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On the power of direct tests for rational expectations against the alternative of constant gain learning

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Abstract

In this paper we study the power of direct tests for rational expectations against the constant gain learning alternative. The investigation is by means of a Monte Carlo study. The tests considered use quantitative expectations data and qualitative survey data that has been quantified. The main finding is that the power of tests for rational expectations against constant gain learning may be very small, making it impossible to distinguish the hypotheses.

Keywords: adaptive learning, tests for rational expectations, quantification methods, constant gain least squares

JEL: D84, D83, C12

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1. Introduction

The rational expectations hypothesis has dominated the macroeconomics literature since the 1970s. However, in many recent models it has been replaced by a more plausible adaptive learning hypothesis assuming that agents form expectations by estimating and updating a forecasting function (for an overview see e.g. Evans, Honkapohja 2001). Applications of adaptive learning have provided new insights to key issues of the monetary policy, business cycles, and asset pricing (Evans, Honkapohja 2009). One important conclusion from the adaptive learning literature is that policies which are optimal under rational expectations may no longer be optimal when agents use a learning mechanism (in the context of the monetary policy see e.g. Orphanides, Williams 2008). Given the central role that expectations play in modern macroeconomic theory it is important to be able to study empirically the way in which expectations are formed.

A popular way to investigate rationality is by means of direct tests (see e.g. Keane, Runkle 1990; Lovell 1986; Pesaran 1987; Zarnowitz 1985 and also Łyziak 2003; Tomczyk 2004; Osińska 2000). This approach is preferred to indirect testing which focuses on cross equation parameter restrictions applied to a particular parametric economic model. The direct approach uses data obtained from consumer and business tendency surveys which can be either quantitative or qualitative, with the latter type prevailing (see Pesaran, Weale 2006). The qualitative data provide an expected direction of change for a given economic variable. For the purpose of empirical studies the qualitative data need to be transformed to figures by means of one of the many conversion procedures which have been proposed (see Batchelor, Orr 1988; Berk 1999; Carlson, Parkin 1975; Pesaran 1987; Seitz 1988; Smith, McAleer 1995). In a recent overview Nardo (2003) summarizes the contradicting results of standard rationality tests when survey data are used in test regression.

The aim of this paper is to study the power of tests for rational expectations when these are applied to expectations data consistent with the adaptive learning hypothesis. Expectations are derived from a forecasting function estimated by constant gain least squares (CGLS). We focus on the efficiency and orthogonality tests. The tests are applied to quantitative and quantified series.

The properties of the rationality tests are analysed by means of Monte Carlo experiments. Although the tests are often applied, their power has not been thoroughly investigated. In particular, there are no studies of the properties of the rationality tests when these are applied to expectations consistent with the constant gain learning hypothesis. Our contribution is quite unique for two more reasons. First, we study the power of rationality tests for the data generated from the process allowing for feedback from expectations to the realizations of the forecast variable. Second, we apply the tests not only to quantitative but also to quantified data. The properties of tests using quantified series were previously studied by Common (1985) who examined the size and power of the serial correlation test against the alternative of adaptive expectations; the quantification methods he considered were the balance statistics method and the Carlson and Parkin (1975) method.

The main finding of the paper is that tests for rational expectations may have very low power against the constant gain learning both for quantitative and quantified data. The tests are hence not well suited for making empirical distinction between the two types of expectations. Low power means that if the null hypothesis of rationality is not rejected it is not safe to conclude that expectations are rational as they might have been generated by learning agents. False conclusion concerning rationality may consequently lead to a non-optimal choice of policy design.

The outline of the paper is as follows. Section 2 describes the types of expectations data and section 3 the quantification procedure used. Section 4 presents the alternative expectations formation schemes. Section 5 contains the description of tests for rational expectations. The design of the Monte Carlo experiments and the results obtained are given in sections 6 and 7 respectively. Conclusions are presented in section 8.

2. The expectations data

Data on expectations are usually obtained from consumer and business tendency surveys. Survey data can be either quantitative or qualitative. In the first case, agents provide a numerical value for the variable and in the second the expected direction of change. While the quantitative data can be directly used in econometric studies, the qualitative responses need to be first converted into figures.

