

Staff Paper

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Abstract

We study a farmer's decision to convert traditional crop land into growing dedicated energy crops, taking in account sunk conversion costs, uncertainties in traditional and energy crop returns, and learning. The optimal decision rules differ significantly from the expected net present value rule, which ignores learning, and from real option models that allow only one way conversions into energy crops. These models also predict drastically different patterns of land conversions into and out of energy crops over time. Using corn-soybean rotations and switchgrass as examples, we show that the model predictions are sensitive to assumptions about stochastic processes of the returns. Government policies might have unintended consequences: subsidizing conversion costs into switchgrass reduces proportions of land in switchgrass in the long run.

Keywords: real options, irreversibility, sunk costs, land conversion, biofuel, cellulosic biomass, dynamic modeling, stochastic process, biofuel policy

JEL codes: Q42 , Q24

Categories: Land Economics/Use, Resource/Energy Economics and Policy, Risk and Uncertainty

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Introduction

Replacing fossil fuel with renewable fuels, including biofuels such as ethanol, has been advocated for contributing to energy independence and mitigating climate change. Currently most of the ethanol produced in the United States comes from corn grain, raising concerns about the negative environmental impacts associated with corn production, upward pressure on food prices, and greenhouse gas emissions due to indirect land use changes as rising food prices induce cultivation of new lands (Searchinger et al. 2008). A promising alternative is cellulosic ethanol, which relies on nonfood feedstocks. The U.S. Energy Independence and Security Act (EISA) of 2007 mandates blending into transportation fuels of 36 billion gallons of renewable liquid fuels annually by 2022, out of which at least 16 billion gallons must be cellulosic ethanol. Significant expansion of cellulosic ethanol production will require more land to grow dedicated energy crops. A recent simulation of potential US switchgrass production implies a need for 71 million acres of crop land to meet the 2007 EISA mandate (Thomson et al. 2008). Yet idle land in the United States, including CRP land, is only about 40 million acres (Lubowski et al 2006). This implies that current production land will need to be converted to grow cellulosic energy crops.

Although large scale production of cellulosic energy crops is not commercially viable at present, its advent could have dramatic effects on land use change and associated economic and environmental impacts. Forecasting the conditions under which such change would occur is an important first step toward evaluating likely outcomes and relevant policy interventions. Forecasting land use change depends critically on farmers' land use decisions, which in turn are driven by several salient features of dedicated energy crops.

First, all the major cellulosic energy crop contenders are perennial. Grass crops, such as switchgrass and miscanthus, as well as short rotation woody crops, such as poplar and willow, all need several years to establish before achieving full yield potential (Powlson et al. 2005). Devoting land to energy crops represents a long term commitment by the farmer and incurs sunk costs. Moreover, converting land back to traditional annual crops also incurs (possibly substantial) costs (e.g., costs of killing persistent perennial rootstocks).

Second, farmers growing cellulosic energy crops face two broad sources of revenue uncertainty compared with traditional crops. Unlike traditional crops, most energy crops are new to American farmers. A 2005 survey conducted in Tennessee found that most respondents had not heard of growing switchgrass for biofuel production (Jensen et al 2007). Hence, farmers will need to invest time in order to learn how best to grow these crops. Not only are farmers unfamiliar with these crops, but also the seed companies that have invested decades of research into current crops have only just started varietal improvement of energy crops. As a result, there is great uncertainty about both how to manage variability in energy crop production and what might be genetically attainable yield levels.

Quantity aside, energy crop prices are largely undetermined but likely to exhibit different volatility patterns from traditional crops. Crops destined for conversion into ethanol will have prices determined in large part by the ethanol market, which is linked to the gasoline market (Tyner, 2008). Energy crop price volatility is likely to be aggravated as ethanol shifts in and out of status as a cost-effective fuel substitute for gasoline, based on the relative prices of petroleum and corn grain, the leading current ethanol feedstock in the United States. Although mandated growth in cellulosic ethanol demand under EISA may mitigate one policy related source of price uncertainty, there remain important uncertainties regarding federal climate-change policy and

state-level renewable energy policies. In sum, energy crop revenue uncertainties are great due to both production and price uncertainties.

Finally, although real options methods exist for modeling stochastic revenue streams and uncovering optimal decision rules, the real options literature typically relies upon mathematically convenient stochastic information processes that are not necessarily consistent with observed variability of energy returns. For instance, policy uncertainties and learning-by-doing in growing energy crops will unlikely be described by a geometric Brownian motion. There is a need for studies that go beyond mathematically convenient processes in modeling information and learning.

In this paper, we study land conversions between traditional crops and energy crops, incorporating these three features of energy crops. We make several contributions to the literature. First, we extend studies based on the net present value (NPV) approaches to allow for uncertainty, sunk costs and learning. In the NPV approach (Walsh et al 2003; Fumasi 2008), a farmer will convert land to energy crops if the expected NPV of the returns from energy crops exceed those from current (traditional) crops. But under uncertainty and sunk costs, the farmer may be more reluctant to convert land into and out of the two uses, similar to the predictions of real option theory (Dixit and Pindyck, 1994).

Second, we extend the real options studies and allow for land use conversion in two directions, so a farmer deciding on converting to energy crops is allowed to take into consideration of the future possibility of converting the land back to traditional crops under plausible market conditions. Real option theory has been widely applied in urban land use decisions since Titman (1985) (e.g., Capozza and Li 1994; Abeyayehu, et al. 1999). A common assumption in this literature is that land conversion is absolutely irreversible. This assumption might be reasonable for urban development, but for agricultural land, a farmer can switch between different uses with costs.

Allowing two-way land conversion is important to capture the flexibility of farmer's land use decisions, and is particularly important for energy crops given the high degrees of uncertainties involved.

Third, we contrast the effects of two alternative stochastic processes for returns from the two competing crop choices, geometric Brownian motion (GBM) and mean reverting (MR). We use both historical market data and simulated agronomic yield data to parameterize and test the stochastic processes and show how optimizing farmer behavior responds to alternative assumptions about the stochastic processes.

Our paper is closely related to the broader real options literature (Dixit and Pindyck, 1994), especially those allowing for two way decisions. Dixit (1989) studies a firm's entry and exit decisions assuming that the output price is the only state variable and evolves according to GBM. Mason (2001) extends this work to examine a mine's decision to start and to shut down, for which not only the output price is uncertain but the resource reserve is limited. This results in the optimal decision rule depending on both the reserve stock level and the price. Dixit (1989) and Mason (2001) both assume that the decision maker only obtains a return in one state (entry or active) and thus only one stochastic state variable (price) governs the optimal action. Our model allows two separate but possibly correlated returns, those from traditional crops and from energy crops. Kassari and Lasserre (2004) examine the optimal abandonment rule between two species, both of which have stochastic values. However, in their study, a species cannot be recovered once it is lost, which is equivalent to switching in one direction only.

A General Land Conversion Model

Consider a risk neutral farmer¹ with a unit of land facing two land use alternatives, $i \in S = \{1, 2\}$, who can convert from alternative i to j with a lump-sum sunk cost C_{ij} . Specifically, $i=1$ denotes growing traditional crops while $i=2$ denotes growing energy crops. The return to alternative i in period t is denoted by $\pi_i(t)$, which is assumed to evolve according to a known stochastic process of the general form:

$$(1) \quad d\pi_i(t) = \alpha_i(\pi_i, t)dt + \sigma_i(\pi_i, t)dz_i$$

where the drift term $\alpha_i(\pi_i, t)$ and the variance term $\sigma_i(\pi_i, t)$ are known nonrandom functions, and dz_i is the increment of a Wiener process. Thus, new information about future returns of the two crops comes in the form of newly observed return levels, which become starting points for the distributions of future returns. Note that we model the returns directly, instead of modeling the price and yield uncertainties separately. This approach simplifies our analysis, and in the empirical section we derive the return processes from the underlying price, yield and cost processes. The correlation coefficient of the two return processes is ρ , i.e., $E(dz_1 dz_2) = \rho dt$. Traditional crop and energy crop returns could be correlated for a variety of reasons, e.g. both are linked with energy prices and are subject to macro-economic shocks. Finally, let r be the farmer's discount rate.

