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# Uniform output subsidies in an economic union with ...rms heterogeneity<sup>a</sup>

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#### Abstract

In this paper we show the importance of cost asymmetry and demand curvature in the exect of a uniform output subsidy policy in an economic union. We consider an economic union formed by two countries each with a single ...rm producing a homogeneous good. We ...nd that when ...rms have dixerent cost, the optimal level of the uniform subsidy can be negative if the demand is concave enough. The low cost ...rm expands its market share if the demand function is suf-...ciently convex whereas in the case of a concave demand function it is the higher cost ...rm which gains market share. This implies that a uniform output subsidy policy may cause a change in production e¢ciency. Finally, we consider how a divergence between private and social costs of public funds may axect the desirability of such a subsidy policy.

Key words: Uniform output subsidy policy, economic union, social welfare, cost di¤erences.

JEL classi...cation: F13, F15

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# 1 Introduction

This paper focuses on the implications of uniform output subsidies as a welfare maximising policy in an imperfectly competitive market. We consider the case of an economic union as an area formed by di¤erent countries or regions in which there is a common economic authority that decides the same level of subsidies for all the ...rms.<sup>1</sup> We assume the existence of two countries each with a single ...rm. Firms produce a homogeneous good with constant return to scale and compete in quantities. The objective of the common economic authority is to maximize the social welfare and we assume that the only instrument that it can use is an output subsidy. The question is how to do it. It is easy to see that a carefully chosen subsidy per unit of output to each ...rm is e¤ective in achieving an optimal outcome. However, it may be politically infeasible (it will be di⊄cult to justify di¤erent output subsidies for each country) or technically di⊄cult to apply di¤erential subsidies.

The use of output subsidies have been widely studied in the context of strategic trade policy by Brander and Spencer (1985), De Meza (1986), Neary (1994) and Collie (1993). Brander and Spencer (1985) were the ...rst to show that, in a Cournot duopoly setting, an export subsidy to a home ...rm is desirable because it raises the ...rm's market share and pro...ts at the expense of its foreign competitor. Further analysis of ...rm asymmetries can be found in De Meza (1986) and Neary (1994). Neary (1994) considers the problem in which the social cost of public funds exceeds unity, ...nding that they are optimal only for surprisingly low values of the social cost of public funds. Van Long and Soubeyran (1997) show that if domestic ...rms do not have identical cots, the optimal trade policy is determined by the interplay between the Her...ndahl index of concentration and the elasticity of demand. For an excellent survey on strategic trade policy see Brander (1995).

In this paper we consider the existence of an economic union with two producing countries with a ...rm in each country. Countries (...rms) 1 and 2, produce a homogeneous good for the common market. We assume that there is no trade of the good with the rest of the world, i. e. all production is consumed in the domestic market, in order to study in pure form the exect of the output subsidy.<sup>2</sup> We assume the existence of a common economic

<sup>&</sup>lt;sup>1</sup>For instance, the European Union has attempted to harmonise all aid being given by the governments of member states to the di¤erent industrial sectors. This was done to eliminate or reduce distortion of competition within the Community.

<sup>&</sup>lt;sup>2</sup>Note that our model is similar to the third-maket model of Brander and Spencer

authority that introduces an output subsidy. The economic authority has no other objective than to maximize the social welfare taking both producers and consumers into account. Hence, in this case, the economic authority chooses a certain level of subsidies, the same for the two ...rms, in terms of welfare maximization. We want to answer the following two questions: i) which is the optimal level of the uniform output subsidy policy? ii) which are the exect of the uniform output subsidy policy on output, market share and payoxs under ...rm heterogeneity?

