

# A Modified Partial Adjustment Model of Aggregate U.S. Agricultural Supply

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Aggregate U.S. agricultural supply response is modeled through a modified partial adjustment model, where the effects of weather and other temporal stochastic effects are structured to be purely static, while the effects of price and technology, or trend, are dynamic. The model is applied to a time series of aggregate U.S. farm output, aggregate U.S. crop production, and aggregate U.S. livestock and livestock products production for several sample periods within the period 1911-1958. The three aggregate output indexes are tested for irreversibilities in supply response, and no evidence of a definitive irreversible supply function is found for any of the dynamic supply models. The use of a nonstochastic difference equation to model the aggregate farm output and crop production equations results in short-run elasticity estimates that are somewhat smaller than previous studies suggest while the long-run elasticities are somewhat larger.

Although there are serious limitations to an aggregate measure of agricultural output in a supply response framework, the concept is frequently used in agricultural economics, and a few attempts have been made to empirically estimate equations from time series data (Griliches; Tweeten and Quance). This paper examines some alternative specifications from the basic Nerlove model used by Griliches and the consequences to empirical estimates of supply elasticities. The various specifications focus on the way in which stochastic components of a dynamic regression equation are treated, and they have implications for time series estimation of supply response equations for individual farm commodities as well as aggregate indices.

Nerlove (1956, 1958) presented the following output adjustment model and applied it to corn, cotton and wheat production in the United States. Consider an

individual producer with output level  $Y_t$  in period  $t$  and a "desired" or "long-run normal" level of output  $Y_t^*$ , which is a function of price  $P_t$  and technology  $T_t$ ,

$$Y_t^* = \alpha + \beta P_t + \theta T_t. \quad (1)$$

Because of adjustment costs and fixed assets, the output adjustment achieved in any period is assumed to be a (constant) fraction of the difference between the desired output and the previous period's output,

$$Y_t - Y_{t-1} = \delta(Y_t^* - Y_{t-1}), \quad (2) \\ 0 < \delta < 1.$$

Direct substitution for  $Y_t^*$  in (2) and solving for  $Y_t$  gives the dynamic supply equation

$$Y_t = \alpha\delta + \beta\delta P_t + \theta\delta T_t + (1 - \delta)Y_{t-1}. \quad (3)$$

The usual estimation procedure is to add a disturbance term to (3) and apply the model to aggregate data.

The first attempt to empirically estimate a dynamic aggregate U.S. agricultural supply function was Griliches' study in which he applied ordinary least squares to a structural equation of the form

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$$\ln Y_t = a + b \ln P_t + c \ln W_t + d \ln Y_{t-1} + gt + u_t \quad (4)$$

where  $Y_t$  is the index of farm output;  $P_t$  is the March index of prices received for all farm products deflated by the March index of prices paid for production items, farm wages, taxes, and interest;  $W_t$  is Stallings' index for the effects of weather on farm output;  $u_t$  is a random disturbance, and  $a$ ,  $b$ ,  $c$ ,  $d$  and  $g$  are constants. Griliches analyzed aggregate farm output and two subaggregates: all crops, and livestock and livestock products.<sup>1</sup>

This model can be interpreted as a modified partial adjustment model in which the adjustment is linear in logarithms. The use of a linear trend in a logarithmic relationship to approximate technological change can be interpreted as a tacit assumption that the supply curve has shifted to the right at a constant (compounded annually) percentage rate.

Two aspects of the approach of Griliches warrant closer attention. First, the implied adjustment equation is in actual output rather than planned or expected output, i.e., the level of output that would prevail given average conditions on the variables not subject to control by farmers. Griliches (p. 291) discussed the problem as follows:

Measured output is not necessarily equal to planned output, due to 'weather' and other random effects. This . . . factor would lead to a downward bias in the estimate of the coefficient of lagged output since the adjustment assumed by the model proceeds from the previously 'planned' output, of which actual output is not an error-free measure.

Because of the dependence of ultimate farm output upon weather and other random factors, it appears that farmers can at best plan for an expected level of output, given an average season in terms of

these uncontrollable factors. Such an "expected" output level is a conceptual, unobservable variable, and the relevant problem is how to specify an empirically estimable adjustment equation in this variable.

