

# Costly Search and Design

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## Abstract

Firms compete by choosing both a price and a design from a family of designs that can be represented as demand rotations. Consumers engage in costly sequential search among firms. Each time a consumer pays a search cost he observes a new offering. An offering consists of a price quote and a new good, where goods might vary in the extent to which they are good matches for the consumer. In equilibrium, only two design-styles arise: either the most niche where consumers are likely to either love or loathe the product, or the broadest where consumers are likely to have similar valuations. In equilibrium, different firms may simultaneously offer both design-styles. We perform comparative statics on the equilibrium and show that a fall in search costs can lead to higher industry prices and profits and lower consumer surplus. Our analysis is related to discussions of how the internet has led to the prevalence of niche goods and the "long tail" phenomenon.

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## 1 Introduction

Firms, through their choices of marketing and product design, have some ability to affect the nature of demand that they face. A growing literature, notably Johnson and Myatt (2006) and Lewis and Sappington (1994), has considered these decisions. More recently, Bar-Isaac, Caruana and Cuñat (2008, 2009) put more emphasis on consumers' information-gathering decisions and highlight that these are co-determined with the firm's pricing and marketing strategies in equilibrium. This literature has focused on monopoly settings, instead this paper is one of the first to extend the analysis to a competitive environment. This leads to a wide variety of results that shed light on the coexistence of niche goods with mass market strategies, the related "long tail" phenomenon, and popular discussions on the effects of the internet.

In order to introduce competition among firms and allow for consumer information-gathering in an analytically tractable way, we build on an established and well-explored model which considers consumers who search both to obtain price-quotes and to learn about the extent to which differentiated goods suit them (Wolinsky, 1986; Bakos, 1997; Anderson and Renault, 1999). Antecedents of these and related models of search have a long history in industrial organization (see for example, Stiglitz, 1989). Recently, and with the perception that the internet should lead to falling search costs, there has been renewed academic and popular interest in consumer search and in such search models.<sup>1</sup>

In particular, Anderson (2004, 2006) sparked widespread interest and discussion of how changes in production and search technologies have changed the pattern of sales and the market shares of the most popular goods as compared to fringe goods in the "long tail". This discussion both builds on previous academic work and has sparked further exploration (see Brynjolfsson, Hu and Smith (2006) for a discussion and further references). Brynjolfsson, Hu and Smith (2006) also suggest that "the Long Tail will change the kinds of products that are profitable". This paper formally explores this idea, contributing to this debate.

Formally, we consider firms that compete by choosing price and "design" along the lines of Johnson and Myatt's (2006) model of a monopoly rotating demand: Here, competitive firms can choose designs from a range which vary between broad market designs that are inoffensive to all consumers, or more niche or quirky designs which are either loved or

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<sup>1</sup>See for example, Bakos (1997) which has been cited more than 1,000 times, and Baye, Morgan and Scholten (2006) who focus, in particular, on the persistence of price dispersion in the online world.

loathed.<sup>2</sup> Consumers search among firms in a way that is standard in models of costly sequential search: Each consumer can pay a small cost to obtain a price-quote from an additional firm and learn about the extent to which the product offered by that firm is well-suited to his tastes.

The model generates a number of simple and interesting results. First, firms choose extremal product designs; that is, either a most broad-based design or a most niche design. Moreover, for sufficiently low search costs, all firms choose the most niche designs available. Further, as is common in search models, but perhaps less noted in popular discussions, search costs and scale effects have very different effects on outcomes.

Perhaps, most striking are the results at intermediate levels of search costs where we illustrate that both kinds of extremal designs might co-exist: Some firms choose a broad-based design and pricing strategy, while all other firms choose a niche strategy. Thus, the model predicts that sales and price distributions should be bi-modal, as suggested by Elberse and Oberholzer-Gee (2006). While other models can generate similar patterns, they do so through assuming differences in the productivity of firms either exogenously as in (Goldmanis et al.; 2009, Elberse and Oberholzer-Gee; 2006) or through a process of technological innovation with different competing vintages (for example, Aghion et al. 2005). Instead, here such patterns arise even when all firms have identical technological opportunities.

Further, by allowing for an endogenous choice of product designs we are able to analyze the combined effects of decreasing search costs on prices, both directly, through increased competition, and indirectly, through a higher prevalence of niche designs. The effect of more niche designs on price can overcome the effect of competition, thus leading to prices, profits and welfare being non monotonic in search costs. There is a clear intuition: With low search costs, and consumers visiting many stores, firms have to offer consumers something very attractive not only in terms of price, but also in terms of the utility that the good provides. This latter consideration leads firms to choose niche designs, but effectively these niche designs differentiate firms and so soften price competition.

Some of these results echo intuitions that have appeared elsewhere in the literature. In particular, Kuksov (2004) presents a duopoly model where consumers know the varieties

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<sup>2</sup>Note that, as in Johnson and Myatt (2006), we need not require a physical design interpretation to induce demand rotations. Firms might similarly induce demand rotations through providing more or less information: in an e-commerce application this might take the form of more or less detailed product descriptions.

available (but not their location) prior to search and different designs come with different costs associated and Cachon, Terwiesch and Xu (forthcoming) focus specifically on multi-product firms, where consumers search costlessly within a firm but at some cost between firms. Our model allows for a continuum of designs and much more general demand specification and, moreover, in marked contrast to these papers, highlights the emergence of asymmetric market structures where broad and niche designs co-exist with no firms that seek intermediate design strategies.

## 2 Model

There is a continuum of firms of measure 1. Each firm produces a single product. There is a continuum of consumers of measure  $m$ . Each consumer,  $l$ , has tastes described by a conditional utility function (not including any search costs) of the form

$$u_{li}(p_i) = -p_i + \varepsilon_{li} \text{ for } i = 1, \dots, N \quad (1)$$

if she buys product  $i$  at price  $p_i$ . The term  $\varepsilon_{li}$  can be interpreted as a match value between consumer  $l$  and product  $i$ . Here  $\varepsilon_{li}$  is the realization of a random variable with distribution function  $F_i$ . We assume that realizations of the  $\varepsilon_{li}$  are independent.<sup>3</sup> Note that in this specification, we assume that the consumer is risk neutral.

A consumer incurs a search cost  $c$  to learn the price  $p_i$  charged by any particular firm  $i$  as well as her match value  $\varepsilon_{li}$  for the product sold by that firm. Consumers search sequentially. The utility of a consumer  $l$  is given by

$$u_{lk}(p_k) - kc, \quad (2)$$

if she buys product  $k$  at price  $p_k$  at the  $k$ th firm she visits.

