of high covariance in currency returns is not due to the impounding of common public news directly into prices without any role for order flow (which is how the covariance is traditionally modeled).

#### 4.5 Implications

The results in Table 3 indicate rather strongly that order flows across the market are a proximate cause of daily exchange rate movements in all our sample currencies. According to our model, this arises because market wide order flows contain information about the aggregate shifts in the public's demand for currency that is valuable to dealers seeking to set market clearing prices (i.e., to establish stock equilibrium). Further, the model identified the conditions under which market-wide order flows would not contain any more price relevant information than the order flow arising from trade in one currency pair. The results in Table 3 indicate that these conditions do not hold. On the contrary, when viewed from the perspective Proposition 5, our empirical results suggest strongly that  $\Sigma_c$  and  $\Sigma_R$  cannot both be diagonal; dealers and the public appear to be solving integrated portfolio problems to determine their currency demands.

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<sup>12</sup> This question is motivated in part by the well-known fact that high return correlation is not necessarily evidence of high market integration. Indeed, even markets in autarky can exhibit high return correlation if the shocks they are exposed to are highly correlated.

Further support for this view comes from Tables 1 and 2. The results in Table 1 indicate that it is hard to predict exchange rates from their history alone. This means that the covariance matrix of the exchange rate changes conditioned on public information (i.e., excluding order flows) is well approximated by the unconditional covariance matrix. The results in Table 2 imply that this matrix has many off-diagonal elements that are significant. Thus, the behavior of exchange rates in our sample suggests strongly that the public's demand for foreign currencies should be derived as the solution to a multi-currency portfolio problem rather than a collection of single-currency problems (as is often presumed in past empirical work using transactions data). Our finding that market-wide order flows account for a significant fraction of daily exchange rate changes is indeed consistent with this view. If there had been no statistically significant evidence of cross-currency order flow effects in Table 3, we would have been unable to reject the hypothesis that demands for currencies by the public are segmented. In view of the strong contemporaneous correlations amongst exchange rates, such a finding would have cast doubt on the degree to which international money markets are informationally integrated. Our findings in Table 3 point in the opposite direction. The strong links between order flows and exchange rates are symptomatic of a high degree of informational integration in international money markets.

## 5. Conclusion

This paper develops a multi-currency model of portfolio allocation in the presence of dispersed information and tests the model's main implication, namely that order flow in one market should directly affect prices in others. The model explains 45 to 78 percent of daily returns in all nine of the currencies we examine. Moreover, the model's prediction that order flow information should be relevant for determining prices in multiple markets is borne out. Order flows in two currencies in particular have substantial informational impact on prices in other markets: the German Mark and the Swiss Franc.<sup>13</sup> This is in keeping with recent empirical

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<sup>13</sup> Under our null hypothesis, these cross-market order flow effects operate directly, i.e., other markets learn from the German Mark and Swiss Franc order flow itself rather than learning from the DM/\$ and SF/\$ exchange rates (which are themselves determined from their own-market order flows). The important point for this paper is that the underlying drivers are the order flows. Exactly how the

results on information geography (8 of our 9 currencies are within European geographic region). Finally, we find that order flow information accounts on average for about 80 percent of the (high) unconditional covariance in currency returns.

Though our introduction distinguishes between speculative, geographic, and informational integration, these three lines of work are by no means independent. Nevertheless, the macro approach to asset pricing has never afforded informational integration much attention because it presumes that all relevant information is public. In those traditional settings, there is no role for aggregation of dispersed information—public information is directly and immediately impounded in asset prices, across the board. In more realistic information environments, however, the notion of informational integration has some traction. Our results suggest that FX markets are such an environment, despite the common belief that private or dispersed information is less relevant in FX than in other security markets. More broadly, it is not unreasonable to view all three of the integration approaches as being, fundamentally, about informational integration. Whether this type of synthesis is meaningful for future work remains to be seen.

One policy area where the above synthesis may be useful is that of central bank intervention. In particular, consider the important issue of whether coordination across central banks makes intervention more effective. If intervention is uncoordinated, it will not take account of the cross-currency information effects that we document here. Interestingly, even if coordinated (in the usual sense of being simultaneous and in the same direction), intervention efforts might still not take full account of the cross-currency information effects documented here. Our results suggest that order flows can act on the full constellation of exchange rates, not simply on in the markets where intervention is executed. In some markets, these indirect effects appear to be significant.