The second characteristic of the Griliches model that must be considered is the fact that the introduction of the lagged dependent variable on the right hand side of the regression equation, without further restrictions on the exogenous variables, imposes the same geometric lag pattern on all exogenous variables. While this is inconsequential if the only regressors are price and linear trend, we would expect that this period's weather would tend to have only a contemporaneous effect. Given this assumption the dynamic effects of the weather index should be eliminated.

The third problem arises when ordinary least-squares regression techniques are applied to a time-series model with the lagged dependent variable included as one of the predetermined explanatory variables. It is well-known that ordinary least-squares estimates obtained from models with lagged dependent variables and serially correlated errors result in biased and inconsistent estimates. In any time-series model, we would expect the residuals to be autocorrelated because of left out explanatory variables that are correlated over time. An approach to handling these last two problems is developed below where it will be seen that they are both part of the same overall problem of dynamic specification.

It is our opinion that rigid behavioral hypotheses about producer behavior are at best crude approximations to aggregate behavior, especially when posited in the usual linear form with aspirations for empirical testing. A priori reasoning produces several equally plausible models, and the data base is incapable of discerning the correct model with a high probability, even if it were within the set tested. Thus, the approach adopted here is to use

<sup>1</sup> The price variable for the livestock model is the previous year's average annual price index, rather than March of the current year.

the rational distributed lag model as a rather general additive approximation to the unknown structure of the aggregate supply function, without formally specifying a behavioral model and deriving a system of product supply and demand equations.

### Alternative Specifications

One question raised in the previous section is whether the adjustment process is in actual quantities produced or in what would be "average" quantities associated with the period. Lagged output under average conditions for random effects on output would be preferable to  $Y_{t-1}$  in (2), implying that  $Y_{t-1}$  in the right-hand side of (3) is an independent variable subject to measurement errors. Note that "planned" output would differ from "long-run normal" output as a result of adjustment costs and fixed assets, and a dynamic adjustment equation is still warranted.

A second issue raised above is the question of purging the effects of a weather variable from the dynamic aspects of supply response. Assuming that we have an index for the effects of weather,  $W_t$ , which enters additively with parameter  $\gamma$ , then by defining the variable  $\tilde{Y}_t = Y_t - \gamma W_t$ , we can rewrite the adjustment equation (2) in terms of output net of weather effects

$$\tilde{Y}_t - \tilde{Y}_{t-1} = \delta(Y_t^* - \tilde{Y}_{t-1}). \quad (2a)$$

The dynamic supply equation (3) then becomes

$$Y_t = \alpha\delta + \beta\delta P_t + \gamma W_t + \theta\delta T_t + (1 - \delta)(Y_{t-1} - \gamma W_{t-1}). \quad (3a)$$

The weather variable in (3a) enters without any distributed lag response; a simple proof is by induction. Clearly,  $\partial Y_t / \partial W_t = \gamma$ , and

$$\begin{aligned} \partial Y_t / \partial W_{t-1} &= (1 - \delta)[\partial Y_{t-1} / \partial W_{t-1} - \gamma] \\ &= (1 - \delta)[\gamma - \gamma] = 0. \end{aligned} \quad (5)$$

Since  $\partial Y_t / \partial W_{t-1} = 0$ , it follows inductively that  $\partial Y_t / \partial W_{t-j} = 0$  for  $j \geq 1$ , so that elimination of weather effects from the partial adjustment equation simultaneously removes the weather variable from any distributed lag response in the derived regression equation (3a).

An operational statistical model is obtained by adding a disturbance term to (3a), but the equation is nonlinear in parameters.<sup>2</sup> Practical estimation could be with nonlinear least squares or a conditional linear least-squares search on  $\delta$ .