A key insight of the real options approach is that when the land is in use i , say in traditional crops, the farmer has the *option* of converting it into energy crops when market conditions are "favorable." Once converted, it is costly to revert it back to traditional crops if the market conditions turn out to be less favorable. Thus, sticking to the current land use (in traditional crops)

¹ We assume risk neutrality for simplicity; similar results can be obtained when the farmer is risk averse.

has an additional value, called option value, derived from the option of converting it into the alternative use (in energy crops). But since the land in energy crops can be further converted back to traditional crops (albeit at a cost), this option value of converting from traditional to energy crops further depends on the option value associated with converting from energy to traditional crops. The mutual dependence of the two option values significantly complicates the solution algorithm.

Let $V^i(\pi_1(t), \pi_2(t))$ be the farmer's period t expected present-value payoff starting with land use i and following optimal land conversion rules. Due to the option of converting into use $j \neq i$, the payoff depends on the distribution of future returns of *both* land uses, the information for which is contained in the two current returns, $\pi_1(t)$ and $\pi_2(t)$. At time t , the farmer chooses between keeping the land in use i and converting it into alternative use j :

$$(2) \quad V^i(\pi_1(t), \pi_2(t)) = \max \left\{ \pi_i(t)dt + e^{-rdt} EV^i(\pi_1(t+dt), \pi_2(t+dt)), V^j(\pi_1(t), \pi_2(t)) - C_{ij} \right\}$$

The first term on the right hand side describes the payoffs if the land is kept in use i : In the infinitesimal period $[t, t+dt]$, the farmer receives profit from land use i at rate $\pi_i(t)$, and at the end of the period, receives the new *discounted expected* payoff $e^{-rdt} EV^i(t+dt)$. The second term on the right hand side describes the payoff if the land is converted into use j : the farmer receives the expected payoff of use j , $V^j(t)$, but incurs the conversion cost C_{ij} .

As shown by Brekke and Oksendal (1994), the decision problem in (2) can be equivalently expressed by a set of complementary slackness conditions, as long as the value functions $V^1(\bullet)$ and $V^2(\bullet)$ are stochastically C^2 .² First, (2) implies that

² This condition is trivially satisfied in our applications.

$$(3) \quad V^i(\pi_1(t), \pi_2(t)) \geq \pi_i(t)dt + e^{-rdt} EV^i(\pi_1(t+dt), \pi_2(t+dt))$$

which, after applying Ito's lemma, can be expressed as $LV^i(\pi_1(t), \pi_2(t)) \geq 0$, where

$$(4) \quad LV^i(\pi_1, \pi_2) \equiv rV^i(\pi_1, \pi_2) - \pi_i(t) - \sum_{i=1}^2 \alpha_i(\pi_i, t) V_{\pi_i}^i - \sum_{i=1}^2 1/2 \sigma_i^2(\pi_i, t) V_{\pi_i \pi_i}^i - \rho \sigma_1(\pi_1, t) \sigma_2(\pi_2, t) V_{\pi_1 \pi_2}^i$$

and the subscripts of V^i denote partial derivatives. Equation (2) also implies that

$V^i(\pi_1(t), \pi_2(t)) \geq V^j(\pi_1(t), \pi_2(t)) - C_{ij}$. Then we know the value functions V^i and V^j have to satisfy:

- (i) $LV^i(\pi_1, \pi_2) \geq 0, \quad i = 1, 2$
- (5) (ii) $V^i(\pi_1, \pi_2) \geq V^j(\pi_1, \pi_2) - C_{ij} \quad i, j \in \{1, 2\} \text{ and } i \neq j$
- (iii) either (i) or (ii) has to hold as a strict equality.

If (i) is an equality, the farmer should keep his land in current use i , and if (ii) is an equality, the farmer should switch the land use to j . If both are equalities (a nongeneric case), the farmer is indifferent between converting and not converting. The optimal conversion decisions (i.e., solutions to (5)) are represented by two *conversion boundaries* in the $\pi_1 - \pi_2$ space, one for each type of current land use, as shown in figure 1. If the current land use is in traditional crops ($i=1$), the conversion boundary, $\pi_2 = b^{12}(\pi_1)$, denotes the returns from traditional and energy crops that the farmer is indifferent between converting to energy crops and sticking to traditional crops. Above this boundary, i.e., when $\pi_2 > b^{12}(\pi_1)$, the returns from energy crops are sufficiently high that it is optimal for the farmer to convert to energy crops. Conversely if $\pi_2 < b^{12}(\pi_1)$, the farmer should stick to growing traditional crops. Intuitively, as shown in figure 1, boundary b^{12} lies above

the 45° line: given the sunk costs and uncertainties, the farmer is reluctant to convert to energy crops even when its return $\pi_2(t)$ slightly exceeds $\pi_1(t)$, the return from traditional crops.

Similarly, if the current land use is in energy crops, the boundary for converting to traditional crops is given by $\pi_2 = b^{21}(\pi_1)$. If the return from energy crops is too low compared with traditional crops so that $\pi_2 < b^{21}(\pi_1)$, the farmer should convert to traditional crops. Otherwise, it is optimal for the farmer to stick to the current use (in energy crops). Again, due to uncertainty and sunk costs, b^{21} lies below the 45° line in figure 1: once the land is already in energy crops, the farmer is reluctant to convert it to traditional crops unless the return from traditional crops is sufficiently high.

Thus, the two boundaries divide the $\pi_1 - \pi_2$ space into three regions: above the boundary b^{12} , it is optimal to convert from traditional crops to energy crops; below the boundary b^{21} , it is optimal to convert from energy crops to traditional crops; and between the two boundaries, it is optimal to keep land in its current use.

Since the two value functions $V^1(\bullet)$ and $V^2(\bullet)$ are interdependent, there are no analytical solutions to (5).³ We follow Miranda and Fackler (2002) and employ the collocation method, which approximates the unknown value functions $V^i(\pi_1, \pi_2)$ using a linear combination of n known basis functions:

$$(6) \quad \hat{V}^i(\pi_1, \pi_2) = \sum_{j_1=1}^{n_1} \sum_{j_2=1}^{n_2} c_{j_1 j_2} \phi_{j_1 j_2}(\pi_1, \pi_2)$$

where $\hat{V}^i(\bullet)$ represents the numerical approximation of $V^i(\bullet)$. Compared to the shooting algorithm and finite difference method that are often used to numerically solve the value functions in

³ If we allow land conversion in one direction only, then analytical solutions exist for special stochastic processes of $\pi_1(t)$ and $\pi_2(t)$, e.g., geometric Brownian motions. Numerical methods must be employed for more general processes even for one way conversion models. For instance, Insley and Rollins (2005) and Conrad and Kotani (2005) formulate (4) as a linear complementarity problem and then use the finite difference approach to solve it numerically.

stochastic dynamic optimization problems, the collocation method is a fast and robust alternative (Dangl and Wirl 2004). The coefficients $c_{j_1 j_2}$ are found by requiring the approximant to satisfy the optimality condition (5) at a set of interpolation nodal points. The threshold returns that induce the farmer to convert land will also be solved at the nodal points.

Data and Parameter Estimation

The empirical analysis focuses on a representative farmer's optimal land conversion decision in the North-central United States,⁴ where corn and soybean are two major crops frequently grown in rotation. We assume that the farmer currently grows corn and soybean in a balanced rotation (with half of his land in each crop). The alternative land use is to grow switchgrass. The model can be easily adapted to other locations and crops.

Construction of Returns to Land Uses

We first estimate the drift and variance parameter functions $\alpha_i(\pi_i, t)$ and $\sigma_i(\pi_i, t)$ for the two land uses, and the correlation coefficient of the two stochastic processes ρ . Typically, these parameters are estimated using historical time series on returns. We use USDA's North-central regional data for 1982 to 2008 (USDA, 2008) to calculate the annual average return to the corn-soybean rotation, deflating the returns by the Producer Price Index (PPI, 1982=100). The data series is plotted in figure 2.