As is standard in the search literature (and will become clear later), a consumer's search and purchase behaviour can be described by a threshold rule  $\underline{U}$ : she buys the current product obtaining  $u_{li}(p_i)$  if this is less or equal than  $\underline{U}$ , and continues searching otherwise.

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<sup>3</sup>Taking these realizations to be independent, while consistent with the previous literature on search models (Wolinsky (1986) and Anderson and Renault (1999)) is not without loss of generality insofar as it does not permit us to model different firms attempting to target different niches. That is, there is no spatial notion of differentiation or product positioning. However, given that we assume a continuum of firms and no ability for consumers to determine location in advance, this assumption may be more reasonable. Some of the outcomes are similar to the ones of a spatial model (see Bakos, 1997).

We introduce the notion of design by supposing that the distribution of consumer tastes at a given firm  $F_i$  is firm specific. The firm picks a design  $s$  from a set  $S = [B, N]$  and a design leads to a distribution of consumer tastes  $F_s(\theta)$  with support on some interval  $(\underline{\theta}_s, \bar{\theta}_s)$  and logconcave densities  $f_s(\theta)$ . Regardless of design, the firm produces goods at a marginal cost of 0.

The strategy for each firm  $i$ , therefore, consists not only of a choice of price  $p_i$  but (in a departure from Wolinsky (1986), Bakos (1997) and Anderson and Renault (1999)) also a choice of a product design  $s_i \in S$ . We suppose that there are no costs associated with choosing different designs  $s$ .

We follow Johnson and Myatt (2006) in supposing that different product designs induce demand rotations. Formally, there is a family of rotation points  $\theta_s^\dagger$  such that  $\frac{\partial F_s(\theta)}{\partial s} < 0$  for  $\theta > \theta_s^\dagger$  and  $\frac{\partial F_s(\theta)}{\partial s} > 0$  for  $\theta < \theta_s^\dagger$ ; further  $\theta_s^\dagger$  is increasing in  $s$ . The notion of a demand rotation essentially is a formal approach to the notion that some designs lead to a wider spread in consumer valuations than others. In particular, a higher value of  $s$  should be interpreted as a more “quirky” product which appeals more to some consumers and less to others.

Note that, although we assume that many firms can choose the same type of product design  $s$ , this is not to say that from consumer’s perspective their products are the same, since at each firm she would get a new realization from the distribution  $F_s$ .

Our notion of equilibrium is Nash in consumer and firm strategies.<sup>4</sup> Specifically, consumers choose a threshold  $\underline{U}$ , while firms choose a pair  $(p, s)$ . Given that all firms are alike, they will all choose the same strategy if there is one that dominates the rest. In case of indifference, we will describe the equilibrium as a mixed strategy one, that is, as if each firm chooses an element from  $\sigma \in \Delta(\mathbb{R} \times [B, N])$ . Note that this is equivalent to having each firm choosing a pure strategy and the distribution of these pure strategies being  $\sigma$ .

Finally, note that there always exist equilibria where consumers do not search and firms choose prohibitively high prices. We do not consider such equilibria if others exist.

## 2.1 Consumer behaviour

Suppose that a consumer expect firms to choose strategy  $\sigma$ . When the consumer currently holds a best offer with utility  $U$ , then if the consumer samples another firm and finds a product with price  $p$  and match value  $\varepsilon$ , she will prefer it only if  $-p + \varepsilon > U$ . In this case

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<sup>4</sup>In particular this implies passive beliefs: That is, if a consumer observes an off-equilibrium price it does not affect her search and purchase rule.

the additional utility obtained is  $\varepsilon - (U + p)$  and so the expected incremental utility from searching one more firm that is expected to have design  $s$  and price  $p$  is

$$g_{s,p}(U) = E_\varepsilon(\varepsilon - p - U | \varepsilon - p > U) \Pr(\varepsilon - p > U) = \int_{U+p}^{\infty} (\varepsilon - U - p) f_s(\varepsilon) d\varepsilon. \quad (3)$$

It is, therefore, worth searching exactly one more firm if and only if the expected value of a search is worth more than the cost. That is, as long as  $E_\sigma [g_{s,p}(U)] \geq c$ , or, equivalently, if  $U < \underline{U}$  where  $\underline{U}$  is implicitly defined by:

$$E_\sigma [g_{s,p}(\underline{U})] = \int_s \int_p \left( \int_{\underline{U}+p}^{\infty} (\varepsilon - \underline{U} - p) f_s(\varepsilon) d\varepsilon \right) \sigma(p, s) dp ds = c. \quad (4)$$

Note that there is at most one solution to (4), since the left hand side is strictly decreasing in  $\underline{U}$ . However, for  $c$  large enough, there is no  $\underline{U}$  that satisfies (4): If  $c$  is sufficiently large, then no consumer would ever continue searching and firms would have full monopoly power (as in Diamond, 1971). Note that even if a consumer would prefer to continue searching given some existing offering, this does not guarantee that the consumer wants to start to search at all. We will later write down the conditions for search to be initiated.

## 2.2 Firm profit maximization

Suppose that consumers are using a  $\underline{U}$  threshold strategy. Consider now the firm's problem of maximizing profits by choosing  $(p, s)$ . Then, a consumer who visited the firm would choose to buy as long as she received a match  $\varepsilon$  such that  $-\varepsilon > \underline{U} - p$ . Thus, the probability of sale is  $1 - F_s(p + \underline{U})$ .

Consider  $\rho$  to be the probability that a consumer who visits another firm  $j \neq i$  would buy from that firm. The expected number of consumers who visit firm  $i$  as a first visit is  $m$ , a further  $m(1 - \rho)$  visit the firm as a second visit,  $m(1 - \rho)^2$  as a third visit, and so on. At each stage, consumers purchase from firm  $i$  and exit the market with probability  $1 - F_s(p_s + \underline{U})$ . We can, therefore, write demand for a firm that chooses a design  $s$  and price  $p$  as

$$D(p, s) = \frac{m}{\rho} (1 - F_s(p + \underline{U})). \quad (5)$$

Then, trivially, firm profits can be written as

$$\Pi = \frac{m}{\rho} p (1 - F_s(p + \underline{U})). \quad (6)$$

It is useful to define  $p_s(\underline{U})$  as the firm's profit-maximizing price when the consumer's threshold is  $\underline{U}$  and the design strategy is  $s$ :

$$p_s(\underline{U}) = \arg \max p_s(1 - F_s(p_s + \underline{U})). \quad (7)$$

This price is implicitly determined as

$$p_s(\underline{U}) = \frac{1 - F_s(p_s(\underline{U}) + \underline{U})}{f_s(p_s(\underline{U}) + \underline{U})}. \quad (8)$$

**Lemma 1** *The profit maximizing price  $p_s(\underline{U})$  associated with a design  $s$ , when a consumer's stopping rule is given by  $\underline{U}$  is uniquely defined and is non-increasing and continuous in  $\underline{U}$ .*

