Doctors' remuneration schemes and hospital competition in two-sided markets with common network externalities¹

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Abstract

This paper uses a two-sided market model of hospital competition to study the implications of different remunerations schemes on the physicians' side. The two-sided market approach is characterized by the concept of *common network externality* (CNE) introduced by Bardey et al. (2010). This type of externality occurs when occurs when both sides value, possibly with different intensities, the same network externality. We explicitly introduce effort exerted by doctors. By increasing the number of medical acts (which involves a costly effort) the doctor can increase the quality of service offered to patients (over and above the level implied by the CNE). We first consider pure salary, capitation or fee-for-service schemes. Then, we study schemes that mix fee-for-service with either salary or capitation payments. We show that salary schemes (either pure or in combination with fee-for-service) are more patient friendly than (pure or mixed) capitations schemes. This comparison is exactly reversed on the providers' side. Quite surprisingly, patients always loose when a fee-for-service scheme is introduced (pure of mixed). This is true even though the fee-for-service is the only way to induce the providers to exert effort and it holds whatever the patients' valuation of this effort. In other words, the increase in quality brought about by the fee-for-service is more than compensated by the increase in fees faced by patients.

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1 Introduction

Health care system in developed countries face a number of challenges. First, there is the achievement or consolidation of universal access. Second, there is a strong concern about the level of health care quality delivered to patients. Third, the increasing health care costs observed in most countries defy the sustainability of health care systems. Consequently, cost control has become a prominent issue. In this paper we deal with the last two issues which, as eloquently pointed out in Newhouse (1996), are often in conflict.

This potential trade-off between quality and cost control has been widely examined in the literature. On one hand, many papers compare the incentives generated by different remuneration schemes for providers. More precisely, it is studied how remuneration schemes affect providers' output, typically measured by health care quality and by the number of patient consultations (Devlin and Sarma, 2010). It is usually recognized that providers are encouraged to provide more services under a fee-for-service scheme, than under other remuneration schemes, such as capitation payment or salary¹. On the other hand, in order to lower the health care costs and to encourage health care quality, different countries have experimented ongoing reforms that stimulate competition (Brekke *et al.*, 2009). Several authors have tried to assess the impact of competition on health care quality. While theoretical studies usually predict a positive impact of competition on health care quality, the empirical literature leads to more mitigated results.²

From a methodological perspective, the first strand of literature analyzes remuneration schemes within a principal-agent framework, while the second uses imperfect competition models to examine the relationship between of competition and health care quality.³ Right now, there exists a gap between these two approaches and the interplay

¹See for instance Carlsen and Grytten (2010).

²See for instance Ma and Burgess (1993), Gravelle (1999) and Brekke, Nuscheler and Straume (2007). The empirical literature is surveyed by Gaynor (2006).

³Empirical studies of remuneration schemes usually take into account some selection effects. In

between competition on the one hand, and the incentive properties of remuneration schemes on the other hand does not appear to be well understood. In this paper, we use a two-sided market approach in order to bridge this gap, at least in part⁴. More precisely, we consider a duopoly situation between two for-profit hospitals where the competition is twofold. On one side, hospitals compete to attract patients; on the other side, to affiliate doctors. Patients are sensitive to price, to the number of consultations and to the health care quality delivered by hospitals. Doctors are also sensitive to the quality delivered to patients, in a general way, to the financial transfer paid by hospitals and more specifically to the number of consultation when hospitals use, at least partially, a fee-for-service scheme.⁵

Our two-sided approach is based on the concept of *common network externality* (CNE) introduced by Bardey *et al.* (2010). It is used to represent the health care quality delivered by a hospital and affect utility of both patients and doctors. It is a well established fact that the quality of health care delivered in hospitals depends on the doctors' "workload". This is documented, for instance, by Tarnow-Mordi *et al.* (2000) who use UK data to show that variations in mortality can be explained in part by excess workload in the intensive care unit. Accordingly, health care quality is frequently related to the doctor/provider ratio; see Mc Gillis Hall (2004). In other words, it increases when the number of health care professionals increases (for a given number of patients), but decreases when the number of patients increases (for a given number of providers). In this paper, we adopt a quite general expression for the quality provided by hospitals. We continue to assume that quality always increases in the number of doctors. However, no assumption about the patients' impact on the CNE

particular, they study how providers select the health care organization they work for according to the remuneration schemes they adopt.

⁴In fact, Ma and Riordan (2003) consider a competitive environment and analyze providers' remuneration schemes. The difference with our paper is that they do not have network externalities that reinforce the interplay between competition and incentives.

⁵In reality many hospitals are not for profit. It would be interesting to extend our analysis to mixed oligopoly (see Conclusion).

is needed. We do not rule out the possibility that it can also increases in the number of patients for low values because of a "learning-by-doing effect". For larger patients' numbers, on the other hand, the congestion effect can be expected to dominate (and we return to the negative relationship between number of patients and quality). In all cases, both sides benefit from a higher quality albeit for different reasons and possibly with different intensity. This is quite obvious on the patients' side, where one can expect a higher quality to translate into a improvement in patients' health state (or at the very least into a reduction in waiting lines for appointments, *etc...*). Physicians benefit from a higher quality through a reduction in their workload,⁶ or indirectly, through their altruism (or simply job satisfaction).⁷ This quality index can also be interpreted as a proxy that indicates how binding is the time constraint of the providers who work for an hospital. Consequently, it can be viewed as a determinant of the consultation duration received by patients.

We depart from the setting of Bardey *et al.* (2010) by introducing effort exerted by doctors. Specifically, this effort is measured by the number of *additional* consultations. The idea is that by increasing the number of medical acts (which involves a costly effort) the doctor can increase the quality of service offered to patients (over and above the level implied by the CNE). More precisely, as doctors must receive a given number of patients in a certain time interval, they can increase the time devoted to each patient by increasing the number of consultation in their extra-time.⁸

The general remuneration scheme we define includes a salary, a capitation payment and a fee-for-service component. On the patients' side, we concentrate on the case with a fixed fee. In a first step, we consider *pure* payment schemes. Not surprisingly, we find that the number of consultation is higher under a fee-for-service scheme than under

 $^{^{6}}$ See for instance Fergusson-Paré (2004) for the nursing workload. Griffin and Swan (2006) also find a strong relationship between nurses' workload and quality of health care.

 $^{^{7}}$ See, Liu and Ma (2010).

⁸Additional consultations play essentially the role of endogenous labor supply in our model.

other schemes. As a matter of fact, when providers are remunerated solely *via* a salary or a capitation payment, they provide the minimum level of effort. Under salary and capitation schemes, hospitals obtain the same profit at equilibrium. Patients pay a lower price and providers are less remunerated when providers receive capitation payments rather than salary schemes. In other words, a capitation payment scheme favor patients while providers are better off under a salary scheme. Next, in spite of the fact that patients value positively the number of consultations, our results suggest that patients are worse off when providers are paid *via* a fee-for-service rather than under a salary scheme. We show this analytically for the case when the number of acts provides only small benefits to patients. For larger levels of benefits, numerical simulations appear to corroborate this result. Surprisingly, we find that hospitals' profit may be higher when providers are remunerated *via* a fee-for-service scheme rather than under a capitation payment or salary.

Second, we consider payment schemes mixing fee-for-service with either salary or capitation payments. We show that in both cases, hospitals set the fee-for-service rate just equal to the patients' valuation of doctors' effort. Consequently, an efficient level of effort is achieved and total welfare is maximum. Nevertheless, the introduction of a fee-for-service along with either a wage or a capitation scheme always reduces patients' welfare, while doctors' welfare is enhanced. Exactly like in the pure remuneration case, the presence of a capitation element favors patients, while a salary term favors doctors.

The paper is organized as follows. Section 2 presents the set-up. Section 3 provides equilibrium conditions under general payment schemes. Pure salary and capitation schemes are considered in Section 4, while a pure fee-for-service system on the providers' side is studied in Section 5. Finally, mixed schemes are considered in Section 6.

2 The model

Consider two hospitals $j = \{1, 2\}$ located at both endpoints of the Hotelling's line. They compete for patients (group P of mass 1) on one side and for physicians or doctors (group D of mass m) on the other side.⁹ Both groups are uniformly distributed over an interval of length 1. The utilities of both groups exhibit quadratic transportation costs with parameters t_P and t_D respectively.¹⁰

Let n_j^i denote the *share* of type i = P, D individuals affiliated with hospital j = 1, 2, while N_j^i denotes the *number* of affiliates. With our normalizations, we have $N_j^P = n_j^P$ and $N_j^D = mn_j^D$. The two-sided market representation we adopt is based on the concept of "common network externality" with the following definition:

Definition 1 (Bardey et al., 2010) A common network externality, described by the function $q_j = \varphi(N_j^P, N_j^D)$, occurs when both sides value, possibly with different intensities, the same network externality.

This network externality is one of the determinants of the quality offered by a hospital. An important feature of this definition is that the functional form φ is the same on both sides (for instance q_j can represent simply the patient/doctor ratio). In other words, patients and doctors agree on the ranking of quality levels but they may differ in the intensity of preferences. We assume $\partial \varphi / \partial N_i^D > 0$ so that quality increases with the number of doctors. Even though doctor/patient ratio illustration is useful for the sake of interpretation, we consider a more general assumption concerning how the number of patients affect the quality. Indeed, we assume $\partial \varphi / \partial N_i^P < 0$ for sufficiently large levels of and N_i^P , but $\partial \varphi / \partial N_i^P > 0$ is not ruled out for small patient numbers. These assumptions on q_j allow us to capture that for sufficiently large patient numbers quality is

 $^{^{9}}$ We shall refer to members of group D indistinctively as (health care) providers, physicians or doctors.

¹⁰None of our results would change if transportation costs were linear rather than quadratic.

negatively related to the doctors' workload, while for small it may be positively related to patient numbers because of a learning-by-doing effect.¹¹

The utility of a patient, located at z, who patronizes hospital j and faces a total bill of K_j (a fixed fee) is given by

$$V = \overline{V} + Q_j - K_j - t_P \left(z - x_j\right)^2, \qquad (1)$$

where \overline{V} is a constant, while Q_j is a quality index.¹² This quality index consists of two elements. The first one is the common network externality which, as explained above, depends on the relative number of patients and providers affiliated to this hospital. We can think about this term as representing the intrinsic quality offered by hospital j. The second element depends on the physician's effort which, for simplicity, is measured by the number of medical acts consumed by each patient. Formally, we have

$$Q_j = \gamma q_j + \zeta e_j,$$

where γ and ζ represent preference intensities for intrinsic quality q_j and for the number of medical acts measured by the effort variable e_j . This "congestion effect" can be understood by taking into account the doctor's time constraint. In a given length of time, a doctor has to see a certain number of patients; this determines the duration of the visit, proxy of the quality q_j provided by hospital j. However, this congestion effect is reduced when the physician increases the number of visits, which in turn requires extra effort (longer working hours). This effect is captured by the variable e_j . Observe that, q_j and e_j are substitutes. In other words, a low level of quality q_j can be compensated by a higher number of medical acts. The value taken of ζ may differ according to the type of disease that is considered. For all non chronic diseases, we can expect ζ to be close to 0. Roughly speaking, for diseases which do not require a special attention from doctors,

¹¹The characterization of the various equilibria we study does not depend on the sign of $\partial \varphi / \partial N_i^P$.

