PAYG pensions, tax-cum-subsidy and optimality

Fanti, Luciano and Gori, Luca
University of Pisa, Department of Economics, University of Pisa, Department of Economics

23. January 2010

PAYG pensions, tax-*cum*-subsidy and optimality

Luciano Fanti[*] and Luca Gori[**]

Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I–56124 Pisa (PI), Italy

Abstract Using a simple OLG small open economy with endogenous fertility we show that the command optimum can be decentralised in a market setting using both a PAYG transfer from the young (old) to the old (young) and a tax-*cum*-subsidy (subsidy-*cum*-tax) policy, to redistribute within the working age generation. The latter instrument, in fact, reduces (increases) the opportunity cost of bearing children and, hence, stimulates (depresses) fertility. The policy implications are straightforward: when PAYG transfers exist and child rearing is time consuming, a tax-*cum*-subsidy (subsidy-*cum*-tax) policy can be used to internalise the externality of children, while also representing a Pareto improvement.

Keywords Overlapping generations; PAYG Pensions; Small open economy; Tax-*cum*-subsidy

JEL Classification H24; H55; J13; J26

[*] *E-mail address*: lfanti@ec.unipi.it; tel.: +39 050 22 16 369; fax: +39 050 22 16 384.

[**] Corresponding author. *E-mail address*: luca.gori@ec.unipi.it; tel.: +39 050 22 16 212; fax: +39 050 22 16 384.

1. Introduction

The debate about the existence of an “interior” optimal population growth rate in the basic two-period neoclassical overlapping generations (OLG) model with exogenous fertility (Samuelson, 1958; Diamond, 1965) is long lasting (e.g. Phelps, 1968; Samuelson, 1975; Deardorff, 1976; Jaeger and Kuhle, 2009). Although the importance of such a debate in macroeconomics is undoubted, only recently Abio (2003), building on a textbook double Cobb-Douglas OLG closed economy with endogenous fertility has shown that an interior golden rule of population growth can exist. Hence, policies aiming at achieving the optimal rate of population growth in a market setting may be highly valuable.

The achievement of a golden rule of procreation in a market setting is even more important in presence of public pension systems (e.g. PAYG pensions), owing to the external effects that children create on society as a whole, as clearly pointed out by Cigno (1993). In fact, if PAYG pensions arrangements exist, the higher the number of children, the higher the tax base and, hence, the higher expected benefit received by old-aged in the future, i.e. the viability of the PAYG system increases along with the number of children. Hence, when an inter-generational transfer from the young to the old is provided by the government on a PAYG basis, children imply a positive externality in the overall economy, which however is not taken into account by each single individual as long as the contribution rate to the PAYG system paid by each member of the current working age generation is shared amongst all members of that generation, that is the benefit of having children is too small to be internalised by each single agent in the market economy.

Therefore, the *laissez-faire* fertility rate may be different from the golden rule of population growth. Hence, the study of policies aiming at eliminating the externalities of children and then favouring the achievement of the command optimum in a market setting is high in both the political and academic debates.

Moreover, population ageing and below-replacement fertility phenomena currently occurring in many developed countries have exacerbated the concerns about offspring externalities and even stimulated some recent analyses of welfare implications of public pensions in an endogenous fertility setting. For instance, in the basic OLG model, and under the small open economy hypothesis, van Groezen et al. (2003) and Fenge and Meier (2005) are positioned. In particular, van Groezen et al. (2003) assumed a fixed cost of children and showed that PAYG taxes (subsidies) and child allowances (taxes) act as Siamese twins to realise the command optimum,[1] while Fenge and Meier (2005), building on a model with time cost of children, compared the substitutability between the child allowance and the child factor[2] (rather then the PAYG tax) as instruments to replicate the second best (rather than the first best) allocation at the steady state.

In this paper we discuss an alternative way to deal with the external effects of children in the same OLG small open economy context used by van Groezen et al. (2003) and Fenge and Meier (2005). In particular, we assess the effectiveness of a traditional instrument in the theory of public finance, namely the tax-*cum*-subsidy (subsidy-*cum*-tax) policy,[3] that can be used – together with a PAYG inter-generational transfer system – to correct the offspring externalities when fertility is the result of a rationale comparison between benefits and costs of children. In particular, different from van Groezen et al. (2003) and similarly with Fenge and Meier (2005), we consider an OLG small open economy with time cost of children and define: (*i*) the tax-*cum*-subsidy (T/S) policy the case in which a wage tax is collected and rebated as a lump-sum subsidy within the same working-age

[1] Van Groezen and Meijdam (2008) showed that PAYG taxes (subsidies) and child allowances (taxes) act as Siamese twins to replicate the steady-state command optimum even in a general equilibrium OLG closed economy with endogenous fertility and fixed cost of children.