**Proof.** First note that since  $f_s(x)$  is logconcave then  $\frac{1-F_s(x)}{f_s(x)}$  is monotone decreasing in  $x$ .<sup>5</sup>

Suppose (for contradiction) that at some value of  $\underline{U}$ ,  $p_s(\underline{U})$  is increasing in  $\underline{U}$ , then also  $p_s(\underline{U}) + \underline{U}$  is increasing in  $\underline{U}$  and so  $\frac{1-F_s(p_s(\underline{U})+\underline{U})}{f_s(p_s(\underline{U})+\underline{U})} = p_s(\underline{U})$  is decreasing in  $\underline{U}$ , which provides the requisite contradiction. ■

Now we can write profits as

$$\Pi = \frac{m}{\rho} p_s(\underline{U})(1 - F_s(p_s(\underline{U}) + \underline{U})), \quad (9)$$

and the firm's problem is to maximize this with respect to its remaining strategic variable  $s$ . Note that neither the optimal price nor the optimal design choice depend on  $m$  or  $\rho$ , as these are just constant factors in profits<sup>6</sup>.

Johnson and Myatt (2006) have shown that in a monopoly model profits are quasi-convex in design and so a firm would choose an extremal design. In our environment, taking the behaviour of all other firms as given, the residual demand that a firm faces is still determined through a demand rotation and since the firm is a monopolist on this residual demand the result still applies and so, in our environment, firms chooses extremal designs.

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<sup>5</sup>See Corollary 2 of Bagnoli and Bergstrom (2005) and more broadly, see this paper for functions which do and do not satisfy this logconcavity assumption.

<sup>6</sup>This highlights that search costs play a qualitatively different role to scale effects, which is of course a central point of Wolinsky (1986) and discussed by Anderson and Renault (1999) who highlight that limits when search costs tend to 0 and when the ratio of firms to consumers increases are quite different.

**Proposition 2** *Firms choose extremal designs, that is either  $s = N$  or  $s = B$ .*

**Proof.** The optimal design is chosen to maximize  $p_s(\underline{U})(1 - F_s(p_s(\underline{U}) + \underline{U}))$ . Now, given that  $p_s - \underline{U}$  is an affine transform of  $p_s$ , it follows that  $D(p_s, s)$  as in (5) are rotation-ordered. The proof then follows immediately from Proposition 1 in Johnson and Myatt (2006), p. 761. ■

To gain some intuition for this result, first consider the case where the optimal price at a given design  $s$  is below the point of rotation so that the profit-maximizing quantity is greater than the quantity at the point of rotation  $1 - F_s(\theta_s^\dagger)$ . Then, decreasing  $s$  (and so “flattening” out demand) will lead to a greater quantity sold even if the price is kept fixed. Therefore decreasing  $s$  must lead to higher profits.

Next consider the case where the optimal price is above the point of rotation so that the profit-maximizing quantity is less than the quantity at the point of rotation  $1 - F_s(\theta_s^\dagger)$ . Then, increasing  $s$  (and so “steepening” demand) will lead to a greater quantity sold even if the price is kept fixed. Therefore increasing  $s$  must lead to higher profits in this case.

Using Proposition 2, we can restrict attention to equilibrium strategies in which firms choose a broad design  $(p_B, B)$  with probability  $\lambda$  and a niche one  $(p_N, N)$  with probability  $(1 - \lambda)$ . where  $p_B$  and  $p_N$  are defined by (8) for  $s = B, N$  respectively.

### 3 Equilibrium

Given all the previous analysis, we can express an equilibrium in this model as a pair  $(\underline{U}, \lambda)$ , where  $\underline{U}$  summarizes the searching and purchase behavior of consumers and  $\lambda$  determines the proportions of firms choosing niche and broad strategies. These two parameters have to satisfy the following conditions. First, rearranging formula (4), consumers optimize their behavior when

$$c = \lambda \int_{\underline{U} + p_B(\underline{U})}^{\bar{\theta}_B} (\varepsilon - \underline{U} - p_B(\underline{U})) f_B(\varepsilon) d\varepsilon + (1 - \lambda) \int_{\underline{U} + p_N(\underline{U})}^{\bar{\theta}_N} (\varepsilon - \underline{U} - p_N(\underline{U})) f_N(\varepsilon) d\varepsilon. \quad (10)$$

Second, as explained above, firms choose either a niche or broad position, or are indifferent between the two:

$$\lambda = \arg \max \lambda p_B(\underline{U})(1 - F_B(p_B(\underline{U}) + \underline{U})) + (1 - \lambda) p_N(\underline{U})(1 - F_N(p_N(\underline{U}) + \underline{U})), \quad (11)$$



and third,  $p_B(\underline{U})$  and  $p_N(\underline{U})$  are determined by (8) as

$$p_B(\underline{U}) = \frac{1 - F_B(p_B(\underline{U}) + \underline{U})}{f_B(p_B(\underline{U}) + \underline{U})}, \quad (12)$$

$$p_N(\underline{U}) = \frac{1 - F_N(p_N(\underline{U}) + \underline{U})}{f_N(p_N(\underline{U}) + \underline{U})}. \quad (13)$$

As we have already pointed out, we are interested in equilibria in which consumers do initiate their search for a product. In order to determine the relevant condition, it is convenient to compute first the equilibrium probability that a consumer purchases at a particular firm:

$$\rho(\lambda, \underline{U}) = \lambda \left( \int_{\underline{U} + p_B(\underline{U})}^{\bar{\theta}_B} f_B(\varepsilon) d\varepsilon \right) + (1 - \lambda) \left( \int_{\underline{U} + p_N(\underline{U})}^{\bar{\theta}_N} f_N(\varepsilon) d\varepsilon \right). \quad (14)$$

Now, we can express the expected value of initiating search  $V(\lambda, \underline{U})$  as

$$V(\lambda, \underline{U}) = \rho(\lambda, \underline{U}) E_\lambda [g_{\lambda, p}(U) | U \geq \underline{U}] - c + (1 - \rho(\lambda, \underline{U})) V(\lambda, \underline{U}). \quad (15)$$

Thus, one needs to check that

$$V(\lambda, \underline{U}) = \frac{1}{\rho(\lambda, \underline{U})} \left[ \lambda \int_{\underline{U} + p_B(\underline{U})}^{\bar{\theta}_B} (\varepsilon - p_B(\underline{U})) f_B(\varepsilon) d\varepsilon + (1 - \lambda) \int_{\underline{U} + p_N(\underline{U})}^{\bar{\theta}_N} (\varepsilon - p_N(\underline{U})) f_N(\varepsilon) d\varepsilon - c \right] \geq 0. \quad (16)$$

In the next sections we characterize the equilibrium in detail depending on whether this is in mixed or pure strategies. We consider first the pure strategy equilibria and then the mixed ones. Note that there are two possibilities to consider in pure strategies. One in which all firms choose a broad design ( $B$ ), and another in which all firms choose a niche one ( $N$ ). We then seek to further understand the effect of a change in search costs  $c$  on the the firm equilibrium strategies, as well as on profits and consumer surplus.