We ...nd that the uniform output subsidy level can be negative (a tax) under ...rm heterogeneity when the demand function is su¢ciently concave. Assuming that the uniform output subsidy level is positive, we also ...nd that the optimal uniform output subsidy policy can a ect ... rms di erently depending on their cost di¤erences and on the curvature of the demand function. We distinguish three possibilities. First, for su¢ciently convex demand functions, the more e¢cient ...rm increases its market share and the output diperential with respect to the less eccient ...rm. Inside this region, for some cases the less e¢cient ...rm decreases its output. Second, for not too convex demand functions, both ...rms increase their output, the output dixerential increases but the less eccient ...rm increases its market share. Therefore, the output increases of the less eccient ...rm is proportionally greater than the one of the more e¢cient ...rm. Third, for concave demand functions, the output di¤erential decreases, the less e¢cient ...rm increases its market share and even, if the demand function is succiently concave, the more eccient ...rm decreases its output. In term of payo¤s, the more e¢cient ...rm always increase its payoxs with the introduction of the uniform output subsidy. The more striking conclusion is that when the cost di¤erences are su¢ciently large and in the case of convex demand functions, there exists situations for which the less eccient ...rm decreases its payo¤s and therefore the less eccient ...rm prefers no output subsidies!

Finally, we consider the (more realistic) possibility that the social cost of public funds exceeds unity. We ...nd that the shadow price of public funds depends positively on the cost di¤erences between the ...rms.

The structure of the paper is as follows. In Section 2 we present the model and the optimal uniform output subsidy in the economic union and its exect in term of output, market share and pro...ts of the two ...rms. Section 3

<sup>(1985)</sup> considering an economic union formed by the two producing countries plus the third (consuming) country.

considers the existence of a cost asymmetry between private and social costs of public funds. Section 4 concludes the paper.

#### 2 The model

We will concentrate on a quantity-setter model. There is a homogenous good produced by ...rms that have no objectives other than pro...ts. It is wellknown that under symmetry, the introduction of a subsidy increases social welfare and ...rms' payo¤. However, in general the level of output subsidies for each ...rm depends on the elasticity, on the degree of concavity of the inverse demand function and on the technology. In order to study when it is social welfare improving to introduce a uniform output subsidy when we have di¤erent ...rms we will assume that we have two ...rms, 1 and 2, with constant marginal cost functions denoted by  $C_1(x_1) = cx_1$  and  $C_2(x_2) = cx_2$ , where c > 0 and  $\bar{}$  1, i. e., without loss of generality we assume that ...rm 1 is more cost competitive than ... rm 2. We will assume that in each country or region it is located a ... rm. Let p be the market price of the product and p(x)be the inverse demand function mapping aggregate output into prices where x is total output. We assume that the decision on subsidies is irreversible and prior to decision of ... rms on output. This situation is model as a two-stage game. In stage 1, the social planner choose a subsidy level per unit of output. In stage 2, ... rms simultaneously choose output levels for the common market. Let s be the uniform subsidy per unit of output received by the ...rms where ...rms i's payo¤ functions is given by

$$U_{i}(x_{i}; x; s) = p(x)x_{i i} C_{i}(x_{i}) + sx_{i}$$
(1)

and the consumer surplus is given by

$$U_3(x) = V(x) i px$$
<sup>(2)</sup>

where V(x) is a strictly increasing function.

De...nition 1 A Cournot equilibrium is a list of outputs  $(x_1^{i_1}; ...; x_n^{i_n})$  such that for all i = 1; 2 we have that

$$p(x^{*})x_{i}^{*} i C_{i}(x_{i}^{*}) + sx_{i}^{*} p(x^{*} i x_{i}^{*} + x_{i})x_{i} i C_{i}(x_{i}) + sx_{i}$$
 8x<sub>i</sub> 2 <<sup>+</sup>

From now on we will assume that the Cournot equilibrium is interior. Social welfare (denoted by W) is the sum of consumer and producers's payo<sup>a</sup> functions net of the value of the subsidy payments

$$W = V(x)_{i} C_{1}(x_{1})_{i} C_{2}(x_{2})$$
(3)

Thus, we can de...ne an optimal allocation as follows:

De...nition 2 A level of subsidy so is optimal if it maximizes

$$V(x(s)) = C_1(x_1(s)) = C_2(x_2(s))$$
(4)

Our ...rst assumption restricts the payo¤ function of each ...rm to be twice continuously di¤erentiable (<sup>32</sup>):

Assumption 1.  $U_i(\mathfrak{c}) \ge 3^2$  for all i = 1; 2.