[2] They defined the child factor as a parameter which captures the relative importance of the individual number of children on PAYG pensions.

[3] See, e.g., Atkinson and Stiglitz (1980). Gahvari (1993) has analysed a T/S policy in a life-cycle growth model closed to international trade. Recently, Fanti and Gori (2007), have analysed the effects of the T/S policy on the steady-state second best allocation in an OLG model without PAYG transfers.

(child-bearing) generation at a balanced budget, and (*ii*) the subsidy-*cum*-tax (S/T) policy the case in which a wage subsidy is financed by a lump-sum tax.

We show that, depending on the mutual relationship between the constant world interest rate and the social discount factor, a PAYG transfer from the young (old) to the old (young) and a T/S (S/T) policy act as Siamese twins to realise the command optimum in a market setting (see van Groezen et al, 2003). This implies that the T/S (S/T) instrument can be used as an alternative to child allowances (taxes) to correct offspring externalities in economies where the public PAYG transfers exist. This result occurs because a T/S (S/T) policy reduces (increases) the opportunity cost of children.

Moreover, while Fenge and Meier (2005) abstract form the Pareto efficiency issue, van Groezen et al. (2003) find that introducing a child allowance in a PAYG-taxed economy may represent a Pareto improvement. In this paper we show that the introduction of a T/S (S/T) policy in an economy with PAYG pensions (transfers from the old to the young) may also represent a Pareto improvement.

This paper contributes to the OLG economic literature with public PAYG pensions and endogenous population by evaluating the conditions under which a tax-*cum*-subsidy policy can be used as an alternative instrument for optimality purposes.

The remainder of the paper is organised as follows. In Section 2 we build on the decentralised economy. In Section 3 we show that the command optimum can be achieved by a government who has at its disposal both a PAYG transfer from the young (old) to the old (young) and a T/S (S/T) instrument in a market setting. In Section 4 we discuss the welfare implications of the T/S (S/T) policy. Section 5 concludes.

2. The market economy

2.1. Firms

Consider a small open economy with perfect capital mobility that faces an exogenously given (constant) interest rate r. Production takes place according to a standard neoclassical constant-returns-to-scale technology $f(k_t, h_t)$, where k_t and h_t represent the per capita stock of capital and the units of time supplied by each member of the working-age generation on the labour market, respectively. Since capital is perfectly mobile, both the capital-labour ratio and the wage rate w are fixed and constant.

2.2. Government

In every period the government runs separately (*i*) a balanced PAYG pension budget (i.e. current pensions are paid out on the basis of current contributions) to redistribute across generations, and (*ii*) a T/S policy to redistribute within the working age generation. It is worth noting that for the sake of notational convenience we build the model with PAYG pensions and T/S policy, rather than the opposite PAYG transfer from old to the young and a S/T policy. The cases PAYG pensions (tax on the old) and T/S (S/T) will be specified when necessary.

The government pension budget at t (in per worker terms) reads as:

$$p_t = \eta n_{t-1}, \tag{1}$$

where $p_t > 0$ is the pension, $0 < \eta < w$ is the fixed contribution rate, and n_{t-1} represents the average rate of fertility in the whole economy at time $t-1$.[4]

The per worker T/S formula at t, instead, reads as:

$$\tau_t = \sigma w h_t, \tag{2}$$

where $\tau_t > 0$ is a lump-sum subsidy, $0 < \sigma < 1$ a fixed wage tax and w the unitary wage income earned workers.[5]

[4] Note that a PAYG subsidy is defined as $\eta < 0$, while a S/T scheme as $\sigma < 0$.

2.3. Individuals

We assume a three-period OLG economy populated by identical individuals. Life is divided into childhood, young-adulthood (working period) and old-age (retirement period). As a child each individual does not make economic decisions. Adult individuals belonging to generation t (N_t) have a homothetic and separable lifetime utility function (U_t) defined over $c_{1,t}$, $c_{2,t+1}$ and n_t, i.e. young-aged and old-aged consumptions and the number of children (see, e.g., Eckstein and Wolpin, 1985; Galor and Weil, 1996).