### 3.1 All broad equilibrium

In this case the equilibrium can be expressed as  $(\underline{U}, \lambda) = (U_B, 1)$ , where  $U_B$  is implicitly characterized by (10), which now becomes:

$$c = \int_{U_B + p_B(U_B)}^{\bar{\theta}_B} (\varepsilon - U_B - p_B(U_B)) f_B(\varepsilon) d\varepsilon, \quad (17)$$

Since the right hand side of (17) is decreasing in  $U_B + p_B(U_B)$ , and  $U_B + p_B(U_B)$  is monotonic in  $U_B$  (following a similar argument to Lemma 1), there is a unique solution for (17). Note that  $U_B$  depends on  $c$ .

In order for this pair  $(U_B, 1)$  to be an equilibrium, one needs to make sure firms do not want to deviate to a design  $N$ , with its corresponding price  $p_N(U_B)$ . That is,

$$p_B(U_B)(1 - F_B(p_B(U_B) + U_B)) \geq p_N(U_B)(1 - F_N(p_N(U_B) + U_B)). \quad (18)$$

It is convenient to define  $\bar{U}$  as the value at which (18) holds with equality; that is, it is implicitly defined by:

$$p_B(\bar{U})(1 - F_B(p_B(\bar{U}) + \bar{U})) = p_N(\bar{U})(1 - F_N(p_N(\bar{U}) + \bar{U})). \quad (19)$$

Up to this point, we have not ruled out that there may be many solutions for  $\bar{U}$ , though we will do so below.

Associated with  $\bar{U}$ , it is convenient to define

$$c_B := \int_{\bar{U} + p_B(\bar{U})}^{\infty} (\varepsilon - \bar{U} - p_B(\bar{U})) f_B(\varepsilon) d\varepsilon. \quad (20)$$

Thus,  $c_B$  is the search cost that induces the consumer behaviour (reservation threshold)  $\bar{U}$  when in equilibrium all firms choose a broad design and price accordingly: When all firms choose a broad design, and the search cost is  $c_B$  then (by definition of  $\bar{U}$ ) the deviation condition (18) is just binding.

Finally, we need to make sure that the consumer wants to start the search process at all. Using, the expression derived in (16), here, this condition is

$$\frac{\int_{\bar{U} + p_B(U_B)}^{\infty} \varepsilon f_B(\varepsilon) d\varepsilon - c}{1 - F_B(U_B + p_B(U_B))} - p_B(U_B) > 0. \quad (21)$$

We can now summarize all the above in the following proposition.

**Proposition 3** *The unique solution to consumer behaviour and its corresponding equilibrium prices, as determined by (17) and (12), constitute an All Broad Equilibrium if and only if  $c \geq c_B$  and it is worthwhile for consumers to initiate search, as captured by (21).*

**Proof.** To prove the existence of the All Broad equilibrium one needs to show that (18)

is satisfied.

First consider some solution to (19), and call this particular solution  $\bar{U}$ .

Note that

$$\begin{aligned} p_B(\bar{U})(1 - F_B(p_B(\bar{U}) + \bar{U})) &= p_N(\bar{U})(1 - F_N(p_N(\bar{U}) + \bar{U})) \\ &\geq p_B(\bar{U})(1 - F_N(p_B(\bar{U}) + \bar{U})) \end{aligned} \quad (22)$$

where the equality follows from the definition of  $\bar{U}$  and the inequality follows from the definition of  $p_N(\cdot)$  as the profit-maximizing price.

It follows that

$$1 - F_B(p_B(\bar{U}) + \bar{U}) \geq 1 - F_N(p_B(\bar{U}) + \bar{U}). \quad (23)$$

Similarly

$$\begin{aligned} p_N(\bar{U})(1 - F_N(p_N(\bar{U}) + \bar{U})) &\geq p_N(\bar{U})(1 - F_B(p_N(\bar{U}) + \bar{U})), \text{ and so} \\ 1 - F_N(p_N(\bar{U}) + \bar{U}) &\geq 1 - F_B(p_N(\bar{U}) + \bar{U}) \end{aligned} \quad (24)$$

We use these fact to show that  $p_N(\bar{U}) > p_B(\bar{U})$  and  $1 - F_B(p_B(\bar{U}) + \bar{U}) > 1 - F_N(p_N(\bar{U}) + \bar{U})$ .

Suppose (for contradiction) that  $p_N(\bar{U}) < p_B(\bar{U})$ . Note that since  $N$  and  $B$  are drawn from a family of demand rotations, it follows that there is some  $\hat{x}$  such that  $1 - F_N(x) > 1 - F_B(x)$  if and only if  $x > \hat{x}$ .

First suppose  $p_B(\bar{U}) + \bar{U} > \hat{x}$  then  $1 - F_N(p_B(\bar{U}) + \bar{U}) > 1 - F_B(p_B(\bar{U}) + \bar{U})$  in contradiction to (23). If instead  $\hat{x} \geq p_B(\bar{U}) + \bar{U} > p_N(\bar{U}) + \bar{U}$ , then (24) is contradicted.

It follows that  $p_N(\bar{U}) > p_B(\bar{U})$  and from (19), trivially  $1 - F_B(p_B(\bar{U})) > 1 - F_N(p_N(\bar{U}))$ .

Next, returning to the maximization problem, we can rewrite  $p_B(\bar{U})$  and  $p_N(\bar{U})$  as the solutions to the maximization problems explicitly and so re-write (19) as:

$$\max_{p_B} p_B(1 - F_B(p_B + \bar{U})) = \max_{p_N} p_N(1 - F_N(p_N + \bar{U})). \quad (25)$$

As usual, by the envelope theorem and the FOC of the previous problems we know that  $\frac{d\pi_i}{dU}|_{\bar{U}} = -p_i f_i(p_i + \bar{U}) = 1 - F_i(p_i(\bar{U}))$  for both  $i = B, N$ . Now, as argued above  $\frac{d\pi_B}{dU}|_{\bar{U}} = 1 - F_B(p_B(\bar{U})) > 1 - F_N(p_N(\bar{U})) = \frac{d\pi_N}{dU}|_{\bar{U}}$ .