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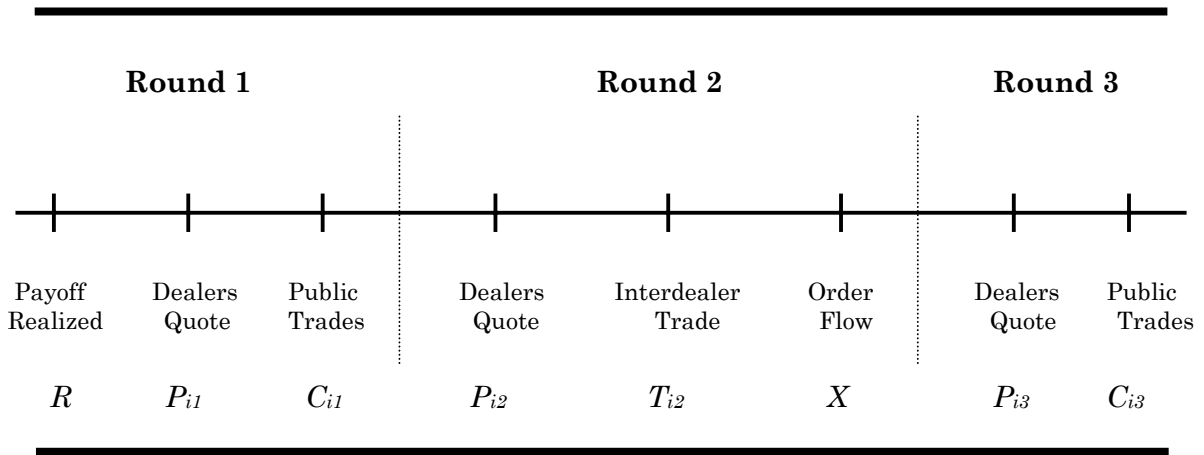
information conveyed by order flows propagates is a question about the transmission mechanism, not the underlying drivers.

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**Figure 1**  
Daily Timing



**Table 1: Sample Statistics**

Currency	Variable	Mean (%)	Sdt. (%)	$\rho_1$	$\rho_2$
Mark	$\Delta p_t$	-0.035	0.396	0.052	-0.019
	$X_t$	0.001	0.137	-0.033	0.099
Yen	$\Delta p_t$	0.033	0.419	0.021	0.032
	$X_t$	0.036	0.099	0.161	0.140
Pound	$\Delta p_t$	-0.043	0.305	0.066	0.109
	$X_t$	0.012	0.049	0.289*	0.102*
B. Franc	$\Delta p_t$	-0.039	0.381	0.089	-0.046
	$X_t$	0.003	0.007	-0.115	-0.095
S Franc	$\Delta p_t$	-0.035	0.528	0.038	-0.023
	$X_t$	-0.015	0.070	0.154	0.073
Krona	$\Delta p_t$	-0.044	0.413	-0.030	0.037
	$X_t$	0.000	0.004	-0.005	0.079
F. Franc	$\Delta p_t$	-0.017	0.351	0.117	-0.094
	$X_t$	0.000	0.028	0.301*	0.247*
Lira	$\Delta p_t$	-0.035	0.264	-0.149	0.054
	$X_t$	0.001	0.015	0.132	0.085
Guilder	$\Delta p_t$	-0.039	0.412	0.008	-0.024
	$X_t$	-0.002	0.006	0.013	0.018

Notes: Sample statistics for:  $\Delta p_t$ , the daily change in the log transaction price (foreign currency/\$); and  $X_t$ , daily order flow measured is the difference between the number of buyer- and seller-initiated trades between currency k and the dollar (in thousands, negative sign denotes net dollar sales).  $\rho_1$  and  $\rho_2$  denote sample autocorrelations at lags one and two. A "\*" denotes significance at the 5 per cent level. Market activity after 4 PM on Friday is aggregated until 4 PM on Monday. Sample: May 1 to August 31, 1996.



**Table 2: Contemporaneous Correlations**

<b>Returns</b>	Mark	Yen	Pound	B. Fr.	S. Fr.	Krona	F. Fr.	Lira
Yen	0.47							
Pound	0.49	0.07						
B. Franc	0.95	0.47	0.47					
S. Franc	0.92	0.55	0.40	0.89				
Krona	0.85	0.42	0.41	0.88	0.80			
F. Franc	0.94	0.48	0.48	0.94	0.91	0.85		
Lira	0.61	0.37	0.35	0.63	0.61	0.52	0.67	
Guilder	0.94	0.49	0.48	0.96	0.88	0.86	0.92	0.66
<b>Order flow</b>	Mark	Yen	Pound	B. Franc	S. Franc	Krona	F. Franc	Lira
Yen	0.18 (0.09)							
Pound	0.28 (0.0)	0.15 (0.16)						
B. Franc	0.18 (0.09)	0.23 (0.03)	-0.09 (0.40)					
S Franc	0.64 (0.00)	0.25 (0.02)	0.34 (0.00)	0.10 (0.33)				
Krona	-0.02 (0.85)	-0.03 (0.79)	0.15 (0.18)	0.09 (0.41)	0.09 (0.43)			
F. Franc	0.56 (0.00)	0.30 (0.01)	0.23 (0.03)	0.13 (0.22)	0.47 (0.00)	0.08 (0.46)		
Lira	0.29 (0.01)	0.28 (0.01)	0.31 (0.01)	0.08 (0.48)	0.41 (0.00)	0.18 (0.10)	0.37 (0.00)	
Guilder	0.17 (0.12)	0.23 (0.04)	-0.12 (0.25)	0.12 (0.26)	0.12 (0.25)	-0.11 (0.31)	0.20 (0.07)	-0.03 (0.77)