Young-adult individuals are endowed with one unit of time. Child rearing activities require a time cost $0 < q < 1$ per child, with q being the exogenous fraction of the time endowment that must be spent raising a child. Therefore, as an adult, each young devotes a fraction $h_t = 1 - qn_t$ of time to work on the labour market with $q n_t$ being the share of time spent raising children.[6]

When old individuals are retired and live on the basis of (*i*) the amount of resources saved when young-adult (s_t) plus the accrued interest at the constant world interest rate r, and (*ii*) the public provided pension benefit (p_{t+1}).

The representative individual entering the working period at time t faces the following problem:

$$\max_{\{c_{1,t}, c_{2,t+1}, n_t\}} U_t(c_{1,t}, c_{2,t+1}, n_t) = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}) + \gamma \ln(n_t), \tag{P}$$

subject to

$$\begin{aligned} c_{1,t} + s_t &= w(1 - q n_t)(1 - \sigma) + \tau_t - \eta \\ c_{2,t+1} &= (1 + r)s_t + p_{t+1} \end{aligned},$$

---

[5] Individuals do not take Eqs. (1) and (2) into account when deciding on the number of children and on material consumption over the life cycle.

[6] The condition $1 - qn_t > 0$ implies $n_t < 1/q$, i.e., the higher the time spent raising a child, the lower the number of children.

where $0 < \beta < 1$ is the individual subjective discount factor and $\gamma > 0$ the taste for children.

The first order conditions for an interior solution are:

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1}{\beta} = 1 + r \ , \tag{3}$$

$$\frac{c_{1,t}}{n_t} \cdot \gamma = q\, w(1-\sigma) . \tag{4}$$

Eq. (3) equates the marginal rate of substitution between young-adult and old-age consumption to the constant world interest rate, whereas Eq. (4) equates the marginal rate of substitution between material consumption when young and the number of children to the marginal cost of raising an additional child. Notice that the wage tax σ acts as a child allowance by reducing the cost of children.

Combining Eqs. (1)-(4) with the lifetime budget constraint gives the demand for children and the working period consumption function, that is:

$$n^* = \frac{\gamma(w-\eta)}{q\, w[(1+\beta)(1-\sigma)+\gamma] - \gamma \frac{\eta}{1+r}} , \tag{5}$$

$$c_1^* = \frac{(w-\eta) q\, w(1-\sigma)}{q\, w[(1+\beta)(1-\sigma)+\gamma] - \gamma \frac{\eta}{1+r}} . \tag{6}$$

From Eq. (5) the following proposition holds:

Proposition 1. *A T/S policy always stimulates individual fertility.*

Proof. The proof uses the following derivative:

$$\frac{\partial n^*}{\partial \sigma} = \frac{\gamma(w-\eta)(1+r)^2 q\, w(1+\beta)}{\{q\, w(1+r)[(1+\beta)(1-\sigma)+\gamma] - \gamma\eta\}^2} > 0 \, , \tag{6.1}$$

for any $0 < \sigma < 1$. **Q.E.D.**

Proposition 1 show that when a time cost of children is considered the introduction of a T/S policy, through wage taxation, reduces the opportunity cost (i.e., the net wage) of losing an additional unit of time spent working to care about children and, hence, stimulates individual fertility by reducing the labour supply. From Eq. (6.1), the opposite result that a S/T policy is always fertility-reducing can easily be derived since, through wage subsidisation, the opportunity cost of losing an additional unit of time spent working is augmented in that case, i.e. the S/T policy increases the labour supply.

In the next section we derive the first best outcome and analyse how the government can use the intra- and inter-generational instruments discussed above to realise the command optimum in a market setting.

3. The first best solution

Following van Groezen et al. (2003), we assume that the social planner maximises the discounted flow of individual lifetime utilities over an infinite horizon

$$W_t = \sum_{i=t}^{+\infty} \delta^{i-t} \cdot U\left(c_{1,i}, c_{2,i+1}, n_i\right), \tag{PP}$$

subject to the economy's resource constraint[7]

$$f\left(k, h_i\right) = c_{1,i} + \frac{c_{2,i}}{n_{i-1}} + n_i k - n_i d_{i+1} + \left(1+r\right) d_i,$$

where d is the amount of per capita foreign debt, $0 < \delta < 1$ is the social discount factor and $h_i = 1 - q n_t$.

Maximisation of (PP) gives the first order conditions for the command optimum:

$$\frac{c_{1,t+1}}{c_{1,t}} = \frac{\delta\left(1+r\right)}{n_t}, \tag{7}$$

[7] We assume that capital totally depreciates at the end of each period.