¹²As most of patients may benefit from an health insurance plan, the price P_j can be interpreted as the patient's out-of-pocket payment to hospital j. Our analysis remains valid under this interpretation as long as there is not too much heterogeneity in health insurance coverage among patients.

patients prefer to benefit from a longer consultation than to have the same duration of consultation split into shorter meetings. On the contrary, for chronic diseases, patients may prefer to benefit from the same duration spread over several consultations.

The utility of a physician, located at y, and working for hospital j is given by

$$U = \overline{U} + \theta q_j + T_j - t_D (y - x_j)^2 - \Psi(E_j), \qquad (2)$$

where \overline{U} is a constant, θ is the preference for quality q_j , while T_j denotes the remuneration paid by hospital j to its providers. The term $\Psi(E_j)$ corresponds to the disutility of effort. For simplicity, we assume a quadratic disutility of effort throughout the paper so that $\Psi(E_j) = E_j^2/2$. A doctor's total effort E_j is given by his effort per patient e_j times the number of patients. The parameters \overline{V} and \overline{U} are assumed to be sufficiently large to ensure full coverage on both sides of the market.

The total remuneration of a physicians working for hospital j, treating $n_j^P/(mn_j^D)$ patients and realizing an effort e_j per patient (and a total effort $E_j = n_j^P/(mn_j^D)e_j$) is given by

$$T_j = w_j + d_j \frac{n_j^P}{m n_j^D} + \frac{c_j n_j^P}{m n_j^D} e_j,$$

In words, it may include a fixed salary $w_j \ge 0$, a capitation payment $d_j \ge 0$ and a fee-for-service rate $c_j \ge 0$.

The strategic players in our setting are the hospitals which simultaneously choose their price structures $(K_j, T_j (w_j, d_j, c_j))$. These prices induce an patient-doctor allocation such that each patient and doctor joins his preferred hospital, and with the doctor's effort, e_j , chosen according to

$$e_j \in \arg\max\frac{c_j n_j^P}{m n_j^D} e_j - \Psi(\frac{n_j^P}{m n_j^D} e_j),$$

which (using the quadratic specification of Ψ) yields

$$e_j^* = \frac{mn_j^D}{n_j^P} c_j. \tag{3}$$

Not surprisingly, e_j^* increases with the fee-for-service rate c_j which is in line with the supply induced demand literature.¹³ Furthermore, $c_j = 0$ implies $e_j^* = 0$. Effort is costly, but does not give any direct benefits to providers. Consequently, a positive effort level can only be achieved through financial incentives.¹⁴ It is worth noticing that our set-up is somewhat biased towards fee-for-service remuneration scheme (at least as long as $\zeta > 0$). This is because, additional consultations contribute positively to the total quality perceived by patients and it is necessary to have a strictly positive fee-for-service rate to ensure additional consultations. It is important to keep this in mind when interpreting our results obtained later on. In particular, we show that, in spite of this optimistic view of the fee-for-service remuneration, the introduction (or addition) of a fee-for-service element always makes patients worse of in a two-sided competition setting.

As both sides are fully covered, demand levels are equivalent to market shares. Defining the quality differential between hospitals as

$$g(n_1^P, mn_1^D) = \varphi(n_1^P, mn_1^D) - \varphi(1 - n_1^P, m(1 - n_1^D)) = q_1 - q_2,$$

in our Hotelling's set-up, the demand functions are

$$\begin{split} n_1^P &= \frac{1}{2} + \frac{1}{2t_P} \left[\gamma g \left(n_1^P, m n_1^D \right) + \zeta \left(e_1 - e_2 \right) - \left(K_1 - K_2 \right) \right], \\ n_1^D &= \frac{1}{2} + \frac{1}{2t_D} \left[\theta g \left(n_1^P, m n_1^D \right) + w_1 - w_2 + d_1 \frac{n_1^P}{m n_1^D} - d_2 \frac{\left(1 - n_1^P \right)}{m (1 - n_1^D)} \right. \\ &\left. + \left[\frac{c_1 n_1^P}{m n_1^D} e_1 - \Psi \left(\frac{n_1^P}{m n_1^D} e_1 \right) - \left(\frac{c_2 (1 - n_1^P)}{m (1 - n_1^D)} e_2 - \Psi \left(\frac{n_1^P}{m n_1^D} e_2 \right) \right) \right] \right]. \end{split}$$

¹³More precisely, the supply induced demand theory states that physicians benefit from asymmetric information (derived from their diagnosis). Consequently, they can select the number of medical acts provided in order to maximize their own utility. In that context, the number of acts increases with the fee-for-service rate when the price effet dominates the income effect. This is the case in our setting because with quasilinear preferences the income effect is zero.

¹⁴Doctors are also sensitive to health care quality delivered to patients through the CNE's component (for altruistic or workload related reasons). However, this effect does not give any incentives to provide effort.

Using (3) to substitute for the effort levels chosen by the providers yields

$$n_{1}^{P} = \frac{1}{2} + \frac{1}{2t_{P}} \left\{ \gamma g\left(n_{1}^{P}, mn_{1}^{D}\right) + \zeta \left(c_{1} \frac{mn_{1}^{D}}{n_{1}^{P}} - c_{2} \frac{m\left(1 - n_{1}^{D}\right)}{1 - n_{1}^{P}}\right) - \left(K_{1} - K_{2}\right) \right\},$$

$$n_{1}^{D} = \frac{1}{2} + \frac{1}{2t_{D}} \left\{ \theta g\left(n_{1}^{P}, mn_{1}^{D}\right) + w_{1} - w_{2} \right\}$$

$$(4)$$

$$+ d_1 \frac{n_1^P}{mn_1^D} - d_2 \frac{(1 - n_1^P)}{m(1 - n_1^D)} + \frac{1}{2} \left[(c_1)^2 - (c_2)^2 \right] \right\}.$$
 (5)

The derivatives of these demand functions with respect to the parameters of the pricing functions, namely K_j on the patients' side and w_j , d_j and c_j are stated in Appendix A. There and in the remainder of the paper subscripts are used for the derivatives of g, which are denoted g_P and g_D . The expressions provided in the Appendix imply that

$$\frac{dn_1^P}{dd_1} = \frac{1}{m} \frac{dn_1^P}{dw_1}, \qquad \frac{dn_1^D}{dd_1} = \frac{1}{m} \frac{dn_1^D}{dw_1}.$$
(6)

In words, equation (6) states that salary and capitation fee affect demand behavior *on* both sides in a similar way; the respective derivatives are simply proportional to each other (according to their relative mass).

3 Equilibrium: general expressions

Our main objective is to compare the implications of different remuneration and pricing schemes. To do so, we shall successively consider the different instruments in isolation or in various combinations. To avoid repetitions, we shall start by considering the general problem obtained when *all* instruments are available. Though somewhat lengthy and tedious the expressions so obtained are convenient to generate the special cases considered in the remainder of the paper.

Hospital j maximizes its profit functions with respect to K_j, c_j, d_j and w_j . Without loss of generality, we concentrate on the program of hospital 1 which consists in maximizing Π_1 with respect to K_1 and T_1 where profit is defined by¹⁵

$$\Pi_{1} = n_{1}^{P} (K_{1}, K_{2}, T_{1}, T_{2}, \phi) (K_{1} - d_{1} - c_{1}e_{1}) - mw_{1}n_{1}^{D} (K_{1}, K_{2}, T_{1}, T_{2}, \phi),$$

$$= n_{1}^{P} (K_{1}, K_{2}, T_{1}, T_{2}, \phi) [K_{1} - d_{1}] - mn_{1}^{D} (K_{1}, K_{2}, T_{1}, T_{2}, \phi) [c_{1}^{2} + w_{1}].$$
(7)

Differentiating with respect to the pricing parameters and setting $n_1^P = n_1^D = 1/2$ in the resulting expressions shows that the following conditions hold in a symmetric equilibrium.

$$\frac{\partial \Pi_1}{\partial K_1} = \frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} \left[K_1 - d_1 \right] - m \frac{\partial n_1^D}{\partial K_1} \left[c_1^2 + w_1 \right] = 0, \tag{8}$$

$$\frac{\partial \Pi_1}{\partial w_1} = \left[K_1 - d_1\right] \frac{\partial n_1^P}{\partial w_1} - \frac{m}{2} - m \frac{\partial n_1^D}{\partial w_1} \left[c_1^2 + w_1\right] = 0,\tag{9}$$

$$\frac{\partial \Pi_1}{\partial c_1} = \frac{\partial n_1^P}{\partial c_1} \left[K_1 - d_1 \right] - m \left[c_1^2 + w_1 \right] \frac{\partial n_1^D}{\partial c_1} - 2c_1 \frac{m}{2} = 0, \tag{10}$$

$$\frac{\partial \Pi_1}{\partial d_1} = -\frac{1}{2} + \frac{dn_1^P}{dd_1} \left[K_1 - d_1 \right] - \left[c_1^2 + w_1 \right] m \frac{dn_1^D}{dd_1} = 0.$$
(11)

Not surprisingly, it follows from (6) that

$$\frac{\partial \Pi_1}{\partial d_1} = \frac{1}{m} \frac{\partial \Pi_1}{\partial w_1}$$

Consequently, if hospitals use simultaneously capitation payment and salary to remunerate doctors, we have a continuum of symmetric equilibria. This issue has been discussed in Bardey *et al.* (2010).¹⁶ In this paper, we refrain from dealing with the complexity of equilibria multiplicity. Instead, we concentrate on studying the equilibrium allocations obtained under different type of providers' remuneration schemes. Our main focus will be on schemes that involve a fee-for-service, possibly in combination with capitation of salary payments. In a first step, we will report the equilibria under (pure) wage or

¹⁵It is worth noticing that even if we would have a regulated price on patients' side, as it can be the case in several health care systems, it does not change the two-sided mechanism competition at work in our model. Indeed, even though we would have a regulated price on patients' side, the patients' demand would still depend on transfers paid to doctors, maintaining the two-sided mechanism.

¹⁶This result also appears in Armstrong (2006) when platforms use two-part tariffs.

capitation scheme which constitute interesting benchmarks. Observe that when there is no fee-for-service (c = 0) we have e = 0 and we return essentially to the setting of Bardey *et al.* (2010) who do already characterize the equilibria under wage and capitation schemes. To make this paper self-contained, we shall restate the main results we need for the purpose of comparison. We shall refrain from repeating the proofs as well as the discussion and intuition except when they are directly relevant for our analysis.

4 Pure salary and capitation schemes

Assume first that the hospitals use a salary scheme for providers, combined with a fixed payment for patients.¹⁷ The symmetric equilibrium is then obtained by solving (8) and (9) after setting c = 0 and using the expressions for the demand derivatives provided in Appendix A. It is described in the following proposition.

Proposition 1 (Bardey et al., 2010) When hospitals use K_j and w_j as sole instruments the symmetric equilibrium is given by

$$K_1^w = t_P - \frac{1}{2} \left(\gamma + m\theta \right) g_P, \qquad (12)$$

$$w_1^w = -t_D + \frac{1}{2} \left(\gamma + m\theta\right) g_D, \tag{13}$$

and hospitals realize a profit of

$$\Pi^{w} = \frac{mt_{D} + t_{P}}{2} - \frac{(\gamma + \theta m)(g_{P} + mg_{D})}{4}.$$
(14)

Observe that g_P and g_D are evaluated at $n_1^P = n_1^D = 1/2$, so that this proposition provides a closed form solution.

Turning to the case where hospitals use K_j and d_j , solving (8) and (11) establishes the following proposition.