Notes: Contemporaneous correlations in daily data. See Table 1 for variable definitions. All return correlations (log price change) are significant at the 1 percent level, save the Pound-Yen correlation (p-value 0.52). Asymptotic p-values for the null hypothesis of a zero correlation are reported in parentheses for order flow correlations (see Hamilton 1994, p. 301). Sample: daily data from May 1 to August 31, 1996.

**Table 3: Price Change Regressions**

Return	Order Flows									Diagnostics		
	Mark	Yen	Pound	B. Franc	S. Franc	Krona	F. Franc	Lira	Guilder	$R^2$	$\chi^2_{\text{serial}}$	$\chi^2_{\text{hetero}}$
Mark	2.28									0.68	0.37	0.01
	(8.74)										0.33	0.05
Yen	1.63	0.16	0.22	-1.55	1.33	1.39	1.24	1.73	4.11	0.76	0.55	0.21
	(7.49)	(0.69)	(0.47)	(0.55)	(3.35)	(0.28)	(1.32)	(1.02)	(1.20)		0.38	0.13
Pound	2.61									0.45	0.07	0.03
	(7.69)										0.38	0.15
B. Franc	-0.11	2.16	0.88	5.49	1.34	-4.10	1.76	0.24	-0.24	0.54	0.02	0.01
	(0.33)	(6.52)	(1.30)	(1.33)	(2.32)	(0.55)	(1.28)	(0.10)	-(0.05)		0.09	0.06
S. Franc	3.41									0.33	0.01	0.20
	(5.16)										0.00	0.70
Krona	0.65	0.04	2.69	-4.54	0.01	-2.18	-0.27	3.17	-0.22	0.45	0.00	0.44
	(2.67)	(0.15)	(5.26)	(1.44)	(0.02)	(0.39)	(0.26)	(1.68)	(0.06)		0.00	0.58
F. Franc	3.11									0.00	0.39	0.82
	(0.73)										0.71	0.95
Lira	1.38	0.18	0.00	-3.26	1.42	2.79	1.87	2.31	4.32	0.78	0.32	0.78
	(6.80)	(0.83)	(0.01)	(1.24)	(3.86)	(0.59)	(2.13)	(1.46)	(1.36)		0.19	0.00
Guilder	5.11									0.53	0.45	0.80
	(6.13)										0.16	0.92
Mark	1.45	0.75	0.53	-4.96	2.89	-8.26	0.90	2.94	4.65	0.70	0.22	0.14
	(4.40)	(2.21)	(0.77)	(1.17)	(4.86)	(1.08)	(0.63)	(1.15)	(0.90)		0.14	0.02
Yen	6.33									0.00	0.75	0.50
	(0.55)										0.56	0.57
Pound	0.91	0.23	-0.68	-2.27	2.19	0.23	1.74	1.36	7.72	0.69	0.23	0.91
	(3.52)	(0.87)	(1.25)	(0.68)	(4.66)	(0.04)	(1.55)	(0.68)	(1.90)		0.01	0.99
B. Franc	7.46									0.40	0.74	0.60
	(4.53)										0.29	0.21
S. Franc	1.13	0.11	-0.06	-1.94	1.02	-6.37	2.17	4.21	5.33	0.75	0.85	0.27
	(5.66)	(0.52)	(0.14)	(0.75)	(2.83)	(1.37)	(2.52)	(2.71)	(1.70)		0.02	0.00
Krona	10.69									0.37	0.15	0.76
	(6.06)										0.46	0.80
F. Franc	0.68	-0.12	1.07	-2.40	0.11	-15.66	0.21	10.90	4.45	0.65	0.09	0.65
	(3.88)	(0.67)	(2.92)	(1.06)	(0.34)	(3.87)	(0.28)	(8.02)	(1.63)		0.27	0.49