Since switchgrass has not been grown commercially as a biofuel feedstock, we do not have historical data for switchgrass returns. We instead construct a hypothetical series of returns for switchgrass grown as an energy crop. The return equals the farmgate price of switchgrass times

⁴ The North-central area includes Illinois, Indiana, Iowa, Michigan, Minnesota, Missouri, Ohio, and Wisconsin.

yield minus the production costs. The farmgate price is determined by ethanol producers' willingness to pay (WTP) as well as government subsidies.⁵ Ethanol producers' WTP is equal to the ethanol price subtracted by the conversion costs from switchgrass to ethanol and the transportation costs from field to a processing facility. The ethanol price (in \$ per gallon) is obtained from Nebraska Energy Office from 1982 to 2008. The estimated conversion cost is assumed to be \$ 0.91 per gallon (DiPardo 2004). Assuming that one ton of switchgrass yields 91 gallons of ethanol (Schmer et al. 2008) and multiplying by this conversion rate, we convert the ethanol price and conversion cost from a per gallon basis into per ton basis. The transportation cost is assumed to be \$8 per ton (Babcock et al. 2007). For government subsidies, we use the \$45/ton matching payment currently provided by USDA to biofuel producers for their costs of collection, harvest, transport and storage of biomass. Switchgrass yield for 1982-2008 in the same North-central states as the corn-soybean returns data is obtained from Thomson et al. (2008), who used the EPIC Model at the 8-digit watershed level that can be aggregated to the state level. The average switchgrass yield is 3.10 tons/acre and the standard deviation is 0.20 tons/acre. The production cost is assumed to be \$194/acre (Duffy and Nanhou 2001). Finally, the nominal returns are deflated to a 1982 base using the PPI. These returns are plotted in figure 2.

Parameter Estimation

We consider two commonly used return processes, geometric Brownian motion (GBM) and geometric mean reversion (MR). The GBM process is widely used in real option studies for its analytical tractability, and is represented as

$$(7) \quad d\pi_i = \alpha_i \pi_i dt + \sigma_i \pi_i dz_i, \quad i = 1, 2$$

⁵ Governmental subsidies are important in the competitiveness of switchgrass. With our baseline parameters assumptions, ethanol prices in some years were so low that the switchgrass revenue was not able to cover the production costs if there had been no governmental subsidy.

where α_i and σ_i are drift and variance parameters respectively. If a return follows a GBM process, the mean and variance of the return rise over time without boundary. Thus it is a nonstationary series. The MR process, on the other hand, assumes that the random variable will revert to a long term average, is stationary and is described by

$$(8) \quad d\pi_i = \eta_i(\bar{\pi} - \pi_i)\pi_i dt + \sigma_i \pi_i dz_i, \quad i = 1, 2$$

where $\bar{\pi}_i$ is the long term average return of land use i , η_i is the speed of reversion and σ_i is the variance rate. The returns revert to a long-run equilibrium $\bar{\pi}_i$ at a speed of $\eta_i \pi_i$. The further the returns divert from the $\bar{\pi}_i$, the quicker the reversion will be.

Theoretically, both processes can be justified in describing how agricultural returns evolve over time. GBM can better reflect a trend which could be positive due to technological advances that boost productivity while a MR process can better reflect the long term equilibrium conditions when technology is unchanging. Statistical tests of the stationarity of corn-soybean and switchgrass returns generate mixed signals. For instance, for the logarithmic corn-soybean returns, the null hypothesis of unit root is rejected at 5% level based on the Dickey-Fuller test and at 1% level based on Phillip-Perron test, but the null hypothesis of stationarity is also rejected at 10% level based on KPSS test. Similar results are found for logarithmic switchgrass returns. Given the nascent state of biofuel crop technology and markets, the dynamic growth implicit in a GBM process seems the more justified of the two. Thus, we use GBM processes in our baseline model, and study how the results change if MR processes are used instead.

To estimate the parameters of the two processes for corn-soybean and switchgrass returns, we first discretize the two continuous time processes. If π_i follows a GBM, then $\ln \pi_i$ follows a simple Brownian motion with drift, which is the limit of a random walk. In particular,

$$(9) \quad \ln \pi_{i,t+1} - \ln \pi_{i,t} = (\alpha_i - 1/2\sigma_i^2) + \sigma_i \varepsilon_{i,t}, \quad i = 1, 2$$

where $\varepsilon_i \sim N(0,1)$. The maximum likelihood estimates of the drift α_i and the standard deviation σ_i are thus $\hat{\alpha}_i = m_i + 0.5s_i^2$ and $\hat{\sigma}_i = s_i$, where m_i and s_i are respectively the mean and standard deviation of the series $\ln \pi_{i,t} - \ln \pi_{i,t-1}$. The estimate of the correlation coefficient ρ is the correlation between series $\ln \pi_{1,t} - \ln \pi_{1,t-1}$ and $\ln \pi_{2,t} - \ln \pi_{2,t-1}$. The parameter estimates for the GBM representation of the corn-soybean and switchgrass returns are presented in Table 1.⁶

The discrete time approximation to the MR process in (8) is as follows:

$$(10) \quad \pi_{i,t} - \pi_{i,t-1} = \eta_i(\bar{\pi}_i - \pi_{i,t-1})\pi_{i,t-1} + \sigma_i\pi_{i,t-1}e_t, \quad i = 1, 2$$

where again $e_i \sim N(0,1)$. Dividing both sides by $\pi_{i,t-1}$, we obtain

$$(11) \quad \frac{\pi_{i,t} - \pi_{i,t-1}}{\pi_{i,t-1}} = a_i + b_i\pi_{i,t-1} + \sigma_i e_{i,t}, \quad i = 1, 2$$

Since the stochastic processes of the returns to corn-soybean and switchgrass have correlated residuals (i.e., e_{1t} and e_{2t} are correlated), we use a seemingly unrelated regression model to estimate the parameters a_i , b_i , σ_i and ρ . Consistent estimates of η_i and $\bar{\pi}_i$ are then obtained as

$$\hat{\eta}_i = -\hat{b}_i, \text{ and } \hat{\bar{\pi}}_i = -\frac{\hat{a}_i}{\hat{b}_i}. \text{ The results are reported in table 1.}$$

⁶ The estimated growth rate of the switchgrass return is 0.17, which is higher than the assumed discount rate of 0.08. For the dynamic optimization problem to have a solution, the expected growth rate cannot exceed the discount rate (otherwise, the expected payoff from switchgrass is infinite). We thus assume that the switchgrass return grows at the same rate as corn-soybean.

Consistent with the observation in figure 2, the return to switchgrass has higher volatility than that to corn-soybean, no matter which type of stochastic processes they follow. The shocks to the two returns have a correlation coefficient of -0.3.

Historical returns to corn-soybean and switchgrass are negatively correlated as indicated by the estimation results. Part of the reason is that switchgrass revenue is simulated as a function of petroleum price and is thus positively correlated with petroleum, whereas until 2005, corn-soybean returns were negatively correlated with petroleum prices since petroleum was used as transportation and fertilizer inputs. However, this pattern of negative correlation could change as more corn and soybean are used to produce biofuels, and as agricultural and petroleum markets become more integrated. Then high petroleum prices may push up corn and soybean prices, increasing their returns. Furthermore, the positive correlation may become stronger as switchgrass or other energy crops expand production and compete with corn-soybean for limited land. In response to this possibility, we test the sensitivity of our results to the effects of positive correlations between the two returns.

Land Conversion Costs

As a perennial crop, switchgrass needs to become established and will not achieve full yield until the third or fourth year after seeding. It needs to be replanted every ten years. We use the NPV of the (infinite) sequence of first-year switchgrass establishment costs as an estimate of land conversion costs from corn-soybean to switchgrass, C_{12} .⁷ Switchgrass establishment costs include seed, chemicals, machinery and labor (e.g. Hallam et al. 2001; Duffy and Nanhou 2001; Khanna et al. 2008; Perrin et al. 2008). The estimated costs vary widely across studies because of different

⁷ The NPV of the decennial establishment cost for switchgrass (\$136/acre) overestimates C_{12} if the farmer does not permanently stay with switchgrass. In this case, we overestimate the farmer's reluctance to convert to switchgrass but not to a large extent (one time establishment cost is \$109/acre).

assumptions, methods employed, and production locations. We use \$109/acre estimated by Khanna et al (2008) because they report the detailed costs categories year by year, which facilitates our calculations. The NPV of the cost sequence is \$136/acre at a discount rate of 8%.

Conversion of land from switchgrass to corn-soybean production requires clearing existing vegetation residue by tillage or herbicides. We use the costs of converting land in Conservation Reserve Program back to crop production as an approximate of the conversion costs from switchgrass to corn-soybean. Higher than normal fertilizer rates may be required for two years after conversion (Blocksome et al. 2008). We assume \$47/acre conversion cost from switchgrass to corn-soybean production, which includes \$17/acre disking operation costs and \$30/acre total additional fertilizer costs for the first two years (Williams et al. 2009).