In the simulations we generate the expectations of N respondents. The data have both the quantitative and quantified form. The qualitative expectations have the shape typical of data on inflationary expectations collected from consumer surveys carried out in the OECD countries. In these surveys respondents state whether they expect prices to rise faster than at present, rise at the same rate, rise more slowly, stay at their present level or go down. In the simulations we assume that survey expectations are formulated with respect to the next period. Below, the qualitative answers collected in period $t-1$ concerning expectations for time t are summarized by the fractions of respondents that answered "rise faster than at present", "rise at the same rate", "rise more slowly", "stay at their present level" and "go down" and are denoted by RF_t, SR_t, RS_t, S_t and D_t, respectively. The qualitative data are then used to derive quantitative measurements of expectations by means of the probability conversion procedure described in section 3.

3. Quantification procedures

The qualitative responses are quantified using a version of the probability approach applicable to data obtained from surveys with five response categories. The probability approach rests on several assumptions which have been thoroughly reviewed in the literature (see e.g. Batchelor, Orr 1988; Berk 1999; 2002; Forsells, Kenny 2004 and Łyziak 2003) and will not thus be described here in detail. In the case of inflationary expectations, the idea of the method is that answers of individual respondents (from $i = 1,..., N$) are formed depending on two sensitivity intervals, one centred on 0 and the other centred on the current perceived inflation rate. Both the perceived inflation rate and the end points of the indifference intervals are assumed to be fixed among the respondents. The perceived rate is further assumed to be known and equal to the current rate of inflation. Then the intervals have the form: $\langle -r, r \rangle$ and $\langle \pi_{t-1} - s, \pi_{t-1} + s \rangle$. The responses are formulated as follows. In the case when the expected inflation for the i-th respondent falls within the interval: $\langle -r, r \rangle$ the respondent reports that prices are going to stay the same. If the expected inflation is smaller than the lower end point of this indifference interval, i.e. $-r$, the expected decrease in prices is reported. For the expectations falling between the values r and $\pi_{t-1} - s$ respondents claim that prices will rise more slowly. The "prices will rise at the same rate" answer is given if the expectations are covered by the second interval $\langle \pi_{t-1} - s, \pi_{t-1} + s \rangle$ and the "rise faster" response reported in the case the expected inflation is larger than $\pi_{t-1} + s$.

Given these assumptions it can be shown that:

$$P\{\pi_t \le -r | \Omega_{t-1}\} = F_{t-1}(-r) = D_t$$
$$P\{-r \le \pi_t \le r | \Omega_{t-1}\} = F_{t-1}(r) - F_{t-1}(-r) = S_t$$
$$P\{r \le \pi_t \le \pi_{t-1} - s | \Omega_{t-1}\} = F_{t-1}(\pi_{t-1} - s) - F_{t-1}(r) = RS_t$$
$$P\{\pi_{t-1} - s \le \pi_t \le \pi_{t-1} + s | \Omega_{t-1}\} = F_{t-1}(\pi_{t-1} + s) - F_{t-1}(\pi_{t-1} - s) = SR_t$$
$$P\{\pi_t \ge \pi_{t-1} + s | \Omega_{t-1}\} = 1 - F_{t-1}(\pi_{t-1} + s) = RF_t$$

where Ω_{t-1} is the union of individual information sets and F_{t-1} is the cumulative distribution function of π_{t-1}. We assume that F_{t-1} is the cumulative standard normal distribution. Then the average expected rate of price changes, π_t^e is given by:

$$\pi_t^e = \pi_{t-1} \frac{\Phi^{-1}(1 - RF_t - SR_t - RS_t) + \Phi^{-1}(D_t)}{\Phi^{-1}(1 - RF_t - SR_t - RS_t) + \Phi^{-1}(D_t) - (\Phi^{-1}(1 - RF_t) + \Phi^{-1}(1 - RF_t - SR_t))} \quad (1)$$

where Φ^{-1} is the inverse of the cumulative standard normal distribution.

In what follows the probability method is applied to survey data generated in the Monte Carlo experiments.

4. Expectations formation schemes

The main focus of the paper is on the power of the rationality tests against constant gain learning. For comparison, the power against adaptive expectations and the size of the tests are also considered. Altogether three different types of expectations series are employed: rational expectations, expectations generated as in CGLS adaptive learning and adaptive expectations.

The rational expectations hypothesis of Muth (1961) assumes that expectations are essentially the same as the predictions of the relevant theory but may be subject to idiosyncratic errors. In the experiments, the rational expectations are generated so that all the prediction errors made by agents are Gaussian white noise. We assume that agents know the form of the equation which generates the actual data and its parameter values.