Since  $U$  is monotonic and decreasing in  $c$ , for higher values of  $c$  we have lower values  $U$ . And for lower values of  $U$  than  $\bar{U}$ ,  $\pi_B$  grows faster than  $\pi_N$ . Thus, the firm's no deviation

condition is satisfied (locally) for  $c \geq c_B$  and violated (locally) for  $c < c_B$ .

Finally, consider the uniqueness of  $\bar{U}$ .

Suppose (for contradiction) that there exists a  $\bar{c}$  for which there are many solutions to (19). Consider the two highest solutions and label them  $\bar{U}_1 > \bar{U}_2$ . We know that locally, (by the continuity of  $\underline{U}$  as defined in (4) in  $c$ ) there exists a search cost  $c'$  just below the search cost  $\bar{c}$  that induces  $U(c') > \bar{U}_2$  and the no deviation condition (18) fails. Similarly, there exists some other  $c''$  such that  $\bar{U}_1 > U(c'')$ . Since all functions are continuous, there must exist some  $U$  such that  $\bar{U}_1 > U(c'') > U > U(c') > \bar{U}_2$  which satisfies (19). But this contradicts that  $\bar{U}_1$  and  $\bar{U}_2$  were the two highest solutions. ■

### 3.2 All niche equilibrium

First, we can directly assure the existence of an All Niche equilibrium for sufficiently small searching costs:

**Proposition 4** *When  $c$  is sufficiently small, then, in equilibrium, all firms choose a most niche product design  $s = N$ .*

**Proof.** We know that  $\underline{U}$  is defined as

$$c = \lambda \int_{\underline{U} + p_B(\underline{U})}^{\bar{\theta}_B} (\varepsilon - \underline{U} - p_B(\underline{U})) f_B(\varepsilon) d\varepsilon + (1 - \lambda) \int_{\underline{U} + p_N(\underline{U})}^{\bar{\theta}_N} (\varepsilon - \underline{U} - p_N(\underline{U})) f_N(\varepsilon) d\varepsilon. \quad (26)$$

and that firms maximize

$$\max_s \frac{m}{\rho(\lambda, \underline{U})} p_s(\underline{U}) (1 - F_s(p_s(\underline{U}) + \underline{U})). \quad (27)$$

Note that as  $c$  tends to zero  $\underline{U} + p_B(\underline{U})$  tends to  $\bar{\theta}_B$  and  $\underline{U} + p_N(\underline{U})$  tends to  $\bar{\theta}_N$ . This implies that  $\rho(\lambda, \underline{U})$ , as defined in (14) tends to zero.

Suppose for contradiction that when  $c$  tends to 0 then  $\underline{U} < T < \bar{\theta}_B$ . Then, since  $\underline{U} + p_B(\underline{U})$  tends to  $\bar{\theta}_B$ , it must be that  $p_B(\underline{U}) > \Delta > 0$  for some constant  $\Delta$ . A firm by choosing  $s = B$  and  $p_B = \Delta$  would achieve unbounded profits yielding a contradiction. In particular this contradiction shows that as  $c$  tends to zero  $\underline{U} > T$  for  $T < \bar{\theta}_B$  arbitrarily close to  $\bar{\theta}_B$ .

Now consider a firm that chooses design  $B$  and price  $p_B$  for all  $c$ , with positive probability. We consider a deviation to a design  $N$  and a price  $p_B$  which we claim gives strictly

higher profits for  $c$  small enough. Here, equilibrium profits are  $p_B \frac{m}{\rho(\lambda, \underline{U})} (1 - F_B(p_B + \underline{U}))$  and deviation profits are  $p_B \frac{m}{\rho(\lambda, \underline{U})} (1 - F_N(p_B + \underline{U}_\sigma))$ .

Since  $\underline{U} \rightarrow \bar{\theta}_B$  then  $\underline{U} > \theta_B^\dagger$  for  $c$  low enough and, given our definition of a demand rotation, then  $(1 - F_N(p_B + \underline{U})) > (1 - F_B(p_B + \underline{U}))$ . So this is a profitable deviation and we reach a contradiction, which leads us to conclude that  $\lambda$  must be equal to 0. ■

Next, we can replicate the analysis above for the all niche equilibrium. The analogous conditions in the niche case are as follows:  $U_N$  is defined implicitly by:

$$c = \int_{U_N + p_N(U_N)}^{\infty} (\varepsilon - U_N - p_N(U_N)) f_N(\varepsilon) d\varepsilon, \quad (28)$$

where  $p_N(U_N)$  is defined in (8)

The no deviation condition now corresponds to deviating to a broad design and is:

$$p_N(U_N)(1 - F_N(p_N(U_N) + U_N)) \geq p_B(U_N)(1 - F_B(p_B(U_N) + U_N)). \quad (29)$$

Note that this holds with equality  $U_N = \bar{U}$ . We can define  $c_N$  as follows

$$c_N := \int_{\bar{U} + p_N(\bar{U})}^{\infty} (\varepsilon - \bar{U} - p_N(\bar{U})) f_N(\varepsilon) d\varepsilon. \quad (30)$$

and the equivalent condition to (21) is:

$$\frac{\int_{x_N}^{\infty} \varepsilon f_N(\varepsilon) d\varepsilon - c}{1 - F_N(p_N(U_N) + U_N)} - p_N(U_N) > 0. \quad (31)$$

Furthermore, analogous to Proposition 3, we obtain the following result.

**Proposition 5** *There exists an All Niche Equilibrium if and only if  $c \leq c_N$  and it is worthwhile for consumers to initiate search, as captured by (31).*

**Proof.** It follows trivially from the proof of 3 that if there is some  $c'$  for which an all niche equilibrium exists then an all niche equilibrium exists if and only if  $c \leq c_N$ ; however, Proposition 4 guarantees that there is some  $c'$  for which an all niche equilibrium exists. ■

### 3.3 Mixed Strategy Equilibria

In a mixed strategy equilibrium, where both  $N$ -designs and  $B$ -designs are chosen with non-trivial probabilities, a firm must be indifferent between the two designs so that

$$p_B(\bar{U})(1 - F_B(p_B(\bar{U}) + \bar{U})) = p_N(\bar{U})(1 - F_N(p_N(\bar{U}) + \bar{U})) \quad (32)$$

This implicitly defines the optimal consumer strategy to the previously defined value of  $\bar{U}$ . Using (20) and (30), we can rewrite (10) as:

$$c = \lambda c_B + (1 - \lambda)c_N, \quad (33)$$

Trivially,  $\lambda = 0$  requires that  $c = c_N$ , and similarly  $\lambda = 1$  requires  $c = c_B$ . In particular (ignoring the participation/Diamond condition) there is at least one equilibrium throughout the whole space of  $c$ . If  $c_N > c_B$  then there is a region of multiplicity where all broad, all mixed and all niche coexist, otherwise, if  $c_N < c_B$  then the mixed strategies equilibrium exactly fills the gap between the regions where all broad and all niche exist. Finally if  $c_N = c_B$  the mixed strategies equilibrium has no mass (this is the outcome with linear demand rotations, as discussed below). Further note that in the mixed strategies region  $\lambda$  is going to be linear in  $c$  and whether it is increasing or decreasing depends on the relationship between  $c_N$  and  $c_B$ .