In order to simplify notation, when the context is clear, we will denote derivatives by primes, i.e.  $\frac{@p(x)}{@x} = p^{0}(x)$ , etc. The next assumption requires, on the one hand, that the inverse demand function be either concave or "not too" convex and, on the other hand, it bounds the degree of economies of scale.

Assumption 2. 2 + R  $_{,}$  0 where R =  $\frac{p^{00}(x)}{p^{0}(x)}x$  is a measure of the degree of concavity (convexity) of the inverse demand function.

Finally, the next assumption means that the inverse demand function p(x) is strictly decreasing, i.e. that V (x) is strictly concave.

Assumption 3.  $U_i(\mathfrak{k})$  is strictly decreasing on x given  $x_1$  and  $x_2$ , for all i = 1; 2.

Di¤erentiating ...rst order condition of pro...t maximization for ...rms with respect to individual output and subsidies we obtain:

$$\frac{dx_{i}}{ds} = i \frac{p^{0}(x)(1 + {}^{\textcircled{R}}_{i}R)\frac{dx_{j}}{ds} + 1}{p^{0}(x)(2 + {}^{\textcircled{R}}_{i}R)} \text{ for } i \notin j \text{ and } i; j = 1; 2$$
(5)

where  $\mathbb{B}_i = \frac{x_i}{x}$  is the market share of ...rm i. Solving the above system of equations, we get:

$$\frac{dx_{i}}{ds} = i \frac{1 + ({}^{\textcircled{B}_{j}} i {}^{\textcircled{B}_{i}}) R}{p^{\textcircled{0}}(x) (3 + R)} \text{ for } i \notin j \text{ and } i; j = 1; 2$$
(6)

and

$$\frac{dx}{ds} = i \frac{2}{p^{0}(x)(3+R)}$$
(7)

From expression (6) it follows that ...rm i's change in output depends on the di¤erences in market share between both ...rms in the case of non-linear demand functions. In the case of convex demand functions, ...rm i's change in output is larger as the market share of this ...rm is greater. In the case of concave demand functions, ...rm i's change in output is lower as the market share of this ...rms is greater. Note also that total output increases with the introduction of the uniform subsidy.

The social welfare, de...ned as the sum of the consumer and producers's payo¤s functions net of the subsidy cost, is given by

$$W = V(x) i C x_1 i C x_2$$
(8)

Di¤erentiating the social welfare with respect to s, we get:

$$\frac{dW}{ds} = i \frac{1}{p^{0}(x)(3+R)} [(2p(x)_{i} c_{i}^{-}c)_{i} R(^{(R_{1}}_{i} ^{(R_{2})}c(^{-}_{i} 1)]$$
(9)

Note that the sign of  $\frac{dW}{ds}$  in expression (9) depends on the sign of the expression in brackets. From the ...rst order condition of pro...t maximization for ...rms we know that the ...rst term of the expression in brackets is positive. Therefore, the change in the social welfare depends on the degree of concavity (convexity) of the inverse demand function and on the di¤erences in cost between the ...rms. Social welfare increases with the introduction of subsidies if the demand function is convex or linear (or for concave demand functions if the ...rms are su¢ciently similar). This is due to the fact that in the case of convex demand functions the gains in consumer surplus from lower prices are large compared to the cost of the subsidy. In the case of concave demands and ...rms su¢ciently di¤erent, we have that the gains in consumer surplus are small relative to the subsidy cost.