In the case of adaptive learning, the economic agents form their expectations on the basis of a forecasting function. The parameters of the function have to be estimated and they are updated when new data become available. The updating proceeds by constant gain least squares. Suppose that expectations for time t are formed at time $t-1$ according to the equation:

$$\pi_t^e = x'_{t-1} \hat{\beta}_{t-1}$$

where π_t^e is an expected value of the variable π_t, x_{t-1} is a vector of predictors and $\hat{\beta}_{t-1}$ is a vector of parameter estimates obtained on the basis of information available at time $t-1$. Then the constant gain least squares parameter updating rule can be written as (see Evans, Honkapohja 2001):

$$\begin{aligned}\hat{\beta}_t &= \hat{\beta}_{t-1} + \gamma R_t^{-1} x_{t-1}(\pi_t - x'_{t-1}\hat{\beta}_{t-1}) \\ R_t &= R_{t-1} + \gamma(x_{t-1}x'_{t-1} - R_{t-1})\end{aligned} \tag{2}$$

The constant gain procedure discounts past observations at a geometric rate 1 – γ. CGLS is reasonable when market participants believe that the economic environment is changing overtime but do not know when the changes occur.

The last type of expectations i.e. adaptive expectations are revised in line with past forecast errors according to:

$$\pi_t^e = \pi_{t-1}^e + \lambda(\pi_{t-1} - \pi_{t-1}^e) \tag{3}$$

where $\lambda \in \langle 0,1 \rangle$.

5. Tests for rational expectations

The hypothesis of rationality is typically investigated by means of the orthogonality test considered to be the most comprehensive test for rational expectations (see Pesaran 1984). The test has been applied both to quantitative and quantified expectations. It consists in regressing the expectations error on information known at time $t-1$ and testing whether the informational variables are significant. The test equation has thus the following form:

$$(\pi_t - \pi_t^e) = \beta_0 + \beta_1 I_{t-1} + \varepsilon_t \tag{4}$$

where π_t^e stands for either quantitative or quantified expectations, I_{t-1} represents informational variables known at time $t-1$ and the null hypothesis of rational expectations is given by $\beta_0 = 0$ and $\beta_1 = 0$.

There is a special case of the orthogonality test concerned with the efficient use of the information contained only in the history of changes in the variable under investigation. This efficiency test is based on the equation:

$$(\pi_t - \pi_t^e) = \beta_0 + \beta_1 \pi_{t-1} + \varepsilon_t \tag{5}$$

and the null hypothesis of rationality is $\beta_0 = 0$ and $\beta_1 = 0$.

Most commonly the above hypotheses are tested on the basis of the ordinary least squares (OLS) estimation of (4) and (5) using the *F*-test. In what follows we report the OLS test results.

6. Design of the Monte Carlo experiments

To analyse the properties of survey-based tests for rational expectations we conduct the Monte Carlo study. The data generating processes (DGPs) used to generate 'actual' realisations of variables are based on structural bivariate models. The variables denoted by π and y can be interpreted as an inflation rate and the deviation of the actual output from its potential level, in which case the model becomes the inverse Lucas supply model (Lucas 1973). Other interpretations are, however, also possible. In the experiments, quantitative expectations with respect to the values of π of $N = 1000$ respondents are generated using the alternative expectations formation schemes described in section 4. In order to obtain the qualitative series, the data are converted into survey answers. Then, the procedure described in section 3 is used to put the qualitative responses into numerical form.

The rationality tests described in section 5 are applied to both the quantitative and quantified expectations. Series of 100, 200 and 400 observations are considered. Samples are generated using random initial values of the variables. In the experiments 1000 replications are used.

The detailed description of the experiments is the following:

1. The pseudo-datasets are generated from the following general process:

$$\begin{aligned} \pi_t &= \alpha + \beta \pi_t^e + \delta y_t + \varepsilon_t \\ y_t &= 0.5 y_{t-1} + \eta_t \end{aligned} \tag{6}$$