### 3.4 Summarizing the characterization

First, for high enough  $c$ , the market breaks down for standard reasons—firms have sufficient monopoly power and, since they cannot commit not to, extract so much surplus that consumers do not consider it worthwhile to search. For lower values, of  $c$  there may be an interval where all firms choose a broad design, and within this region prices and profits are decreasing and consumer surplus is increasing as  $c$  declines. Further, there always exists an interval of values of  $[0, c_N]$  for some  $c_N$  where all firms choose a niche design and within this region prices and profits are decreasing as  $c$  declines while consumer surplus rises as  $c$  declines. However, changes in  $c$  can lead to a shift from one of these intervals to another and as a result, profits, prices and consumer surplus may be non-monotonic in  $c$ . An important intuition of the paper is that two counteracting forces affect prices. Lower  $c$  increases price competition, but price competition induces more niche designs (analogous to increasing differentiation) that leads to lower price competition. Such changes need not only arise as

discrete regime changes, but lower search costs can smoothly decrease consumer surplus in the region where the mixed strategy equilibrium exists.

The partial characterization of this section does not demonstrate that there are in fact equilibria where all firms choose broad designs. We illustrate this possibility by fully characterizing the case of linear demand rotations in Appendix A. Moreover, this example, highlights that profits and consumer surplus can be non-monotonic in search costs. Secondly, we show that mixed strategy equilibria can arise in Appendix B.

## 4 Conclusions and Extension

We briefly summarize the discussion and relate it to the introductory motivation before highlighting a number of further considerations and extensions.

**Summary** This paper presents a simple and tractable model integrating consumer search and firms' strategic product design choices. Equilibrium can be characterized relatively simply insofar as firm and consumer behaviour can be separately analyzed and the firms' strategic interactions arise only through consumer behaviour. Since there is a continuum of firms, each one has only a negligible effect on consumer behaviour. Thus, we are able to make significant progress in characterizing the equilibrium even without having to impose much structure on the functional form for demand.

The characterization is of some considerable interest in itself. Even though, all firms are ex-ante identical, an asymmetric industry outcome can arise where firms take very different approaches—some taking a “broad-market” strategy, seeking a very broad design and choosing a relatively low price and others taking a “niche” strategy with quirky products priced high to take advantage of the (relatively few) consumers who are well-matched to the product. The contrast between broad-market and niche strategies has been explored elsewhere, notably Johnson and Myatt (2006), in the earlier work of Lewis and Sappington (1994) and, more recently, Bar-Isaac, Caruana and Cuñat (2009); however, these models focus on monopolies. Instead, here we present a competitive model in a market with search frictions where these different strategies can coexist. The characterization of equilibrium suggests that in examining empirical distributions of sales volumes, one might anticipate bimodal distributions with a proportion of firms clustered around high sales and others around low sales.

This observation, and the comparative statics of the equilibrium with respect to consumer search costs can be brought to bear in considering demand-side explanations for

the “long tail” effect of the internet. As search costs fall, a greater proportion of firms choose the “niche” strategy, and, in part, due to the different industry structure, but in part also since it is cheaper for consumers to more easily seek better-suited products, niche firms account for a larger proportion of the industry’s sales. Note, that in contrast to much discussion surrounding scale or production cost effects, we assume that production technologies do not vary and are identical in terms of costs.

In addition, the comparative statics results are interesting in highlighting that prices (and profits) can be non-monotonic in consumer search costs. There is an intuitive rationale: As search costs fall, then as long as the product designs remain unchanged, prices fall. However, at ever lower prices, the “broad-market” strategy becomes less appealing to firms, some of whom adopt a “niche” strategy, charging a high price to the (few) consumers who are well-matched for the product. Moreover, the firms’ choosing to adopt a niche strategy effectively impose a positive externality on other firms, since this choice of a niche strategy effectively acts as a form of differentiation that softens price competition.

**Endogenous Firm Entry** One can endogenize the proportion of consumers per firm, that is  $m$  by assuming an entry cost and allowing for a free entry condition. As discussed on page 7, scale (as captured by  $m$ ) has no effect at all on firms’ equilibrium prices and designs. So, as a consequence, the characterization of the equilibrium strategies (and of consumer surplus) is identical to the analysis in Section (3). The sole effect of endogenizing entry would be that net profits for firms would be zero and rather than characterizing equilibrium profits, we would characterize the number of firms (that is, the ratio of firms per consumer) in the industry. Note in particular that high profits in the case of an exogenous number of firms corresponds to a high ratio of firms per consumer when entry is endogenous.

**Coordinated industry behaviour** In the model, firms take their actions separately but their choices have consequences for all other firms in the industry. There is, therefore, a rationale to try to coordinate on industry-level responses and attempt to internalize the externalities that arise.

In particular, since profits can be non-monotonic in search costs, as search costs fall exogenously the industry might benefit from further reducing search costs. Thus an industry response to the internet (which we may plausibly consider as an exogenous fall in consumer search costs) is to provide additional enabling technologies (such as industry-sponsored comparison sites) that further reduce search costs for consumers.



**Prominence and search order** A small and recent literature has explored the effect of prominence and search order (Arbatskaya; 2007, and Armstrong, Vickers and Zhou, 2008). The model can easily be adapted to suppose that some firms are more prominent in the sense that the order of search is not identically distributed across firms, but, instead, some particular firms are more likely to be visited sooner in a consumers search process than others.

Such a change would have no effect whatsoever on the equilibrium decision of firms, such a notion of “prominence” is similar to a scale effect. In the notation of the model, this would act as some firms facing a high value of  $m$  and others facing a low value of  $m$ . However, scale (as captured by  $m$ ) has no effect at all on firms’ equilibrium prices and designs. This echoes the result of Armstrong, Vickers and Zhou (2008) that with a continuum of firms, prices are unaffected by supposing one firm is always visited first. Prominence in their model then only plays a role when a significant number of consumers search through every one of the products on offer and consider revisiting a firm that they have already visited. One could consider adapting the model of this paper in this way; however, for many applications it seems an unrealistic assumption, and would lead to a much less analytically tractable model.