¹⁷Recall that with c = e = 0, a fee-for-service on the patients' side is of no relevance.

Proposition 2 (Bardey et al., 2010) When hospitals use K_j and d_j as sole instruments the symmetric equilibrium is given by

$$K_{j}^{d} = \frac{mt_{D}}{2} + t_{P} - \frac{1}{4} \left(\gamma + \theta m\right) \left(2g_{P} + mg_{D}\right), \tag{15}$$

$$d_{j}^{d} = -\frac{mt_{D}}{2} + \frac{1}{4}m(\gamma + \theta m)g_{D}.$$
 (16)

and hospitals realize a profit of

$$\Pi^{d} = \frac{mt_{D} + t_{P}}{2} - \frac{\left(\gamma + \theta m\right)\left(g_{P} + mg_{D}\right)}{4}.$$

Notice that d_j^d is exactly equal to $mw_j^w/2$. In other words, the total remuneration received by providers $T_j^d = d_j^d/m$ is half of the remuneration achieved in the salary game, namely $T_i^w = w_i^w$. To understand why capitation payment leads to lower compensations, let us start from the equilibrium salary w^w . By definition, this salary level is such that no hospital can gain by decreasing its salary given the salary offered by the other platform. Now, when the capitation payment level is the strategic variable, a decrease in a say d_1 induces (for a given level of d_2) a reduction in compensation offered by hospital 2 (because some providers move to hospital 2). This implies that a reduction in d_1 (given d_2) is beneficial, even though a reduction in w_1 (given w_2) is not. Interestingly, the price level is also smaller with the capitation payment scheme. To see this, combine (15) and (12) to obtain $K_j^d = K_j^w - (m/2)w^w$. Intuitively, we can once again start from the equilibrium under salary schemes. By definition, hospital 1 cannot gain by decreasing its price given K_2 and w_2 . Under the capitation payment regime, a reduction in K_1 brings about a reduction in the compensation (per provider) paid by hospital 2 (because some patients move from hospital 2 to hospital 1). This in turn mitigates the negative effects of a decrease in the price and implies that a unilateral price decrease is beneficial when d_2 is held constant even though it was not beneficial when w_2 was constant.

The main features of the comparison between salary and capitation scheme are summarized in the following proposition.¹⁸

Proposition 3 Comparing the equilibria achieved under salary and capitation payments shows that

i) $e^w = e^d = 0$: in both cases, doctors have no incentives to exert effort (increase the number of medical acts) and set e at its minimum level.

ii) $T_j^d < T_j^w$ and $K_j^d < K_j^w$: patients pay a lower price and providers a lower remuneration under a capitation payment than under a salary scheme.

iii) $V^d > V^w$ and $U^d < U^w$ for all z: all patients are better off and all providers worse off under capitation then under wage schemes.

iv) $\Pi_i^w = \Pi_i^d$: hospitals' profits are the same under both remuneration schemes.

The heath economics literature has extensively dealt with the relative merits of payment schemes and specifically their incentive properties. A point that is often made is that flat payment schemes (as opposed to fee-for-service schemes) have the advantage of providing stronger incentives for cost reduction.¹⁹ Our results are in line with this conventional wisdom albeit in a somewhat trivial way. Specifically, we find that both payment schemes provide the same incentives to limit the number of medical acts as much as possible. Finally, since a switch from salary to capitation scheme decreases both patients' pay and doctors' remuneration, the impact on hospitals' profits is *a priori* ambiguous. In our specific setting the two effects happen to cancel out exact so that profits are the same under the two schemes; we simply have a transfer of rents from providers to patients. This result is due to the assumption the market fully covered on both sides which implies that hospitals compete in a "business stealing" model. When the CNE is simply determined by the doctor/patient ratio, the term $g_P + mg_D$ is equal

¹⁸Items *i*), *ii*) and *iv*) follow directly from Propositions (1) and (2). Item *iii*) follows from *i*) and *ii*), making use of (1) and (2), the specification of patients' and doctors' utilities.

¹⁹See, for instance, Gosden *et al.* (1999) for a review of the literature on the remuneration of health care providers.

to 0 and hospitals' profits do not depend on the common network externalities (see Bardey *et al.*, 2010). This is because the negative externality generated by patients and the positive one due to doctors exactly cancel each other out. In other words, their profits are be the same as in a Hotelling game without network externalities.

5 Pure fee-for-service schemes

We now turn to the case where hospitals use a fee-for-service rate on the doctors' side, while patients continue to pay a fixed fee. The hospital's relevant first-order conditions are now equations (8) and (16). The symmetric equilibrium achieved in the case is described in Proposition 4, which is established in Appendix B.

Proposition 4 When hospitals use K_j and c_j as sole instruments the symmetric equilibrium is described by

$$K_{1}^{c} = t_{P} - \frac{1}{2} \left[(\gamma + 2m\theta) g_{P} - 4mc_{1}^{c}\zeta \right] + \frac{\theta g_{P}m\zeta}{2c_{1}^{c}},$$

$$(c_{1}^{c})^{2} = -2t_{D} + \frac{1}{2} \left[(\gamma + 2m\theta) g_{D} + 4c_{1}^{c}\zeta \right] + \frac{\zeta}{c_{1}^{c}} \left(t_{D} - \frac{\theta mg_{D}}{2} \right),$$

and hospitals realize a profit of

$$\Pi^{c} = \frac{1}{2} \left[t_{P} + 2mt_{D} - \frac{1}{2} \left[(\gamma + 2m\theta) \left(g_{P} + mg_{D} \right) \right] - \frac{m\zeta}{c_{1}^{c}} \left(t_{D} - \frac{\theta \left(mg_{D} + g_{P} \right)}{2} \right) \right].$$

While we were able to obtain closed-form solutions under wage and capitation schemes, this is no longer possible with a pure fee-for-service scheme. Accordingly, the prices reported in Proposition 4 are implicitly defined as functions of c_1^c . This makes their interpretation more difficult. An observation that can easily be made at this point is that hospitals' profits increase with the fee-for-service rate. However, this is a relationship between two endogenous variables which has to be interpreted with care. Closed form solutions continue to be available in the special case where $\zeta = 0$. In this situation, e can be interpreted as a pure induced demand effect. Indeed, with c > 0and $\zeta = 0$, doctors exert a positive level of effort (increasing the number of acts) to increase their remuneration, but this does not induce any benefits to patients. The equilibrium in this case is stated in the following corollary, which follows directly from Proposition 4.

Corollary 1 Assume $\zeta = 0$. When hospitals use K_j and c_j as sole instruments the symmetric equilibrium is described by

$$K_1^c = t_P - \frac{1}{2} (\gamma + 2m\theta) g_P,$$
 (17)

$$(c_1^c)^2 = -2t_D + \frac{1}{2}(\gamma + 2m\theta)g_D, \qquad (18)$$

and hospitals realize a profit of

$$\Pi^{c} = \frac{1}{2} \left[t_{P} + 2mt_{D} - \frac{1}{2} \left[(\gamma + 2m\theta) \left(g_{P} + mg_{D} \right) \right] \right].$$

On the patients' side, as usual, the price charged depends positively on the transportation cost t_P . Moreover, the negative externality generated by patients increases the price. Comparing K_1^c defined by (17) with K_1^w specified by (12) shows that this effect is stronger when doctors are remunerated via a fee-for-service than under a salary scheme. Consequently we have $K_1^c (\zeta = 0) > K_1^w$. Intuitively, the fee-for-service induces a higher level of e, which increases the hospitals' cost. This cost increase is shifted, at least to some degree, to patients. In the same way, on the doctors' side, hospitals take more advantage of the transportation cost t_D when they use a fee-for-service scheme due to the positive number of events. The positive externality generated by doctors favor them in comparison with a salary payment. Moreover, it can be noticed that we obtain a symmetric equilibrium that ensures a positive fee-for-service-rate only if $4t_D \leq (\gamma + 2m\theta) g_D$. In words, the positive externality generated by doctors must be high enough to outweigh the transportation cost that reduces their remuneration. We will now compare patients' and providers' welfare and hospitals' profits achieved under fee-for-service and under the other remuneration schemes. We will concentrate on the comparison with the salary regime. The comparisons will make use of the following lemma which is established by substituting the equations provided in Propositions 1 and 4 into the definitions of V, U and Π and by rearranging the resulting expressions.

Lemma 1 Welfare and profit variations between wage and fee-for-service regimes are given by

$$\Delta V = V^c - V^w = \zeta m c_1^c - K_1^c - (0 - K_1^w) = m \left[-\zeta c_1^c + \frac{1}{2} \theta g_P \left(\frac{c_1^c - \zeta}{c_1^c} \right) \right], \quad (19)$$

$$\Delta U = U^{c} - U^{w} = (c_{1}^{c})^{2} \left(1 - \frac{m^{2}}{2}\right) - w_{1}^{w}$$
(20)

$$= -mc_1^c \left(\frac{mc_1^c - 4\zeta}{2}\right) - \left[t_D - \frac{\theta mg_D}{2}\right] \left(\frac{c_1^c - \zeta}{c_1^c}\right),\tag{21}$$

$$\Delta \Pi = \Pi^c - \Pi^w = \frac{m}{2} \left[t_D - \frac{\theta \left(mg_D + g_P \right)}{2} \right] \left(\frac{c_1^c - \zeta}{c_1^c} \right).$$
(22)

These expressions are rather complex. The only obvious result is that $\Delta V < 0$ for $\zeta = 0$. Intuitively, the fee-for-service increases the number of acts (we have e > 0). As discussed above, this results in higher payments for patients but does not give them any extra benefits. The other expressions are ambiguous, even for $\zeta = 0$. When $t_D \leq \theta m g_D/2$, we have $\Delta U < 0$, so that providers are also better off with a salary scheme. This is because they receive a higher payment and do not incur any disutility of effort. However, when $t_D > \theta m g_D/2$ these two effects go in opposite directions. Regarding $\Delta \Pi$, we have an explicit expressions for $\zeta = 0$.²⁰ Consequently, some results can be obtained for that case. For instance, when the CNE is determined by the doctor/patient ratio (a function homogeneous of degree 0, which implies $m g_D + g_P = 0$), $\Delta \Pi$ is necessarily positive. It appears that hospitals take advantage of the fee-for-service to charge twice the transportation cost on doctors, allowing them to increase their profit (compared to salary or capitation scheme).

 $^{^{20}}$ The second factor on the RHS of (22) is then equal to 1.

When $\zeta > 0$, only few analytical results can be obtained. They make use of the following Lemma (established in Appendix C) which studies the comparative statics properties of c_1^c and K_1^c with respect to ζ .

Lemma 2 The variations of c_1^c and K_1^c with respect to ζ satisfy the following properties.

i) In the neighborhood of $\zeta = 0$, $\theta m g_D \ge 2t_D$ ensures that $dc_1^c/d\zeta \ge 1$ which in turn implies $c_1^c \ge \zeta$.

ii)

$$\frac{dK_1^c}{d\zeta} = m \left[4c_1 + \left(-2c_1 + \frac{\theta g_P}{2c_1} \right) \left(1 - \varepsilon \right) \right],$$

where

$$\varepsilon = \frac{\zeta}{c_1} \frac{dc_1^c}{d\zeta}$$

The variation of the total fee paid by patients with respect to ζ is ambiguous and mainly depends on the elasticity of the fee-for-service rate with respect to ζ . Situations in which this elasticity is higher than 1 can be interpreted as an induced demand effect. In such a case, the fee-for-service rate paid to providers increases faster than the patients' valuation of the number of medical acts. Then, the fixed price paid by patients increases faster than their valuation of the number of events. On the contrary, for values of this elasticity inferior to 1, we have two countervailing forces at work and the overall effect is ambiguous.