Several values of the β parameter are investigated including 0, 0.3, 0.5, 0.7 and 0.9 resulting in five alternative DGPs. Each value of β implies a different weight with which expectations influence the actual values of π. For $\beta = 0$ there is no feedback from expectations to actual realizations of π. The other parameter values are chosen so that in each case the model corresponds to the same reduced form under the rational expectations hypothesis. Hence α is set to $4(1 - \beta)$ and δ is put equal to $1 - \beta$. The errors are assumed to be normally distributed with the covariance matrix

$$\begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

2. Expectations of 1000 agents are then generated. The parameters of the functions used to generate expectations consistent with the CGLS adaptive learning and adaptive expectations i.e. γ and λ are set equal to 0.025 and 0.1 respectively. Similar values are commonly employed in theoretical studies of many economic processes. Sargent (1999) applies these values in his simulation study of inflationary expectations. Orphanides and Williams (2008) consider $\gamma = 0.02$ to be a 'reasonable benchmark'. In an empirical study, analysing US quarterly data Branch and Evans (2006) obtain $\hat{\gamma} = 0.007$ for the GDP growth rate and $\hat{\gamma} = 0.062$ for the CPI inflation. To examine the power of the rationality tests more fully we additionally consider a much higher value of γ equal to 0.1. This value, meaning that past observations are discounted geometrically with the rate 0.9, would imply that agents perceive the economic environment as often changing.

The rational expectations of the i-th agent, $i = 1,..., 1000$ for are generated as:

$$\pi_{it}^e = 4 + 0.5 y_{t-1} + u_{it}, \quad u_{it} \sim N(0,6)$$

These rational expectations are determined by the forecasts of π_t in the rational expectations equilibrium conditional on information available at $t-1$. To obtain predictions of individual respondents we augment these forecasts with individual errors u_{it}.

For CGLS adaptive learning expectations the following equation is used:

$$\pi_{it}^e = \hat{\alpha}_0 + \hat{\alpha}_1 y_{t-1} + u_{it}, \quad u_{it} \sim N(0,6)$$

where $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are parameter estimates obtained for the equation $\pi_t = \alpha_0 + \alpha_1 y_{t-1} + \xi_t$ for observations up to period $t-1$ using (2) with γ equal to 0.025 or 0.1. These estimates do not converge to the parameter values of the rational expectations equilibrium.

Finally, adaptive expectations are obtained on the basis of:

$$\pi_{it}^e = \pi^e{}_{i(t-1)} + 0.1(\pi_{t-1} - \pi^e{}_{i(t-1)}) + u_{it}, \quad u_{it} \sim N(0,6)$$

3. The quantitative average expectations data at time t are calculated as

$$\pi_t^e = 1/N \sum_{i=1}^{N} \pi_{it}^e$$

4. The qualitative survey expectations data are obtained for the indifference intervals: $\langle -0.3, 0.3 \rangle$ and $\langle \pi_{t-1} - 0.3, \pi_{t-1} + 0.3 \rangle$. The conversion is done in the following manner: all the individual expectations smaller than -0.3 are expressed as "go down" answers, values falling into the interval $\langle -0.3, 0.3 \rangle$ as "stay at their present level", values between 0.3 and $\pi_{t-1} - 0.3$ as "rise more slowly", values covered by $\langle \pi_{t-1} - 0.3, \pi_{t-1} + 0.3 \rangle$ as "rise at the same rate" and finally, values exceeding $\pi_{t-1} + 0.3$ as "rise faster than at present" responses.

The quantitative data are then summarized by calculating the proportions of each response at time $t-1$, i.e. by obtaining the RF_t, SR_t, RS_t, S_t and D_t statistic values.

5. The qualitative data are converted to figures by means of the probability method.

6. The 5% orthogonality and efficiency tests are then applied to both quantitative and quantified expectations data. In the orthogonality test π_{t-1} and y_{t-1} are used as the informational variables.

7. The rejection rate of the null hypothesis in the rationality tests in 1000 replications is calculated.

7. Monte Carlo results

The results of the Monte Carlo experiments are given in the Appendix. These are presented as proportions of 1000 Monte Carlo replications in which the rational expectations hypothesis was rejected by the efficiency and orthogonality tests at the 5% significance level. The test outcomes are given for the quantitative expectations and for the series quantified by means of the probability method. Results are reported for five DGPs differing by the value of β equal to 0, 0.3, 0.5, 0.7 or 0.9 and three sample sizes of 100, 200 and 400 observations.

Table 1 shows the results concerning the estimated size of the alternative tests for rational expectations. Under rational expectations the five DGPs are identical and so only one set of results is given for each sample size. It can be seen that in each case the estimated size of the tests is reasonably close to the nominal value.

Table 2 reports the power of the rationality tests against the adaptive expectations alternative. The ability of the tests to reject the null hypothesis is very good in the majority of cases. The power of the orthogonality test is much better than that of the efficiency test. For the smallest sample size the probability of rejecting the null hypothesis by at least one of the tests, for $\beta \neq 0$ exceeds 0.7, and in the case of $\beta = 0$ is of the order of 0.5. For samples of 200 observations the power is larger than 0.8 for all the cases and in samples of 400 observations the false null hypothesis is almost always rejected. The tests using quantified data perform similarly to the tests using quantitative data.