Results and Sensitivity Analysis

Given the baseline parameter values, we solve the optimality condition in (5) using OSSOLVER (Fackler 2004), implemented with CompEcon Toolbox in Matlab (Miranda and Fackler 2002). The same solver was employed by Nøtbakken (2006) to solve her model of a fleet's optimal decision to enter or to leave a fishery. The family basis function we use is a piecewise linear spline. For each state variable (i.e., π_1 and π_2), the nodal points are evenly spaced over the revenue interval [0, 5] (in hundred dollars) with an increment of 0.1.

Figure 3a shows the two boundaries (the solid lines) for conversions from corn-soybean to switchgrass (b^{12}) and from switchgrass to corn-soybean (b^{21}) assuming that both returns follow GBM. The boundaries indicate significant hysteresis in land conversion decisions. For instance, the real average annual returns based on 2008 prices in 1982 dollars are $\pi_1 = \$119/\text{acre}$ for corn-soybean and $\pi_2 = \$175/\text{acre}$ for switchgrass. If the land is currently in corn-soybean, the minimum

switchgrass return for converting the land to switchgrass is $b^{12}(119) = \$345/\text{acre}$, which is significantly higher than $\$175$. Thus, the land will be kept in corn-soybean rotation even though $\pi_2 > \pi_1$. Conversely, if the land is already in switchgrass, the required minimum corn-soybean return for converting into corn-soybean is about $\$340/\text{acre}$. Thus, the land will not be converted either.

Given the two boundaries, we calculate the expected probabilities that a piece of land in corn-soybean will be in switchgrass for each year during a 30 year period, with the 2008 returns as the initial (time zero) returns: $\pi_1(0) = \$119/\text{acre}$ and $\pi_2(0) = \$175/\text{acre}$. Given this starting point, we draw $N (=1000)$ sample paths of the joint return processes for 30 years according to (7) and the parameter values in table 1.⁸ Each sample path of the two returns, $\{(\pi_1(t), \pi_2(t)), t = 1, \dots, 30\}$, is then compared with the conversion boundaries, $(b^{12}(\bullet), b^{21}(\bullet))$, to decide whether the land is kept in its current use or should be converted to the alternative use. For instance, in year 1, when the land is still in corn-soybean, the realized returns on a particular sample path, $(\pi_1(1), \pi_2(1))$, are compared with boundary b^{12} . If the realized returns are in the “no action zone” (e.g., if $\pi_2(1) < b^{12}(\pi_1(1))$ according to the optimal decision rule), the land is kept in corn-soybean, and similar comparisons are made in year 2. If, on the other hand, the realized returns are in the “conversion zone” (i.e., if $\pi_2(1) \geq b^{12}(\pi_1(1))$), the land is converted to switchgrass, and in year two, the realized returns $(\pi_1(2), \pi_2(2))$ will be compared with boundary b^{21} to decide whether the land should be converted into corn-soybean. Finally, for each period we count the number of sample paths on which the land is in switchgrass. Dividing this number by N , we obtain the proportion of land in switchgrass for each period.

⁸ The two correlated stochastic processes $\{(\pi_1(t), \pi_2(t)), t \in [0, 30]\}$ are approximated by the Euler method and implemented using Matlab’s Econometric toolbox.

Figure 3b illustrates the proportion of land in switchgrass for the 30 year period (solid line). Since the starting level of switchgrass return (at \$175/acre) is much higher than that of corn-soybean (at \$119/acre), more land is gradually converted into switchgrass, peaking at 30% of the total land area. However, the switchgrass return also has a higher level of uncertainty, and eventually some land in switchgrass is converted back to corn-soybean, stabilizing at about 14% of the land area.

Comparison with NPV and One Way Conversion Rules

We next compare the two conversion boundaries found above with those based on the NPV rule. According to the NPV decision rule, the farmer will switch from corn-soybean to switchgrass when the expected NPV of switching is higher than staying in corn-soybean, i.e., when

$E \int_0^{\infty} \pi_2(t) e^{-rt} dt - C_{12} \geq E \int_0^{\infty} \pi_1(t) e^{-rt} dt$. Similarly, the farmer will convert from switchgrass to corn-

soybean when $E \int_0^{\infty} \pi_1(t) e^{-rt} dt - C_{21} \geq E \int_0^{\infty} \pi_2(t) e^{-rt} dt$. Given that both $\pi_1(t)$ and $\pi_2(t)$ follow

GBM, we use (7) and obtain two NPV conversion boundaries: $b_{NPV}^{12}(\pi_1) = \pi_1 \frac{r - \alpha_2}{r - \alpha_1} + (r - \alpha_2)C_{12}$ for

conversion from corn-soybean to switchgrass, and $b_{NPV}^{21}(\pi_1) = \pi_1 \frac{r - \alpha_2}{r - \alpha_1} - (r - \alpha_2)C_{21}$ for conversion

from switchgrass to corn-soybean. The two NPV boundaries (in dash lines, based on GBM process) are shown in figure 3a.

As illustrated in figure 3a, the NPV rule predicts that the farmer will convert between land uses far more readily than under the dynamically optimal real options rule. For instance, if the corn-soybean return is \$130/acre, the average historical return during 1975 - 2007, the farmer who grows corn-soybean will convert to switchgrass if the switchgrass return exceeds \$135/acre, and the farmer who grows switchgrass will convert to corn-soybean if the switchgrass return is less

than \$128/acre. But the real option rule, given by $b^{12}(\bullet)$ and $b^{21}(\bullet)$, indicates that the corn-soybean farmer will convert to switchgrass only if the switchgrass return exceeds \$365/acre, which is 2.7 times the NPV threshold. The switchgrass farmer will convert to corn-soybean only if the switchgrass return is lower than \$55/acre, 57% lower than the corresponding NPV threshold.

The different conversion boundaries under these two decision rules, $(b^{12}(\bullet), b^{21}(\bullet))$ and $(b_{NPV}^{12}(\bullet), b_{NPV}^{21}(\bullet))$, imply different amounts of land converted between the two uses. Figure 3b compares the proportions of land in switchgrass under the real option and NPV rules. The NPV rule predicts that land will be quickly converted into switchgrass (peaking at 73% of total land area), followed by a gradual decline, and eventually stabilizing at about 57%. The predicted proportion of land in switchgrass is consistently higher than the predictions of the dynamically optimal model.

A real options model that only allows one way conversion will predict significantly greater farmer reluctance to convert than a two-way conversion model, as shown in figure 4a.⁹ For instance, the threshold return for converting from corn-soybean to switchgrass, b_{ow}^{12} , doubles b^{12} , the threshold boundary when two way conversion is accounted for. Similarly, the corn-soybean return threshold for a farmer to convert from switchgrass to corn-soybean is twice as high under the one-way conversion model compared to the two way conversion model. Because of the increased hysteresis, the one way real options model predicts much lower proportions of land in switchgrass, as shown in Figure 4b.

Effects of Different Stochastic Processes

⁹ The two corresponding conversion boundaries, b_{ow}^{12} and b_{ow}^{21} , are obtained by imposing a prohibitively large cost of reverting back the earlier conversion.

We next investigate the effects of assuming different stochastic processes by comparing the conversion boundaries and switchgrass proportion under GBM and MR processes. We first “anchor” the two processes so that they are comparable by estimating the parameter values of the two processes using the *same* time series data for π_1 and π_2 . The parameter estimates for the two processes are presented in table 1. This anchoring approach implies that the parameter values may not be completely comparable. For instance, although the variance rate for the corn-soybean return under the GBM assumption (at 0.29) is roughly the same as that under the MR assumption (at 0.30), the variance rate for switchgrass return under the GBM assumption (at 0.62) is estimated to be much smaller than that under the MR assumption (at 0.97).

The two-way conversion boundaries for corn-soybean and switchgrass returns follow distinct patterns according to whether the underlying stochastic processes follow GBM or MR parameters, as illustrated in figure 5a. The solid lines define the optimal land conversion boundaries assuming GBM. Under a GBM process, the conversion pattern mainly depends on the relative volatility and conversion costs. This yields a fairly symmetric boundary pair, albeit with a lower threshold for conversion from switchgrass to corn-soybean at low returns than vice-versa. Under a MR process, the pattern is asymmetric, with a lower threshold for conversion to switchgrass and a much higher threshold to corn-soybean for low return rate than under GBM. But this pattern at low return levels is reversed at higher returns, with declining tendency to convert to switchgrass and rising tendency to convert to corn-soybean. The difference in conversion boundary patterns arises from three distinct effects of MR as compared to GBM processes: the relative effects of uncertainty, distant time horizon and mean reversion.