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1}{\beta} = 1 + r \,, \tag{8}$$

$$\frac{c_{1,t}}{n_t} \cdot (\gamma + \beta) = q\,w + k - d_{t+1} \,. \tag{9}$$

Comparison of Eqs. (4) and (9) makes clear the reason why the privately chosen number of children may differ from golden rule of population growth: the planner takes into account both the *inter-generational transfer effect* (the marginal benefit) and the *capital dilution effect* (the marginal cost) of raising children, which, instead, are not taken into account by individuals. The former effect is captured by the subjective discount factor β in Eq. (9): it implies that the number of children in the market economy may be too low as compared with the social optimum. The latter effect describes how fertility affects savings: it is measured by the term $k - d$ in Eq. (9) and implies that individuals in a market economy may decide to have a too high number of children. Only when these two opposite forces both exactly cancel out the individual demand for children is optimal.

Exploiting Eqs. (7)-(9) and the economy's resource constraint at the steady-state, we get the optimal amount of per capita foreign debt, that is:

$$d^{**} = k - q\,w \cdot \left[\frac{\delta + \beta + \delta(\gamma + \beta)}{(\gamma + \beta)(1 - \delta) - \delta - \beta} \right] + \frac{w}{1 + r} \cdot \frac{\gamma + \beta}{(\gamma + \beta)(1 - \delta) - \delta - \beta} \,. \tag{10}$$

Combining Eqs. (9) and (10), and using Eq. (7) at the steady-state, we obtain the golden rule of population growth and the optimal amount of young-adult consumption, respectively:

$$n^{**} = \delta(1 + r) \,, \tag{11}$$

$$c_1^{**} = \frac{\delta\, w[1 - q(1 + r)]}{\delta(1 + \beta + \gamma) - \gamma} \,. \tag{12}$$

From Eq. (12) it can readily be seen $c_1^{**} > 0$ if

- *Case (a):* $r < \bar{r}$ *and* $\bar{\delta} < \delta < 1$

- *Case (b):* $r > \bar{r}$ *and* $0 < \delta < \bar{\delta}$,

where $\bar{r} := (1-q)/q$ and $\bar{\delta} := \gamma/(1+\beta+\gamma)$. Moreover, using Eq. (11), the condition $n < 1/q$ implies that $\delta < \hat{\delta} := 1/q(1+r)$ must hold to ensure the existence of a positive supply of labour, i.e. $h > 0$.

Let

$$\bar{\bar{\delta}} := \hat{\delta} \cdot \bar{\delta} , \tag{13}$$

$$\bar{\bar{r}} := (q \cdot \bar{\delta})^{-1} - 1, \tag{14}$$

and

$$\hat{r} := (1+\bar{\bar{r}}) \cdot (\bar{\delta})^2 - 1, \tag{15}$$

be a threshold value of the social discount factor and two threshold values of the constant world interest rate, respectively, where $\hat{r} < \bar{r} < \bar{\bar{r}}$ always holds. Moreover, $\hat{\delta} < \bar{\delta}$ ($\hat{\delta} > \bar{\delta}$) for any $r > \bar{\bar{r}}$ ($r < \bar{\bar{r}}$). Then the following proposition holds:

Proposition 2. *Let Case (a) {Case (b)} hold. Then* $\hat{\delta} > 1$ *and* $\bar{\bar{\delta}} > \bar{\delta}$ *{*$\hat{\delta} < 1$ *and* $\bar{\bar{\delta}} < \bar{\delta}$ *}. Therefore, for any* $r < \bar{r}$ *{*$\bar{r} < r < \bar{\bar{r}}$ *[*$r > \bar{\bar{r}}$*]} the command optimum can be decentralised by a government in a market economy:*