From expression (6) and the ...rst order condition of the welfare maximization problem we get the following expression for the optimal level of the uniform subsidy:

$$s = *p(x) \frac{1_{i} (\mathbb{R}_{1 i} \mathbb{R}_{2})^{2} R}{2} > 0 \qquad i^{\alpha} \qquad R < \frac{1}{(\mathbb{R}_{1 i} \mathbb{R}_{2})^{2}} \qquad (10)$$

where  $* = i \frac{p^0(x)}{p(x)}x$  is the elasticity of the inverse demand function. As it can be observed, the equilibrium uniform subsidy is positive for convex or linear demand functions. Note also that the uniform subsidy is always positive if ...rms are symmetric in costs. However, if the demand is concave (R > 0), and there are dimerences in costs such as  $(\mathbb{R}_1 i \mathbb{R}_2)^2 R > 1$ , expression (10) is negative (a tax)

Figure 1 shows the locus for a zero output subsidy as a function of the di¤erences in cost and the curvature of the demand function. If the ...rms are su¢ciently similar, the optimal level of subsidy is positive but for very concave demand function. On the contrary, if the ...rms are su¢ciently di¤erent the optimal level of subsidy is negative (a tax) for positive low values of R.

#### [Insert here Figure 1]

We now study the exects of the uniform output subsidy policy over the ...rms in terms of outputs, market shares and pro...ts under the assumption that the optimal level of subsidy is positive, i.e. either the demand function is not too concave or the cost dixerences between the ...rms are not too large or both. This policy shifts out the best-response functions of both ...rms, and then, total output increases, prices are driven downwards and therefore the consumer surplus increases. However, if ...rms have dixerent cost functions we can expect an asymmetric exect of this common policy. In fact, the introduction of the subsidies may increase the output of the less eCcient ...rm and even increase its market share. In this case, the economy as a whole would produce less eCciently. The next proposition presents the results in terms of the output of the ...rms.<sup>3</sup>

**Proposition 3** In the case of convex demand functions (R<0), the introduction of the uniform subsidy provokes an increase on the output of the lower cost ...rm and an increase on the di¤erences in output between the ...rms. In the case of concave demand functions (R>0), the introduction of the subsidy provokes an increase on the output of the higher cost ...rms and a decrease on the di¤erences in output between the ...rms. For linear demands (R=0), the introduction of the subsidy increases the output of the two ...rms by the same amount.

<sup>&</sup>lt;sup>3</sup>The same result was obtained by Van Long and Soubeyran (1997) when analysing the optimal trade policy in the case of domestic ...rms with non-identical unit costs.

**Proof.** From expression (6) we have that the change in output of ...rm 1 (the low cost ...rm) is given by:

$$\frac{dx_1}{ds} = i \frac{1 + \binom{\mathbb{R}_2}{2} i \frac{\mathbb{R}_1}{\mathbb{R}_1} R}{p^0(x) (3 + R)}$$
(11)

From A.3 and A.2', the denominator in the above expression is negative. Since  $({}^{\textcircled{B}_2}{}_i {}^{\textcircled{B}_1}) < 0$ , the above expression is positive for convex or linear demand functions. In the case of concave demand functions, expression (11) is also positive for  $R({}^{\textcircled{B}_1}{}_i {}^{\textcircled{B}_2}) < 1$ . Similarly, for ...rm 2 (the high cost ...rm) we have:

$$\frac{dx_2}{ds} = i \frac{1 + (\mathbb{B}_1 i \mathbb{B}_2) R}{p^0(x) (3 + R)}$$
(12)

where the above expression is positive for concave or linear demand functions. In the case of convex demand functions, expression (12) is positive for  $R(\mathbb{B}_{2i} = \mathbb{B}_1) < 1$ . From the ...rst order conditions of pro...t maximization for ...rms we have that the di¤erence in output between the ...rms is given by:

$$x_1 i \quad x_2 = i \quad \frac{c (-i 1)}{p^0 (x)}$$
 (13)

Di¤erentiating the output di¤erential with respect to the level of subsidy we get:

$$\frac{d(x_1 i x_2)}{ds} = \frac{c(-i 1)}{p^0(x)} R \frac{dx}{ds}$$
(14)

Since  $\frac{dx}{ds} = \frac{2}{p^0(x)(3+R)} > 0$  and  $p^0(x) < 0$ , the sign of the above expression depends on the curvature of the inverse demand function. The output di¤erential increases (decreases) for convex (concave) demand functions and it does not change for linear demand functions.