**Search on price and product attributes** As in previous literature, such as Wolinsky (1986) and Anderson and Renault (1999), “search” in this model is a combined process whereby consumers learn all characteristics of a product (notably price and its match with their taste) simultaneously. One could conceive of situations where it may be possible for consumer to rationally make separate search decisions on particular product attributes, learning price first and later considering attributes, or considering some attributes first along the lines of Bar-Isaac, Caruana and Cuñat (2008). Indeed, there is considerable discussion of these kind of search procedures as heuristics in psychology and marketing; Payne, Bettman and Johnson (1993) in an important contribution, also provide a wide-ranging summary. An extension that allowed for such multi-dimensional search would be substantive but of great interest.

**Ex-ante firm heterogeneity and superstar effects** An interesting and relatively straightforward extension is to allow for ex-ante differences in firms in terms of their “natural” appeal or vertical quality. Thus, before any product tailoring or design choices, some products are simply naturally better products, with the potential to be superstars.

As discussed above, Anderson (2006) and other commentators on the effects of the inter-

net on sales concentration have highlighted “long tail” effects; there is a parallel discussion, notably Elberse and Oberholzer-Gee (2006) arguing online retailing, has led to increased “superstar” effects, with the best-selling products becoming ever-more successful.<sup>7</sup>

Extending the model to allow for ex-ante vertical product differences, can incorporate this effect. When search costs are very high, however, consumers cannot locate these superior products, as search costs fall, consumers should be able to more easily find them—suggesting a superstar effect. Note, however, that as search costs fall, the competition from the long tail may become more severe, as inferior products choose a niche strategy yielding more attractive options for some well-matched consumers. Indeed, it can be shown that both the “superstar” and the “long tail” phenomena can coexist as inferior firms gradually switch from broad-based to niche strategies and consumers more easily find superior firms.

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<sup>7</sup>See also Hervas-Drane (2009) for further references and a model that contrasts two different channels (sequential search and ex-ante recommendations) through which the internet might affect superstar and long tail effects.

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## A Example: Linear Demands

We analyze the particular case where demand is linear. We have shown already that we only need to worry about extremal product designs so we can restrict the analysis to the two linear demand functions that correspond with the most broad design ( $B$ ) and the most niche design ( $N$ ). Without loss of generality, any possible family of linear demand function can be a result of having consumer types  $\varepsilon$  uniformly distributed with the following structure  $\varepsilon \sim U[0, \alpha_s]$  with probability  $\beta_s$  and  $\varepsilon = 0$  with probability  $(1 - \beta_s)$ .<sup>8</sup>

In particular it is convenient to define the extremal designs ( $N$  and  $B$ ) as follows: When product design is  $N$  then  $\varepsilon \sim U[0, \alpha_N]$  with probability  $\beta_N$  and  $\varepsilon = 0$  otherwise. When product design is  $B$  then  $\varepsilon \sim U[0, \alpha_B]$  with probability  $\beta_B$  and  $\varepsilon = 0$  otherwise. We impose that  $\alpha_N > \alpha_B$  and  $\beta_N < \beta_B$  in order to ensure that these are demand rotations as defined above (i.e. the demand curves cross once) and the  $N$  design is the most niche one. This allows us to write

$$F_N(x) = \frac{\beta_N}{\alpha_N}x + (1 - \beta_N) \text{ if } x \in [0, \alpha_N], \text{ and} \quad (34)$$

$$F_B(x) = \frac{\beta_B}{\alpha_B}x + (1 - \beta_B) \text{ if } x \in [0, \alpha_B].$$

### A.1 Characterization

We begin by characterizing  $p_B(U)$  and  $p_N(U)$ , as in (8)

$$p_B(U) = \arg \max_p p(\beta_B - \frac{\beta_B}{\alpha_B}(p + U)) = \frac{\alpha_B - U}{2}, \text{ and} \quad (35)$$

$$p_N(U) = \arg \max_p p(\beta_B - \frac{\beta_B}{\alpha_B}(p + U)) = \frac{\alpha_N - U}{2}. \quad (36)$$

We turn to characterize  $U_B(c)$  as implicitly defined in (17). This must satisfy

$$c = \int_{U_B + p_B(U)}^{\infty} (\varepsilon - U_B - p_B(U)) f_B(\varepsilon) d\varepsilon \quad (37)$$

$$= \int_{\frac{U_B + \alpha_B}{2}}^{\alpha_B} (\varepsilon - \frac{U_B + \alpha_B}{2}) \frac{\beta_B}{\alpha_B} d\varepsilon = \frac{1}{8\alpha_B} \beta_B (\alpha_B - U_B)^2 \quad (38)$$

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<sup>8</sup>Further, that the distributions have the same means requires an additional restriction that  $\frac{\alpha_s \beta_s}{2} = \frac{\alpha_{s'} \beta_{s'}}{2} \forall s, s'$ .

so that

$$U_B(c) = \alpha_B - \sqrt{\frac{8\alpha_B c}{\beta_B}}, \text{ and, similarly,} \quad (39)$$

$$U_N(c) = \alpha_N - \sqrt{\frac{8\alpha_N c}{\beta_N}}. \quad (40)$$

Next, note (32) is given by  $\frac{\alpha_B - \bar{U}}{2}(\beta_B - \frac{\beta_B(\alpha_B + \bar{U})}{\alpha_B}) \geq \frac{\alpha_N - \bar{U}}{2}(\beta_N - \frac{\beta_N(\alpha_N + \bar{U})}{\alpha_N})$ , which can be rewritten as

$$\bar{U} = \frac{\alpha_N - \alpha_B}{\alpha_N \beta_B - \beta_N \alpha_B} \sqrt{\alpha_N \beta_N \alpha_B \beta_B} - \frac{\alpha_N \alpha_B (\beta_N - \beta_B)}{\alpha_N \beta_B - \beta_N \alpha_B}. \quad (41)$$

It follows that  $c_N$ , which is defined implicitly by  $U_N(c_N) = \bar{U}$ , and  $c_B$ , which is defined implicitly by  $U_B(c_B) = \bar{U}$  are identical and given by

$$c_N = c_B = \frac{1}{8} \beta_N (\alpha_N - \alpha_B)^2 \frac{(\alpha_N \beta_B - \sqrt{\alpha_N \beta_N \alpha_B \beta_B})^2}{\alpha_N (\alpha_N \beta_B - \beta_N \alpha_B)^2} = \frac{1}{8} \beta_B (\alpha_N - \alpha_B)^2 \frac{(\beta_N \alpha_B - \sqrt{\alpha_N \beta_N \alpha_B \beta_B})^2}{\alpha_B (\alpha_N \beta_B - \beta_N \alpha_B)^2}. \quad (42)$$

For the case where all firms choose a broad design the condition for the consumer to ever visit a second firm once she has visited a first one is the condition for the expected gain of visiting an additional firm exceeding  $c$  when the outcome at the first firm is the worst possible match.

$$c < \int_0^{\alpha_B} (\varepsilon - 0) f_B(\varepsilon) d\varepsilon = \frac{\alpha_B \beta_B}{2} \quad (43)$$

Note that this condition is equivalent to  $U_B + p_B(U_B) > 0$ .