**Table 3: Price Change Regressions (Cont.)**

Guilder										15.77	0.06	0.81	0.53
										(1.57)		0.44	0.73
	1.36	0.18	0.23	-1.85	1.61	1.02	1.83	3.68	6.18	0.75	0.56	0.36	
	(5.77)	(0.74)	(0.47)	(0.61)	(3.76)	(0.19)	(1.80)	(2.01)	(1.68)		0.36	0.37	

Notes: The table reports GLS coefficient estimates (x100) with absolute t-statistics in parentheses. The sample spans four months (May 1 to August 31, 1996), which is 89 trading days.  $\chi^2_{\text{serial}}$  is the p-value of a chi-squared test for residual first-order (top row) and fifth-order (bottom row) serial correlation.  $\chi^2_{\text{hetero}}$  is the p-value of a chi-squared test for first-order (top row) and fifth-order (bottom row) ARCH in the estimated residuals. Sample: daily data from May 1 to August 31, 1996.

**Table 4: Covariance Decomposition**

	Mark	Yen	Pound	B. Franc	S. Franc	Krona	F. Franc	Lira
Yen	0.69 (0.00)							
Pound	0.79 (0.01)	1.06 (0.94)						
B. Franc	0.81 (0.00)	0.71 (0.00)	0.86 (0.08)					
S Franc	0.76 (0.00)	0.68 (0.00)	0.82 (0.10)	0.80 (0.00)				
Krona	0.82 (0.00)	0.78 (0.05)	0.93 (0.49)	0.82 (0.00)	0.84 (0.00)			
F. Franc	0.78 (0.00)	0.69 (0.00)	0.83 (0.05)	0.81 (0.00)	0.76 (0.00)	0.82 (0.00)		
Lira	0.85 (0.01)	0.78 (0.08)	0.78 (0.10)	0.84 (0.00)	0.83 (0.01)	0.90 (0.17)	0.86 (0.01)	
Guilder	0.80 (0.00)	0.72 (0.00)	0.78 (0.01)	0.79 (0.00)	0.80 (0.00)	0.82 (0.00)	0.80 (0.00)	0.81 (0.00)

Notes: The upper entry in each cell reports the fraction of the covariance between  $\Delta p_t^j$  and  $\Delta p_t^i$  for column currency  $j$  and row currency  $i$ , attributable to the order flows:  $1 - Cov(\hat{v}_t^j, \hat{v}_t^i) / Cov(\Delta p_t^j, \Delta p_t^i)$ , where  $\hat{v}_t^k$  for  $k = i, j$ , is the estimated residual from regression (14). The lower entry reports in parenthesis the asymptotic p-value for the null hypothesis that the fraction equals unity (i.e.,  $Cov(\hat{v}_t^j, \hat{v}_t^i) = 0$ ).

## Appendix

Portions of these proofs are matrix equivalents of proofs contained in Evans and Lyons (1999) and Cao, Evans, and Lyons (2002); we refer readers to those earlier papers for additional detail.

### *Proof of Propositions 1 and 2*

No arbitrage requires that all dealers post a common vector of quotes in any given trading round. Common prices require that quotes be conditioned on commonly observed information only. In rounds one and two, this includes the previous day's vector of round-three prices, plus the public-information innovation at the beginning of day  $t$ ,  $\Delta R_t$ . (Dealer  $i$ 's round-two quote therefore cannot be conditioned on his realization of  $C_{i1}$ .)

The equations that pin down the levels of these prices embed the dealer and customer trading rules. These trading rules must be consistent with equilibrium price. This implies the following key relations:

$$E [C_{i1}|\Omega_{P1}] + E [D_{i2}(P_1)|\Omega_{P1}] = 0, \tag{A1}$$

$$E [C_{i1}|\Omega_{P1}] + E [D_{i2}(P_2)|\Omega_{P1}] = 0, \tag{A2}$$

$$E [C_1|\Omega_{P3}] + E [C_3(P_3)|\Omega_{P3}] = 0, \tag{A3}$$

where  $C_1 = \sum_{i=1}^N C_{i1}$ . The first two equations state that  $P_1$  and  $P_2$  must be set such that customer demand and dealer demand are expected to offset (and that these prices are conditioned on public information). The third equation states that  $P_3$  must be set such that customer demand in round's one and three are expected to offset (i.e., at the round three price the public willingly absorbs the random round one aggregate portfolio shift). These equations pin down equilibrium price because any price except that which satisfies each would generate net excess demand in round-2 interdealer trading, which cannot be reconciled since dealers trade among themselves.