There are many sources of random variation other than weather effects which lead to a disturbance term in (3), including aggregation over individual farms, omitted variables, and other minor specification errors. If the disturbance term added to (3) or (3a) to get a statistical model is dominated by factors that do not reflect changes in output capacity, then the lagged dependent variable would be better defined as the lagged expectation of the regression equation. This statistical model for (3a) would be

$$Y_t = \alpha\delta + \beta\delta P_t + \gamma W_t + \theta\delta T_t + (1 - \delta)(\eta_{t-1} - \gamma W_{t-1}) + u_t, \quad (6)$$

where  $\eta_t = E(Y_t | W_t)$  and  $Y_t = \eta_t + u_t$ . The partial adjustment model associated with (6) is

$$\tilde{\eta}_t - \tilde{\eta}_{t-1} = \delta(Y_t^* - \tilde{\eta}_{t-1}), \quad (2b)$$

where  $\tilde{\eta}_t = \eta_t - \gamma W_t$ .

The three versions of partial adjustment, (2), (2a), and (2b), represent different numbers of random components purged from the measure of output used as the dependent variable in a dynamic regression equation. Nothing is removed in (2); weather effects are purged in (2a); while in (2b), both weather effects and the entire disturbance term of the regression equation are purged.

A priori reasoning suggests going at least

<sup>2</sup> There is, however, sufficient identification to estimate  $\alpha$ ,  $\beta$ , and  $\theta$  from a linear regression by combining estimates of  $\delta$ ,  $\alpha' = \alpha\delta$ ,  $\beta' = \beta\delta$ , and  $\theta' = \theta\delta$ .

as far as (2a) by purging weather effects from output, but moving all the way to (2b) might remove some components that indirectly reflect the fixed assets and management inertia which are important in specification of the partial adjustment hypothesis. The essential question is just what the disturbance term comprises, and it must be recognized that this term is a construction of the statistician with many vagaries as to its actual source.

In an aggregate supply equation, we would expect much of the disturbance term to stem from aggregation problems in the dependent variable and from the single aggregate price index used to subsume all individual commodity prices. It makes sense to purge the former components, but not the latter from the partial adjustment equation. In a sense, the limitations of using a single price index can be viewed as specification error, given the definition of the dependent variable, in that a more complete vector of prices would better explain aggregate output. Some of our empirical results suggest this to be the case.

If  $u_t$  in (6) has the classical properties, then replacing  $\eta_{t-1}$  with  $Y_{t-1}$  would produce a disturbance following a first-order moving average process,  $u_t - (1 - \delta)u_{t-1}$ . Ordinary least squares estimates would then produce biased and inconsistent estimators.<sup>3</sup> The disturbance term is likely to be autocorrelated in any specification, but there is no a priori reason to suppose that it will be restricted to a first-order Markov process with parameter  $-(1 - \delta)$ . The structure of dynamic models must be discovered from the data with the help of economic theory in most applications, and in these situations there is a distinct advantage in not having the lagged dependent variable serving as an independent

variable in the regression model. If the disturbance term in (6) is misspecified, and the remainder of the equation is correctly specified, least-squares estimates are still consistent, which is not the case when  $\eta_{t-1}$  is replaced by  $Y_{t-1}$ .

Least-squares estimates of the unknown parameters in (6) can be calculated with nonlinear least-squares algorithms by treating the  $\{\eta_t\}$  as unobservable variables, but nevertheless, as subject to least-squares estimation because each is implicitly a function of the parameters  $(\alpha, \beta, \gamma, \theta, \delta)$  plus the initial condition parameter  $\eta_0$ . This latter parameter can be given an a priori estimate of  $Y_0$ .<sup>4</sup> The computational algorithm used in this study is a modified Marquardt nonlinear least-squares routine much the same as that commonly used in time-series packages to deal with a moving average error process. The estimation procedure yields estimates that are asymptotically equivalent to maximum likelihood under normality, and with one independent variable is equivalent to the method of Maddala and Rao except for the handling of initial conditions. Additional details on the practical estimation of models of this type in (6) are given in Burt.

Another advantage in using (6) is the simplicity with which general distributed lags on the independent variables can be approximated by the rational lag model and superimposed on the partial adjustment model of (2).<sup>5</sup> Let  $L$  be the lag operator where  $LX_t = X_{t-1}$  and

<sup>4</sup> A Monte Carlo study by Schmidt found that using  $Y_0$  for  $\eta_0$  was about as efficient as estimating  $\eta_0$  simultaneously with the other parameters.