In the neighborhood of $\zeta = 0$, we have $c_1^c \ge \zeta$ which, from (19) implies $\Delta V < 0$ so that patients are worse off when the doctors' remuneration is switched from wage to fee-for-service. Intuitively, the positive level of c implies that doctors exert some effort. However, the valuation of this effort is low and it is more than outweighed by the increase in the patients' payments.

6 Mixed fee-for-service schemes

We now consider the case where the different types of remuneration can be combined (on the doctors' side). First, we study a scheme involving both a salary and a fee-forservice. Then, we consider a combination of capitation and fee-for-service. As to the patients, we continue to consider only fixed fees.

6.1 Fee-for-service and salary

The relevant first-order conditions are now (8), (10) and (9). The resulting equilibrium is stated in the following proposition, which is established in Appendix D.

Proposition 5 When hospitals use a fixed fee K_j on the patients' side, while combining wage w_j and fee-for-service c_j on the doctors' side,

(i) the symmetric equilibrium is given by

$$K_1^{wc} = t_P - \frac{1}{2} (\gamma + m\theta) g_P + 2m\zeta^2,$$
 (23)

$$w_1^{wc} = -t_D + g_D \frac{1}{2} (\gamma + m\theta) + \zeta^2,$$
 (24)

$$c_1^{wc} = \zeta, \tag{25}$$

and hospitals realize a profit of

$$\Pi^{wc} = \Pi^{w} = \Pi^{d} = \frac{1}{2} \left[t_{P} + mt_{D} - \frac{1}{2} \left(\gamma + m\theta \right) \left(g_{P} + mg_{D} \right) \right].$$
(26)

(ii) the induced effort level $e_i^* = m\zeta$ is efficient (maximizes total surplus).

Interestingly, the mixed payment case turns out to be simpler to solve than the pure fee-for-service case and we obtain closed form solutions like in Section 4. The proposition shows that the introduction of a fee-for-service (on top of the salary) only makes a difference when $\zeta > 0$, *i.e.*, when effort (number of acts) is valued positively by patients. For $\zeta = 0$, the extra instrument is not used in equilibrium and both the

patients' bill as well as the wage remain at the same levels as under a pure wage scheme (we have $K_1^{wc} = K_1^w$ and $w_1^{wc} = w_1^w$). Now, when $\zeta > 0$, hospitals use a positive fee-for-service in equilibrium, and it is just equal to ζ (the marginal benefit to patients).

The shifting pattern of this extra fee is quite interesting. One could have expected some kind of crowding out (or substitution) between remuneration schemes, but we find exactly the opposite result: salary but also prices increase such that patients loose and doctors win. This result only comes from the role played by the number of consultation and does not interact with the quality captured by the common network externalities. To understand this, consider a slightly different game in which the fee-for-service rate is exogenously set at its efficient level $c_1 = c_2 = \zeta$, while the number of consultations per patient is *fixed* and given by $e = m\zeta$ (the level of effort implied by (3) when $c_1 = c_2 = \zeta$ and when $n_1^P = n_1^D = 1/2$.²¹ In this game, when hospitals compete in price and salary as in Section 4, it can easily be shown that, symmetric equilibrium, prices and salaries are equal to $K_1^w + m\zeta^2$ and w_1^w respectively (where K_1^w and w_1^w are defined by expressions (12) and (13) in Proposition 1. Now, let us continue to assume that the fee-for-service is given by $c_1 = c_2 = \zeta$, but that the number of consultations per patient is determined by (3) which depends on the number of patients and doctors effectively affiliated with both hospitals. Further assume that prices and wages in hospital 2 are given by $K_2^w + m\zeta^2$ and w_2^w respectively.²² An increase in the price or the wage in hospital 1 now brings

$$n_1^P = \frac{1}{2} + \frac{1}{2t_P} \left(\gamma g \left(n_1^P, n_1^D \right) - [K_1 - K_2] \right), n_1^D = \frac{1}{2} + \frac{1}{2t_D} \left(\theta g \left(n_1^P, n_1^D \right) + [w_1 - w_2] \right).$$

 22 The demand system for hospital 1 becomes

$$n_{1}^{P} = \frac{1}{2} + \frac{1}{2t_{P}} \left(\gamma g\left(n_{1}^{P}, n_{1}^{D}\right) + \zeta^{2} \left(\frac{mn_{1}^{D}}{n_{1}^{P}} - \frac{m\left(1 - n_{1}^{D}\right)}{1 - n_{1}^{P}}\right) - (K_{1} - K_{2}) \right),$$

$$n_{1}^{D} = \frac{1}{2} + \frac{1}{2t_{D}} \left(+ \theta g\left(n_{1}^{P}, n_{1}^{D}\right) + [w_{1} - w_{2}] \right).$$

 $^{^{21}\}mathrm{In}$ this game, the demand functions for hospital 1 are given by

about an increase in the number of consultations per patient in hospital 1 (because some patients move from hospital 1 to hospital 2, while doctors move from hospital 2 to hospital 1). This in turn mitigates the negative effects of an increase in the price and the wage and implies that a unilateral price and wage increase is beneficial when w_2, K_2 and c_2 are held constant. This leads to higher symmetric equilibrium prices and wages which, as shown by (23) and (24) are given by $K_1^w + 2m\zeta^2$ and $w_1^w + m\zeta^2$. To sum up, the introduction of the fee-for-service component along with a salary scheme leads to higher prices on the patients' side and higher wages on doctors' side. Observe, that this has no adverse effect on hospitals' profits; the extra compensation paid to doctors is exactly shifted to patients. A patient's bill increase by $2m\zeta^2$, which is equal to the sum of the fee-for-service $(mc_1^{wc}\zeta = m\zeta^2)$ and the extra salary $(m\zeta^2)$.²³

Welfare comparisons are also much simpler than in the pure fee-for-service case. With the closed form solutions reported in Propositions 1 and 5, it is straightforward to compare patients' and doctors' welfare.

Proposition 6 When a fee-for-service component is introduced into a pure salary scheme, the welfare variations are:

- i) on the patients' side, $\Delta V = V^{wc} V^w = -m\zeta^2 < 0;$
- *ii)* on the doctors' side, $\Delta U = U^{wc} U^w = (3/2) \zeta^2 > 0$.

To sum-up, patients loose, doctors win and (as shown by 26) hospitals are indifferent. Patients do benefit from the increase in e (which they value when $\zeta > 0$), but this benefit is more than offset by the increase in fees.

6.2 Fee-for-service and capitation payment

The relevant first-order conditions are now (8), (10) and (11). The solution is derived in Appendix E, and presented in the following proposition.

 $^{^{23}\}mathrm{The}\ m$ appears in the expressions because there are m doctors per patient in a symmetric equilibrium.

Proposition 7 When hospitals use a fixed fee K_j as the sole instruments on the patients' side, while capitation payment d_j and fee-for-service c_j on the doctors' side,

(i) the symmetric equilibrium is given by

$$d_1^{dc} = -\frac{1}{2}mt_D + (\theta m + \gamma)\frac{mg_D}{4} + \frac{1}{2}m\zeta^2, K_1^{dc} = t_P + \frac{1}{2}mt_D - \frac{(\gamma + m\theta)(2g_P + mg_D)}{4} + \frac{3}{2}m\zeta^2, c_1^{dc} = \zeta,$$

and hospitals realize a profit of

$$\Pi^{dc} = \Pi^{wc} = \Pi^{w} = \Pi^{d} = \frac{1}{2} \left[t_P + mt_D - \frac{1}{2} \left(\gamma + m\theta \right) \left(g_P + mg_D \right) \right],$$

(ii) the induced effort level $e_j^* = m\zeta$ is efficient (maximizes total surplus).

As in the previous case, for $\zeta = 0$, at a symmetric equilibrium hospitals do not use a fee-for-service rate and consequently we obtain exactly the same equilibrium as the one obtained in a pure capitation payment on doctors' side. Now, when $\zeta > 0$, the fixed price paid by patients is increased. This rent paid by patients is totally transferred to doctors as hospitals' profit remain the same as in the previous cases (except the case when hospitals only remunerate providers *via* a fee-for-service scheme). The intuition is exactly the same as when the fee-for-service was combined with the salary. As in that previous setting, there is no crowding out between remuneration schemes, *i.e.*, the capitation payment received by doctors increases simultaneously with the fee-for-service rate. As in the previous section, prices are wages are increased because the number of consultations per patient is increasing in the number of doctors and decreasing in the number of patients.

Proposition 8 When a fee-for-service component is introduced in a pure capitation payment scheme, the welfare variations are:

- i) on the patients' side, $\Delta V = V^{dc} V^d = -m\zeta^2/<0;$
- ii) on the providers' side, $\Delta U = U^{dc} U^d = \zeta^2 > 0$.

The introduction of a fee-for-service component in a capitation payment scheme unambiguously decreases the patients' welfare and increases the doctors' utility. As in the previous case *i.e.* in the salary case, the fee-for-service introduction favors doctors while patients are worse off. However, the capitation scheme remains more "patient friendly" exactly like under pure (salary of capitation) remuneration schemes; see Proposition 3.

7 Numerical illustration

We now provide a numerical example which illustrates our analytical results and provides a basis of comparisons for the cases where analytical results are ambiguous. Table 1 reports the results for the following example: $\varphi\left(N_j^C, N_j^P\right) = \left(N_j^C/N_j^P\right)$, $t_P = 4$, $t_D = 1$, $\gamma = 2$, $\theta = 1$, m = 0.3, $\overline{V} = 10$ and $\overline{U} = 0$. We consider different levels of ζ including 0 (the case for which we have a full set of analytical results). This uses the simplest meaningful specification for quality by assuming that the CNE depends on the patient-doctor ratio.

For the most part this example simply illustrates the earlier results and there is no point reviewing them here. However, there are some extra features which supplement the analytical results. First, we find that a fee-for-service is bad for patients' welfare, even for levels of ζ beyond the neighborhood of $\zeta = 0$. As ζ increases, patients put a higher value on the doctors' effort and only a fee-for-service can induce this effort. This effect tends to make the fee-for-service remuneration attractive to patients. However, this comes at a price. As competition for doctors intensifies, their total compensation increases in a significant way and this extra cost is more then fully shifted to the patients. Overall, it turns out that the increase in fees more then outweighs the benefits patients derive from the higher effort.

