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LOCAL STRONG *d*-MONOTONICITY OF THE KALAI-SMORODINSKY AND NASH BARGAINING SOLUTION

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Local strong d-monotonicity of the Kalai-Smorodinsky and Nash bargaining solution

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Abstract

In this paper we investigate how the Kalai-Smorodinsky and Nash bargaining solution respond to a change in the disagreement point d. We call a bargaining solution locally strong d-monotonic at the disagreement point d if an infinitesimal increase of d_i, while for each $j \neq i$, d_j remains constant, then agent i is the only one who's payoff increases. We present sufficient conditions for the Pareto frontier under which the Kalai-Smorodinsky and Nash bargaining solution satisfy this property. It turns out that in general the local strong d-monotonicity property of the Kalai-Smorodinsky solution is a stronger requirement than that of the Nash bargaining solution.

1 Introduction

Thomson investigated in [8] how three well-known bargaining solutions, NB, the Nash bargaining solution [5], KS, the Kalai-Smorodinsky solution [4] and the egalitarian solution [3] respond to certain changes in the disagreement point d for a fixed feasible set. He shows for a general class of N-person bargaining problems that all three solutions satisfy the d-monotonicity property. This property states that, given some agent i, if d_i increases while d_j remains constant for all $j \neq i$ then agent i's payoff increases (or at least not decreases). The stronger requirement, that not only agent i's payoff does not decrease but also the payoffs of none of the other agents increases is called strong d-monotonicity. For this general class of bargaining problems Thomson shows that strong d-monotonicity only holds for the Egalitarian solution. Both, for the NB

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and KS-solution, a counterexample is given.
This notion of strong d-monotonicity is a global property in the sense that this property should hold for every positive increment of d_i at every threatpoint d.
In fact it is also interesting to see under which conditions for a fixed d this property holds locally. That is, to see what the gains/losses will be for the other players if one (arbitrarily chosen) player unilaterally changes his threatpoint something. If this player is the only one who gains from such a small (positive) deviation we call the bargaining solution local strong d-monotonic at the threatpoint d. Given the threatpoint d and the corresponding bargaining point, this notion tells us something about the stability of the realized bargaining point. This, in the following sense. If the bargaining point is local strong d-monotonic at d then whenever one player likes to deviate from his threatpoint unilaterally, this action will be disapproved by all other players. So, the player who likes to change his threatpoint has to have rather good reasons before he will suggest to reopen the bargaining process. This, in contrast to the case that such a change in the threatpoint is benefitial for some other player(s) too. In that case it is rational for that (those) other player(s), at least, to be not against such a change in the threatpoint. So, a less number of players will be against a reopening of the bargaining process in such a case. In this sense, the threshold to reopen the bargaining process will be lower, and the bargaining point is called less stable. Note that one may expect that coalitions will be formed under these conditions. Thomson argues in [8], under such conditions the eventual acceptance of the compromise value will be less likely. Information on local d-monotonicity at the disagreement point in general will tell us, therefore, something about the realizability of the bargaining point.
In this paper we investigate those bargaining problems where the Pareto frontier can be described by a smooth strictly concave function. This class of problems (for larger classes of bargaining problems, see e.g., [6] or [9]) is particular popular in applied economic sciences (see e.g. the literature on policy coordination [2], [7], [10], [1]). For this class of problems we show that in case the Pareto frontier is rather "flat" then both the Kalai-Smorodinsky (KS-solution) and Nash bargaining (NB-solution) solution are local strong d-monotonic at every threatpoint, which implies that for those frontiers both solutions are even strong d-monotonic. Under those conditions there are therefore no reasons to expect a priori difficulties whether these solution can be realized, whatever the threatpoint will be. However, in case the frontier is less "flat", e.g. if it is described by a Cobb-Douglas function, then the local strong d-monotonicity property does not hold any longer at every threatpoint for the KS- solution. This, in contrast to the NB-solution which, as we will see, still (under some mild conditions) has this property. So, from this point of view one might cautiously conclude that generally the realizibality of the Nash-bargaining solution will be enhanced over the Kalai-Smorodinsky one.
In fact, we will first present general formulas on how both compromise values react on a change in the threatpoint before considering the specific local strong d-monotonicity case. These general formulas are also interesting on their own.
The paper is organized as follows. In section two we present some preliminaries. Then, in section three we consider the local strong d-monotonicity of the KS-solution for the

3-person case. In its first subsection we present a sufficient condition on the shape of the Pareto-frontier, under which this property holds. In the second subsection we treat two examples not satisfying the sufficient shape condition. In case the Pareto-frontier is described by a quadratic function it turns out that the KS-solution is local strong d-monotonic at every threatpoint, whereas for a Cobb-douglas function in general the local strong d-monotonicity property does not hold. In section four we consider the same issues, but now w.r.t. the NB-solution. The general N-person case is discussed in section five. The paper ends with some concluding remarks.

2 Preliminaries

Following Thomson, we define an N-person bargaining problem to be a pair (S, d), where $S \subset \mathbb{R}^N$ is called the feasible set, $\mathbb{R}^N$ the utility space and d the disagreement point. If the agents unanimously agree on a point $J \in S$, they obtain J. Otherwise, they obtain d. In this paper we are interested in the effect of changes in the disagreement point on the point of agreement for a fixed feasibility set. Therefore, we will be considering not just one single bargaining problem but a whole class of bargaining problems obtained by varying the threatpoint. So it is natural to consider the next notion of solution for a class of bargaining problems. Given a class of N-person bargaining problems, a solution is a function F associating with every (S, d) in this class the point of agreement $F(S, d) \in S$. Note that, since we will consider here a fixed feasibility set, the dependence of F on S can be omitted. F is called the Nash solution if for every fixed (S, d), $F(S, d)$ is assigned the point where the product $\Pi(J_i - d_i)$ is maximized for $J \in S$ with $J \geq d$ (here we use the vector inequality notation); F is called the Kalai-Smorodinsky solution if for every fixed (S, d), $F(S, d)$ is assigned the maximal point of S on the segment connecting the disagreement point d and the so-called ideal point $J^I(S, d)$, where for each i $J_i^I(S, d) := \max\{J_i | J \in S; J \geq d\}$.
Thomson considers two classes of bargaining problems: 1) $\overline{\Sigma}^N$, where the feasibility set $\bar{S}$ is assumed to be convex, compact and such that there exists a $J \in \bar{S}$ with $J > d$; and 2) Σ^N, which is a subclass of $\overline{\Sigma}^N$, the so-called class of comprehensive bargaining problems. This subclass is obtained by considering just those elements in $\bar{S}$ satisfying the additional property that whenever $J \in S$ and $d < \overline{J} < J$, then $\overline{J} \in S$.
We will consider in this paper a subclass Σ_P^N of Σ^N. We assume that the (fixed) feasibility set in this subclass Σ_P^N satisfies the additional requirement that the set of (weak) Pareto optimal solutions can be described by a smooth strictly concave function.
The property of local strong d-monotonicity with respect to the disagreement point d is formulated as follows:

Definition: If, for a fixed (S, d), $\frac{\partial F_j(S,d)}{\partial d_i} \leq 0$ for all $j \neq i$, and $\frac{\partial F_i(S,d)}{\partial d_i} \geq 0$, then the bargaining solution $F(S, d)$ is called ***local strong d-monotonic*** at d.

Note that the counterexamples given by Thomson can be approximated arbitrarily close by a bargaining problem from our class Σ_P^N. So, using continuity arguments, it is clear that the NB-solution and the KS-solution will not satisfy strong d-monotonicity for this particular class too if $N > 2$. On the other hand it is obvious that the local strong d-monotonicity requirement does hold for both outcomes for the two-player case, for an arbitrarily chosen threatpoint.

Another remark concerns the interpretation of the matrix $DF(S,d)$ with entries the partial derivatives $\frac{\partial F_j(S,d)}{\partial d_i}$. Assume that the set S is fixed, and that the compromise value $F(S,d)$ is a differentiable function of d. Then, neglecting higher order derivatives we have that in the neighbourhood of the threatpoint d_0, this compromise value can be approximated by

$$F(S,d) = F(S,d_0) + DF(S,d_0)(d - d_0).$$

3 The KS-solution: 3-person case

Let $J^I = (J_1^I J_2^I J_3^I)$ represent the ideal point, $J^{KS} = (J_1^{KS} J_2^{KS} J_3^{KS})$ the Kalai-Smorodinsky solution and P be the set of Pareto optimal solutions that can be represented by a twice differentiable, decreasing, strictly concave function. That is, to be more precisely, assume that there exists a twice differentiable function φ such that each pair $(J_1, J_2, J_3) \in P$, can be written as $J_3 = \varphi(J_1, J_2)$, where all partial derivatives of φ, denoted in the sequel by $\varphi_i^{'}$, are negative and the hessian of φ, $\varphi^{''}$, is negative definite. The KS-solution can now be derived from the equations:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} + \lambda \begin{pmatrix} d_1 - J_1^I \\ d_2 - J_2^I \\ d_3 - J_3^I \end{pmatrix} = \begin{pmatrix} J_1^{KS} \\ J_2^{KS} \\ \varphi(J_1^{KS}, J_2^{KS}) \end{pmatrix}, \tag{1}$$

where the ideal point $J^I = (J_1^I, J_2^I, J_3^I)$ is determined by:

$$\begin{aligned} d_3 &= \varphi(J_1^I, d_2), &\quad \text{or} \quad& -d_3 + \varphi(J_1^I, d_2) = 0, \\ d_3 &= \varphi(d_1, J_2^I), &\quad \text{or} \quad& -d_3 + \varphi(d_1, J_2^I) = 0, \\ J_3^I &= \varphi(d_1, d_2). && \end{aligned}$$

Since $\varphi_i^{'} < 0, i = 1,2$ it is easily verified, using the implicit function theorem, that these equations imply that J_1^I and J_2^I are implicitly given by a function of (d_1, d_2, d_3). Suppose that

$$\begin{pmatrix} J_1^I \\ J_2^I \end{pmatrix} = \begin{pmatrix} \tilde{f}_1(d_1, d_2, d_3) \\ \tilde{f}_2(d_1, d_2, d_3) \end{pmatrix} = \tilde{f}(d_1, d_2, d_3),$$

Then, according the implicit function theorem,

$$\frac{\partial(J_1^I, J_2^I)}{\partial(d_1, d_2, d_3)} = \frac{\partial \tilde{f}}{\partial(d_1, d_2, d_3)} = \begin{pmatrix} 0 & -\frac{\varphi'_2}{\varphi'_1}(J_1^I, d_2) & \frac{1}{\varphi'_1}(J_1^I, d_2) \\ -\frac{\varphi'_1}{\varphi'_2}(d_1, J_2^I) & 0 & \frac{1}{\varphi'_2}(d_1, J_2^I) \end{pmatrix} \tag{2}$$

From (1) follows now that:

$$d_3 + \lambda(d_3 - J_3^I) = \varphi(J_1^{KS}, J_2^{KS}) \qquad \text{or} \qquad \lambda = \frac{J_3^{KS} - d_3}{d_3 - J_3^I}.$$

Therefore, J_1^{KS}, J_2^{KS} are implicitly determined by:

$$\begin{cases} J_1^{KS} - d_1 - \frac{J_3^{KS} - d_3}{d_3 - J_3^I}(d_1 - J_1^I) = 0 \\ J_2^{KS} - d_2 - \frac{J_3^{KS} - d_3}{d_3 - J_3^I}(d_2 - J_2^I) = 0 \end{cases}$$

or, substituting the formulas for J_3^{KS}, J_1^I and J_2^I:

$$\begin{cases} g_1 := J_1^{KS} - d_1 + \frac{d_3 - \varphi(J_1^{KS}, J_2^{KS})}{d_3 - \varphi(d_1, d_2)}(d_1 - \tilde{f}_1(d_1, d_2, d_3)) = 0 \\ g_2 := J_2^{KS} - d_2 + \frac{d_3 - \varphi(J_1^{KS}, J_2^{KS})}{d_3 - \varphi(d_1, d_2)}(d_2 - \tilde{f}_2(d_1, d_2, d_3)) = 0 \end{cases} \tag{3}$$

To show that $g := (g_1, g_2)$ determines implicitly (J_1^{KS}, J_2^{KS}) as a function of (d_1, d_2, d_3), consider

$$J_g := \left(\frac{\partial(g_1, g_2)}{\partial(J_1^{KS}, J_2^{KS})}, \frac{\partial(g_1, g_2)}{\partial(d_1, d_2, d_3)} \right).$$