<sup>5</sup> Technology is ignored momentarily to simplify the discussion. A separate lag structure on  $T_t$  could be included in the same fashion as for price. If  $T_t$  is approximated by a linear trend as in most empirical models, the dynamic specification is of no consequence except for the small effects of initial conditions. This assertion follows from the special nature of the linear trend and the linear lag operator  $L$ .

<sup>3</sup> Of course, a search over  $\delta$  with the structure of the error term accounted for, using a criterion of conditional OLS given  $\delta$  produces asymptotic maximum likelihood estimators which are the same as nonlinear LS estimators from (6).

$$\beta(L) = \beta_0 + \beta_1 L + \dots + \beta_k L^k$$

$$\omega(L) = 1 - \omega_1 L - \dots - \omega_m L^m.$$

Dropping the technology variable for simplicity, (1) is generalized to

$$Y^*_t = \alpha + \beta(L)P_t/\omega(L) \quad (7)$$

which on substitution into the partial adjustment equation (2b) in "expected" output levels, net of weather effects, yields

$$\eta_t = \alpha\delta + \beta(L)\delta P_t/\omega(L) + \gamma W_t \quad (8)$$

$$+ (1 - \delta)(\eta_{t-1} - \gamma W_{t-1}).$$

Multiplying by  $\omega(L)$  and rearranging yields

$$\eta_t = \alpha' + \beta(L)\delta P_t + \gamma_0 W_t + \dots + \gamma_n W_{t-n} \quad (9)$$

$$+ \lambda_1 \eta_{t-1} + \dots + \lambda_n \eta_{t-n},$$

where  $n = m + 1$  and the  $\{\gamma_j\}$  and  $\{\lambda_j\}$  are functions of  $\delta$ ,  $\gamma$ , the  $\{\beta_j\}$ , and the  $\{\omega_j\}$ . Although it is not obvious in (9) that there are no dynamic weather effects, an inductive proof like that given below (3a) is straightforward.

### The Data and Some Limitations

In view of the obvious limitations of using aggregate output and price indices, the statistical models used here must be recognized as rather crude and subject to considerable specification error. This problem is compounded by various government programs designed to alter agricultural production and income which are too numerous and varied to handle with concomitant variables in the regression equation. Rapid technological change during the post World War II period also greatly complicates modeling supply response when a smooth trend is used to measure the effects of technology.

Output is measured by the USDA index of farm output for the aggregate farm output model, the index of output of all crops for the all crops model, and the index of output of livestock and livestock products for the livestock model. The out-

put price variables used are the March price index for all farm output, the March price index for all crops, and the annual price index for livestock and livestock products lagged one year.<sup>6</sup> The aggregate output price and the all-crops price were deflated by the annual index of prices paid for production items, farm wage rates, interest and taxes lagged one year. The livestock price was deflated by the annual average price paid by farmers for feed, lagged one year.<sup>7</sup> The weather variable is Stallings' index for the appropriate aggregate index, a series available for the years 1900-57. The only serious attempt to update the deflator was by Kost, but the extension was only through 1962.

Although the data are available back to 1910, our results suggested that there was probably a serious deficiency in the weather index somewhere before 1913. This conclusion was reached on the basis of model sensitivity to inclusion of these earliest three years in the sample with and without the weather index as an explanatory variable. Specifications emanating from (6) were particularly helpful in this regard because  $\eta_0$  can be estimated as a parameter and compared to  $Y_0$  as an a priori approximation to  $\eta_0$ . This is essentially a least squares "backcast" to check for specification error.

The importance of weather on agricultural output, the considerations in the above paragraph, the problems of government programs, and a surge in technological change in the 1950s lead us to lean heavily on the sample period 1914-51, using 1913 as an initial condition ob-

<sup>6</sup> The choice of periods for prices was nearly the same as that used by Griliches; the only exception was that we used a lagged annual deflator on March price instead of the current March deflator.

<sup>7</sup> Data for 1910-56 are summarized in USDA Agriculture Handbook No. 118, Vol. 1, 1957. Sample means of the variables for 1914-51 are: weather index .996, trend 32.5, aggregate price .957, crop price .943, livestock/feed price 1.113, total output .568, and crop output .644.

TABLE 1. Estimated Equations for Aggregate Agricultural Supply Response.