Turning to the mixed schemes, we know from the analytical part that patients' welfare decreases as a fee-for-service element is introduced along with a salary or capitation payment. The numerical example also shows what happens when a wage element is in-

| | $\zeta = 0$ | $\zeta = 0.5$ | $\zeta = 1$ | $\zeta = 2$ |
|---------------------------------------|-------------|---------------|-------------|-------------|
| T^w | 3.6 | 3.6 | 3.6 | 3.6 |
| T^d | 1.8 | 1.8 | 1.8 | 1.8 |
| T^c | 3.2 | 5.66 | 9.49 | 22.22 |
| T^{wc} | 3.6 | 4.1 | 5.6 | 11.6 |
| T^{dc} | 1.8 | 2.15 | 3.21 | 7.44 |
| K^w | 5.38 | 5.38 | 5.38 | 5.38 |
| K^d | 4.84 | 4.84 | 4.84 | 4.84 |
| K^c | 5.56 | 6.23 | 7.35 | 11.14 |
| K^{wc} | 5.38 | 5.53 | 5.98 | 7.78 |
| K^{dc} | 4.84 | 4.95 | 5.29 | 6.64 |
| Π^c | 2.30 | 2.26 | 2.25 | 2.23 |
| $\Pi^w = \Pi^c = \Pi^{wc} = \Pi^{dc}$ | 2.15 | 2.15 | 2.15 | 2.15 |
| V^w | 4.62 | 4.62 | 4.62 | 4.62 |
| V^d | 5.16 | 5.16 | 5.16 | 5.16 |
| V^c | 4.44 | 4.13 | 3.58 | 1.69 |
| V^{wc} | 4.62 | 4.55 | 4.32 | 3.42 |
| V^{dc} | 5.16 | 5.13 | 5.01 | 4.56 |
| U^w | 3.6 | 3.6 | 3.6 | 3.6 |
| U^d | 1.8 | 1.8 | 1.8 | 1.8 |
| U^c | 3.05 | 5.40 | 9.06 | 21.22 |
| U^{wc} | 3.6 | 4.08 | 5.55 | 11.42 |
| U^{dc} | 1.8 | 2.16 | 3.25 | 7.62 |

Table 1: Equilibrium under different remuneration schemes when $\varphi(N_j^P, N_j^D) = (N_j^P/N_j^D)$, $t_P = 4$, $t_D = 1$, $\gamma = 2$, $\theta = 1$, m = 0.3, $\overline{V} = 10$, $\overline{U} = 0$ for different levels of ζ .

troduced in a fee-for-service scheme. For the considered parameter values, this leads to an increase in patients' welfare. More interestingly, it has an ambiguous effect on doctors' welfare. It increases when ζ is small, but decreases for larger levels of ζ . In other words, when ζ is sufficiently large, doctors would prefer a pure fee-for-service scheme.

8 Conclusion

This paper represents an attempt to study the interplay between hospitals' competition and doctors' remuneration schemes properties via a two-sided market approach that includes common network externalities. In a first step, we consider *pure* wage, capitation of fee-for-service payment schemes. We find that the number of consultations, and consequently the level of quality delivered, is higher under a fee-for-service scheme than under other schemes. As a matter of fact, when doctors are remunerated solely via a salary or a capitation payment, they provide the minimum level of effort. Under salary and capitation schemes, hospitals obtain the same profit at equilibrium. Patients pay a lower price and doctors are less remunerated when they receive capitation payments rather than salary schemes are used. In other words, a capitation payment scheme favor patients while patients are better off under a salary scheme. Next, even though our set-up can be considered as biased in favor of fee-for-service schemes, our results suggest that patients are worse off when doctors are paid via a fee-for-service rather than under a salary scheme. We show this analytically for the case when the number of acts provides only small benefits to patients. For larger levels of benefits, numerical simulations appear to corroborate this result.

Second, we consider payment schemes mixing fee-for-service with either salary or capitation payments. We show that in either case, hospitals set the fee-for-service rate just equal to the patients' valuation of the number of consultations. Both type of mixed schemes yield the same profit for hospitals as under pure capitation fee or salary schemes. Moreover, the two mixed schemes imply the same overall welfare even though they differ in their implications for patients and doctors. Exactly like in the pure remuneration case, the presence of a capitation element favors patients, while a salary term favors doctors. Finally, our results show that the introduction of a fee-for-service component in a capitation or salary scheme always favor doctors whereas patients are worse off, in spite of the quality increase.

This paper can be extended in several directions. First, it would be interesting to consider outcome where the market for patients is not completely covered. From a theoretical perspective this would actually simplify the model. However, it would make it more interesting from an applied policy perspective as access to health care is a major problem in practice. Second, both from a theoretical and from a practical perspective, it would be useful to study mixed oligopolies (with public or non profit hospitals).

References

- Armstrong M, 2006, "Competition in Two-Sided Markets", Rand Journal of Economics, 37(3), 668–691.
- [2] Brekke K., R. Nuscheler and O. Straume, 2006, "Quality and Location Choices Under Price Regulation", Journal of Economics & Management Strategy, 15, 1, 207-227.
- [3] Brekke K., L. Siciliani and O. Straume, 2009, "Hospital competition and quality with regulated prices", CESifo working paper 2635, 2009, forthcoming in *Scandinavian Journal of Economics*.
- [4] Choné P and C.A Ma, 2010, "Optimal Health Care Contracts under Physician Agency", Annales d'Economie et de Statistiques, forthcoming.

- [5] Cutler D. and R. Zeckhauser, 2000, The Anatomy of Health Insurance, in Anthony Culyer and Joseph Newhouse, eds., Handbook of Health Economics, Volume IA, Amsterdam: Elsevier, 563–643.
- [6] Fergusson-Paré M, 2004, "ACEN Position Statement: Nursing Workload-A Priority for Healthcare", Nursing Leadership, 17(2), 24–26.
- [7] Fortin B., N. Jacquemet and B. Shearer, 2008, "Policy Analysis in the Health-Services Market: Accounting for Quality and Quantity", CIRPEE working paper n°807.
- [8] Gaynor M and M V. Pauly, 1990, "Compensation and Productive Efficiency in Partnerships: Evidence from Medical Groups Practice" *Journal of Political Econ*omy, 98, no. 3, 544–573.
- [9] Gosden T, Pedersen L and D. Torgerson, 1999, "How should we pay doctors? A systematic review of salary payments and their effect on doctor behaviour", QJ Med, 92, 47–55.
- [10] Griffin K and B.A Swan, 2006, "Linking Nursing Workload and Performance Indicators in Ambulatory Care", Nursing Economics, 24(1), 41–44.
- [11] Ma C. A., 1994, "Health Care Payment Systems: Cost and Quality Incentives", Journal of Economics & Management Strategy, 3, 93–112.
- [12] Ma C.A. and J. F. Burgess, 1993, "Quality competition, welfare and regulation", *Journal of Economics*, 58, 2, 153-173.
- [13] Ma C.A and M. Riordan, 2002, "Health Insurance, Moral Hazard, and Managed Care", Journal of Economics & Management Strategy, Vol. 11, 81-107.
- [14] Mc Gillis Hall L, 2004, "Quality Work Environments for Nurse and Patient Safety", ISBN-13: 9780763728809.

- [15] Newhouse J.P, 1996, Reimbursing Health Plans and Health Providers: Efficiency in Production Versus Selection, *Journal of Economic Literature*, 34, 1236-1263.
- [16] Pezzino M and Pignatoro G., 2008, "Competition in the Health Care Markets: a Two-Sided Approach", Working Paper University of Manchester.
- [17] Rochet J-C. and Tirole J., 2003, "Platform Competition in Two-Sided Markets", Journal of the European Economic Association, 1, 990–1029.
- [18] Rochet J-C. and Tirole J., 2006, "Two-Sided Markets: A Progress Report", Rand Journal of Economics, 37, 645–667.
- [19] Tarnow-Mordi W.O, C Hau, A Warden and A.J Shearer, 2000, "Hospital Mortality in Relation to Staff Workload: a 4-Year Study in an Adult Intensive Care Unit", *The Lancet*, 356, 185-189..

Appendix

A Properties of the demand functions

Differentiating (4) and (5), rearranging and solving yields

$$\begin{aligned} \frac{dn_1^P}{dK_1} &= \frac{-1}{4t_D t_P |B|} \left[\left(2t_D - \left(\theta m g_D - \frac{4}{m} d_1 \right) \right) \right], \\ \frac{dn_1^D}{dK_1} &= \frac{-1}{4t_D t_P |B|} \left[\left(\theta g_P + \frac{4}{m} d_1 \right) \right], \\ \frac{dn_1^P}{dw_1} &= \frac{1}{4t_D t_P |B|} \left[m \left[\gamma g_D + 4c_1 \zeta \right] \right], \\ \frac{dn_1^D}{dw_1} &= \frac{1}{4t_D t_P |B|} \left[2t_P - \left(\gamma g_P - 4mc_1 \zeta \right) \right], \\ \frac{dn_1^P}{dc_1} &= \frac{m}{4t_P t_D |B|} \left[\zeta \left(2t_D - \left(\theta m g_D - 4 \left(\frac{d_1}{m} + c_1^2 \right) \right) \right) + c_1 \gamma g_D \right], \\ \frac{dn_1^D}{dc_1} &= \frac{1}{4t_P t_D |B|} \left[\left(2t_P - \left[\gamma g_P - 4mc_1 \zeta \right] \right) c_1 + \left(\theta g_P + \frac{4}{m} d_1 \right) m \zeta \right], \\ \frac{dn_1^P}{dd_1} &= \frac{1}{4t_D t_P |B|} \left[\gamma g_D + 4c_1 \zeta \right] = \frac{1}{m} \frac{dn_1^P}{dw_1}, \\ \frac{dn_1^D}{dd_1} &= \frac{1}{4mt_D t_P |B|} \left[2t_P - \left(\gamma g_P - 4mc_1 \zeta \right) \right] = \frac{1}{m} \frac{dn_1^D}{dw_1}, \end{aligned}$$

where

$$|B| = \frac{1}{4t_P t_D} \left[4t_P t_D - 2t_P \theta m g_D - \gamma g_P 2t_D + \frac{4}{m} d_1 \left(2t_P - \gamma \left(g_P + m g_D \right) \right) + 4m c_1 \zeta \left[2t_D - \theta \left(m g_D + g_P \right) \right] \right].$$

B Proof of Proposition 4

The demand functions properties are in this case:

$$\begin{aligned} \frac{dn_1^P}{dK_1} &= \frac{-1}{4t_D t_P |B|} \left[(2t_D - \theta m g_D) \right], \\ \frac{dn_1^D}{dK_1} &= \frac{-1}{4t_D t_P |B|} \left[\theta g_P \right], \\ \frac{dn_1^P}{dc_1} &= \frac{m}{4t_P t_D |B|} \left[\zeta \left(2t_D - \left(\theta m g_D - 4 \left(c_1 \right)^2 \right) \right) + c_1 \gamma g_D \right], \\ \frac{dn_1^D}{dc_1} &= \frac{1}{4t_P t_D |B|} \left[(2t_P - \left[\gamma g_P - 4mc_1 \zeta \right]) c_1 + \theta g_P m \zeta \right], \end{aligned}$$

where

$$|B| = \frac{1}{4t_P t_D} \left[4t_P t_D - 2t_P \theta m g_D - \gamma g_P 2t_D + 4mc_1 \zeta \left[2t_D - \theta \left(m g_D + g_P \right) \right] \right].$$

The first-order conditions reduce to

$$\frac{\partial \Pi_1}{\partial K_1} = \frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} K_1 - mc_1^2 \frac{\partial n_1^D}{\partial K_1} = 0,$$

$$\frac{\partial \Pi_1}{\partial c_1} = \frac{\partial n_1^P}{\partial c_1} K_1 - mc_1^2 \frac{\partial n_1^D}{\partial c_1} - mc_1 = 0.$$

From (8) and (10), we have

$$\left(\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} K_1\right) \frac{\partial n_1^D}{\partial c_1} = \frac{\partial n_1^D}{\partial K_1} \left(\frac{\partial n_1^P}{\partial c_1} K_1 - mc_1\right)$$

or,

$$K_1 = \frac{\frac{\partial n_1^D}{\partial c_1} + 2mc_1\frac{\partial n_1^D}{\partial K_1}}{2\left(\frac{\partial n_1^D}{\partial K_1}\frac{\partial n_1^P}{\partial c_1} - \frac{\partial n_1^P}{\partial K_1}\frac{\partial n_1^D}{\partial c_1}\right)}.$$