The power of the rationality tests against the constant gain learning alternative with $\gamma = 0.025$ is given in Table 3. The power of all the tests for all sample sizes is generally smaller than the nominal size. These results indicate that using the rationality tests it is not possible to distinguish rational expectations from expectations formed by agents using this particular form of CGLS estimation.

The prospects of rejecting the null hypothesis for larger values of γ can be evaluated by looking at results from Table 4 corresponding to $\gamma = 0.1$. For $T = 200$, the adaptive learning expectations are still undistinguishable from the rational expectations. The power of all the tests is close to the nominal size. For $T = 200$ the probabilities of rejecting the null hypothesis are still quite low and do not exceed 0.22. For the largest sample size the prospects of rejecting the null hypothesis depend on the value of the β parameter. For values of β equal 0.3, 0.5 or 0.7 the power is higher than 0.5 while for $\beta = 0$ or $\beta = 0.9$ it is smaller than 0.3.

The results show that the power of the tests against constant gain learning is low both for the case in which expectations influence the realisations of the forecast variable and for the no-feedback case. The results of the tests based on quantified data reproduce those obtained for quantitative series quite well.

8. Conclusions

Expectations play a major role in modern macroeconomic analysis. Conclusions from policy studies often depend crucially on the particular assumptions concerning the way in which expectations are formed i.e. depend on whether the rational expectations hypothesis, the adaptive expectations hypothesis or the adaptive learning hypothesis is used.

Survey data are often used for testing the rational expectations hypothesis. So-called direct tests are based on quantitative expectations data or quantified qualitative data. The tests investigate the optimal properties of rational expectations and they do not specify a particular alternative. In this paper we have studied the power of such tests when the alternative is constant gain learning.

The main finding of the paper is that tests for rational expectations may have very low power against the CGLS adaptive learning for values of the learning parameter typically considered in the literature. There is a disturbing implication, viz. that if the null hypothesis of rationality is not rejected it is not safe to conclude that expectations are rational for they might have been generated by learning agents. As the way in which agents form expectations matters, e.g. for the optimal policy design, false conclusion concerning rationality may have serious consequences.

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Appendix

Table 1

Estimated size of the 5% efficiency and orthogonality tests for rational expectations

Quantitative data		Quantified data	
Efficiency test	**Orthogonality test**	**Efficiency test**	**Orthogonality test**
	***T* = 100**		
0.039	0.048	0.041	0.044
	***T* = 200**		
0.050	0.048	0.042	0.048
	***T* = 400**		
0.046	0.042	0.043	0.047

Notes: Estimated size of the 5% efficiency and orthogonality tests is given for both quantitative expectations and expectations quantified by means of the probability method. The true indifference intervals are given by: $\langle -0.3, 0.3\rangle$ and $\langle \pi_{t-1} - 0.3,\ \pi_{t-1} + 0.3\rangle$. Results are reported for sample sizes T of 100, 200 and 400 observations.

Table 2

Estimated power against adaptive expectations with $\lambda = 0.1$ of the 5% efficiency and orthogonality tests for rational expectations

	Quantitative data		Quantified data	
	Efficiency test	**Orthogonality test**	**Efficiency test**	**Orthogonality test**
		***T* = 100**		
$\beta = 0$	0.227	0.510	0.222	0.506
$\beta = 0.3$	0.320	0.720	0.300	0.698
$\beta = 0.5$	0.362	0.868	0.381	0.833
$\beta = 0.7$	0.514	0.952	0.494	0.899
$\beta = 0.9$	0.731	0.885	0.642	0.735
		***T* = 200**		
$\beta = 0$	0.380	0.839	0.385	0.818
$\beta = 0.3$	0.432	0.955	0.422	0.944
$\beta = 0.5$	0.423	0.989	0.415	0.980
$\beta = 0.7$	0.593	0.996	0.568	0.989
$\beta = 0.9$	0.864	0.979	0.743	0.872
		***T* = 400**		
$\beta = 0$	0.693	0.990	0.702	0.989
$\beta = 0.3$	0.654	1.000	0.657	1.000
$\beta = 0.5$	0.485	1.000	0.486	1.000
$\beta = 0.7$	0.682	1.000	0.630	1.000
$\beta = 0.9$	0.964	1.000	0.878	0.980

Notes: Estimated power against adaptive expectations with $\lambda = 0.1$ of the 5% efficiency and orthogonality tests is given for both quantitative expectations and expectations quantified by means of the probability method. The true indifference intervals have the form: $\langle -0.3, 0.3\rangle$ and $\langle \pi_{t-1} - 0.3,\ \pi_{t-1} + 0.3\rangle$. Results are reported for five DGPs given by (6) with values of β of 0, 0.3, 0.5, 0.7 and 0.9 and sample sizes T of 100, 200 and 400 observations.