Using the following notation,

$$I_i := \frac{d_i - J_i^I}{d_3 - J_3^I}, \qquad K_i := \frac{d_i - J_i^{KS}}{d_3 - J_3^I}$$

for $i = 1, 2, 3$, we get from (3) that

$$\frac{\partial g}{\partial(J_1^{KS}, J_2^{KS})} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \varphi'(J_1^{KS}, J_2^{KS}) \tag{4}$$

where $\varphi' := (\varphi'_1 \;\; \varphi'_2)$ is the Jacobian of φ, and $\frac{\partial g}{\partial(d_1, d_2, d_3)} =$

$$\begin{pmatrix} -1 + K_3(1 + I_1\varphi'_1(d_1, d_2)) & K_3(\frac{\varphi'_2}{\varphi'_1}(J_1^I, d_2) + I_1\varphi'_2(d_1, d_2)) & I_1 - K_3(I_1 + \frac{1}{\varphi'_1(J_1^I, d_2)}) \\ K_3(\frac{\varphi'_1}{\varphi'_2}(d_1, J_2^I) + I_2\varphi'_1(d_1, d_2)) & -1 + K_3(1 + I_2\varphi'_2(d_1, d_2)) & I_2 - K_3(I_2 + \frac{1}{\varphi'_2(d_1, J_2^I)}) \end{pmatrix},$$

which can be rewritten as

$$E + K_3 V, \tag{5}$$

where

$$E = \begin{pmatrix} -1 & 0 & I_1 \\ 0 & -1 & I_2 \end{pmatrix}$$

and

$$V = \begin{pmatrix} 1 + I_1\varphi'_1(d_1, d_2) & \frac{\varphi'_2}{\varphi'_1}(J_1^I, d_2) + I_1\varphi'_2(d_1, d_2) & -(I_1 + \frac{1}{\varphi'_1(J_1^I, d_2)}) \\ \frac{\varphi'_1}{\varphi'_2}(d_1, J_2^I) + I_2\varphi'_1(d_1, d_2) & 1 + I_2\varphi'_2(d_1, d_2) & -(I_2 + \frac{1}{\varphi'_2(d_1, J_2^I)}) \end{pmatrix}.$$

Now, since (4) is always non-singular, we can use the implicit function theorem to conclude that

$$\frac{\partial(J_1^{KS}, J_2^{KS})}{\partial(d_1, d_2, d_3)} = -\{\frac{\partial g}{\partial(J_1^{KS}, J_2^{KS})}\}^{-1}\{\frac{\partial g}{\partial(d_1, d_2, d_3)}\} \tag{6}$$

where the inverse of the matrix in (4) is:

$$\{\frac{\partial g}{\partial(J_1^{KS}, J_2^{KS})}\}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{\det}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}\varphi'(J_1^{KS}, J_2^{KS})$$

and det, the determinant of $\frac{\partial g}{\partial(J_1^{KS}, J_2^{KS})}$, equals $1 - \{I_1\varphi'_1 + I_2\varphi'_2\}(J_1^{KS}, J_2^{KS})$.
To complete the picture of $\frac{\partial J_i^{KS}}{\partial d_j}$ we still have to consider $\frac{\partial J_3^{KS}}{\partial d_j}$. Since $J_3^{KS} = \varphi(J_1^{KS}, J_2^{KS})$, we have that

$$\frac{\partial J_3^{KS}}{\partial d_j} = \begin{pmatrix} \varphi'_1(J_1^{KS}, J_2^{KS}) & \varphi'_2(J_1^{KS}, J_2^{KS}) \end{pmatrix}\begin{pmatrix} \frac{\partial J_1^{KS}}{\partial d_j} \\ \frac{\partial J_2^{KS}}{\partial d_j} \end{pmatrix}.$$

Now, let $L := \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \varphi'_1 & \varphi'_2 \end{pmatrix}\{\frac{\partial g}{\partial(J_1^{KS}, J_2^{KS})}\}^{-1}$. Using (6) it is then easily verified that

$$\frac{\partial(J_1^{KS}, J_2^{KS}, J_3^{KS})}{\partial(d_1, d_2, d_3)} = -L\{E + K_3V\}. \tag{7}$$

Moreover, elementary calculation shows that $L = \frac{1}{\det}\begin{pmatrix} 1 - I_2\varphi'_2 & I_1\varphi'_2 \\ I_2\varphi'_1 & 1 - I_1\varphi'_1 \\ \varphi'_1 & \varphi'_2 \end{pmatrix}$, where all partial derivatives φ'_i are taken at (J_1^{KS}, J_2^{KS}). This yields our first conclusion:

<u>Theorem 1:</u>
Under the stated assumptions on the Pareto frontier, the KS-solution is local strong d-monotonic if and only if matrix $-L\{E + K_3V\}$ has the following sign-matrix: $\begin{pmatrix} + & - & - \\ - & + & - \\ - & - & + \end{pmatrix}$.

Note that since matrix L is not full rank, the kernel of this matrix is always nontrivial. We will next show that the diagonal entries of the above matrix have always the appropriate sign (i.e. are always positive), this, irrespective of the choice of the threatpoint d. To show this, we reconsider the factorization of $\frac{\partial g}{\partial(d_1,d_2,d_3)}$ in (5). The following factorization is also possible

$$\frac{\partial g}{\partial(d_1,d_2,d_3)} = K_3E_2 + V_2,$$

where

$$E_2 = \begin{pmatrix} I_1\varphi'_1(d_1,d_2) & I_1\varphi'_2(d_1,d_2) & -\frac{1}{\varphi'_1(J_1^I,d_2)} \\ I_2\varphi'_1(d_1,d_2) & I_2\varphi'_2(d_1,d_2) & -\frac{1}{\varphi'_2(d_1,J_2^I)} \end{pmatrix}$$

and

$$V_2 = \begin{pmatrix} -1+K_3 & K_3\frac{\varphi'_2}{\varphi'_1}(J_1^I,d_2) & I_1(1-K_3) \\ K_3\frac{\varphi'_1}{\varphi'_2}(d_1,J_2^I) & -1+K_3 & I_2(1-K_3) \end{pmatrix}$$