The denominator can be succesively rearranged as follows

$$2\left(\frac{\partial n_{1}^{D}}{\partial K_{1}}\frac{\partial n_{1}^{P}}{\partial c_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{\partial n_{1}^{D}}{\partial c_{1}}\right)$$

$$= 2\left(\frac{-1}{4t_{D}t_{P}|B|}[m\theta g_{P}]\frac{1}{4t_{P}t_{D}|B|}\left[\zeta\left(2t_{D} - \left(\theta mg_{D} - 4\left(c_{1}\right)^{2}\right)\right) + c_{1}\gamma g_{D}\right]\right]$$

$$+ \frac{1}{4t_{D}t_{P}|B|}\left[\left(2t_{D} - \theta mg_{D}\right)\right]\frac{1}{4t_{P}t_{D}|B|}\left[\left(2t_{P} - \left[\gamma g_{P} - 4mc_{1}\zeta\right]\right)c_{1} + \theta g_{P}m\zeta\right]\right)$$

$$= \frac{2c_{1}}{\left(4t_{D}t_{P}|B|\right)^{2}}\left(-4\zeta\left(c_{1}\right)m\theta g_{P} + 2t_{D}2t_{P} - 2t_{D}\gamma g_{P} - \theta mg_{D}2t_{P}\right)$$

$$+ 4mc_{1}\zeta\left(2t_{D}\right) - \theta mg_{D}4mc_{1}\zeta\right)$$

$$= \frac{2c_{1}}{\left(4t_{P}t_{D}|B|\right)}.$$

Consequently, we obtain

$$K_{1} = \frac{(2t_{P} - [(\gamma + 2m\theta) g_{P} - 4mc_{1}\zeta]) c_{1} + \theta g_{P}m\zeta}{2c_{1}},$$

$$= t_{P} - \frac{1}{2} [(\gamma + 2m\theta) g_{P} - 4mc_{1}\zeta] + \frac{\theta g_{P}m\zeta}{2c_{1}}.$$

Moreover, we have

$$\frac{\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} K_1}{\frac{\partial n_1^D}{\partial K_1}} = mc_1^2$$

leading to

$$\frac{1}{2} \left(4t_P t_D - 2t_P \theta m g_D - \gamma g_P 2t_D + 4mc_1 \zeta \left[2t_D - \theta \left(mg_D + g_P\right)\right]\right) \\ - \left[2t_D - \theta m g_D\right] \left[t_P - \frac{1}{2} \left(\gamma + 2m\theta\right) g_P + 2mc_1 \zeta + \frac{\theta g_P m \zeta}{2c_1}\right] \\ = -\theta g_P m c_1^2,$$

implying

$$c_1^2 = 2c_1\zeta - 2t_D + \frac{g_D}{2}\left(\gamma + 2m\theta\right) + \frac{\zeta}{c_1}\left(t_D - \frac{\theta m g_D}{2}\right).$$

Finally, evaluating hospitals' profits at this equilibrium yields

$$\Pi^{c} = \frac{1}{2} \left[t_{P} - \frac{1}{2} \left[(\gamma + 2m\theta) g_{P} - 4mc_{1}\zeta \right] + \frac{\theta g_{P}m\zeta}{2c_{1}} - m \left(2c_{1}\zeta + \left(-2t_{D} - t_{D}\frac{\zeta}{c_{1}} + \frac{g_{D}}{2} \left[(\gamma + 2m\theta) \right] - g_{D}\frac{\theta m\zeta}{2c_{1}} \right) \right) \right],$$

$$= \frac{1}{2} \left[t_{P} + 2mt_{D} - \frac{1}{2} \left[(\gamma + 2m\theta) \left(g_{P} + mg_{D} \right) \right] + \frac{m\zeta}{c_{1}} \left(t_{D} - \frac{\theta \left(g_{P} + mg_{D} \right)}{2} \right) \right].$$

C Proof of Lemma 2

Prices in a symmetric equilibrium are given by:

$$K_1^{***} = t_P - \frac{1}{2} \left[(\gamma + 2m\theta) g_P - 4mc_1 \zeta \right] + \frac{\theta g_P m \zeta}{2c_1},$$

$$(c_1^{***})^2 = -2t_D + \frac{1}{2} \left[(\gamma + 2m\theta) g_D + 4c_1 \zeta \right] + \frac{\zeta}{c_1} \left(t_D - \frac{\theta m g_D}{2} \right).$$

Differentiation with respect to ζ gives:

$$\begin{pmatrix} 1 & -2m\zeta + \frac{\theta_{BP}m\zeta}{2c_1^2} \\ 0 & 2\left(c_1 - \zeta\right) + \frac{\zeta}{c_1^2}\left(t_D - \frac{\theta_{mg_D}}{2}\right) \end{pmatrix} \begin{pmatrix} dK_1 \\ dc_1 \end{pmatrix} = - \begin{pmatrix} -2mc_1 - \frac{\theta_{mg_P}}{2c_1} \\ -2c_1 - \frac{1}{c_1}\left(t_D - \frac{\theta_{mg_D}}{2}\right) \end{pmatrix} d\zeta$$

So, the Cramer's rule gives:

$$\begin{aligned} \frac{dc_1}{d\zeta} &= \left. \frac{1}{|\Upsilon|} \right| \left| \begin{array}{c} 1 & 2mc_1 + \frac{\theta mg_P}{2c_1} \\ 0 & 2c_1 + \frac{1}{c_1} \left(t_D - \frac{\theta mg_D}{2} \right) \end{array} \right|, \\ &= \left. \frac{1}{|\Upsilon|} \left[2c_1 + \frac{1}{c_1} \left(t_D - \frac{\theta mg_D}{2} \right) \right], \end{aligned}$$

with

$$|\Upsilon| = 2\left(c_1 - \zeta\right) + \frac{\zeta}{c_1^2}\left(t_D - \frac{\theta m g_D}{2}\right).$$

Moreover, we have

$$\frac{dK_{1}}{d\zeta} = \frac{1}{|\Upsilon|} \begin{vmatrix} 2mc_{1} + \frac{\theta mg_{P}}{2c_{1}} & -2m\zeta + \frac{\theta g_{P}m\zeta}{2c_{1}^{2}} \\ 2c_{1} + \frac{1}{c_{1}}\left(t_{D} - \frac{\theta mg_{D}}{2}\right) & 2\left(c_{1} - \zeta\right) + \frac{\zeta}{c_{1}^{2}}\left(t_{D} - \frac{\theta mg_{D}}{2}\right) \end{vmatrix} \\
= \frac{1}{|\Upsilon|} \left[\left(2mc_{1} + \frac{\theta mg_{P}}{2c_{1}} \right) \left(2\left(c_{1} - \zeta\right) + \frac{\zeta}{c_{1}^{2}}\left(t_{D} - \frac{\theta mg_{D}}{2}\right) \right) \\ - \left(2c_{1} + \frac{1}{c_{1}}\left(t_{D} - \frac{\theta mg_{D}}{2}\right) \left(-2m\zeta + \frac{\theta g_{P}m\zeta}{2c_{1}^{2}} \right) \right) \right], \\
= 2mc_{1} + \frac{\theta mg_{P}}{2c_{1}} - \frac{dc_{1}}{d\zeta} \left(-2m\zeta + \frac{\theta g_{P}m\zeta}{2c_{1}^{2}} \right), \\
= m \left[2c_{1} + \frac{\theta g_{P}}{2c_{1}} - \frac{\zeta}{c_{1}}\frac{dc_{1}}{d\zeta} \left(-2c_{1} + \frac{\theta g_{P}}{2c_{1}} \right) \right], \\
= m \left[4c_{1} + \left(-2c_{1} + \frac{\theta g_{P}}{2c_{1}} \right) \left(1 - \epsilon \right) \right],$$

with

$$\epsilon = \frac{\zeta}{c_1} \frac{dc_1}{d\zeta}.$$

Finally, we have

$$\frac{dc_1}{d\zeta} \geq 1$$

$$\Leftrightarrow$$

$$2c_1 - \frac{1}{c_1} \left(t_D - \frac{\theta m g_D}{2} \right) \geq 2 \left(c_1 - \zeta \right) - \frac{\zeta}{c_1^2} \left(t_D - \frac{\theta m g_D}{2} \right).$$

A sufficient condition to ensure this last inequality is $\theta m g_D \ge 2t_D$.

D Proof of Proposition 5

$$\begin{split} \frac{\partial \Pi_1}{\partial K_1} &= \frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} K_1 - m \left[c_1^2 + w_1 \right] \frac{\partial n_1^D}{\partial K_1} = 0, \\ \frac{\partial \Pi_1}{\partial c_1} &= \frac{\partial n_1^P}{\partial c_1} K_1 - m \left[c_1^2 + w_1 \right] \frac{\partial n_1^D}{\partial c_1} - m c_1 = 0, \\ \frac{\partial \Pi_1}{\partial w_1} &= K_1 \frac{\partial n_1^P}{\partial w_1} - \frac{m}{2} - m \left[c_1^2 + w_1 \right] \frac{\partial n_1^D}{\partial w_1} = 0. \end{split}$$

From (8) and (10), we have respectively that

$$\frac{\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} K_1}{\frac{\partial n_1^D}{\partial K_1}} = m \left[c_1^2 + w_1 \right]$$

and,

$$\frac{\frac{\partial n_1^P}{\partial c_1}K_1 - mc_1}{\frac{\partial n_1^D}{\partial c_1}} = m \left[c_1^2 + w_1\right].$$

It gives

$$\left(\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} K_1\right) \frac{\partial n_1^D}{\partial c_1} = \left(\frac{\partial n_1^P}{\partial c_1} K_1 - mc_1\right) \frac{\partial n_1^D}{\partial K_1}$$

or,

$$K_1 = \frac{\frac{\partial n_1^D}{\partial c_1} + 2mc_1\frac{\partial n_1^D}{\partial K_1}}{2\left(\frac{\partial n_1^P}{\partial c_1}\frac{\partial n_1^D}{\partial K_1} - \frac{\partial n_1^P}{\partial K_1}\frac{\partial n_1^D}{\partial c_1}\right)}.$$

From (8) and (9), we have respectively that

$$\frac{\frac{1}{2} + \frac{\partial n_1^T}{\partial K_1} K_1}{\frac{\partial n_1^D}{\partial K_1}} = m \left[c_1^2 + w_1 \right]$$

and,

$$\frac{K_1\frac{\partial n_1^P}{\partial w_1} - \frac{m}{2}}{\frac{\partial n_1^D}{\partial w_1}} = m\left[c_1^2 + w_1\right].$$

It gives that

$$\left(\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} K_1\right) \frac{\partial n_1^D}{\partial w_1} = \frac{\partial n_1^D}{\partial K_1} \left(K_1 \frac{\partial n_1^P}{\partial w_1} - \frac{m}{2}\right)$$

or,

$$K_1 = \frac{\frac{\partial n_1^D}{\partial w_1} + m \frac{\partial n_1^D}{\partial K_1}}{2\left(\frac{\partial n_1^D}{\partial K_1} \frac{\partial n_1^P}{\partial w_1} - \frac{\partial n_1^P}{\partial K_1} \frac{\partial n_1^D}{\partial w_1}\right)}.$$

