Contracts for experts with opposing interests*

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Abstract

We study the problem of optimal contract design in an environment with an uninformed decision maker and two perfectly informed experts. We characterize optimal contracts and observe that consulting two experts rather than one is always beneficial; this is so even if the bias of a second expert is arbitrary large and this expert would have no value in a cheap talk environment. We also provide conditions under which these contracts implement the first best outcome; our sufficient condition is weaker than the conditions in the literature on the environments without commitment. In order to derive optimal contracts, we prove a "constant-threat" result that states that one can restrict attention to contracts in which the action implemented in case of a disagreement among the experts is *independent* of their reports. A particular implication of this result is that an optimal contract is constant for a large set of experts' preferences and hence is robust to mistakes in their specification.

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1 Introduction

A growing body of literature studies optimal contracting between a decision maker and a single biased expert who has decision relevant information.[1] This paper focuses on environments in which the decision maker can obtain a second opinion from another expert. We are interested in the following questions: What is the value of a second opinion and what is the structure of optimal contracts?

We consider a model with an uninformed decision maker and two perfectly informed experts. The set of actions available to the decision maker is a unit interval. The experts are strategic and biased in different directions.[2] The experts' information is not verifiable, i.e., communication is cheap talk. The decision maker can commit to an action rule that is contingent on the reports of the experts. (This commitment assumption distinguishes our model from the literature on cheap talk communication.)[3]

Our first result is that adding a second expert is always valuable for the decision maker: In our model, the optimal contract improves the payoff of the decision maker relative to what she would obtain with one expert (Proposition 3). This holds *regardless* of the magnitude of the biases of the experts. Hence, there is a clear sense in which two experts are *complementary*.[4] In particular, this observation is true even if the biases of the experts are sufficiently large and the experts are not valuable without commitment, i.e., no information can be obtained from the experts through cheap-talk communication.[5]

[1]Holmström (1977, 1984), Melumad and Shibano (1991), Martimort and Semenov (2006), Alonso and Matouschek (2008), Goltsman et al. (2009), Kovac and Mylovanov (2009), Armstrong and Vickers (2008), Koessler and Martimort (2009), Li and Li (2009), and Lim (2009) study optimal contracts in environments in which contingent monetary transfers are not feasible. In Baron (2000), Krishna and Morgan (2008), Bester and Krähmer (2008), Raith (2008), and Ambrus and Egorov (2009), the optimal contracts are characterized for environments in which the decision maker can commit to monetary payments that are contingent on the expert's recommendation.

[2]We discuss the case of similarly biased experts in Section 5.

[3]Crawford and Sobel (1982) is the seminal reference on cheap talk communication with one expert. Cheap talk communication with two experts has been studied in Krishna and Morgan (2001a,b), Battaglini (2002, 2004), Ambrus and Takahashi (2008); Ambrus and Lu (2009), and Li (2008, 2009)

[4]Krishna and Morgan (2001b) consider sequential cheap talk communication with two experts and show that there is a value to a second expert; however, this is so only if at least one of the experts has a relatively small bias. In a model with two experts, discrete outcome, action spaces, Li (2008) demonstrates that the value of the second expert may be negative.

[5]It is well known from Battaglini (2002) and Ambrus and Takahashi (2008) that full revelation of experts information can be achieved if the biases of the experts are not too large relative to the outcome

Similarly to the cheap talk models, existence of a second expert allows the decision maker to implement the *first best* outcome in some environments. We provide conditions (Proposition 2 and Remark 1) for the first best to be implementable; the conditions bound the size of bias of each expert. These conditions are related but *weaker* than those required to implement the first best outcome in cheap talk environments (Krishna and Morgan (2001a), Battaglini (2002), and Ambrus and Takahashi (2008)).

Our characterization of optimal contracts relies on the "constant-threat" result (Proposition 1) that states that we can restrict attention to constructions in which any disagreement between experts results in a lottery that is independent of their reports. This result significantly simplifies the design problem, and we view it as the main technical contribution of the paper. The commitment assumption in our model is essential for this result. By contrast, the constructions of, e.g., first best outcomes in cheap talk environments might have to implement actions after a disagreement between experts that depend non-trivially on their reports.

A particular implication of the constant-threat result is that an optimal mechanism might be constant across a large set of environments that differ in preferences of the experts and the distribution of private information (Corollary 1). This observation is valuable if the decision maker is concerned about robustness of the optimal mechanism with respect to these details of the environment.

In this paper, we assume that both experts are perfectly informed. This assumption is common in the related literature. Yet, it is an important assumption in that it allows the decision maker to check the reports of the experts against each other, and inconsistent reports do not occur on the equilibrium path. The question of robustness to noise has been addressed in Battaglini (2004) within a model with multiple experts, a multidimensional environment, and noisy signals. In particular, the paper shows that if the decision maker has some commitment power, it becomes possible to achieve the first best outcome in the limit as the number of experts increases. We discuss robustness of our results to noise in Section 5.

The problem of optimal contracting with two experts has also been studied in Martimort and Semenov (2008). Our models and approaches are quite different. Martimort and Semenov (2008) consider two experts who hold different independently distributed private information and, unlike in our paper, are biased, at least in expectation, in the same

space. By contrast, in the model we study no information transmission is possible through cheap talk if the biases of the experts are sufficiently large.

direction. Furthermore, their paper employs dominant strategy implementation whereas our solution concept is Nash equilibrium. Among their results, Martimort and Semenov (2008) demonstrate impossibility of the first best outcome and show that a sufficiently high bias renders the experts not valuable for the decision maker.

The remainder of the paper is organized as follows. Section 2 describes the model. The constant-threat principle is derived in Section 3. Applying this principle, we characterize the optimal contracts in Section 4. In Section 5, we discuss the robustness of the contracts with respect to noise and describe the optimal contract for the environment with similarly biased experts.