Following the previous analysis we then have that $\frac{\partial(J_1^{KS},J_2^{KS},J_3^{KS})}{\partial(d_1,d_2,d_3)} = -L\{K_3E_2 + V_2\}$. This can be rewritten as

$$-\frac{K_3}{\det}\begin{pmatrix} I_1\varphi'_1(d_1,d_2) & I_1\varphi'_2(d_1,d_2) & -(1-I_2)a_1 - I_1a_2 \\ I_2\varphi'_1(d_1,d_2) & I_2\varphi'_2(d_1,d_2) & -I_2a_1-(1-I_1)a_2 \\ \varphi'_1(d_1,d_2)(1-\det) & \varphi'_2(d_1,d_2)(1-\det) & -a_1-a_2 \end{pmatrix} - LV_2, \quad (8)$$

where we used for simplicity of notation,

$$a_1 := \frac{\varphi'_1(J_1^{KS},J_2^{KS})}{\varphi'_1(J_1^I,d_2)} \qquad a_2 := \frac{\varphi'_2(J_1^{KS},J_2^{KS})}{\varphi'_2(d_1,J_2^I)}.$$

We will now consider the sign-matrix of the different matrices that appear in (8). Thereto, we first note that $\frac{1}{I_1}$ equals the slope of the line connecting the points (J_1^I,d_2,d_3) and (d_1,d_2,J_3^I) (a similar remark holds w.r.t. $\frac{1}{I_2}$). So, the following inequalities hold: $\frac{-1}{\varphi'_1(J_1^I,d_2)} \leq I_1 \leq \frac{-1}{\varphi'_1(d_1,d_2)}$. From this immediately the next inequalities result: $1+I_i\varphi'_i(d_1,d_2) \geq 0$, $I_1 + \frac{1}{\varphi'_1(J_1^I,d_2)} \geq 0$ and $I_2 + \frac{1}{\varphi'_2(d_1,J_2^I)} \geq 0$. Furthermore, we have that K_3 satisfies $0 < K_3 \leq 1$ and $I_i > 0$. Using this, we see that in (8) the sign-structure is as follows:

$$-\begin{pmatrix} - & - & ? \\ - & - & ? \\ + & + & - \end{pmatrix} - \begin{pmatrix} + & - \\ - & + \\ - & - \end{pmatrix}\begin{pmatrix} - & + & + \\ + & - & + \end{pmatrix}.$$

By elaborating this sign scheme, we see that the sign of the diagonal entries of matrix $\frac{\partial(J_1^{KS},J_2^{KS},J_3^{KS})}{\partial(d_1,d_2,d_3)}$ are always positive (independent of the choice of the threatpoint d). This result is in line with the result of Thomson, i.e.,

Corollary 2:
The KS-solution satisfies d-monotonicity.

Now, reconsider the representation (7) for the partial derivatives of J^{KS} w.r.t. d_i. It is easily verified that the sign scheme for matrix L is $\begin{pmatrix} + & - \\ - & + \\ - & - \end{pmatrix}$. Furthermore, since

$\phi_i' \leq 0$ and $0 < K_3 \leq 1$, the sign scheme of $E + K_3V$ is $\begin{pmatrix} - & ?_1 & + \\ ?_2 & - & + \end{pmatrix}$. Here $?_1$ denotes the sign of $\frac{\varphi'_2}{\varphi'_1}(J_1^I, d_2) + I_1\varphi'_2(d_1, d_2)$, and $?_2$ the sign of $\frac{\varphi'_1}{\varphi'_2}(d_1, J_2^I) + I_2\varphi'_1(d_1, d_2)$, which are both indetermined. Now, consider $\frac{\partial J_1^{KS}}{\partial d_3}$. From (5) and (7) we have that this derivative equals $\frac{-1}{\det}\left(\begin{pmatrix} 1 & 0 \end{pmatrix} + \varphi'_2\begin{pmatrix} -I_2 & I_1 \end{pmatrix}\right)\begin{pmatrix} \frac{\partial g_1}{\partial d_3} \\ \frac{\partial g_2}{\partial d_3} \end{pmatrix}$. Elementary calculation shows that this derivative can be rewritten as $\frac{-1}{\det}\{\frac{\partial g_1}{\partial d_3} + K_3\varphi'_2(J_1^{KS}, J_2^{KS})h\}$, where $h := \frac{-I_2}{\varphi'_1(J_1^I, d_2)} + \frac{I_1}{\varphi'_2(d_1, J_2^I)}$. Similarly it can be shown that $\frac{\partial J_2^{KS}}{\partial d_3} = \frac{-1}{\det}\{\frac{\partial g_2}{\partial d_3} - K_3\varphi'_1(J_1^{KS}, J_2^{KS})h\}$. Using this and the stated result from corollary 2, we have that (see (7))

$$\operatorname{signum} \frac{\partial(J_1^{KS}, J_2^{KS}, J_3^{KS})}{\partial(d_1, d_2, d_3)} = \begin{pmatrix} + & --?_1 & -+?_3 \\ --?_2 & + & --?_3 \\ -+?_2 & -+?_1 & + \end{pmatrix}, \tag{9}$$

where $?_3$ denotes the sign of h.
From this sign scheme we observe that if all entries denoted with an "?" are zero, then the KS-solution will be local strong d-monotonic, whereas if this is not the case it becomes dubious whether this monotonicity property will hold. In any case it is clear that to investigate whether the KS-solution will be local strong d-monotonic we only have to verify the sign of three partial derivatives. This, since if e.g. the sign of $?_1$ is negative, it is clear from the above sign matrix that the sign of $\frac{\partial J_3^{KS}}{\partial d_2}$ will be negative too. So, it suffices in that case to verify in the second column of this sign matrix only the sign of the first entry $\frac{\partial J_1^{KS}}{\partial d_2}$. We like to emphasize the following result:

Theorem 3:
If the next equalities hold,
i) $\frac{\varphi'_2}{\varphi'_1}(J_1^I, d_2) + I_1\varphi'_2(d_1, d_2) = 0$,
ii) $\frac{\varphi'_1}{\varphi'_2}(d_1, J_2^I) + I_2\varphi'_1(d_1, d_2) = 0$,
iii) $\frac{-I_2}{\varphi'_1(J_1^I, d_2)} + \frac{I_1}{\varphi'_2(d_1, J_2^I)} = 0$,
then the KS-solution is local strong d-monotonic.