In the next proposition, we present the results in terms of the market shares of the ...rms.

**Proposition 4** The introduction of the uniform subsidy increases the market share of the low cost ...rm if R < i 1, increases the market share of the high cost ...rm if R > i 1 and does not change the market share of the ...rms for R = i 1.

**Proof.** From the ...rst order conditions of pro...t maximization for ...rms we have that the di¤erence in market share between the ...rms is given by:

<sup>®</sup><sub>1</sub> i <sup>®</sup><sub>2</sub> = i 
$$\frac{c(-i)}{p^{0}(x)x}$$
 (15)

Di¤erentiating the market share di¤erential with respect to the level of subsidy we get:

$$\frac{d\left({}^{\textcircled{R}}_{1} \right)}{ds} = \frac{c\left(\bar{1} \right)}{p^{0}(x)x^{2}} [R+1] \frac{dx}{ds}$$
(16)

Since  $\frac{dx}{ds} = i \frac{2}{p^0(x)(3+R)} > 0$  and  $p^0(x) < 0$ , the sign of the above expression depends on the curvature of the inverse demand function. The market share di¤erential increases (decreases) for R < i 1 (R > i 1) demand functions and it does not change for R = i 1.

The above two propositions shows that the optimal uniform output subsidy policy can a¤ect ...rms di¤erently depending on their cost di¤erences and on the curvature of the demand function. In Figure 2 we present the changes in output and market share of the ...rms as a function of the curvature of the demand function. We distinguish three regions. For R < i 1, the more e¢cient ...rm increases its market share and the output di¤erential with respect to the less e¢cient ...rm. Inside this region, if the demand function is su¢ciently convex, the less e¢cient ...rm decreases its output. For i 1 < R < 0, both ...rms increase their output, the output di¤erential increases but the less e¢cient ...rm is proportionally greater than the one of the more e¢cient ...rm. For R > 0, the output di¤erential decreases, the less e¢cient ...rm increases its market share and even, if the demand function is su¢ciently concave, the more e¢cient ...rm decreases its output.

#### [Insert here Figure 2]

**Proposition 5** The introduction of the uniform subsidy increases the payo¤s of the low cost ...rm, whereas the payo¤s of the high cost ...rm may decrease for su¢ciently convex demand functions if the ...rms are su¢ciently di¤erent.

**Proof.** The change in the payo¤s of ...rm 1 (the low cost ...rm) with respect to the level of subsidy is given by:

$$\frac{\mathrm{d}\mathsf{U}_1}{\mathrm{d}\mathsf{s}} = \frac{{}^{@}\mathsf{U}_1}{{}^{@}x_1}\frac{\mathrm{d}x_1}{\mathrm{d}\mathsf{s}} + \frac{{}^{@}\mathsf{U}_1}{{}^{@}x_2}\frac{\mathrm{d}x_2}{\mathrm{d}\mathsf{s}} + \frac{{}^{@}\mathsf{U}_1}{{}^{@}\mathsf{s}} \tag{17}$$

From the ...rm 1's ...rst order condition of pro...t maximization we have that  $\frac{@U_1}{@x_1} = 0$  and provided that  $\frac{@U_1}{@x_2} = p^{0}(x)x_1$  and  $\frac{@U_1}{@s} = x_1$ , we get:

$$\frac{dU_1}{ds} = x_1 \cdot \frac{2 + R + R(\mathbb{B}_2 | \mathbb{B}_1)}{3 + R} > 0$$
(18)

where the above expression is always positive for any value of R. Therefore, the payo¤s of the low cost ...rm always increase with the introduction of the uniform subsidy. In the case of ...rm 2 (the high cost ...rm) we get:

$$\frac{dU_2}{ds} = x_2 \frac{2 + R + R(\mathbb{B}_1 | \mathbb{B}_2)}{3 + R} \mathbf{Q} 0$$
(19)

where this expression can be negative if the demand function is su ciently convex and the cost di¤erences between the ...rm are su ciently high.