2 The Model

There are two experts $i = 1, 2$ and a decision maker. The decision maker has to select an action from set $X = [0, 1]$ of feasible actions. The most preferred action for the decision maker (the *state*), $x \in X$, is a realized value of a random variable $\tilde{x}$ with support on X. The decision maker is uninformed about x and believes that the distribution of $\tilde{x}$ is represented by a cumulative distribution function F.

The experts know the value of x. The decision maker can ask them for recommendations and commit to take an action that is contingent on their reports.

Let y denote an action. The payoff function of the decision maker is $u_0(x, y)$ and the payoff functions of expert $i = 1, 2$ is $u_i(x, y)$. We assume that for every $x \in X$ each function $u_i(x, y)$, $i = 0, 1, 2$, is strictly concave in y.

The decision maker's payoff function is maximized at the action equal to the state,

$$\arg\max_{y \in X} u_0(x, y) = x, \;\; x \in X.$$

For every $x \in X$ we define $y_i^*(x) = \arg\max_{y \in X} u_i(x, y)$, $i = 1, 2$. We assume that the experts have *opposing interests*:

$$y_1^*(x) < x < y_2^*(x), \quad \text{for every } x \in X. \tag{1}$$

Some of our results are obtained for the environment with quadratic preferences and fixed biases, which is standard in the literature on experts: $u_0(x, y) = -(x - y)^2$, $u_1(x, y) = -((x - b_1) - y)^2$ and $u_2(x, y) = -((x + b_2) - y)^2$, where $b_1, b_2 > 0$.

Let $\mathcal{X}$ denote the set of distributions on X (*randomized actions*). Identifying point distributions with points we have $X \subset \mathcal{X}$. We extend the definition of u_i to $X \times \mathcal{X}$ via the statistical expectation:

$$u_i(x,\mu) = \int u_i(x,y)\lambda(dy), \quad x \in X,\ \lambda \in \mathcal{X}.$$

A contract is a measurable function

$$\mu: \quad X^2 \to \mathcal{X}, \quad (x_1,x_2) \mapsto \mu(x_1,x_2),$$

where $\mu(x_1,x_2)$ is a randomized action that is contingent on the experts' reports (x_1,x_2). A contract induces a *game* (a direct mechanism), in which after observing x the experts simultaneously make reports $x_1,x_2 \in X$ and the outcome $\mu(x_1,x_2)$ is implemented.

A contract μ is *incentive compatible* if truth-telling, $x_1 = x_2 = x$, is a Nash equilibrium: for all $x,x' \in X$

$$\begin{aligned} u_1(x,\mu(x,x)) &\geq u_1(x,\mu(x',x)), \\ u_2(x,\mu(x,x)) &\geq u_2(x,\mu(x,x')). \end{aligned} \tag{2}$$

By the revelation principle, any equilibrium outcome of the experts' interaction in a game whose space of outcomes is X or $\mathcal{X}$ can be represented by the truth-telling equilibrium outcome in some incentive compatible contract. In what follows, we will consider only incentive compatible contracts.

A contract μ is *optimal* if it maximizes the expected payoff of the decision maker,

$$v^\mu = \int_X u_0(x,\mu(x,x))dF(x),$$

among all incentive compatible contracts. Since the set of incentive compatible contracts is compact in weak topology and v^μ is continuous in μ, an optimal contract exists.

3 The Constant-threat Principle

In any contract, the main incentive issue is to motivate each expert to agree with the other expert who is expected to tell the truth. Therefore, the contract must punish disagreements. The difficulty here is that (i) *a priori* it is unclear which expert, if any, tells the truth, and (ii) since the experts have opposing interests, a punishment that is more severe for one of the experts tends to benefit the other expert. As a result, a threat lottery that results after a disagreement may depend non-trivially on the experts' reports.

We now prove our main technical result, the constant-threat principle, which allows us to characterize optimal contracts. It states that one can restrict attention to contracts in which the lottery implemented after a disagreement has support on extreme actions 0 and 1 and is *independent* of the reports. This result reduces the problem of finding optimal contracts to the problem of finding actions that are implemented if the experts report their information truthfully, $\mu(x, x)$, and the probability of implementing $y = 1$ after a disagreement. Thus, it drastically decreases complexity of the design problem because we avoid the optimization problem in which we search on a continuum of lotteries with support on X that are implemented after a disagreement (one threat lottery for each pair of reports $x_1, x_2 \in X, x_1 \neq x_2$).

The idea behind the constant threat principle is as follows. First, by concavity of the experts' payoff functions, any lottery over actions implemented after a disagreement can be replaced, using a mean-preserving spread, by a lottery between actions 0 and 1 without affecting the experts' incentives to report truth.

Now, let μ be a contract in which a disagreement always results in a lottery between 0 and 1. The crucial step in the the proof is to observe that, say, expert 1 (who is left-biased) always prefers action x to the extreme right action 1. Hence, in all states where a disagreement lottery is better than x, his payoff from action 0 must be strictly greater than that from x. It follows that in these states his expected payoff from that lottery must be *decreasing* in the probability assigned on action 1 (similarly, the payoff of expert 2 from a disagreement lottery must be *increasing* in the probability assigned on action 1). Let $\underline{r}$ be the lottery that achieves the highest payoff for expert 1 among the lotteries that can be achieved by the best deviations of expert 1 in various states $x \in X$. Denote by $\underline{p}$ the probability this lottery assigns to action 1. Define $\overline{p}$ for expert 2 analogously. The result now follows from the observation that $\underline{p} \leq \overline{p}$, which is nothing else than the argument that minimax is larger than or equal to maximin. Hence, there exists a lottery r^c that assigns probability p^c to action 1, where $\underline{p} \leq p^c \leq \overline{p}$, such that replacing every threat lottery with r^c does not violate the incentive constraints of the experts.

Let $\mathcal{X}^*$ be the set of probability distributions with support on $\{0, 1\}$. We say that a contract $\mu = (\mu, z_1, z_2)$ is *constant-threat* if

(C) there exists $c \in \mathcal{X}^*$ such that $\mu(x_1, x_2) = c$ whenever $x_1 \neq x_2$.