(1) without intervention if $\delta = \bar{\bar{\delta}}$*;*

(2) using both a PAYG tax (tax on the old) $\eta = \eta_{GR}(\delta) > 0$ *(*< 0*) to redistribute across generations, and a T/S (S/T) policy* $\sigma = \sigma_{GR}(\delta) > 0$ *(*< 0*) to redistribute within the working-age generation, if* $\bar{\bar{\delta}} < \delta < 1$ *(*$\bar{\delta} < \delta < \bar{\bar{\delta}}$*) {if* $0 < \delta < \bar{\bar{\delta}}$ *(*$\bar{\bar{\delta}} < \delta < \bar{\delta}$ *) [*$0 < \delta < \bar{\bar{\delta}}$ *(*$\bar{\bar{\delta}} < \delta < \hat{\delta}$*)]}, where*

$$\eta_{GR}(\delta) := \frac{w[\delta q(1+r)(1+\beta+\gamma)-\gamma]}{\delta(1+\beta+\gamma)-\gamma} < w, \tag{16}$$

$$\sigma_{GR}(\delta) := \frac{\delta q(1+r)(1+\beta+\gamma)-\gamma}{q(1+r)[\delta(1+\beta+\gamma)-\gamma]} < 1. \tag{17}$$

Proof. See Appendix A.

Given the parametric structure as regards preferences (β, γ) and the child-bearing technology (q), and depending on the mutual relationship between the social discount factor (δ) and the world interest rate (r), Proposition 2 reveals that the first best outcome can be realised by a government in a market setting using appropriately both an instrument to redistribute across generations, i.e. a PAYG tax (tax on the old) and a T/S (S/T) policy to redistribute within the working age generation. In particular, if the interest rate is relatively low, $r < \bar{r}$, *Case (a)*, then when lifetime utilities are discounted at too high (low) a rate by the planner, i.e., $\delta > \bar{\bar{\delta}}$ ($\delta < \bar{\bar{\delta}}$), the command optimum is achieved with both a PAYG tax (tax on the old) to transfer resources from the young (old) to the old (young) and a T/S (S/T) system to promote (disincentive) fertility, since in that case the individual demand for children is too low (high) as compared with the golden rule of population growth, i.e., the inter-generational transfer effect dominates (is dominated by) the capital dilution effect. By contrast, if the interest rate is relatively high, $r > \bar{r}$, *Case (b)*, then, different from *Case (a)* the government must use a PAYG tax (tax on the old) together with a T/S (S/T) policy to incentive (disincentive) fertility when the social discount factor is sufficiently low (high), i.e., $\delta < \bar{\bar{\delta}}$ ($\delta > \bar{\bar{\delta}}$). In the special case $\delta = \bar{\bar{\delta}}$, the market solution automatically coincides with the first best and no government intervention is required. In other words, the recommended policy is symmetrical depending on the mutual relationship between the social discount factor and the interest rate.

We note that if social discount factor is relatively high, i.e., $\delta > \bar{\bar{\delta}}$, then only in *Case (a):* $r < \bar{r}$ and $\bar{\delta} < \delta < 1$, a T/S policy should be used together with a PAYG tax to redistribute from the young to the old to realise the fist best, while in the opposite *Case (b):* $r > \bar{r}$ and $0 < \delta < \bar{\delta}$, the condition $\delta > \bar{\bar{\delta}}$ implies that the government should adopt a S/T policy along with a PAYG tax on the old.

Therefore, when fertility is an endogenous economic variable and child bearing activities are time-consuming, a PAYG tax (tax on the old) and a T/S (S/T) policy act like Siamese twins.

3.1. The special case $\delta = 1$

In the case $\delta = 1$, i.e. the social planner treats all generations symmetrically, the first best outcome at the steady-state can be realised in a market setting by means exclusively of an intra-generational T/S (S/T) policy as long as the world interest rate is high (low) enough, i.e., $\hat{r} < r < \bar{r}$ ($r < \hat{r}$), while it is realised with no government intervention if and only if $r = \hat{r}$, irrespective of whether a PAYG pension system is in place or not. When $r \neq \hat{r}$, a too high (low) interest rate implies that the number of children in a market economy is too low (high) as compared with the golden rule of population growth. Therefore, fertility may be stimulated (discouraged) – and thus the command optimum may be replicated – simply by making the time-child-rearing activities less (more) costly with an appropriate use of the T/S (S/T) instrument.

Proposition 3. *Let* $r < \bar{r}$ *and* $\delta = 1$ *hold. Then the command optimum can be decentralised by a government in a market setting:*

(1) without intervention if $r = \hat{r}$*;*

(2) by means exclusively of a T/S (S/T) policy $\sigma = \sigma_{GR}(1) > 0$ *(*< 0*) if* $\hat{r} < r < \bar{r}$ *(*$r < \hat{r}$*), for any* $-\infty < \eta < w$.

Proof. See Appendix B.

In the next section we look at the welfare effects of the T/S policy, i.e. a wage tax rebated as a lump-sum subsidy when the government redistribute across generations through a PAYG system.