The uncertainty effect follows from the higher relative volatility of the switchgrass return in MR process than in GBM case (3 vs. 2 times the corresponding corn-soybean uncertainty). Hence, the

optimizing farmer is more reluctant to convert land into switchgrass as well as out of it.¹⁰ Due to the higher relative volatility for MR, the uncertainty effect raises the conversion boundary from corn-soybean to switchgrass and lowers the boundaries in both directions.

The distant time horizon effect arises from the higher projected long term average return from switchgrass as compared to corn-soybean. This effect lowers the conversion boundary from corn-soybean to switchgrass and raises the conversion boundary from switchgrass to corn-soybean compared to GBM case.

While the previous two effects hold true for low as well as high return levels, the mean reversion effect on the conversion boundaries behaves differently at low return levels than at high ones. When both corn-soybean and switchgrass returns are high, mean reversion pulls them downward towards the long term average. However, the switchgrass return reverts to its mean more slowly than the corn-soybean return because it has both a smaller reversion speed parameter and a smaller absolute difference between the current return and the long term average. At high return levels, these effects lower the conversion boundary from corn-soybean to switchgrass and raises the boundary from switchgrass to corn-soybean. By contrast, when the corn-soybean and switchgrass returns are low, mean reversion causes them to rise. If the switchgrass return reverts more slowly than the corn-soybean return, it raises the boundary from corn-soybean to switchgrass and lowers the boundary from switchgrass to corn-soybean. When the order of the reversion speed is reversed, the net effect is ambiguous because the switchgrass return has a lower reversion speed parameter but a higher difference between current return and long term average.

¹⁰ The reluctance to convert out of switchgrass as the switchgrass return becomes more uncertain is a feature of the real option argument: as σ_2 increases, it is more likely that future return π_2 is high, which implies that the farmer should not convert out of switchgrass. In response, the farmer has more incentive to wait until π_2 is low relative to π_1 before converting out. This prediction is opposite to that of standard risk aversion arguments, and has been used by Schatzki (2003) to test real option versus risk aversion assumptions.

The asymmetric pattern of the MR returns in figure 5a arises from the interaction of the three effects. For conversion boundary b^{12} , when corn-soybean returns are low the distant time horizon effect dominates the other two effects, lowering the boundary. But when corn-soybean returns are high, the uncertainty effect dominates and the boundary is raised. For boundary $bH21$, the three effects work together to lower the boundary when the corn-soybean returns are low, and the mean reversion effect dominates at high corn-soybean return levels.

Consistent with the conversion boundaries in figure 5a, figure 5b shows that the predicted proportion of land in switchgrass is higher under MR for the first 4 years, since the conversion boundary into switchgrass is low initially. As the returns grow over time, the proportion of land in switchgrass declines and becomes lower than under GBM processes. Eventually the switchgrass land proportion under both processes stabilizes around 13%.

Effects of Conversion Costs

Figure 6a shows how halving the cost of conversion costs affects the optimal conversion rule under the GBM assumption. In the top panel, reducing the conversion costs from corn-soybean to switchgrass (C_{12}) creates the desired incentive by making the corn-soybean grower less reluctant to make the conversion. However, it also has the indirect effect of making the switchgrass grower more prone to convert (back) to corn-soybean, because although the farmer currently growing switchgrass will not directly benefit from the subsidy for conversion to switchgrass, its existence reduces the expected cost of converting from corn-soybean back to switchgrass, thereby reducing the implied cost of switching back to corn-soybean. Thus it indirectly increases his incentive to convert land to corn-soybean. The direct effect of lowering C_{12} is greater than the indirect effect. The reduction in C_{12} lowers the boundary from corn-soybean to switchgrass more than the

boundary from switchgrass to corn-soy. Similarly, the reduction in C_{21} lowers both of the conversion boundaries but lowers the boundary from switchgrass to corn-soybean more than the boundary from corn-soybean to switchgrass. These results also hold under the MR assumption.

An important insight from this two-way model is that a policy to subsidize conversion to dedicated energy crops, such as the USDA Biomass Crop Assistance Program, can have the joint effect of encouraging conversion both into and away from the biomass crop. Figure 6b illustrates how the two effects interact over time. Reducing the conversion cost from corn-soybean to switchgrass (C_{12}) leads to higher proportions of land in switchgrass for the first 11 years. But after that, land in switchgrass is in fact lower, due to the higher incentive to switch back to corn-soybean. Further, lowering conversion cost into corn-soybean, C_{21} , in fact *promotes* conversion into switchgrass for the first seven years. Finally, in the long run, lowering C_{12} and lowering C_{21} have almost the same effects on land in switchgrass.

Effects of Uncertainties

As discussed earlier, higher uncertainties in either corn-soybean or switchgrass returns will cause the farmer to be more reluctant to take any conversion action. Figure 7a shows that doubling the variance parameter of corn-soybean return σ_1 (or switchgrass return σ_2) significantly raises the conversion boundary from switchgrass (or corn-soybean) to corn-soybean (or switchgrass), b^{21} (or b^{12}), and slightly raise the conversion boundary from corn-soybean to switchgrass b^{12} (or b^{21}).

As argued by Sarkar (2003), high uncertainties do not automatically translate into fewer conversions: although conversion is undertaken only with “more extreme” returns with the higher boundaries, higher uncertainty levels also mean that “extreme returns” occur more frequently.

Figure 7b shows that doubling σ_1 and doubling σ_2 have strikingly different impacts: the

proportion of land in switchgrass increases significantly as σ_1 doubles, but decreases to nearly zero (at 0.05) in the long run as σ_2 doubles.

Conclusion and discussion

This study develops a real options framework to analyze the farmer's land use decision between traditional annual crops and perennial energy crops. The study innovates from existing models of optimal conversion under the assumption of irreversible decisions by introducing a model for two-way conversion. The possibility of costly reversibility in crop production is illustrated using an annual corn-soybean crop rotation and perennial switchgrass representative alternative crop systems. Consistent with real options theory, the option value of sticking to the current land use delays converting land into switchgrass as well as converting out of it. By comparison with the real options results, an NPV model predicts that an optimizing farmer would be much more prone to convert land to switchgrass compared. A one-way real option model characterizing the land conversion decision as irreversible predicts much greater reluctance to convert land from annual corn-soybean to a perennial switchgrass energy crop, also implying lower accumulated land under energy crops over a 30-year time horizon. We further show how two alternative stochastic process assumptions affect the optimal conversion rule and the proportion of land devoted to the dedicated energy crop.

From a policy perspective, this model offers two important insights. First, compared to deterministic break-even analyses (e.g., Tyner, 2008; James et al., 2010), it highlights the significant option value of delaying land conversion even when a static net present value threshold is passed. The illustrative case here suggests that returns from dedicated energy crops may have to exceed double the breakeven NPV level before becoming a dynamically optimal choice.

Second, compared with past real options models that assume complete irreversibility of decisions, this two-way model reveals that conversion subsidies to encourage biofuel crop planting have a two-edged impact. The effect of reducing conversion costs from corn-soybean to an energy crop (switchgrass) is to lower the conversion threshold revenue levels in both directions, meaning that not only is it easier to convert from corn-soybean into switchgrass, but it also is easier to convert the other way. Compared to the case of no subsidy, the predicted proportion of land planted to the switchgrass energy crop is higher with the subsidy in the intermediate period but actually becomes lower toward the latter part of a 30-year time horizon.