We say that two incentive compatible contracts, μ and μ', are *equivalent* if they implement the same action whenever the reports of the experts coincide, i.e., $\mu(x, x) = \mu'(x, x)$ for all $x \in X$. Thus, two equivalent contracts implement identical actions *in equilibrium*, but may implement different actions off-equilibrium.

Proposition 1 (Constant-threat principle) *For every optimal contract there exists an equivalent constant-threat contract.*

Note that a constant-threat contract which is equivalent to some optimal contract must be optimal as well, since it implements the same actions in equilibrium.

Proof. Let μ be an optimal contract. Observe that by concavity of $u_i(x, y)$ in y, $i = 1, 2$, for any measure λ,

$$\int u_i(x,y)\lambda(dy) \geq \left(\int y\lambda(dy)\right) u_i(x,0) + \left(1 - \int y\lambda(dy)\right) u_i(x,1), \quad x \in X.$$

Hence, replacing $\mu(x_1, x_2)$, $x_1 \neq x_2$, by a lottery that puts probability $\int y\mu(x_1, x_2)(dy)$ on action 0 and the complementary probability on action 1 will not violate the incentive constraints of the experts. Therefore, there exists an equivalent contract μ' in which every threat lottery implemented after a disagreement has support on $\{0, 1\}$.

We now show that there exists a constant threat contract μ^c equivalent to μ'. For every pair of different reports, $x_1, x_2 \in X, x_1 \neq x_2$, let $p(x_1, x_2)$ be the probability that $\mu(x_1, x_2)$ in μ' assigns to 1 after a disagreement. We extend the definition of $p(\cdot, \cdot)$ to X^2 by setting $p(x, x) = 1 - \int y\mu(x, x)(dy)$ for all $x \in X$. Define

$$\mathcal{P}_1(x) = \{p(x', x) | x' \in X\} \text{ and } \mathcal{P}_2(x) = \{p(x, x') | x' \in X\}.$$

Let

$$D_i(x,p) = \max\{0, pu_i(x,1) + (1-p)u_i(x,0) - u_i(x, \mu(x,x))\}, \quad p, x \in X, i = 1, 2.$$

By construction, a deviation by expert i in state x leading to a lottery $p' \in X$ is non-profitable iff $D_i(x, p') = 0$. Furthermore, by definition of $p(x, x)$,

$$D_i(x, p(x,x)) = 0, \quad x \in X, i = 1, 2.$$

Thus, incentive constraints (2) can be written as

$$D_i(x,p) = 0, \quad x \in X, \ p \in \mathcal{P}_i(x), \ i = 1, 2. \tag{IC}$$

We now show that

$$\begin{aligned} &D_1(x,p) \text{ is non-increasing in } p \text{ for every } x \in X; \\ &D_2(x,p) \text{ is non-decreasing in } p \text{ for every } x \in X. \end{aligned} \tag{*}$$

We start by showing that we can restrict attention to contracts that on the equilibrium path are deterministic and implement actions that are bounded by the experts' most preferred actions,

$$\mu(x,x) \in [0,1], \quad y_1^*(x) \leq \mu(x,x) \leq y_2^*(x), \quad x \in X. \tag{P_3}$$

To see why this is true, fix some $x' \in X$ and suppose first that $\mu(x',x')$ is a proper lottery. Then, concavity of the payoff functions implies that replacing $\mu(x',x')$ with the expected value of this lottery improves the payoffs of all players without violating any incentive constraints. Next, let $\mu(x',x') = y' \in X, y' > y_2^*(x')$ for some $x' \in X$. Since $y_2^*(x')$ is closer than y' to the most preferred alternatives of all players, concavity of the payoff functions implies that setting $\mu(x',x') = y_2^*(x')$ improves the payoffs of all parties on the equilibrium path without violating incentive constraints.

Since $u_1(x,y)$ is concave in y and $y_1^*(x) \leq \mu(x,x)$ by (P_3), it follows that $u_1(x,y)$ is decreasing in y on $[\mu(x,x),1]$ for every x, and hence

$$u_1(x,\mu(x,x)) \geq u_1(x,1).$$

If, in addition, $u_1(x,\mu(x,x)) \geq u_1(x,0)$, then, $D_1(x,p) = 0$ for every $p \in [0,1]$. On the other hand, if $u_1(x,\mu(x,x)) < u_1(x,0)$, then $u_1(x,1) < u_1(x,0)$ and, hence $u_1(x,p)$ and $D_1(x,p)$ are decreasing in p. This establishes the first statement in (*). The argument for the second statement is analogous.

Next, let

$$\begin{aligned} a_1(x) &= \inf \mathcal{P}_1(x), \quad x \in X; \\ a_2(x) &= \sup \mathcal{P}_2(x), \quad x \in X. \end{aligned}$$

By (IC) and continuity of u_i, we have $D_i(x,a_i(x)) = 0$ for $x \in X$. By (*),

$$\begin{aligned} D_1(x,p) = &\ 0, \quad p \geq a_1(x), \ x \in X; \\ D_2(x,p) = &\ 0, \quad p \leq a_2(x), \ x \in X. \end{aligned} \tag{3}$$

Define

$$\begin{aligned} \underline{p} &= \sup_{x \in X} a_1(x) = \sup_{x \in X} \inf P_1(x) = \sup_{x \in X} \inf_{x' \in X} p(x',x); \\ \overline{p} &= \inf_{x \in X} a_2(x) = \inf_{x \in X} \sup P_2(x) = \inf_{x' \in X} \sup_{x \in X} p(x',x). \end{aligned}$$

Then, there exists p^c such that $\underline{p} \leq p^c \leq \overline{p}$. By (3),

$$D_i(x, p^c) = 0, \quad x \in X, i = 1, 2.$$

The result now follows from (IC). ■

The result in Proposition 1 can be generalized. We say that an incentive compatible contract is *undominated* if there does not exist another incentive compatible contract that yields to all players a greater (equilibrium) payoff in every state and a strictly greater payoff in some state. The arguments behind Proposition 1 are not affected if we consider undominated contracts instead of optimal contracts.