Of course the strength of this theorem lies in the facts that, first, the above conclusion continues to hold in case these equalities are approximately satisfied and, second, the presented conditions are independent of the location of the KS-point.
As a special case, we consider the case that φ represents a plane. From our previous analysis then immediately the following equalities follow: $\frac{-1}{\varphi'_1(J_1^I, d_2)} = I_1 = \frac{-1}{\varphi'_1(d_1, d_2)}$. As a result we have that all equalities in the above theorem are satisfied. A more close analysis in fact shows that matrix V in (7) equals the zero matrix. Substitution of this into, and next spelling out of, (7) gives then that

$$\frac{\partial(J_1^{KS}, J_2^{KS}, J_3^{KS})}{\partial(d_1, d_2, d_3)} = \frac{1}{\det}\begin{pmatrix} 1 - I_2\varphi'_2 & I_1\varphi'_2 & -I_1 \\ I_2\varphi'_1 & 1 - I_1\varphi'_1 & -I_2 \\ \varphi'_1 & \varphi'_2 & -(I_1\varphi'_1 + I_2\varphi'_2) \end{pmatrix} \tag{10}$$

where all partial derivatives are taken again at (J_1^{KS}, J_2^{KS}). From this it is easily verified that the sign-matrix of this matrix does not depend on the choice of the threatpoint d. So, we conclude that

Corollary 4:
The KS-solution is strong d-monotonic in case the Pareto-frontier is described by a plane.

Another example of a situation in which the conditions of theorem 3 are satisfied is if the Pareto frontier is represented by the quadratic function $\varphi(x,y) = r - \frac{1}{2}ax^2 - \frac{1}{2}by^2$, where r, a, b are some positive constants. With $d = (0,0,0)$ it is easily verified that the first two conditions of theorem 3 are satisfied. To verify the third condition, we note that $I_1 = \frac{J_1^I}{J_3^I}$, and $I_2 = \frac{J_2^I}{J_3^I}$. Using the facts that $J_1^I = \sqrt{\frac{r}{a}}$ and $J_2^I = \sqrt{\frac{r}{b}}$, direct substitution of this into the third condition shows that this one is also satisfied. So, the KS-solution is local strong d-monotonic at $d = (0,0,0)$ for this quadratic function.

3.1 Two examples

In this subsection we first elaborate the quadratic case by which we concluded the previous section in some more generality and show that the KS-solution is strong d-monotonic for that particular case.

Example 5:
Let φ be represented by the quadratic function $\varphi(x) = r + b^T x + \frac{1}{2}x^T Ax$, where b, x are two-dimensional vectors, A is a negative definite 2x2 diagonal matrix and all entries of b are negative.
Then, $\varphi'_1(x) = (b^T + x^T A)\begin{pmatrix}1\\0\end{pmatrix}$ and $\varphi'_2(x) = (b^T + x^T A)\begin{pmatrix}0\\1\end{pmatrix}$. Before we consider the different entries of (7) we first note that for this particular case $K_3 \le \frac{1}{2}$. Without going into mathematical details we note that this can be shown by substitution of the relation $J^{KS} = (1-K_3)d + K_3 J^I$ into the equation $J_3^{KS} = \varphi(J_1^{KS}, J_2^{KS})$. This gives a quadratic equation in K_3 from which, after some tedious calculation, the inequality can be verified.
Due to the facts that A is diagonal and $\frac{-1}{\varphi'_1(J_1^I, d_2)} \le I_1$ it is easily verified that $\frac{\varphi'_2}{\varphi'_1}(J_1^I, d_2) + I_1\varphi'_2(d_1, d_2) \le 0$. So, (see (9)) $\frac{\partial J_3^{KS}}{\partial d_1} \le 0$. Next, consider $\frac{\partial J_2^{KS}}{\partial d_1}$. Using the notation $P := \left(1 - I_1\varphi'_1(J_1^{KS}, J_2^{KS})\right) K_3 \frac{\varphi'_1}{\varphi'_2}(d_1, J_2^I)$, we have from (7) that

$$\begin{aligned}\frac{\partial J_2^{KS}}{\partial d_1} &= \frac{-1}{\det}\{P + I_2(K_3 - 1)\varphi'_1(J_1^{KS}, J_2^{KS}) + I_2 K_3 \varphi'_1(d_1, d_2)\} \\ &= \frac{-1}{\det}\{P + I_2(K_3 - 1)(b^T + J^{KS^T} A)\begin{pmatrix}1\\0\end{pmatrix} + I_2 K_3 (b^T + d^T A)\begin{pmatrix}1\\0\end{pmatrix}\}\end{aligned}$$

$$
\begin{aligned}
&= \frac{-1}{\det}\{P + I_2(K_3-1)(b^T + (d + K_3(J^I - d))A)\begin{pmatrix}1\\0\end{pmatrix} + I_2K_3(b^T + d^TA)\begin{pmatrix}1\\0\end{pmatrix}\} \\
&= \frac{-1}{\det}\{P + I_2(2K_3-1)(b^T + d^TA)\begin{pmatrix}1\\0\end{pmatrix} - I_2K_3(1-K_3)(J^I-d)^TA\begin{pmatrix}1\\0\end{pmatrix}\} \\
&\leq 0.
\end{aligned}
$$

Using the same arguments it can be shown that also both $\frac{\partial J_1^{KS}}{\partial d_2}$ and $\frac{\partial J_3^{KS}}{\partial d_2}$ are negative. Finally, consider the partial derivatives $\frac{\partial J_1^{KS}}{\partial d_3}$. Under the assumption that $h := \frac{-I_2}{\varphi'_1(J_1^I, d_2)} + \frac{I_1}{\varphi'_2(d_1, J_2^I)} < 0$, we have from (9) that $\frac{\partial J_1^{KS}}{\partial d_3} \leq 0$. Furthermore, we can rewrite $\frac{\partial J_2^{KS}}{\partial d_3}$ as:

$$
\frac{\partial J_2^{KS}}{\partial d_3} = \frac{-1}{\det}\{I_2(1 - K_3 - K_3\frac{\varphi'_1(J_1^{KS}, J_2^{KS})}{\varphi'_1(J_1^I, d_2)}) - \frac{K_3}{\varphi'_2(d_1, J_2^I)} + K_3I_1\frac{\varphi'_1(J_1^{KS}, J_2^{KS})}{\varphi'_2(d_1, J_2^I)}\}.
$$

Elementary calculation shows that $\frac{\varphi'_1(J_1^{KS}, J_2^{KS})}{\varphi'_1(J_1^I, d_2)} \leq 1$, which immediately implies that $\frac{\partial J_2^{KS}}{\partial d_3} \leq 0$. Obviously, the same arguments can be used to show that the partial derivatives have the appropriate sign in case $h > 0$.
So, we conclude that under these conditions, the KS-solution is strong d-monotonic.