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Table 1. Baseline Parameters for Numerically Solving the Dynamic Optimal Land

Conversion Rule

<i>Parameters of the stochastic processes of the returns to corn-soy and switchgrass</i>					
		Returns to corn-soy		Returns to switchgrass	
GBM	Drift parameter	$\hat{\alpha}_1$	0.04	$\hat{\alpha}_2$	0.04
	Variance parameter	$\hat{\sigma}_1$	0.29	$\hat{\sigma}_2$	0.62
	Correlation parameter	$\hat{\rho}$	-0.30		
<hr/>					
	Long-run production profit	$\hat{\pi}_1$	133	$\hat{\pi}_2$	219
MR	Reverting speed	$\hat{\eta}_1$	0.009	$\hat{\eta}_2$	0.004
	Variance parameter	$\hat{\sigma}_1$	0.30	$\hat{\sigma}_2$	0.97
	Correlation parameter	$\hat{\rho}$	-0.31		
<hr/>					
<i>Land conversion costs</i>					
Corn-soy to switchgrass C_{12} :		136 \$/acre	Switchgrass to corn-soy C_{21} : 47\$/acre		
<hr/>					
<i>Discount factor</i> r		0.08			
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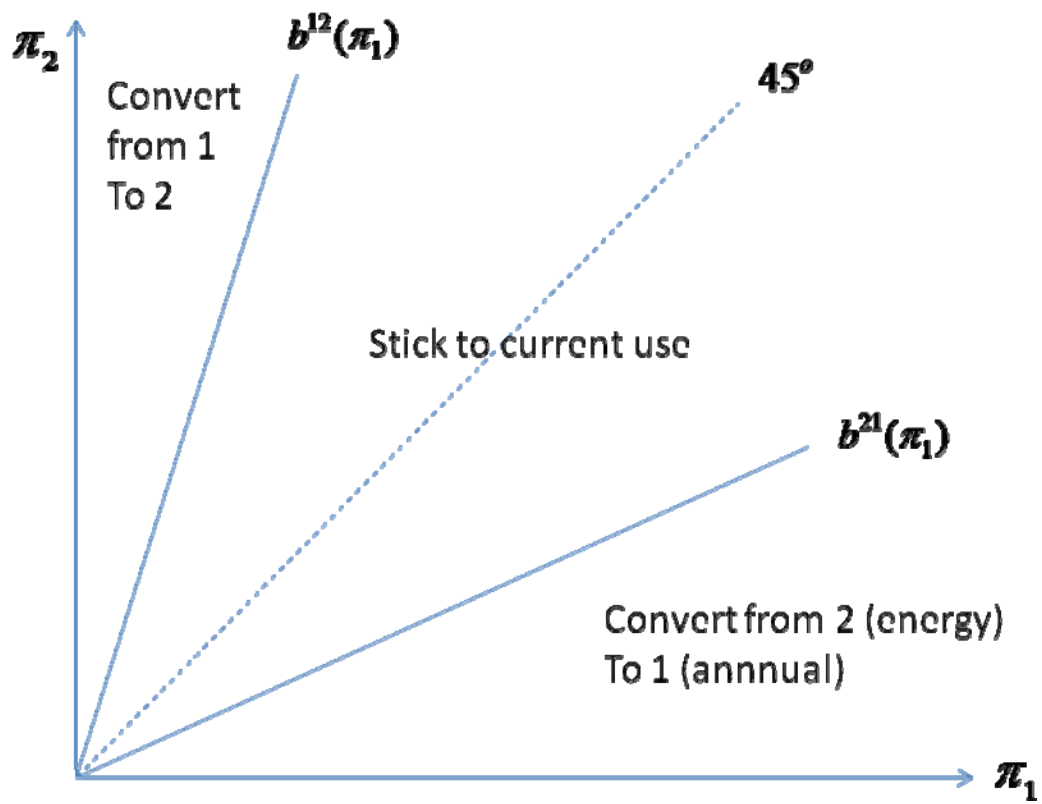


Figure 1. Conversion Boundaries

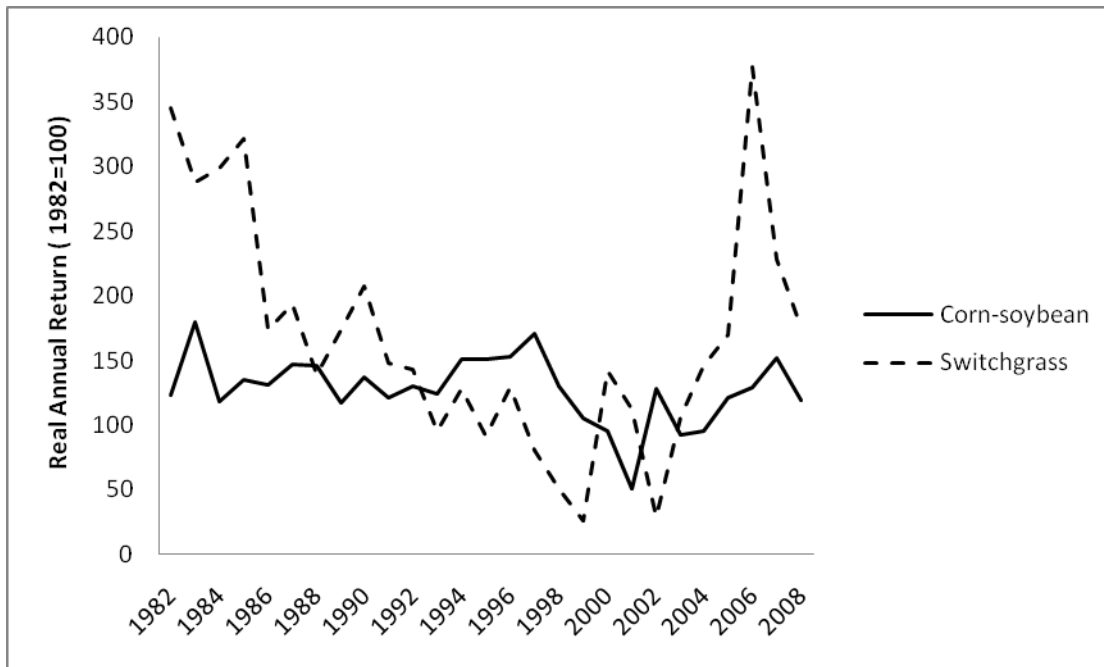


Figure 2. Average returns to corn-soybean and switchgrass in North-central U.S. (1982 - 2008, in 1982 dollars).

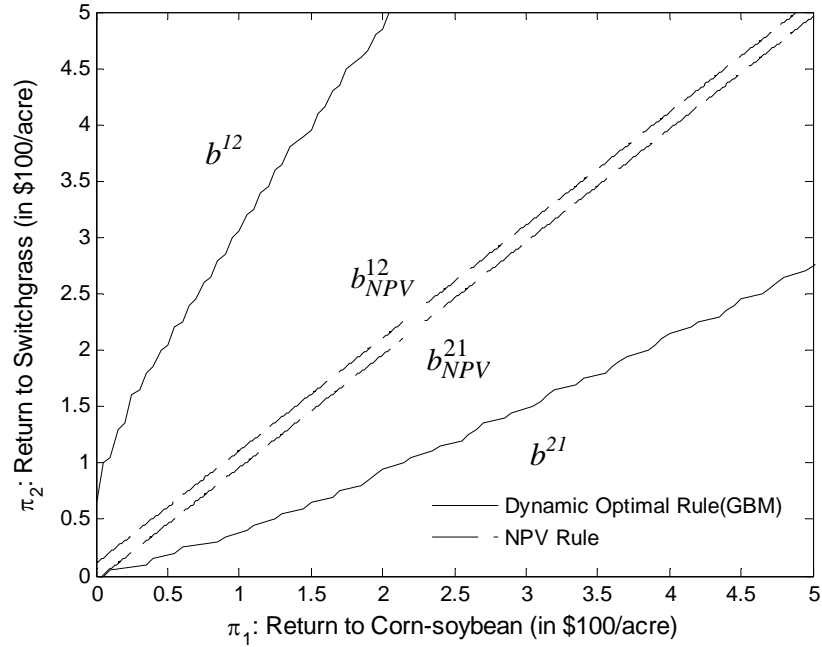


Figure 3a. NPV vs. Dynamic optimal (under GBM): conversion boundaries

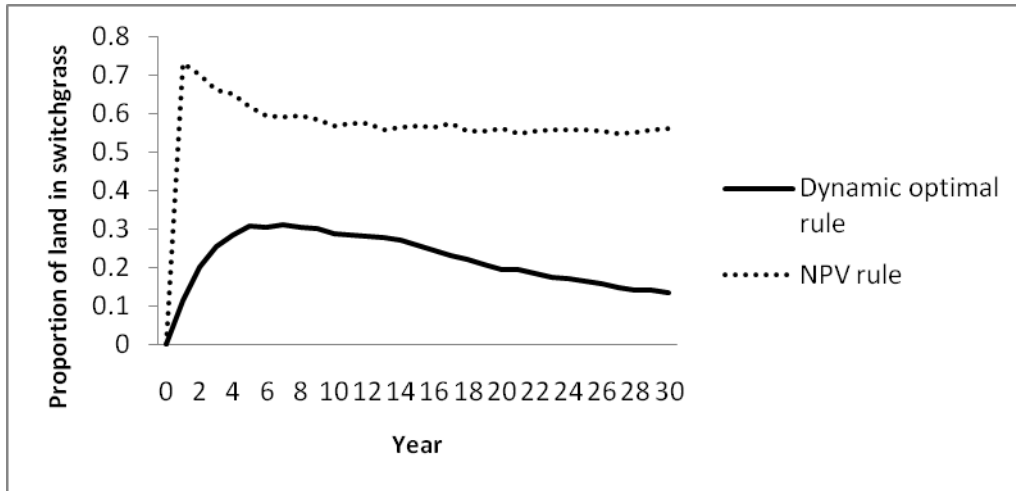


Figure 3b. NPV vs. Dynamic optimal rule (under GBM): proportion of land in switchgrass.