In the remainder of the paper, we will study optimal contracts in the set of constant threat contracts. Typically, however, there exist contracts that induce the same equilibrium outcome and are not constant threat.

Finally, we would like to remark on the multiplicity of equilibria in the constant threat contracts. In this paper, we focus on the truthtelling equilibria; this is justified by the revelation principle. At the same time, there can be many other equilibria in a given contract. For example, consider a constant threat contract in which a disagreement results in a lottery that mixes between 0 and 1 with equal probability and which implements $y = 1/2$ if both experts report x'. For this contract, it is an equilibrium for the experts to report $x_1 = x_2 = x'$ regardless of the state.

4 Optimal Contracts

Let $\mathcal{C}$ be the set of incentive compatible constant-threat contracts. By Proposition 1, there exists an optimal contract in $\mathcal{C}$. In this section, we characterize these contracts.

4.1 First Best Contracts

We start our analysis of optimal contracts by identifying conditions under which they implement the most preferred alternative of the decision maker. A contract in $\mathcal{C}$ that in each state implements the most preferred action for the decision maker, if it exists, is called *first best*.

We assume that each expert's utility depend only on the difference between her most preferred action and the implemented action: for each $i = 1, 2$

$$u_i(x, y) = -d_i(y - (x - b_i(x))), \tag{4}$$

where $b_i : X \mapsto \mathbb{R}$, $b_2(x) < 0 < b_1(x)$ for all $x \in X$, and $d_i : \mathbb{R} \mapsto \mathbb{R}$ is a convex differentiable function that achieves its minimum at 0, $i = 1, 2$. The point $x - b_i(x)$ is the most preferred action of i in state x. The values of b_1 and b_2 reflect the conflict of preferences between the experts and the decision maker and are called the experts' *biases.*

The next result provides a sufficient condition for existences of the first best contract under these assumptions.

Proposition 2 *Let (4) hold. There exists the first best contract if* $\sup_{x\in X, i=1,2} |b_i(x)| \leq 1/2$.

Proof. There exists the first best contract if and only if there is $p \in [0, 1]$ such that for each expert $i = 1, 2$ and for every $x \in X$,

$$u_i(x, x) \geq (1 - p)u_i(x, 0) + pu_i(x, 1). \tag{5}$$

By convexity of d_i, we have for $i = 1, 2$ and for every $x \in X$,

$$\begin{aligned} \frac{d_i(x - b_i(x))}{2} + \frac{d_1(1 - x + b_i(x))}{2} &\geq d_1\left(\frac{x - b_i(x)}{2} + \frac{1 - x + b_i(x)}{2}\right) \\ &= d_i(1/2) \geq d_i(b_i(x)), \end{aligned} \tag{6}$$

where the second inequality follows from the assumption that $\sup_{x\in X, i=1,2} |b_i(x)| \leq 1/2$. Observe that (6) is equivalent to (5) with $p = 1/2$, which implies existence of the first best contract with the treat lottery that puts equal probabilities on 0 and 1. ■

The first best contract constructed in the proof of Proposition 2 uses as a threat point the lottery that mixes with equal probability between 0 and 1. The logic behind the construction is straightforward: if the experts' biases are not too large, they are better off under the decision maker's most preferred alternative rather than the threat lottery. It is interesting to note that the optimal threat lottery is symmetric even if the experts' biases are not equal and their payoff functions are not symmetric.

Under some additional structure on the payoff functions, the sufficient condition in Proposition 2 becomes necessary.

Remark 1 Assume that d_i is symmetric around 0 and that $b_i(x) = \tilde{b}_i$ is constant, $i = 1, 2$. Then, there does not exist the first best contract whenever $\max |\tilde{b}_i| > 1/2$.

Proof. Assume that $\tilde{b}_1 > 1/2$. First, let $p < 1$. Then,

$$(1 - p)d_1(1 - \tilde{b}_1) + pd_1(\tilde{b}_1) < (1 - p)d_1(\tilde{b}_1) + pd_1(\tilde{b}_1) = d_1(\tilde{b}_1),$$

which contradicts (5) for $x = 1$ and $i = 1$. On the other hand, if $p = 1$, then for $x = \max\{1 + \tilde{b}_2, 0\}$,

$$d_2(1 - (x - \tilde{b}_2)) < d_2(\tilde{b}_2),$$

which contradicts (5) for $i = 2$. The argument for $\tilde{b}_2 < -1/2$ is symmetric. ■

The above results are related to Krishna and Morgan (2001a), Battaglini (2002), and Ambrus and Takahashi (2008) who study cheap talk communication with two experts. For the environment considered in Remark 1, Proposition 1 in Battaglini (2002) establishes that a necessary and sufficient condition for a fully revealing cheap talk equilibrium is that the sum of the absolute values of the experts' biases is less than half of the measure of the action space.[6] Proposition 2 and Remark 1 complement this result by providing necessary and sufficient conditions for the first best outcome under commitment. Our condition is weaker and it bounds the size of *each* expert's bias rather than their *sum*; interestingly, the value of the bound is the same in both environments.

The construction of fully revealing equilibria in cheap talk and our construction of a first best contract are analogous but not identical. In a cheap talk environment, for any pair of disagreeing reports there is a threat action such that an expert who can induce this pair of reports prefers the first best outcome to the threat action. This threat action is supported by (out-of-equilibrium) beliefs that make it optimal. The proof then verifies that for each pair of states (reports) there exists a threat action that satisfies a number of inequalities that depend on biases of the experts; in equilibrium, the threat action might have to depend non-trivially on the reports of the experts.

By contrast, in our model a contract can use lotteries as threat actions that cannot be supported in a cheap talk model, even out of equilibrium.[7] The proof of the possibility of the first best in our environment employs a constant threat lottery that mixes equally between the extreme actions and makes use of concavity property of payoff functions. Furthermore, the proof of the necessary condition relies on the fact that it is sufficient to consider report-independent threat lotteries.