From Proposition 5 it can be drawn two conclusions. The more e¢cient ...rm always increase its payo¤s with the introduction of the uniform output subsidy. The more striking conclusion is that when the cost di¤erences are su¢ciently large and in the case of convex demand functions, there exists situations for which the less e¢cient ...rm prefers no output subsidies!

Finally, Propositions 3, 4 and 5 provide a simple explanation of the change in social welfare due to the introduction of the subsidy. The uniform output subsidy policy in the economic union produces two exects. On the one hand, the so-called policy coordination exect, which implies that total output increases, prices are driven downwards and therefore the consumer surplus increases. On the other hand, the so-called homogenization exect, which implies that the introduction of the output subsidy changes the output differential between the ...rms and their market shares changing the level of production ecciency in the economy as a whole. From Proposition 4 we know that if R > 1, the introduction of the subsidy makes the less e¢cient ...rm to increase his market share. Therefore, the economy as a whole produces less e $\bigcirc$  ciently. On the contrary if R < i 1 the more e $\bigcirc$  cient ... rm expands its market share and then, the optimal subsidy policy causes an increase in the total ecciency level of the union. If the cost dimerences between ...rms are low, the technological exect disappears. As the cost dixerences become larger, the technological exects also becomes more important. The sum of these two exects explains why the optimal policy in the economic union is a

tax when the demand is su¢ciently concave and cost asymmetry is large. In this situation an output subsidy causes a reduction in the total production e¢ciency level of the economic union together with the fact that the gains in consumer surplus are small relative to the subsidy cost when the demand is concave. Our results in terms of the optimal subsidy policy are di¤erent to the ones obtained by Van Long and Soubeyran (1997) due to the fact that consumers surplus is a component of our de...nition of the social welfare.

## 3 Private versus social cost of subsidies

In the preceding analysis private and social costs of public funds have been treated as equivalent. However, in the real world, raising subsidy revenue imposes distortionary costs on the economy, implying that the opportunity cost of a dollar of public funds might exceed 1. Neary (1994) considers the problem of a pro...t-shifting export subsidy in the third-market model in which the social cost of public funds exceeds unity. He ...nds that export subsidies are optimal only for surprisingly low values of the social cost of public funds (4/3 for a linear demand, with an upper limit of 2 for non-linear demands) and, if subsidies are justi...ed, they should be higher the more cost competitive the domestic ...rms are.

Following Neary (1994) we consider a weight parameter  $\pm$ , to the subsidy payments that may exceed unity, that is, the government places a greater weight on subsidy expenses than on private pro...t generation. Therefore, the social welfare of the union is de...ned as:

$$W = V(x)_{i} Cx_{1i} Cx_{2i} (\pm i) Sx_{1i} (\pm i) Sx_{2}$$
(20)

where  $\pm$  1; is the relative weight of subsidy cost. As we can observe, as  $\pm$  increases, the cost in term of welfare of the output subsidy increases. If  $\pm$  is su $\oplus$  ciently high, the optimal policy would be a tax instead of an output subsidy. Several arguments regarding this kind of asymmetry can be considered. First, Gruenspecht (1988) argues that it may re‡ect the deadweight cost of raising taxes elsewhere in the economy.<sup>4</sup> Brander (1995) notes that

<sup>&</sup>lt;sup>4</sup>Browning (1987) and Carmichael (1991) give estimations of the parameter  $\pm$  for the US economy. Browning (1987) under the assumption that subsidies are ...nanced by tax on labour earnings obtains a value of  $\pm$  between 1.10 and 4.03, with preferred estimates lying between 1.32 and 1.47. Carmichael (1991) obtains an estimate of 1.34 for credit subsidies for the export of Boeing 737-200 aircraft.

this will be the case when the government puts less weight on shareholders' welfare than on taxpayer's welfare for income distribution or other reasons. Second, the asymmetry between the private and social cost of subsidies can arise from the fact that domestic ...rms can be foreign-owned.<sup>5</sup> Neary (1991) gives a third explanation of  $\pm$  as the limited budget available to a public agency charged with allocating subsidies between a number of ...rms.