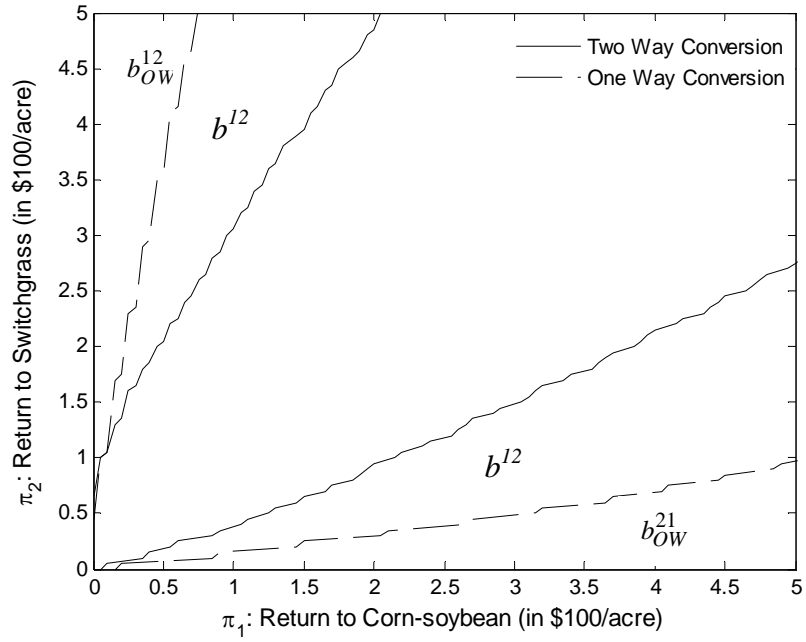


Figure 4a. Two way vs. one way conversion (under GBM): conversion boundaries

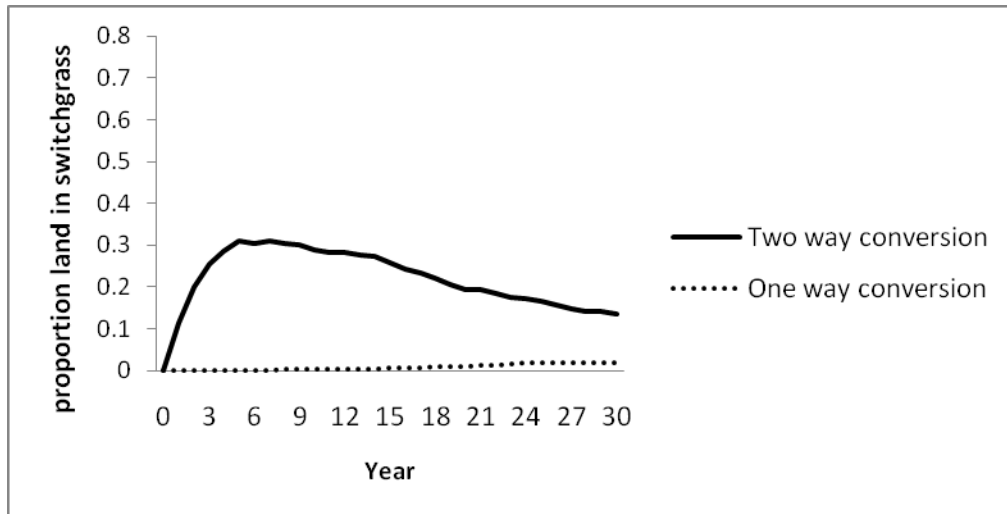


Figure 4b. Two way vs. one way conversion (under GBM): proportion of land in switchgrass