[6] Krishna and Morgan (2001a) provide a sufficient condition for a fully revealing cheap talk equilibria in an environment with constant and equal opposing biases that each expert's bias is less than 1/4.

[7] The concavity of payoff functions implies that a lottery cannot be a best response for the decision maker.

4.2 Robustness of the First Best Contract

An interesting implication of the above results is that the lottery that mixes between 0 and 1 with equal probability is the most effective threat lottery for implementing the first best if the payoff functions are symmetric and the biases are not too large. This is so even if the experts' biases are *not* symmetric.

Corollary 1 *Let the conditions in Proposition 2 be satisfied. Then, the first best contract is constant in the preferences of the experts.*

The constancy of the optimal contract is a useful feature if the decision maker is concerned about robustness of the contract with respect to her knowledge of the environment. In particular, if the optimal contract is constant, then the decision maker need not possess correct knowledge about the magnitude and the direction of the experts' biases, or the distribution of their information.

4.3 Second Best Contracts

What are the properties of an optimal contract if the first best outcome cannot be implemented? In what follows, we characterize optimal contracts that, given the threat lottery, maximize the payoff of the decision maker in each state.[8]

Observe that any contract in $\mathcal{C}$ can be identified by a pair

$$(p, g): \quad p \in [0,1], \quad g : X \to X,$$

where p is the probability of action 1 after a disagreement and $g(x)$ is the action implemented on the equilibrium path.

Let us pick a constant-threat contract (p, g) in $\mathcal{C}$. By concavity of payoff functions, both experts prefer $y = x$ in state $x = p$ to the treat lottery,

$$u_i(p, p) \geq p u_i(p, 1) + (1 - p) u_i(p, 0).$$

This implies that an optimal contract implements the most preferred alternative for the decision maker, $g(x) = x$, at least in state $x = p$. In addition, if the experts' payoff functions are strictly concave, we obtain $g(x) = x$ for a proper interval containing p.

[8] Trivially, there also exists a continuum of other contracts that deliver the same expected payoff for the decision maker but do not have this property for a set of states of measure zero.

Proposition 3 *An optimal contract implements the most preferred alternative of the decision maker in a non-empty set of states. If the experts' payoffs are strictly concave, this set is not a singleton.*

This observation highlights the value of two experts for the decision maker. Two experts are always valuable because there exists a contract which implements the most preferred action of the decision maker at least in some states. This is true regardless of the degree of conflict of preferences between the experts and the decision maker.

We now describe the structure of an optimal contract in states where the first best outcome is not incentive compatible. For a given probability p of action 1 in an optimal constant-threat contract, let $\tilde{X}_i^p$ be the set of states in which expert i weakly prefers the threat lottery to the decision maker's most preferred action,

$$\tilde{X}_i^p = \{x \in [0,1] \,:\, u_i(x,x) < \bar{u}_i(x,p)\},$$

where $\bar{u}_i(x,p)$ is expert i's expected payoff from the threat lottery p,

$$\bar{u}_i(x,p) = (1-p)u_i(x,0) + pu_i(x,1).$$

Hence, $\tilde{X}_1^p \cup \tilde{X}_2^p$ is the set of states where implementing the most preferred action is not incentive compatible.

We now show that at any state x in $\tilde{X}_1^p \cup \tilde{X}_2^p$ the incentive constraint of only one of the experts is violated, i.e., $\tilde{X}_1^p \cap \tilde{X}_2^p = \varnothing$. By assumption, the experts have opposing interests, i.e., $y_1^*(x) < x < y_2^*(x)$. If $p > x$, then expert 1 prefers action x to action $y = p$ and hence to the threat lottery. Otherwise, expert 2 prefers x to the threat lottery. Hence, at least one expert prefers x to the threat lottery, implying that $\tilde{X}_1^p \cap \tilde{X}_2^p = \varnothing$.

It follows immediately that whenever $x \in \tilde{X}_i^p$ for some expert i, an optimal contract will stipulate to choose action $g(x)$ that is the "closest" point to x (from the perspective of the decision maker) subject to the incentive constraint for i,

$$g(x) \in \operatorname*{arg\,max}_{y \,:\, u_i(x,y) \geq \bar{u}_i(x,p)} u_0(x,y).$$

4.4 Quadratic preferences and constant biases

We can obtain stronger results if we impose additional structure on the preferences of the experts. Specifically, we make the assumption, which is standard in the literature, that the expert's preferences can be represented by a quadratic payoff function with a constant bias,

$$u_i(x, y) = -(y - (x - b_i))^2, \ i = 1, 2, \tag{7}$$

where $b_1 > 0$ and $b_2 < 0$. Assume also $u_0(x, y) = -(y - x)^2$.

In order to determine the set $\tilde{X}_i^p$ of states where expert i prefers threat lottery p to the most preferred action x for the decision maker, we solve the inequality $u_i(x, x) < \bar{u}_i(x, p)$. Using (7) we obtain

$$(1 - p)(x - b_i)^2 + p(1 - (x - b_i))^2 < b_i^2. \tag{8}$$

In order to state the result, the following definitions are in order. For any $p \in X$, let $D_i = b_i^2 - p(1 - p)$ and let

$$\underline{x}_i^p = p - b_i - \sqrt{D_i}, \ \ \overline{x}_i^p = p - b_i + \sqrt{D_i}. \tag{9}$$

In addition, for $|b_1|$ and $|b_2|$ below 1/2 define

$$\underline{p}^* = \frac{1 - \sqrt{1 - 4b_2^2}}{2}, \ \ \overline{p}^* = \frac{1 + \sqrt{1 - 4b_1^2}}{2}.$$

It is easy to verify that the solution of (8) is the interval $(\underline{x}_i^p, \overline{x}_i^p)$, and hence $\tilde{X}_i^p = (\underline{x}_i^p, \overline{x}_i^p) \cap [0, 1]$. Note that $\tilde{X}_1^p$ is nonempty if and only if $b_1 > 1/2$ or $p > \overline{p}^*$; symmetrically, $\tilde{X}_2^p$ is nonempty if and only if $b_2 < -1/2$ or $p < \underline{p}^*$.