Proposition 1 follows directly from the fact that expected value of  $C_{i1}$  conditional on public information  $\Omega_{P1}$  is a vector of zeros, and expected speculative dealer demand  $D_{i2}$  is also a vector of zeros at this public-information-unbiased price. To be more precise, this statement postulates that the dealer's demand  $D_{i2}$  has this property; we show below in the derivation of the optimal trading rule that this is the case.

Proposition 2 follows from the fact that  $X_t$  is a sufficient statistic for the period's aggregate portfolio shift  $\sum_{i=1}^N C_{i1}$ . Given the aggregate portfolio shift must be absorbed by the public in round 3,  $P_{3t}$  must adjust to induce the necessary public demand. Specifically, the round-3 price must satisfy:

$$C_3(P_{3t}) + C_1 = 0.$$

Given the optimal rule for determining  $T_{i2}$  ( $T_{i2} = \alpha C_{i1}$  which we establish below), we can write  $C_1$  in terms of interdealer order flow  $X_t$  as:

$$C_1 = \alpha^{-1} X_t$$

and since the specification for the public's demand is:

$$C_3 = \Gamma (E [P_{3t+1} + R_{t+1} | \Phi_t] - P_{3t}),$$

this implies a market-clearing round-3 price of:

$$P_{3t} = E [P_{3t+1} + R_{t+1} | \Phi_t] + (\alpha\Gamma)^{-1} X_t.$$

Ruling out bubble solutions, the rational expectations solution to this expression can be written as:

$$P_{3t} = \sum_{\tau=1}^t (\Delta R_\tau + \Lambda X_\tau)$$

with  $\Lambda = (\alpha\Gamma)^{-1}$ . This sum is the expected payoff on the risky asset (the  $\Delta R_\tau$  terms), adjusted for a risk premium, which is determined by cumulative portfolio shifts (the  $X_\tau$  terms). This yields equation (13) in the text, and together with Proposition 1 gives equation (9).

#### *Equilibrium Trading Strategies*

An implication of common interdealer quotes  $P_2$  is that in round 2 each dealer receives a share  $1/(N-1)$  of every other dealer's interdealer trade. This order corresponds to the position disturbance  $T_{i2}^*$  in the dealer's problem.

Given the quoting strategy described in Propositions 1 and 2, the trading strategy described in Proposition 3 is optimal and corresponds to symmetric linear equilibrium:

#### *Proof of Proposition 3:*

As noted above, because returns are independent across periods, with an unchanging stochastic structure, the dealers' problem collapses to a series of independent trading problems, one for each period (see Cao, Evans and Lyons (2002) for further details).. Because there are only  $N$  dealers, however, each dealer acts strategically in the sense that his speculative demand depends on the impact his trade will have on subsequent prices.

It is well known that if a random variable  $W$  is distributed  $N(\mu, \sigma^2)$  and the utility function  $U(W) = -\exp(-\theta W)$ , then  $E[U(W)] = -\exp(-\theta(\mu - \frac{\theta}{2}\sigma^2))$ . Maximizing  $E[U(W)]$  is therefore equivalent to maximizing  $(\mu - \frac{\theta}{2}\sigma^2)$ . This result allows us to write the dealers speculative-demand problem as:

$$Max_{\{D_{i2}\}} D'_{i2} (E[P_3 | \Omega_{T_{i2}}] - P_2) - \frac{\theta}{2} D'_{i2} \Xi D_{i2}, \tag{A4}$$

where the information set  $\Omega_{T_{i2}}$  is defined in the text, and  $\Xi$  denotes the variance of  $P_3$  conditioned on  $\Omega_{T_{i2}}$ .

Now, from Proposition 2, we can write:

$$E[P_3|\Omega_{T_{i2}}] - P_2 = E[\Lambda X_t|\Omega_{T_{i2}}]. \quad (\text{A5})$$

And from the definitions of  $\Omega_{T_{i2}}$  and  $X_t$  we know that:

$$E[\Lambda X_t|\Omega_{T_{i2}}] = \Lambda T_{i2}. \quad (\text{A6})$$

The expected value of the other dealers' trades in  $X_t$  is 0 under our specification because: (i) customer trades are mean-zero and independent across dealers, and (ii) there is no information in the model other than customer trades to motivate speculative demand. This fact also implies that dealer  $i$ 's trade in round 2,  $T_{i2}$  from equation (7), is equal to:

$$T_{i2} = D_{i2} + C_{i1}.$$

Therefore, we can write the dealer's problem as:

$$\text{Max}_{\{D_{i2}\}} D'_{i2} \Lambda (D_{i2} + C_{i1}) - \frac{\theta}{2} D'_{i2} \Xi D_{i2}. \quad (\text{A7})$$