Table 3

Estimated power against CGLS adaptive learning with $\gamma = 0.025$ of the 5% efficiency and orthogonality tests for rational expectations

	Quantitative data		Quantified data	
	Efficiency test	**Orthogonality test**	**Efficiency test**	**Orthogonality test**
		***T* = 100**		
$\beta = 0$	0.011	0.012	0.017	0.014
$\beta = 0.3$	0.016	0.018	0.014	0.023
$\beta = 0.5$	0.016	0.023	0.019	0.022
$\beta = 0.7$	0.024	0.033	0.025	0.031
$\beta = 0.9$	0.033	0.043	0.030	0.038
		***T* = 200**		
$\beta = 0$	0.006	0.010	0.007	0.013
$\beta = 0.3$	0.014	0.016	0.013	0.017
$\beta = 0.5$	0.019	0.028	0.018	0.028
$\beta = 0.7$	0.035	0.039	0.028	0.038
$\beta = 0.9$	0.036	0.048	0.033	0.046
		***T* = 400**		
$\beta = 0$	0.007	0.012	0.007	0.013
$\beta = 0.3$	0.009	0.023	0.011	0.021
$\beta = 0.5$	0.018	0.041	0.019	0.039
$\beta = 0.7$	0.041	0.057	0.027	0.054
$\beta = 0.9$	0.050	0.047	0.037	0.048

Notes: Estimated power against CGLS adaptive learning with $\gamma = 0.025$ of the 5% efficiency and orthogonality tests is given for both quantitative expectations and expectations quantified by means of the probability method. The true indifference intervals have the form: $\langle -0.3, 0.3 \rangle$ and $\langle \pi_{t-1} - 0.3, \pi_{t-1} + 0.3 \rangle$. Results are reported for five DGPs given by (6) with values of β of 0, 0.3, 0.5, 0.7 and 0.9 and sample sizes T of 100, 200 and 400 observations.

Table 4

Estimated power against CGLS adaptive learning with $\gamma = 0.1$ of the 5% efficiency and orthogonality tests for rational expectations

	Quantitative data		**Quantified data**	
	Efficiency test	**Orthogonality test**	**Efficiency test**	**Orthogonality test**
		$\boldsymbol{T = 100}$		
$\beta = 0$	0.027	0.027	0.028	0.027
$\beta = 0.3$	0.045	0.050	0.047	0.053
$\beta = 0.5$	0.056	0.063	0.050	0.066
$\beta = 0.7$	0.051	0.054	0.047	0.056
$\beta = 0.9$	0.035	0.044	0.033	0.039
		$\boldsymbol{T = 200}$		
$\beta = 0$	0.097	0.076	0.098	0.075
$\beta = 0.3$	0.169	0.152	0.155	0.150
$\beta = 0.5$	0.201	0.210	0.189	0.202
$\beta = 0.7$	0.171	0.183	0.158	0.173
$\beta = 0.9$	0.068	0.066	0.059	0.066
		$\boldsymbol{T = 400}$		
$\beta = 0$	0.375	0.283	0.370	0.273
$\beta = 0.3$	0.572	0.501	0.549	0.474
$\beta = 0.5$	0.637	0.621	0.611	0.599
$\beta = 0.7$	0.543	0.540	0.489	0.503
$\beta = 0.9$	0.138	0.129	0.111	0.115

Notes: Estimated power against CGLS adaptive learning with $\gamma = 0.1$ of the 5% efficiency and orthogonality tests is given for both quantitative expectations and expectations quantified by means of the probability method. The true indifference intervals have the form: $\langle -0.3, 0.3 \rangle$ and $\langle \pi_{t-1} - 0.3, \pi_{t-1} + 0.3 \rangle$. Results are reported for five DGPs given by (6) with values of β of 0, 0.3, 0.5, 0.7 and 0.9 and sample sizes T of 100, 200 and 400 observations.