The next result describes the structure of an optimal contract.

Proposition 4 *Let (p, g) be an optimal constant-threat contract. Then,*

$$g(x) = \begin{cases} x + |b_1| - \sqrt{-\bar{u}_1(x, p)}, & \text{if } x \in \tilde{X}_1^p; \\ x - |b_2| + \sqrt{-\bar{u}_2(x, p)}, & \text{if } x \in \tilde{X}_2^p, \\ x, & \text{otherwise.} \end{cases}$$

Proof. If $x \notin \tilde{X}_1^p \cup \tilde{X}_2^p$, then the first best action is incentive compatible, $g(x) = x$.

Let $x \in \tilde{X}_1^p$ (the argument for $x \in \tilde{X}_2^p$ will be analogous). In an optimal contract the decision maker implements an action $g(x)$ that minimizes the distance to x, subject to the incentive constraint for expert 1, that is,

$$g(x) \in \arg\min_{y \in [0,1]} (y - x)^2$$

subject to

$$(y - (x + b_1))^2 \leq (1 - p)(x + b_1)^2 + p(1 - (x + b_1))^2.$$

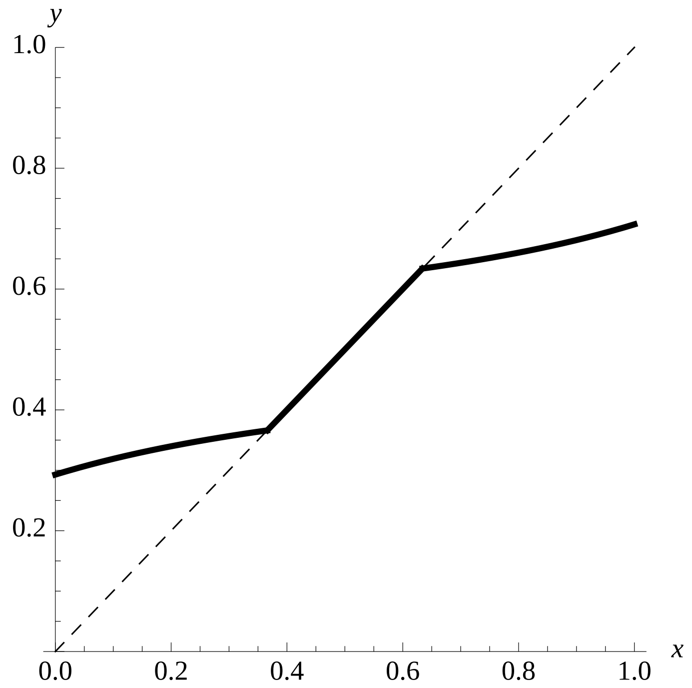

Figure 1: An Optimal Contract with Quadratic Preferences, $|b_1| = |b_2| = 1$, $p = 1/2$.

Solving the above inequality for y we obtain

$$y \in [0,1] \backslash \left(x + b_1 - \sqrt{-\bar{u}_1(x,p)}, x + b_1 + \sqrt{-\bar{u}_1(x,p)} \right).$$

Since $x \in \tilde{X}_1^p$, the above constraint must be binding. As $b_1 > 0$ by assumption, the closest action to x is $g(x) = x + b_1 - \sqrt{-\bar{u}_1(x,p)}$. It is straightforward to verify that in this case $g(x) \in \tilde{X}_1^p$. As $\tilde{X}_1^p \cap \tilde{X}_2^p = \varnothing$, the incentive constraint for expert 2 is satisfied as well. ■

In the case where both biases are greater than $1/2$, an optimal contract looks as follows (Fig. 1). Note that in this case $\tilde{X}_1^p = [0, \overline{x}_1^p)$ and $\tilde{X}_2^p = (\underline{x}_2^p, 1]$. For the "moderate states" in $[\overline{x}_1^p, \underline{x}_2^p]$, both experts prefer the decision maker's most preferred action to the threat lottery, and the first best outcome is achieved (the points along the 45° line on Fig. 1). For the "extreme left" states in $[0, \overline{x}_1^p)$, expert 2 strictly prefers the threat lottery to x, and hence the decision maker implements an action that is closer to expert 2's most preferred action. The distortion for the "extreme right" states is analogous.

The result in Proposition 4 allows us to transfer the problem of finding an optimal contract into a one dimensional optimization problem over value of the threat point:

$$\min_{p \in [0,1]} \int_0^1 (g_p(x) - x)^2 dF(x), \tag{10}$$

where g_p, with some abuse of notation, is given by Proposition 4.

The value of the threat point in the optimal contract depends on the distribution of the state x. In general, there is no closed form solution for optimal threat points. Nevertheless, under additional assumptions, we obtain the following result.

Proposition 5 *Let the experts' biases be opposing and equal,* $b_1 = -b_2 = b$*, and distribution of* x *be symmetric, i.e.,* $F(1-x) = 1 - F(x)$*,* $x \in [0,1]$*. Then there exists an optimal contract with* $p = 1/2$*.*

Proof. The proof is in the Appendix. ■

5 Discussion

5.1 Similarly biased experts

Throughout the paper we have considered the environment in which the experts are biased in different directions (c.f., (1)). Let us now assume that the experts are biased in the same direction, e.g., they always prefer an action higher than the decision maker's optimal action. Then, the experts have the same least preferred action $y = 0$, which is the optimal threat. Therefore, the contract that threatens the experts to implement $y = 0$ whenever they disagree can implement the first best outcome.