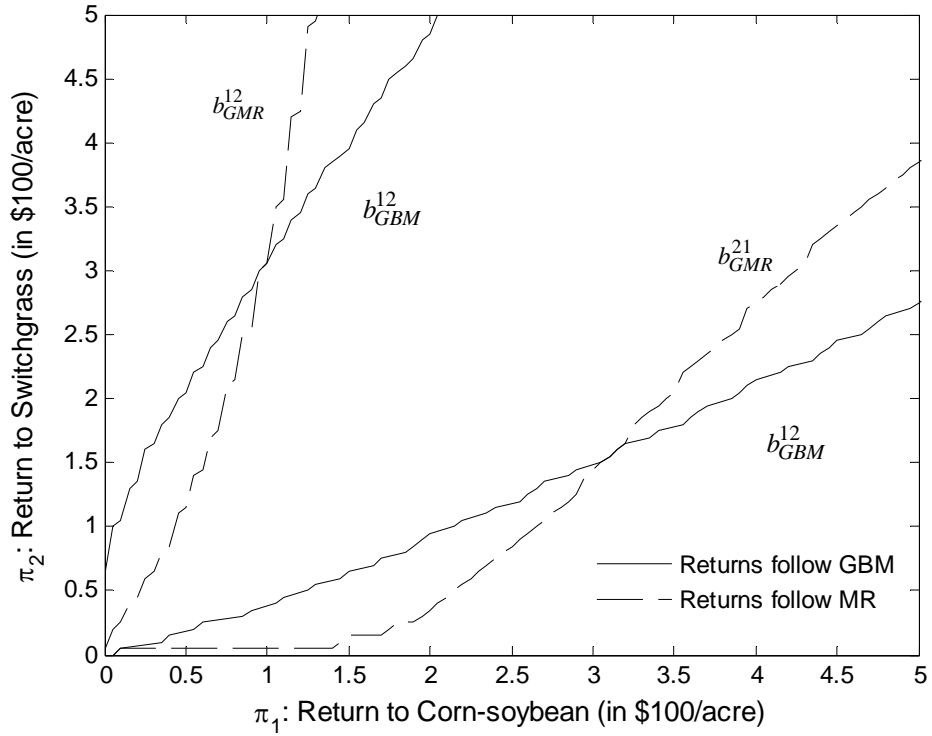


Figure 5a. GBM vs. MR: conversion boundaries

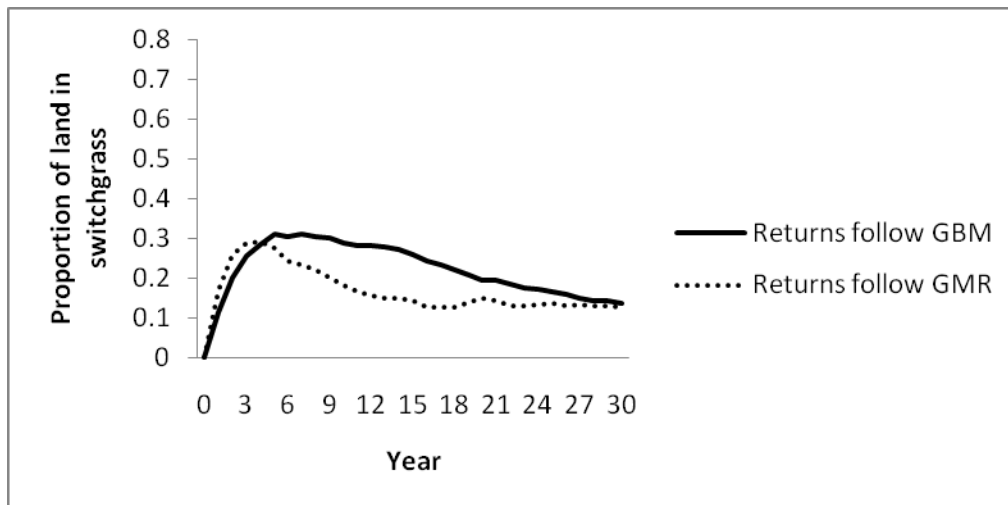


Figure 5b. GBM vs. MR: proportion of land in switchgrass

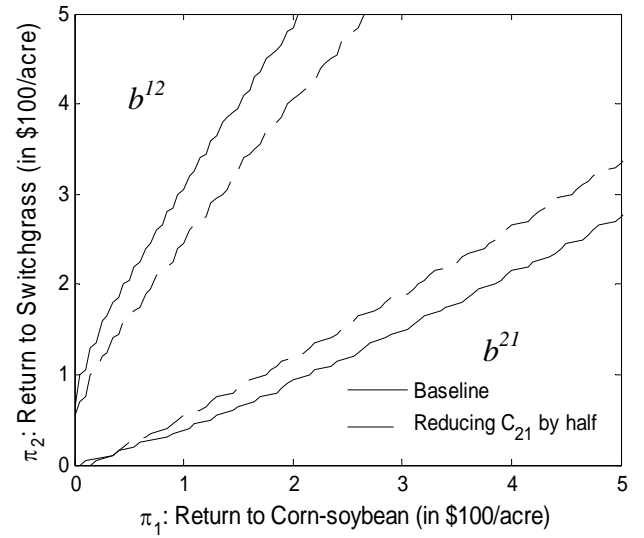
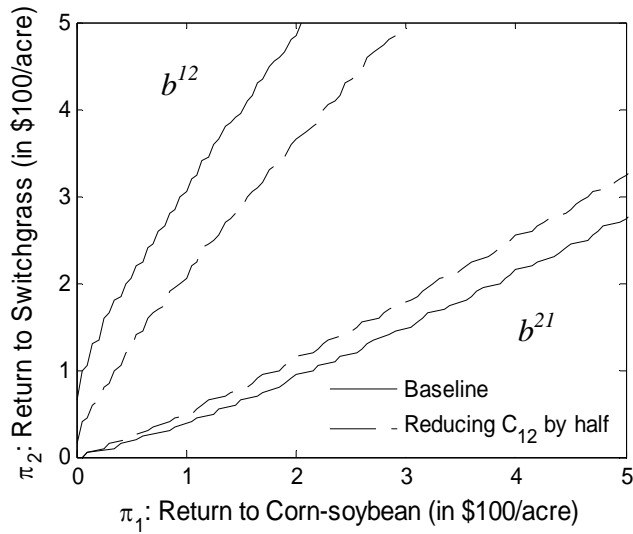


Figure 6a. Reducing conversion costs (under GBM): conversion boundary

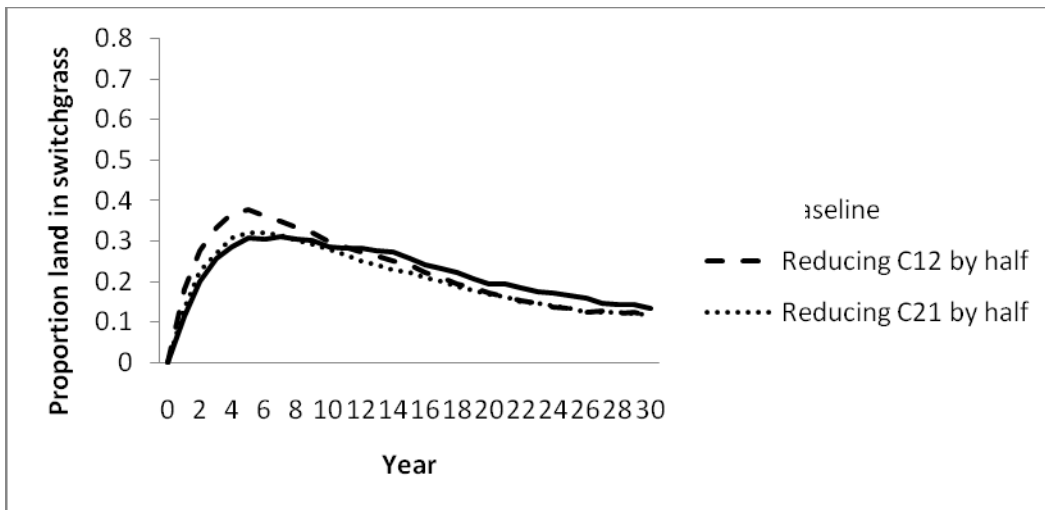


Figure 6b. Reducing conversion costs (under GBM): proportion of land in switchgrass

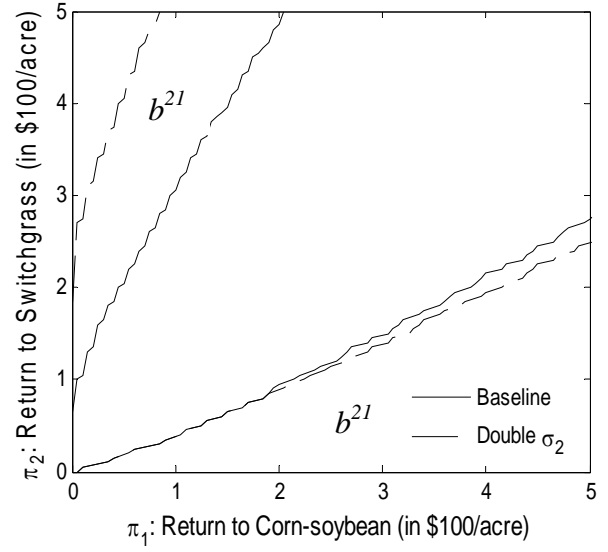
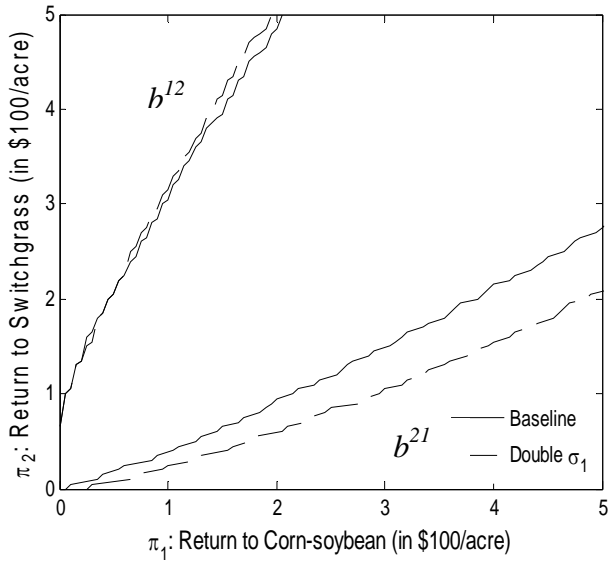


Figure 7a. Return volatility (under GBM): conversion boundary

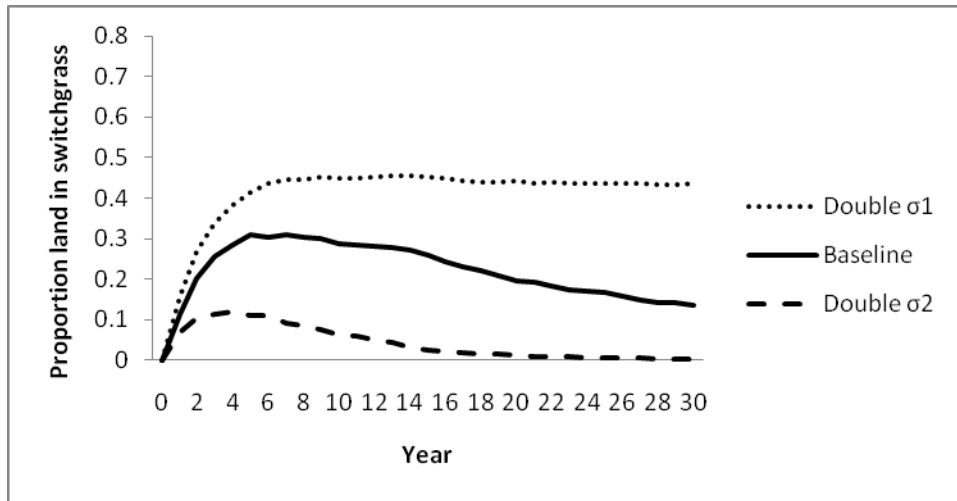


Figure 7b. Return volatility (under GBM): proportion of land in switchgrass