The first-order condition of this problem is:

$$(\Lambda + \Lambda') D_{i2} + \Lambda C_{i1} - \theta \Xi D_{i2} = 0, \quad (\text{A8})$$

which implies a speculative demand of:

$$D_{i2} = (\theta \Xi - (\Lambda + \Lambda'))^{-1} \Lambda C_{i1}. \quad (\text{A9})$$

This demand function, and the fact that  $T_{i2} = D_{i2} + C_{i1}$ , imply:

$$D_{i2} = \left[ (\theta \Xi - (\Lambda + \Lambda'))^{-1} \Lambda + I \right] C_{i1} = \alpha C_{i1}. \quad (\text{A10})$$

*Proof of Proposition 4:*

With the riskless asset in zero net supply, the optimization problem facing the public can be written as

$$\begin{aligned} & \text{Max}_{\{C_{3t}\}} E[-\exp(-\theta V_{t+1}) | \Phi_t] \\ & \text{s.t. } V_{t+1} = C'_{3t} (P_{3t+1} + R_{t+1} - P_{3t}). \end{aligned}$$

Under our assumptions, and following from the results in Propositions 1 and 2, the conditional distribution of the public's wealth,  $V_{t+1}$ , is normal with mean  $C'_{3t} (E[P_{3t+1} + R_{t+1} | \Phi_t] - P_{3t})$ , and variance

$C_3' Var (P_{3t+1} + R_{t+1} | \Phi_t) C_3$ . The first order condition for the maximization problem is therefore

$$E [P_{3t+1} + R_{t+1} | \Phi_t] - P_{3t} - \theta \Sigma_P C_{3t} = 0.$$

Rearranging this equation and aggregating over the continuum of agents that comprise the public gives equation (12). Although the assumed zero net supply of the riskless asset simplifies the optimization problem, this assumption is not necessary to derive equation (12). In particular, Cao, Evans, and Lyons (2002) show that the value function associated with the public's optimization problem is exponential in wealth even when the public's equilibrium holdings of the riskless asset are non-zero. And, as a result, the first order condition for  $C_3$  takes the same form as the equation above.

*Proof of Proposition 5*

Part A: Show that diagonality of  $\Lambda$  implies diagonality of  $\Sigma_P$  and  $\Sigma_R$ . If  $\Lambda$  diagonal then  $\Lambda = \Lambda'$ . From the definitions above we have

$$\Lambda = \theta \Sigma_P (\theta \Xi - \Lambda')^{-1} (\theta \Xi - \Lambda - \Lambda') \quad (\text{A11})$$

so if  $\Lambda$  is diagonal,

$$\Lambda = \theta \Sigma_P (\theta \Xi - \Lambda)^{-1} (\theta \Xi - 2\Lambda) = \theta (\theta \Xi - 2\Lambda) (\theta \Xi - \Lambda)^{-1} \Sigma_P = \Lambda'$$

since  $\Xi$  and  $\Sigma_P$  are by definition symmetric. Now vectorize the inner two terms:

$$vec \left( \theta \Sigma_P (\theta \Xi - \Lambda)^{-1} (\theta \Xi - 2\Lambda) \right) = vec \left( \theta (\theta \Xi - 2\Lambda) (\theta \Xi - \Lambda)^{-1} \Sigma_P \right)$$

which can be rewritten as

$$\theta ((\theta \Xi - 2\Lambda) \otimes \Sigma_P) vec \left( (\theta \Xi - \Lambda)^{-1} \right) = \theta (\Sigma_P \otimes (\theta \Xi - 2\Lambda)) vec \left( (\theta \Xi - \Lambda)^{-1} \right).$$

Clearly, this condition can only hold when

$$\begin{aligned} I &= ((\theta \Xi - 2\Lambda) \otimes \Sigma_P)^{-1} (\Sigma_P \otimes (\theta \Xi - 2\Lambda)) \\ &= ((\theta \Xi - 2\Lambda)^{-1} \otimes \Sigma_P^{-1}) (\Sigma_P \otimes (\theta \Xi - 2\Lambda)) \\ &= ((\theta \Xi - 2\Lambda)^{-1} \Sigma_P) \otimes (\Sigma_P^{-1} (\theta \Xi - 2\Lambda)) \\ &= \Psi^{-1} \otimes \Psi \end{aligned}$$