5.2 Optimal contracts with one expert

In this subsection, we comment on the difference between optimal contracts in our model and in a model with one expert only. Without a second expert, the recommendations to the decision maker by the first expert remain unchecked. Therefore, the relevant incentive constraints are with respect to other actions that can be induced by the expert's reports rather than with respect to the outcome resulting from a disagreement with another expert. Consequently, optimal contracts have a number of differences: There is bunching of implemented actions across states with one agent (Proposition 3 in Alonso and Matouschek (2008), and Proposition 1 in Kovac and Mylovanov (2009)) and no bunching with two experts (Proposition 4). With one agent, optimal contracts do not implement first best actions because this cannot be made incentive compatible (see, e.g., Proposition 1 in Kovac and Mylovanov (2009)). This is not so with two experts: there is always a nonempty subset of states where the first best outcome is implemented (Propositions 2–3 in this paper). Furthermore, in the model with one expert the optimal contract implements the *expert's* most preferred action for a positive measure of states (Proposition 3 in Alonso and Matouschek (2008) and Proposition 1 in Kovac and Mylovanov (2009)). Again, this is not so with two experts (Propositions 2 and 4).

5.3 Discontinuity and robustness to noise

In this paper, we assume that both experts are perfectly informed. This assumption is common in the literature that studies cheap talk communication with two experts in payoff environments similar to the one in this paper. It has been made, for example, in Gilligan and Krehbiel (1989), Krishna and Morgan (2001a,b), Battaglini (2002), Levy and Razin (2007), Ambrus and Takahashi (2008), and Li (2008, 2009).[9] Yet, this assumption is important. It allows the decision maker to check the reports of the experts against each other, and inconsistent reports do not occur on the equilibrium path. The issue of robustness to noise was pointed out by Battaglini (2002, 2004) in the context of fully revealing equilibria in cheap talk that rely on implausible out-of-equilibrium beliefs. Although there is no issue of out-of-equilibrium beliefs in our model due to commitment assumption, the contracts identified in this paper are discontinuous in the reports. As a result, one might wonder if they are not robust with respect to, for example, small amount of exogenous noise added either to signals or to reports (as in Blume et al. (2007)), or minor mistakes of the experts.

Ambrus and Lu (2009) discuss our construction of first best contracts and demonstrate its robustness to a specific type of noise used in Battaglini (2002) and studied in their paper. In this section, we take a different approach and show that a constant threat contract can be modified and made continuous in the experts' reports. In the environments with a small amount of noise, the modified contract achieves payoffs close to the payoffs in the original contract in the environment without noise. The modified contract is incentive compatible in the original environment but may not be so in noisy environments. This approach to robustness is analogous to the approach taken in Ambrus and Takahashi (2008) for a multidimensional cheap talk environment. One interpretation of this approach

[9] The experts are imperfectly informed in the models of Austen-Smith (1993), Wolinsky (2002), and Battaglini (2004). See also Li and Suen (2009) for a survey of work on decision making in committees; this literature often assumes that different members of the committee hold distinct pieces of information.

is that the experts (incorrectly) believe that they make no mistakes and their reports are not distorted during transmission to the decision maker.

Assume that u_i is twice continuously differentiable and strictly concave in the implemented action, $i = 1, 2$. Fix a small constant $\varepsilon > 0$. Consider the set of constant threat contracts $\mathcal{C}_\varepsilon$ where the incentive constraints are satisfied as strict inequalities with the margin at least ε, i.e., $(p, g) \in \mathcal{C}_\varepsilon$ if

$$u_i(x, g(x)) \geq \bar{u}_i(x, p) + \varepsilon, \quad \text{for all } x \in [0, 1], \ i = 1, 2 \tag{11}$$

We will refer to a constant threat contract that maximizes the expected payoff of the decision maker subject to (11) as an *ε-optimal* contract.

Let (p, g) be an ε-optimal contract. Denote by L the bound on $du_i(x, g(x_i))/dx_i$, $i = 1, 2$. We now construct a modified contract μ' as follows. Define

$$\eta(x_1, x_2) = \min\left\{\frac{L}{\varepsilon}|x_1 - x_2|, 1\right\}.$$

For any two reports $x_1, x_2 \in X$, we define $\mu'(x_1, x_2)$ to be the lottery that chooses the threat lottery (that assigns probability p on decision 1) with probability $\eta(x_1, x_2)$ and action $g((x_1 + x_2)/2)$ with the complementary probability. That is, the probability that the threat lottery is chosen is a linearly increasing function of the distance between the reports, x_1 and x_2.

Thus constructed, the contract μ' is continuous in the experts' reports. Furthermore, if both experts report their information truthfully, the decision maker's payoff in μ' for a small amount of noise is close to the decision maker's payoff in (p, g) in the environment without noise.

We now show that μ' is incentive compatible in the environment without noise; furthermore, the incentive constraints are satisfied with strict inequality. By construction, expert i's payoff in (p, g) as a function of the reports and the state can be written as

$$v_i(x, x_1, x_2) = \begin{cases} u_i(x, g(x_i)), & \text{if } x_1 = x_2, \\ \bar{u}_i(x, p), & \text{if } x_1 \neq x_2. \end{cases}$$

It follows then that i's payoff in μ' is equal to

$$v_i'(x, x_1, x_2) = (1 - \eta(x_1, x_2))u_i\left(x, g\left((x_1 + x_2)/2\right)\right) + \eta(x_1, x_2)\bar{u}_i(x, p).$$

Define $\Delta v_1'(x, x_1) = v_1'(x, x_1, x) - v_i'(x, x, x)$. If $\frac{L}{\varepsilon}|x - x_1| \geq 1$, then $\eta(x_1, x) = 1$, and by (11)

$$\Delta v_1'(x, x_1) \leq \bar{u}_1(x, p) - u_1(x, g(x)) \leq -\varepsilon < 0.$$

Next, let $\frac{L}{\varepsilon}|x - x_1| < 1$. Then $\eta(x_1, x) = \frac{L}{\varepsilon}|x - x_1|$, and we obtain

$$\begin{aligned}
\Delta v_1'(x, x_1) &= (1 - \eta(x_1, x))u_1(x, g((x_1 + x)/2)) + \eta(x_1, x)\bar{u}_1(x, p) - u_1(x, g(x)) \\
&\leq (1 - \eta(x_1, x))[u_1(x, g((x_1 + x)/2)) - u_1(x, g(x))] - \eta(x_1, x)\varepsilon \\
&< |u_1(x, g((x_1 + x)/2)) - u_1(x, g(x))| - L|x - x_1| \\
&\leq L|x - x_1| - L|x - x_1| = 0,
\end{aligned}$$

where the first inequality is obtained by (11) and the last inequality is obtained by applying the Taylor expansion to $u_1(x, g((x_1 + x)/2))$ with respect to x_1 and that the derivative $du_1(x, g((x_1 + x)/2))/dx_1$ is bounded by L.