Combining with (D) yields

$$\frac{\frac{\partial n_1^D}{\partial c_1} + 2mc_1\frac{\partial n_1^D}{\partial K_1}}{2\left(\frac{\partial n_1^P}{\partial c_1}\frac{\partial n_1^D}{\partial K_1} - \frac{\partial n_1^P}{\partial K_1}\frac{\partial n_1^D}{\partial c_1}\right)} = \frac{\frac{\partial n_1^D}{\partial w_1} + m\frac{\partial n_1^D}{\partial K_1}}{2\left(\frac{\partial n_1^D}{\partial K_1}\frac{\partial n_1^P}{\partial w_1} - \frac{\partial n_1^P}{\partial K_1}\frac{\partial n_1^D}{\partial w_1}\right)}$$

or,

$$\begin{bmatrix} \frac{\partial n_1^D}{\partial c_1} + 2mc_1 \frac{\partial n_1^D}{\partial K_1} \end{bmatrix} \left(\frac{\partial n_1^D}{\partial K_1} \frac{\partial n_1^P}{\partial w_1} - \frac{\partial n_1^P}{\partial K_1} \frac{\partial n_1^D}{\partial w_1} \right)$$
$$= \left(\frac{\partial n_1^D}{\partial w_1} + m \frac{\partial n_1^D}{\partial K_1} \right) \left[\frac{\partial n_1^P}{\partial c_1} \frac{\partial n_1^D}{\partial K_1} - \frac{\partial n_1^P}{\partial K_1} \frac{\partial n_1^D}{\partial c_1} \right].$$

It gives

$$2mc_{1} = \frac{\left(\frac{\partial n_{1}^{D}}{\partial w_{1}} + m\frac{\partial n_{1}^{D}}{\partial K_{1}}\right)\left[\frac{\partial n_{1}^{P}}{\partial c_{1}}\frac{\partial n_{1}^{D}}{\partial K_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{\partial n_{1}^{D}}{\partial c_{1}}\right] - \frac{\partial n_{1}^{D}}{\partial c_{1}}\left(\frac{\partial n_{1}^{D}}{\partial K_{1}}\frac{\partial n_{1}^{P}}{\partial w_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{\partial n_{1}^{D}}{\partial w_{1}}\right)}{\left(\frac{\partial n_{1}^{D}}{\partial K_{1}}\frac{\partial n_{1}^{P}}{\partial w_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{\partial n_{1}^{D}}{\partial w_{1}}\right)\frac{\partial n_{1}^{D}}{\partial K_{1}}},$$

implying that

$$2mc_{1} = \frac{\left(\frac{\partial n_{1}^{D}}{\partial w_{1}} + m\frac{\partial n_{1}^{D}}{\partial K_{1}}\right)\left[\frac{\partial n_{1}^{P}}{\partial c_{1}}\frac{\partial n_{1}^{D}}{\partial K_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{\partial n_{1}^{D}}{\partial c_{1}}\right] - \frac{\partial n_{1}^{D}}{\partial c_{1}}\left[\frac{\partial n_{1}^{D}}{\partial K_{1}}\frac{\partial n_{1}^{P}}{\partial w_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{\partial n_{1}^{D}}{\partial w_{1}}\right]}{\left(\frac{\partial n_{1}^{D}}{\partial K_{1}}\frac{\partial n_{1}^{P}}{\partial w_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{\partial n_{1}^{D}}{\partial w_{1}}\right)\frac{\partial n_{1}^{D}}{\partial K_{1}}}{\frac{\partial n_{1}^{D}}{\partial c_{1}}\left(\frac{\partial n_{1}^{D}}{\partial w_{1}} + m\frac{\partial n_{1}^{D}}{\partial K_{1}}\right) - \frac{\partial n_{1}^{D}}{\partial c_{1}}\left(m\frac{\partial n_{1}^{P}}{\partial K_{1}} + \frac{\partial n_{1}^{P}}{\partial w_{1}}\right)}{\frac{\partial n_{1}^{D}}{\partial K_{1}}\frac{\partial n_{1}^{P}}{\partial w_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{\partial n_{1}^{D}}{\partial w_{1}}}.$$

The numerator gives

$$\begin{split} &\frac{\partial n_1^{P}}{\partial c_1} \left[\frac{\partial n_1^{D}}{\partial w_1} + m \frac{\partial n_1^{1}}{\partial K_1} \right] - \frac{\partial n_1^{P}}{\partial c_1} \left[m \frac{\partial n_1^{P}}{\partial K_1} + \frac{\partial n_1^{P}}{\partial w_1} \right] \\ &= \frac{m}{(4t_D t_P |B|)^2} \left[\left[2\zeta t_D - \theta \zeta m g_D + 4\zeta c_1^2 + c_1 \gamma g_D \right] \left[2t_P - (\gamma + \theta m) g_P + 4mc_1 \zeta \right] \right] \\ &+ \left[-2t_P c_1 + \gamma g_P c_1 - 4\zeta m c_1^2 - \theta g_P m \zeta \right] \left[-2t_D + (\gamma + \theta m) g_D + 4c_1 \zeta \right] \right] \\ &= \frac{m}{(4t_D t_P |B|)^2} \left(4t_P t_D \zeta - 2\zeta t_D (\gamma + \theta m) g_P + 2\zeta t_D 4mc_1 \zeta - \theta \zeta m g_D 2t_P \right. \\ &+ \theta \zeta m g_D (\gamma + \theta m) g_P - \theta \zeta m g_D 4mc_1 \zeta \\ &+ 4\zeta (c_1)^2 2t_P - 4\zeta (c_1)^2 (\gamma + \theta m) g_P + 4\zeta (c_1)^2 4mc_1 \zeta \\ &+ 4t_P t_D c_1 - 2t_P c_1 (\gamma + \theta m) g_D - 2t_P c_1 4c_1 \zeta \\ &+ 4t_P t_D c_1 - 2t_P c_1 (\gamma + \theta m) g_D - 2t_P c_1 4c_1 \zeta \\ &+ 4\zeta m c_1^2 2t_D - 4\zeta m c_1^2 (\gamma + \theta m) g_D - 4\zeta m c_1^2 4c_1 \zeta \\ &+ \theta g_P m \zeta 2t_D - \theta g_P m \zeta (\gamma + \theta m) g_D - \theta g_P m \zeta 4c_1 \zeta), \\ &= \frac{m}{(4t_D t_P |B|)^2} \left(\left(\zeta + c_1 \right) \left[4t_P t_D - 2t_D \gamma g_P - \theta m g_D 2t_P + 4mc_1 \zeta \left(2t_D - \theta \left(m g_D + g_P \right) \right) \right] \right), \end{split}$$

Therefore, we obtain

$$2mc_1 = \frac{\frac{\partial n_1^P}{\partial c_1} \left[\frac{\partial n_1^D}{\partial w_1} + m \frac{\partial n_1^D}{\partial K_1} \right] - \frac{\partial n_1^D}{\partial c_1} \left[m \frac{\partial n_1^P}{\partial K_1} + \frac{\partial n_1^P}{\partial w_1} \right]}{\frac{\partial n_1^D}{\partial K_1} \frac{\partial n_1^P}{\partial w_1} - \frac{\partial n_1^P}{\partial K_1} \frac{\partial n_1^D}{\partial w_1}}{\frac{\partial n_1^D}{\partial w_1}},$$

$$= m \left(\zeta + c_1 \right).$$

Then, we have

$$c_1 = \zeta.$$

Moreover, patients' price becomes

$$\begin{split} K_1 &= \frac{\frac{\partial n_1^D}{\partial w_1} + m \frac{\partial n_1^D}{\partial K_1}}{2\left(\frac{\partial n_1^D}{\partial K_1} \frac{\partial n_1^P}{\partial w_1} - \frac{\partial n_1^P}{\partial K_1} \frac{\partial n_1^D}{\partial w_1}\right)},\\ &= \frac{2t_P - \left(\gamma g_P - 4mc_1\zeta\right) - m\theta g_P}{2},\\ &= t_P - \frac{1}{2}\left(\gamma + m\theta\right)g_P + 2mc_1\zeta,\\ &= t_P - \frac{1}{2}\left(\gamma + m\theta\right)g_P + 2m\zeta^2. \end{split}$$

The salary is determined by

$$\begin{split} mw_1 &= \frac{\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} K_1}{\frac{\partial n_1^D}{\partial K_1}} - mc_1^2, \\ &= \frac{1}{-\theta g_P} \left(2t_D t_P - t_D \gamma g_P - t_P \theta m g_D + 2\zeta mc_1 \left(2t_D - \theta \left(m g_D + g_P \right) \right) \right. \\ &- \left[2t_D - \theta m g_D \right] \left(t_P - \frac{1}{2} \left(\gamma + m\theta \right) g_P + 2m\zeta^2 \right) \right) - m\zeta^2. \end{split}$$

Therefore, we obtain

$$w_1 = -t_D + \frac{g_D}{2} \left(\gamma + m\theta\right) + \zeta^2.$$

Hospitals' profit becomes:

$$\Pi_{1} = \frac{1}{2} \left[\hat{K}_{1} - m \left[(\hat{c}_{1})^{2} + \hat{w}_{1} \right] \right],$$

$$= \frac{1}{2} \left[t_{P} - \frac{1}{2} \left(\gamma + m\theta \right) g_{P} + 2m\zeta^{2} - m \left[\zeta^{2} - t_{D} + \frac{g_{D}}{2} \left(\gamma + m\theta \right) + \zeta^{2} \right] \right],$$

$$= \frac{1}{2} \left[t_{P} + mt_{D} - \frac{1}{2} \left(\gamma + m\theta \right) \left(g_{P} + mg_{D} \right) \right].$$

E Proof of Proposition 7

From the set of relevant first order conditions, we have:

$$\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} [K_1 - d_1] = mc_1^2 \frac{\partial n_1^D}{\partial K_1},$$

$$\frac{\partial n_1^P}{\partial c_1} [K_1 - d_1] - mc_1 = mc_1^2 \frac{\partial n_1^D}{\partial c_1},$$

$$-\frac{1}{2} + \frac{dn_1^P}{dd_1} [K_1 - d_1] = mc_1^2 \frac{dn_1^D}{dd_1}.$$

It gives

$$\frac{\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} \left[K_1 - d_1 \right]}{\frac{\partial n_1^D}{\partial K_1}} = mc_1^2$$

and,

$$\frac{-\frac{1}{2} + \frac{dn_1^P}{dd_1} \left[K_1 - d_1 \right]}{\frac{dn_1^D}{dd_1}} = mc_1^2,$$

therefore, we have

$$\left(\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1} \left[K_1 - d_1\right]\right) \frac{dn_1^D}{dd_1} = \frac{\partial n_1^D}{\partial K_1} \left(-\frac{1}{2} + \frac{dn_1^P}{dd_1} \left[K_1 - d_1\right]\right)$$

or,

$$K_1 - d_1 = \frac{\frac{dn_1^D}{dd_1} + \frac{\partial n_1^D}{\partial K_1}}{2\left(\frac{\partial n_1^D}{\partial K_1}\frac{dn_1^P}{dd_1} - \frac{\partial n_1^P}{\partial K_1}\frac{dn_1^D}{dd_1}\right)}.$$

Moreover, we have

$$\frac{\frac{\partial n_1^P}{\partial c_1} \left[K_1 - d_1 \right] - mc_1}{\frac{\partial n_1^D}{\partial c_1}} = mc_1^2.$$