The previous example illustrated that in general the conditions presented in theorem 3 are not sufficient. Next we will consider another type of Pareto function which often occurs in economic literature and which in general does not satisfy the local strong d-monotonicity property.

<u>Example 6:</u>
Assume that the Pareto frontier is described by the Cobb-douglas function $\varphi(x,y) = \sqrt{(a-x)(b-y)}$, where a, b are some positive constants. Moreover, assume that the threatpoint d is the origin $(0,0,0)$. (Note that in principle this choice is not correct if we want to use our theory since we get problems when using the implicit theorem. However, it is not difficult to show that by choosing $d = (0,0,d_3)$ for a very small d_3, we get approximately the formulas stated below. Therefore, for the sake of easy exposition we take d to be the origin).
It is now easily verified that, $J_1^I = a$, $J_2^I = b$, $J_3^I = \sqrt{ab}$ and $J_i^{KS} = K_3J_i^I$, $i = 1,2,3$. So, $I_1 = \sqrt{\frac{a}{b}}$ and $I_2 = \sqrt{\frac{b}{a}}$. Furthermore, $\varphi'_1(x,y) = \frac{-1}{2}\frac{\varphi(x,y)}{a-x}$ and $\varphi'_2(x,y) = \frac{-1}{2}\frac{\varphi(x,y)}{b-y}$. In particular we obtain that $\varphi'_1(J_1^{KS}, J_2^{KS}) = \frac{-1}{2}\sqrt{\frac{b}{a}}$ and $\varphi'_2(J_1^{KS}, J_2^{KS}) = \frac{-1}{2}\sqrt{\frac{a}{b}}$.
Now, consider $\frac{\partial J_1^{KS}}{\partial d_2}$. From (7) we have that

$$
\begin{aligned}
\frac{\partial J_2^{KS}}{\partial d_1} &= \frac{-1}{\det}\{(1 - I_2\varphi'_2)K_3\left(\frac{\varphi'_2}{\varphi'_1}(J_1^I, d_2) + I_1\varphi'_2(d_1,d_2)\right) + I_1\varphi'_1(-1 + K_3(1 + I_2\varphi'_2(d_1,d_2)))\} \\
&= \frac{-1}{\det}\{(1 - \sqrt{\frac{b}{a}}\frac{-1}{2}\sqrt{\frac{a}{b}})K_3\left(0 + \sqrt{\frac{a}{b}}\frac{-1}{2}\sqrt{\frac{a}{b}}\right) + \sqrt{\frac{a}{b}}\frac{-1}{2}\sqrt{\frac{b}{a}}\left(-1 + K_3(1 + \sqrt{\frac{b}{a}}\frac{-1}{2}\sqrt{\frac{a}{b}})\right)\} \\
&= \frac{-1}{\det}\{\frac{-3}{4}\frac{a}{b}K_3 + \frac{-1}{2}(-1 + \frac{1}{2}K_3)\}
\end{aligned}
$$

Now, since $K_3 \geq \frac{1}{3}$ (see e.g. Thomson (1987)), it is clear that there exist choices of a and b such that φ is not local strong d-monotonic at the origin.

4 The Nash bargaining solution: 3-person case

Since, by assumption, $J_3 = \varphi(J_1, J_2)$ the Nash bargaining solution $J^{NB} := (J_1^{NB}, J_2^{NB}, J_3^{NB})$ is determined by the argument that solves the maximization problem

$$\max_{J_1, J_2}(J_1 - d_1)(J_2 - d_2)(\varphi(J_1, J_2) - d_3)$$

This maximization problem has, according to Nash, exactly one solution. Obviously, this solution lies not on the edge of P, i.e., it is an internal element of P. Thus, the first order conditions yield that the Nash bargaining solution is uniquely determined by:

$$\varphi(J_1^{NB}, J_2^{NB}) - d_3 + \varphi'_1(J_1^{NB} - d_1) = 0 \tag{11}$$

$$\varphi(J_1^{NB}, J_2^{NB}) - d_3 + \varphi'_2(J_2^{NB} - d_2) = 0. \tag{12}$$

Furthermore, using the shorthand notation $p := \{\varphi'_2 + \varphi''_{12}(J_1^{NB} - d_1)\}(J_2^{NB} - d_2) + \varphi(J_1^{NB}, J_2^{NB}) - d_3 + \varphi'_1(J_1^{NB} - d_1)$, the second order conditions give that matrix:

$$\begin{pmatrix} (2\varphi'_1 + \varphi''_{11}(J_1^{NB} - d_1))(J_2^{NB} - d_2) & p \\ p & (2\varphi'_2 + \varphi''_{22}(J_2^{NB} - d_2))(J_1^{NB} - d_1) \end{pmatrix}$$

is semi-negative definite.
Using the first order conditions, we can rewrite this matrix as

$$H := \begin{pmatrix} (2\varphi'_1 + \varphi''_{11}(J_1^{NB} - d_1))(J_2^{NB} - d_2) & (\varphi'_2 + \varphi''_{12}(J_1^{NB} - d_1))(J_2^{NB} - d_2) \\ (\varphi'_1 + \varphi''_{21}(J_2^{NB} - d_2))(J_1^{NB} - d_1) & (2\varphi'_2 + \varphi''_{22}(J_2^{NB} - d_2))(J_1^{NB} - d_1) \end{pmatrix}$$

Next, we follow the same procedure as in the proof of the Kalai Smorodinsky solution. That is, consider

$$\begin{cases} g_1 = \varphi(J_1^{NB}, J_2^{NB}) - d_3 + \varphi'_1(J_1^{NB} - d_1) = 0 \\ g_2 = \varphi(J_1^{NB}, J_2^{NB}) - d_3 + \varphi'_2(J_2^{NB} - d_2) = 0 \end{cases}$$