Equation No.	Period	Model	Weather <sup>a</sup>	Aggregate Price	Crop Price	Livestock/Feed Price	Linear Trend	Lagged Output	Auto-regressive Error	R <sup>2</sup>	Short-Run Elasticity	Long-Run Elasticity
1.1a	1914-51	Y <sub>t-1</sub>	.148 (.025)	.050 (.016)			.0025 (.0008)	.720 (.097)	-.392 (.184)	.9500	.09	.30
1.1b	1921-51	Y <sub>t-1</sub>	.191 (.035)	.050 (.021)			.0017 (.0011)	.780 (.112)	-.397 (.210)	.9450	.08	.35
1.2a	1914-51	E(Y <sub>t-1</sub> )	.172 (.034)	.047 (.019)			.0022 (.0009)	.760 (.114)	.327 (.153)	.9467	.08	.33
1.2b	1921-51	E(Y <sub>t-1</sub> )	.201 (.045)	.055 (.022)			.0014 (.0009)	.805 (.109)	.312 (.171)	.9467	.09	.44
1.3a	1914-51	Y <sub>t-1</sub>	.186 (.027)		.053 (.011)	.123 (.031)	.0040 (.0006)	.447 (.074)	-.415 (.174)	.9676	.19	.34
1.3b	1921-51	Y <sub>t-1</sub>	.231 (.035)		.035 (.015)	.123 (.034)	.0046 (.0009)	.407 (.098)	-.409 (.217)	.9669	.15	.25
1.4a	1914-51	E(Y <sub>t-1</sub> )	.168 (.026)		.046 (.009)	.162 (.026)	.0029 (.0005)	.550 (.067)	-.223 (.158)	.9692	.20	.45
1.4b	1921-51	E(Y <sub>t-1</sub> )	.227 (.033)		.035 (.011)	.152 (.025)	.0028 (.0006)	.575 (.076)	-.183 (.177)	.9710	.17	.39
1.5	1914-51	E(Y <sub>t-1</sub> )	.167 (.021)		.043 (.021)	.142 (.045)		.779 (.139)	.910 (.067)	.9243	.18	.83
1.6	1921-57	Y <sub>t-1</sub> and logs <sup>b</sup>	.333 (.057)	.082 (.034)			.0061 (.0023)	.597 (.132)		.9562	.08	.20
1.7	1921-57	Y <sub>t-1</sub> and logs <sup>b</sup>	.326 (.052)	.069 (.026)			.0027 (.0019)	.800 (.110)	-.376 (.192)	.9587	.07	.35

Note: Numbers in parentheses are asymptotic standard errors.  
<sup>a</sup> The weather variable is excluded from the distributed lag in all equations.  
<sup>b</sup> All variables in natural logarithms except trend.

servation. We also estimated the sample period 1921–51 and 1921–57 for comparison with Griliches' results and because of some question about the accuracy of output and price indices prior to 1920.

## Results

### *Aggregate Output Response*

Results of fitting several first order difference equations to aggregate farm output are given in Table 1 (numbers in parentheses are asymptotic standard errors). The two sample periods for each equation are paired, one above the other, with an a-b designation on the equation number to denote the periods beginning in 1914 and 1921. The model designation  $Y_{t-1}$  versus  $E(Y_{t-1})$  refers to the assumptions associated with partial adjustment equation (2a) and (2b), respectively. Reported elasticities are at sample means, and adjusted R-square includes the explanatory value of the autoregressive error structure.

Equations using  $E(Y_{t-1})$  as the lagged output measure were estimated with  $E(Y_0) = \eta_0$  treated as an additional parameter, but the estimate is not reported. The estimate of  $\eta_0$  was always very close to  $Y_0$ , and the first observation was saved in first-order autoregressive error specifications because  $u_0 = Y_0 - \eta_0$  provides an initial condition for the error process.

In equations 1.3 and 1.4 in Table 1, we use an index of total farm output for the dependent variable and separate price indices for the crop and livestock sector responses. The improved fit, as measured by adjusted R-square, and the reduced standard errors on the regression coefficients imply that separate price indices reduce the specification error associated with the weakness of a single aggregate price index. The better fit of equations 1.4 compared to 1.3 suggests that the partial adjustment equation (2b) is superior to (2a) when the regression equation is well specified.