The second condition is that the ex-ante expected surplus of starting the search process is positive. Still within the broad case we can define the consumer surplus as  $V_B$  and express it as

$$V_B = -c + \int_{U_B + p_B(U_B)}^{\alpha_B} (\varepsilon - p_B) f_B(\varepsilon) d\varepsilon + F_B(x_B) V_B = \frac{\int_{U_B + p_B(U_B)}^{\alpha_B} (\varepsilon - p_B(U_B)) f_B(\varepsilon) d\varepsilon - c}{1 - F_B(U_B + p_B(U_B))}. \quad (44)$$

Note that  $V_B > 0$  implies (43) as  $\int_{U_B + p_B(U_B)}^{\alpha_B} (\varepsilon - p_B(U_B)) f_B(\varepsilon) d\varepsilon - c > 0$  implies  $\int_0^{\alpha_B} (\varepsilon - 0) \frac{\beta_B}{\alpha_B} d\varepsilon - c = \frac{\alpha_B \beta_B}{2} - c > 0$ .

Substituting for  $p_B$  and  $U_B$  and simplifying then  $V_B > 0$  is equivalent to

$$\frac{\alpha_B \beta_B}{8} > c. \quad (45)$$

Analogously for the “all niche” equilibrium to exist is necessary that

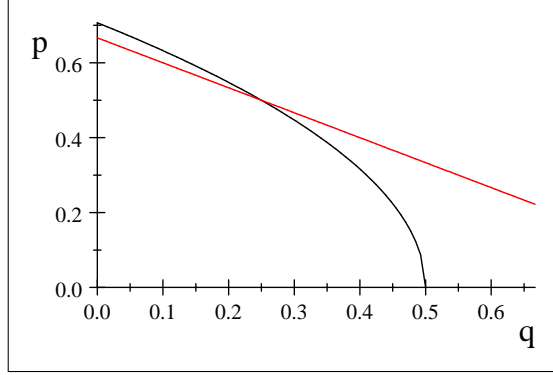
$$\frac{\alpha_N \beta_N}{8} > c. \quad (46)$$

Note that it is necessarily the case that  $\frac{\alpha_N \beta_N}{8} > c_N = c_B$ . Suppose that  $\frac{\alpha_B \beta_B}{8} > c_N = c_B$  then we can summarize the discussion above as follows.

**Proposition 6** *If  $c > \frac{\alpha_B \beta_B}{8}$  the unique equilibrium outcome is no search by consumers and no sales for firms. For  $\frac{\alpha_B \beta_B}{8} > c > c_N = c_B$ , the equilibrium has all firms choose the broad strategy and a price of  $\sqrt{2 \frac{\alpha_N}{\beta_N} c}$  and earn profits  $m \sqrt{2 \frac{\alpha_N}{\beta_N} c}$  and consumer surplus is  $\alpha_N - 2 \sqrt{\frac{2c\alpha_N}{\beta_N}}$ . Finally, for  $c_N = c_B > c$ , the equilibrium has all firms choose the niche strategy and a price of  $\sqrt{2 \frac{\alpha_B}{\beta_B} c}$ , earning profits  $m \sqrt{2 \frac{\alpha_B}{\beta_B} c}$  while consumer surplus is  $\alpha_B - 2 \sqrt{\frac{2c\alpha_B}{\beta_B}}$ .*

## B Example: Design Dispersion

Although we have a family of demand rotations, it is only the extremal ones that play a role when there are no design costs, so we describe only the extreme designs. We suppose that the niche  $N$  designs are such that  $F_N(x) = \frac{1}{2} + x^2$  for  $x \in (0, \frac{1}{\sqrt{2}})$  and  $F_B(x) = \frac{3}{2}x$  for  $x \in (0, \frac{2}{3})$ . In particular the two distributions implied by these designs are logconcave. Implied demands for monopolist firms are illustrated in the figure below.



Monopoly demand functions for broad  
and niche designs.

We begin by characterizing  $p_N(U)$  and  $p_B(U)$  as in (8). Specifically,

$$p_N(U) = \arg \max p\left(\frac{1}{2} - (p + U)^2\right) = \frac{1}{6}\sqrt{2}\sqrt{2U^2 + 3} - \frac{2}{3}U \quad (47)$$

$$p_B(U) = \arg \max p\left(1 - \frac{3}{2}(p + U)\right) = \frac{2 - 3U}{6} \quad (48)$$

Next, we solve for  $\bar{U}$ . As in (32),  $\bar{U}$  is implicitly defined by

$$p_N(\bar{U})\left(\frac{1}{2} - (p_N(\bar{U}) + \bar{U})^2\right) = p_B(\bar{U})\left(1 - \frac{3}{2}(p_B(\bar{U}) + \bar{U})\right). \quad (49)$$

The solution is  $\bar{U} = 0.29793$  and the associated  $p_N(\bar{U}) = 0.22153$  and  $p_B(\bar{U}) = 0.18437$ .

Next we can consider  $c_N$ , and  $c_B$  as in (30) and (20) to obtain  $c_N = 2.2695 \times 10^{-2}$  and  $c_B = 2.5494 \times 10^{-2}$  and so here  $c_B > c_N$ .

It remains to check the conditions (31) and (21) to ensure that the “interesting action” occurs in a feasible range. Consider, first, (31). This is given by

$$\frac{\int_{p_N(\bar{U})+\bar{U}}^{\frac{1}{\sqrt{2}}} \varepsilon 2\varepsilon d\varepsilon - c}{1 - \frac{1}{2} - (p_N(\bar{U}) + \bar{U})^2} - p_N(\bar{U}) > 0, \text{ or, equivalently, } c < 9.1266 \times 10^{-2}, \quad (50)$$

and, similarly, (21) is given by

$$\frac{\int_{p_B(\bar{U})+\bar{U}}^{\frac{2}{3}} 24\varepsilon^3 d\varepsilon - c}{1 - \frac{3}{2}(p_N(\bar{U}) + \bar{U})} - p_B(\bar{U}) > 0, \text{ or, equivalently, } c < 0.10789. \quad (51)$$

So the interesting action (where  $c_B$  and  $c_N$  are defined) is indeed in an interesting range and the mixed strategy equilibrium described in Section 3.3 does indeed arise, as the unique equilibrium for  $c_B > c > c_N$ , and in this range prices increase and consumer surplus decrease as  $c$  falls, though outside this range the comparative statics are the more intuitive ones.