Let $g := (g_1\ g_2)^T$. Then,

$$\frac{\partial g}{\partial(J_1^{NB}, J_2^{NB})} = \begin{pmatrix} 2\varphi'_1 + \varphi''_{11}(J_1^{NB} - d_1) & \varphi'_2 + \varphi''_{12}(J_1^{NB} - d_1) \\ \varphi'_1 + \varphi''_{21}(J_2^{NB} - d_2) & 2\varphi'_2 + \varphi''_{22}(J_2^{NB} - d_2) \end{pmatrix}. \tag{13}$$

Now, suppose that H is invertible (which is of course generically true). Then, from the negative-definiteness of H, we have that the determinant of H is positive. Straightforward calculations show then that also the determinant of the above matrix in (13), which we will denote by det, is positive. So, we can use the implicit function theorem to conclude that g implicitly defines (J_1^{NB}, J_2^{NB}) as a function of (d_1, d_2, d_3) and that the derivative of this function is given by

$$\frac{\partial(J_1^{NB}, J_2^{NB})}{\partial(d_1, d_2, d_3)} = -\{\frac{\partial g}{\partial(J_1^{NB}, J_2^{NB})}\}^{-1}\{\frac{\partial g}{\partial(d_1, d_2, d_3)}\}. \tag{14}$$

It is easily verified that

$$\frac{\partial g}{\partial(d_1, d_2, d_3)} = \begin{pmatrix} -\varphi'_1 & 0 & -1 \\ 0 & -\varphi'_2 & -1 \end{pmatrix}, \tag{15}$$

and, using (13),

$$\{\frac{\partial g}{\partial(J_1^{NB}, J_2^{NB})}\}^{-1} = \frac{1}{det}\begin{pmatrix} 2\varphi'_2 + \varphi''_{22}(J_2^{NB} - d_2) & -\varphi'_2 - \varphi''_{12}(J_1^{NB} - d_1) \\ -\varphi'_1 - \varphi''_{21}(J_2^{NB} - d_2) & 2\varphi'_1 + \varphi''_{11}(J_1^{NB} - d_1) \end{pmatrix}. \tag{16}$$

To complete the picture of $\frac{\partial J_i^{NB}}{\partial d_j}$ we still have to consider $\frac{\partial J_3^{NB}}{\partial d_j}$. Like for the KS-solution this is achieved by noting that $J_3^{NB} = \varphi(J_1^{NB}, J_2^{NB})$. Consequently,

$$\frac{\partial J_3^{NB}}{\partial d_j} = \begin{pmatrix} \varphi'_1(J_1^{NB}, J_2^{NB}) & \varphi'_2(J_1^{NB}, J_2^{NB}) \end{pmatrix} \begin{pmatrix} \frac{\partial J_1^{NB}}{\partial d_j} \\ \frac{\partial J_2^{NB}}{\partial d_j} \end{pmatrix}.$$

Now, similarly as for the KS-solution introduce $L := \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \varphi'_1 & \varphi'_2 \end{pmatrix}$. Then, it is easily verified that

$$\frac{\partial(J_1^{NB}, J_2^{NB}, J_3^{NB})}{\partial(d_1, d_2, d_3)} = -L\{\frac{\partial g}{\partial(J_1^{NB}, J_2^{NB})}\}^{-1}\{\frac{\partial g}{\partial(d_1, d_2, d_3)}\}, \tag{17}$$

where the entries can be calculated from (15) and (16), respectively, and all partial derivatives are taken at the NB-point.
So, we have

Theorem 7:
Under the stated assumptions on the Pareto frontier and the assumption that matrix H is negative definite, the NB-solution is local strong d-monotonic if and only if the above mentioned matrix (17) has the sign-matrix:

$$\begin{pmatrix} + & - & - \\ - & + & - \\ - & - & + \end{pmatrix}.$$

For the same reason as in the KS-case, the above matrix is singular. As already mentioned in section 2 this observation is important in case one likes to construct a consistent

path of threatpoints leading to the compromise value.
In particular note that, independent of the location of the threatpoint, always $\frac{\partial J_i^{NB}}{\partial d_i} > 0$, $i = 1,2$. By choosing a different parametrization of φ it is easily verified that also $\frac{\partial J_3^{NB}}{\partial d_3} > 0$, which is in line with the d-monotonicity result of Thomson [8]

Corollary 8:
The NB-solution satisfies d-monotonicity.

From the first order conditions (11,12) we have that $\varphi'_1(J_1^{NB} - d_1) = \varphi'_2(J_2^{NB} - d_2)$. Using this, simple calculations show that

Theorem 7 (continued):
The NB-solution is local strong d-monotonic if and only if the next three inequalities are satisfied at the NB-point:
i) $-\varphi'_1 - \varphi''_{21}(J_2^{NB} - d_2) \geq 0$
ii) $\varphi'_1\varphi'_2 + (J_2^{NB} - d_2)(\varphi'_1\varphi''_{22} - \varphi'_2\varphi''_{21}) \geq 0$
iii) $\varphi'_1\varphi'_2 + (J_1^{NB} - d_1)(\varphi'_2\varphi''_{11} - \varphi'_1\varphi''_{21}) \geq 0$

Note that always at least one of the above conditions is satisfied. Furthermore, we immediately deduce from this theorem that

Theorem 9:
The NB-solution is strong d-monotonic if the following three conditions are satisfied:
i) $\varphi''_{21} \leq 0$
ii) $\varphi'_1\varphi''_{22} - \varphi'_2\varphi''_{21} \geq 0$
iii) $\varphi'_2\varphi''_{11} - \varphi'_1\varphi''_{21} \geq 0$

Note that the above conditions are, e.g., satisfied if $\varphi''_{21} = 0$. In particular this implies that in case φ is described by the quadratic surface as described in example 5, the NB-solution is strong d-monotonic. Furthermore it is easily verified that in case φ is described by the Cobb-Douglas function given in example 6, $\varphi''_{21} = \frac{1}{4}\frac{\varphi}{a-x}$. Obviously, $\varphi''_{21} > 0$, so the conditions ii) and iii) of theorem 9 are trivially satisfied. On the other hand, we have that

$$\begin{aligned}
-\varphi'_1 - \varphi''_{21}(J_2^{NB} - d_2) &= \frac{1}{2}\frac{\varphi}{a - J_1^{NB}} - \frac{1}{4}\frac{J_2^{NB} - d_2}{\varphi} \\
&= \frac{1}{2}\frac{1}{\varphi}\{b - J_2^{NB} - \frac{1}{2}(J_2^{NB} - d_2)\}.
\end{aligned}$$