It is straightforward to check that the last equality only holds when  $\Psi = \Sigma_P^{-1} (\theta \Xi - 2\Lambda) = sI$  for some non-zero scalar  $s$ . Rearranging this condition, gives  $\Lambda = \frac{1}{2} (\theta \Xi - s \Sigma_P)$ . Combining this expression with (A11)



gives

$$\begin{aligned}
\Lambda &= s\theta\Sigma_P(\theta\Xi - \Lambda)^{-1}\Sigma_P \\
&= s\theta\Sigma_P(s\Sigma_P + \Lambda)^{-1}\Sigma_P \\
&= s\theta\Sigma_P(\Sigma_P(sI + \Sigma_P^{-1}\Lambda))^{-1}\Sigma_P \\
&= s\theta\Sigma_P(sI + \Sigma_P^{-1}\Lambda)^{-1}
\end{aligned}$$

Rearranging this equation, gives

$$s\Theta + \Theta\Theta = s\theta I \quad (\text{A12})$$

where  $\Theta = \Sigma_P^{-1}\Lambda$ . Transposing both sides of this equation gives

$$s\Theta' + \Theta'\Theta' = s\theta I$$

which is exactly the same form as (A12) with  $\Theta'$  replacing  $\Theta$ . Any solution to the equation must therefore be symmetric. This means that  $\Sigma_P^{-1}\Lambda = \Lambda(\Sigma_P^{-1})'$ , a condition that is only satisfied when  $\Lambda$  is diagonal if  $\Sigma_P$  is diagonal. Finally, recall that  $\Lambda = \frac{1}{2}(\theta\Xi - s\Sigma_P)$ , so  $\Xi = \frac{1}{\theta}(2\Lambda + s\Sigma_P)$  must be diagonal. By definition,  $\Xi = \text{Var}(P_{3,t}|\Omega_{Ti2}) = \Lambda\text{Var}(X_t|c_{it})\Lambda'$  where  $\text{Var}(X_t|c_{it}) = \frac{N-1}{N}\alpha\Sigma_c\alpha'$ . Hence  $\Xi = \frac{N-1}{N}\Lambda\alpha\Sigma_c\alpha'\Lambda' = \frac{N-1}{N}\Sigma_P\Sigma_C\Sigma_P$ . Rearranging the last expression gives,  $\Sigma_C = \frac{N}{N-1}\Sigma_P^{-1}\Xi\Sigma_P^{-1}$  so  $\Sigma_C$  must also be diagonal.

To complete part A of the proof, we need to show that the diagonality of  $\Sigma_P$  and  $\Sigma_C$  implies that  $\Sigma_R$  is also diagonal. Equations (9) and (10) imply that

$$P_{3t+1} = P_{3t} + \Delta R_{t+1} + \Lambda X_{t+1}.$$

So, if members of the public hold rational expectations,

$$\Sigma_P \equiv \text{Var}(P_{3t+1}|\Phi_t) = \Sigma_R + \Lambda\text{Var}(X_{t+1}|\Phi_t)\Lambda' \quad (\text{A13})$$

From Proposition 3,  $X_{t+1} = \alpha\sum_{i=1}^N C_{i1t+1}$  where the  $C_{i1t+1}$  vectors are distributed independently across the  $N$  dealers, with covariance  $\Sigma_C$ . Hence,  $\text{Var}(X_{t+1}|\Phi_t) = N\alpha\Sigma_c\alpha'$ . Substituting this expression in (A13) and simplifying gives

$$\Sigma_P = \Sigma_R + N\theta^2\Sigma_P\Sigma_C\Sigma_P. \quad (\text{A14})$$

This equation implicitly defines the relation between  $\Sigma_R$ ,  $\Sigma_C$  and  $\Sigma_P$  when members of the public hold rational expectations. It follows trivially from this equation that  $\Sigma_R$  must be diagonal if  $\Sigma_P$  and  $\Sigma_C$  are diagonal.

Part B: Now we show that diagonality of  $\Sigma_C$  and  $\Sigma_R$  implies diagonality of  $\Lambda$ . First we demonstrate that  $\Lambda$  is diagonal if  $\Sigma_C$  and  $\Sigma_P$  are diagonal. We then show that diagonality of  $\Sigma_C$  and  $\Sigma_R$  implies diagonality

of  $\Sigma_P$ .