4 Welfare effects of the T/S policy

In almost all industrialised countries retired people's consumption plans are based essentially on the generosity of the social security system, which has been characterised in the last decades – especially in Anglo-Saxon countries – on unfunded PAYG pension schemes. Our objective in this section is to assess the welfare properties of the T/S (S/T) policy when the government redistribute across generations through a PAYG system.

We will show that in a PAYG-taxed economy introducing a T/S policy can represent a Pareto improvement.

Proposition 4. *Let ($\eta \leq 0$) $0 < \eta < w$ hold. Then introducing a T/S policy (is always Pareto-worsening) $0 < \sigma < \hat{\sigma}$ is Pareto-improving and*

$$\sigma = \sigma^* := \frac{\eta}{q\,w(1+r)}, \tag{18}$$

is Pareto efficient.

Proof. See Appendix C.

Therefore, if a PAYG inter-generational transfer is absent ($\eta = 0$) a T/S policy is always Pareto-worsening because it will actually promote fertility while reducing monotonically consumption over the life cycle. By contrast, in a PAYG-taxed economy, a wage tax $0 < \sigma < \hat{\sigma}$ rebated as a lump-sum subsidy within the same working-age (child-bearing) generation makes all individuals better off and thus implies a Pareto improvement as compared with the case in the absence of such a policy. Moreover, a T/S policy $\sigma = \sigma^*$ is Pareto efficient, because it is not possible to make the current as well as all subsequent generations better off by either increasing or decreasing the wage tax σ. In particular, the optimal T/S policy is obtained as the present value of the PAYG tax divided by the cost of raising a child.

The following figure clearly depicts in a stylised way the welfare gains obtained by all generations when the government introduces a T/S policy in a PAYG-taxed economy. The solid line refers to the welfare level in an economy with PAYG pensions and T/S policy, the dashed line to the *laissez faire* economy. Introducing a T/S policy in the range $0 < \sigma < \hat{\sigma}$ represents a Pareto improvement, while σ^* corresponds to the Pareto efficient allocation.

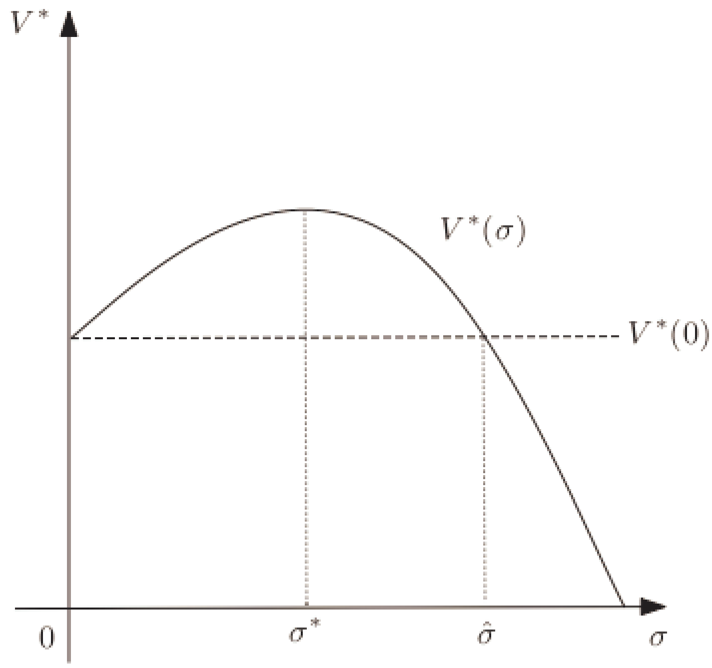

Figure 1. The lifetime welfare as a function of σ in a PAYG-taxed economy (i.e., $0 < \eta < w$).

Finally, we note that the introduction of a S/T policy $0 < \sigma < \left|\hat{\hat{\sigma}}\right|$ represents a Pareto improvement with $\sigma = \left|\sigma^*\right|$ being Pareto efficient for any $\eta < 0$, i.e. the indirect lifetime utility index of all generations can be maximised even with the S/T instrument when a PAYG transfer from the old to the young is realised. However, this appears as a less realistic case. For this reason and for economy of space, we abstract from a formal and graphical analyses of the S/T policy.[8]

5. Conclusions

Using an overlapping generations small open economy with endogenous fertility and time cost of children, we show that the government can let the market solution coincide with the command optimum using both a PAYG tax (tax on the old) to redistribute across generations and tax-*cum*-subsidy (subsidy-*cum*-tax) policy: if the government redistribute across generations through a PAYG tax (tax on the old), and child rearing activities are time consuming, a T/S (S/T) policy, through a reduction (increase) in the net wage earned by the members of the working age generation, reduces (increases) the opportunity cost of losing an additional working unit of time to take care of children, and, hence, acts as a fertility-enhancing (fertility-reducing) device allowing to eliminate the external effects of children on society as a whole. Moreover, in a PAYG-taxed (subsidised) economy a T/S (S/T) scheme can be Pareto efficient.