From the first order condition (12) we deduce that

$$\frac{1}{2}(J_2^{NB} - d_2) = \frac{\varphi - d_3}{\varphi}(b - J_2^{NB})$$

Substitution of this into the righthandside of the above expression shows that $-\varphi'_1 - \varphi''_{21}(J_2^{NB} - d_2)$ is positive. Note that this is independent of the choice of the threatpoint

d. So, we conclude from theorem 7 that for this example the NB-solution is strong d-monotonic too. In fact a similar reasoning shows that this conclusion holds for a general Cobb-Douglas function $\varphi(x,y) = (a-x)^{\alpha}(b-y)^{1-\alpha}$.

5 The N-person case

The analysis of local strong d-monotonicity in the N-person case can be done along the lines of the 3-player case. First, write for every $J_1, ..., J_N \in P, J_N = \varphi(J_1, ..., J_{N-1})$. Next, we introduce the notation $d_{/N} := (d_1, .., d_{N-1})$, $J^I_{/N} := (J^I_1, .., J^I_{N-1})$, $J^{KS}_{/N} := (J^{KS}_1, .., J^{KS}_{N-1})$ and $d_{/N,i} := (d_1, .., d_{i-1}, J^I_i, d_{i+1}, ..d_{N-1})$. Along the lines of section three we first note that, since all partial derivatives of φ are smaller than zero, we can apply the implicit function theorem to conclude that $J_i, i = 1, ..N-1$, can be viewed as a function of d and that more in particular we have:

$$\frac{(\partial J_1, ..., J_{N-1})}{\partial(d_1, ..., d_N)} = \begin{pmatrix} 0 & -\frac{\varphi'_2}{\varphi'_1}(d_{/N,1}) & \cdots & -\frac{\varphi'_{N-1}}{\varphi'_1}(d_{/N,1}) & \frac{1}{\varphi'_1}(d_{/N,1}) \\ -\frac{\varphi'_1}{\varphi'_2}(d_{/N,2}) & 0 & \cdots & -\frac{\varphi'_{N-1}}{\varphi'_2}(d_{/N,2}) & \frac{1}{\varphi'_2}(d_{/N,2}) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{\varphi'_1}{\varphi'_{N-1}}(d_{/N,N}) & \cdots & \cdots & 0 & \frac{1}{\varphi'_2}(d_{/N,N}) \end{pmatrix}.$$

Since

$$\frac{\partial g}{\partial(J^{KS}_1, ..., J^{KS}_{N-1})} = I - \begin{pmatrix} I_1 \\ \vdots \\ I_{N-1} \end{pmatrix} \left(\varphi'_1(J^{KS}_{/N}), \cdots, \varphi'_{N-1}(J^{KS}_{/N})\right),$$

where I is the identity matrix, we immediately see that this matrix is invertible and that the inverse of this matrix equals:

$$\left\{\frac{\partial g}{\partial(J^{KS}_1, ..., J^{KS}_{N-1})}\right\}^{-1} = I + \frac{\begin{pmatrix} I_1 \\ \vdots \\ I_{N-1} \end{pmatrix} \left(\varphi'_1(J^{KS}_{/N}), \cdots, \varphi'_{N-1}(J^{KS}_{/N})\right),}{1 - (\varphi'_1(J^{KS}_{/N}), \cdots, \varphi'_{N-1}(J^{KS}_{/N})) \begin{pmatrix} I_1 \\ \vdots \\ I_{N-1} \end{pmatrix}}$$

From this, using the same arguments as in the 3-player case, it is then possible to calculate $\frac{\partial g}{\partial d}$ as $E + K_N V$ (see (5)) and next analyze $\frac{\partial J^{KS}_{/N}}{\partial d}$. Here matrix E and V are given by, respectively

$$E = \left(-I \;\middle|\; \begin{pmatrix} I_1 \\ \vdots \\ I_{N-1} \end{pmatrix} \right)$$

and

$$V = \left(I + \begin{pmatrix} I_1 \\ \vdots \\ I_{N-1} \end{pmatrix} \left(\varphi'_1(d_{/N}), \cdots, \varphi'_{N-1}(d_{/N})\right) \mid \begin{pmatrix} -I_1 \\ \vdots \\ -I_{N-1} \end{pmatrix} \right) - \frac{(\partial J_1, \ldots, J_{N-1})}{\partial(d_1, \ldots, d_N)}.$$

To analyze the general case for the NB-solution, we need a general expression for $\frac{\partial g}{\partial J_i^{NB}}$ (see (14)). It is easily verified that this derivative is given by the matrix

$$(I + \text{ones})\text{diag}(\varphi'_1, \ldots, \varphi'_{N-1}) + \text{diag}(J_1^{NB} - d_1, \ldots, J_{N-1}^{NB} - d_{N-1})\varphi'',$$

where 'ones' denotes the matrix which entries are all 1, and diag is the shorthand notation for diagonal matrix. Unfortunately, the inverse of this matrix can not be calculated in general as easily as in the KS-case, which complicates a more detailed analysis of this case.

6 Concluding Remarks

In this paper we introduced the notion of local strong d-monotonicity and studied how this property can be examined for the Kalai-Smorodinsky and Nash bargaining solution in detail. The analysis was mainly performed in a 3-player context. For the general N-player case we just indicated how these properties can be verified.
Though it is difficult to compare the conditions on the Pareto frontier under which the KS-solution and the NB-solution are local strong d-monotonic, it seems that in general this property will be more often satisfied by the NB-solution than by the KS-solution. In particular we saw that in case the Pareto-frontier is represented by a quadratic surface, both the NB and KS solution satisfy the local d-monotonicity property, whereas this is not the case if the Pareto-frontier is represented by a Cobb-Douglas function. In the latter case the KS-solution does not satisfy this property, in general, anymore. So, a preliminary cautious conclusion may be that the NB solution in general seems to be better realizable than the KS solution.
The performed analysis requires knowledge of the derivative of the bargaining solution w.r.t. the threatpoint, which, in general, is singular.

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