Di¤erentiating the social welfare function with respect to the subsidy yields:

$$\frac{dW}{ds} = \frac{@V}{@x}\frac{dx}{ds}_{i} c\frac{dx_{1}}{ds}_{i} - c\frac{dx_{2}}{ds}_{i} (\pm_{i} 1)x_{1i} (\pm_{i} 1)s\frac{dx_{1}}{ds}_{i} (\pm_{i} 1)x_{2i} (\pm_{i} 1)s\frac{dx_{2}}{ds} = 0$$
(21)

or equivalently:

$$[p_{i} c_{i} (\pm_{i} 1)s] \frac{dx_{1}}{ds} + [p_{i} c_{i} (\pm_{i} 1)s] \frac{dx_{2}}{ds} (\pm_{i} 1)x_{1i} (\pm_{i} 1)x_{2} = 0$$
(22)

Substituting expression (6) in equation (22) and from the ...rst order condition of pro...t maximization for ...rms we get:

$$\begin{bmatrix} i \ p^{0}(x)x_{1} \ i \ \pm S \end{bmatrix} \frac{\mathbf{h}_{p^{0}(x)(3+R)}^{\mathbf{i}} \mathbf{i}}{p^{0}(x)(3+R)} + \begin{bmatrix} i \ p^{0}(x)x_{2} \ i \ \pm S \end{bmatrix} \frac{\mathbf{h}_{p^{0}(x)(3+R)}^{\mathbf{i}} \mathbf{i}}{p^{0}(x)(3+R)}$$

$$= \begin{bmatrix} i \ 1 \end{bmatrix} \begin{bmatrix}$$

After some simpli...cations, we obtain the following general expression for the optimal level of subsidy:

$$s = i p^{0}(x)x \frac{4 i 3 \pm R i \pm R i R(\mathbb{R}_{1} i \mathbb{R}_{2})^{2}}{2 \pm}$$
(24)

If  $\pm = 1$ , the above expression reduces to expression (10). Therefore, under symmetry between the social and private costs of subsidies, the optimal subsidy level in an economic union is positive, except if the demand is too concave and the costs di¤erences are su¢ciently high. However, we can observe that as  $\pm$  rises above unity, the optimal output subsidy in the economic

 $<sup>^{5}</sup>$ Lee (1990) and Dick (1993) examine export subsidies with partly foreign-owned ...rms, showing that the optimal export subsidy is always smaller in the presence of cross ownership.

union is more likely to be negative. Following Neary (1994), we consider ...rst the linear case. In the case of a linear demand R = 0: The optimal subsidy reduces to:

$$s = i p^{0}(x)x \frac{4i 3\pm}{2\pm}$$
 (25)

which is positive if and only if  $\pm < 4=3$ . Therefore, in the linear case we obtain the same result as Neary (1994) for the non-cooperative export subsidies. In Neary's (1994) terminology, a subsidy is only justi...ed for ...rms which are more than 75 percent owned by domestic residents.

For non-linear demand, we can show that the threshold value of  $\pm$  at which the optimal subsidy switches from a positive to a negative value is:

$$\pm^{\alpha} = \frac{4 + R_{i} R(\mathbb{B}_{1 i} \mathbb{B}_{2})^{2}}{3 + R}$$
(26)

Comparing  $\pm^{*}$  with the threshold value in the linear case (4=3), we obtain:

$$\pm^{\pi} i \frac{4}{3} = \frac{i R i 3R(\mathbb{B}_{1} i \mathbb{B}_{2})^{2}}{9 + 3R}$$
(27)

The denominator is always positive since  $2 + R_{\odot} = 0$ . The numerator will be negative for concave demand and positive for convex demand. From this set of results we obtain the following two propositions:

**Proposition 6** The threshold value of  $\pm$  is lower than 4=3 for concave demand (R > 0) and greater than 4=3 for convex demand (R < 0).

**Proof:** In the case of a concave demand, the numerator of expression (27) is always negative. On the other hand, in the case of a convex demand, the numerator is always positive.  $\blacksquare$ 

**Proposition 7** In the case of symmetric production costs ( $^{-} = 1$ ), the maximum value for  $\pm$  is 2. As asymmetries in production costs increases, the value increases, with a maximum value for  $\pm$  of 4.