The policy implications are straightforward: when individuals take into account the effects of child rearing activities on their labour supply decisions a government can avoid to implement a child allowance (tax) system to internalise offspring externalities: it is sufficient to use a traditional instrument in the theory of public finance, namely the T/S (S/T) policy.

Appendix A

[8] The proof is available on request.

Proof of Proposition 2.

Let $\delta = \bar{\bar{\delta}}$ hold. Substituting out $\sigma = \eta = 0$ into Eqs. (5) and (6) gives $n^* = n^{**}$, $c_1^* = c_1^{**}$ for any r.

Let $\delta \neq \bar{\bar{\delta}}$ hold. Combining (5) with (11) and (6) with (12) and solving for σ and η gives

$$\sigma = \sigma(\eta), \tag{A1}$$

$$\eta = \eta(\sigma), \tag{A2}$$

respectively. From Eqs. (A1) and (A2) we get $\eta_{GR}(\delta)$ and $\sigma_{GR}(\delta)$, see Eqs. (16) and (17) in the main text.

If $r < \bar{r}$ and $\bar{\delta} < \delta < 1$ {$r > \bar{r}$ and $0 < \delta < \bar{\delta}$}, then $0 < \eta_{GR}(\delta) < w$ (< 0) and $0 < \sigma_{GR}(\delta) < 1$ (< 0) if and only if $\bar{\bar{\bar{\delta}}} < \delta < 1$ ($\bar{\delta} < \delta < \bar{\bar{\bar{\delta}}}$) {$0 < \delta < \bar{\bar{\bar{\delta}}}$ ($\bar{\bar{\bar{\delta}}} < \delta < \bar{\delta}$) [$0 < \delta < \bar{\bar{\bar{\delta}}}$ ($\bar{\bar{\bar{\delta}}} < \delta < \hat{\delta}$)]}. Substituting out $\eta = \eta_{GR}(\delta) > 0$ (< 0) and $\sigma = \sigma_{GR}(\delta) > 0$ (< 0) into Eqs. (5) and (6) gives $n^* = n^{**}$, $c_1^* = c_1^{**}$ for any $\bar{\bar{\bar{\delta}}} < \delta < 1$ ($\bar{\delta} < \delta < \bar{\bar{\bar{\delta}}}$) and $r < \bar{r}$ {$0 < \delta < \bar{\bar{\bar{\delta}}}$ ($\bar{\bar{\bar{\delta}}} < \delta < \bar{\delta}$) and $\bar{r} < r < \bar{\bar{r}}$ [$0 < \delta < \bar{\bar{\bar{\delta}}}$ ($\bar{\bar{\bar{\delta}}} < \delta < \hat{\delta}$) and $r > \bar{\bar{r}}$]}. **Q.E.D.**

Appendix B

Proof of Proposition 3.

Let $r < \bar{r}$ and $\delta = 1$ hold. Combining either (5) with (11) or (6) with (12) and solving for σ gives $\sigma = \sigma_{GR}(1)$.

If $r = \hat{r} < \bar{r}$, then substituting out $\sigma = 0$ into Eqs. (5) and (6) gives $n^* = n^{**}$, $c_1^* = c_1^{**}$ for any $-\infty < \eta < w$.

If $r \neq \hat{r}$, then $0 < \sigma_{GR}(\delta) < 1$ (< 0) if and only $\hat{r} < r < \bar{r}$ ($r < \hat{r}$). Substituting out $\sigma_{GR}(1) > 0$ (< 0) into Eqs. (5) and (6) gives $n^* = n^{**}$, $c_1^* = c_1^{**}$ for any $\hat{r} < r < \bar{r}$ ($r < \hat{r}$). **Q.E.D.**

Appendix C

Proof of Proposition 4.