The argument for $i = 2$ is analogous.

6 Conclusions

In this paper, we study optimal contract design in an environment with an uninformed decision maker and two perfectly informed experts. Our main insight that allows characterizing optimal contracts is the "constant-threat" result that states that one can restrict attention to contracts in which the action implemented in case of a disagreement among the experts is *independent* of their reports. This result simplifies the design problem and makes it possible to characterize optimal contracts. We describe optimal contracts and provide conditions under which these contracts implement the first best outcome. Finally, we remark that, unlike in the models with cheap talk communication, there is a complementarity among experts for the decision maker, that is, adding a second expert is always valuable.

Appendix

Proof of Proposition 5. For $b \leq 1/2$ the statement holds trivially, since the first-best contract can be constructed (see Proposition 2 and its proof).

Assume $b > 1/2$. Let (p, g_p) be a constant threat contract, where g_p is described in Proposition 4. By an argument presented in Section 4.4, if both biases are greater than

$1/2$, then $\tilde{X}_1^p = [0, \overline{x}_1^p)$ and $\tilde{X}_2^p = (\underline{x}_2^p, 1]$. We can now write the expected payoff of the decision maker as

$$\begin{aligned} v^{(p,g_p)} &\equiv \int_0^1 \left[-(g_p(x) - x)^2\right] dF(x) \\ &= -\int_0^{\overline{x}_1^p} (b - \sqrt{-\overline{u}_1(x,p)})^2 dF(x) - \int_{\underline{x}_2^p}^1 (b - \sqrt{-\overline{u}_2(x,p)})^2 dF(x). \end{aligned}$$

Recall that $\overline{u}_i(x,p) = -(1-p)(x-b_i)^2 - p(1-(x-b_i))^2$ and, by (9), $\overline{x}_1^p = p - b_1 + \sqrt{b_1^2 - p(1-p)}$ and $\underline{x}_2^p = p - b_2 - \sqrt{b_2^2 - p(1-p)}$. Using the symmetry assumption $b_1 = -b_2 = b$, we obtain that $\underline{x}_2^p = 1 - \overline{x}_1^{1-p}$ and $\overline{u}_2(x,p) = \overline{u}_1(1-x, 1-p)$, and hence

$$v^{(p,g_p)} = -\int_0^{\overline{x}_1^p} (b - \sqrt{-\overline{u}_1(x,p)})^2 dF(x) - \int_{1-\overline{x}_1^{(1-p)}}^1 (b - \sqrt{-\overline{u}_1(1-x,1-p)})^2 dF(x).$$

Next, using the assumption $F(x) = 1 - F(1-x)$ that entails $dF(x) = dF(1-x)$, after the substitution $x' = 1 - x$ we obtain

$$v^{(p,g_p)} = -\int_0^{\overline{x}_1^p} (b - \sqrt{-\overline{u}_1(x,p)})^2 dF(x) - \int_0^{\overline{x}_1^{(1-p)}} (b - \sqrt{-\overline{u}_1(x',1-p)})^2 dF(x').$$

Let us now differentiate $v^{(p,g_p)}$ with respect to p. Observe that $\bar{u}_1(\bar{x}_1^p, p) = -b^2$, and

$$\frac{\partial v^{(p,g_p)}}{\partial \bar{x}_1^p} = -(b - \sqrt{-\overline{u}_1(x,p)})^2\Big|_{x=\bar{x}_1^p} = -(b-b)^2 = 0.$$

Note that $d\bar{x}_1^p/dp$ exists for all $p \in [0,1]$. Hence, the value of the expression $\frac{\partial v^{(p,g_p)}}{\partial \bar{x}_1^p} \cdot \frac{d\bar{x}_1^p}{dp}$ is well defined and equal to zero. An analogous statement holds for $\bar{x}_1(1-p)$. Thus, derivatives w.r.t. bounds of integration are ignored, and after defining $h(x) = \frac{\partial \overline{u}_1(x,p)}{\partial p} = 2(x+b) - 1$ we obtain

$$\frac{\partial}{\partial p} v^{(p,g_p)} = -\int_0^{\overline{x}_1^p} \left[\frac{b}{\sqrt{-\overline{u}_1(x,p)}} - 1\right] h(x) dF(x) \tag{12}$$

$$+ \int_0^{\overline{x}_1^{1-p}} \left[\frac{b}{\sqrt{-\overline{u}_1(x,1-p)}} - 1\right] h(x) dF(x). \tag{13}$$

It is straightforward to check that $\frac{\partial}{\partial p} v^{(p,g)}|_{p=\frac{1}{2}} = 0$. We now verify that $v^{(p,g_p)}$ is concave in p, thus $p = 1/2$ is a maximum. By $b > 1/2$, we have $h(x) = \frac{\partial \overline{u}_1(x,p)}{\partial p} = 2(x+b) - 1 > 0$. Hence, the expression

$$\left[\frac{b}{\sqrt{-\overline{u}_1(x,p)}} - 1\right] \cdot h(x)$$

is nondecreasing in p. Furthermore, since $\frac{b}{\sqrt{-\overline{u}_1(\overline{x}_1^p, p)}} = 1$, the above expression is nonnegative for all $x \leq \overline{x}_1^p$. Thus, the right-hand side term in (12) is nonincreasing in p. A similar argument shows that the term in (13) is nonincreasing in p as well. It follows that $v^{(p,g_p)}$ is concave in p. ■

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