It gives that

$$\left(\frac{\partial n_1^P}{\partial c_1}\left[K_1 - d_1\right] - mc_1\right)\frac{\partial n_1^D}{\partial K_1} = \frac{\partial n_1^D}{\partial c_1}\left(\frac{1}{2} + \frac{\partial n_1^P}{\partial K_1}\left[K_1 - d_1\right]\right)$$

or,

$$K_1 - d_1 = \frac{\frac{\partial n_1^D}{\partial c_1} + 2mc_1\frac{\partial n_1^D}{\partial K_1}}{2\left(\frac{\partial n_1^P}{\partial c_1}\frac{\partial n_1^D}{\partial K_1} - \frac{\partial n_1^D}{\partial c_1}\frac{\partial n_1^P}{\partial K_1}\right)}$$

Then, we have

$$\frac{\frac{dn_1^D}{dd_1} + \frac{\partial n_1^D}{\partial K_1}}{\left(\frac{\partial n_1^D}{\partial K_1} \frac{dn_1^P}{dd_1} - \frac{\partial n_1^P}{\partial K_1} \frac{dn_1^D}{dd_1}\right)} = \frac{\frac{\frac{\partial n_1^D}{\partial c_1} + 2mc_1\frac{\partial n_1^D}{\partial K_1}}{\left(\frac{\partial n_1^P}{\partial c_1} \frac{\partial n_1^D}{\partial K_1} - \frac{\partial n_1^D}{\partial c_1} \frac{\partial n_1^P}{\partial K_1}\right)}$$

or,

$$\begin{bmatrix} \frac{dn_1^D}{dd_1} + \frac{\partial n_1^D}{\partial K_1} \end{bmatrix} \left(\frac{\partial n_1^P}{\partial c_1} \frac{\partial n_1^D}{\partial K_1} - \frac{\partial n_1^D}{\partial c_1} \frac{\partial n_1^P}{\partial K_1} \right)$$
$$= \begin{bmatrix} \frac{\partial n_1^D}{\partial c_1} + 2mc_1 \frac{\partial n_1^D}{\partial K_1} \end{bmatrix} \left(\frac{\partial n_1^D}{\partial K_1} \frac{dn_1^P}{dd_1} - \frac{\partial n_1^P}{\partial K_1} \frac{dn_1^D}{dd_1} \right).$$

It can also be written

$$2mc_{1} = \frac{\left[\frac{dn_{1}^{D}}{dd_{1}} + \frac{\partial n_{1}^{D}}{\partial K_{1}}\right]\left(\frac{\partial n_{1}^{P}}{\partial c_{1}}\frac{\partial n_{1}^{D}}{\partial K_{1}} - \frac{\partial n_{1}^{D}}{\partial c_{1}}\frac{\partial n_{1}^{P}}{\partial K_{1}}\right) - \frac{\partial n_{1}^{D}}{\partial c_{1}}\left(\frac{\partial n_{1}^{D}}{\partial K_{1}}\frac{dn_{1}^{P}}{dd_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{dn_{1}^{D}}{dd_{1}}\right)}{\frac{\partial n_{1}^{D}}{\partial K_{1}}\left(\frac{\partial n_{1}^{D}}{\partial K_{1}}\frac{dn_{1}^{P}}{dd_{1}} - \frac{\partial n_{1}^{P}}{\partial K_{1}}\frac{dn_{1}^{D}}{dd_{1}}\right)}$$

or,

$$2mc_1 = \frac{\frac{\partial n_1^P}{\partial c_1} \left[\frac{dn_1^D}{dd_1} + \frac{\partial n_1^D}{\partial K_1} \right] - \frac{\partial n_1^D}{\partial c_1} \left[\frac{\partial n_1^P}{\partial K_1} + \frac{dn_1^P}{dd_1} \right]}{\left(\frac{\partial n_1^D}{\partial K_1} \frac{dn_1^P}{dd_1} - \frac{\partial n_1^P}{\partial K_1} \frac{dn_1^D}{dd_1} \right)}.$$

Let us now simplify the denominator. We have

$$\begin{aligned} &\frac{\partial n_1^D}{\partial K_1} \frac{dn_1^P}{dd_1} - \frac{\partial n_1^P}{\partial K_1} \frac{dn_1^D}{dd_1} \\ &= \frac{-1}{4t_D t_P |B|} \left[\left(\theta g_P + \frac{4}{m} d_1 \right) \right] \frac{1}{4t_D t_P |B|} \left[\gamma g_D + 4c_1 \zeta \right] \\ &+ \frac{1}{4t_D t_P |B|} \left[2t_D - \left(\theta m g_D - \frac{4}{m} d_1 \right) \right] \frac{1}{4m t_D t_P |B|} \left[2t_P - \left(\gamma g_P - 4m c_1 \zeta \right) \right], \\ &= \frac{1}{m \left(4t_D t_P |B| \right)}. \end{aligned}$$

The numerator gives:

$$\begin{aligned} \frac{\partial n_{1}^{P}}{\partial c_{1}} \left[\frac{dn_{1}^{D}}{dd_{1}} + \frac{\partial n_{1}^{D}}{\partial K_{1}} \right] &- \frac{\partial n_{1}^{D}}{\partial c_{1}} \left[\frac{\partial n_{1}^{P}}{\partial K_{1}} + \frac{dn_{1}^{P}}{dd_{1}} \right] \\ &= \frac{1}{(4t_{P}t_{D} |B|)^{2}} \left[\left(2\zeta t_{D} - \theta\zeta mg_{D} + \frac{4\zeta d_{1}}{m} + 4\zeta (c_{1})^{2} + c_{1}\gamma g_{D} \right) (2t_{P} - \gamma g_{P} + 4mc_{1}\zeta - \theta mg_{P} - 4d_{1}) \right. \\ &+ \left(2t_{P}c_{1} - \gamma g_{P}c_{1} + 4mc_{1}^{2}\zeta + \theta m\zeta g_{P} + 4\zeta d_{1} \right) \left(2t_{D} - (\theta m + \gamma) g_{D} + \frac{4}{m}d_{1} - 4c_{1}\zeta \right) \right], \\ &= \frac{(\zeta + c_{1})}{(4t_{P}t_{D} |B|)^{2}} \left(4t_{P}t_{D} - 2t_{D}\gamma g_{P} - \theta mg_{D}2t_{P} + 4mc_{1}\zeta (2t_{D} - \theta (mg_{D} + g_{P})) \right. \\ &+ \left. \frac{4}{m}d_{1} \left(2t_{P} - \gamma \left(g_{P} + mg_{D} \right) \right) \right), \\ &\left(\zeta + c_{1} \right) \end{aligned}$$

$$= \frac{(3+1)}{(4t_P t_D |B|)}.$$

Therefore, we have

$$2mc_1 = \frac{\frac{\partial n_1^P}{\partial c_1} \left[\frac{dn_1^D}{dd_1} + \frac{\partial n_1^D}{\partial K_1} \right] - \frac{\partial n_1^D}{\partial c_1} \left[\frac{\partial n_1^P}{\partial K_1} + \frac{dn_1^P}{dd_1} \right]}{\left(\frac{\partial n_1^D}{\partial K_1} \frac{dn_1^P}{dd_1} - \frac{\partial n_1^P}{\partial K_1} \frac{dn_1^D}{dd_1} \right)},$$

implying that $c_1 = \zeta$.

According to that

$$\begin{split} &\frac{\partial n_{1}^{P}}{\partial c_{1}}\frac{\partial n_{1}^{D}}{\partial K_{1}} - \frac{\partial n_{1}^{D}}{\partial c_{1}}\frac{\partial n_{1}^{P}}{\partial K_{1}} \\ &= \frac{1}{4t_{P}t_{D}|B|} \left[2\zeta t_{D} - \theta\zeta mg_{D} + 4\zeta \left(\frac{d_{1}}{m} + (c_{1})^{2}\right) + c_{1}\gamma g_{D} \right] \frac{-1}{4t_{D}t_{P}|B|} \left[\theta mg_{P} + 4d_{1} \right] \\ &+ \frac{1}{4t_{P}t_{D}|B|} \left[\left(2t_{P} - \left[\gamma g_{P} - 4mc_{1}\zeta\right]\right)c_{1} + \left(\theta g_{P} + \frac{4}{m}d_{1}\right)m\zeta \right] \frac{1}{4t_{D}t_{P}|B|} \left[\left(2t_{D} - \theta mg_{D} + \frac{4}{m}d_{1}\right) \right], \\ &= \frac{c_{1}}{\left(4t_{P}t_{D}|B|\right)^{2}} \left[2t_{P}2t_{D} - 2t_{D}\gamma g_{P} - 2t_{P}\theta mg_{D} + 4mc_{1}\zeta \left(2t_{D} - \theta \left(mg_{D} + g_{P}\right)\right) \right. \\ &+ \frac{4}{m}d_{1} \left(2t_{P} - \gamma \left(g_{P} + mg_{D}\right)\right) \right], \end{split}$$

we have

$$\begin{split} K_1 - d_1 &= \frac{\frac{dn_1^D}{dd_1} + \frac{\partial n_1^D}{\partial K_1}}{2\left(\frac{\partial n_1^D}{\partial K_1} \frac{dn_1^P}{dd_1} - \frac{\partial n_1^P}{\partial K_1} \frac{dn_1^D}{dd_1}\right)},\\ &= t_P - \frac{(\gamma + m\theta)\,g_P}{2} + 2mc_1\zeta - 2d_1. \end{split}$$

Moreover, combining (8) and (11), we have

$$\left(\frac{\partial n_1^P}{\partial K_1} + \frac{dn_1^P}{dd_1}\right) \left[K_1 - d_1\right] = mc_1^2 \left(\frac{\partial n_1^D}{\partial K_1} + \frac{dn_1^D}{dd_1}\right)$$

or,

$$\left(-\left(2t_D - \left(\theta m g_D - \frac{4}{m}d_1\right)\right) + \gamma g_D + 4c_1\zeta\right) [K_1 - d_1]\right)$$
$$= mc_1^2 \left(-\left(\theta g_P + \frac{4}{m}d_1\right) + \frac{1}{m}\left(2t_P - (\gamma g_P - 4mc_1\zeta)\right)\right).$$

Therefore, we obtain that

$$\tilde{d}_1 = -\frac{1}{2}mt_D + (\theta m + \gamma)\frac{mg_D}{4} + m\zeta^2$$

and,

$$\tilde{K}_1 = t_P + \frac{1}{2}mt_D - \frac{(\gamma + m\theta)(2g_P + mg_D)}{4} - \frac{1}{2}m\zeta^2.$$

At a symmetric equilibrium, hospitals' profit are:

$$\begin{split} \tilde{\Pi}_{1} &= \frac{1}{2} \left[\tilde{K}_{1} - \tilde{d}_{1} - mc_{1}^{2} \right] \\ &= \frac{1}{2} \left[t_{P} - \frac{(\gamma + m\theta) g_{P}}{2} + 2mc_{1}\zeta \right] \\ &- 2 \left(-\frac{1}{2}mt_{D} + (\theta m + \gamma) \frac{mg_{D}}{4} + m\zeta^{2} \left(1 - \frac{1}{2}m \right) \right) - m\zeta^{2} \right], \\ &= \frac{1}{2} \left[t_{P} + mt_{D} - \frac{(\gamma + m\theta) (g_{P} + mg_{D})}{2} \right] = \Pi_{1}^{*} = \Pi_{1}^{**}. \end{split}$$