Coefficients estimated for each separate price index must be interpreted as partial effects on the total aggregate output index, holding the other price index constant. Reported elasticities for equations 1.3 and 1.4 were computed from an aggregation formula as follows. Two main assumptions were used to derive the formula: (1) the index of prices paid for feed used to deflate the livestock price index can be interpreted as an exogenous input price, and (2) the net effect on total output of each separate price index is restricted to its own component of output.<sup>8</sup>

By definition of the index of prices received for all farm products,

$$P_t = w_1 P_{Ct} + w_2 P_{Lt}, \\ w_1 + w_2 = 1,$$

where  $P_t$  is the index of prices received for all farm products in year  $t$ ,  $P_{Ct}$  is the index of prices received for all crops,  $P_{Lt}$  is the index of prices received for livestock and livestock products,  $w_1$  is the relative importance of crops, and  $w_2$  is the relative importance of livestock and livestock products during a given weight-base period for the construction of the aggregate prices received index. Feed consumed is calculated as a fixed proportion of livestock production (USDA, Handbook No. 118, Vol. 2, p. 34), and seed crops are calculated as a percentage of the average value of all crops during a given weight-base period. Therefore, total output is defined as

$$Q_t = v_1 Q_{Ct} + v_2 Q_{Lt}, \\ 0 \leq v_1, v_2 \leq 1,$$

where  $Q_t$  is total farm output in year  $t$ ,  $Q_{Ct}$  is all crop production,  $Q_{Lt}$  is total livestock production,  $v_1$  is the percentage of

<sup>8</sup> The logical basis for calculating a single price elasticity is compromised somewhat by the prices paid for feed index appearing in the livestock price variable, creating a negative correlation between the two price variables. This negative correlation will tend to bias the elasticity upward when using the formula derived here.

all crop production not used for seed, and  $v_2$  is the percentage of livestock production net of feed consumed.

For aggregated indices not usually much different in magnitude, it seems reasonable to assume that for small changes in the price indices,

$$\partial Q_t / \partial P_{Ct} \cong \partial Q_{Ct} / \partial P_{Ct}$$

and

$$\partial Q_t / \partial P_{Lt} \cong \partial Q_{Lt} / \partial P_{Lt},$$

allowing the approximation

$$\partial Q_t / \partial P_t \cong v_1 \beta_1 + v_2 \beta_2,$$

where  $\beta_1$  and  $\beta_2$  are coefficients on the crop and livestock price indices, respectively. This leads to an estimated aggregate price elasticity of

$$\epsilon \cong (v_1 \beta_1 + v_2 \beta_2)(w_1 P_{Ct} + w_2 P_{Lt}) / Q_t.$$

The weights used to calculate the elasticities in Table 1 are based on 1937-41 and 1935-39 for prices and quantities, respectively. The actual weights are  $w_1 = .422$ ,  $w_2 = .578$ ,  $v_1 = .9935$ , and  $v_2 = .4018$ .

In an attempt to delineate an upper bound on the long-run price elasticity, equation 1.5 in Table 1 was estimated with trend omitted, which imputes the effects of technological change to price effects. Since this is a two-price equation, the elasticity has an additional upward bias from the aggregation procedure (see footnote 8).

The impact of the error specification is illustrated by a log-linear equation for 1921-57 (basically Griliches' equation 1.5)<sup>9</sup> which are equations 1.6 and 1.7 in Table 1 with and without an autoregressive error. The long-run elasticity estimate increased from .20 to .35 by introducing the autoregressive error.

Many of the same equations were estimated with the variables in logarithms ex-

cept for trend, but the linear models consistently gave a better fit, and the parameter estimates were more stable across sample periods. We also tried an exponential trend in the linear models, but it did not improve the fit or change the parameter estimates much except for samples going through 1957.

Several rational lag specifications were tried on the price variable jointly with the partial adjustment equation. For example, a partial adjustment/adaptive-expectations model was considered quite plausible on a priori grounds. A second-order difference equation resulted in a marginally significant coefficient on  $E(y_{t-2})$ , but the significance was traced to observations for 1934 and 1936, which are the worst drought years in the series; consequently, the second order model was not given serious credence. Aggregation over all prices probably removes the opportunity to model any distributed lag price expectations, even if they are important for some individual commodities.