If  $\Sigma_C$  and  $\Sigma_P$  are diagonal,  $\Xi$  and  $\Lambda\alpha$  diagonal. Hence

$$\Lambda\alpha = \Lambda(\theta\Xi - \Lambda - \Lambda')^{-1}(\theta\Xi - \Lambda') = (\theta\Xi - \Lambda)(\theta\Xi - \Lambda - \Lambda')^{-1}\Lambda' = \alpha'\Lambda'.$$

Now vectorize the inner two terms:

$$\text{vec}(\Lambda(\theta\Xi - \Lambda - \Lambda')^{-1}(\theta\Xi - \Lambda')) = \text{vec}((\theta\Xi - \Lambda)(\theta\Xi - \Lambda - \Lambda')^{-1}\Lambda')$$

which can be rewritten as

$$((\theta\Xi - \Lambda) \otimes \Lambda) \text{vec}((\theta\Xi - \Lambda - \Lambda')^{-1}) = (\Lambda \otimes (\theta\Xi - \Lambda)) \text{vec}((\theta\Xi - \Lambda - \Lambda')^{-1}).$$

Clearly, this condition can only hold when

$$\begin{aligned} I &= ((\theta\Xi - \Lambda) \otimes \Lambda)^{-1}(\Lambda \otimes (\theta\Xi - \Lambda)) \\ &= ((\theta\Xi - \Lambda)^{-1} \otimes \Lambda^{-1})(\Lambda \otimes (\theta\Xi - \Lambda)) \\ &= ((\theta\Xi - \Lambda)^{-1}\Lambda) \otimes (\Lambda^{-1}(\theta\Xi - \Lambda)) \\ &= \Upsilon^{-1} \otimes \Upsilon \end{aligned}$$

As above, it is straightforward to check that the last equality only holds when  $\Upsilon = \Lambda^{-1}(\theta\Xi - \Lambda) = sI$  for some non-zero scalar  $s$ . Hence,  $\Lambda = \frac{\theta}{1+s}\Xi$ , a diagonal matrix.

Finally, we show that diagonality of  $\Sigma_C$  and  $\Sigma_R$  implies diagonality of  $\Sigma_P$  when members of the public hold rational expectations. For this purpose write the rational expectations solution for  $\Sigma_P$  as  $\Sigma_P^D + \Sigma_P^O$  where  $\Sigma_P^D$  is a diagonal matrix and  $\Sigma_P^O$  is a symmetric matrix with zeros on its leading diagonal. By definition,  $\Sigma_P^D$  solves (A14): i.e.,  $\Sigma_P^D = \Sigma_R + N\theta^2\Sigma_P^D\Sigma_C\Sigma_P^D$  when  $\Sigma_C$  and  $\Sigma_R$  are diagonal. To establish that  $\Sigma_P$  is diagonal, we substitute  $\Sigma_P^D + \Sigma_P^O$  for  $\Sigma_P$  in (A14) and simplify the result to obtain

$$\Sigma_P^O = N\theta^2(\Sigma_P^D\Sigma_C\Sigma_P^O + \Sigma_P^O\Sigma_C\Sigma_P^D + \Sigma_P^O\Sigma_C\Sigma_P^O). \quad (\text{A15})$$

By construction, there are zeros on the leading diagonal of  $\Sigma_P^O$ . Moreover, since  $\Sigma_P^D\Sigma_C = \Sigma_C\Sigma_P^D$  are diagonal, it is straightforward to check that there must be zeros on leading diagonal of the matrix products  $\Sigma_P^D\Sigma_C\Sigma_P^O$  and  $\Sigma_P^O\Sigma_C\Sigma_P^D$ . Hence, (A15) implies that the leading diagonal of  $\Sigma_P^O\Sigma_C\Sigma_P^O$  must also contain all zeros if  $\Sigma_P = \Sigma_P^D + \Sigma_P^O$  satisfies (A14). This requirement implies that

$$\iota_k \Sigma_P^O \Sigma_C \Sigma_P^O \iota_k' = 0, \quad i = 1, 2, \dots, K, \quad (\text{A16})$$

where  $\iota_k$  as a  $1 \times K$  vector with  $k'$ th. element equal to one, and all the other elements equal to zero. Now

we use the fact that

$$\begin{aligned} \iota_k \Sigma_P^O \Sigma_C \Sigma_P^O \iota_k' &= \text{trace} (\iota_k \Sigma_P^O \Sigma_C \Sigma_P^O \iota_k') \\ &= \text{trace} (\Sigma_C \Sigma_P^O \iota_k' \iota_k \Sigma_P^O) \end{aligned}$$

Evaluating the trace in the last line (using the fact that  $\Sigma_C$  is diagonal), and substituting the result back into (A16) gives

$$\sum_{j=1}^K [\Sigma_C]_{jj} \left( [\Sigma_P^O]_{jk} \right)^2 = 0, \quad i = 1, 2, \dots, K,$$

where  $[\cdot]_{j,i}$  denotes element  $j,i$  of the matrix. Since  $[\Sigma_C]_{jj} \geq 0$ , this condition can only be met if  $[\Sigma_P^O]_{jk} = 0$  for all  $j$  and  $k$ . Hence,  $\Sigma_P^O$  must be a matrix of zeros, and  $\Sigma_P$  is diagonal.