Assume that the government introduces an intra-generational T/S scheme ($0 < \sigma < 1$) in period t to redistribute within the working age (child bearing) generation. Then, the welfare of the current (time-t) elderly is not affected by such a policy. Therefore, the indirect lifetime utility index of the time-t working age generation as well as the welfare levels of the generations born in all subsequent periods are exactly the same, i.e., $V_t = V^*$. This implies that, for any given value of η, the maximisation of V^* with respect to σ gives the maximum social welfare as well. Therefore, assume the government faces the following problem:

$$\max_{\{\sigma\}} V^* = \ln\left(\mathrm{H} \cdot (1-\sigma)^{1+\beta} \cdot \{q\,w(1+r)[(1+\beta)(1-\sigma)+\gamma] - \gamma\eta\}^{-(1+\beta+\gamma)}\right), \tag{C1}$$

where $\mathrm{H} := \beta^{\beta} \cdot \gamma^{\gamma} \cdot (q\,w)^{1+\beta} \cdot (1+r)^{1+2\beta+\gamma} \cdot (w-\eta)^{1+\beta+\gamma}$, for any η.

Differentiating Eq. (C1) with respect to σ gives:

$$\frac{\partial V^*(\sigma)}{\partial \sigma} = \frac{\gamma(1+\beta)[\eta - \sigma\,q\,w(1+r)]}{(1-\sigma)\{q\,w(1+r)[(1+\beta)(1-\sigma)+\gamma] - \gamma\eta\}}. \tag{C2}$$

- If $\eta \leq 0$ then $\dfrac{\partial V^*(\sigma)}{\partial \sigma} < 0$ for any $0 < \sigma < 1$.
- If $0 < \eta < w$ then

$$\frac{\partial V^*(\sigma)}{\partial \sigma} \gtreqless 0 \Leftrightarrow \sigma \lesseqgtr \sigma^*, \tag{C3}$$

with $\sigma = \sigma^*$ (see Eq. 18 in the main text) being an interior global maximum. Since $V^*(\sigma)$ is a positive (negative) monotonic function of σ for any $0 < \sigma < \sigma^*$ ($\sigma^* < \sigma < 1$) and $\lim_{\sigma \to 1} V^*(\sigma) = -\infty$, then there always exists a threshold value $\hat{\sigma} > \sigma^*$ such that $V^*(\hat{\sigma}) = V^*(0)$ for any $0 < \eta < w$. Hence, $V^*(\sigma) > V^*(0)$ ($V^*(\sigma) < V^*(0)$) for any $0 < \sigma < \hat{\sigma}$ ($\hat{\sigma} < \sigma < 1$). **Q.E.D.**

References

Atkinson A.B., Stiglitz, J.E., 1980. Lectures on Public Economics. McGraw-Hill, London.

Cigno, A., 1993. Intergenerational transfers without altruism. European Journal of Political Economy 9, 505–518.

Deardorff, A.V., 1976. The optimum growth rate for population: comment. International Economic Review 17, 510–515.

Diamond, P.A., 1965. National debt in a neoclassical growth model. American Economic Review 55, 1126–1150.

Eckstein, Z., Wolpin, K.I., 1985. Endogenous fertility and optimal population size. Journal of Public Economics 27, 93–106.

Fanti, L., Gori, L., 2007. Long-run output, welfare and fertility in a neoclassical OLG growth model: the effects of intra-generational tax policies. Studi Economici 93, 53–70.

Fenge, R., Meier, V., 2005. Pensions and fertility incentives. Canadian Journal of Economics 38, 28–48.

Gahvari, F., 1993. Taxation and government expenditure in a life-cycle growth model. Public Finance 48, 33–56.

Galor, O., Weil, D.N., 1996. The gender gap, fertility, and growth. American Economic Review 86, 374–387.

Groezen, B. van, Meijdam, L., 2008. Growing old and staying young: population policy in an ageing closed economy. Journal of Population Economics 21, 573–588.

Groezen, B van, Leers, T., Meijdam, L., 2003. Social security and endogenous fertility: pensions and child allowances as Siamese twins. Journal of Public Economics 87, 233–251.

Jaeger, K., Kuhle, W., 2009 The optimum growth rate for population reconsidered. Journal of Population Economics 22, 23–41.

Phelps, E.S., 1968. Population increase. Canadian Journal of Economics 1, 497–518.

Samuelson, P.A., 1958. An exact consumption-loan model of interest with or without the social contrivance of money. Journal of Political Economy 66, 467–482.

Samuelson, P.A., 1975. The optimum growth rate for population. International Economic Review 16, 531–538.