The primary differences in the specifications presented here and those of Griliches are: (1) the deflator for March price, (2) exclusion of weather from a distributed lag response, (3) a first order autoregressive error structure, and (4) linear instead of log-linear equations.

### Aggregate Crop Response

Results for the crops index are quite similar to aggregate output with approximately the same long-run price elasticities as the single price models in Table 1. Several first order difference equations for the 1914-51 period are given in Table 2.

Equations 2.1 and 2.2 illustrate the sensitivity of the lag structure to specification of the error term when the lagged dependent variable is a regressor. Equations 2.1 and 2.3 yield almost identical long-run elasticities, and there is little basis for choosing between the stochastic versus nonstochastic difference equation models.

<sup>9</sup> Griliches' equation estimated by OLS did not exclude the weather index from the distributed lag and resulted in an even smaller estimate of the coefficient on  $Y_{t-1}$ , i.e., 0.298, and a smaller long-run elasticity of 0.14.

**TABLE 2. Estimated Equations for Aggregate Crops Response (1914-51).**

Equation No.	Model	Weather <sup>a</sup>	Crop Price	Linear Trend	Lagged Output	Auto-regressive Error	R <sup>2</sup>	Short-Run Elasticity	Long-Run Elasticity
2.1	$Y_{t-1}$	.214 (.031)	.066 (.017)	.0023 (.0006)	.669 (.098)	-.583 (.157)	.872	.10	.29
2.2	$Y_{t-1}$	.234 (.043)	.097 (.023)	.0043 (.0009)	.337 (.130)		.845	.14	.22
2.3	$E(Y_{t-1})$	.231 (.044)	.055 (.016)	.0019 (.0005)	.739 (.095)	-.049 (.162)	.862	.08	.31
2.4	$E(Y_{t-1})$ and $\log s^b$	.371 (.069)	.077 (.024)	.0027 (.0008)	.730 (.102)	-.049 (.162)	.842	.08	.28

Note: Numbers in parentheses are asymptotic standard errors.

<sup>a</sup> The weather variable is excluded from the distributed lag in all equations.

<sup>b</sup> All variables in natural logarithms except a linear trend.

As in Table 1 for aggregate output, the former tends to have a smaller coefficient on the measure of lagged response. Equation 2.4, which is linear in logarithms of all variables except trend, has nearly the same dynamic structure as the linear model in 2.3, but the fit is not as good.

#### *Aggregate Livestock Response*

Since the weather index is unimportant in livestock output, it might be expected that our results would be close to those in Griliches' study because much of the discrepancy found in the aggregate output and crops models seems to be due to dynamic specification on the weather index. But another important aspect is specification of the structure of the disturbance term; recall the estimated negative first-order autoregressive structure for the lagged dependent variable equations in Tables 1 and 2.

In the livestock equation the autoregressive parameter estimate is positive, and with the lagged dependent variable in the equation, it is not significant for samples truncated before about 1960. The equations estimated with the lagged expectation of the dependent variable suggested a stochastic difference equation with a small positive autoregressive error param-

eter for samples within the period 1911-58. A recent Monte Carlo study of models with a "trended" variable and lagged dependent variable as regressors suggests that ordinary least squares is superior to generalized least squares when the autoregressive error parameter is between zero and 0.5 [Maeshiro].

We concluded that little improvement could be made over Griliches' estimates of aggregate livestock response during 1911-58. Attempts to estimate a relationship for the post World War II period were discouraging. Regression coefficients on trend and the lagged dependent variable were too confounded to draw conclusions about long-run price response in recent years. Apparently, technological change has been too sporadic to capture with a smooth trend. Although we do not report any equations here, the most consistent estimates in our analysis across various models were for the period 1921-51 with a short-run elasticity of 0.3 and with 0.7 to 0.8 for the long-run.<sup>10</sup>

#### *Irreversibility Tests*

The Wolfram technique for estimating irreversible functions in time series, which

<sup>10</sup> Detailed results including the fitted equations are given in LaFrance.