Proof: In the symmetric case,  ${}^{\mathbb{B}}{}_1 = {}^{\mathbb{B}}{}_2 = 1{=}2,$  so the expression reduces to

$$\frac{i R}{9 + 3R}$$
(28)

Taking the limit when R goes to  $_i$  2, we obtain an upper limit value of  $\pm$  of 2. As  $\mathbb{B}_1 \stackrel{\text{$\mathbb{R}}}{_1}$  goes to one, the expression reduces to:

$$\frac{i}{9+3R} \frac{4R}{9+3R}$$
(29)

For a value of R = i 2, the upper limit of  $\pm$  is 4, that is, the output subsidy is positive when the government values a dollar of its revenue between one and four dollars of private payo<sup>a</sup>. This value is twice that obtained by Neary (1994) for the non-cooperative pro...t-shifting export subsidy case.

Figure 3 shows the threshold value of the social cost of public funds as a function of the di¤erences in costs for some particular values of the curvature of the demand function. If the demand is concave, the threshold value decreases as di¤erences in costs increases. If the demand is convex, the threshold value increases as di¤erences in costs increases. Therefore, in our case the output subsidy is likely to be more positive than in the standard non-cooperative game when the demand is convex and ...rms have di¤erent costs. This high value for the social cost of public funds is explained by the fact that when di¤erences in costs are very high and the demand is convex, the subsidies cause large gains in consumer surplus relative to the subsidy cost. Ballard et al. (1985) suggest an opportunity cost in the range of 1.17 to 1.56 per dollar raised for the US economy. The maximum value that we obtain is larger than the empirically plausible values.

[Insert here Figure 3]

### 4 Conclusions

In this paper we show the importance of cost asymmetry and demand elasticity in the exect of a uniform output subsidy policy for the case of an economic union in which there are two ...rms producing a homogeneous good. We assume that the union is self-su¢cient in the good and that there is a common economic authority whose objective is to maximize social welfare. In so doing, it takes account of the interests of both ...rms and consumers in the union, using the sole instrument at its disposal: an output subsidy. We ...nd that the optimal output subsidy level may be negative (i.e. a tax) if cost dixerences are important and the demand is concave.

This policy causes two exects: the homogenization exect and the policy coordination exect. The ...rst exect causes an increase in total production

in the union, a fall in price and then, an increase in consumer surplus. The second exect implies that the uniform output subsidy policy in the economic union causes a change in the production e¢ciency level of the economy of the union as a whole. We ...nd that when  $R <_i 1$ , the low cost ...rm expands its market share whereas for  $R >_i 1$  it is the higher cost ...rm which expands its market share. Therefore, this policy axects the level of production e¢ciency in the economic union. However, we show that, in general, the two ...rms increase their pro...ts, except when the demand is convex and cost dixerences are important. In the latter case, the uniform subsidy policy causes a reduction in the pro...ts of the less e¢cient ...rm. This set of results shows the importance of ...rms heterogeneity across countries and the curvature of the demand functions in the design of an output subsidy policy in an economic union (i. e. this can be the case of the European Union).

Finally, we consider an asymmetry between private and social costs, that is, we take into account that the social cost of public funds can be larger than unity. We ...nd that in the case of a linear demand the threshold shadow price of public funds for the output subsidy to be positive is 4/3. This value increase as asymmetries in costs are larger and the demand is more convex, with a maximum value of 4. This value is larger than the empirical estimated values, suggesting that even after taking account of the distortionary exect of raising taxes, an output subsidy policy could be welfare improving in an imperfectly competitive market in the economic union context.

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**Figure 1.** Locus for a zero uniform output subsidy as a function of the curvature of the inverse demand function and market share differences.



Figure 2. Effects on output and market shares of the uniform output subsidy policy.



**Figure 3.** Threshold values of the social cost of the uniform output subsidy as a function of market share differences.