**TABLE 3. Comparison of Griliches' Price Elasticities with the New Approach.**

Output Aggregate	Griliches		LaFrance and Burt	
	Short-Run Elasticity	Long-Run Elasticity	Short-Run Elasticity	Long-Run Elasticity
Total Output	0.10	0.15	0.08	0.3 ± 0.10 <sup>a</sup>
All Crops	0.16	0.23	0.08	0.28 ± 0.13 <sup>b</sup>
Livestock and Products	0.20	0.70	0.30	0.70-0.80

<sup>a</sup> Taking 0.30 as a basic "unbiased" estimate of the long-run elasticity, this is an approximate 95% confidence interval based on asymptotic distribution theory and the two-price model.

<sup>b</sup> This is an approximate 95% confidence interval based on asymptotic distribution theory and equation 2.4.

was later simplified by Houck for its operational application, was applied to the three output indices. In all cases, the trend variable was confounded with the two price variables, one each for increasing and decreasing prices. A positive trend in the reversible specification changed to a negative trend when the single price was replaced by the two separate price series to allow for irreversibilities. Adjusted R-square was essentially the same in the irreversibility specification with trend deleted as in the reversible equation with trend included. These results suggest that the two variables for rising and falling prices are almost a perfect substitute for a single price variable jointly with linear trend.

The model developed by Traill, Coleman and Young for irreversible supply response was also tested, but their method turned out to be rather infeasible because the absolute maximum price appears very early in the data for the total output and crops indices. This confounded their "maximum price" variable with the intercept. We tried to remedy this by subjecting the previously experienced maximum price to an exponential decay,

$$P_t \lambda^j, 0 < \lambda < 1,$$

where  $P_t$  is price in period  $t$  and  $t + j$  is the period in which a maximum with respect to  $t$  is sought for the variable  $P_t \lambda^j$ . A search was made on  $\lambda$  using a conditional least squares criterion, but the revised

method did not improve the earlier results.

We concluded that there is little evidence of irreversibilities in aggregate supply response insofar as current econometric methodology can detect. Although these methods did not reveal a definitive irreversible supply function, we suspect the outcome is a weakness in the methods. The simple deductive economic argument for irreversibilities in agricultural supply, especially in the aggregate, is most convincing (Johnson and Pasour).

### Conclusions

A comparison of Griliches' elasticity estimates with those obtained in this study is presented in Table 3. The refinements in specification of partial-adjustment equations for supply response tend to produce higher long-run and lower short-run price elasticities than a straightforward use of the lagged output variable, as in Griliches' model. However, the differences in elasticities are small when an autoregressive error is specified in the equations containing the lagged dependent variable and the weather variable is excluded from the distributed lag. Differences between the linear and log-linear specifications account for some of the discrepancy in estimated elasticities; our logarithmic equations gave long-run elasticities about 20 percent below the same linear equations.

With the caveat that an average price elasticity over the entire domain of a supply equation is at best a crude indicator of supply response behavior, our results suggest point estimates of .1 and .3 in the short-run and long-run, respectively, for each total output and crops indices over the period 1914–51. We could not improve upon Griliches' estimates of price elasticities for livestock and livestock products which were .2 and .7 in the short-run and long-run, respectively.

The lagged expectation measure of output yields an error term which appears to obey the classic assumptions for the crop index, while actual lagged output gives such an error term for the livestock and livestock products index. These results are consistent with lagged output being an *indirect* estimate of the investment in fixed assets specialized to crop production while it is more of a *direct* estimate of such assets used in livestock production. The dominant role of breeding stock in livestock production (particularly beef) suggests that lagged output would be a more direct measure of fixed assets in livestock production than in crops production. We note that the livestock output index is for production, not marketings, and reflects changes in inventories. These results substantiate Griliches' assertion that the partial adjustment model he used employed a lagged output variable for the partial adjustment equation which was subject to measurement error, at least in the aggregate output and all crops models.

Partitioning various terms in the dynamic regression equation out of the measure used for lagged output should be useful in modeling supply response of individual commodities. The specification of (2b) where the regression disturbance term is purged from the lagged dependent variable appears to be most appropriate in well specified models, which are more likely for individual commodities than for an aggregate index of output.

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